GUN PRICE REGULATION AND THE GUN CONTROL DEBATE

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Abstract. This article studies gun control as price regulations on gun sales. In the gun control debate there are two opposing sides: pro-control advocates argue that restricting the amount of guns inevitably reduces gun violence while anti-control advocates argue that gun control disarms law-abiding citizens rather than criminals. A model is constructed which explains the basis for either argument. It is shown that under certain parameter values, gun control backfires as it increases gun-carrying costs for armed non-criminals more than for criminals. Under other parametric circumstances, gun control disarms both criminals and armed non-criminals as it increases the costs for both groups in tandem.

Keywords: gun control, gun violence and crime
JEL-codes: D62, D74, K42.

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1. Introduction

Gun control advocates point to high levels of gun ownership as the cause of much, if not most, gun violence in the United States and argue that gun control will inevitably reduce it by reducing the number of guns in circulation. Anti control advocates argue on the other hand that high levels of gun ownership are a response to, not a cause of, violent crimes, and that gun control disarms law-abiding citizens, not criminals. Gun control advocates argue that this is at best a vicious cycle: more than 500,000 reported firearm thefts are reported every year in the United States where 90% of these were taken from armed households and most of which were used for criminal purposes.

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Under these two opposite views there is hardly any surprise that gun control policies are valued very differently by both sides. What are typically termed gun control policies, not including punishments and severe sentences for gun related crimes, are a set of policies that basically can be divided in three types: those that restrict the access to guns for certain subpopulation groups through screening mechanisms, like the Brady act, those that restrict the usage of a gun in certain environments like gun-free environments, schools, bars, stadiums, libraries etc. and those that affect the price of guns through government regulations like taxes. Anti gun control advocates for example argue that a gun permit is just like a driving license: people have the right to carry one if they comply with basic norm regulations. Naturally, under this view an individual that misuses a gun is subject to having his or her license revoked. It is fair then to argue that both sides seem to agree, up to some extent, that certain people should not be allowed to carry a gun because they simply do not comply with norm regulation or are ill-suited to carry one, for example, patients with mental problems, ex convicted offenders and youths. It is true that there is no ideal screening mechanism to determine perfectly who may or may not be allowed to carry a gun but if this mechanism would to exist both sides might agree up to some extent on implementing it. Nonetheless I do not focus on the first two types of gun policies, I only focus on gun price regulations as a mechanism to control gun violence since it is a more standard approach in economics as a way of controlling negative externalities.

The effect that regulation of gun prices has in a society depends fundamentally on the way we think aggregate behavior of armed individuals will be affected. Gun control advocates seem to believe that the higher the price of a gun is less amount of violently inclined individuals would be willing to buy them which in turn lowers gun violence. Therefore gun control is seen as a way of coordinating society towards a state in which there is an overall lower amount of guns that generates less gun accidents and gun rage interactions. On the other hand, anti-gun control advocates argue that a gun can deter effectively an armed delinquent and therefore gun control is seen as a way of disarming potential victims, increasing the fraction of the population
to be preyed upon by gun criminals.

The paper builds a model that allows to understand both sides of the debate. Usually the debate is highly ideological and either side makes use of different models with distinct assumptions to argue their point. In this process it is not clear a priori for readers to figure out the key assumptions from the less essential ones that drive the conclusions. Therefore the aim of this paper is to state at the very least the assumptions that are required to model each side of this debate which has the advantage of clarifying the key assumptions underlying each approach. It is important to emphasize that the case for gun control can only be decided empirically not theoretically. Nonetheless understanding the logic of each side within a simple model proves useful for empirical research. The main finding is that if gun violence comes mainly from criminals that seek material rewards then gun control backfires as it increases gun-carrying costs for armed non criminals more than for armed criminals. On the other hand, if gun violence comes not only from delinquency but also from violent behavior among non criminals then gun control can disarm both criminals and armed non criminals as it increases the costs for both groups in tandem.

2. The model

The model focuses on criminal gun interactions that have an economic motive behind them, e.g. robberies and property theft, and non criminal gun interactions that may arise due to violent behavior. Even though violent behavior with a gun in every day usage can be viewed as a criminal assault I consider that if it is not a behavior seeking a material reward then it is simply termed violent behavior and not a criminal behavior.

2.1. Preferences and Criminal Behavior. Assume a mass of individuals normalized to one where each is endowed with the same disposable income level $W > 0$ subject to capture by delinquents. Let the utility of any individual depend on a consumption good denoted $C$, with normalized price equal to one, and a service called "private security", denoted $S$, in a linear utility form $u = C + S$. The production
function of private security is simply $S = vG$ where $G$ is an indicator function that
takes the value one if the individual owns a gun and $v$ represents innate ability of
providing private security when acquiring a gun. Types in the population are dis-
tributed according to a continuous and strictly increasing cumulative distribution
function c.d.f. $\Phi$ defined on the range $[0, \infty)$. I assume explicitly that any individual
would decide to own at most one gun if he or she wishes to do so since the use of
owning more than one gun for either self-defense or criminal activities is nil.

Individuals that decide to become criminals have buy a gun and endure a psy-
chological cost $a > 0$. For example, a pre-meditated aggressive criminal attack on
someone in a gun encounter can have a deep psychological effect on an attacker, even
though a criminal might get use to it the more he or she does it. In principle this cost
does not have to be constant since it can be thought to be heterogenous in the popu-
lation, nonetheless I assume it to be constant for simplicity. The crucial assumption
that I will make is that this cost is a strictly decreasing function in $v$, which amounts
to assume that types of individuals that are more able to manipulate a gun will at-
tenuate this cost if they choose a criminal career. Therefore the functional form of
the psychological cost is $\frac{a}{v} > 0$. The reduced form utility of an individual is then

$$u(v) = C + \left( v - \frac{a}{v} P \right) G$$

where $P$ takes the value one if the individual decides to prey on others with a gun for
material rewards. I assume explicitly that a criminal can only seek material rewards
if he or she owns a gun.

