"Tropical" Real Business Cycles? A Bayesian Exploration\textsuperscript{1}.

Andres Fernandez\textsuperscript{2}

Abstract

The use of frictionless small open economy models to account for business cycles in developing countries is controversial. Using Bayesian methods, we explore the role of permanent shocks to the technology trend in accounting for business cycles in these economies and we test the performance of such model against alternative models that incorporate a variety of other real impulses such as procyclical fiscal policies; terms of trade fluctuations; and perturbations to the foreign interest rate. We compare the models' performance to the data from a relatively unexplored developing -and "tropical"- country, Colombia. Our findings leave us skeptics that business cycles in developing economies can be modeled with a standard neoclassical model driven \textit{solely} by technology shocks as has recently been argued in other studies.

\textit{Keywords:} Business Cycles; Developing economies; dynamic stochastic general equilibrium models; small open economy models; Bayesian estimation.

\textit{JEL classification:} E32, F41, F47, C11.

\textsuperscript{1}This paper is part of the requirements in my Ph.D. studies. I wish to thank the comments by my advisor Roberto Chang, Rodrigo Fuentes, Philipo Ochino, John Landon-Lane, Levan Mahadeva, Bruce Mizrach, Paulina Restrepo, Norman Swanson, Eugene White and other participants to the Macroeconomics Study Group at Rutgers University in January and July 2007 and the LACEA-LAMES session on Business Cycles in Bogota, Colombia on October 2007. Of course, any errors and omissions are mine alone.

\textsuperscript{2}Contact: afernandez@economics.rutgers.edu.
1 INTRODUCTION

Understanding business cycle regularities in developing countries is a crucial step in the process of designing appropriate stabilization policies and sound macroeconomic management in these countries. A first step toward this understanding must take into account the differences on the business cycles properties in developing countries relative their developed counterparts. As can be observed in Table I.1, three dimensions in which these differences manifest are: (i) observed business cycles in emerging countries are more volatile; (ii) the trade balance-to-output ratio is more countercyclical in emerging countries than in developed countries; and, (iii) consumption appears to be more volatile than output at business-cycle frequencies. These stylized facts, among others, have been widely documented in Mendoza (1995), Agenor et.al (2000), Rand and Tarp (2002), Neumayer and Perri (2004), Gita and Gopinath (2007a) and
Garcia-Cicco et al. (2007). Explaining these contrasts between emerging and industrialized economies is, in the words of Uribe (2007), "at the top of the research agenda in small-open-economy macroeconomics".

Despite these important differences a brief review of the literature on emerging markets business cycles modelling does not show a consensus on the best approach to account for them. One strand of the literature has tried to explain business cycles in developing economies within a neoclassical growth framework augmented by real driving forces in addition to technology shocks. Mendoza (1995) expands a real business cycle model to account for tradable/non-tradable goods in which the terms of trade are an additional driving force. Since emerging countries typically specialize in exports of a few primary commodities for which they are small players in the world markets for the goods they export or import, it follows that the terms of trade can be regarded as an exogenous source of aggregate fluctuations. Mendoza (1995) finds they account for 45 to 60 percent of the observed variability of GDP. The argument of stronger real shocks has been extended to financial markets. The idea is that developing economies exhibit low levels of aggregate savings forcing them to rely heavily on foreign investment, via capital inflows. Uribe and Yue (2006) explore the significant correlation between the business cycles in emerging markets and the interest rate that these countries face in international financial markets. They find that one third of business cycles in emerging economies is explained by disturbances in external financial variables (e.g. the foreign interest rate and the spread). Moreover, they find evidence of a further increase in the volatility of domestic variables because of the presence of feedback from domestic variables to country spreads. Similarly, Neumayer and Perri (2005) find that eliminating country risk lowers Argentine output volatility by 27%. Another explanation for some of the stylized facts of the business cycles in developing economies explores the role of macroeconomic policies in amplifying the cycle (i.e. procyclical policies) as documented by Agenor et al. (2000) and Kaminsky et al. (2004).

Still within the neoclassical framework, but on a more orthodox strand, some authors claim to properly account for the business cycle in developing economies by relying exclusively on pure technology forces in the line of the real business cycle school of thought. Kydland and Zagra (1997, 2002) argue that nominal factors do not seem to be able to account for any significant fraction of the
business cycles of Latin American countries, in general. They argue that, in the case of Argentina, the predictions of a standard neoclassical growth model conforms rather well with the observations during the Argentinean ‘lost decade’ years (90’s). More recently, Aguiar and Gopinath (2004, 2007a), claim that accounting for possible regime switches giving rise to changes in the long-run growth trend in these economies is enough to account for the business cycle stylized facts. Their underlying premise is that emerging markets are characterized by frequent regime switches motivated mainly by dramatic reversals in economic policy. In their words: "shocks to trend growth are the primary source of fluctuations in these [emerging] markets as opposed to transitory fluctuations around a trend". Thus, the higher volatility of consumption can be explained as agents, seeking to smooth their consumption levels, observe changes in the permanent component of the trend. Aguiar and Gopinath’s conclusion is driven by an estimated volatility of the technological growth process in the Mexican economy four to five times higher than the volatility of the transitory technology shock. In another paper, Aguiar and Gopinath (2007b) find this result to be robust under the presence of stochastic interest rate shocks.

However, the findings of Aguiar and Gopinath (2007a) are rejected by Garcia-Cicco et.al. (2007) when a longer dataset for the Argentinian economy is used. These authors argue that in order to properly estimate the parameters of the stochastic trend, in particular the volatility of the permanent component, long time series are needed. Accordingly, they estimate the Aguiar and Gopinath model on a yearly dataset for Argentina covering over a century of aggregate data and find that the model performs poorly. They estimate the volatility of the technological growth process to be only 50% higher than the volatility of the transitory technology shock, far from the wider difference found by Aguiar and Gopinath (2007a) of 450%. More importantly, they find that this difference is not enough to properly account for the main moments in the Argentinian macroeconomic data, in particular the higher volatility of consumption relative to output, nor to properly trace back the trade balance autocorrelation function.

We observe, nonetheless, two shortcomings when using a real business cycle model to test whether the Aguiar and Gopinath hypothesis -i.e. that fluctuations are driven by trend shocks instead of fluctuations around a stable trend-holds or not in the data. First, in the event that it doesn’t hold, as in Garcia-Cicco et.al. (2007), one is left wondering if the failure of the model to replicate the data is to be blamed on the driving forces -i.e. the permanent/transitory
technology shocks- or on the model’s propagation mechanism. And, second, in the event that it does hold, we are still left wondering about its relative performance against alternative models. Indeed, as pointed by Hartley et. al. (1998), the fact that the model is highly stylized implies that the actual data alone provide, at best, a weak standard and that more important than simply fitting the data is the relative performance against alternative models.

The goal of this paper is to test the Aguiar and Gopinath hypothesis in the same spirit of Garcia-Cicco et.al. (2007) while using a methodological approach that addresses these two shortcomings explicitly. To address the first one, we allow for potential model misspecification in the Aguiar and Gopinath model, arising from omitting real driving forces other than technology shocks. We thus focus our attention to the impulse forces driving the fluctuations rather than the transmission mechanism. This strategy is motivated by the findings in previous studies that the propagation mechanism embedded in this type of models adds relatively little to the pattern of fluctuations beyond what is implicit in the driving forces themselves (see Cogley and Nason, 1995; Hartley et.al. 1998). Based on the literature surveyed above, we include separately three structural driving forces to the standard neoclassical framework: (i) a procyclical government spending process; (ii) terms of trade fluctuations; and (iii) shocks to the foreign interest rate. In each of the three cases the baseline Aguiar and Gopinath model is modified by replacing the growth shock by each of the three alternative driving forces, while keeping the transitory technology shock.

To address the second shortcoming we use a Bayesian-likelihood-based method to estimate the benchmark Aguiar and Gopinath model as well as the alternative models driven by the other real processes and compare their empirical plausibility based upon assessments of their relative conditional probabilities. Thus, we take each model as provider of a complete statistical characterization of the data in the form of a likelihood function with which we test how plausible the benchmark model is relative to the alternative models, given the available data.

