Beyond Expected Utility in the Economics of Health and Longevity

Juan Carlos Córdoba*

and

Marla Ripoll^{†‡}

September, 2013

Abstract

The expected utility framework is the workhorse model used to study issues of health and longevity. We show that this framework has important limitations for this purpose. The underlying assumption of expected utility is that individuals are indifferent between early and late resolution of uncertainty. This assumption generates a number of predictions regarding the economic value of life that are inconsistent with available evidence. In addition, cross-country calibrated versions of the expected utility framework with a standard CRRA utility predict that life is a bad rather than a good for a sizable fraction of the world population; and that for an even larger fraction, the value of life is lower than the present value of foregone income, a natural lower bound. We show that a non-expected utility framework à la Epstein-Zin-Weil in which both the elasticity of intertemporal substitution and the coefficient of relative risk aversion are below one can overcome the limitations of expected utility. We illustrate the quantitative implications of our model by evaluating the welfare effects of changes in life expectancy and per capita income across countries and time.

Keywords: life expectancy, value of statistical life, mortality risk aversion, Epstein-Zin-Weil preferences, welfare, AIDS. *JEL Classification: 115, J17.*

^{*}Department of Economics, Iowa State University, 279 Heady Hall, Ames IA 50010, e-mail: cordoba@iastate.edu.

[†]Department of Economics, University of Pittsburgh, 4532 W.W. Posvar Hall, Pittsburgh PA 15260, e-mail: ripoll@pitt.edu.

[‡]For helpful comments we thank participants at the 2012 Macro Brown Bag at the University of Pittsburgh, the 2012 Growth and Development Conference at the University of Toronto, the 2012 Economics Seminar at Universidad Javeriana, the 2012 Midwest Macroeconomics Meetings, and the 2012 Society for Economic Dynamics Meeting.

1 Introduction

There is a growing interest in applying dynamic-style macro models to investigate issues of health and longevity. Recent examples include Murphy and Topel (2006) and Hall and Jones (2007), who study the economic value of health improvements and the reasons for the secular increase in health spending in the US; and Becker et al. (2005) and Jones and Klenow (2011), who estimate the economic gains associated to lower mortality rates around the world. A common feature of this literature is the use of the expected-utility model. Although this type of framework has been used to study a variety of macroeconomic and finance issues, it is not clear that it is also justified in the study of longevity issues.

This paper documents a number of limitations in using the standard expected utility model to study health and longevity, and proposes an alternative model that overcomes them. We identify five main limitations. The expected utility model predicts: (i) that individuals are indifferent between late and early resolution of uncertainty, but when it comes to longevity, a variety of studies suggest that many individuals prefer late resolution, particularly in cases of incurable diseases; (ii) a constant marginal benefit of survival, rather than the more intuitive diminishing marginal benefits; (iii) a positive relationship between the ratio of the willingness to pay for life extensions to income, and both income and longevity; existing data, however, suggests a negative relationship. Furthermore, calibrated expected utility models rely on non-homothetic preferences to replicate plausible values of the willingness to pay for life extensions by relatively rich individuals. As a result, the model predicts that: (iv) a sizable fraction of the world population would regard life as a bad rather than as a good; and (v) an even larger fraction of the world population, mostly poor to middle income individuals, would require an imputed compensation upon death that is lower than the present value of foregone income.

We consider non-expected utility models along the lines proposed by Kreps and Porteus (1978), Epstein and Zin (1989, 1991) and Weil (1990) and show that they can overcome the five limitations of the expected utility model. The key feature of the non-expected utility model considered here is that it disentangles the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion (CRRA). In our context, risk aversion refers to risk associated with a very specific gamble, that of life or death. We find that when EIS is below the inverse of the CRRA, individuals prefer late to early resolution of uncertainty regarding life-death lotteries, and the marginal benefits of survival are diminishing. Moreover, if the EIS and the CRRA are lower than one, then the model predicts a negative relationship between longevity and the ratio of the willingness to pay for life extensions to income. Since longevity and income are positively correlated in the data, it also predicts that this ratio decreases with income, as suggested by the evidence. Finally, our homothetic model can match the willingness to pay for life extensions by rich individuals without implying a value of life below the present value of foregone income for poor to middle income individuals.

Consider the five predictions of the standard expected-utility model and the corresponding ones in our non-expected utility model. First, expected utility models predict that individuals are indifferent about the timing of the resolution of uncertainty as formalized by Kreps and Porteus (1978). To fix ideas, suppose a single coin toss. If heads, the consumption stream for periods 1 and 2 is 5 and 5. If tails, the consumption stream is 5 and 0. Since consumption at time 1 is identical in both cases, expected-utility predicts that the individual must be indifferent between tossing the coin at period 1 or at period 2. The prediction that individuals are indifferent about the timing of uncertainty resolution is particularly implausible in the context of longevity issues. Evidence suggests that many individuals prefer late resolution of uncertainty, what is sometimes called "protective ignorance," in cases of incurable diseases. For example, studies regarding predictive genetic testing for the Huntington's disease find that a sizable portion of the population at risk prefers not to know (Kessler, 1994; van der Steenstraten et al., 1994; Tibben et al., 1993; Yaniv et al., 2004). Individuals cite as the major reasons to avoid being tested "fear of adverse emotional effects after an unfavorable diagnosis, such as deprivation of hope, life in the role of a patient, obsessive searching for symptoms and inability to support one's spouse (Yaniv et al. 2004, p. 320)." Wexler (1979) describes the results of 35 interviews with individuals at risk for the disease as follows: "All of the interviewers were painfully aware that the disease is terminal, but for them termination comes not at the moment of death but at the moment of diagnosis. Most fantasize the period following diagnosis to be a prolonged and unproductive wait on death row" (p. 199-220). Studies of HIV testing avoidance also find that many individuals exhibit some type of protective ignorance (Kellerman et al., 2002; Day et al., 2003; Weiser et al., 2006). For example, Day et al. (2003, p. 665) conclude that the major barriers to voluntary counselling and testing were "fear of testing positive for HIV and the potential consequences, particularly stigmatization, disease and death."

Non-expected utility models can explain why individuals prefer late resolution of uncertainty. It occurs when the concavity associated to temporal deterministic fluctuations of consumption is larger than the concavity associated to a temporal random fluctuations, that is, when EIS < (1/CRRA). The intuition is the following. If uncertainty is resolved early, then consumption fluctuations become deterministic early on and the EIS plays the key role in determining utility. If uncertainty is resolved late, then consumption fluctuations remain random and the CRRA plays the central role. When EIS < (1/CRRA) individuals prefer late resolution because it is a way to choose a less concave utility function to evaluate consumption fluctuations. Individuals who feel that death comes at the moment of diagnosis are rationalized by the model as an individual with an EIS < 1, so that lifetime utility is the minimum if any consumption is equal to zero. Preference for late resolution differentiates longevity issues from other economic issues such as the allocation of assets. For example, studies of the equity premium puzzle usually find that EIS > (1/CRRA) is required in order to explain the large equity premium implying that, in the context of financial markets, individuals prefer early to late resolution (Donaldson and Mehra, 2008). Putting these inequalities together, our paper suggests that individuals are more averse to financial risk than to health risks. We show that in an extension of our model with both types of risks, a preference for early resolution of financial risks can coexist with a preference for late resolution of health risks (Section 7).

The second issue identified above is that the standard model predicts a constant marginal utility

of survival. This means that a patient in the model values equally a procedure that provides one additional percentage point of survival regardless of whether the chances of survival without the procedure are 5% or 95%. This prediction conflicts with the standard economic intuition that individuals value more a good when it is scarce than when it is abundant. A constant marginal benefit of survival is a natural consequence of expected-utility formulations which are linear in the probabilities. In contrast, our non-expected utility formulation implies linearity in probabilities only in the special case EIS = (1/CRRA). However, if EIS < (1/CRRA) then the marginal utility of survival is decreasing in survival. The intuition for this results is tied to why individuals prefer late resolution. In our model, the time discount factor is proportional to the survival rate raised to a power, π^a . If individuals are indifferent between early and late resolution then a = 1and the discount factor is proportional to π . Individuals who prefer late resolution are intrinsically more patient in the sense that $\pi^a > \pi$ or, since $\pi \in (0, 1)$, when a < 1. The fact that the discount factor increases at a decreasing rate with π is what explains the diminishing marginal utility of survival in our model.

The standard model also predicts that the value of statistical life (VSL) relative to income increases with both income and longevity. The VSL is an estimate of the social willingness to pay to save one life. For example, the Environmental Protection Agency employs a VSL of \$6.3 million for cost-benefit analysis. Below we review various empirical estimates and their methods. The prediction that the VSL-to-income ratio increases with income follows because life is a superior good in the expected utility model, as stressed by Hall and Jones (2007). The prediction that the VSLto-income ratio increases with longevity follows from the fact that the value of life increases with survival π for any income level, which increases the willingness to pay for life extensions. However, the first prediction is inconsistent with available evidence surveyed in Viscusi and Aldy (2003) for a cross-section of countries and a cross-section of individuals in the US. As we document below, this evidence suggests that the VSL-to-income ratio is decreasing in income: poorer individuals seem to value life relative to their annual income more than richer people do.

The non-expected utility model, on the other hand, can better replicate the cross-country pattern of a decreasing VSL-to-income ratio. The key channel is the ability of the model to generate a decreasing willingness to pay for survival as survival increases. According to the model, the high value of life relative to income in poorer countries can be explained by the fact that life is typically shorter in those countries and therefore more valuable. Specifically, we show that when the EIS < 1, the typical case considered in quantitative macro, and the CRRA < 1, then the VSL-to-income ratio decreases with survival. Since survival and income are strongly positively correlated in the data, the model can replicate a pattern consistent with the evidence documented in Viscusi and Aldy (2003). The model can also explain the time-series pattern of increasing health expenditures over GDP studied by Hall and Jones (2007). In their separable model, this pattern results from life being a superior good. In our model, it results from the aging of the population, which means that average remaining life span is shorter and therefore more valuable.

A fourth issue with the expected utility model is that, under the standard CRRA utility representation, it may imply a negative VSL for poor countries and individuals. To illustrate this issue suppose $W = \pi u(c) + (1 - \pi) u(\omega)$ is the expected utility of an (alive) individual: it weights the utility of being alive u(c), and utility of being dead $u(\omega)$, by the probabilities of each event. The value of life in consumption units can be defined as $VSL = (W - u(\omega))/u'(c)$ in this example. This formula shows that ω , which can be interpreted as the minimum consumption level, is the key parameter to match the VSL in the expected utility model, and more importantly, that ω is typically large. For example, if $u(c) = \ln c$, as is commonly assumed in macro, and $\omega = 0$ then the imputed utility of dying is $-\infty$ and as a result $VSL = \infty$. To match a plausible VSL, such as \$6.3 million, it is necessary to assume a sufficiently high ω . The need for a large ω is not exclusive of the log utility formulation but, as we show, generalizes to separable CRRA utilities, as well as other forms. This need is particularly acute when EIS < 1, which is the typical case in macro, but it remains even when EIS > 1. The expected utility model therefore has the prediction that additional years of life are a bad rather than a good for those individuals whose consumption is below ω . The reason is that when income is lower than the calibrated ω , as it may be the case in poor countries or for poor individuals, then the value of life becomes negative. For example, Becker et al. (2005) estimate ω to be slightly below one dollar a day, which would imply that a sizable fraction of the world population, over one billion people, don't value life. But the evidence on the VSL surveyed in Viscusi and Aldy (2003) indicates that in the poorest country for which data is available, India, the VSL is positive. Based on wage data for Indian manufacturing workers in 1990, whose average annual income was \$778 (2000 US dollars), estimates of the VSL are around \$1 million, and they go up to \$4 million when correcting for self-selection. Life seems to be worth living everywhere. Although it is always possible to set the minimum level of consumption so that life is worth living for all people around the world, this can only be done assuming a VSL that is outside of the range of available estimates from micro data for rich countries, specifically between \$4 and \$9 million for the US (Viscusi and Aldy, 2003, p. 6). We also show that departures from CRRA that could alleviate the presence of negative VSL would counterfactually require lower degrees of risk aversion for poor individuals, and furthermore do not overturn the prediction of the expected utility model that the VSL-to-income ratio is increasing in income.

A final issue with the expected utility model under the standard CRRA utility representation is that the imputed compensation upon death for poor and middle income individuals is below the present value of foregone income. The non-expected utility model, on the other hand, predicts that regardless of their level of income, the VSL exceeds the present value of foregone income. The issue with the standard model is that a single parameter controls both risk aversion and aversion to non-risky fluctuations of consumption. We show that by disentangling the two parameters it is possible to set a standard curvature of the per-period utility function, so that EIS < 1 is possible, but without making dying infinitely costly even when $\omega = 0$. In our non-expected utility model, the VSL is a positive function of the CRRA but it is mostly independent of the EIS. As a result, we are able to set the minimum consumption equal to zero, so that life is worth living everywhere, set any plausible value for EIS within the range of existing estimates, and calibrate the CRRA to match a plausible value of statistical life in the US. This is not possible with the expected utility model which, as a result, provides counterfactual predictions for questions of health and longevity. We investigate the quantitative predictions of our non-expected utility model by carefully calibrating it and using it to assess the economic values of longevity changes in a panel of 144 countries and for the period 1970 to 2005. We compare our results to those of Becker et al. (2005) and Jones and Klenow (2011), both of which use the expected utility framework, and document significant differences. Figure 1 displays life expectancy at birth in 2005 relative to 1970 for a sample of 144 countries. Except in the cases in which war or AIDS have shortened life spans, there have been significant gains everywhere else, with some poorer countries gaining as much as 26 years of life, and richer countries gaining between 7 and 10 years. In contrast, per capita income differences have been quite persistent since 1970. With the exception of some growth miracles, income disparities remain quite stable. Figure 2 portrays life expectancy and per capita income relative to the US in 2005. The figure suggests that while there is substantial cross-sectional inequality in income, with a number of countries with less than 10% of US per capita income, there is less inequality in life spans, with no country having less than 50% of life expectancy in the US.