Guns are produced in the economy through a perfectly competitive market in
which producers have a technology with constant returns to scale. Therefore the
equilibrium price of a gun, denoted $q$, is simply $q = c + t$ where $c$ is the constant
marginal cost of producing a gun and $t$ is the tax the government charges per gun
sold in the market. Increases in $q$ come from increases in gun taxes. I assume that all
guns are identical and therefore no distinction is made between legal and illegal gun
markets. Therefore an individual that wants to buy (or sell) a gun can purchase (sell)
it at price $q$ in the market. More realistically there are markets for legal and stolen guns which can have very different market structures e.g. the former competitive market while the latter a thin imperfect market in the sense that there are few buyers and sellers. Nonetheless, the assumption is less restrictive than what it is thought to be at first glance if gun regulations that increase the price of legal guns also increase stolen gun prices. Moreover, the assumption of identical types of guns allows me to simplify this complicated issue significantly at the cost of not considering other gun control policies designed to regulate stolen guns, e.g. serial numbers on guns that make it harder to sell stolen guns.

2.2. Strategic Interactions. Most of the gun control debate has to do with the behavior of criminals as a response to non-criminal disarmament and therefore the analysis is carried out with an eye on strategic behavior. To do so let me assume that the choice set for an individual consists of three actions: to buy a gun and prey on others for material rewards ($P$), to buy a gun and defend oneself with it ($D$) and not to buy a gun at all ($N$). Types that choose $D$ or $N$ are called "non criminals" while types that choose $P$ are called criminals (individuals with guns that seek an economic reward). Since guns are expensive durable goods it is reasonable to assume that criminals can steal the gun from potentially armed victims and would be able to sell them in the market at the prevailing price.

The effectiveness of a gun attack and its associated potential lethality depend on the joint actions chosen by individuals. I assume that an individual that is armed or unarmed incurs in an expected injury cost $F > 0$ if matched with an armed individual. Moreover, let $r$ represent the expected probability that a criminal has of succeeding in a gun attack. By symmetry this probability is a half if the two individuals are criminals. In this case if a criminal does not get the upper hand in the interaction he loses his disposable wealth and his gun, while if successful he captures $W > 0$ and the gun of his victim worth $q$. If the criminal is matched with an unarmed victim then he always gets his way i.e. $r = 1$, capturing $W$. Now if the criminal interacts
with an armed victim then $0.5 < r < 1$, which captures the idea that a gun serves for self-defense and can deter, even if imperfectly, a gun attack.\footnote{Moreover a criminal would have the advantage of surprise which explains why it is greater than a half.} In this case if the criminal is successful he captures $W$ and the gun of his victim worth $q$ while if he is unsuccessful he gets nothing. I assume explicitly that $F$ and $r$ are the same across the population.

Violent human behavior without a material reward is outside the rational decision paradigm. Nonetheless this behavior can occur in many circumstances without there being an explicit material reward attached to it.\footnote{The several public gun shootings that the United States has witnessed in the last two decades can hardly be reconciled with the idea that gun attackers were moved by material rewards.} I simplify matters as much as possible by assuming that violent interactions among non criminals can occur with a constant probability $0 < \varphi < 1$ which is the same across types.\footnote{The exogeneity is artificial of course since it could depend on several factors which might reflect both cultural and idiosyncratic differences in the way individuals get into violent interactions. The assumption is only justified in terms of mathematical tractability.} If two non criminals are matched no income is redistributed, since there is no economic motive for the violent behavior, nonetheless expected injury costs arise if at least one of them has a gun. If two self-defense gun owners are matched and there is a violent interaction among them then each has the same probability (a half) of getting an upper hand in the confrontation while if a self-defense gun owner is matched with a non gun owner and a violent interaction arises the gun owner gets her way and inflicts an expected injury of $F$ to the other individual.

Criminals in principle can also face police persecution and therefore could be caught and convicted with a certain probability and sentenced to a penalty.\footnote{See Mialon and Wiseman (2005) for an analysis of these policy variables to control gun violence while respecting the right to bear arms.} Even though this has a deterrent effect on criminal incentives and could easily be incorporated in the analysis I will abstract from this issue since I am interested in the strategic nature of gun interactions. Formally speaking the model constructed here is compatible with the set up of a zero probability for a criminal of being apprehended.
and convicted. All ex ante expected payoffs for any given match between a pair of strategies drawn from the population are given then by

\[
U[P, P] = \frac{1}{2} [2W + q] - F + v - \frac{a}{v} - q \\
U[P, D] = r [2W + q] + (1 - r) W - F + v - \frac{a}{v} - q \\
U[P, N] = 2W + v - \frac{a}{v} - q \\
U[D, P] = (1 - r) W - F + v - q \\
U[D, D] = W - \varphi F + v - q \\
U[D, N] = W + v - q \\
U[N, P] = -F \\
U[N, D] = W - \varphi F \\
U[N, N] = W
\]

Individuals are randomly matched to play the symmetric and simultaneous game (2) in a pairwise fashion.

\[
\begin{array}{ccc}
P & D & N \\
\end{array}
\]  

(2)

It is important to emphasize that the type of guns consistent with the simultaneous nature of the random matching is more likely to be a hand gun since these are easily concealed from the opponent up to the point in which they are withdrawn.\(^6\) For other type of guns the interactions can occur in a sequential fashion due to the difficulty of concealing them, say, because of their size.

2.3. Equilibrium. This section studies equilibrium of the model. I focus only on pure strategy equilibrium neglecting the possible mixed strategy equilibria that might arise. Every type \(v\) is assumed to choose between \(P\), \(D\) and \(N\) such that expected utility is maximized given the conjecture each \(v\) has on the fraction of criminals \(\alpha\).

\(^6\)Most of the debate on whether gun laws serve as a deterrent mechanism against crime take the form of concealed hand-gun laws which match well the present simultaneous game played in any given match.
and self-defense gun owners $\beta$ in the population. Therefore let me define what a pure strategy equilibrium formally is in the model.

**Definition 1.** A pure strategy Nash equilibrium is an action $b_v : [0, \infty) \to \{P, D, N\}$ for every $v$ in $[0, \infty)$ such that: i) $b_v = \arg \max_{b \in \{P, D, N\}} \pi (b, v; \alpha, \beta)$ where

$$
\pi (b, v; \alpha, \beta) = \alpha U [b, P] + \beta U [b, D] + (1 - \alpha - \beta) U [b, N].
$$

(3)

and ii) $\alpha$ is the proportion of $v$'s such that $b_v = P$ while $\beta$ is the proportion of $v$'s such that $b_v = D$.