For the empirical purpose of the paper we use data from a developing - and "tropical"- economy: Colombia. Our baseline dataset is quarterly ranging from 1977:Q1 to 2007:Q2, and, because we take seriously the criticism raised by Garcia-Cicco et.al. (2007) concerning the use of short time series on this type of

---

3This is acknowledged by Garcia-Cicco et.al. (2007) themselves in their concluding remarks.
estimation, we also report results on an annual dataset from 1925 to 2006. Table I.2 plots the main macroeconomic moments in the Colombian economy across the two available dataset. As can be seen, Colombian data is characterized by some of the main stylized facts from the developing economies highlighted in Table I.1. There is a higher macroeconomic volatility; investment is much more volatile than income and the trade balance share is not only highly volatile but significantly more countercyclical. There is, however, ambiguous evidence regarding the relative volatility of consumption. While the yearly dataset exhibits a consumption volatility twice as big as the one in income, the quarterly evidence is less clear. Moreover, when we exclude durable goods consumption from aggregate consumption and include it on investment (which unfortunately can be done only from 1994) as other studies have done (see Cooley and Prescott, 1996) the relative consumption falls well bellow one. This fact highlights one potential pitfall when dealing with less reliable data on consumption on developing economies, specially going further back in time. We will overcome this obstacle by reporting robustness results across the two samples as well as the small subsample with properly-measured consumption data.

While we think the alternative models that we use are highly simplified and more elaborated models should be considered when modeling business cycles in developing economies, we are nonetheless surprised by the results we get by comparing them to the performance of the baseline Aguiar and Gopinath model. In fact, not only our findings suggest that growth shocks may not be as relevant in explaining business cycles but that the data rejects the baseline model driven solely by technology shocks and favors virtually all the alternative models where real driving forces other than these impulses come into play. We are thus skeptics that business cycles in developing economies can be modeled with a standard neoclassical model driven solely by technology shocks.

The paper is divided into six sections, including this introduction. The second lays out formally the Aguiar and Gopinath model and the three alternative models based on separate driving forces, and illustrates how potential misspecification may lead to spurious identification of structural shocks. The third section describes the Bayesian estimation approach. The fourth and fifth sections present the baseline results and a series of robustness analyses, respectively. Concluding remarks are given in the sixth section. Most of the technical
details are given in a technical supplement available upon request.

2 SMALL, OPEN-AND "TROPICAL"- ECONOMY BUSINESS CYCLES MODELS

This section lays out formally the Aguiar and Gopinath model. We then investigate the conditions under which this model can be used to identify permanent and transitory components in the technology process. To do so we consider model misspecification explicitly by including alternative driving forces other than the two technological processes in the benchmark model. We illustrate that, under reasonable parameters, omitting the presence of other driving forces that are relevant in explaining business cycles might lead to spurious identification of the structural shocks in the two technological processes.

2.1 Benchmark Model (\(M_0\))

Consider a small open economy populated by an infinite number of households with preferences described by a generic utility function which has aggregate consumption (\(C\)) and labor (\(h\)) as its arguments

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, h_t)
\]

where we use upper case letters to denote variables that exhibit growth in equilibrium. There is a single good in the economy which is produced with a technology that takes the generic form

\[
Y_t = a_t F(K_t, X_t h_t)
\]

where \(K\) is the stock of capital which dynamics are governed by the law of motion:

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]

and \(\delta \in [0, 1)\) is the depreciation rate of capital; \(a_t\) is a stochastic total factor
productivity shock that follows an AR(1) stationary process in logs:

\[ \ln a_{t+1} = \rho_a \ln a_t + \epsilon_{t+1}^a ; \text{ where } \epsilon_{t+1}^a \sim i.i.d (0; \sigma_a^2) \] (4)

The period by period budget constraint of the representative household is given by the evolution of foreign debt, \( D_t \):

\[ D_t = (1 + r_{t-1}) D_{t-1} - Y_t + C_t + I_t + \Phi (K_{t+1}, K_t) \] (5)

where \( r_t \) denotes the interest rate at which domestic residents can borrow in international markets and \( \Phi (K_{t+1}, K_t) \) is a capital adjustment cost function. Note that in equilibrium, the budget constraint can be rewritten as:

\[ D_t = (1 + r_{t-1}) D_{t-1} - TB_t \] (6)

where \( TB \) is the trade balance.

The productivity trend \( X_t \) grows according to

\[ \gamma_t = \frac{X_t}{X_{t-1}} \] (7)

where the logarithm of \( \gamma_t \) follows a first-order autoregressive process of the form

\[ \ln(\gamma_{t+1}/\gamma_X) = \rho_{\gamma} \ln(\gamma_t/\gamma_X) + \epsilon_{t+1}^\gamma; \quad \epsilon_{t+1}^\gamma \sim N IID(0, \sigma_\gamma^2) \] (8)

and the parameter \( \gamma_X \) measures the long-run deterministic growth rate of the technological progress.

To ensure that the deterministic steady state is independent of the initial asset position we follow Schmidt-Grohe and Uribe (2003) and close the model by assuming a debt elastic interest rate

\[ r_t = r^* + p(D_t) \] (9)

where \( r^* \) denotes the constant foreign interest rate, \( p(D_t) \) is a country-specific interest rate premium assumed to be strictly increasing in its argument, \( D_t \), the aggregate level of foreign debt.

The optimality conditions associated to this problem are:

\[ U_c(C_t, h_t) = \lambda_t \] (10)
\[ \lambda_t = \beta \frac{1 + r_t}{\gamma_t} E_t \lambda_{t+1} \]  
\[ -U_h(C_t, h_t) = \lambda_t a_t F_h(K_t, X_t h_t) \]  
\[ \lambda_t [1 + \Phi(K_{t+1}, K_t)] = \beta \frac{1 + r_t}{\gamma_t} E_t \left\{ \frac{a_{t+1} F_h(K_{t+1}, h_{t+1})}{(1 - \delta) - \Phi(K_{t+1}, K_{t+2}, K_{t+1})} \right\} \]  

where \( \lambda \) is the Lagrangean multiplier associated to the problem\(^4\).

A stationary equilibrium along a balanced growth path can be characterized by transforming the non-stationary variables into stationary by dividing them by \( X_{t-1} \): \( z_t \equiv Z_t / X_{t-1} \). Thus, the stationary equilibrium in the model is given by the process of \( \{c_t, y_t, i_t, k_{t+1}, d_t, h_t, r_t, \lambda_t, t_{bt_t}, t_{bt_y}\}_{t=0}^{\infty} \), given the exogenous process for \( \{X_t, \gamma_t, a_t\}_{t=1}^{\infty} \), and the initial conditions for \( \{D_0, K_0, X_0, a_0\} \) satisfying equations 2-13.

We assume the following parameterization. In the instantaneous utility function we assume GHH-type preferences

\[ u(C_t, h_t) = \left[ C_t - \omega^{-1} X_t h_t^{\gamma} \right]^{1-\gamma} - 1 \]  

where \( \omega \) and \( \gamma \) are the parameters governing the labor supply and intertemporal elasticities, respectively.

The production function will adopt a Cobb-Douglas technology

\[ F(K_t, X_t h_t) = K_t^{\alpha} (X_t h_t)^{1-\alpha} \]  

where \( \alpha \) and \( 1 - \alpha \) are the relative shares of capital and labor.

We adopt a quadratic function as the capital-adjustment cost function:

\[ \Phi(K_{t+1}, K_t) = \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - \gamma_X \right)^2 ; \quad \phi > 0 \]  

Finally, following Schmidt-Grohe and Uribe (2003), the risk premium function is defined as:

\[ p(D_t) = \psi \left[ \exp \left( D_t / X_{t-1} - d \right) - 1 \right] \]  

where \( d \) is the steady-state level of debt.