We use calibrated versions of both the expected and non-expected utility models to compute full measures of income that adjust for differences in life expectancy, and compare them to standard measures of income. While parameters are assumed to be common across countries, we let countries differ on their per capita income and life expectancy at birth. Common parameters across countries are either exogenously set to standard values used in the literature, or calibrated to relevant data on the VSL in the US. For the non-expected utility model, the two key parameters are the EIS $1/\sigma$, and the CRRA γ . We exogenously set $\sigma = 1.25$ following Murphy and Topel (2006) and calibrate γ to match a VSL of \$4.5 million in the United States, one within the range of available estimates. The calibration results in $\gamma = 0.57$, so individuals are averse to the risk of dying, but the CRRA is lower than one. As we discuss below, the calibrated value of γ is smaller than one even for VSL outside the \$4 to \$9 million estimated for the US. For instance, while $\gamma = 0.74$ is consistent with a VSL of \$9 million, $\gamma = 0.85$ implies a VSL of about \$36 million. In other words, data on the VSL provide a tight identification of mortality risk aversion, one that supports $\gamma < 1$. Our calibration of $\gamma = 0.57$ implies that in 2005, the minimum VSL was the one computed for Zimbabwe at \$26,000, while the maximum was \$7.4 million in Luxembourg. However, the implied VSL-to-income ratio is 153 in Zimbabwe and 105 in Luxembourg.

The non-expected utility model yields novel predictions on the welfare effects of changes in life expectancy. Consider first the welfare changes across time between 1970 and 2005. The expected utility model implies that countries that lost life expectancy during this period experienced a welfare gain. This is the case because VSL is negative in these countries, so that a shorter life span increases welfare. In contrast, the non-expected utility model predicts the opposite: a full measure of income that incorporates both changes in income and life expectancy indicates a welfare loss for these, mostly poor, countries. Regarding countries that gained life expectancy between 1970 and 2005, which are mainly poor and middle-income countries, we show that the expected utility model predicts a welfare *loss* for at least some these countries. The reason is that this model heavily penalizes gains in life expectancy of even as much as 20 to 25 years because per capita income remained mostly stagnant in these countries. In contrast, as the VSL is positive in all countries under the non-expected utility model, large gains in life expectancy do show up as welfare gains even for those countries whose per capita income remained stagnant over the 1970-2005 period. The differences between the two models in evaluating the welfare gains across time are substantial.

Consider now the cross-country welfare measures in 2005. In this case we compare each country's income and full income against that of the US in 2005. We show that in a cross-section, the expected utility model implies a "full income" world inequality that is lower than the income inequality. In other words, once life expectancy is taken into account, poor countries fair better compared to the US than when only per capita income is considered. The reason is that even though life expectancy is much lower in these countries, under the expected utility model life is also worth less there (shorter life spans are preferred). The predictions of the non-expected utility model are different. Specifically, under non-expected utility poorer countries fair even worse relative to the US when a full measure of income is taken into account because life expectancy there is too low, while the VSL-to-income ratio is high.

The model we propose can also be used to assess the welfare effects of positive events like the end of wars and devastating events like AIDS. For this purpose we compute full measures of income to compare 1990 and 2005, the relevant dates to the AIDS pandemic. Countries that experience end of wars, like Rwanda, Bhutan and Nepal gained 16, 12 and 11 years of life respectively between 1990 and 2005. From the perspective of the expected utility model, these relatively sizeable increases in life expectancy do not change welfare as much. For instance, even though Rwanda's per capita income increased over this period, this model implies that welfare actually decreased there, a result that reflects the implied negative value of life in this country. The predictions of the non-expected utility model are quite different: a longer life span directly adds to welfare. Similarly, this model highly penalizes shorter life spans, as it is the case countries affected by AIDS: Central Africa lost 3 years of life expectancy between 1990 and 2005, while South Africa lost 9, Botswana 13 and Zimbabwe 19.

The remainder of the paper is organized as follows. Section 2 shows the mechanics of the non-expected utility framework using a two-period model. Relevant empirical evidence on the VSL is reviewed in Section 3. Section 4 describes the standard expected utility model used in the literature, calibrates the model, and derives its various counterfactual predictions. Section 5 describes the general non-expected utility model, calibrates it and discusses how its advantages. Section 6 uses both models to compute the welfare effects of the observed changes in life expectancy since 1970, as well as to evaluate the effects of ending wars and the AIDS pandemic since 1990. Section 7 presents various robustness checks for our results, and concluding comments are in Section 8.

2 A two-period model

This section presents the basic mechanism of the paper in the context of a two-period model. It also illustrates the limitations of the expected utility model and the advantages of a non-expected utility approach. Consider an individual who lives for one period and may live for a second period with probability π . Life expectancy is $1 + \pi$. Consumption in the first period is c_0 while consumption in the second period \tilde{c}_1 is a random variable equal to c_1 with probability π or ω with probability $1 - \pi$. Parameter ω corresponds to the perceived consumption upon dying. The resolution of uncertainty in this example may occur in the first or second period, what is referred to as early versus late resolution. Early (late) resolution means that the uncertainty about the second-period life-death outcome is resolved in the first (second) period.

The lifetime utility of the individual is represented by the function:

$$V(\mathbf{c},\pi) = E_0 \left[u(c_0) + \beta u \left(v^{-1} \left(E_1 \left[v(\tilde{c}_1) \right] \right) \right) \right]$$
(1)

where $\mathbf{c} = (c_0, c_1)$, E_t is the mathematical expectation conditional on information available at time t = [0, 1], and u and v are strictly increasing strictly concave functions. In our example $E_t = E$ because the probability of dying is constant. Function $V(\mathbf{c}, \pi)$ is of the non-expected utility type but includes as a special case the following expected utility type,

$$V^{EU}(\mathbf{c},\pi) = u(c_0) + \beta E\left[u(\widetilde{c}_1)\right],\tag{2}$$

where superscript EU denotes expected utility. This case occurs when either u = v, or when uncertainty is resolved early. Specifically, when uncertainty is resolved early we have

$$V(\mathbf{c},\pi) = \pi \left[u(c_0) + \beta u \left(v^{-1} \left(v(c_1) \right) \right) \right] + (1-\pi) \left[u(c_0) + \beta u \left(v^{-1} \left(v(\omega) \right) \right) \right] = u(c_0) + \beta E \left[u(\widetilde{c}_1) \right],$$

which implies that under early resolution, it is the concavity of function u the one that determines the risk aversion associated to the second-period lottery.

When $u \neq v$ and uncertainty is resolved late, $V(\mathbf{c}, \pi)$ becomes:

$$V(\mathbf{c},\pi) = u(c_0) + \beta u\left(v^{-1}\left(E\left[v(\tilde{c}_1)\right]\right)\right).$$
(3)

In this formulation the degree of risk aversion is determined by the concavity of v, while u determines aversion to intertemporal consumption fluctuations. To see this more clearly, it is convenient to define the certainty equivalent associated to the lottery of life as $\bar{c}_1(v) \equiv v^{-1}(E[v(\tilde{c}_1)])$. Given $\bar{c}_1(v)$, lifetime utility can then be written as a deterministic flow of utilities $u(c_0) + \beta u(\bar{c}_1(v))$. The only role of v is to determine the certainty equivalent: the more risk averse the individual is, as defined by the curvature of v, the lower the certainty equivalent is. For example, linear v implies risk neutrality, case in which $\bar{c}_1(v) = \pi c_1 + (1 - \pi) \omega$.

Comparing (3) and (2) it follows that the individual prefers late to early resolution of uncertainty whenever $v^{-1}(E[v(\tilde{c}_1)]) > u^{-1}(E[u(\tilde{c}_1)])$ or, equivalently, $\bar{c}_1(v) > \bar{c}_1(u)$. In words, the individual prefers late resolution when v is less concave (has less curvature) than u. This preference for late resolution is consistent with the evidence presented in the introduction according to which individuals avoid testing for fatal diseases, afraid that termination of life really comes at the moment of diagnosis rather than at the moment of death. In this case, "protective ignorance" is preferred. The expected utility model, by assuming u = v cannot be consistent with this preference, while the non-expected utility model can.

Suppose further that u and v are of the CRRA type: $u(c) = c^{1-\sigma}/(1-\sigma)$ and $v(c) = c^{1-\gamma}/(1-\gamma)$ where $\sigma \ge 0$, $\sigma \ne 1$ and $\gamma \ge 0$, $\gamma \ne 1$. In this representation, the EIS is $1/\sigma$ while the CRRA is γ . In the current context γ captures aversion to the risk of dying. The restriction $\gamma \ge 0$ is required for risk aversion, although $\gamma < 0$ is also possible if the individual is a risk lover. The expected-utility model is obtained in the special case $\sigma = \gamma$. If uncertainty is resolved early, then σ determines both intertemporal substitution and risk aversion, while when uncertainty is resolved late, then γ determines risk aversion. According to our discussion above, late resolution of uncertainty is preferred whenever $\sigma > \gamma$.

Given the assumed functional forms for u and v, (1) becomes:

$$V(\mathbf{c},\pi) = \frac{c_0^{1-\sigma}}{1-\sigma} + \beta \frac{(\pi c_1^{1-\gamma} + (1-\pi)\omega^{1-\gamma})^{\frac{1-\sigma}{1-\gamma}}}{1-\sigma}.$$
(4)

The following proposition summarizes four key properties of the model regarding longevity. Notice that life expectancy is linear in π . Therefore increases in life expectancy, or longevity, are analogous to increases in π .

Proposition 1 - Longevity under non-expected utility. Consider the non-expected utility representation à la Epstein-Zin-Weil in equation (4). Then: (i) $V_{\pi}(\mathbf{c},\pi) > 0$ if and only if $c_1 > \omega$; (ii) $V_{\pi\pi}(\mathbf{c},\pi) = 0$ if $\sigma = \gamma$; (iii) $V_{\pi\pi}(\mathbf{c},\pi) < 0$ if and only if $\sigma > \gamma$; (iv) late resolution of uncertainty is preferred if and only if $\sigma > \gamma$.

Proof. The marginal utility of survival is given by:

$$V_{\pi}(\mathbf{c},\pi) = \beta (\pi c_1^{1-\gamma} + (1-\pi) \,\omega^{1-\gamma})^{\frac{\gamma-\sigma}{1-\gamma}} \frac{c_1^{1-\gamma} - \omega^{1-\gamma}}{1-\gamma}.$$

This is positive if and only if $c_2 > \omega$. Moreover,

$$V_{\pi\pi}(\mathbf{c},\pi) = \beta \left(\gamma - \sigma\right) \left(\pi c_1^{1-\gamma} + (1-\pi) \,\omega^{1-\gamma}\right)^{\frac{\gamma-\sigma}{1-\gamma}-1} \left(\frac{c_1^{1-\gamma} - \omega^{1-\gamma}}{1-\gamma}\right)^2.$$

Thus, $V_{\pi\pi}(\mathbf{c},\pi) = 0$ if $\gamma = \sigma$, the separable case, or negative only if $\sigma > \gamma$. Part *(iv)* follows from the discussion above.

Part (i) of Proposition 1 states that survival, or life expectancy, is a good if and only if $c_1 > \omega$. As we show below, this simple restriction is difficult to satisfy in calibrated versions of the expected utility model. Part (ii) states that the marginal utility of survival is constant in the expected utility model: longevity does not exhibit diminishing marginal utility. Part *(iii)* shows that the nonexpected utility model exhibits a diminishing marginal utility of survival when EIS < (1/CRRA)meaning that the individual is relative more averse to non-random fluctuations of consumption than to random fluctuations. If, for example, the individual is risk neutral ($\gamma = 0$) then $V_{\pi\pi}(\mathbf{c},\pi) < 0$ for any $\sigma > 0$. The intuition for the diminishing marginal returns is that the added benefits of survival, in terms of higher future utility, are partly downplayed by the added gap between present consumption and future certainty equivalent consumption, a gap that is more costly the larger the σ .

Consider next the predictions of the model regarding the willingness to pay for life extensions. Suppose the individual has initial assets a_0 , incomes y in each period, markets are complete and the interest rate is r. The life-time budget constraint of the individual is

$$c_0 + \pi (1+r)^{-1} c_1 = a_0 + y + \pi (1+r)^{-1} y.$$

The willingness of an individual to pay for a procedure that increases the chances of survival by $\Delta \pi$ is:

$$WTP = \left| \frac{\partial a_0}{\partial \pi} \right| \Delta \pi = \frac{\partial V / \partial \pi}{\partial V / \partial a_0} \Delta \pi$$

The overall willingness to pay by a population of size N is $N \cdot WTP$, while the number of lives saved by the procedure is $N \cdot \Delta \pi$. The value of statistical life (VSL) is defined as the overall willingness to pay to save a life:

$$VSL = \frac{N \cdot WTP}{N \cdot \Delta \pi} = \frac{\partial V / \partial \pi}{\partial V / \partial a_0}.$$

It is important to point out that this model-generated definition of VSL is consistent with its empirical counterpart, as we discuss in Section 3. Empirical estimates of the VSL are based on the wage differentials from marginal changes in the survival probabilities across occupations.

The equation above implies that a positive VSL requires $c_1 > \omega$. Using (1), the VSL in this two-period can be written as:

$$VSL = \theta(\omega/c_1, \gamma) \cdot c_1/(1+r).$$
(5)

where

$$\theta(x,\gamma) \equiv \frac{1-x^{1-\gamma}}{1-\gamma}.$$
(6)

Term $c_1/(1+r)$ in equation (5) is the present value of loss consumption in case of death. Coefficient $\theta(\omega/c_1, \gamma)$ is an adjustment over this present value, a premium if $\theta > 1$, or a discount if $\theta < 1$. We call θ the coefficient of (relative) mortality aversion (CMA). If $\theta > 1$ the individual exhibits mortality aversion in the sense that the imputed compensation in the event of death is higher than the present value of loss consumption. In contrast, if $\theta < 1$ this imputed compensation is below the present value of loss consumption. The case $\theta > 1$ is more plausible since most empirical estimates

of the VSL are consistently higher than the present value of loss consumption, as we review below. The following proposition characterizes some properties of the CMA.