In words: a pure strategy Nash equilibrium is a collection of actions such that each type $v$ in the population chooses an action that maximizes her expected utility and there is a consistency of beliefs for every type about the expected fractions of criminals, self-defense gun owners and non gun owners.

To solve for pure strategy Nash equilibrium I take the approach of first conjecturing how the solution might be and then verifying that the conjecture is in fact correct.

**Conjecture 1.** a) If type $v$ chooses $P$ then all types $v'$ such that $v' > v$ choose also $P$; b) If type $v$ chooses $N$ then all types $v'$ such that $v' < v$ choose also $N$.

Let

$$v_P \equiv \inf \{v : v \text{ chooses } P\} \quad \text{and} \quad v_D \equiv \sup \{v : v \text{ chooses } N\}.$$

The conjecture comes from (1) since higher types get a higher utility just from owning a gun (choosing $D$ or $P$) i.e. $u' (v)|_{b \in \{D, P\}} = 1$ while among gun owners higher types get a lower psychological cost of choosing $P$ i.e. $u' (v)|_{b = P} = 1 + \frac{a}{v^2}$ hence the conjecture reduces to $v_P > v_D$.

Under the conjecture there is a "bunching" equilibrium structure where types choose optimally in the following way: all types below $v_D$ choose $N$, types between $v_P$ and $v_D$ choose $D$ and types above $v_P$ choose $P$. The intuition is straightforward:
since higher types get a higher utility for owning a gun marginal type \( v_D \) breaks the population in gun owners and non gun owners while among types that choose to buy a gun higher types incur in a lower psychological cost if they choose crime, hence \( v_P \) is no less than \( v_D \) and breaks the population of gun owners in self-defense gun owners (types lower than \( v_P \) but greater than \( v_D \)) and criminals (types above \( v_P \)). This structure implies that the fractions of criminals and self-defense gun owners are given by

\[
\alpha = \frac{1 - \Phi(v_P)}{\Phi(v_P) - \Phi(v_D)} \quad \text{and} \quad \beta = \Phi(v_P) - \Phi(v_D)
\]

where \( \Phi(v) \) denotes the fraction of the population that has a value for a gun strictly below \( v \).

a) Under the conjecture it must be that marginal type \( v_P \) is indifferent between choosing \( P \) or \( D \) therefore \( \pi(P, v_P; \alpha, \beta) = \pi(D, v_P; \alpha, \beta) \). This indifference condition can be written as an equilibrium implicit equation

\[
\alpha \left( \frac{q}{2} + rW \right) + \beta (r (W + q) + \varphi F) + (1 - \alpha - \beta) W - \frac{a}{v_P} = 0 \quad (4)
\]

where \( \frac{a}{v_P} \) is the marginal cost of becoming a criminal and it involves the psychological cost \( \frac{a}{v_P} \) while the expected marginal benefits for type \( v_P \) are \( (\frac{q}{2} + rW), (r (W + q) + \varphi F) \) and \( W \) respectively. Replacing \( \alpha \) and \( \beta \) in equation (4) and rearranging I get an equilibrium implicit equation relating \( v_P \) with \( v_D \)

\[
h^{PD}(v_P, v_D) \equiv \frac{q}{2} + rW + \Phi(v_P) \Gamma + \Phi(v_D) (W (1 - r) - rq - \varphi F) - \frac{a}{v_P} = 0 \quad (5)
\]

where \( \Gamma \equiv (r - \frac{1}{2}) q + \varphi F > 0 \) since \( r > 0.5, q > 0, \varphi F \geq 0 \). The term \( \Phi(v_P) \) is the mass of non criminals while \( \Phi(v_D) \) is the mass of non gun owners. Notice that \( W (1 - r) - rq - \varphi F \) has an ambiguous sign depending on the values of \( \varphi \) and \( q \). Let me denote by \( \bar{\varphi} \) the value of \( \varphi \) that satisfies \( W (1 - r) - rq - \varphi F = 0 \) i.e.

\[
\bar{\varphi} \equiv \max \left\{ 0, \frac{W (1 - r) - rq}{F} \right\} \quad (6)
\]

so that values of \( \varphi \) such that \( \bar{\varphi} < \varphi \) implies \( W (1 - r) - rq - \varphi F < 0 \). This threshold \( \bar{\varphi} \) is a non increasing function of \( q \) and \( r \) while non decreasing in \( W \) for given \( F \). Moreover, \( \bar{\varphi} \in (0, 1) \) if \( r \in \left( \frac{1}{2}, \bar{r} \right) \) where \( \bar{r} \equiv \frac{W}{q + W} \in \left( \frac{1}{2}, 1 \right). \)
b) On the other hand, under the conjecture it must be that marginal type $v_D$ is indifferent between $D$ and $N$ and therefore satisfies $\pi(D, v_D; \alpha, \beta) = \pi(N, v_D; \alpha, \beta)$ which after replacing in the payoffs and $(\alpha, \beta)$ yields another equilibrium equation that relates $v_P$ and $v_D$

$$g_{DN}^{\alpha} (v_P, v_D) \equiv v_D - q + (1 - r) W (1 - \Phi (v_P)) = 0. \quad (7)$$

Naturally, a pure strategy equilibrium holds if the two equilibrium equations (5) and (7) hold simultaneously which determine equilibrium marginal types $(v_P^*, v_D^*)$ and the equilibrium proportions $(\alpha^*, \beta^*)$ from $\alpha^* = 1 - \Phi (v_P^*)$ and $\beta^* = \Phi (v_D^*) - \Phi (v_P^*)$.