\(^4\) See Technical Supplement for more details.
2.2 Misspecification and Alternative Real Driving Forces

The desire of the representative agent to smooth consumption, embedded in the optimality conditions of the above model, will determine different responses in consumption whether the nature of the shock is transitory or permanent. This difference is used by Aguiar and Gopinath (2007a) to identify the persistence of the technology shocks, or, to use the notation in the model, the relative importance of $\sigma_\gamma$ vis a vis $\sigma_a$. In their own words: "if we observe in the data a large response of consumption to income and a corresponding large deterioration of net exports, the standard business cycle model will identify the underlying shock as a change in trend. If, however, for the same increase in output, consumption rises by less and net exports drop only slightly (or improve), the shock will be identified as a transitory shock".

This claimed is corroborated in the first three row-panels of Fig.II.1 where, using a baseline parameterization, we plot the response of three key macroeconomic ratios following a similar perturbation to each of the shocks \{\sigma_\gamma = 0.001; \sigma_a = 0.001\}. As can be observed, following a growth shock, there is a pronounced countercyclical response of the trade balance share; an initial positive response of the consumption to income ratio; and a strong increase of relative investment. On the other hand, after a transitory shock the net exports share drops only slightly; the consumption share drops and investment reacts only mildly.

While this identification strategy is logically correct, because the model is highly stylized it rests upon a strong "ceteris paribus" argument that takes as given all other possible sources of perturbations to these economies. For example, it obviates any dynamics introduced by other potential exogenous driving forces, such as exogenous movements income via terms of trade fluctuations, foreign interest rates or any exogenous wedges to private consumption induced by, say, movements in the government spending. But what if these other dynamics are in fact relevant to account for business cycles in developing economies? To what extent would this kind of model misspecification lead to spurious results in the identification of the permanent and transitory components of the technology shocks? To answer these questions, we allow for the presence of three additional and separate driving forces in the standard neoclassic model following the literature surveyed briefly in the previous section. First we introduce a procyclical government spending process; second we differentiate between foreign and home goods to introduce a relative price -the terms of trade- that we
model as an exogenous process; and, third, we allow for perturbations to the foreign interest rate. To be concrete, we will develop three separate alternatives of the neoclassic framework in which the only technology process is transitory—there are no growth shocks—but we allow for an additional real driving force.

2.2.1 *Procyclical Fiscal Policy Model* (M₁)

Following the literature that has identified role of procyclical fiscal policies in amplifying the cycle in emerging economies, we extend the standard neoclassic framework to include a government spending process as in Christiano and Eichenbawn (1992) but we introduce a procyclical coefficient in the process, by which current increases (decreases) in output lead to future increases (decreases) in government spending.

The period-by-period budget constraint includes now a government spending term, \( g \):

\[
d_t = (1 + r_{t-1}) d_{t-1} - y_t + c_t + \Phi (k_{t+1}, k_t) \]

(18)

The novelty comes from modelling the process for the government spending shock. Following Canova (2007) we assume the following process for the deviations of the government spending process from steady-state values:

\[
\ln \left( \frac{g_{t+1}}{g_t} \right) = \rho_g \ln \left( \frac{g_t}{g} \right) + \Phi (y_t, y) + \epsilon_{t+1} \quad \log \epsilon_t \sim (0; \sigma^2_G) \]

(19)

where \( \rho > 0 \) captures the procyclicality of fiscal policy.\(^6\)

2.2.2 *Terms of Trade Model* (M₂)

The inclusion of terms of trade in a neoclassic framework follows a simplified version of Mendoza (1995).\(^7\) The main difference lies now is the consumption good, \( c_t \), which is now a composite of home and foreign goods assumed to have

\(^5\)Note that in this model as well as the next two models we use lower case variables because the variables are already stationary given our assumption of no growth shocks.

\(^6\)For the sake of brevity we omit the first order conditions as well as the definition of the competitive equilibrium in all the three alternative models. These are specified in a technical supplement available upon request.

\(^7\)The simplification lies in that we don’t distinguish between tradables/non-tradables as in Mendoza (1995).
a Cobb-Douglas form:

\[ c_t = \left( c_t^F \right)^\vartheta \left( c_t^H \right)^{1-\vartheta} \left( \vartheta \right)^\vartheta \left( 1 - \vartheta \right)^{1-\vartheta} \]  

(20)

with associated consumption-based price index, in terms of foreign goods:

\[ p_t = q_t^{1-\vartheta} \]  

(21)

where we have taken the price of foreign goods as numeraire, thus making \( q_t \) the relative price of home goods in terms of foreign goods, or the terms of trade. Therefore, it can be shown that the optimal demand for each of the two goods can be expressed as:

\[ c_t^F = \vartheta p_t c_t \]  

\[ c_t^H = (1 - \vartheta) \frac{p_t}{q_t} c_t \]  

(22)

Aggregate investment is assumed to be done in foreign goods and evolves according to the standard law of motion.

\[ k_{t+1} = k_t (1 - \delta) + i_t \]  

(23)

The period-by-period budget constraint can be written as

\[ d_t = (1 + r_{t-1}) d_{t-1} - q_t y_t + i_t + \Phi (k_{t+1} - k_t) + p_t c_t \]  

(24)

The technology to produce of home goods is a standard function of the labor hours and the capital stock:

\[ y_t = a_t F (k_t, h_t) \]  

(25)

where \( a_t \) is an aggregate productivity level.

The two driving forces of the economy are assumed to be the aggregate technology process and the exogenous terms of trade process where we follow Mendoza (1995) in allowing for the comovement between domestic and global shocks by letting domestic productivity and terms of trade to be correlated:

\[ \ln q_{t+1} = \rho_q \ln q_t + \varepsilon^q_{t+1}; \quad \varepsilon^q_t \sim i.i.d(0; \varSigma_q^2) \]  

(26)
\[ \ln a_{t+1} = \rho_a \ln a_t + \varepsilon_{t+1}^a; \]

\[ \varepsilon_t^a = \psi_a \varepsilon_t^q + v_t^a \]

\[ v_t^a \sim i.i.d(0; \sigma_v^a); E(\varepsilon_t^q, v_t^a) = 0 \]

2.2.3 Foreign Interest Rate Model \((M_3)\)

Following Neumeyer and Perri (2005) and Uribe and Yue (2006), the final dimension we explore incorporates to the standard neoclassic framework the real effect of perturbations to the real interest rate. The models suggested by these two works, however, incorporate financial frictions that are beyond the scope of this paper. We take a simpler approach by adding a driving force in the foreign interest rate.

The debt elastic interest rate is modified to account for a stochastic foreign interest rate:

\[ R_t = R_t^* + p(\tilde{d}_t) \]

where we assume the deviations of \( R_t^* \) form its steady state to follow an AR(1) process estimated earlier as deviations from steady-state levels:

\[ \ln (R_{t+1}^*/R^*) = \rho_R \ln (R_t^*/R^*) + \varepsilon_{t+1}^{R^*}; \varepsilon_t^{R^*} \sim i.i.d(0; \sigma_{R^*}^2) \]

Insert Fig II.1 here

We are now equipped to analyze the potential consequences of model misspecification in the identification of the permanent and transitory components of the technological process. To do so we perturb each one of the alternative models \( \{M_1 - M_3\} \) by the additional driving forces embedded on them and compare them to the benchmark model. This is done in the last three rows of Fig. II.1.

The second row panels show that a similar pattern as the one obtained with permanent shocks to trend might arise if a transitory shock is accompanied by a transitory negative foreign interest rate shock in model \( M_2 \). This effect is the result of a combined intertemporal substitution of consumption and investment that worsen the trade balance share. A dynamic qualitatively similar to the one plotted in the first row of Fig. II.1.
The third row-panels show that similar dynamics might be generated by a positive transitory terms of trade shock in model $M_3$. Following this perturbation, the boost in consumption of foreign goods drives up total consumption share and increases (foreign) investment goods. As a result, we observe a steeper fall in the trade balance to income ratio, similar as the one observed after a permanent technology shock in the benchmark model. A key assumption in this result comes from assuming a negative correlation between the terms of trade shocks and the domestic technology process, $\psi_a < 0$, as the one calibrated in Mendoza (1995). Lastly, the fourth row depicts the response of a combined impulse to the transitory technology shock and a negative shock to government expenditure in model $M_3$. While, in this case the response of investment is a bit stronger and consumption increases, the trade balance improves.