Proposition 2 - Coefficient of mortality aversion. (i)
$$\theta(1, \gamma) = 0$$
, $\theta(0, \gamma) = 1/(1 - \gamma)$ and $\theta_x(x, \gamma) < 0$; (ii) $\theta\left(\gamma^{\frac{1}{1-\gamma}}, \gamma\right) = 1$; (iii) $\theta(\omega/c, \gamma) \geqq 1$ if $c \geqq \omega \gamma^{\frac{1}{\gamma-1}}$.

Part (i) of Proposition 2 states that the CMA could be as low as zero if consumption if the dead and alive states are identical, or as high as $1/(1 - \gamma)$. If x = 0, the CMA is constant and a monotonic transformation of γ , the CRRA. In addition, the higher the consumption imputed upon death, the lower the premium θ . Part (ii) describes the level of ω/c at which $\theta(\omega/c, \gamma) = 1$, so that the VSL exactly coincides with the present value of loss consumption. That level is $\gamma^{1/(1-\gamma)}$. This implies that in the case of risk neutrality ($\gamma = 0$), then $\theta = 1$ only if $\omega/c = 0$. Part (iii) states that depending on the consumption level, the VSL may be larger, equal or lower than the present value of loss consumption. If $\omega = 0$, then the VSL is always higher than the present value of loss consumption. In this case the CMA maps one-to-one with the CRRA γ . This is our preferred case, as it is consistent with the intuitive idea that the VSL should be bounded below by the present value of loss consumption.¹ In contrast, if $\omega > 0$, then poorer individuals with consumption in the interval $[\omega, \omega \gamma^{\frac{1}{\gamma-1}})$ have positive VSL, but value life below the present value of loss consumption. Finally, if the individual is risk neutral ($\gamma = 0$) but $\omega > 0$, then $\theta(x, \gamma) < 1$ for all $x \ge 0$.

To derive further intuition, suppose $a_0 = 0$ and that the interest rate is such that consumption is constant: $c_0 = c_1 = y^2$. The value of life in the expected utility case then be written as:

$$VSL^{EU}(y,\pi,\omega,\sigma) = \beta y \theta(\omega/y,\sigma).$$
(7)

The following proposition characterizes the VSL for the expected utility case.

Proposition 3 - VSL under expected utility. Consider the expected utility representation with standard CRRA utility, which corresponds to assuming $\sigma = \gamma$ in equation (4). Then: (i) $VSL^{EU}(y, \pi, y, \sigma) = 0$; (ii) $VSL^{EU}(y, \pi, 0, \sigma) = \infty$ if $\sigma > 1$ or $VSL^{EU}(y, \pi, 0, \sigma) = \beta y (1 - \sigma)^{-1}$ if $\sigma \in (0, 1)$; (iii) $VSL^{EU}(\lambda y, \pi, \omega, \sigma) > \lambda VSL^{EU}(y, \pi, \omega, \sigma)$ for $\lambda > 0$ if $\omega > 0$; and (iv) $VSL^{EU}(y, \pi, 0, \sigma) = 0$.

$$V_0(a_0, \pi_0) = V_0(a_0 - WTP(\epsilon), \pi_0 + \epsilon),$$

so that the $WTP(\epsilon)$ corresponds to a consumption equivalent measure. As discussed above, the WTP is used to calculate the VSL when $\epsilon \to 0$.

²The required interest rate is:

$$1 + r = (\pi + (1 - \pi) (\omega/y)^{1 - \gamma})^{\frac{\gamma - \omega}{1 - \gamma}} / \beta$$

In the expected utility case, $\sigma = \gamma$, this expression simplifies to $1 + r = \beta^{-1}$. In the non-expected utility case with $\omega = 0$ and $\gamma \in (0, 1)$, the expression simplifies to $1 + r = \pi^{\frac{\sigma - \gamma}{1 - \gamma}} / \beta$. We focus on these two cases in the paper. The main results of the paper do not depend on this particular formulation of prices.

¹Notice that the VSL is ultimately a consumption equivalent measure. To see this, let $V_0(a_0, \pi_0)$ be the indirect life time utility of an individual with initial assets a_0 and π_0 probability of surviving to the next period. Let $WTP(\epsilon)$ be the willigness to pay, in time-0 consumption units, to increase the survival probability in ϵ percental points. Then the $WTP(\epsilon)$ is defined as follows:

Parts (i) and (ii) of the proposition describe how VSL^{EU} depends on ω : it spans the whole real line when $\sigma > 1$ but it has a finite upper bound when $\sigma \in (0, 1)$. The upper bound is reached when $\omega = 0$. In particular $VSL^{EU} = \infty$ if EIS < 1 and $\omega = 0$. The VSL is infinite in this case because dying is an unbearable state: it gives $-\infty$ utility. Since EIS < 1 is an empirically important case, this property is key to understand why the expected utility model requires a sufficiently large value of ω/y to match a plausible VSL for the US, a requirement that remains important even when EIS > 1. But a large value of ω/y results in a counterfactual prediction: it means that some individuals, those with consumption below ω , do not value life. As shown below, calibrated expected utility models predict that a sizeable fraction of the world population would not value life.

Part *(iii)* of the Proposition 3 implies that the ratio of VSL to income increases with income. For example, if all consumptions double $(\lambda = 2)$, so that income doubles, the VSL more than doubles. As we show next section, this prediction is inconsistent with available evidence suggesting a VSLto-income ratio that decreases with income. Finally, part *(iv)* implies that the VSL is independent of π . This property is exclusive of the two period model. As we show below in a more general expected-utility model, $VSL_{\pi}^{EU}(y, \pi, \omega, \sigma) > 0.^3$ According to the last two properties, the expected utility model predicts that the VSL-to-income ratio is increasing in both income and longevity. The evidence suggests otherwise.

In contrast, the non-expected utility model does not hinge on a positive value of ω in order to match a plausible VSL. Suppose $\omega = 0$ so that VSL is always positive for all consumption levels. In this case equation (5) becomes:

$$VSL^{NE}(y,\pi,\gamma) = \pi^{\frac{\gamma-\sigma}{1-\gamma}}\beta y\theta(0,\gamma) \text{ for } \gamma \in (0,1).$$
(8)

The restriction $\gamma \in (0,1)$ is needed when $\omega = 0$ to prevent $Ev(\tilde{c}_1) = 0$. The key parameter determining the value of life in the non-expected utility model is γ . The following proposition characterizes the VSL for the non-expected utility case when $\omega = 0$.

Proposition 4 - VSL under non-expected utility. Consider the non-expected utility representation à la Epstein-Zin-Weil in equation (4), and assume that $\omega = 0$. Then: (i) $VSL^{NE}(y, \pi, 0) = y/(1+r)$, $VSL^{NE}(y, \pi, 1) = \infty$, and $VSL_{\gamma}(y, \pi, 1) > 0$ for any $\sigma \ge 0$ and $1 > \gamma > 0$; (ii) $VSL^{NE}(\lambda y, \pi, \gamma) = \lambda VSL(y, \pi, \gamma)$; and (iii) $VSL_{\pi}(y, \pi, \gamma) < 0$ if $\sigma > \gamma$.

Part (i) of Proposition 4 describes how the VSL depends on γ . Matching a finite VSL now requires a large enough value of γ , but since $\omega = 0$ has been assumed, the model predicts that all individuals value life. Part (ii) implies that the VSL-to-income ratio is independent of income when $\omega = 0$, while part (iii) implies that the VSL decreases with survival (and longevity).⁴ This

³The two period model misses the full impact of survival many periods. We show below an adjustment of the order $\frac{1}{1-\beta\pi}$ is needed.

⁴The two period model misses the full gains of surviving for many periods. As we show below, the adjustment needed is proportional to $\left(1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}\right)^{-1}$.

property will allow us to show that the non-expected utility model is consistent with the empirical evidence regarding the VSL-to-income ratio.

In conclusion, the expected and the non-expected utility models provide a very different rationalization of the value of life. In the expected utility model ω plays the key role, while under non-expected utility γ does. As a result, the models have very different predictions regarding how the VSL responds to changes in income and survival. While in the non-expected utility model all individuals, poor or rich, require a compensation in the event of death that is higher than the present value of loss consumption, in the expected utility model poorer individuals discount this compensation. This discount implies that the VSL falls below the natural lower bound implied by the value of loss consumption. The models also differ in the implied preferences for the temporal resolution of uncertainty.

3 Evidence on the value of statistical life

In this section we review the large literature estimating the VSL. Estimation is often based on wage differential across occupations with different mortality risks, or from market prices for products that reduce fatal injuries. Both of these approaches have produced similar estimates of the VSL. To illustrate this concept, consider a worker who requires an annual premium of \$500 per year in order to accept an increase in the annual probability of accidental death of 1/10,000. In a pool of 10,000 workers, one worker is expected to die and the aggregate compensation for such death is $VSL = \$500 \times 10,000 = \5 million. Actual estimates of the VSL in the US range between \$4 to \$9 million in 2004 dollars for a 40 year old male (Viscusi, 1993; Viscusi and Aldy, 2003). These estimates have important policy implications and are used for policy evaluations. For instance, the Environmental Protection Agency has used \$6.3 million in cost-benefit analysis since 1993. Different values have been used in calibrated quantitative models. Murphy and Topel (2006) use a value of \$6.3 million in assessing the value of health and longevity, while Hall and Jones (2007) calibrate their benchmark model to a lower value of \$3 million. The calibrated model in BPS implies a VSL for developed countries of between \$1.5 and \$2 million.

There are some estimates of the VSL in countries other than the US. Viscusi and Aldy (2003) report the VSL and the average income from 21 different studies around the world published since 1982 (see their Table 4, p. 27-28). Countries represented in these studies include richer countries such as Australia, Austria, Canada, Japan, and the United Kingdom, and some developing countries in Asia: Hong Kong, India, South Korea, and Taiwan. Although this international evidence tends to produce estimates of the VSL that are lower than in the US, the order of magnitude is very similar despite the quite different labor market conditions across these countries. One may be worried that the estimates of the VSL are too high in developing countries, but as Viscusi and Aldy argue, this is likely not the case. What lends credibility to these estimates is that in developing countries on-the-job mortality risks are 3 to 5 times greater than the average ones in richer countries, while average earnings are 2 to 4 times lower (p. 29). In situations when risks are very high, the estimated VSL in the US is low. Thus, it is most likely the case that the estimates of the VSL in developing

countries are on the low rather than on the high side.

Figure 3 plots the ratio of VSL to income as a function of income (in 2000 US dollars) estimated from the international studies documented in Viscusi and Aldy (2003). The correlation between the two is -0.47 indicating that as as income increases, the VSL increases less than proportionally. As it is apparent from the graph, this negative correlation is driven by the 3 studies in India, the poorest country in the sample (average income of \$778). There are no other studies for countries with average income between \$1,000 and \$5,000 that would provide more estimates of the VSL in these poorer countries. Fortunately, there are many more studies for the US that span some variation in average income, at least between \$20,000 and \$50,000. Figure 4 plots the ratio of VSL to income as a function of income from studies for the US as documented by Viscusi and Aldy (2003) on Table 2 (p. 19-21) and Table 3 (p. 25). The correlation between the two variables is -0.39 in this case, confirming a pattern similar to the cross-country findings from Figure 3.

The evidence presented above indicates that the VSL/y ratio is decreasing in income. In addition, the positive correlation between income and longevity illustrated in Figure 2, suggests that the ratio VSL/y decreases with longevity. The expected utility model predicts a positive relationship between the VSL/y ratio and income, as stated in part *(iii)* of Proposition 3. The positive relationship is particularly strong for poorer countries or individuals with consumption closer to ω . As discussed above, the model also predicts a positive relationship between the VSL/yratio and longevity, which is also counterfactual. In contrast, the non-expected utility model predicts no relationship between the VSL/y ratio and income (part (ii) of Proposition 4), but it predicts a negative relationship between this ratio and longevity when $\sigma > \gamma$. Since income and longevity are positively correlated in the data, the model is consistent with evidence of a VSL/yratio that is both decreasing with income and longevity.

4 The expected utility model

In this section we set up the general multiperiod expected-utility model of perpetual youth, explore its predictions, and discuss its limitations.

4.1 A model of perpetual youth

The model in this section is in the spirit of Yaari (1965), Usher (1973), Blanchard (1985), Rosen (1988), Murphy and Topel (2006) and particularly Becker, Phillipson and Soares (2005) –BPS henceforth. An individual has a probability S_t of surviving to period t. Preferences are of the expected utility type:

$$E\left[\sum_{t=0}^{\infty} \beta^t u(\widetilde{c}_t)\right] = \sum_{t=0}^{\infty} \beta^t \left[S_t u(c_t) + (1 - S_t) u(\omega)\right]$$

where $E[\cdot]$ is the mathematical expectation, $\beta < 1$ is a pure discount factor, $u(\cdot)$ is a standard per-period utility function satisfying $\lim_{c\to 0} u'(c) = \infty$, and c_t is consumption at time t if alive. Notice that survival, S_t , is a good if and only if $c_t > \omega$. As described by Kreps and Porteus (1978), individuals endowed with these preferences are indifferent about the time of resolution of uncertainty.