**Existence.** This section studies the existence and uniqueness of a pure strategy Nash equilibrium. From equation (7) let me solve for $v_D$ and replace it in (5) which yields

$$H (v_P) \equiv \frac{q}{2} + W + \Phi (v_P) \Gamma + \Phi \left[ q - (1 - r) W (1 - \Phi (v_P)) \right] F \left[ \varphi - \varphi \right] - \frac{a}{v_P} \quad (8)$$

where $F [\varphi - \varphi] \equiv W (1 - r) - rq - \varphi F$. Notice that $H (\bullet)$ is a continuous function of $v_P$. The following proposition gives conditions for the existence of a pure strategy Nash equilibrium.

**Proposition 1.** Under the assumptions of the model a pure strategy Nash equilibrium exists.

**Proof:** It is sufficient to show that $H (0) < 0$ and $H (\infty) > 0$ since the continuity of $H (\bullet)$ implies the existence of $v_P^* \in (0, \infty)$ such that $H (v_P^*) = 0$ by the mean value theorem. Notice first $H (0) < 0$ since when $v_P$ approaches zero the term $-\frac{a}{v_P}$ goes to minus infinity while the rest of the terms are bounded given that $\Phi (z) = \Phi (0) = 0$ for any $z \leq 0$. Second notice that once $\Gamma$ is replaced in and terms are regrouped yields

$$H (\infty) = r (W + q) + \Phi (q) F \varphi + \varphi F (1 - \Phi (q)) > 0$$

since $r (W + q) > 0$, $\Phi (\infty) = 1$, $0 < \Phi (q) < 1$ and $F \varphi \geq 0$. $\Box$
It is interesting to verify that in equilibrium it must be that \( v^*_P > v^*_D \).

**Lemma 1.** In any pure strategy Nash equilibrium \( v^*_P > v^*_D \).

**Proof.** Consider a pure strategy Nash equilibrium \((v^*_P, v^*_D)\) and suppose that \( v^*_P \leq v^*_D \). From equation (5) in equilibrium it must be that \( v^*_P > 0 \) or otherwise the term \( a/v^*_P \) would be infinite and the equality would not hold. Moreover from equation (7) it is easy to see that \( v^*_D < q \) because \((1-r)W[1-\Phi(v^*_P)] > 0 \) hence \( v^*_P < q \). Now consider a sequence \( \{r_n\} \) such that \( r_n \in (\frac{1}{2}, 1) \) with \( \lim_{n \to \infty} r_n = 1 \). For every value or \( r_n \) there is a corresponding sequence \( v^*_D, r_n \) such that \( \lim_{n \to \infty} v^*_D, r_n = q \) or (7) would not hold. From (7) and \( \Phi \) strictly increasing the inverse of \( \Phi \) exists and therefore

\[
v^*_P = \Phi^{-1} \left( 1 + \frac{v^*_D - q}{(1-r)W} \right)
\]

holds. Given that it must be \( 0 < v^*_P \leq v^*_D \) then \( v^*_D, r_n \) must converge faster to \( q \) than \( r_n \) to one because if not then \( v^*_P = 0 \) since the term \( 1 + \frac{v^*_D - q}{(1-r)W} \) would tend to minus infinity and \( \Phi^{-1} (-z) = 0 \) for any \( z < 0 \). Therefore in the limit it must be that \( v^*_P = \Phi^{-1} (1) = \infty \) By continuity of (7) this analysis still holds for \( r \) close to one and therefore \( v^*_P > q \) which contradicts \( v^*_P \leq v^*_D \). \( \Box \)

Figure 1 illustrates a unique equilibrium in which the population is partitioned in criminals (fraction \( \alpha^* = 1 - \Phi (v^*_P) \)), self-defense gun owners (fraction \( \beta^* = \Phi (v^*_P) - \Phi (v^*_D) \)) and non gun owners (fraction \( 1 - \alpha^* - \beta^* = \Phi (v^*_D) \)). The following proposition establishes a sufficient condition for uniqueness.

**Proposition 2.** Let \( \varphi \in [0, \bar{\varphi}] \) where \( \bar{\varphi} \equiv \max \left\{ 0, \frac{W(1-r)-rq}{F} \right\} \) then there is a unique pure strategy Nash equilibrium.
Proof. It suffices to establish that $H'(v_P) > 0$ for all $v_P \in [0, \infty)$. To see this notice that taking the derivative of (8) with respect to $v_P$ yields

$$H'(v_P) = \Phi'(v_P) \Gamma + \frac{a}{(v_P)^2} + (1 - r) WF [\bar{\nu} - \nu] \Phi'(v_D) \Phi'(v_P) \quad (9)$$

which is positive if $\bar{\nu} - \nu \geq 0$ since $\Phi' > 0$ and $a > 0, r < 1, \Gamma > 0$. \(\Box\)

Figure 2 shows a numerical example for an exponential c.d.f. $\Phi(x) = 1 - \exp(-x)$ and a set of parameter values $W = 1, e = 0, a = 1, r = 0.6, \varphi = 0, q = 0.5$.

Corollary 1. A necessary condition for multiple pure strategy equilibria is $\varphi \in (\bar{\nu}, 1)$. Moreover, there is at most an odd finite number of equilibria.

Proof: The slope $H'(v_P)$ can become negative for a certain range of $v_P$ only if $\bar{\nu} - \nu < 0$. It is a necessary condition for multiple equilibria but not sufficient since the first and second terms in (9), which are positive, can dominate depending on the slope of $\Phi$. Moreover, if there are multiple pure strategy Nash equilibria the continuity of $H(\bullet)$ implies that (8) is at most zero a finite number of times and has to be odd since $H(0) < 0$ and $H(\infty) > 0$. \(\Box\)
Marginal Rates of Substitution. This section focuses on the behavior of the equilibrium functions to examine later how changes in $q$ affect both $v^*_D$ and $v^*_P$. To do so consider the marginal rates of substitution of the equilibrium implicit functions (5) and (7). It is easy to see from (7) that
\[
\frac{dv^*_D}{dv^*_P}_{DN} = -\frac{\partial g^{DN}}{\partial v^*_P}/\partial v^*_D = (1 - r) W \Phi'(v^*_P) \tag{10}
\]
and from (5) that
\[
\frac{dv^*_P}{dv^*_D}_{PD} = -\frac{\partial h^{PD}}{\partial v^*_D}/\partial v^*_P = -\frac{F(\tilde{\varphi} - \varphi) \Phi'(v^*_P)}{\left(\frac{a}{\tilde{v}^*_P}\right)^2} + \Gamma \Phi'(v^*_P) \tag{11}
\]
where $\tilde{\varphi}$ is defined above in (6).