To sum up, this simple illustration shows that potential misspecification in the benchmark model coming from excluding other real driving forces, may lead to a poor identification of the parameters of the underlying productivity process when using moments from macroeconomic aggregates such as consumption, investment and net exports. For instance, a researcher that observes a strong countercyclical trade balance along with increases in consumption an investment shares may be lead to identify, under the baseline model $M_0$, a permanent shock as a cause of these dynamics, while in fact they may be the result of transitory shocks to the terms of trade and/or the interest rates. Moreover, these dynamics might also be affected by wedges in consumption induced by government expenditure perturbation.

To overcome this obstacle in assessing the role of trend growth in developing countries business cycles dynamics we follow a full-information analysis that takes each of the models considered as providing a complete statistical characterization of the data. This way the performance of each model will be assessed on the basis of its entire likelihood function and not just on a subset of implications conveyed by the collection of moments. More specifically, by comparing the benchmark model ($M_0$) to the three alternatives on a full-information basis we can have a sense of how far will this model take us in explaining the dynamics of the data observed in developing countries. And, to what extent do the dynamics implied by other forces help us fit the data better. The next section describes the estimation methodology to be used.
3 ESTIMATION STRATEGY

We follow a Bayesian estimation strategy that has been increasingly used in the estimation of dynamic stochastic general equilibrium models. Since our final goal is to compare the performance of the benchmark model against the three alternatives considered, to be consistent in the comparison we chose to fix the "deep parameters" depicting preferences and technology across all models and use Bayesian techniques to estimate the parameters governing the driving forces. The following sections briefly describe the estimation technique; the data used; as well as the baseline calibration for the deep parameters and the choice of priors for the driving forces’ parameters.

3.1 A Bayesian Estimation Approach

Following Schmidt-Grohe and Uribe (2004) each of the models above, in a log-linearized version, can be written in this canonical form:

\[
\begin{align*}
    x_{1,t+1} &= M(\Theta) x_{1,t} + v_{t+1} \\
    x_{2,t} &= C(\Theta) x_{1,t}
\end{align*}
\]

where \( \{x_1, x_2\} \) are the state and control variables vectors, respectively; \( v_{t+1} \) is a vector of structural perturbations; and the matrices \( M(\Theta) \) and \( C(\Theta) \) are a function of the parameters, \( \Theta \). This system can be compactly written as a law of motion equation:

\[
    \Psi_{t+1} = \Phi(\Theta)\Psi_t + Bu_{t+1}
\]

On the other hand, having data on a vector \( X_t \), this can be expressed as a non-invertible linear combination of the state variables in a measurement equation:

\[
    X_t = \Gamma\Psi_t
\]

where \( \Gamma \) is a conformable matrix that maps the observable time series of the observable elements \( X_t \) to their theoretical counterparts in \( \Psi_t \). The two equations are the starting point for a time invariant Kalman filter with which
one can recursively construct the likelihood function over the $T$ data points of $X_t$:

$$L(X | \Theta) = \prod_{t=1}^{T} L(X_t | \Theta)$$

(33)

From a Bayesian perspective, the observation of $X$ is taken as given and inferences regarding $\Theta$ center on statements regarding probabilities associated with alternative specifications on $\Theta$ conditional on $X$. By satisfying the likelihood principle, the Bayesian approach uses all the information from the data to make the probability statements on $\Theta$. Bayes Theorem is used to update our beliefs about $\Theta$. Formally:

$$p(\Theta | X) \propto p(\Theta) \cdot L(X | \Theta)$$

(34)

where $p(\Theta)$ represent the prior distribution placed over the driving forces’ parameters of the model, before observing the data. The posterior distribution then allows us to make probability statements regarding the unknown parameters in our model.

In order to report posterior statistics we need to be able to make random draws from the posterior distribution for which we will make use of advances in Monte Carlo Markov Chain (MCMC) theory to get dependent draws from the posterior distribution, $p(\Theta | X)$. A particularly simple and useful algorithm for this purpose is the random walk Metropolis Hastings (RWMH) algorithm (see Ann and Schorfheide (2006) for a detailed explanation) which has been shown to efficiently draw from the ergodic distribution of $p(\Theta | X)$.

Once $p(\Theta | X)$ is approximated, point estimates as well as confidence intervals of the parameters in the driving forces can be obtained from the generated draws. Given that one of our goals is to assess the relative role of permanent shocks, it will be of particular interest to obtain the posterior marginal distribution of the two parameters associated to the technological process in the benchmark model, $M_0$:

$$p(\sigma_\gamma | X, M_0) : p(\sigma_a | X, M_0)$$

More importantly, given that another goal is to assess the performance of the benchmark model $M_0$ against the other alternatives, it will be of particular
interest to obtain the marginal data densities of each model considered

\[ p(X \mid M_j) = \int_{\theta_j} p(\theta_j \mid M_j) \cdot p(X \mid \theta_j, M_j) \, d\theta_j; \quad \text{for } j = 0, 1, 2, 3. \]

which represents the out-of-sample predictive performance of each model over the observed sample, or the likelihood values of each model given the data. Forming a Bayes factor as a simple ratio:

\[ BF_j = \frac{p(X \mid M_j)}{p(X \mid M_0)} \]

one can thus assess the relative performance of the alternative models with respect to \( M_0 \).

### 3.2 Data Description

As mentioned in the Introduction, we use two dataset in our estimations. First a quarterly dataset is constructed from 1977:Q1 to 2007:Q2 with the main macroeconomic aggregates: GDP, consumption, investment, government spending and net exports. Since data on this variables was not collected by a single government agency along the entire quarterly period, we used the method by Hill and Fox (1997) to splice the series\(^{10}\).

Second, to seriously take the criticism raised by Garcia-Cicco et.al. (2007) concerning the use of short time series on this type of estimation, we also report results on an annual dataset from 1925 to 2006 taken from two sources. For the national accounting data, the Estadisticas Historicas de Colombia dataset was used\(^{11}\). The rest of the yearly dataset was taken from GRECO (2004). This dataset ranges up to 2000 so our dataset was updated using various official sources.

The left panels in Figure III.1 plot the main macroeconomic variables, in per-capita levels, for the two samples. In addition, the right panels in Figure III.1 plot the time series of the other driving forces that we had been mentioning on a theoretical basis so far: the terms of trade and the foreign (ex-post) real

---

\(^{10}\)Macroeconomic data was collected by the Departamento Nacional de Planeacion, a central planning government agency between the period 1977:1 1997:IV. Since 1994 another government agency, the Departamento Administrativo Nacional de Estadisticas, the statistical bureau, is in charged of collecting the macroeconomic data. The information provided by the two agencies can be accessed on line at http://www.dnp.gov.co/ and http://www.dane.gov.co/.

\(^{11}\)This data could be accessed on-line at http://www.dnp.gov.co/
interest rate. These were taken from GRECO(2000), the International Financial Statistics and Global Financial Data.

Insert Fig III.1 here

Because the benchmark model, $M_0$, as well as the other alternative models, $M_1 - M_3$ are driven by two exogenous and independent driving processes - a transitory technology shock and an additional real driving force-, to avoid stochastic singularity we can only use two time series to estimate them. Thus, as our baseline estimation we choose to use the time series of consumption and trade balance shares: $X = \{C/Y; TBY\}$, because of their rich informational content emphasized by Aguiar and Gopinath (2007). We will, nonetheless, test the robustness of our results using other variables as observables.

3.3 Baseline Calibration a Prior Selection

As mentioned above because our final goal is to compare the performance of the benchmark model against the alternatives considered, to be consistent in the comparison we chose to fix the "deep parameters" depicting preferences and technology across all models.