Assuming complete markets, the lifetime budget constraint is:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t S_t c_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t S_t y_t$$

where y_t is yearly income. It is useful to consider a recursive representation of the problem. Let $\pi_t = S_{t+1}/S_t$ be the conditional probability of surviving up to period t+1 given that the individual has survived up to period t. The individual's problem can be written as

$$V_t(a_t; \pi_t) = \max_{a_{t+1}} \left\{ u(y_t + a_t - \pi_t (1+r)^{-1} a_{t+1}) - u(\omega) + \beta \pi_t V_{t+1}(a_{t+1}, \pi_{t+1}) \right\}$$
(9)

where a_t are the initial asset holdings at time t, $\pi_t (1+r)^{-1}$ is the price of bonds under complete markets, and $a_0 = 0$. The marginal utility of survival is given by

$$\partial V_t / \partial \pi_t = -u'(c_t^*) a_{t+1}^* (1+r)^{-1} + \beta V_{t+1}(a_{t+1}^*)$$

where c_t^* and a_{t+1}^* are the optimal choices. The first term represent the higher cost of saving as higher survival rates affect complete market prices. The second term is added utility upon surviving. The willingness to pay for a $\Delta \pi$ increase in survival and the value of statistical life are given, respectively, by

$$WTP_t = \left| \frac{\partial a_t}{\partial \pi_t} \right| \Delta \pi = \left| \frac{\partial V_t / \partial \pi_t}{\partial V_t / \partial a_t} \right| \Delta \pi$$

and

$$VSL_t = \frac{WTP_t}{\Delta \pi_t} = \beta \frac{V_{t+1}(a_{t+1}^*; \pi_{t+1})}{u'(c_t^*)} - \frac{a_{t+1}^*}{1+r}.$$
(10)

The expression above is quite intuitive. The VSL at time t is then given by the utility from consumption in the remaining life expressed in terms of goods, $\beta V_{t+1}(a_{t+1}^*, \pi_{t+1})/u'(c_t^*)$, minus the cost of obtaining that utility, $a_{t+1}^*/(1+r)$. Both of these components are expressed in present value since the increase in survival $\Delta \pi_t$ affects the individual starting in period t + 1.

To further simplify the problem assume a constant probability of surviving $\pi_t = \pi$, constant income $y_t = y$, and a gross interest rate 1 + r equal to $1/\beta$. In this case the optimal solution is a constant consumption path $c_t = y$, and no savings $a_{t+1}^* = 0$ for all t. We adopt these simplifying assumptions in order to recreate a steady-state situation in which the representative individual in each country receives the per capita income of the country every year of his life, and faces the survival probability of his country as in BPS. Substituting these results in (9) provides the steady state utility

$$V^{EU}(y,\pi,\omega) \equiv \frac{u(y) - u(\omega)}{1 - \beta\pi}.$$
(11)

Given that savings are zero, the marginal utility of survival is constant and the value of statistical life becomes

$$VSL^{EU}(y,\pi,\omega) \equiv \frac{\beta}{1-\beta\pi} \frac{u(y)-u(\omega)}{u'(y)}.$$
(12)

The intuition for this expression is the following: the individual enjoys a utility flow of being alive of $u(y) - u(\omega)$ per period, which expressed in terms of goods corresponds to $(u(y) - u(\omega)) / u'(y)$. The $VSL^{EU}(y, \pi, \omega)$ is the present value of these flows discounted at rate $\beta\pi$. Notice that a positive VSL requires $y > \omega$.

For the case of CRRA preferences $u(c) = c^{1-\sigma}/(1-\sigma)$, equation (12) reads

$$VSL^{EU}(y,\pi,\omega) = \frac{y/(1+r)}{1-\beta\pi}\theta(\omega/y,\sigma),$$
(13)

where $\theta(\omega/y, \sigma)$ is the CMA defined in (6) and characterized in Proposition 2. Equation (13), a generalized version of (7), shows that the value of life is equal to the present value of income, or human wealth $\frac{y/(1+r)}{1-\beta\pi}$ times the CMA. The following proposition, analogous to Propositions 1 and 3 above, summarizes the main theoretical predictions of the expected utility model. Define $\phi^{EU}(y, \pi, \omega) \equiv VSL^{EU}(y, \pi, \omega)/y$, the VSL-to-income ratio.

Proposition 5 - Properties of the expected utility model. Consider the expected utility representation with standard CRRA utility. Let $\pi_t = \pi$, $y_t = y$, $(1 + r) = 1/\beta$, and $u(c) = c^{1-\sigma}/(1-\sigma)$. Then: (i) the individual is indifferent between early and late resolution of uncertainty; (ii) the marginal utility of survival is constant; (iii) $\phi_{\pi}^{EU}(y,\pi,\omega) > 0$ and $\phi_{y}^{EU}(y,\pi,\omega) > 0$ if $\omega > 0$, or $\phi_{y}^{EU}(y,\pi,0) = 0$ if $\omega = 0$; (iv) $VSL^{EU}(y,\pi,0) = \infty$ and $VSL^{EU}(y,\pi,y) = 0$ if $\sigma > 1$; (v) $\theta(\omega/y,\sigma) \geqq 1$ if $y \geqq \omega \sigma^{\frac{1}{\sigma-1}}$.

Proposition 5 describes the five predictions of the expected utility model that limit its use in studying longevity issues. These are the predictions discussed in the introduction. Parts (i) and (ii) are self explanatory, while part (iii) implies that there is a positive relationship between the VSL-to-income ratio and both income and longevity. Part (iv) implies that to match a plausible VSL for rich countries and/or for rich individuals, a possibly large value of ω is needed, which results in negative VSL for poor countries and individuals. As we show next section, this is the case not only when $\sigma > 1$, the value most commonly used in quantitative macro, but also when $\sigma < 1$. Finally, part (v) implies that individuals with income below $\omega \sigma^{\frac{1}{\sigma-1}}(>\omega)$ require a compensation in the event of death that is lower than the present value of loss income.

4.2 Quantitative predictions

In order to investigate the quantitative predictions of the expected utility model, the following parameters and functional forms are needed: β , c, π , $u(\cdot)$ and ω . We set the risk free rate to 3%, which implies $\beta = 0.97$, and let $u(c) = c^{1-\sigma}/(1-\sigma)$. As in BPS, the spirit of the exercise is to think of the average individual in a country as receiving a constant income y in every period, and facing a constant survival probability as implied by the life expectancy of the country. For income data we use 2005 per-capita income in PPP prices from the Penn World Tables Version 7.0, and for survival probabilities we use the inverse of the 2005 life expectancy as reported in the World Development Indicators. The values for the US are y = \$42,535 and $\pi = 98.7\%$ which corresponds to a life expectancy of 78 years.

The only two remaining parameters to be set are σ and ω . Estimates of $EIS = 1/\sigma$ range from close to zero (for example, Hansen and Singleton, 1983; Hall, 1988; and Campbell and Mankiw, 1989) to around 1.2 (Browning, Hansen, and Heckman, 1999). Estimates of ω , the level of income at which individuals in the model are indifferent between being alive or dead, are unavailable. An upper bound for ω is \$456 which corresponds to the \$1.25 per day used by the World Bank to define the poverty line in 2005 (Chen et al., 2008). Such upper bound is unlikely to bind because around 20% of the world population, or 1.4 billion people, was below this poverty line in 2005. A more stringent upper bound for ω is \$169 which corresponds to the income of the poorest country, Zambia. The percentage of world population below such level was around 2.5% in 2005.⁵

Table 1 reports the VSL predicted by the expected utility model (equation 13) in the US for various combinations of the EIS $(1/\sigma)$ and ω . For these same parameters, Table 2 reports the level of per capita income at which $\theta(\omega/y, \sigma) = 1$, i.e., the level of per capita income at which the VSL is exactly equal to the present value of loss income (consumption). We denote this level of income y_{\min} , as incomes below this imply a VSL that falls below the natural lower bound, i.e., below the present value of loss income. According to part (v) of Proposition 5, $y_{\min} \equiv \omega \sigma^{\frac{1}{\sigma-1}}$. As seen in Table 1, the expected utility model predicts unrealistically high VSL for the range of most commonly used values of the EIS, those below 0.85, and for any plausible value of ω . Predicting realistic VSL is key for health and longevity models in order to avoid overstating the importance of health and survival issues. The expected utility model can match a realistic VSL when the EIS is below one and ω is sufficiently large, or when EIS is larger than one. There are issues with both alternatives. The first has the counterfactual implication that since per capita income is below \$500 for the 1.4 billion people below the poverty line in 2005, a significant fraction of the world population would prefer to be dead according to the model. The second alternative requires an EIS higher than the standard used in quantitative macro. Murphy and Topel (2006) use the first alternative, while BPS used the second.

Table 2 shows that depending on the choice of parameters σ and ω , the VSL for a considerable fraction of the world population would be below the natural lower bound implied by the present

 $^{^{5}}$ Calculations were done using the World Bank interactive website at (last accesed 12/31/2012):

http://iresearch.worldbank.org/PovcalNet/index.htm?1.

value of loss income. For example, consider the combination EIS = 1.25 and $\omega = \$500$, similar to the one used by BPS, which implies a VSL of around 3 million for the US, close to three times the present value of average incomes.⁶ In addition to the relatively high EIS, this parametrization implies that individuals with incomes below \$1, 526, or around 75% of the world population, require a compensation upon death that is below the present value of their average incomes. In other words, while calibrated versions of the expected utility model predict an accidental death compensation of around three times the value of forgone income in the US, it predicts a compensation below forgone earnings for around 75% of the world population. This model also predicts a VSL of around \$77,000for India, or 1.35 times the present value of their earnings, which is implausibly low given existing estimates. For example, Viscusi and Aldy (2003) report a VSL of around \$1 million for Indian manufacturing workers in 1990.

Figure 5 portrays the VSL-to-income ratio VSL/y for 144 countries in our sample for EIS = 1.25 and $\omega = \$500$. The figure confirms that VSL/y increases with income, as stated in Proposition 5, but also shows that the predicted ratio sharply increases with income. This robust prediction of the expected utility model is nonetheless inconsistent with the empirical evidence reported in Figures 3 and 4 which instead suggests a decreasing ratio. Second, the model predicts a negative value of life for 5 countries in the sample, all of them African countries. As discussed above, it is in principle feasible to choose a lower value of ω so that the VSL is positive for all countries but at the cost of obtaining implausible high values for the US and other rich countries. Furthermore, this would only shift up the curve in Figure 5 without changing its increasing shape.

5 A non-expected utility model

This section shows that non-expected utility models, along the lines of the Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989), can solve the limitations of the expected utility model documented in the previous section. These models offer the natural starting point because they maintain the axioms of von Neumann-Morgensten preferences except the reduction of compounded temporal lotteries, breaking the indifference to temporal lotteries. Therefore, non-expected utility can naturally account for the evidence suggesting that individuals prefer late resolution of uncertainty in certain medical situations. Surprisingly, a preference for late resolution of uncertainty also fixes some other limitations of the expected utility model in studying longevity.

Epstein and Zin (1989) and Weil (1989) provide a simple parametric class of Kreps and Porteus (1978) preferences characterized by a constant EIS and a constant, but unrelated, CRRA. We refer to this parametric class as the EZW preferences. While the EIS determines the degree of aversion to temporal deterministic consumption fluctuations, the CRRA determines the degree of aversion to atemporal stochastic consumption fluctuations. Late resolution of uncertainty is obtained when EIS < (1/CRRA). The added flexibility of the EZW preferences allows us to simultaneously choose an EIS below one, as is the typical case in many macro applications, and $\omega = 0$ so that life is worth living everywhere, without the unappealing implication that the VSL is infinite, as would

⁶The implied value of ω in BPS is \$353 in 1990 dollars, which corresponds to \$493 in 2005 dollars.

be case with the expected utility model. Instead, the VSL is the target that allow us to identify the CRRA. Finally, the model predicts that ratio VSL/y decreases with longevity, and since longevity and income are positively correlated, the model is consistent with the pattern displayed in Figures 3 and 4.

5.1 The model

Consider the following parametric version of Kreps and Porteus (1978) preferences formulated by Epstein and Zin (1989):

$$W_{t} = (1 - \gamma)^{-1} \left[c_{t}^{1-\sigma} + \beta [(1 - \gamma) E_{t} \widetilde{W}_{t+1}]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1-\gamma}{1-\sigma}},$$
(14)

where $\sigma \geq 0$ and $\gamma \geq 0$. The main feature of these preferences is that they separate intertemporal substitution from risk aversion: the EIS is $1/\sigma$ while γ is the CRRA. The expected utility case is obtained when $\sigma = \gamma$. While the formulation above has been used in the context of financial risk, the specific lottery we are interested in is a more extreme one, namely a life-death lottery. It is important to keep in mind that while existing applications of equation (14) associate γ with aversion to financial risk, here we associate γ with aversion to mortality risk.

Kreps and Porteus (1978) show that agents exhibit a preference for late (early) resolution of uncertainty depending on whether lifetime utility is concave (convex) in $E_t \widetilde{W}_{t+1}$. In the case of equation (14), preference for late resolution requires $\sigma > \gamma$, or equivalently EIS < (1/CRRA). In the context of financial risk, Vissing-Jorgenson and Attanasio (2003) present evidence to support a preference for early resolution of uncertainty, namely EIS > (1/CRRA). In contrast, the medical evidence discussed above and the calibration exercise we present next section suggest that in the context of a life-death lottery, late resolution of uncertainty is preferred. Taken together, these observations suggest that individuals are more willing to accept a large consumption jump upon dying, than to accept consumption jumps while alive. Moreover, since $E_t \widetilde{W}_{t+1}$ is linear in the survival probability, then the concavity required for late resolution of uncertainty also implies diminishing marginal benefits of surviving.

In what follows we use a more compact formulation of (14), one used by Epstein and Zin (1989). Defining $V \equiv (1 - \gamma) W^{\frac{1}{1-\gamma}}$, (14) can be written as

$$V_t = \left[c_t^{1-\sigma} + \beta [E_t \widetilde{V}_{t+1}^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}.$$
 (15)

This formulation is not only compact but also convenient because $V \in [0, \infty]$ while $W \in [-\infty, \infty]$, so that the minimum lifetime utility is V = 0.