Definition 2. Strategic substitutability in gun interactions is a situation such that $\frac{dv^*_D}{dv^*_P}_{DN} > 0$ and $\frac{dv^*_P}{dv^*_D}_{PD} \leq 0$. Strategic complementarity in gun interactions is a situation such that $\frac{dv^*_D}{dv^*_P}_{DN} > 0$ and $\frac{dv^*_P}{dv^*_D}_{PD} > 0$.

The following proposition is straightforward.
Proposition 3. Strategic substitutability in gun interactions arises if \( \varphi \leq \bar{\varphi} \) while strategic complementarity arises if \( \varphi > \bar{\varphi} \).

**Proof:** Notice \( \frac{dv^*_r}{dv^*_p} \bigg|_{DN} > 0 \) since \( \Phi' > 0 \) and \( r < 1 \) while the sign of \( \frac{dv^*_r}{dv^*_p} \bigg|_{PD} \) depends only on the sign of \( \bar{\varphi} - \varphi \) since the rest of the terms are positive. If \( \varphi \) is less than (respectively greater than) \( \bar{\varphi} \) then the marginal rate of substitution of \( h^{PD} = 0 \) is negative (respectively positive). \( \Box \)

That the marginal rate of substitution of \( g^{DN} = 0 \) is positive means that as crime increases in society (\( v^*_p \) decreases) the incentives for marginal type \( v^*_D \) of buying a gun for self-defense rises increasing self-defense gun ownership (\( v^*_D \) decreases). Moreover, that the marginal rate of substitution of \( h^{PD} = 0 \) can be either positive or negative depends crucially on the size of \( \varphi \) relative to the threshold \( \bar{\varphi} \). Consider \( \bar{\varphi} \geq \varphi \geq 0 \) in which case a gun basically serves to fend off a criminal since violent gun interactions are relatively low among non criminals. So as \( \varphi \) approaches zero more non criminals choose not to buy a gun (\( v^*_D \) increases) and this increases the incentives for threshold type \( v^*_P \) to choose \( P \) over \( D \) since more unarmed potential victims are available (\( v^*_P \) decreases). Consider now \( \bar{\varphi} < \varphi \) in which case besides fending off criminals a gun serves also for violent interactions among non criminals. As \( \varphi \) increases non gun owners have the incentive to acquire a gun since a violent interaction with another non criminal is more likely (\( v^*_D \) decreases). Hence marginal type \( v^*_P \) has a lower incentive to choose \( P \) over \( D \) because there are now more armed non criminals (\( v^*_P \) also decreases).

Recall that \( \bar{\varphi} \) is a decreasing function of \( r \) and \( q \) (weakly) while increasing in \( W \) for given \( F \). Therefore if \( r, q \) or \( F \) are relatively large, ceteris paribus, then \( \bar{\varphi} \) tends to be small and the strategic complementarity case is more likely to arise. On the other hand, if \( W \) is relatively large, ceteris paribus, then \( \bar{\varphi} \) tends to be large and the strategic substitutability case is more likely to arise. From proposition 2 it is straightforward to see that strategic substitutability is associated with a unique equilibrium. Figure 3 illustrates in the space \((v_P, v_D)\) the same set up as in figure 2.
where the upward sloping curve is the implicit equilibrium function $g^{DN}(v_P, v_D) = 0$ while the downward sloping curve is $h^{PD}(v_P, v_D) = 0$. The 45 degree line is included to verify that in equilibrium $v_P > v_D$. Figure 4 illustrates strategic complementarity in gun interactions for the same set of parameters as in figure 2 but now with $\varphi = 0.1$ and $F = 10$. The curve $h^{PD}(v_P, v_D) = 0$ has a positive slope since for this set of parameter values $\varphi \approx 0.01$.

According to corollary 1 strategic complementarity can generate in principle multiple equilibrium since both implicit equilibrium functions are upward sloping. Nonetheless, this possibility is not automatically implied and depends on the shape of c.d.f. $\Phi$ and parameter values. Numerical simulations showed that for a sufficiently concave c.d.f. $\Phi$, like the exponential c.d.f., a unique equilibrium arises under strategic complementarity. On the other hand for a c.d.f. sufficiently "S-shaped" multiple equilibria can arise. For example, under a $(0.8,1,30)$ Weibull c.d.f. $\Phi(x) = 1 - \exp(-(x - 0.8)^{30})$, which has a sharp S-shape, it is possible to get multi-
ple equilibria.\footnote{A Weibull \((\alpha, \beta, \gamma)\) c.d.f. \(G(x)\) is zero for all \(x < \alpha\), while for \(x \geq \alpha\):
\[
G(x) = 1 - \exp \left( - \left( \frac{x - \alpha}{\beta} \right) ^\gamma \right).
\]
Notice that an exponential c.d.f. is simply a \((0,1,1)\) Weibull c.d.f.} Figure 5 illustrates this possibility where the implicit equilibrium functions \(g^{DN} = 0\) and \(h^{PD} = 0\) look like the thin and bold curves respectively generated numerically using a \((0.8,1,30)\) Weibull c.d.f. and a set of parameter values \(W = 1, e = 0, a = 1, r = 0.8, \varphi = 0.5, q = 0.23\) and \(F = 15\).

2.4. Gun Injury Costs and Social Welfare. The whole gun control debate has to do with the negative externality that firearms bring to a society. Guns are objects that are inherently dangerous, designed and used to inflict injuries on victims, it is the externality that they bring to human interactions that has justified in some contexts the regulation of guns. Hence, I shall focus specifically on gun injury costs and social welfare to analyze whether an increase in the price of guns can lower the former while increasing the latter.