The models share eight "deep parameters" in common, \{\(r^*\), \(\beta\), \(\gamma\), \(\omega\), \(\delta\), \(d\), \(\psi\), \(\phi\)\}, determining the long run level of interest rate, the discount factor, the elasticities of labor supply and the consumption through time, the depreciation rate of capital and the adjustment cost and premium functions. We calibrate \(r^*\) from the long-run time series available. Taking the average of the two active real interest rates in Colombia reported by GRECO (2004) from 1925-2000, yields a yearly value of 0.06679. Compared to the long run average of the US real interest rate reported also by GRECO, and equal to 0.0487, this figure shows a long-run yearly premium of roughly 2%. In steady state, \(r^*\) pins down the discount factor \(\beta\) except for the benchmark model with balanced growth where we calibrate the long run aggregate growth, \(\gamma X\), from the time series of output per capita, which, using the expanded dataset for Colombia between 1925-2005 is equal to 1.0181 .

The parameter \(\gamma\) governing the intertemporal elasticity of substitution, \(\frac{1}{\gamma}\), has been traditionally calibrated between 1 and 2. As noted by Mendoza (1991), however, point estimates are controversial, specially in the case of developing countries given the lack of reliable data. While Prescott (1986) argued that \(\gamma\)
is not likely to be greater than 1, for the case of developing countries different studies have found it to be higher: 1.72 (Ogaki, et.al., 1996), 2.61 (Ostry and Reinhart, 1992) and even 5 for the case of Argentina (Reinhart and Vegh, 1995). Thus, indicating the presence of relatively interest-inelastic consumption growth rates. We will use a value of 2, following Aguiar and Gopinath (2007a). The parameter $\omega$, governing the elasticity of labor supply, $\frac{1}{\omega - 1}$, is perhaps the hardest one to calibrate for developing countries given the virtual inexistence of systematic labor market databases. In developed countries studies, this parameter has been set to match the variability of hours measured in micro-level studies (Mendoza, 1991). In these studies a point estimate used, for example, has been $\omega = 1.455$ implying an elasticity of 2.2. While for the Colombian case, no study that we know of has tried to calibrate or estimate this elasticity, studies for other developing countries have set a lower elasticity, perhaps motivated by the higher degree of labor market imperfections observed in these economies. For Argentina, Neumeyer and Perri (2002) set the elasticity at 1.51 and, in their calibration of the Mexican economy, Aguiar and Gopinath (2004) set it at 1.66. We arbitrarily follow the last study and set $\omega = 1.6$.

The lack of reliable data on the aggregate capital stock unable us to get systematic values for capital depreciation and shares. We therefore use a yearly value of $\delta = 0.1$, which is somewhat standard in both the literature on developed and developing countries (see Mendoza 1991, 1995, respectively). Following Schmidt-Grohe and Uribe (2003) we calibrate the steady-state level of debt, $d$, so as to match the long-run trade balance-to-income ratio, $tby$, equal to 2.8% using our yearly dataset. We follow the same authors in calibrating $\psi = 0.001$, measuring the sensitivity of the country interest-rate premium to deviations of external debt from trend$^{12}$.

The value of the adjustment-cost parameter, $\phi$, has been traditionally calibrated so as to moderate the variability of capital accumulation in order to match the standard deviation of investment (Mendoza, 1991). In this case, that strategy is problematic as the volatility of investment in developing countries is not moderate. Perhaps for this reason, other studies focusing on developing countries have used another approach by estimating this parameter. Aguiar and

---

$^{12}$To be concrete, in the case of the Canadian economy studied by Schmidt-Grohe and Uribe (2003), these authors suggest a criterion for calibrating $\psi$ as to match the sample volatility of current account-to-GDP. However, as it was documented in the Introduction, a salient characteristic of developing countries is the high volatility of this variable, which cannot be properly matched via a calibration of this parameter. In their study of the Argentinian and Mexican economies, Garcia-Cicco et.al. (2007) and Aguiar and Gopinath (2004), respectively, calibrate this parameter to be $\psi = 0.001$, in the same way that we proceed.
Gopinath (2004) and Garcia-Cicco et.al (2007), for example, estimate it in the order of 3.79 and 0.90. But these estimates are hard to reconcile with values calibrated for developed countries where the observed volatility of investment is lower than in developing countries. Moreover, sensitivity analysis on the choice of this parameter value while generating the impulse responses of Fig. II.1 revealed that small variations of this parameter may have non trivial implications for the dynamics of the overall economy. We therefore choose to calibrate it a low and identical value on all models and for that purpose we use Mendoza (1991) which sets it at $\phi = 0.028$.

Labor share in Colombian data was measured by Hamann and Riascos (1998) to be 0.67 implying a value of $\alpha = 0.33$. Lastly, in the model with terms of trade $\vartheta$ is calibrated by using the long-run value of imports share which, from our yearly dataset is 0.12. A summary of the calibration is depicted in Table 5.

Insert Table III.1 Here

With respect to the parameters of the driving forces to be estimated, Table III.2 summarizes the prior distributions chosen. These priors are guided by several considerations that reflect beliefs about the value of the parameters coming from previous empirical studies or by economic theory. In addition, the degree of certainty about the predetermined beliefs are set by the tightness of the priors.

In selecting the priors for the benchmark model, in accordance to the previous discussion, the most relevant parameters are the degree of volatility of the transitory and permanent shocks, $\{\sigma_\gamma; \sigma_a\}$. As was mentioned in the Introduction, previous studies have set the ratio $\sigma_\gamma/\sigma_a$ to range from $3/2$, as in Garcia-Cicco et.al. (2007) to $4 - 5$ as in Aguiar and Gopinath (2007a). We choose a more conservative ratio of $4/3$ and use a rather loose priors. For that we use a Gamma function defined on $\mathbb{R}^+$ as is common in the literature. With respect to the AR coefficients in the two technology processes $\{\rho_\gamma; \rho_A\}$ the evidence is more ambiguous. While Aguiar and Gopinath (2007a) estimate them to be $\{0.001; 0.95\}$, Garcia-Cicco et.al. (2007) find estimates of $\{0.399; 0.006\}$. Given this conflicting previous evidence, we chose to set identical and very loose prior centered around 0.5 within the unit interval on both of them using the Beta function.

In model $M_1$ we chose the same priors for the parameters in the transitory
technology shock as we did for the priors in $M_0$. With respect to the parameters governing the government expenditure process, $\{\sigma_G; \rho_G; p\}$, we form our priors for the first two from Canova (2007) who estimated a similar process for the U.S. and found a significantly higher volatility in the government spending process with respect to the technology process, as well as a moderate AR(1) persistence coefficient of 0.4. Canova found, however, a negative $p$ implying a countercyclical process, but, as discussed previously, studies on developing countries have identified a strong procyclical expenditure, which we use as the basis for a positive prior on this parameter, as indicated in Table III.2.

In model $M_2$, again, we chose the same priors for the parameters in the transitory technology shock as we did for the priors in $M_0$ and $M_1$. With respect to the parameters governing the terms of trade process, $\{\sigma_q; \rho_q\}$, we use Mendoza (1995) as a source of information to form our beliefs. He calibrates the AR coefficient to be around 0.4 and a high volatility of 0.01 in developing countries. Given this and the significant volatility the terms of trade exhibit in the particular case of Colombia (see Fig.III.1) we set our priors on a higher volatility as well as a higher persistence of the process. With respect to the key coefficient capturing the correlation between the technology and terms of trade processes, $\psi_A$, we set a prior centered around −0.15, a calibrated value for developing countries by Mendoza (1995) but with a relatively loose normal distribution.

Lastly, in model $M_3$, we chose the same priors for the parameters in the transitory technology shock as we did for the all the other models. We use the estimated AR(1) process for the world interest rate by Uribe and Yue (2006) to guide us in forming the priors for the two parameters in the foreign interest rate $\{\sigma_{R^*}; \rho_{R^*}\}$ where a high persistence and low volatility are estimated.