As in the previous section, let ω be the perceived consumption upon dying. Then (15) can be written as

$$V_t = \left[c_t^{1-\sigma} + \beta [\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t) D^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}},$$
(16)

where

$$D = \left[\omega^{1-\sigma} + \beta [\pi_t D^{1-\gamma} + (1-\pi_t) D^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}}\right]^{\frac{1}{1-\sigma}} = \omega (1-\beta)^{\frac{1}{\sigma-1}}.$$

Equation (16) implies that V_t is increasing in π_t as long as $V_{t+1} > D$. From now on we assume $\omega = 0$ (or D = 0) so that for any individual with positive consumption life is always a good rather than a bad. In the expected utility model of the previous section such assumption would restrict the EIS to be larger than one in order to avoid lifetime utility to become minus infinite and the VSL to become infinite. In the non-expected utility model the assumption $\omega = 0$ does not restrict the EIS, which can take standard values, but instead it requires $\gamma < 1$. Otherwise, if $\gamma > 1$ and $\omega = 0$ then $V_t = 0$ if $\sigma > 1$, or $V_t = c_t$ if $\sigma \in (0, 1)$. This means that with $\gamma > 1$ death becomes an unbearable state as it destroys any value of living either in all periods, if $\sigma > 1$, or in the future, if $\sigma \in (0, 1)$, driving the VSL to infinite. As we discuss below, the restriction on γ is not binding because $\gamma < 1$ is required to match the observed VSL. Finally, the assumption $\omega = 0$ is also convenient because it reduces notation significantly without major costs: any plausible calibration would require ω to be small anyway. Letting $\omega = 0$ in (16) results in:

$$V_t = \left[c_t^{1-\sigma} + \tilde{\beta}(\pi_t)V_{t+1}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$
(17)

where $\tilde{\beta}(\pi) \equiv \beta \pi^{\frac{1-\sigma}{1-\gamma}}$. Notice that all the effect of uncertainty is fully captured by the adjusted discount factor $\tilde{\beta}(\pi)$, as in the standard perpetual youth model. The preference for late resolution of uncertainty is reflected in the fact that $\tilde{\beta}(\pi) > \beta$ when $\sigma > \gamma$ meaning that mortality risk makes the individual more patient.

Individuals choose a consumption path to maximize utility subject to a lifetime budget constraint. Assuming complete markets and a constant interest rate, the individual's problem can be written recursively as:

$$V_t(a_t; \pi_t) = \max_{a_{t+1}} \left\{ \left(y_t + a_t - \pi_t \left(1 + r \right)^{-1} a_{t+1} \right)^{1-\sigma} + \tilde{\beta} \left(\pi_t \right) \left(V_{t+1}(a_{t+1}, \pi_{t+1}) \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}$$

where a_t are the initial asset holdings at time t and $\pi_t (1+r)^{-1}$ is the price of bonds under complete markets.

The implied optimal consumption is given by

$$c_t = c_0 \left(\beta \pi^{\frac{\gamma - \sigma}{1 - \gamma}} \left(1 + r\right)\right)^{t/\sigma}.$$
(18)

Notice that when $\sigma = \gamma$ this condition reduces to the standard optimal consumption obtained under expected utility. The equation shows that consumption grows faster the more the individual cares about the future, the larger the return on savings, 1+r, but also the larger $\pi^{\frac{\gamma-\sigma}{1-\gamma}}$. If $\sigma = \gamma$ then the survival probability does not affect consumption growth because it affects equality the marginal utility of consumption and the marginal cost, as prices reflect changes in the survival probability. If $\gamma < \sigma$, which is the case we stress, then consumption growth is larger than in the expected utility case as individuals are effectively more patient.

5.2 The value of statistical life

The value of statistical life in the non-expected utility model is given by:

$$VSL_{t} = \left| \frac{\partial V_{t} / \partial \pi_{t}}{\partial V_{t} / \partial a_{t}} \right| = \tilde{\beta}'(\pi_{t}) \frac{V_{t+1}^{1-\sigma}(a_{t+1}^{*}; \pi_{t+1})}{c_{t}^{*-\sigma}(1-\sigma)} - \frac{a_{t+1}^{*}}{1+r}.$$

This expression is analogous to (10).⁷ The VSL at time t is the change in the weight that the individual gives to the future, $\tilde{\beta}'(\pi_t)$, times the future lifetime utility expressed in terms of goods, $V_{t+1}^{1-\sigma}(a_{t+1}^*;\pi_{t+1})c_t^{*\sigma}(1-\sigma)^{-1}$, minus the cost of obtaining that utility, $a_{t+1}^*/(1+r)$. As in the previous section and in order to further simplify the problem assume a constant probability of surviving $\pi_t = \pi$, constant income $y_t = y$, and a gross interest rate 1 + r equals to $\pi^{\frac{\sigma-\gamma}{1-\gamma}}/\beta$. In this case the optimal solution is a constant consumption path $c_t = y$, and no savings $a_{t+1}^* = 0$ for all t. Substituting these restrictions into (17) results in the steady state utility:

$$V(y,\pi) \equiv y / \left[1 - \tilde{\beta}(\pi)\right]^{1/(1-\sigma)}.$$
(19)

In addition, the value of statistical life simplifies to:

$$VSL(y,\pi) = \frac{y/(1+r)}{1-\tilde{\beta}(\pi)}\theta(0,\gamma).$$
(20)

where $\theta(0, \gamma)$ is the CMA and the present value of lifetime income is given by $\frac{y/(1+r)}{1-\hat{\beta}(\pi)}$. Expression (20) is similar to (13), the one for the expected utility model. Both formulas are in fact identical when $\sigma = \gamma < 1$ and $\omega = 0$ but there are some key differences otherwise. First, the VSL is finite in the non-expected utility model for any $\sigma \ge 0$. Recall that the expected utility model predicts an infinite VSL when $\sigma > 1$ and $\omega = 0$. This is a key improvement over the expected utility model because it eliminates all the issues discussed above when $\omega = 0$ and $\sigma > 1$, a quite reasonable set of parameters. Second, γ is the key parameter determining the VSL, not σ . Since the expected utility model forces $\gamma = \sigma$, then σ becomes crucial in that model, but once this assumption is relaxed then σ losses its key importance and the weight of the prediction lies on γ . Third, the CMA is always larger than one meaning that the VSL is always larger than the present value of incomes, the natural lower bound. Fourth, the VSL in equation (13) is proportional to income

$$VSL_{t} = \tilde{\beta}'(\pi_{t}) \frac{\frac{1-\gamma}{1-\sigma}W_{t+1}^{\frac{1-\gamma}{1-\gamma}}(a_{t+1}^{*};\pi_{t+1})}{c_{t}^{*-\sigma}} - \frac{a_{t+1}^{*}}{1+r}.$$

Expression (10) is obtained when $\sigma = \gamma$.

⁷To obtain an expression that is directly comparable with expression (10), remember that $V \equiv (1 - \gamma) W^{\frac{1}{1-\gamma}}$. Therefore,

y. This implies that richer countries have a higher VSL than poorer countries, a prediction also true in the expected utility model. However, different from that model, equation (13) implies that the ratio $VSL(y,\pi)/y$ is independent of income, but it decreases with π under certain conditions. The following proposition, analogous to Proposition 5 above, summarizes the main theoretical predictions of the non-expected utility model. Define $\phi(y,\pi) \equiv VSL(y,\pi)/y$.

Proposition 6 - Properties of the non-expected utility model. Consider the non-expected utility representation à la Epstein-Zin-Weil in equation (17), where $\omega = 0$ has been assumed. Let $\pi_t = \pi$, $y_t = y$, $1 + r = \pi^{\frac{\sigma - \gamma}{1 - \gamma}}/\beta$ and $\sigma > \gamma$ in the non-expected utility model. Then: (i) the individual prefers late resolution of uncertainty; (ii) there are diminishing marginal benefits to survival; (iii) $\phi_y(y,\pi) = 0$ and $\phi_{\pi}(y,\pi) < 0$ if $\sigma > 1 > \gamma$, or $\phi_{\pi}(y,\pi) > 0$ if $1 > \gamma \ge \sigma$; (iv) $VSL(y,\pi) > 0$ for any y > 0 and $VSL(y,\pi) < \infty$ for any $\sigma \ge 0$; and (v) $\theta(0,\gamma) > 1$ for any y and σ .

Proof. Proofs were given in the text above except the characterization of $\phi_{\pi}(y,\pi)$. Notice that

$$\phi_{\pi}(y,\pi) = \frac{1}{(1-\gamma)^2} \beta \pi^{\frac{1-\sigma}{1-\gamma}} \left(1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}\right)^{-1} \left[(\gamma - \sigma) + (1-\sigma) \beta \pi^{\frac{\gamma-\sigma}{1-\gamma}} \left(1 - \beta \pi^{\frac{1-\sigma}{1-\gamma}}\right)^{-1} \right], \quad (21)$$

so that if $\sigma > 1 > \gamma$ then $\phi_{\pi}(y, \pi) < 0$ while if $1 > \gamma \ge \sigma$ then $\phi_{\pi}(y, \pi) > 0$.

Proposition 6 implies that the predictions of the non-expected utility model regarding the ratio $VSL(y,\pi)/y$ are quite different from the ones obtained with the expected utility model. As stated in part (iii), $VSL(y,\pi)/y$ is independent of y and it only depends on π . However, since in the cross-country data survival π (or life expectancy) is positively correlated with y, increases in π can be thought of as increases in y. In light of this, the main insight of Proposition 6 is that if $\sigma > 1 > \gamma$, which is our preferred case, the non-expected utility model is consistent with the decreasing pattern of VSL/y portrayed in Figures 3 and 4. It is not possible to generate this pattern with the separable model. To understand the intuition of this key result, consider the other cases in Proposition 6. First, if $1 > \gamma = \sigma$, then the model reduces to the separable framework with $\omega = 0$ and equation (21) implies that $VSL(y,\pi)/y$ is increasing in π . This case is described in Proposition 5. Second, if $1 > \gamma > \sigma$, then we again have that $VSL(y,\pi)/y$ is increasing in π . Thus, these two cases imply that as long as $\gamma \geq \sigma$ the ratio VSL/y does not behave as implied in Figures 3 and 4. The reason is that when $1 > \gamma > \sigma$, then mortality risk aversion is relatively higher. This implies that relative to their income, the value of life in countries where high life expectancy is higher than in countries with shorter life spans. The opposite holds when $\sigma > 1 > \gamma$, case in which mortality risk aversion is relatively lower. In this case the model implies that relative to their income, the value of life in countries with already high life expectancy is lower than in countries where life is scarce (π is low).

Most quantitative macro models assume that the EIS is low ($\sigma > 1$). If in addition, mortality risk aversion is low in the sense that $\sigma > 1 > \gamma$, then we have a framework in which: (i) life is worth living everywhere ($\omega = 0$); and (ii) relative to income, the value of life is higher where life is scarce, which is in poorer countries. These two observations are consistent with the available evidence on the VSL. In addition, they originate from a model in which individuals are not indifferent to the timing in which death uncertainty is resolved, a quite reasonable implication for a dramatic event such as death. We now turn to calibrate the non-expected utility model and to quantitatively compare its predictions with the expected utility one.

5.3 Calibration and results

In this section we calibrate the non-expected utility model and explore its quantitative implications. We set ω to zero and r = 3%. We follow Murphy and Topel (2006) and set $\sigma = 1.25$. As in section 4.2, we use country specific values for π and y using the World Development Indicators and the Penn World Tables 7.0. The only new parameter is γ . We calibrate this parameter to target a VSL of \$4.5 million in the US in 2005. We select a conservative value in the low end of the \$4 to \$9 million interval reported in Viscusi and Aldy (2003). This results in $\gamma = 0.57$. Larger targets for the VSL result in larger calibrated values of γ , but it is always the case that $\gamma < 1$. For instance, if the VSL is set to \$9 million in the US, the calibrated γ equals 0.74. In addition, a $\gamma = 0.85$ is consistent with a VSL of about \$36 million, well above the range of available estimates. In sum, the data on the VSL strongly suggest $\gamma < 1$. We prefer the conservative value of $\gamma = 0.57$, but the main message of our exercise also holds for other calibrations with $\gamma < 1$.

In order to compare the expected and non-expected utility models, we recalibrate the former to be consistent with the same VSL of \$4.5 million in the US. Setting $\sigma = 1.25$, the value of ω consistent with this VSL is $\omega = \$2,000$. This value of ω implies that in 41 countries out of our sample of 144 countries the VSL is negative. Notice this result is different from the one portrayed in Figure 5, where only 5 countries exhibit a negative VSL as in BPS. The reason for this difference is that with a standard value of $\sigma > 1$, a higher value of ω is needed to match a VSL of \$4.5 million for the US. The calibration in Figure 5 corresponds to that in BPS, with a VSL of around \$3 million in the US, a value below the range estimated in the literature. In Section 7 we discuss modifications of the standard CRRA representation that could avoid obtaining negative VSL for poorer countries. However, those modifications do not overturn the prediction that the ratio $VSL(y,\pi)/y$ increases with income under expected utility, as we now turn to discuss.

Figure 6 displays the ratio $VSL(y, \pi)/y$ for both the non-expected and the expected utility models for all countries in the sample in 2005. Results are quite different for both models. Different from the expected utility case, there are no negative VSL for any country under the non-expected utility model. More importantly, consistent with Proposition 6, the pattern of ratio $VSL(y,\pi)/y$ is quite different: increasing in y under the expected utility model and decreasing in y under nonexpected utility. Although $VSL(y,\pi)/y$ is decreasing under non-expected utility, it is important to recall that the VSL is proportional to income. For instance, the VSL in Zimbabwe, the poorest country in the sample with a per capita income of \$169, is of \$26,000 under non-expected utility. Similarly for Chile, the country with the average per capita income of the sample of \$11,219, the VSL is \$1.2 million. The richest country is the US with a per capita income of \$42,535 and a VSL of \$4.5 million. Under expected utility the VSL in Zimbabwe is negative \$6,000, and that for Chile is \$565,000.

Figure 6 suggests that for countries with a per capita income above \$10,000 in 2005, about 25% of the per capita income in the US, the ratio $VSL(y,\pi)/y$ converges to 100. This is the case because as shown in Figure 2, life expectancy is quite similar among those countries. Most of the dispersion in the ratio $VSL(y,\pi)/y$ occurs for countries below \$10,000 of per capita income. Although $VSL(y,\pi)/y$ converges to 100 in Figure 6, higher ratios could be obtained selecting higher calibration targets for the VSL in the US. The micro evidence reported in Figures 3 and 4 suggest a ratio of about 300, but we have calibrated our model taking a more conservative level for the VSL. In sum, our calibrated non-expected utility model is consistent with the negative correlation between $VSL(y,\pi)/y$ and y displayed in Figures 3 and 4, while the expected utility model is not. The correlation between $VSL(y,\pi)/y$ and y under the non-expected utility model in Figure 6 is -0.55.