As is well known social welfare requires some type of interpersonal utility compa-
rability criteria. In principle there are several criteria to choose from. Even though a Utilitarian social welfare function is more common in the literature I believe that a Rawlsian social welfare function can be more simple and useful in the context of gun interactions. In the present context the "worst off" member of society should be a non gun owner type since she is incapable of defending herself if attacked with a gun. Therefore the Rawlsian social welfare function is simply the expected utility of this type \( \pi(N, v; \alpha^*, \beta^*) \) i.e.

\[
SW^R = W (1 - \alpha^*) - F (\alpha^* + \beta^* \varphi).
\]  

(12)

Welfare is composed then by two components: a) \( W (1 - \alpha^*) \) is the amount of disposable income net of the "implicit tax" on disposable income due to predation, b) \( IC = F (\alpha^* + \beta^* \varphi) \) is the injury cost due to gun violence since \( \alpha^* + \beta^* \varphi \) is the amount of gun violence. Notice that \( IC \) is an increasing function of both the fraction of criminals and self-defense gun owners for given \( \varphi > 0 \) since in the model there is no other way of getting injured.

I shall focus on analyzing if a tax increase on gun prices can lower \( IC \) and increase
welfare measured by the Rawlsian social welfare function. Moreover, I shall restrict the analysis to only small changes in taxes to regulate gun prices since in some societies it may be unfeasible for a social planner or policymaker to increase gun taxes severely due to political reasons. For example, there are strong anti gun control lobbying parties in some countries (e.g. National Rifle Association in the United States) that have a strong influence on government decisions about regulating gun prices. Naturally, a ban on firearms could be rationalized in the present model by considering a sufficiently high level for \( q \).

2.5. Comparative Statics. This section relates the behavior of the implicit equilibrium functions \( g^{DN} = 0 \) and \( h^{PD} = 0 \) under small changes in \( q \) with the arguments of both the anti and pro gun control advocates.

Anti Gun Control Argument. The typical anti gun control argument runs as follows: gun control through gun or ammunition taxes disarms only self-defense gun owners while increasing the fraction of criminals. Moreover advocates of this view tend to focus on the benefits that non gun owners enjoy if there is an increase in self-defense gun ownership in the population.\(^8\) To illustrate this argument let me consider the strategic substitutability case characterized by having \( \varphi \geq \varphi \geq 0 \). In this case, as shown above, the implicit equilibrium function \( h^{PD} = 0 \) has a negative slope and a unique pure strategy Nash equilibrium is like the one illustrated in Figure 3.

Proposition 4. (Anti Gun Control Argument) Consider a pure strategy Nash equilibrium where there is strategic substitutability in gun interactions \( \varphi \in [0, \bar{\varphi}] \) then for \( r \) arbitrarily close to one a small increase in \( q \) increases crime and decreases self-defense gun ownership.

Proof: There are two part of the proof: a) From \( g^{DN} = 0 \) solve for \( v_D \) and replace this in equation (5) to get an implicit function only in \( v_P \)

given by
\[
\left\{ \frac{q}{2} + rW + \Phi (v_P) \Gamma - \frac{a}{v_P} \right. \\
\left. + \Phi \left[ q - (1 - r) \right] W (1 - \Phi (v_P)) \right\} F \left[ \varphi - \varphi \right] = 0 \tag{13}
\]

This defines implicitly the equilibrium function \( v_P^* = v_P (q, \theta) \) where \( \theta = (W, F, r, \Phi) \), which determines also \( v_D^* = v_D (v_P (q, \theta)) \) from (7). Equation (13) can be written as \( h(v_P (q, \theta); q) \equiv 0 \) and therefore differentiating with respect to \( q \) and rearranging yields
\[
\frac{dv_P (q, \theta)}{dq} = - \frac{\partial h/\partial q}{\partial h/\partial v_P} \bigg|_{v_P^*} \tag{14}
\]

\[
= - \frac{\frac{a}{v_P^*} + r \beta^* + F \left[ \varphi - \varphi \right] \Phi' (v_D (v_P^*))}{\left( \frac{a}{v_P^*} \right)^2 + \Phi' (v_P^*) \left[ \Gamma + F \left[ \varphi - \varphi \right] (1 - r) W \Phi' (v_D (v_P^*)) \right]} < 0
\]

The sign of (14) is negative because the denominator is positive since \( \Phi' > 0, \Gamma > 0 \) and \( \varphi \geq \varphi \) and the term in square brackets in the numerator is also positive under \( \varphi \geq \varphi \); b) Replacing \( v_P (q, \theta) \) in (7) and differentiating with respect to \( q \) yields
\[
\frac{dv_D (v_P (q, \theta))}{dq} = 1 + \Phi' (v_P^*) (1 - r) W \frac{dv_P (q, \theta)}{dq}. \tag{15}
\]

Take the limit case in which \( r = 1 \) and \( \varphi = \varphi = 0 \). Then part a) still holds and the sign of (15) is positive. Moreover, the function (15) is continuous in \( r \) and therefore this still holds true for \( r \) close to one and \( \varphi \) close to zero such that \( \varphi > \varphi \). Hence, a small increase in \( q \) increases \( v_D^* \) and reduces \( v_P^* \).

Figure 6 shows the anti-control argument in the space \((v_P, v_D)\). The solid lines are the same as in Figure 3 while the dashed lines are the two equilibrium functions for a small increase in \( q \) (from \( q = 0.5 \) to \( q = 0.6 \)). Clearly, \( v_P^* \) decreases while \( v_D^* \) increases. This illustrates well the anti-control argument because as \( q \) is increased, even though now both criminals and self-defense gun owners find it more costly to
acquire and maintain a gun, the latter are disarmed relatively more compared to the former, hence predators have more unarmed victims to prey upon increasing their net benefits. In this case the anti gun control argument matches very well one of their slogans "when guns are outlawed only outlaws will have guns".

The policy recommendation under this view is clear: given that \( \varphi \in [0, \bar{\varphi}] \) is low enough (say zero), society should not regulate the price of guns making \( q \) close enough to its marginal cost i.e. \( q = c \) which increases \( \beta^* \) and decreases \( \alpha^* \). I call this the vigilante policy\(^9\) and the following proposition states when this policy is welfare enhancing under the Rawlsian social welfare function.