4 RESULTS

In this section we present our baseline results using the quarterly dataset from 1977:Q1 to 2007:Q2. We use the time series of consumption and trade balance shares in the vector of observables, $X = \{C/Y; TBY\}$. We also chose to filter
the data by taking log differences to $C/Y$ and linear differences to $TBY$.

Insert Table IV.1 Here

Table IV.1 reports the prior and posterior means and 90% posterior probability intervals for the parameter estimates in the benchmark model as well as in the other three models. The posterior results were taken from a sample of 500,000 draws from the posterior distribution of each model using the RWMH algorithm\(^{13}\). Out of this sample 10% was burned and to reduce the degree of autocorrelation of the draws we took every 100\(^{th}\) draw leaving us with a sample of 4,500 draws. Convergence of the Markov chain was tested on each parameter by calculating Geweke’s separated mean test, whose statistics are reported in the last column of Table IV.1.

The most important result is given in the first two rows where, as can be seen, the estimated volatility of the permanent component is lower than that of the transitory component yielding a ratio $\sigma_\gamma/\sigma_a = 4/5$ even lower than the one observed by Garcia-Cicco et.al. and in sharp contrast to the predominant role given to the permanent component in Aguiar and Gopinath (2007a, 2007b). A closer look at this result is given in Figure IV.1 where the prior and posterior distributions for both parameters are plotted. As can be observed, the mode of the posterior distribution for $\sigma_a$ is located to the right of the one for $\sigma_\gamma$ despite the fact that the priors were set the other way around, indicating a large degree in which the data, not the prior choices, is driving the estimation. Nonetheless, this is not what happened for the estimation on the AR coefficients of the two shocks since the posterior pretty much reproduces the prior.

Insert Fig. IV.1 Here

In order to assess the performance of the benchmark model $M_0$ against the other alternatives, Table IV.2 presents the marginal densities of each model considered. A surprising result is that the benchmark model yields a consistently lower likelihood values given the data when compared to each of the other three alternatives, while model $M_1$ is the one with the best relative performance.

\(^{13}\)The first order approximation and solution of the models are all done by adapting the MATLAB routines provided by Stephanie Schmitt-Grohe and Martin Uribe. The RWMH algorithm was performed in MATLAB adapting the routines generously provided by John Landon-Lane. The codes used in this research as well as the entire dataset are available upon request.
To sum up, this section found two important results. First, a full-information based estimation of the neoclassic growth model augmented by growth shocks does not attribute the predominance of these shocks that Aguiar and Gopinath (2007a, 2007b) find. Second, the data does not seem to favor this model when it is compared to other alternatives where real driving forces other than transitory and permanent technology shocks come into play. The next section verifies the robustness of these two results.

5 ROBUSTNESS ANALYSIS

The robustness analysis performed in this section is done in four dimensions. First, we investigate whether the results of the previous section hold if the models are estimated on longer or shorter periods. Second, we use other macroeconomic time series in the vector of observables. Third, we modify the choice of priors to get a sense of how much they might potentially be driving the posteriors densities. Fourth, we run the baseline estimation with Hodrick-Prescott-filtered data to verify that the results are not driven by very high frequency fluctuations. Lastly, we conjecture a hypothesis as to why the baseline model fails to reproduce the data.

5.1 Alternative Dataset Range

As a first robustness check we take seriously the criticism raised by Garcia-Cico et.al. (2007) that in order to get a more accurate estimation of the relative weights of the growth component one should estimate the model with longer dataset. For that purpose we estimate the model on the yearly dataset covering the period 1925-2006. As an additional robustness check on our quarterly results and in order to estimate the model on a non spliced dataset (see Data Description section), we also run the model on our quarterly dataset period 1994:Q1-2007:Q2.

As can be observed in Figure V.1, the results still do not attribute the predominance of the growth shocks, implying a ratio $\sigma_\gamma/\sigma_n < 1$, regardless of
the sample period, and even lower in the yearly long sample. In addition to this, the model comparison results presented in Table V.1, show that any of the two alternative dataset does not favor the benchmark model when it is compared to other alternatives. Interestingly, while model $M_1$ continues to outperform the other models on the shorter quarterly dataset, it is model $M_2$ that fits the data better when the longer period is considered. This results goes in line with previous studies that have attributed a relevant role of term of trade shocks in accounting for the business cycle in Colombia (see Cardenas, 1992).

Insert Table V.1 Here

5.2 Alternative Observable Vector

As mentioned earlier, a limitation of full-information analysis is the problem of stochastic singularity which we avoided by using only two time series as our baseline estimation of consumption and trade balance shares: $X = \{C/Y; TBY\}$. We will now check the robustness of our baseline results using additional macroeconomic data on GDP, aggregate consumption, and investment. With these data, we form four pairs of time series, $\{C; TBY\}$, $\{C; Y\}$, $\{I; Y\}$, $\{I; C\}$, on which we run the models. Once more, all variables a rendered stationary by log-differencing them.

As can be observed from the results on model comparison presented in Table V.2, the model with interest rate perturbations outperforms the other models when investment data is used in the observables vector. More importantly, the second baseline result still holds as the benchmark model $M_0$ continues to be rejected by the data in favor of the other alternatives.

Insert Table V.2 Here

5.3 Alternative Priors

The first of the two baseline results, that the estimated volatility of the permanent component is lower than that of the transitory component, i.e. $\sigma_\gamma/\sigma_a = 4/5$, might potentially be driven by the prior choices. Indeed, the prior over this ratio was set at $4/3$, which might be too conservative compared to the value of
roughly 4-5 found by Aguiar and Gopinath (2007a). To investigate if the conservative choice of priors might be driving this result we set as prior a ratio of 4. Also, we set the prior of the AR coefficients in the two technology processes that these authors find. A summary of the new prior distributions is given in Table V.3.

The benchmark model is then re-estimated across the three dataset we have been used so far, using consumption and trade balance shares as observable time series: \( X = \{C/Y; TBY\} \). The results for the big quarterly dataset are reported in the right half of Table V.3 and the plot of the prior and posterior distributions for the two volatilities are given in Fig. V.2. As can be observed, the result that the estimated volatility of the permanent component is lower than that of the transitory component is pervasive. In addition, the model continues to be outperformed by the alternative models, as is presented in Table V.4.

5.4 Alternative Filtering Technique

Up to this point all non stationary data used as observables in \( X \) has been transformed by log-differences. This transformation is used in order to allow as much information on the low frequency components of the data as possible. However, it may also be including much noise from very high frequency data than what is commonly assumed at business cycle frequencies. If this is the case then it may be biasing the results in favor of a high transitory component. To evaluate this potential bias we run the baseline estimation on Hodrick-Prescott filtered data. The results of this robustness check are presented in table V.5 and, as can be observed, no differences emerge with respect to the baseline case.
5.5 Why does the Benchmark Model fail? A Conjecture

The analysis made in this section confirms the robustness of the finding that the neoclassic growth model augmented by growth shocks performs poorly when it is compared to other alternatives where real driving forces other than transitory and permanent technology shocks come into play. Which indicates that growth shocks are not receiving the predominance that the baseline model by Aguiar and Gopinath (2007) assigns. But why does this baseline model fail so badly?

Given that the alternative models we use are highly simplified and that we believe more elaborated models should be considered when modeling business cycles in developing economies, a thorough analysis of this question goes beyond the scope of this work. Nonetheless, we try to offer a conjecture based upon observing the time series on the two main driving forces embedded in the alternative models favored by the data. Fig. V.3 plots the annual data on the fiscal deficit share and the terms of trade index from 1925 in Colombia taken from Junguito and Rincon (2007) and GRECO (2004). As can be observed, a significant volatility of the terms of trade was present throughout the century until the mid 70s, while, simultaneously a progressive deterioration of the public sector deficit has been increasing ever since, mainly driven by increasing public spending (Junguito and Rincon, 2007). A bold conjecture is, therefore, that the business cycle in Colombia may have been influenced by these two dynamics. This influence, in turn, may have generated dynamics in the aggregate macroeconomic time series that are better captured by model 1, on a quarterly dataset, and by model 2 on a yearly and longer dataset. We leave the study of this conjecture for future research.