Figure 7 is similar to Figure 6 but it displays $VSL(y, \pi)/y$ as a function of life expectancy. Overall, Figures 6 and 7 suggest that under the non-expected utility model, longer life is valued all around the world, including poorer countries. In fact, as a percentage of per capita income, the non-expected utility model implies that life is relatively more valued where it is more scarce: in poorer countries. It would appear as if this conclusion implies that poor countries should spend a higher share of income on health than rich countries do, which would contradict the data. This is however not the case. In the expected utility framework of Hall and Jones (2007) health expenditure is high in richer countries because health is a superior good. Our non-expected utility framework offers an alternative explanation that does not rely on life being a superior good, namely that richer countries spend more in health because of their demographics. As the share of elderly rises, health expenditure becomes higher because the elderly are the ones who face a shorter expected life span and hence, value life relatively more.

6 World inequality

In this section we use the calibrated expected and the non-expected utility models in order to analyze two issues. First, we evaluate welfare across countries and time by jointly accounting for the changes in per capita income and life expectancy. This welfare evaluation is micro-founded in a model in which individuals value both income and length of life, and provides a theory-based perspective of world inequality. Second, since the models we calibrated above imply a VSL in each country, they have implications for the welfare cost of war and AIDS. We illustrate these predictions for a number of countries in which either wars have ended or AIDS has become widespread.

6.1 Welfare across countries and time

How does world inequality looks like when per capita income measures are adjusted to include differences in life expectancy? Even though per capita income has remained stagnant in a number of poor countries between 1970 and 2005, there have been large gains in life expectancy during the same period. This suggest world inequality has effectively fallen. This section uses the models presented above to compute welfare measures across time and across countries. Specifically, we are interested in using the formula for $V(y,\pi)$ in order to calculate a "full measure" of income adjusted for changes in life expectancy, $T = 1/\pi$. Let $V_0 \equiv V(y_0, \pi_0)$ be the welfare in a benchmark situation and $V_i \equiv V(y_i, \pi_i)$ the welfare in another situation *i*. For welfare measures across time, or growth calculations, the subscripts 0 and *i* refer to two different years for a given country, while for cross-country comparisons they refer to two different countries in a given year. The ratio of per capita income is $R_i = y_i/y_0$ and it is the typical way to measure proportional welfare differences between both situations.

We now define a more comprehensive ratio of incomes that includes an imputed value for differences in life expectancy. We denote this ratio R_i^F where F stands for "full" income ratio which is defined implicitly by

$$V(R_i^F y_0, \pi_0) = V(y_i, \pi_i)$$
(22)

so R_i^F is the proportional change in y_0 required to equate welfare in both situations. Notice that $R_i^F = R_i$ if $\pi_0 = \pi_i$, and $R_i^F \leq R_i$ if $\pi_i \leq \pi_0$.

 R_i^F for the expected (*EU*) and non-expected utility cases can be easily obtained using (11) and (19) for the CRRA utility case. The solutions are given by:

$$R_i^{F-EU} = \left[\frac{1-\beta\pi_0}{1-\beta\pi_i} \left(\frac{y_i}{y_0}\right)^{1-\sigma} + \left(1-\frac{1-\beta\pi_0}{1-\beta\pi_i}\right) \left(\frac{\omega}{y_0}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$
(23)

and

$$R_{i}^{F} = \frac{y_{i}}{y_{0}} \left(\frac{1 - \beta \pi_{0}^{\frac{1 - \sigma}{1 - \gamma}}}{1 - \beta \pi_{i}^{\frac{1 - \sigma}{1 - \gamma}}} \right)^{\frac{1}{1 - \sigma}}.$$
(24)

The solution for the expected utility case is a CES function between the situations in the alive and dead states with a weight, $(1 - \beta \pi_0) / (1 - \beta \pi_i)$ that measures the relative change in "effective mortality rates". The larger the change in mortality the higher the weight assigned to the alive state. Moreover, the lower the σ the more substitutable is the consumption between the two states and the larger the value imputed to mortality changes.

For welfare calculations across time, we use as situation 0 the year 1970 and situation *i* 2005 for each country. Figure 8 displays our relative full income measures across time for both the expected and the non-expected utility models. Specifically, the figure shows R_i^{F-EU}/R_i and R_i^F/R_i as a function of the changes in life expectancy between 1970 and 2005. In other words, the vertical axis measures the proportional change in welfare, in consumption equivalent units, due to changes in longevity. Points agglomerated around the value of one would mean that changes in longevity did not contribute to welfare changes. Notice that countries to the far right of the figure are those that substantially gained life expectancy, generally poorer countries. Those on the far left of the figure are also poorer countries, but they lost life expectancy most likely due to war and

AIDS. As richer countries had more modest gains in life expectancy, they are concentrated around zero. Figure 8 indicates that the expected and non-expected utility models have similar predictions for richer countries. In addition, ratios R_i^{F-EU}/R_i and R_i^F/R_i are closer to one for this set of countries. For the rest of the countries, however, the differences between the models displayed in Figure 8 are substantial. An interesting feature of Figure 8 is that for those countries that lost life expectancy between 1970 and 2005, $R_i^{F-EU}/R_i > 1$ while $R_i^F/R_i < 1$. This is the case because since under the expected utility model the VSL is negative in these countries $(y < \omega)$, then shorter life spans across time increase welfare in a full measure of income. Turning now to countries that gained life expectancy between 1970 and 2005, consider those that gained at least 15 years of life: while R_i^{F-EU}/R_i is well above 1.5 for these countries, R_i^F/R_i is either slightly above or in some cases below one. These calculations suggest that the expected utility model heavily penalizes the welfare gains of non-trivial increases in life expectancy in cases in which per capita income remains stagnant. In contrast, as the VSL is positive in all countries under the non-expected utility model, gains in life expectancy do show up as welfare gains even for those countries whose per capita income remained stagnant over the 1970-2005 period. This time series analysis implies a decrease in world income inequality between 1970 and 2005, one that is specially pronounced under the non-expected utility model.

For welfare calculations across countries in 2005, we label the US as 0 and each of the other countries as i in equations (23) and (24). Figure 9 reports the results of the cross-country welfare calculations. Specifically, the figure plots R_i^{F-EU}/R_i and R_i^F/R_i against life expectancy in 2005. Points agglomerated around the value of one would mean that adding longevity to the measure of welfare did not change the ranking the country had relative to the US when only income was considered. The remarkable feature of this figure is that R_i^{F-EU}/R_i (expected utility model) exhibits a decreasing pattern, while R_i^F/R_i (non-expected utility) is increasing. A high and larger than one R_i^{F-EU}/R_i for poor countries means that once life expectancy is taken into account into a full measure of income, poor countries fair better compared to the US than when only per capita income is considered. The reason is that life expectancy is much lower in these countries, but in the expected utility model life is also worth less there (the ratio VSL/y is low). Things are almost the opposite according to the non-expected utility model. For almost all countries $R_i^F/R_i < 1$ and R_i^F/R_i is an increasing function of life expectancy. In this case, poorer countries fair worse relative to the US when a full measure of income is taken into account because life expectancy there is too low. In sum, Figure 9 implies that while world inequality in the cross-section is lower than previously thought in the expected utility model, it is actually higher in the non-expected utility model. Despite the observed gains in life expectancy since 1970, life is still too short in poorer countries. These are the places where, relative to income, the non-expected utility model predicts a higher value of life.

6.2 Wars and AIDS

We now explore the differences between the expected and non-expected utility models in assessing the welfare effects of positive events like the end of wars and devastating events like AIDS. Table 3 compares the predictions of both models for selected countries. We compute welfare across time using equations (23) and (24), selecting year 1990 as situation 0 and 2005 as situation *i* 2005 for each country. These dates are relevant to the AIDS pandemic. Countries in Table 3 are classified into two groups according to whether they gained or lost life expectancy between 1990 and 2005. An interesting pattern emerges from the table. Countries like Rwanda, Bhutan and Nepal gained 16, 12 and 11 years of life respectively. From the perspective of the expected utility model, these relatively sizeable increases in life expectancy do not change welfare as much. Specifically, the ratio of per capita income R is very similar to the full income ratio R^{Fs} . In fact, in the case of Rwanda, $R^{Fs} < R$ implying that even a life-span gain of 16 years in highly penalized under the expected utility framework. This is the case even if Rwanda experience an 8% increase in per capita income over this period. The predictions of the non-expected utility model are quite different: a longer life span directly adds to welfare.

Consider now countries that lost years of life, mostly due to AIDS: Central Africa (3 years), South Africa (9), Botswana (13) and Zimbabwe (19). Both Central Africa and Zimbabwe not only lost years of life, but also per capita income (respective drops of 26 and 30%). The table indicates that the non-expected utility model highly penalizes these shorter life spans, much more than the expected utility model does. Take for instance the case of Zimbabwe. For this country, $R^{F-EU} > R$ implying that relative to a welfare measure with only income, 19 less years of life turn out to be welfare improving! In contrast, the non-expected utility model implies Zimbabwe's full measure of welfare is simply abysmal ($R^F = 0.2$). Table 3 illustrates that our non-expected utility model implies very high welfare costs of AIDS in Africa.

7 Robustness analysis

In this section we illustrate the robustness of our results by extending the non-expected utility framework proposed above to include financial risk, and by showing how modifications to the standard CRRA expected utility model cannot fully overcome the limitations discussed in this paper.

7.1 Financial risk

Although health evidence supports a preference for late resolution of uncertainty, asset market evidence from the financial literature, such as the equity premium puzzle, suggests preferences for early resolution.⁸ As we show next, these seemingly conflicting evidence can be reconciled with an extension of our model that allows individuals to differentiate mortality risk from other types of

⁸A summary of this literature is provided in Donaldson and Mehra (2008).

consumption risks. Consider the two period example in Section 2. Assume that $\sigma > 1 > \gamma > 0$, as argued above. Under this parameterization the individual prefers late resolution of uncertainty regarding mortality lotteries. Assume further that $\omega = 0$. In this case, the preferences in equation (4) simplify to

$$V(\mathbf{c},\pi) = \frac{c_0^{1-\sigma}}{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} \frac{c_1^{1-\sigma}}{1-\sigma},$$

where c_1 is consumption at t = 1 if alive. Different from Section 2, suppose now that consumption if alive is not deterministic but random. For example, suppose this second type of risk is unemployment risk. To incorporate this second type of risk into the Kreps-Porteus framework, reinterpret c_1 as the consumption equivalent of the employment lottery. More specifically, let $c_1 = w^{-1} (Ew(\hat{c}_1))$ be the consumption equivalent associated to this lottery where function w controls the degree of aversion to unemployment risk. Assume $w(c) = c^{1-\eta}/(1-\eta)$, so η represents the coefficient of unemployment risk aversion. It is now easy to check that the individual prefers early resolution of unemployment risk when $\eta > \sigma$. Utility under early resolution is given by

$$V\left(\mathbf{c},\pi\right) = \frac{c_{0}^{1-\sigma}}{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} E\left[\frac{\widehat{c}_{1}^{1-\sigma}}{1-\sigma}\right],$$

while utility under late resolution is

$$V\left(\mathbf{c},\pi\right) = \frac{c_{0}^{1-\sigma}}{1-\sigma} + \beta \pi^{\frac{1-\sigma}{1-\gamma}} \frac{\left(E\left[\widehat{c}_{1}^{1-\eta}\right]^{\frac{1}{1-\eta}}\right)^{1-\sigma}}{1-\sigma}.$$

If $\sigma = \eta$, both utilities are equal and the individual is indifferent to the timing of resolution of uncertainty. Comparing the second term on the right-hand-side of the last two equations one can conclude that if $\eta > \sigma$ early resolution of unemployment risk is preferred. The intuition is that $V(\mathbf{c}, \pi)$ is increasing in the certainty equivalent c_1 , which is larger under early resolution when σ determines the concavity. In sum, if $\eta > \sigma > 1 > \gamma > 0$ then we have a framework in which the individual preference for early resolution of unemployment risk coexists with a preference for late resolution of mortality risk.⁹ Our paper can be viewed as one in which there is no unemployment risk and therefore the coefficient of unemployment risk aversion plays no role.

7.2 Alternatives to CRRA under expected utility

Our criticism of the expected utility model assumes a CRRA utility function. In this section we show that even representations that depart from CRRA are still subject to the limitations discussed above. Two alternatives to CRRA utility may appear to overturn some of these limitations, particularly the prediction that the VSL is negative for poorer countries. First, suppose one can depart from CRRA by assuming that utility is a function of both market and non-market consumption.

⁹We developed a fully dynamic version of this set up with an application to welfare measures in OECD countries in Boarini et al. (2013).

For instance, assume that utility is given by $(c + z)^{1-\sigma}/(1 - \sigma)$, where z > 0 is the non-market consumption. Introducing this non-homotheticity implies that even at low levels of market consumption c, the value of life could still be positive due to the presence of z > 0. Although this is certainly possible, we find that one would have to accept large values of z in order to obtain a reasonable calibration for the VSL. For instance, for the calibration presented in section 5.3, where $\omega = \$2,000$ is consistent with a VSL of \$4.5 million in the US, z would have to be of similar order of magnitude to ω in order to ensure the VSL is positive in poorer countries. But this results in non-market consumption being between 65 and 95% of total consumption in these countries. For non-market consumption to be a lower fraction of total consumption, the value of ω would need to be lower, but that would require accepting a VSL much larger than the one estimated in the data. The most important point is that even if one is willing to accept that non-market consumption is a very large fraction of total consumption in very poor countries, this specification still has the implication that the VSL-to-income ratio is increasing in income, one that conflicts with the evidence discussed in section 3.