**Proposition 5. (Vigilante Policy)** Consider a pure strategy Nash equilibrium and assume strategic substitutability in gun interactions \( \varphi \in [0, \bar{\varphi}] \) with \( \varphi \) close enough to zero and \( r \) close to one then the policy of a small reduction in \( q \) decreases injury costs due to gun violence and is welfare enhancing.

Figure 7: Equilibrium correspondence between $q$ and $v_P^*$ under strategic substitutability.

**Proof:** From (12) if $\varphi \simeq 0$ then as $q$ is reduced $\alpha^*$ and $IC$ are reduced hence it increases social welfare. By continuity this still holds for $\varphi > 0$ but close to zero. \(\blacksquare\)

This makes a lot of sense: if gun violence comes mainly from crime then increasing self-defense gun ownership under strategic substitutability reduces the injury costs because it deters criminals as they face less unarmed individuals in the population.

Figure 7 constructs the correspondence between the equilibrium threshold $v_P^*$ and the regulated price of guns $q$ by replacing in equation (5) equation (7) to eliminate $v_D^*$ and have a graph in $(v_P^*, q)$ space. This diagram is generated using the same parameter specification as in Figure 3 but letting $q$ vary from zero (for $c = 0$ for simplicity) to one the wealth subject to capture (since $W = 1$ is the normalized value). As seen the marginal type $v_P^*$ in equilibrium increases as $q$ decreases and therefore crime decreases.
Pro Gun Control Argument. Pro gun control advocates tend to justify their case as a way to reduce gun violence irrespective if it comes from criminals or violent interactions among non criminals.\textsuperscript{10} The basic argument is that gun control can disarm violently inclined individuals that carry a gun decreasing gun violence in general. Therefore the gun control argument is understood in this set up if overall injury costs due to gun violence are reduced as gun prices are increased while not increasing crime.

To make sense of pro gun control policies let me consider the strategic complementarity case characterized by having $\bar{\varphi} < \varphi$. In this case, recall that the function $h^{PD} = 0$ has a positive slope and an equilibrium is like the one illustrated in either Figure 4 or 5.

**Proposition 6. (Pro Gun Control Argument)** Consider a pure strategy Nash equilibrium and assume strategic substitutability in gun interactions $\varphi > \bar{\varphi}$ where $F$ is greater than a certain threshold $\bar{F} > 0$ then for $r$ arbitrarily close to one a small increase in $q$ decreases gun ownership and crime.

**Proof:** It is sufficient to show that $\frac{dv_p(q,\theta)}{dq} \geq 0$ for small changes in $q$ since the sign of $\frac{dv_p(v_p)}{dq}$ is positive in this case. Notice again that

$$\frac{dv_p(q,\theta)}{dq} = -\left[\frac{q^*}{\varphi_p^*} + F [\bar{\varphi} - \varphi] \Phi'(v_D(v_p^*))\right] - \frac{\alpha}{(v_p^*)^2} + \Phi'(v_p^*) \left[\Gamma + F [\bar{\varphi} - \varphi] (1 - r) W \Phi'(v_D(v_p^*))\right].$$

Take the limit case in which $r = 1$ the denominator is still positive while the numerator is positive under $\bar{\varphi} < \varphi$ for $F > \bar{F}$ where $\bar{F} \equiv \Phi'(v_D(v_p^*))_{[\varphi - \bar{\varphi}]} > 0$.\textsuperscript{10}

Clearly, increases in $q$ increase the cost of acquiring and maintaining a gun for all potential gun owners. Nonetheless, under strategic complementarity it is more

Figure 8: An increase in $q$ under strategic complementarity in gun interactions.

difficult to disarm gun owners relative to the strategic substitutability case. Since $\tilde{\varphi}$ is a decreasing function of $q$ then as the cost of guns increase the term $-F[\tilde{\varphi} - \varphi]$ increases ceteris paribus (more the greater $F$ is assumed) and now less non criminal gun owners are disarmed relative to the strategic substitutability case. This in turn lowers the incentives for type $v_P$ because the increase cost in $q$ is not outweighed by the increase in non gun owners. Hence both $v^*_P$ and $v^*_D$ increase which allows to decrease gun ownership as well as crime since it would increase proportionally the costs for both gun owner subpopulations.

Figure 8 illustrates the pro control argument in the space $(v_P, v_D)$. The solid lines are the same under the specification in Figure 4, where there is a unique equilibrium, while the dashed lines are the two equilibrium functions for a small increase in $q$ (from $q = 0.5$ to $q = 0.6$). Notice that both $v^*_D$ and $v^*_P$ increase. The policy recommendation is therefore to increase $q$ so that injury costs decrease and social welfare is enhanced. I call this the pro gun control policy.

\footnote{The curve $h^{PD} = 0$ does not shift much with the increase in $q$ and therefore the dashed line is virtually on top of the initial $h^{PD} = 0$ bold curve.}
Proposition 7. (Pro gun control policy) Consider a pure strategy Nash equilibrium and assume strategic complementarity in gun interactions \( \varphi \in (\varphi, 1) \) for \( F > F \) and \( r \) close to one. The policy of a small increase in \( q \) decreases injury costs and is welfare enhancing.

**Proof:** Strategic complementarity in gun interactions \( \varphi > \varphi \) implies that a small increase in \( q \) increases both \( v_p^* \) and \( v_D^* \). Notice that since \( \alpha^* = 1 - \Phi(v_p^*) \) then \( \alpha^* \) decreases and \( W(1 - \alpha^*) \) increases as \( q \) is increased. Furthermore, gun ownership is given by \( \alpha^* + \beta^* = 1 - \Phi(v_D^*) \) and increases in \( v_D^* \) decrease gun ownership. Hence IC falls because if \( \alpha^* + \beta^* \) decreases then \( \alpha^* + \varphi \beta^* \) also decreases. Therefore social welfare increases according to (12).

Figure 9 illustrates the pro gun control policy by representing again the equilibrium correspondence between \( v_p^* \) and \( q \) under strategic complementarity with the same parameter specification as in Figure 4 with \( \varphi = 0.1, F = 10 \) and \( c = 0 \). Given this parameter specification increases in \( q \) increase the equilibrium threshold \( v_p^* \) reducing gun predation. This illustrates well the gun control argument and matches one of their slogans "less guns, less gun violence".

The change from strategic substitutability to complementarity in gun confrontations when the probability of a violent interaction \( \varphi \) increases among non criminals should be summarized in an intuitive way here. Even though in principle an increase in gun prices does increase the cost of owning and maintaining a gun for all gun owners there are two opposite effects that take place in gun confrontations in the population which conditions the way gun control reduces the fraction of gun owners. On the one hand, there is a strategic substitutability in confrontations among criminals and self-defense gun owners since the former would like to be matched with unarmed victims. Therefore small increases in \( q \) disarm self-defense gun owners (relatively more the lower \( \varphi \) is) increasing the amount of unarmed victims on average that criminals face which in turn counteracts the price increase for them. This is
what increases the amount of predation for \( \varphi \) low enough (less than \( \bar{\varphi} \)). On the other hand, there is also a strategic complementarity effect in gun confrontations among non criminals, since the benefit of having a gun increases with \( \varphi \) and which lowers the incentives for criminals to prey. This is what allows gun control to reduce crime as well as self-defense gun ownership.

3. Discussion

This section focuses on several issues in the gun control debate as well as some extensions and limitations of the analysis.

a) After every massacre that has occurred in the United States or any part of the world the debate about gun control is reopened and both sides of the debate come out to defend their positions. The model can allow us to understand better the kind of implicit assumptions each side is willing to make.

Anti-gun control advocates tend to view gun violence in societies as a feature that comes mainly from interactions between "good" guys (non criminals) and "bad" guys (criminals). On the other hand violent human behavior as in school massacres
are viewed as being unusual or not the main violent behavior in gun interactions in society. In terms of the model this view reduces to assuming that \( \varphi \approx 0 \) i.e. gun violence does not occur usually among non criminals. Precisely because of this implicit assumption anti-gun control advocates tend to enhance the benefits that non gun owners would enjoy from increases in gun ownership among non criminals, since they are viewed as the "good" guys that deter the "bad" guys. This reduces to the vigilante policy in the model.

Pro-gun control advocates on the other hand tend to view gun violence as coming from violent individuals that have a gun at their disposal. Under this view there are no "good" and "bad" guys just people that use a gun in criminal and violent interactions. Moreover, the type of violent behavior that has increased in public shootings in the United States has made the point that violent behavior (among non criminals) coupled with lethal weapons can generate tragedies. In terms of the model this view reduces to assuming that \( \varphi \) and \( F \) are sufficiently high, because of the potential lethality of guns, such that \( \varphi > \bar{\varphi} \) i.e. gun violence occurs also among non criminals. Therefore the policy of restricting gun access (either by banning or any other mechanism like taxes) is viewed as restraining gun violence in general.

b) Even though political equilibrium is outside the scope of this article the model suggests possible multiple *political economy* equilibrium. For instance, Australia and Europe have strong gun control policies, \( q \) is high and therefore \( \bar{\varphi} (q) \) is low, making strategic complementarity more likely (at least locally), thus providing support for more gun control. On the other hand, the United States has less gun control, \( q \) is low and \( \bar{\varphi} (q) \) is high, which makes strategic substitutability prevalent and the anti-control argument justified there. This multiplicity suggests that in a more general model that involves political economy equilibrium based on majority voting a society can be locked in one extreme depending on the initial levels of gun control. It is interesting to compare two extreme cases. Australia had a spate of mass public shooting in the 1980s and 1990s, culminating in 1996, when an individual opened fire at the Port Arthur Historical Site in Tasmania killing 35 people. Within two
weeks the government had enacted strict gun control laws that included a ban on semiautomatic rifles. There has not been a mass shooting in Australia since. On the other hand, the United States has had much looser gun control policies even though mass public shootings have increased in the U.S. since the 1960’s when an individual climbed a tower on the University of Texas campus and started taking people down.

c) The model does not distinguish between legal and illegal gun ownership which may be a crucial distinction in assessing gun control policies. Some evidence by Stolzenberg and D’Alessio (2000) show that increases in illegal gun ownership is associated with higher crime rates while for legal ownership it does not.

d) The static nature of the model does not allow to analyze the role of the stock of guns in society. Given that guns are durable goods the role of the stock of guns in a dynamic setting would pose an additional problem for a gun control policy of increasing the price of guns since it would have to be complemented at least with a "buy back program" so that guns can be taken out of private hands when gun owners decide to sell their old guns. Moreover, the static nature of the model only allows to capture some of the short-run costs that predation brings to a society i.e. the deadweight loss that gun violence creates, but it fails to capture long run costs associated with gun predation i.e. more predation in society lowers the return to effort of honest people who create wealth which declines a society’s income per capita.

4. Literature Review

To my knowledge the anti-control argument was modeled theoretically by Taylor (1995), McDonald (1995) and Ghatak (2001). These theoretical papers show in different ways how gun control can increase gun predation and thus back fire as a policy. On the other hand, Glaeser and Glendon (1998) and Chaudhri and Geanakoplos (1998) have suggested a strategic complementarity in gun interactions. The former one is an empirical paper but does go on to suggest that complementarities can arise in gun interactions while the later takes the possibility of strategic complementarity in gun interactions to rationalize gun control. Nonetheless, the main contribution of the current paper is to show a way in which both arguments could be understood in
the same setup for different assumptions on some key parameters.

5. Conclusions

Gun control has been a hot public debate in the United States in the last couple of decades. Two opposing sides in the debate have suggested two very radical and different gun policies. Anti control advocates have argued that gun control disarms only non-criminal gun owners while not decreasing crime. They propose consequently looser gun control regulations, specially lowering taxes so that higher self-defense gun ownership could serve as an effective deterrent mechanism against gun predation. On the other side of the debate gun control advocates have argued that gun ownership is positively related to gun violence in general. Therefore restricting guns in society can reduce gun violence. In this article I have constructed a simple model in which I conceptualize both sides of the debate under different assumptions on key parameters. Namely, if violent behavior among non-criminals is high enough then an overall strategic complementarity effect in gun interactions arises which allows gun control to reduce gun violence through price regulations. If this violent behavior among non-criminals is low enough then gun control can back-fire as the anti control advocates have emphasized.

Bibliography