Insert Fig. V.3 Here

6 CONCLUDING REMARKS

There exists a consensus regarding the differences in the business cycle patterns in developing and developed economies. Where a consensus does not seem to be emerging is on the proper model that can account for these differences. While some studies argue that a standard RBC model, expanded to allow the presence of permanent shocks to the technology trend, is enough to properly model business cycles in developing economies, others present conflicting evidence based
on dataset covering longer periods and/or stress the role of other real driving forces.

We contribute to this debate by exploring the business cycle properties of a developing -and "tropical"- economy. Our approach is more ambitious in the sense that not only we test for the presence of permanent shocks to the technology trend but we also compare the performance across other modifications to the benchmark case by incorporating other potential real impulses. In addition to this, our test is based on full-information analysis, unlike previous studies.

Our results can be summarized in two parts. First, we don't find the growth shocks to be as relevant in explaining business cycles as other studies have found. Second, the data does not seem to favor a model driven solely by technology shocks when compared to other alternatives where real driving forces other than these impulses come into play. These two results are robust to modifications in the range and type of macroeconomic data used, as well as the choice of priors, and filtering techniques.

We are thus skeptical as to whether business cycles in developing economies can be modeled with a standard RBC model augmented solely by permanent shocks to trend. We hope that our findings help stimulate more research into more elaborated models of the business cycles that incorporate a variety of other real driving forces with the dynamics necessary to better fit the data we observe in these economies.

REFERENCES (incomplete)


Kydland and Zarazaga (1997) Is the business cycle of Argentina different, Dallas FED.


Talvi and Vegh (2005) Tax base variability and procyclical ...scal policy in developing countries, Journal of Development Economics Vol 78 1

Uribe, Martin (2007). Lectures in Open Economy Macroeconomics, Mimeo, Duke University


TABLES AND FIGURES
Table I.1: Moments in Emerging and Developed Markets

<table>
<thead>
<tr>
<th></th>
<th>Emerging Markets</th>
<th>Developed Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.d.(Y)</td>
<td>2.74</td>
<td>1.34</td>
</tr>
<tr>
<td>s.d.(C)/s.d.(Y)</td>
<td>1.45</td>
<td>0.94</td>
</tr>
<tr>
<td>s.d.(I)/s.d.(Y)</td>
<td>3.91</td>
<td>3.41</td>
</tr>
<tr>
<td>s.d.(TB/Y)</td>
<td>3.22</td>
<td>1.02</td>
</tr>
<tr>
<td>corr(TB/Y,Y)</td>
<td>-0.51</td>
<td>-0.17</td>
</tr>
<tr>
<td>corr(C,Y)</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td>corr(I,Y)</td>
<td>0.77</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Source: Aguiar and Gopinath (2007a). The pool of emerging markets does not include Colombia. The s.d.(X) and corr(X) stand for the standard deviation and correlation. Y stands for income, C for consumption, I for investment and TB for the trade balance.

Table I.2: Moments in Colombian Economy

<table>
<thead>
<tr>
<th></th>
<th>Quarterly Data:</th>
<th>Yearly Data:</th>
<th>Quarterly Data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.d.(Y)</td>
<td>1.63</td>
<td>2.60</td>
<td>1.78</td>
</tr>
<tr>
<td>s.d.(C)/s.d.(Y)</td>
<td>1.02</td>
<td>2.02</td>
<td>1.07</td>
</tr>
<tr>
<td>s.d.(Cnd)/s.d.(Y)</td>
<td>n.a</td>
<td>n.a</td>
<td>0.69</td>
</tr>
<tr>
<td>s.d.(I)/s.d.(Y)</td>
<td>8.19</td>
<td>6.04</td>
<td>6.38</td>
</tr>
<tr>
<td>s.d.(Id)/s.d.(Y)</td>
<td>n.a</td>
<td>n.a</td>
<td>5.04</td>
</tr>
<tr>
<td>s.d.(TB/Y)</td>
<td>2.59</td>
<td>2.94</td>
<td>1.59</td>
</tr>
<tr>
<td>corr(TB/Y,Y)</td>
<td>-0.36</td>
<td>-0.53</td>
<td>-0.77</td>
</tr>
<tr>
<td>corr(C,Y)</td>
<td>0.80</td>
<td>0.61</td>
<td>0.91</td>
</tr>
<tr>
<td>corr(I,Y)</td>
<td>0.52</td>
<td>0.58</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Source: See Data Appendix. "Cnd" and "Id" stand for Consumption of non-durable goods and Investment plus durable goods consumption.
### Table III.1: Benchmark Calibrated Parameters (Yearly Freq.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^*$</td>
<td>0.067</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.000</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.600</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.100</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.028</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.028</td>
</tr>
<tr>
<td>$\gamma_X$</td>
<td>1.018</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.120</td>
</tr>
</tbody>
</table>

### Table III.2: Priors

<table>
<thead>
<tr>
<th>Model</th>
<th>Name</th>
<th>Domain</th>
<th>Density</th>
<th>Para(1)</th>
<th>Para(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_0$</td>
<td>$\sigma_A$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.5171</td>
<td>$5.8020e-004$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\gamma$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.7116</td>
<td>$5.6215e-004$</td>
</tr>
<tr>
<td></td>
<td>$\rho_A$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td></td>
<td>$\rho_\gamma$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$\sigma_A$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.5171</td>
<td>$5.8020e-004$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_G$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>15.2027</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>$\rho_A$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td></td>
<td>$\rho_G$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>7.4216</td>
<td>18.7293</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>3.4563</td>
<td>4.0919</td>
</tr>
<tr>
<td>$M_2$</td>
<td>$\sigma_A$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.5171</td>
<td>$5.8020e-004$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_q$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>23.6692</td>
<td>0.0026</td>
</tr>
<tr>
<td></td>
<td>$\rho_A$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td></td>
<td>$\rho_q$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>31.2913</td>
<td>17.3506</td>
</tr>
<tr>
<td></td>
<td>$\psi_A$</td>
<td>$\mathbb{R}$</td>
<td>Normal</td>
<td>$-0.1560$</td>
<td>0.0500</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$\sigma_A$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.5171</td>
<td>$5.8020e-004$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{R^*}$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>5.6683</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>$\rho_A$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td></td>
<td>$\rho_{R^*}$</td>
<td>[0, 1)</td>
<td>Beta</td>
<td>50.7791</td>
<td>6.9310</td>
</tr>
</tbody>
</table>
### Table IV.2: Bayes Model Comparison

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Model (0)</th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Densities</td>
<td>324.67</td>
<td>616.87</td>
<td>513.87</td>
<td>531.61</td>
</tr>
<tr>
<td>(Log of) Bayes Factor</td>
<td>0.00</td>
<td>292.20</td>
<td>189.19</td>
<td>206.93</td>
</tr>
</tbody>
</table>