A second modification of the CRRA utility that could in principle overturn the prediction that the VSL is negative in poorer countries consists of introducing a threshold level of consumption below which utility is not concave but linear. For instance, assume that

$$u(c) = \begin{cases} ac \text{ for } c < z\\ az + \frac{c^{1-\sigma}}{1-\sigma} - \frac{z^{1-\sigma}}{1-\sigma} \text{ for } c \ge z \end{cases}$$

where $a = z^{-\sigma}$. The specification above guarantees that the function is continuous and differentiable. In addition, assume that $z \geq \omega$ so that that at least some, if not all, individuals below the threshold z would prefer to be alive. Notice that it is still true that individuals with $c < \omega$ would prefer to die. The linearity of utility at levels of consumption below z implies that it is now possible to choose lower values of ω without making the VSL very high, an issue with the CRRA utility function. In other words, in the absence of concavity at low levels of consumption, the drop in marginal utility from death is not as severe as when utility is concave. One would hope that by selecting an appropriate value for z, one could obtain a calibration in which $\omega < c$ for all countries so that the VSL is always positive, while at the same time the VSL in the US is still within a reasonable range. It turns out that while it is indeed possible to choose z = \$2,000 and $\omega = \$100$ so that $c > \omega$ for all countries, one would have to accept a larger value for the VSL in the US of around \$6.5 million. Although this value is within the range of estimates for the US, this threshold model still suffers from the limitation that the VSL-to-income ratio is increasing, contradicting the evidence presented in section 3. In addition, available evidence suggests that if anything, risk aversion is much higher for poorer countries, while the threshold model assumes zero risk aversion for countries with c < z.¹⁰

From the analysis above we can conclude that departures from CRRA utility functions under the expected utility framework could potentially resolve the issue that life is a bad in poorer countries,

 $^{^{10}}$ See Yesuf and Bluffstone (2009).

but are still left with the counterfactual prediction that the VSL-to-income ratio is increasing. Moreover, it is still the case that these specifications assume that individuals are indifferent to the timing of resolution of uncertainty, an assumption that also conflicts with the evidence documented above.

8 Concluding comments

Death and terminal illnesses are extreme events. Many health decisions implicitly or explicitly have to deal with the possibility of dying, or facing daunting incurable illnesses such as Alzheimer or dementia. Expected utility models predict that, in the absence of potential treatments, individuals should be indifferent about the timing of resolution of uncertainty. The non-expected utility model explored in this paper predicts instead that individuals would prefer late resolution. Our model is consistent with micro-economic evidence supporting preferences for late resolution, and provides a number of predictions that are consistent with aggregate evidence. We document that the expected utility model cannot account for some of that evidence.

Key to the non-expected utility framework is the distinction between the parameters that govern the EIS and the mortality risk aversion. This distinction allows the mortality risk aversion to be identified directly from the evidence on the VSL. Our non-expected utility model predicts that, although the monetary value of (statistical) life is lower in poorer than richer countries, the VSLto-income ratio decreases with income. This pattern is consistent with available international evidence, as well as the cross-sectional empirical evidence within the US, as documented by Viscusi and Aldy (2003).

We use our model to assess the economic value of changes in longevity around the world for the period 1970 to 2005 as well as measure inequality in welfare across countries in 2005. According to our model, gains in life expectancy in poorer countries during the period 1970 to 2005 were particularly valuable in terms of welfare because longevity in poor countries tend to be lower and therefore more valuable. As a result, the systematic increase in life expectancy in most countries around the world since 1970 has decreased world welfare inequality in the last forty years to a larger extent than expected utility models would have predicted, or what is suggested by simple per capita income figures. On the other hand, according to our model, world inequality in 2005 is significantly larger than inequality in per capita incomes. In other words, according to the model the relatively short life span in many poor countries represents a significant economic loss that adds to their already low per capita income. Overall, the non-expected utility framework significantly changes the evaluation of welfare changes across countries and time relative to the standard expected utility model.

Our model can be used in a number of other contexts in which the monetary value of additional years of life is an important part of policy evaluation. We have in mind the literature at the intersection between demographics, health and macroeconomics, one that has gained strength in recent years. Within this literature, understanding the trends in health expenditures as populations age, as well as the trade-offs governments with limited resources face, are examples of contexts in which the framework we propose may be useful. In addition, our model has stark implications for the analysis of health inequality within a country. Our non-expected utility model implies that health programs targeted at raising life expectancy for those who lag behind can deliver sizeable welfare gains, much above those predicted under expected utility. We leave these questions for future work.

References

- Becker, G., T. Philipson and R. Soares. 2005. "The Quantity and Quality of Life and the Evolution of World Inequality." American Economic Review, 95 (1): 277-291.
- [2] Blanchard, O. 1985. "Debt, Deficits and Finite Horizons." Journal of Political Economy, 93: 223-247.
- [3] Boarini, R., J. C. Cordoba, F. Murtin and M. Ripoll. 2013. "Beyond GDP: Is There a Law of One Shadow Price?." OECD Working Paper.
- [4] Browning, M., L. P. Hansen and J. J. Heckman. 1999. "Micro data and general equilibrium models." In J. B. Taylor and M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 8, pp. 543-633, Elsevier.
- [5] Campbell, J. and G. Mankiw. 1989. "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence," NBER Chapters, in: NBER Macroeconomics Annual 1989, Volume 4, 185-246.
- [6] Chen, S., M. Ravallion and P. Sangruala. 2008. "Dollar a day revisited", World Bank Policy Working Paper 4620, September.
- [7] Day, J. H., K. Miyamura, A. D. Grant, A. Leeuw, J. Munsamy, R. Baggaley and G. J. Churchyard. 2003. "Attitudes to HIV voluntary counseling and testing among mineworkers in South Africa: Will availability of antiretroviral therepy encourage testing?" AIDS CARE: Psychological and Socio-medical Aspects of AIDS/HIV, 15 (5): 665-672.
- [8] Donaldson, J. and R. Mehra. 2008. "Risk-Based Explanations of the Equity Premium." In R. Mehra (ed.), Handbook of the Equity Risk Premium, chapter 2, pp. 37-99, Elsevier.
- [9] Epstein, L., and S. E. Zin. 1989. "Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns: A Theoretical Framework." Econometrica, 57 (4), 937-969.
- [10] Epstein, L. and S. E. Zin. 1991. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis." Journal of Political Economy, 99 (2), 263-286.
- [11] Jones, C. and P. Klenow. 2011. "Beyond GDP? Welfare Across Countries and Time." NBER Working Paper No. 16352.

- [12] Hall, R. 1988. "Intertemporal Substitution in Consumption," Journal of Political Economy 96(2), 339-357.
- [13] Hall, R. and C. Jones. 2007. "The Value of Life and the Rise in Health Spending." Quarterly Journal of Economics, 122 (1): 39-72.
- [14] Hansen, L. P, and K. Singleton. 1983. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," Journal of Political Economy 91(2), 249-265.
- [15] Kellerman, S. E., J. S. Lehman, A. Lansky, M. R. Stevens, F. M Hecht, A. B. Bindman and P. M. Wortley. 2002. "HIV Testing Within At-Risk Populations in the United States and the Reasons for Seeking or Avoiding HIV Testing." Journal of Acquired Immune Deficiency Syndromes, 31 (2): 202-210.
- [16] Kessler, S. 1994. Predictive testing for HD: A psychologist's view. American Journal of Medical Genetics 54, 161-166.
- [17] Kreps, D. and E. Porteus. 1978. "Temporal Resolution of Uncertainty and Dynamic Choice Theory." Econometrica, 46 (1), 185-200.
- [18] Murphy, K. and R. Topel. 2006. "The Value of Health and Longevity." Journal of Political Economy, 114 (5): 871-904.
- [19] Rosen, S. 1988. "The Value of Changes in Life Expectancy." Journal of Risk and Uncertainty, 1 (3): 285-304.
- [20] Tibben, A., P. G. Frets, J. J. P. van de Kamp, M. F. Niermeijer, M. Vegter-van der Vlis, R. A. C. Roos, G. B. van Ommen, H. J. Duivenvoorden and F. Verhage. 1993. "Presymptomatic DNA-testing for Hungtington disease: Pretest attitudes and expectations of applicants and their partners in the Dutch program." American Journal of Medical Genetics 48, 10-16.
- [21] Usher, D. 1973. "An Imputation to the Measure of Economic Growth for Changes in Life Expectancy," in Milton Moss, ed., The measurement of economic and social performance, Studies in Income and Wealth, Vol. 38. NBER Conference of Research in Income and Wealth. New York: Columbia University Press, pp. 193-225.
- [22] van der Steenstraten, I.M., A. Tibben, R. A. C. Roos, J. J. P. van de Kamp and M. F. Niermeijer. 1994. "Predictive testing for Huntington disease: Nonparticipants compared with participants in the Dutch program." American Journal of Human Genetics 55: 618-625.
- [23] Viscusi, W.K. 1993. "The Value of Risks to Life and Health." Journal of Economic Literature, 31 (4): 1912-1946.
- [24] Viscusi, W. K. and J. Aldy. 2003. "The Value of a Statistical Life: A Critical Review of Market Estimates Throughout the World." The Journal of Risk and Uncertainty, 27 (1): 5-76.

- [25] Vissing-Jorgensen, A. and O. Attanasio. 2003. "Stock-Market Participation, Intertemporal Substitution, and Risk Aversion." American Economic Review Papers and Proceedings, 383-391.
- [26] Weil, P. 1990. "Nonexpected Utility in Macroeconomics." Quarterly Journal of Economics, 105 (1): 29–42.
- [27] Weiser, S. D., M. Heisler, K. Leiter, F. P. Korte, S. Thou, S. DeMonner, N. Phaladze, D. R. Bangberg, and V. Lacopino. 2006. "Routine HIV Testing in Botswana: A Population-Based Study on Attitudes, Practices, and Human Rights Concerns." PLoS Med 3 (7): 1013-1022.
- [28] Wexler, N. S. 1979. "Genetic 'Russian Roulette': The Experience of Being At Risk for Huntington's Disease." In Genetic Counseling: Psychological Dimensions. S. Kessler (ed.). New York Academic Press.
- [29] Yaari, M. 1965. "Uncertain Lifetime, Life Insurance and the Theory of the Consumer." Review of Economic Studies, 32: 137-150.
- [30] Yaniv, I., D. Benador and M. Sagi. 2004. "On not wanting to know and not wanting to inform others: choices regarding predictive genetic testing." Risk Decision and Policy, 9: 317-336.
- [31] Yesuf, M. and R. Bluffstone. 2009. "Poverty, Risk Aversion, and Path Dependence in Low-Income Countries: Experimental Evidence from Ethiopia." American Journal of Agricultural Economics, 91 (4): 1022-1037.

ω	\$100	\$200	\$300	\$400	\$500
EIS					
0.45	1324.8	567.4	345.3	242.7	184.6
0.65	46.1	31.2	24.7	20.9	18.3
0.85	10.7	8.9	7.9	7.2	6.7
1.05	5.2	4.7	4.4	4.2	4.0
1.25	3.5	3.3	3.1	3.0	2.9

Table 1 – Value of statistical life (millions of U\$) Expected utility model

Notes: Value of statistical life (VLS) is computed as the marginal rate of substitution between assets and survival according to the expected utility model. The VSL corresponds to the overall willingness to pay to save a life. EIS is the elasticity of intertemporal substitution and parameter $\boldsymbol{\omega}$ is the minimum level of consumption.

Table 2 – Lower bound income y_{min} (U\$) Expected utility model

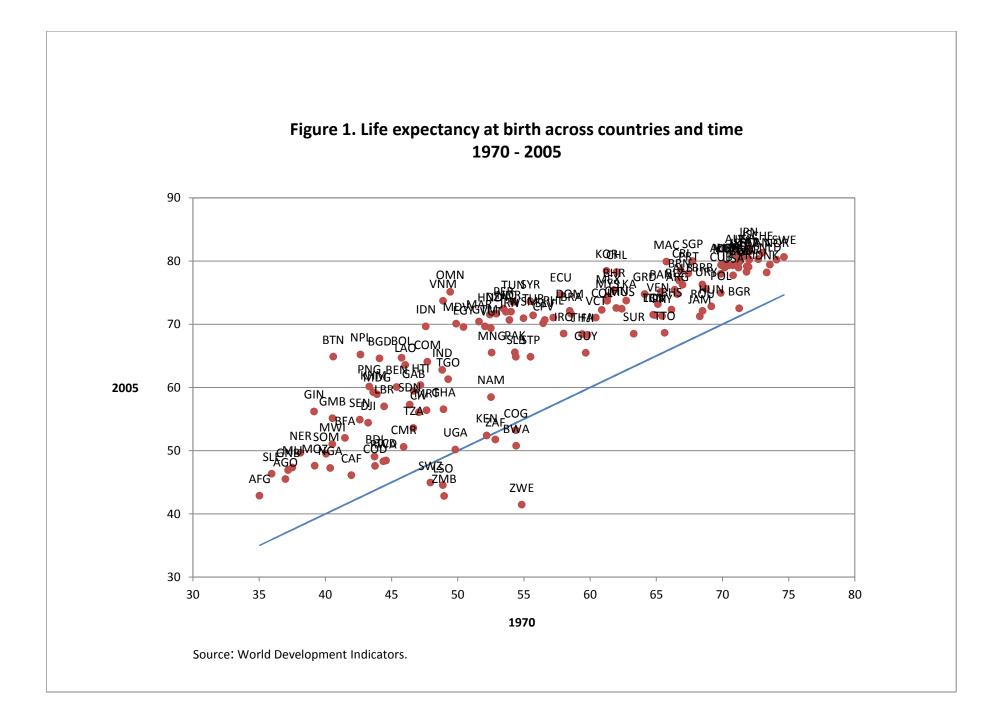
ω	\$100	\$200	\$300	\$400	\$500
EIS					
0.45	192	384	577	769	961
0.65	223	445	668	890	1113
0.85	251	502	753	1005	1256
1.05	279	557	836	1114	1393
1.25	305	610	916	1221	1526

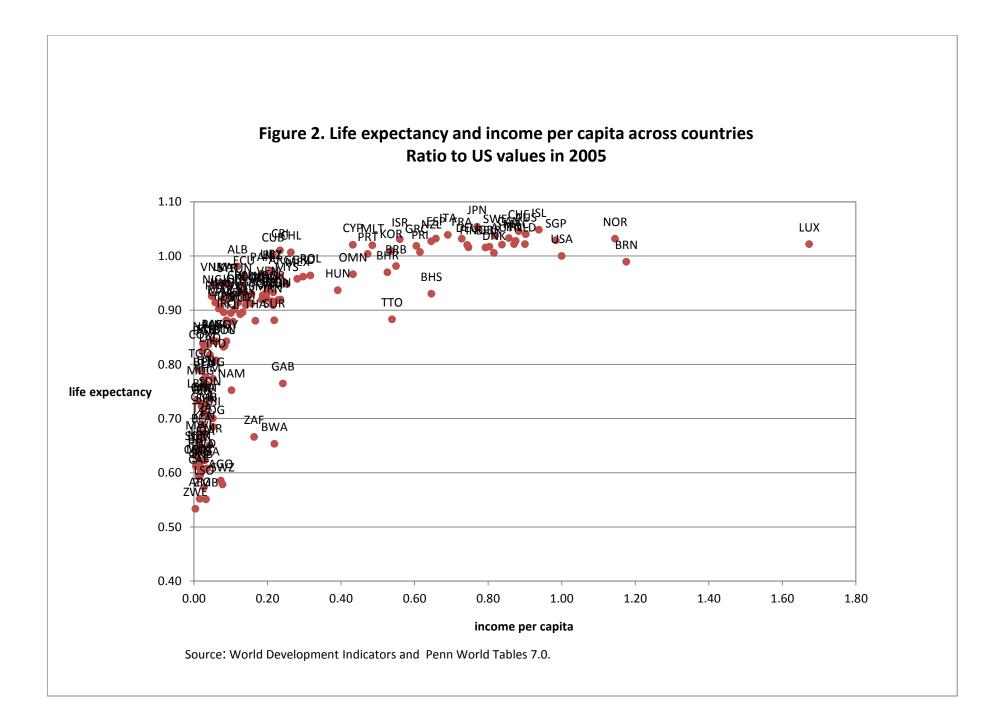
Notes: Lower bound income y_{min} in the expected utility model corresponds to the level of per capita income at which the value of statistical life is exactly equal to the present value of foregone income. For per capita incomes below this lower bound the model predicts that the overall willingness to pay to save a life falls below the present value of foregone income, a natural lower bound. EIS is the elasticity of intertemporal substitution and parameter $\boldsymbol{\omega}$ is the minimum level of consumption.

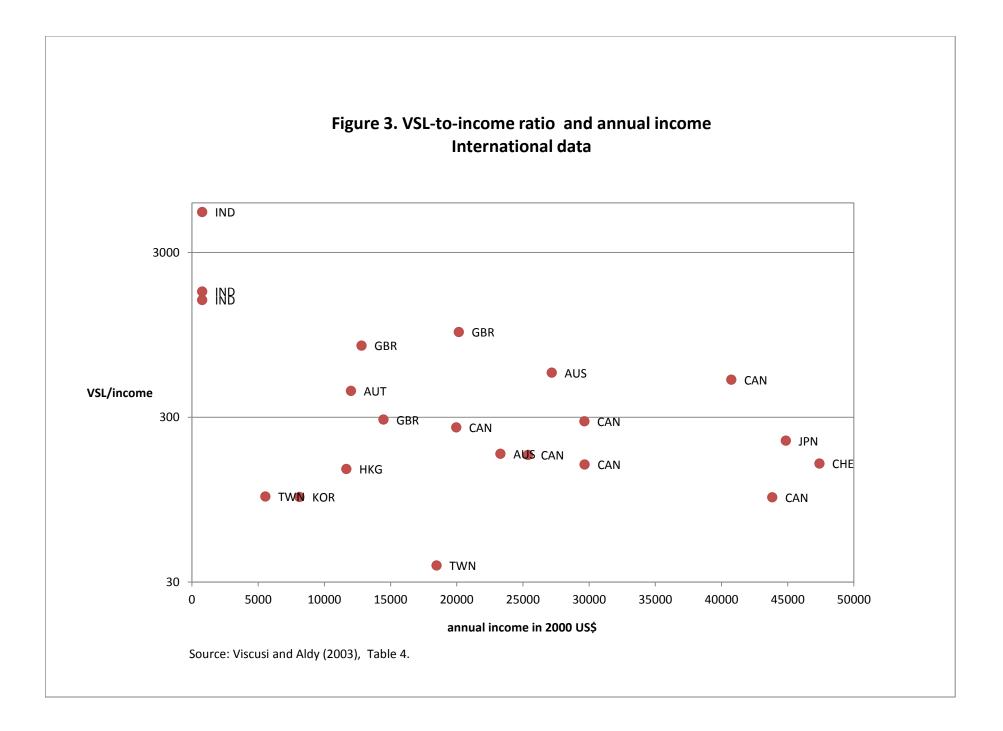
	LE 2005	Δ LE	R	Rf-EU	Rf		
Gains in life expectancy							
Rwanda	48	16	1.08	0.93	5.63		
Bhutan	65	12	2.16	2.27	3.33		
Nepal	65	11	1.20	1.15	1.76		
Losses in life expectancy							
Central Africa	46	-3	0.74	0.76	0.61		
South Africa	52	-9	1.29	1.18	0.89		
Botswana	51	-13	1.61	1.37	0.98		
Zimbabwe	41	-19	0.70	0.92	0.25		

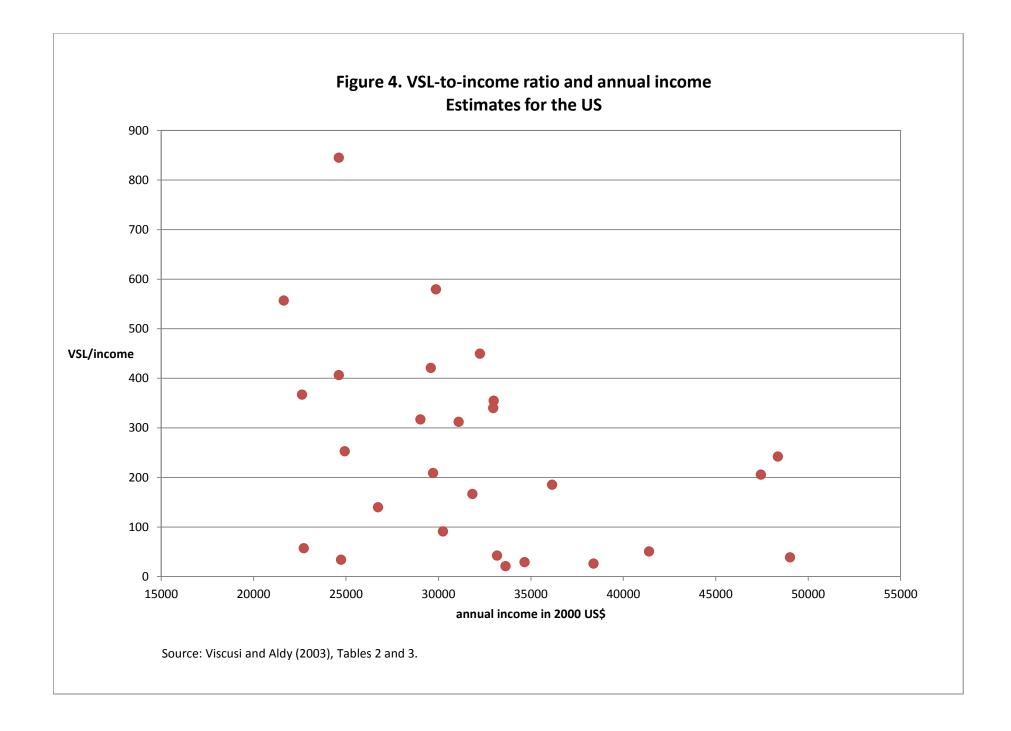
Table 3 – End of wars versus AIDS: 1990-2005

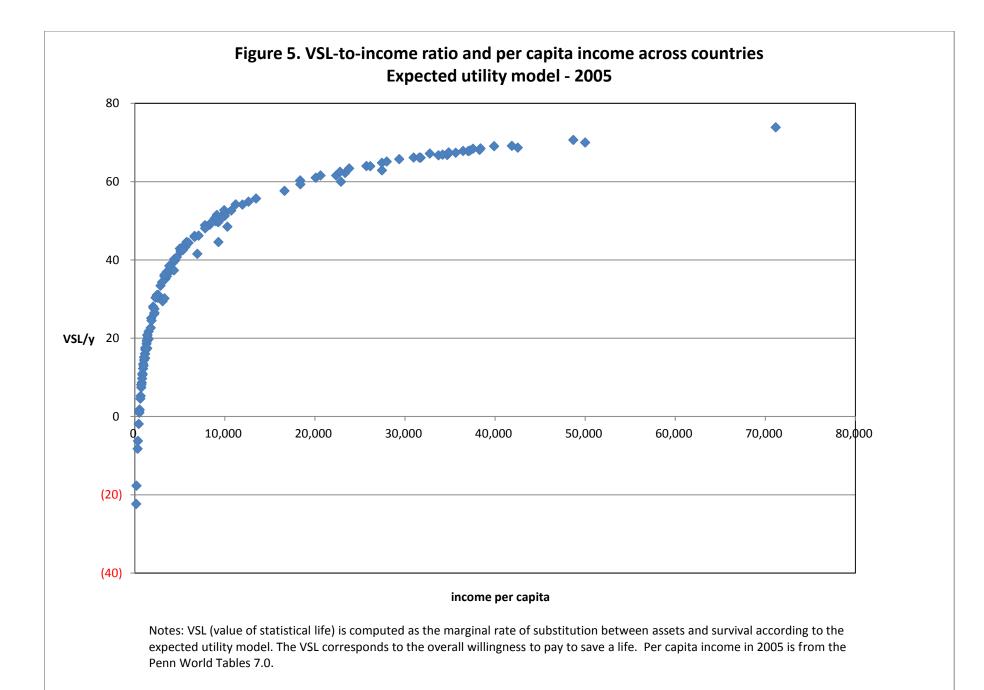
Notes: **LE 2005** denotes life expectancy in 2005 according to the World Development Indicators. **\Delta LE** denotes the difference between life expectancy in 2005 and 1990. **R** is per capita income in 2005 relative to 1990 according to Penn World Tables 7.0. **Rf** refers to full income in 2005 relative to 1990. Full income includes an adjustment for the value of longevity according to the non-expected utility model. **Rf-EU** is the ratio of full incomes but using the expected utility model.

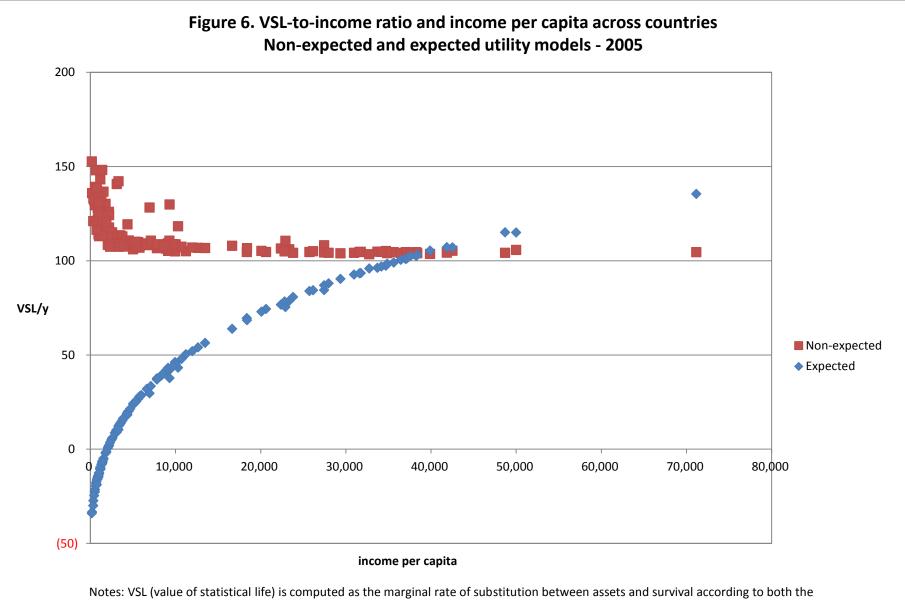




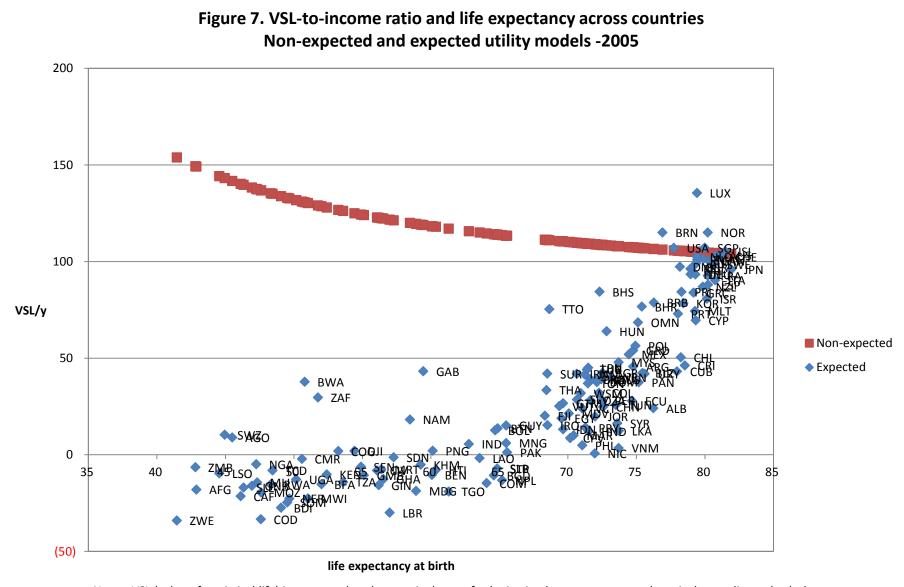








Notes: VSL (value of statistical life) is computed as the marginal rate of substitution between assets and survival according to both the expected and non-expected utility models. The VSL corresponds to the overall willingness to pay to save a life. Per capita income in 2005 is from the Penn World Tables 7.0.



Notes: VSL (value of statistical life) is computed as the marginal rate of substitution between assets and survival according to both the expected and non-expected utility models. The VSL corresponds to the overall willingness to pay to save a life. Per capita income in 2005 is from the Penn World Tables 7.0, and life expectancy is from World Development Indicators.