### Table IV.1: Prior & Posterior Distributions

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_A$</td>
<td>0.0003 [0.00000 0.0011]</td>
<td>0.0005 [0.0004 0.0007]</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>0.0004 [0.0000 0.0014]</td>
<td>0.0004 [0.0004 0.0005]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.5000 [0.1341 0.8655]</td>
<td>0.5069 [0.0458 0.9492]</td>
</tr>
<tr>
<td>$\rho_\gamma$</td>
<td>0.5000 [0.1341 0.8655]</td>
<td>0.5009 [0.0468 0.9497]</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0003 [0.0000 0.0011]</td>
<td>0.0005 [0.0004 0.0006]</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.0290 [0.0179 0.0421]</td>
<td>0.0453 [0.0379 0.0538]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.5000 [0.1341 0.8655]</td>
<td>0.2263 [0.1894 0.2616]</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.2835 [0.1533 0.4329]</td>
<td>0.1635 [0.0902 0.2457]</td>
</tr>
<tr>
<td>$\rho_\gamma$</td>
<td>0.4569 [0.1795 0.7475]</td>
<td>0.4443 [0.1421 0.7704]</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0003 [0.0000 0.0011]</td>
<td>0.0005 [0.0002 0.0011]</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>0.0613 [0.0424 0.0840]</td>
<td>0.0006 [0.0005 0.0008]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.5000 [0.1341 0.8655]</td>
<td>0.2275 [0.1803 0.2710]</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>0.6442 [0.5291 0.7518]</td>
<td>0.5541 [0.4466 0.6673]</td>
</tr>
<tr>
<td>$\psi_A$</td>
<td>-0.1567 [-0.238 -0.075]</td>
<td>-0.1525 [-0.2295 -0.0748]</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0003 [0.0000 0.0011]</td>
<td>0.0004 [0.0003 0.0005]</td>
</tr>
<tr>
<td>$\sigma_R^*$</td>
<td>0.0091 [0.0039 0.0158]</td>
<td>2.2e-009 [1.7e-009 8e-009]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.5000 [0.1341 0.8655]</td>
<td>0.0098 [0.0055 0.0140]</td>
</tr>
<tr>
<td>$\rho_R^*$</td>
<td>0.8805 [0.8045 0.9417]</td>
<td>0.9439 [0.9051 0.9781]</td>
</tr>
</tbody>
</table>

### Table V.3: Alternative Priors for $M_0$ & Posterior Distributions

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_A$</td>
<td>0.0003 [0.0000 0.0016]</td>
<td>0.0006 [0.0004 0.0007]</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>0.0014 [0.0000 0.0038]</td>
<td>0.0004 [0.0004 0.0005]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.9500 [0.9186 0.9745]</td>
<td>0.9557 [0.9263 0.9785]</td>
</tr>
<tr>
<td>$\rho_\gamma$</td>
<td>0.0100 [0.0047 0.0169]</td>
<td>0.0086 [0.0037 0.0151]</td>
</tr>
</tbody>
</table>
### Table V.1: Bayes Model Comparison. Alternative Dataset Range

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>72.63</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>74.46</td>
<td>1.83</td>
</tr>
<tr>
<td>Model (2)</td>
<td>228.68</td>
<td>156.05</td>
</tr>
<tr>
<td>Model (3)</td>
<td>185.78</td>
<td>113.15</td>
</tr>
</tbody>
</table>

X = \{ C/Y ; TBY \} ; Yearly Data: 1925-2006

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>174.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>312.81</td>
<td>138.78</td>
</tr>
<tr>
<td>Model (2)</td>
<td>211.00</td>
<td>36.97</td>
</tr>
<tr>
<td>Model (3)</td>
<td>203.88</td>
<td>29.86</td>
</tr>
</tbody>
</table>

X = \{ C/Y ; TBY \} ; Quarterly Data: 1994:I-2007:II

### Table V.2: Bayes Model Comparison. Alternative Dataset

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>216.48</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>617.65</td>
<td>401.18</td>
</tr>
<tr>
<td>Model (2)</td>
<td>496.69</td>
<td>280.21</td>
</tr>
<tr>
<td>Model (3)</td>
<td>524.81</td>
<td>308.33</td>
</tr>
</tbody>
</table>

X = \{ C ; TBY \} ; Quarterly Data: 1977:I-2007:II

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>555.87</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>581.83</td>
<td>25.96</td>
</tr>
<tr>
<td>Model (2)</td>
<td>522.25</td>
<td>-33.62</td>
</tr>
<tr>
<td>Model (3)</td>
<td>511.73</td>
<td>-44.14</td>
</tr>
</tbody>
</table>

X = \{ C ; Y \} ; Quarterly Data: 1977:I-2007:II

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>-565.40</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>-53.85</td>
<td>511.55</td>
</tr>
<tr>
<td>Model (2)</td>
<td>263.57</td>
<td>828.97</td>
</tr>
<tr>
<td>Model (3)</td>
<td>325.16</td>
<td>890.56</td>
</tr>
</tbody>
</table>

X = \{ I ; Y \} ; Quarterly Data: 1977:I-2007:II

<table>
<thead>
<tr>
<th>Model</th>
<th>Marginal Densities</th>
<th>(Log of) Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (0)</td>
<td>54.72</td>
<td>0.00</td>
</tr>
<tr>
<td>Model (1)</td>
<td>-150.15</td>
<td>-204.87</td>
</tr>
<tr>
<td>Model (2)</td>
<td>287.23</td>
<td>232.51</td>
</tr>
<tr>
<td>Model (3)</td>
<td>321.98</td>
<td>267.27</td>
</tr>
</tbody>
</table>

X = \{ I ; C \} ; Quarterly Data: 1977:I-2007:II
### Table V.4: Bayes Model Comparison. Alternative Priors

<table>
<thead>
<tr>
<th>Model(_{(0)})</th>
<th>Model(_{(1)})</th>
<th>Model(_{(2)})</th>
<th>Model(_{(3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = { C/Y; TBY } ; \text{Quarterly Data: 1977:I-2007:II})</td>
<td>Marginal Densities</td>
<td>323.65</td>
<td>616.87</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Yearly Data: 1925-2006})</td>
<td>(Log of) Bayes Factor</td>
<td>0.00</td>
<td>293.22</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Quarterly Data: 1994:I-2007:II})</td>
<td>Marginal Densities</td>
<td>83.49</td>
<td>74.46</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Yearly Data: 1925-2006})</td>
<td>(Log of) Bayes Factor</td>
<td>0.00</td>
<td>-9.03</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Quarterly Data: 1977:I-2007:II})</td>
<td>Marginal Densities</td>
<td>172.61</td>
<td>312.81</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Yearly Data: 1925-2006})</td>
<td>(Log of) Bayes Factor</td>
<td>0.00</td>
<td>140.20</td>
</tr>
</tbody>
</table>

### Table V.5: Bayes Model Comparison. HP-Filtered Data

<table>
<thead>
<tr>
<th>Model(_{(0)})</th>
<th>Model(_{(1)})</th>
<th>Model(_{(2)})</th>
<th>Model(_{(3)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = { C/Y; TBY } ; \text{Quarterly Data: 1977:I-2007:II})</td>
<td>Marginal Densities</td>
<td>439.41</td>
<td>672.71</td>
</tr>
<tr>
<td>(X = { C/Y; TBY } ; \text{Yearly Data: 1925-2006})</td>
<td>(Log of) Bayes Factor</td>
<td>0.00</td>
<td>233.30</td>
</tr>
</tbody>
</table>
Fig. II.1: Impulse Response Functions

Response of TB/Y

Response of C/Y

Response of I/Y

Model M(0)

-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
1 2 3 4 5 6 7 8 9 10

Transitory Shock

Permanent Shock

Model M(0)

-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
1 2 3 4 5 6 7 8 9 10

Model M(3)

-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
1 2 3 4 5 6 7 8 9 10

Tech. Shock

Tech & R Shocks

Model M(3)

-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
1 2 3 4 5 6 7 8 9 10

Model M(2)

-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
1
2
3
4
5
6
7
8
9
10

Tech. Shock

ToT Shock

Model M(2)

-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
1 2 3 4 5 6 7 8 9 10

Model M(1)

-0.5
0
0.5
1
1.5
2
1 2 3 4 5 6 7 8 9 10

Tech Shock

Tech & G Shocks

Model M(1)

-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
1 2 3 4 5 6 7 8 9 10

35
Figure III.1: Yearly and Quarterly Colombian Data

[Diagram showing yearly and quarterly data with various metrics and time periods]
Figure IV.1 Quarterly Prior and Posterior Densities (77:I-07:II):

Transitory Shock

Permanent Shock
Figure V.1: Yearly and Quarterly Prior and Posterior Densities:
Figure V.2: Prior and Posterior Densities with alternative priors:

Transitory Shock

Permanent Shock
Figure V.3: Terms of trade and Public Deficit in Colombia, 1925-2006: