Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default*

Pablo D'Erasmo
Federal Reserve Bank of Philadelphia

Enrique G. Mendoza
University of Pennsylvania, NBER and PIER

July 11, 2015

Abstract

Europe's debt crisis resembles historical episodes of outright default on domestic public debt about which little research exists. This paper proposes a theory of domestic sovereign default based on distributional incentives affecting the welfare of risk-averse debt- and non-debt holders. A utilitarian government cannot sustain debt if default is costless. If default is costly, debt with default risk is sustainable, and debt falls as concentration of debt ownership rises. A government favoring bond holders can also sustain debt, with debt rising as ownership becomes more concentrated. These results are robust to adding foreign investors, redistributive taxes, or a second asset.

Keywords: Public debt, sovereign default, European debt crisis.
JEL Classifications: E6, E44, F34, H63

*We acknowledge the generous support of the National Science Foundation under awards 1325122 and 1324740. Comments by Jesse Schreger, Fabrizio Perri, Vincenzo Quadrini and by participants at presentations in Columbia University, the 2014 Winter Meetings of the Econometric Society, and the NBER Conference on Sovereign Debt and Financial Crises are also gratefully acknowledged. We thank Jingting Fan for excellent research assistance. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

1 Introduction

The seminal study by Reinhart and Rogoff (2011) identified 68 episodes in which governments defaulted outright (i.e. by means other than inflation) on their domestic creditors in a cross-country database going back to 1750. These domestic defaults occurred via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspensions of payments. Reinhart and Rogoff also documented that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about $2/3^{\text{rds}}$ on average), and that domestic defaults were associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses and full-blown economic crises. Despite of these striking features, they also found that domestic sovereign default is a "forgotten history" that remains largely unexplored in economic research.

The ongoing European debt crisis also highlights the importance of studying domestic sovereign default. In particular, four features of this crisis make it more akin to a domestic default than to the typical external default that dominates the literature on public debt default. First, countries in the Eurozone are highly integrated, with the majority of their public debt denominated in their common currency and held by European residents. Hence, from an European standpoint, default by one or more Eurozone governments means a suspension of payments to "domestic" agents, instead of external creditors. Second, domestic public debt-GDP ratios are high in the Eurozone in general, and very large in the countries at the epicenter of the crisis (Greece, Ireland, Italy, Spain and Portugal). Third, the Eurozone's common currency and common central bank rule out the possibility of individual governments resorting to inflation as a means to lighten their debt burden without an outright default. Fourth, and perhaps most important from the standpoint of the theory proposed in this paper, European-wide institutions such as the ECB and the European Commission are weighting the interests of both creditors and debtors in assessing the pros and cons of sovereign defaults by individual countries, and creditors and debtors are aware of these institutions' concern and of their key role in influencing expectations and default risk.[1] Hall and Sargent (2014) document a similar situation in the process by which the U.S. government handled the management of its debt in the aftermath of the Revolutionary War.

Table 1 shows that the Eurozone fiscal crisis has been characterized by rapid increases in public debt ratios and sovereign spreads that coincided with rising government expenditure ratios. The Table also shows that debt ownership, as proxied by Gini coefficients of wealth distributions, is unevenly distributed in the seven countries listed, with mean and median Gini

[1]The analogy with a domestic default is imperfect, however, because the Eurozone is not a single country, and in particular there is no fiscal entity with tax and debt-issuance powers over all the members.

coefficients of around 2/3rds.[2]

Table 1: Euro Area: Key Fiscal Statistics and Wealth Inequality

	Gov. Debt		Gov. Exp.		Spreads		Gini
Moment (%)	Avg.	2011	Avg.	"crisis peak"	Avg.	"crisis peak"	Wealth
France	34.87	62.72	23.40	24.90	0.08	1.04	0.73
Germany	33.34	52.16	18.80	20.00	-	-	0.67
Greece	84.25	133.09	18.40	23.60	0.37	21.00	0.65
Ireland	14.07	64.97	16.10	20.50	0.11	6.99	0.58
Italy	95.46	100.22	19.40	21.40	0.27	3.99	0.61
Portugal	35.21	75.83	20.00	22.10	0.20	9.05	0.67
Spain	39.97	45.60	17.60	21.40	0.13	4.35	0.57
Avg.	48.17	76.37	19.10	21.99	0.22	7.74	0.64
Median	35.21	64.97	18.80	21.40	0.17	5.67	0.65

Note: Author's calculations based on OECD Statistics, Eurostat, ECSB and Davies, Sandstrm, Shorrocks, and Wolff (2009). "Gov. Debt" refers to Total General Government Net Financial Liabilities (avg 90-07); "Gov. Exp." corresponds to government purchases in National Accounts (avg 00-07); "Sov Spreads" correspond to the difference between interest rates of the given country and Germany for bonds of similar maturity (avg 00-07). For a given country i, they are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$. "Crisis Peak" refers to the maximum value observed during 2008-2012 using data from Eurostat. "Gini Wealth" are Gini wealth coefficients for 2000 from Davies, Sandstrm, Shorrocks, and Wolff (2009) Appendix V.

Taken together, the history of domestic defaults and the risk of similar defaults in Europe pose two important questions: What explains the existence of domestic debt ratios exposed to default risk? And, can the concentration of the ownership of government debt be a determinant of domestic debt exposed to default risk?

This paper aims to answer these questions by proposing a framework for explaining domestic sovereign defaults driven by distributional incentives. This framework is motivated by the key fact that a domestic default entails substantial redistribution across domestic agents, with all of these agents, including government debt holders, entering in the payoff function of the sovereign. This is in sharp contrast with what standard models of external sovereign default assume, particularly those based on the classic work of Eaton and Gersovitz (1981).

We propose a tractable two-period model with heterogeneous agents and non-insurable aggregate risk in which domestic default can be optimal for a government responding to distributional incentives. A fraction γ of agents are low-wealth (L) agents who do not hold government debt,

[2] In Section A.1 of the Appendix, we present a more systematic analysis of the link between debt and inequality and show that government debt is increasing in inequality when inequality is low but decreasing for high levels of inequality.

and a fraction $1-\gamma$ are high-wealth (H) agents who hold the debt. The government finances the gap between exogenous stochastic expenditures and endogenous taxes by issuing non-state-contingent debt, retaining the option to default. In our benchmark case, the government is utilitarian,, so the social welfare function assigns the weights γ and $1-\gamma$ to the welfare of L and H agents respectively.

If the government is utilitarian and default is costless, the model cannot support an equilibrium with debt. This is because for any given level of debt that could have been issued in the first period, the government always attains the second period's socially efficient levels of consumption allocations and redistribution by choosing to default, and if default in period two is certain the debt market collapses in the first period. An equilibrium with debt under a utilitarian government can exist if default entails an exogenous cost in terms of disposable income. When default is costly, repayment becomes optimal if the amount of period-two consumption dispersion that the competitive equilibrium with repayment supports yields higher welfare than the default equilibrium net of default cost.

Alternatively, we show that an equilibrium with debt can be supported if the government's payoff function displays a "political" bias in favor of bond holders, even if default is costless. In this case, the government's weight on H-type agents is higher than the actual fraction of these agents in the distribution of bond holdings. In this extension, the debt is an *increasing* function of the concentration of debt ownership, instead of *decreasing* as in the utilitarian case. This is because incentives to default get weaker as the government's weight on L-type agents falls increasingly below γ. The model with political bias also yields the interesting result that agents that do not hold public debt may prefer a government that weighs bond holders more than a utilitarian government. This is because the government with political bias has weaker default incentives, and can thus sustain higher debt at lower default probabilities, which relaxes a liquidity constraint affecting agents who do not hold public debt by improving tax smoothing.

We also explore other three important extensions of the model to show that the main result of the benchmark model, namely the existence of equilibria with domestic public debt exposed to default risk, is robust. We examine extensions opening the economy so that a portion of the debt is held by foreign investors, introducing taxation as another instrument for redistributive policy, and adding a second asset as an alternative vehicle for saving.

This work is related to various strands of the large literature on public debt. First, studies on public debt as a self-insurance mechanism and a vehicle that alters consumption dispersion in heterogeneous agents models without default, such as Aiyagari and McGrattan (1998), Golosov and Sargent (2012), Azzimonti, de Francisco, and Quadrini (2014), Floden (2001) and

Heathcote (2005).[3]

A second strand is the literature on external sovereign default in the line of the Eaton and Gersovitz (1981) model (e.g. Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2012), Yue (2010)).[4] Also in this literature, and closely related, Aguiar and Amador (2013) analyze the interaction between public debt, taxes and default risk and Lorenzoni and Werning (2013) study the dynamics of debt and interest rates in a model where default is driven by insolvency and debt issuance follows a fiscal rule.

A third strand is the literature on political economy and sovereign default, which also focuses mostly on external default (e.g. Amador (2003), Dixit and Londregan (2000), D'Erasmo (2011), Guembel and Sussman (2009), Hatchondo, Martinez, and Sapriza (2009) and Tabellini (1991)). A few studies like those of Alesina and Tabellini (1990) and Aghion and Bolton (1990) do focus on political economy aspects of government debt in a closed economy, including default, and Aguiar, Amador, Farhi, and Gopinath (2013) examine optimal policy in a monetary union subject to self-fulfilling debt crises.

A fourth important strand of the literature focuses on the consequences of default on domestic agents, the role of secondary markets, discriminatory v. nondiscriminatory default and the role of domestic debt in providing liquidity (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), Mengus (2014) and Di Casola and Sichlimiris (2014)).[5] As in most of these studies, default in our setup is non-discriminatory, because the government cannot discriminate across any of its creditors when it defaults. Our analysis differs in that default is driven by distributional incentives.[6]

Finally, there is also a newer literature that is closer to our work in that it studies the trade-offs between distributional incentives to default on domestic debt and the use of debt in infinite-horizon models with heterogeneous agents (see, in particular, D'Erasmo and Mendoza (2014) and Dovis, Golosov, and Shourideh (2014)). In D'Erasmo and Mendoza, debt is determined by

[3]A related literature initiated by Aiyagari, Marcet, Sargent, and Seppala (2002) studies optimal taxation and public debt dynamics with aggregate uncertainty and incomplete markets, but in a representative-agent environment. Pouzo and Presno (2014) extended this framework to incorporate default and renegotiation.

[4]See Panizza, Sturzenegger, and Zettelmeyer (2009), Aguiar and Amador (2014), and Tomz and Wright (2012) for recent reviews of the sovereign debt literature. Some studies in this area have examined models that include tax and expenditure policies, as well as settings with foreign and domestic lenders, but always maintaining the representative agent assumption (e.g. Cuadra, Sanchez, and Sapriza (2010)), Vasishtha (2010) and more recently Dias, Richmond, and Wright (2012) have examined the benefits of debt relief from the perspective of a *global* social planner with utilitarian preferences.

[5]Motivated by the recent financial crisis and extending the theoretical work of Gennaioli, Martin, and Rossi (2014), a set of recent papers focuses on the interaction between sovereign debt and domestic financial institutions, such as Sosa-Padilla (2012), Bocola (2014), Boz, D'Erasmo, and Durdu (2014) and Perez (2015).

[6]Andreasen, Sandleris, and der Ghote (2011), Ferriere (2014) and Jeon and Kabukcuoglu (2014) study environments in which domestic income heterogeneity plays a central role in the determination of external defaults.

a fiscal rule, while in this paper we model public debt as an optimal choice and derive analytical expressions that characterize equilibrium prices and the solution of the government's problem. Our work differs from Dovis et al. in that they assume complete domestic asset markets, so their analysis abstracts from the role of public debt in providing social insurance, while the non-state-contingent nature of public debt plays a central role in the distributional incentives we examine here and in the endogenous default costs studied in D'Erasmo and Mendoza (2014). In addition, Dovis et al. focus on the solution to a Ramsey problem that supports equilibria in which default is not observed along the equilibrium path, while in our work default is an equilibrium outcome.[7]

2 Model Environment

Consider a two-period economy inhabited by a continuum of agents with aggregate unit measure. Agents differ in their initial wealth position, which is characterized by their holdings of government debt at the beginning of the first period. The government is represented by a social planner with a utilitarian payoff who issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents.

2.1 Household Preferences & Budget Constraints

All agents have the same preferences, which are given by:

$$u(c_0) + \beta E[u(c_1)], \qquad u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where $\beta \in (0,1)$ is the discount factor and c_t for $t = 0, 1$ is individual consumption. The utility function $u(\cdot)$ takes the standard CRRA form.

All agents receive a non-stochastic endowment y each period and pay lump-sum taxes τ_t, which are uniform across agents. Taxes and newly issued government debt are used to pay for government consumption g_t and repayment of outstanding government debt. The initial supply of outstanding government bonds at $t = 0$ is denoted B_0. Given B_0, the initial wealth distribution is defined by a fraction γ of households who are the L-type individuals with initial bond holdings b_0^L, and a fraction $(1-\gamma)$ who are the H-types and hold $b_0^H > b_0^L$. These initial bond holdings satisfy market clearing: $\gamma b_0^L + (1-\gamma) b_0^H = B_0$, which given $b_0^H > b_0^L$ implies that $b_0^H > B_0$ and $b_0^L < B_0$.

[7]See also Golosov and Sargent (2012) who study debt dynamics without default risk in a similar environment.

The budget constraints of the two types of households in the first period are given by:

$$c_0^i + q_0 b_1^i = y + b_0^i - \tau_0 \quad \text{for } i = L, H. \tag{1}$$

Agents collect the payout on their initial holdings of government debt (b_0^i), receive endowment income y, and pay lump-sum taxes τ_0. This net-of-tax resources are used to pay for consumption and purchases of new government bonds b_1^i at price q_0. Agents are not allowed to take short positions in government bonds, which is equivalent to imposing the no-borrowing condition often used in heterogeneous-agents models with incomplete markets: $b_1^i \geq 0$.

The budget constraints in the second period differ depending on whether the government defaults or not. If the government repays, the budget constraints take the standard form:

$$c_1^i = y + b_1^i - \tau_1 \quad \text{for } i = L, H. \tag{2}$$

If the government defaults, there is no repayment on the outstanding debt, and the agents' budget constraints are:

$$c_1^i = (1 - \phi(g_1))y - \tau_1 \quad \text{for } i = L, H. \tag{3}$$

As is standard in the sovereign debt literature, we can allow for default to impose an exogenous cost that reduces income by a fraction ϕ. This cost is usually modeled as a function of the realization of a stochastic endowment income, but since income is constant in this setup, we model it as a function of the realization of government expenditures in the second period g_1. In particular, the cost is a non-increasing, step-wise function: $\phi(g_1) \geq 0$, with $\phi'(g_1) \leq 0$ for $g_1 \leq \overline{g}_1$, $\phi'(g_1) = 0$ otherwise, and $\phi''(g_1) = 0$. Hence, $\overline{g}_1$ is a threshold high value of g_1 above which the marginal cost of default is zero.[8]

2.2 Government

At the beginning of $t = 0$, the government has outstanding debt B_0 and can issue one-period, non-state contingent discount bonds $B_1 \in \mathcal{B} \equiv [0, \infty)$ at the price $q_0 \geq 0$. Each period, the government collects lump-sum revenues τ_t and pays for g_t. Since g_0 is known at the beginning of the first period, the relevant uncertainty with respect to government expenditures is for g_1,

[8]This formulation is analogous to the step-wise default cost as a function of income proposed by Arellano (2008) and now widely used in the external default literature, and it also captures the idea of asymmetric costs of tax collection (see Barro (1979) and Calvo (1988)). Note, however, that for the model to support equilibria with debt under a utilitarian government all we need is $\phi(g_1) > 0$. The additional structure is useful for the quantitative analysis and for making it easier to compare the model with the standard external default models. In external default models, the non-linear cost makes default more costly in "good" states, which alters default incentives to make default more frequent in "bad" states, and it also contributes to support higher debt levels.

which is characterized by a well-defined probability distribution function with mean μ_g. We do not restrict the sign of τ_t, so $\tau_t < 0$ represents lump-sum transfers.

At equilibrium, the price of debt issued in the first period must be such that the government bond market clears:

$$B_t = \gamma b_t^L + (1-\gamma) b_t^H \quad \text{for } t = 0, 1. \tag{4}$$

This condition is satisfied by construction in period 0. In period 1, however, the price moves endogenously to clear the market.

The government has the option to default at $t = 1$. The default decision is denoted by $d_1 \in \{0, 1\}$ where $d_1 = 0$ implies repayment. The government evaluates the values of repayment and default as a benevolent planner with a social welfare function. In the rest of this Section we focus on the case of a standard utilitarian social welfare function: $\gamma u(c_1^L) + (1-\gamma)u(c_1^H)$. The government, however, cannot discriminate across the two types of agents when setting taxation, debt and default policies.

At $t = 0$, the government budget constraint is

$$\tau_0 = g_0 + B_0 - q_0 B_1. \tag{5}$$

The level of taxes in period 1 is determined after the default decision. If the government repays, taxes are set to satisfy the following government budget constraint:

$$\tau_1^{d_1=0} = g_1 + B_1. \tag{6}$$

Notice that, since this is a two-period model, equilibrium requires that there are no outstanding assets at the end of period 1 (i.e. $b_2^i = B_2 = 0$ and $q_1 = 0$). If the government defaults, taxes are simply set to pay for government purchases:

$$\tau_1^{d_1=1} = g_1. \tag{7}$$

3 Equilibrium

The analysis of the model's equilibrium proceeds in three stages. First, we characterize the households' optimal savings problem and determine their payoff (or value) functions, taking as given the government debt, taxes and default decision. Second, we study how optimal government taxes and the default decision are determined. Third, we examine the optimal choice of debt issuance that internalizes the outcomes of the first two stages.

3.1 Households' Problem

Given B_1 and γ, a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$
\begin{aligned}
v^i(B_1,\gamma) \;=\; & \max_{b_1^i} \Big\{ u(y + b_0^i - q_0(B_1,\gamma) b_1^i - \tau_0) + \\
& \beta E_{g_1}\Big[(1 - d_1(B_1,g_1,\gamma)) u(y + b_1^i - \tau_1^{d_1=0}) + d_1(B_1,g_1,\gamma) u(y(1-\phi(g_1)) - \tau_1^{d_1=1}) \Big] \Big\},
\end{aligned} \tag{8}
$$

subject to $b_1^i \geq 0$. The term $E_{g_1}[.]$ represents the expected payoff across the repayment and default states in period 1.[9]

The first-order condition, evaluated at the equilibrium level of taxes, yields the following Euler equation:

$$
u'(c_0^i) \geq \beta(1/q_0(B_1,\gamma)) E_{g_1}\left[u'(y - g_1 + b_1^i - B_1)(1 - d_1(B_1,g_1,\gamma)) \right], \;=\; \text{if} \;\; b_1^i > 0 \tag{9}
$$

In states in which, given (B_1, γ), the value of g_1 is such that the government chooses to default $(d_1(B_1, g_1, \gamma) = 1)$, the marginal benefit of an extra unit of debt is zero.[10] Thus, conditional on B_1, a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the expected marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes.[11]

Given that income and taxes are homogeneous across agents, it has to be the case at equilibrium that $b_1^H > b_1^L$, which therefore implies that $H-$types are never credit constrained. In contrast, whether $L - types$ are credit constrained or not depends on parameter values. This is less likely to happen the higher b_0^L, B_0 or B_1. Whenever the $L-$types are constrained, the $H-$types are the marginal investor and their Euler equation can be used to derive the equilibrium price. For the remainder of the paper we focus on equilibria in which $L-$types are constrained (i.e. $b_0^L = b_1^L = 0$), in order to capture the feature of heterogeneous agents models with incomplete markets that a fraction of agents is always credit constrained endogenously, and public debt has the social benefit that it contributes to reduce the tightness of this constraint.

The equilibrium bond price is the value of $q_0(B_1, \gamma)$ for which, as long as consumption for all agents is non-negative and the default probability of the government is less than 1, the following

[9] Notice in particular that the payoff in case of default does not depend on the level of individual debt holdings (b_1^i), reflecting the fact that the government cannot discriminate across households when it defaults.

[10] Utility in the case of default equals $u(y(1 - \phi(g_1)) - g_1)$, and is independent of b_1^i.

[11] Note also that from the agents' perspective, their bond decisions do not affect $d_1(B_1, g_1, \gamma)$.

market-clearing condition holds:

$$B_1 = \gamma b_1^L(B_1,\gamma) + (1-\gamma) b_1^H(B_1,\gamma), \tag{10}$$

where B_1 in the left-hand-side of this expression represents the supply of public debt, and the right-hand-side is the aggregate government bond demand.

It is instructive to analyze further the households' problem assuming logarithmic utility ($u(c) = \log(c)$), because under these assumptions we can solve for $q_0(B_1,\gamma)$ in closed form, and use the solution to establish some important properties of bond prices and default risk spreads. We show in Section A.4 of the Appendix that the equilibrium bond price is:

$$q_0(B_1,\gamma) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \Pi(B_1,\gamma)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \Pi(B_1,\gamma),} \tag{11}$$

where $\Pi(B_1,\gamma) \equiv E_{g_1}\left[\frac{1-d(B_1,g_1,\gamma)}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1}\right]$ is the expected marginal utility of $H-$type agents for the second period, which weights only non-default states because the marginal benefit of debt is zero in default states. Since, as also shown in the Appendix (Section A.4), $\frac{\partial \Pi(B_1,\gamma)}{\partial B_1} < 0$, it follows that $\frac{\partial q_0(B_1,\gamma)}{\partial B_1} < 0$ for $c_0^H > 0$. Moreover, since bond prices are decreasing in B_1, it follows that the "revenue" the government generates by selling debt, $q_0(B_1,\gamma)B_1$, behaves like the familiar *debt Laffer curve* of the Eaton-Gersovitz models derived by Arellano (2008).[12] This Laffer curve will play a key role later in determining the government's optimal debt choice. In particular, the government internalizes that higher debt eventually produces decreasing revenues, and that in the decreasing segment of the Laffer curve revenues fall faster as the debt increases, and much faster as default risk rises sharply.

A similar expression can be obtained for the "risk-free" price (i.e., the bond price that arises in a model with full commitment), to show that the risk premium is non-negative, and it is strictly positive if there is default in equilibrium. The premium is increasing in B_1 since the default set is increasing in B_1. We also show in Section A.4 of the Appendix that the spread is a multiple of $1/\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right)$. As a result, the total date-0 resources available for consumption of the H-types ($y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0$) have a first-order *negative* effect on default risk spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. Changes in γ have a similar impact

[12]It is straightforward to show that revenue $R(B_1) = q_0(B_1,\gamma)B_1$ follows a Laffer curve in the $[0, B_1^{max}]$ interval, where B_1^{max} is the upper bound of debt such that the government chooses default for any realization of g_1 and thus $q_0(B_1^{max},\gamma) = 0$. Since $R(0) = 0$ with $R'(0) = q(0,\gamma) > 0$, and $R(B_1^{max}) = 0$ with $R'(B_1^{max}) = q_0'(B_1^{max},\gamma)B_1^{max} < 0$, it follows by Rolle's theorem that $R(B_1)$ has at least one local maximum in $(0, B_1^{max})$.

on this ratio also pushing spreads downward. However, as γ increases default incentives also strengthen, as the welfare of debt holders is valued less. Thus, in principle the response of spreads to increases in the concentration of debt ownership is ambiguous.

3.2 Government's Problem

3.2.1 Government Default Decision at $t = 1$

At $t = 1$, the government chooses to default or not by solving this optimization problem:

$$\max_{d \in \{0,1\}} \left\{ W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1) \right\}, \tag{12}$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1)$ denote the values of the social welfare function at the beginning of period 1 in the case of repayment and default respectively. Using the government budget constraint to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government's utilitarian payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1) + (1-\gamma)u(y - g_1 + b_1^H - B_1) \tag{13}$$

and

$$W_1^{d=1}(g_1) = u(y(1 - \phi(g_1)) - g_1). \tag{14}$$

Notice that all households lose g_1 of their income to government absorption regardless of the default choice. Moreover, debt repayment reduces consumption and welfare of L types and rises them for H types (since $(b_1^L - B_1) \leq 0$ and $(b_1^H - B_1) \geq 0$), whereas default implies the same consumption and utility for both types of agents.

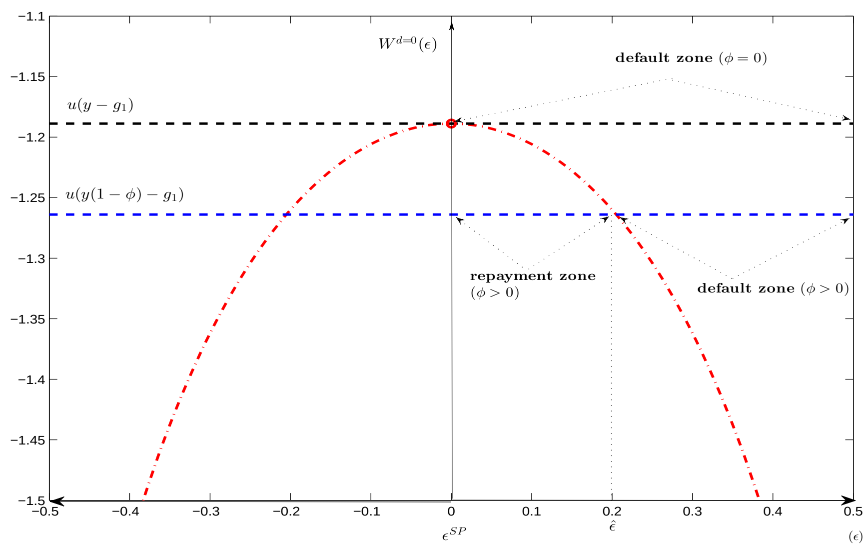
The distributional mechanism determining the default decision can be illustrated by means of a graphical tool. To this end, it is helpful to express the values of optimal debt holdings as $b_1^L = B_1 - \epsilon$ and $b_1^H(\gamma) = B_1 + \frac{\gamma}{1-\gamma}\epsilon$, for some hypothetical decentralized allocation of debt holdings given by $\epsilon \in [0, B_1]$. Consumption allocations under repayment would therefore be $c_1^L(\epsilon) = y - g_1 - \epsilon$ and $c_1^H(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon$, so ϵ also determines the decentralized consumption dispersion. The efficient dispersion of consumption that the social planner would choose is characterized by the value of ϵ^{SP} that maximizes social welfare under repayment, which satisfies this first-order condition:

$$u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right) = u'\left(y - g_1 - \epsilon^{SP}\right). \tag{15}$$

Hence, the efficient allocations are characterized by zero consumption dispersion, because equal marginal utilities imply $c^{L,SP}{=}c^{H,SP} = y - g_1$, which is attained with $\epsilon^{SP} = 0$.

Consider now the government's default decision when default is costless ($\phi(g_1) = 0$). This scenario is depicted in Figure 1, which plots the social welfare function under repayment as a function of ϵ as the bell-shaped curve, and the social welfare under default (which is independent of ϵ), as the black dashed line.

Figure 1: Default Decision and Consumption Dispersion



Clearly, the maximum welfare under repayment is attained when $\epsilon = 0$ which is also the efficient amount of consumption dispersion ϵ^{SP}.[13] Given that the only policy instruments the government can use, other than the default decision, are non-state contingent debt and lump-sum taxes, it is straightforward to conclude that default is always optimal. This is because default produces identical allocations in a decentralized equilibrium as the socially efficient ones, since default produces zero consumption dispersion with consumption levels $c^L{=}c^H = y - g_1$. This outcome is invariant to the values of B_1, g_1 and γ. This result also implies that the model without default costs cannot support equilibria with domestic debt subject to default risk, because default is always optimal.

The outcome is very different when default is costly. With $\phi(g_1) > 0$, default still yields zero consumption dispersion, but at lower levels of consumption and therefore utility, since consumption allocations in the default state become $c^L{=}c^H = (1 - \phi(g_1))y - g_1$. This does not alter the result that the first-best social optimum is $\epsilon^{SP} = 0$, but what changes is that default

[13] Recall also that we defined the relevant range of decentralized consumption dispersion for $\epsilon > 0$, so welfare under repayment is decreasing in ϵ over the relevant range.

can no longer support the consumption allocations of the first best. Hence, there is now a threshold amount of consumption dispersion in the decentralized equilibrium, $\widehat{\epsilon}(\gamma)$, which varies with γ and such that for $\epsilon \geq \widehat{\epsilon}(\gamma)$ default is again optimal, but for lower ϵ repayment is now optimal. This is because when ϵ is below the threshold, repayment produces a level of social welfare higher than the one that default yields. Figure 1 also illustrates this scenario.

3.2.2 Government Debt Decision at $t = 0$

We can now examine how the government chooses the optimal amount of debt to issue in the initial period. Before studying the government's optimization problem, it is important to emphasize that in this model debt is a mechanism for altering consumption dispersion across agents, both within a period and across periods. In particular, since $b_0^L = b_1^L = 0$, consumption dispersion in each period and repayment state can be written as: $c_0^H - c_0^L = \frac{1}{1-\gamma}\left[B_0 - q(B_1,\gamma)B_1\right]$, $c_1^{H,d=0} - c_1^{L,d=0} = \frac{1}{1-\gamma}B_1$, and $c_1^{H,d=1} - c_1^{L,d=1} = 0$. These expressions make it clear that, given B_0, issuing at least some debt ($B_1 > 0$) reduces consumption dispersion at $t = 0$ compared with no debt ($B_1 = 0$), but increases it at $t = 1$ if the government repays (*i.e.,* $d = 0$). Moreover, the debt Laffer curve that governs $q_0(B_1,\gamma)B_1$ limits the extent to which debt can reduce consumption dispersion at $t = 0$. Starting from $B_1 = 0$, consumption dispersion in the initial period falls as B_1 increases, but there is a critical positive value of B_1 beyond which it increases with debt.

At $t = 0$, the government chooses its debt policy internalizing the above effects, including the dependence of bond prices on the debt issuance choice. The government chooses B_1 so as to maximize the "indirect" social welfare function:

$$W_0(\gamma) = \max_{B_1}\left\{\gamma v^L(B_1,\gamma) + (1-\gamma)v^H(B_1,\gamma)\right\}. \tag{16}$$

where v^L and v^H are the value functions obtained from solving the households' problems defined in the Bellman equation (8) taking into account the government budget constraints and the equilibrium pricing function of bonds.

We can gain some intuition about the solution of this maximization problem by deriving its first-order condition and re-arranging it as follows (assuming that the relevant functions are differentiable):

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta}{q(B_1,\gamma)\gamma}\left\{\beta E_{g_1}\left[\Delta d \Delta W_1\right] + \gamma\mu^L\right\} \tag{17}$$

where

$$
\begin{aligned}
\eta &\equiv q(B_1,\gamma)/\left(q'(B_1,\gamma)B_1\right) < 0, \\
\Delta d &\equiv d(B_1+\delta, g_1,\gamma) - d(B_1, g_1,\gamma) \geq 0, \quad \text{for } \delta > 0 \text{ small}, \\
\Delta W_1 &\equiv W_1^{d=1}(g_1,\gamma) - W_1^{d=0}(B_1, g_1,\gamma) \geq 0, \\
\mu^L &\equiv q(B_1,\gamma)u'(c_0^L) - \beta E_{g_1}\left[(1-d^1)u'(c_1^L)\right] > 0.
\end{aligned}
$$

In these expressions, η is the price elasticity of the demand for government bonds, $\Delta d \Delta W_1$ represents the marginal distributional benefit of a default, and μ^L is the shadow value of the borrowing constraint faced by L-type agents.

If both types of agents were unconstrained in their bonds' choice, so that in particular $\mu_L = 0$, and if there is no change in the risk of default (or assuming commitment to remove default risk entirely), so that $E_{g_1}[\Delta d \Delta W_1] = 0$, then the optimality condition simplifies to $u'(c_0^H) = u'(c_0^L)$. Hence, in this case the social planner issues debt so as to equalize marginal utilities of consumption across agents at date 0, which requires simply setting B_1 to satisfy $q(B_1,\gamma)B_1 = B_0$.

If H-type agents are unconstrained and L-types are constrained (i.e. $\mu_L > 0$), which is the scenario we are focusing on, and still assuming no change in default risk or a government committed to repay, the optimality condition reduces to $u'(c_0^H) = u'(c_0^L) + \frac{\eta\mu^L}{q(B_1,\gamma)}$. Since $\eta < 0$, this result implies $c_0^L < c_0^H$, because $u'(c_0^L) > u'(c_0^H)$. Thus, the government's debt choice sets B_1 as needed to maintain an optimal, positive level of consumption dispersion. Moreover, since optimal consumption dispersion is positive, we can also ascertain that $B_0 > q(B_1,\gamma)B_1$, which using the government budget constraint implies that the government runs a primary surplus at $t = 0$. The government borrows resources, but less than it would need in order to eliminate all consumption dispersion (which requires zero primary balance).

The intuition for the optimality of issuing debt can be presented in terms of tax smoothing and savings: Date-0 consumption dispersion without debt issuance would be $B_0/(1-\gamma)$, but this is more dispersion than what the government finds optimal, because by choosing $B_1 > 0$ the government provides tax smoothing (i.e. reduces date-0 taxes) for everyone, which in particular eases the L-type agents credit constraint, and provides also a desired vehicle of savings for H types. Thus, positive debt increases consumption of L types (since $c_0^L = y - g_0 - B_0 + q(B_1,\gamma)B_1$), and reduces consumption of H types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)(B_0 - q(B_1,\gamma)B_1)$). But issuing debt (assuming repayment) also increases consumption dispersion a $t = 1$, since debt is then paid with higher taxes on all agents, while H agents collect also the debt repayment. Thus, the debt is being chosen optimally to trade off the social costs and benefits of reducing (increasing) date-0 consumption and increasing (reducing) date-1 consumption for rich (poor) agents.

In the presence of default risk and if default risk changes near the optimal debt choice, the term $E_{g_1}[\Delta d \Delta W_1]$ enters in the government's optimality condition with a positive sign, which means the optimal gap in the date-0 marginal utilities of the two agents widens even more. Hence, the government's optimal choice of consumption dispersion for $t = 0$ is greater than without default risk, and the expected dispersion for $t = 1$ is lower, because in some states of the world the government will choose to default and consumption dispersion would then drop to zero. Moreover, the debt Laffer curve now plays a central role in the government's weakened incentives to borrow, because as default risk rises the price of bonds drops to zero faster and the resources available to reduce date-0 consumption dispersion peak at lower debt levels. In short, default risk reduces the government's ability to use non-state-contingent debt in order to reduce consumption dispersion.

3.3 Competitive Equilibrium with Optimal Debt & Default Policy

For a given value of γ, a **Competitive Equilibrium with Optimal Debt and Default Policy** is a pair of household value functions $v^i(B_1, \gamma)$ and decision rules $b^i(B_1, \gamma)$ for $i = L, H$, a government bond pricing function $q_0(B_1, \gamma)$ and a set of government policy functions $\tau_0(B_1, \gamma)$, $\tau_1^{d \in \{0,1\}}(B_1, g_1, \gamma)$, $d(B_1, g_1, \gamma)$, $B_1(\gamma)$ such that:

1. Given the pricing function and government policy functions, $v^i(B_1, \gamma)$ and $b_1^i(B_1, \gamma)$ solve the households' problem.

2. $q_0(B_1, \gamma)$ satisfies the market-clearing condition of the bond market (equation (10)).

3. The government default decision $d(B_1, g_1, \gamma)$ solves problem (12).

4. Taxes $\tau_0(B_1, \gamma)$ and $\tau_1^d(B_1, g_1, \gamma)$ are consistent with the government budget constraints.

5. The government debt policy $B_1(\gamma)$ solves problem (16).

4 Quantitative Analysis

In this Section, we study the model's quantitative predictions based on a calibration using European data. The goal is to show whether a reasonable set of parameter values can produce an equilibrium with debt subject to default risk, and to study how the properties of this equilibrium change with the model's key parameters. Since the two-period model is not well suited to account for the time-series dynamics of the data, we see the results more as an illustration of the potential relevance of the model's argument for explaining domestic default rather than as an evaluation of the model's general ability to match observed public debt dynamics.[14]

4.1 Calibration

The model is calibrated to annual frequency, and most of the parameter values are set so that the model matches moments from European data. The calibrated parameter values are summarized in Table 2. The details of the calibration are available in Section A.5 of the Appendix. Note also that we assume a log-normal process for g_1, so that $\ln(g_1) \sim N\left((1-\rho_g)\ln(\mu_g) + \rho_g \ln(g_0), \frac{\sigma_e^2}{(1-\rho_g^2)}\right)$.[15] and the cost of default takes the following functional form: $\phi(g_1) = \phi_0 + (\overline{g}_1 - g_1)/y$.

Table 2: Model Parameters

Parameter		Value
Discount Factor	β	0.96
Risk Aversion	σ	1.00
Avg. Income	y	0.79
Low household wealth	b_0^L	0.00
Avg. Gov. Consumption	μ_g	0.18
Autocorrel. G	ρ_g	0.88
Std Dev Error	σ_e	0.017
Initial Gov. Debt	B_0	0.35
Output Cost Default	ϕ_0	0.004

Note: Government expenditures, income and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain and Portugal.

We abstain from setting a calibrated value for γ and instead show results for $\gamma \in [0,1]$. Data from the United States and Europe suggest that the empirically relevant range for γ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of γ is useful we use $\gamma = 0.7$, which is the mid point of the plausible range.[16]

[14]We solve the model following a similar backward-recursive strategy as in the theoretical analysis. First, taking as given a set of values $\{B_1, \gamma\}$, we solve for the equilibrium pricing and default functions by iterating on (q_0, b_1^i) and the default decision rule d_1 until the date-0 bond market clears when the date-1 default decision rule solves the government's optimal default problem (12). Then, in the second stage we complete the solution of the equilibrium by finding the optimal choice of B_1 that solves the government's date-0 optimization problem (16). It is important to recall that, as explained earlier, for given values of B_1 and γ, an equilibrium with debt will not exist if either the government finds it optimal to default on B_1 for all realizations of g_1 or if at the givenB_1 the consumption of L types is non-positive. In these cases, there is no finite price that can clear the debt market.

[15]This specification allows us to control the correlation between g_0 and g_1 via ρ_g, the mean of the shock via μ_g and the variance of the unpredicted portion via σ_e^2. Note that if $g_0 = \mu_g$, $\ln(g_1) \sim N(\ln(\mu_g), \frac{\sigma_g^2}{(1-\rho_e^2)})$.

[16]In the United States, the 2010 Survey of Consumer Finances indicates that only 12 percent of households

4.2 Results

We examine the quantitative results in the same order in which the backward solution algorithm works. We start with the second period's utility of households under repayment and default. We then move to the first period and examine the equilibrium bond prices. Finally, we study the optimal government debt issuance B_1 for a range of values of γ.

4.2.1 Second period default incentives for given (B_1, g_1, γ)

Using the agents' optimal choice of bond holdings, we compute the equilibrium utility levels they attain at $t = 1$ under repayment v. default for different triples (B_1, g_1, γ). Since we are looking at the last period of a two-period model, these compensating variations reduce simply to the percent changes in consumption across the default and no-default states of each agent:[17]

$$\alpha^i(B_1, g_1, \gamma) = \frac{c_1^{i,d=1}(B_1, g_1, \gamma)}{c_1^{i,d=0}(B_1, g_1, \gamma)} - 1 = \frac{(1-\phi(g_1))y - g_1}{y - g_1 + b_1^i - B_1} - 1$$

A positive (negative) value of $\alpha^i(B_1, g_1, \gamma)$ implies that agent i prefers government default (repayment) by an amount equivalent to an increase (cut) of $\alpha^i(\cdot)$ percent in consumption.

The individual welfare gains of default are aggregated using γ to obtain the utilitarian representation of the social welfare gain of default:

$$\overline{\alpha}(B_1, g_1, \gamma) = \gamma\alpha^L(B_1, g_1, \gamma) + (1-\gamma)\alpha^H(B_1, g_1, \gamma).$$

A positive value indicates that default induces a social welfare gain and a negative value a loss. The default decision is directly linked to the values of $\overline{\alpha}(B_1, g_1, \gamma)$. In particular, the repayment region of the default decision $(d(B_1, g_1, \gamma) = 0)$ corresponds to $\overline{\alpha}(B_1, g_1, \gamma) < 0$ and the default region $(d(B_1, g_1, \gamma) = 1)$ to $\overline{\alpha}(B_1, g_1, \gamma) > 0$.

---

hold savings bonds and 50.4 percent have retirement accounts (which are very likely to include government bonds). These figures would suggest values of γ ranging from 0.5 to 0.88. In Europe, comparable statistics are not available for several countries, but Davies, Sandstrm, Shorrocks, and Wolff (2009) document that the wealth distribution is highly concentrated with Gini coefficients ranging between 0.55 and 0.85. In our model, since $b_0^L = 0$, the Gini coefficient of wealth is equal to γ.

[17] These calculations are straightforward given that in the equilibria we solve for $b_1^L = 0$, and hence $b_1^H = B_1/(1-\gamma)$. The same formula would apply, however, even if these conditions do not hold, using instead the policy functions $b_1^i(B_1, g_1, \gamma)$ that solve the households' problems for any given pair of functions $d_1(B_1, g_1, \gamma)$ and $q_0(B_1, \gamma)$, including the ones that are consistent with the government's default decision and equilibrium in the bond market.

Figure 2: Social Welfare Gain of Default $\overline{\alpha}(B_1, g_1, \gamma)$

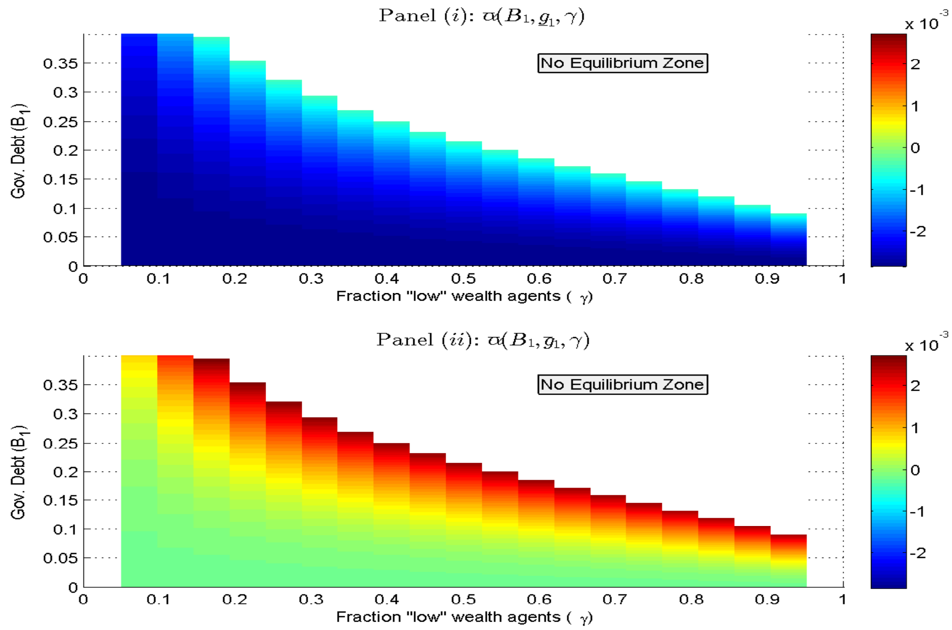


Figure 2 shows two intensity plots of the social welfare gain of default for the ranges of values of B_1 and γ in the vertical and horizontal axes respectively. Panel (i) is for a low value of government purchases, $\underline{g}_1$, set 3 standard deviations below μ_g, and panel (ii) is for a high value $\overline{g}_1$ set 3 standard deviations above μ_g. Figure A.2 in the Appendix shows the default decision rules that correspond to these two plots. The intensity of the color or shading in these plots indicates the magnitude of the welfare gain according to the legend shown to the right of each. The regions shown in white color and marked as "No Equilibrium Zone", represent values of (B_1, γ) for which the debt market collapses and no equilibrium exists. In this zone, there is no equilibrium because, at the given γ, the government chooses to default on the given B_1 for all values of g_1.[18]

The area in which the social welfare gains of default are well defined in these intensity plots illustrates two of the key mechanisms driving the government's distributional incentives to default: First, fixing γ, the welfare gain of default is higher at higher levels of debt, or conversely the gain of repayment is lower. Second, keeping B_1 constant, the welfare gain of

[18]There is another potential "No Equilibrium Zone" that could arise if the given (B_1, γ) would yield $c_0^L \leq 0$ at the price that induces market clearing, and so the government would not supply that particular B_1. This happens for low levels of B_1 relative to B_0. To determine if $c_0^L \leq 0$ at some (B_1, γ) we need $q_0(B_1, \gamma)$, since combining the budget constraints of the L types and the government yields $c_0^L = y - g_0 - B_0 + q_0 B_1$. Hence, to evaluate this condition we take the given B_1 and use the H types Euler equation and the market clearing condition to solve for $q_0(B_1, \gamma)$, and then determine if $y - g_0 - B_0 + q_0 B_1 \leq 0$, if this is true, then (B_1, γ) is in the lower no equilibrium zone.

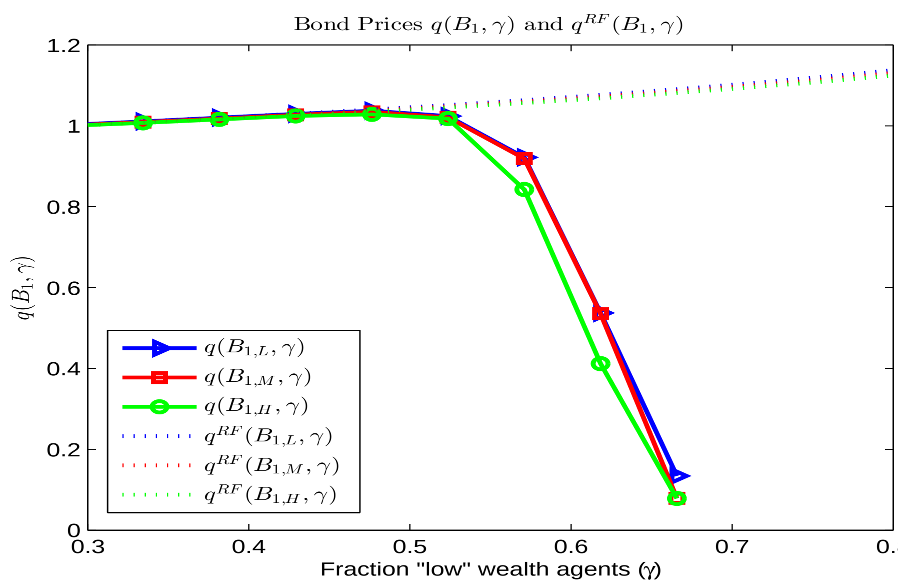
default is also increasing in γ (i.e. higher concentration of debt ownership increases the welfare gain of default). This implies that lower concentration of debt ownership is sufficient to trigger default at higher levels of debt.[19] For example, for a debt of 20 percent of GDP ($B_1 = 0.20$) and $g_1 = \overline{g}_1$, social welfare is higher under repayment if $0 \leq \gamma \leq 0.10$ but it becomes higher under default if $0.10 < \gamma \leq 0.6$, and for higher γ there is no equilibrium because the government prefers default not only for $g_1 = \overline{g}_1$ but for all possible g_1. If instead the debt is 35 percent of GDP, then social welfare is higher under default for all the values of γ for which an equilibrium exists.

The two panels in Figure 2 differ in that panel (ii) displays a well-defined transition from a region in which repayment is socially optimal ($\overline{\alpha}(B_1, g_1, \gamma) < 0$) to one where default is optimal ($\overline{\alpha}(B_1, g_1, \gamma) > 0$) but in panel (i) the social welfare gain of default is never positive, so repayment is always optimal. This reflects the fact that higher g_1 also weakens the incentives to repay.

4.2.2 Bond prices for given (B_1, γ)

Figure 3 shows $q_0(B_1, \gamma)$ as a function of γ for three values of B_1 ($B_L < B_M < B_H$) and a comparison with the prices from the model with the government committed to repay q^{RF}.The bond price functions are truncated when the equilibrium does not exist.

Figure 3: Equilibrium Bond Price



[19]Note that the cross-sectional variance of initial debt holdings is given by $Var(b) = B^2 \frac{\gamma}{1-\gamma}$ when $b_0^L = 0$. This implies that the cross-sectional coefficient of variation is equal to $CV(b) = \frac{\gamma}{1-\gamma}$, which is increasing in γ for $\gamma \leq 1/2$.

Figure 3 illustrates three key features of public debt prices discussed in Section 3:
(**i**) *The equilibrium price is decreasing in* B_1 *for given* γ (the pricing functions shift downward as B_1 rises). This follows from a standard demand-and-supply argument: For a given γ, as the government borrows more, the price at which households are willing to demand the additional debt falls and the interest rate rises. This effect is present even without uncertainty, but it is stronger in the presence of default risk.[20]
(**ii**) *Default risk reduces the price of bonds below the risk-free price and thus induces a risk premium.* Prices are either identical or nearly identical for the values shown of B_1 when $\gamma \leq 0.5$ since the probability of default is either zero or very close to zero. As γ increases above 0.5, however, the risk premium becomes nontrivial and bond prices subject to default risk fall sharply below the risk free prices.
(**iii**) *Bond prices are a non-monotonic function of* γ: When default risk is sufficiently low, bond prices are increasing in γ, but eventually they become a steep decreasing function of γ. Whether bond prices are increasing or decreasing in γ depends on the relative strength of a demand composition effect v. the effect of increasing γ on default incentives. The composition effect results from the fact that, as γ increases, H-type agents become a smaller fraction of the population and wealthier in per-capita terms, and therefore a higher $q_0(B_1, \gamma)$ is needed to clear the market. On the other hand, higher concentration of debt ownership strengthens distributional incentives to default, which pushes for lower bond prices. This second effect starts to dominate for $\gamma > 0.5$, producing bond prices that fall sharply as γ rises, while for lower γ the composition effect dominates and prices rise gradually with γ.[21]

4.2.3 Optimal Debt Choice and Competitive Equilibrium

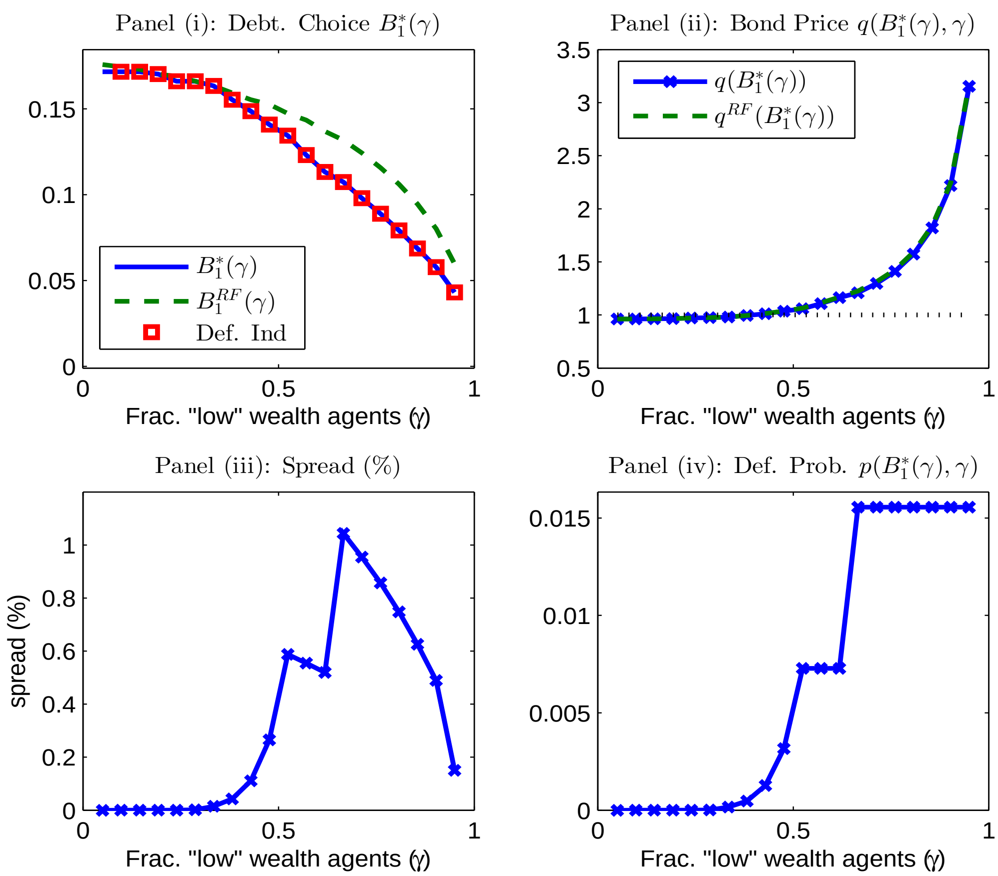
Given the solutions for household decision rules, tax policies, bond pricing function and default decision rule, we finally solve for the government's optimal choice of debt issuance in the first period (i.e. the optimal B_1 that solves problem (16)) for a range of values of γ. Given this optimal debt, we can go back and identify the equilibrium values of the rest of the model's endogenous variables that are associated with the optimal debt choices.

Figure 4 shows the four main components of the equilibrium: Panel (i) plots the optimal first-period debt issuance in the model with default risk, $B_1^*(\gamma)$, and in the case when the government is committed to repay so that the debt is risk free, $B_1^{RF}(\gamma)$; Panel (ii) shows the equilibrium debt prices that correspond to the optimal debt of the same two economies; Panel

[20]Sections A.3 and A.4 of the Appendix provide proofs showing that $q'(B_1, \gamma) < 0$ in the log-utility case. In Figure 3, the scale of the vertical axis is too wide to make the fall in $q(B_1, \gamma)$ as B_1 rises visible for $\gamma < 0.5$.
[21]Sections A.3 and A.4 of the Appendix provide further details, including an analysis of the bond demand decision rules that validates the intuition provided here.

(iii) shows the default spread (the difference in the inverses of the bond prices); and Panel (iv) shows the probability of default. Since the government that has the option to default can still choose a debt level for which it prefers to repay in all realizations of g_1, we identify with a square in Panel (i) the equilibria in which $B_1^*(\gamma)$ has a positive default probability. This is the case for all but the smallest value of γ considered ($\gamma = 0.05$), in which the government sets $B_1^*(\gamma)$ at 20 percent of GDP with zero default probability.

Figure 4: Competitive Equilibrium with Optimal Debt Policy



It is evident from Panel (i) of Figure 4 that optimal debt falls as γ increases in both the economy with default risk and the economy with a government committed to repay. This occurs because in both cases the government seeks to reallocate consumption across agents and across periods by altering the product $q(B_1, \gamma)B_1$ optimally, and in doing this it internalizes the response of bond prices to its choice of debt. As γ rises, this response is influenced by the stronger default incentives and demand composition effect. At equilibrium, the latter dominates in this quantitative experiment, because panel (ii) shows that the equilibrium bond prices rise with γ.

Hence,the government internalizes that as γ rises the demand composition effect strengthens demand for bonds, pushing bond prices higher, and as a result it can actually attain a higher $q(B_1, \gamma)B_1$ by choosing lower B_1. This is a standard Laffer curve argument: In the upward slopping segment of this curve, increasing debt increases the amount of resources the government acquires by borrowing in the first period.

Although the Laffer curve argument and the demand composition effect explain why both $B_1^*(\gamma)$ and $B_1^{RF}(\gamma)$ are decreasing in γ, default risk is not innocuous. As Panel (i) shows, the optimal B_1 choices of the government that cannot commit to repay are lower than those of the government that can. This reflects the fact that the government optimally chooses smaller debt levels once it internalizes the effect of default risk on the debt Laffer curve and its distributional implications. The negative relationship between B_1 and γ is in line with the empirical evidence on the negative relationship between public debt ratios and wealth Gini coefficients at relatively high levels of inequality noted in the Introduction and documented in Section A.1 of the Appendix.

Panels (iii) and (iv) show that, in contrast with standard models of external default, in this model the default spread is neither similar to the probability of default nor does it have a monotonic relationship with it.[22] Both the spread and the default probability start at zero for $\gamma = 0.05$ because $B_1^*(0.05)$ has zero default probability. As γ increases up to 0.5, both the spread and the default probability of the optimal debt choice are similar in magnitude and increase together, but for the regions where the default probability is constant (for $\gamma > 0.5$) the spread falls with γ.[23] These results are in line with the findings of the theoretical analysis in Section 3.

The determination of the optimal debt choice and the relationship between the four panels of Figure 4 can be illustrated further as follows. Define a default-threshold value of γ, $\hat{\gamma}(B_1, g_1)$, as the one such that the government is indifferent between defaulting and repaying for a given (B_1, g_1). The government chooses to default if $\gamma \geq \hat{\gamma}$. Figure 5 shows the optimal debt choice $B_1^*(\gamma)$ together with curves representing $\hat{\gamma}(B_1, g_1)$ for several realizations of g_1. The curves for the lowest ($\underline{g}$), highest ($\overline{g}$), and mean (μ_g) realizations are identified with labels.

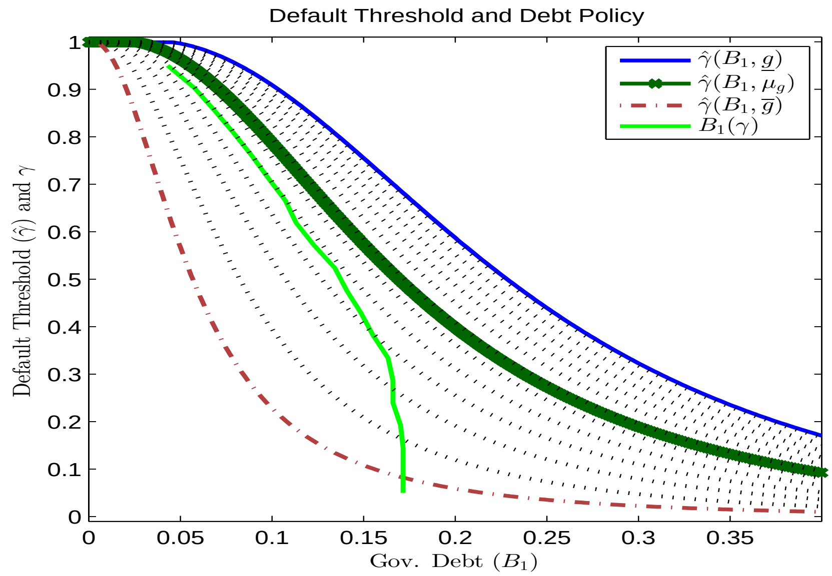
Figure 5 shows that, because of the stronger default incentives at higher γ and higher realizations of g_1, the default-threshold curves are decreasing in B_1 and g_1. There are, therefore, two key "border curves." First, for pairs (B_1, γ) below $\hat{\gamma}(B_1, \overline{g}_1)$ repayment can be expected to occur for sure, because the government will repay even if the highest realization of g_1 is observed. Second, for pairs (B_1, γ) above $\hat{\gamma}(B_1, \underline{g}_1)$ default can be expected to occur for sure, because the

[22] In the standard models, the two are similar and a monotonic function of each other because of the arbitrage condition of a representative risk-neutral lender.

[23] As we explained before, this is derived from the composition effect that strengthens the demand for bonds and results in increasing prices (with default risk and without) as γ increases. This result disappears in the extension of the model that introduces foreign lenders (see Section 5).

government will choose default even if the lowest realization of g_1 is observed.

Figure 5: Default Threshold, Debt Policy and Equilibrium Default



Note: $\underline{g}$ and $\overline{g}$ are the smallest and largest possible realizations of g_1 in the Markov process of government expenditures, which are set to -/+ 3 standard deviations off the mean respectively. The dotted lines correspond to a set of selected thresholds for different values of g_1.

It follows from the above that, for equilibria with debt exposed to default risk to exist, the optimal debt choice $B_1^*(\gamma)$ must lie in between the two borders (if it is below $\hat{\gamma}(B_1, \overline{g}_1)$ the debt is issued at zero default risk, and if it were to be above $\hat{\gamma}(B_1, \underline{g}_1)$ there is no equilibrium). Moreover, the probability of default is implicitly determined as the cumulative probability of the value of g_1 corresponding to the highest debt-threshold curve that $B_1^*(\gamma)$ reaches. This explains why the default probability in Panel (iv) of Figure 4 shows constant segments as γ rises above 0.5. As Figure 5 shows, for $\gamma \leq 0.5$ the optimal debt is relatively invariant to increases in γ, starting from a level that is actually in the region of risk-free debt and then moving into the region exposed to default risk. In this segment, the optimal debt falls slightly and the probability of default rises gradually as γ rises. For γ just a little about 0.5 to 0.6, the optimal debt falls but along the same default threshold curve (not shown in the plot), and hence the default probability remains constant at about 0.007. For $\gamma > 0.6$, the debt choice falls gradually but always along the default-threshold curve associated with a default probability of 0.015.

The above findings suggests that the optimal debt is being chosen seeking to sell the "most debt" that can be issued while keeping default risk low. In turn, the "most debt" that is optimal to issue responds to the incentives to reallocate consumption across agents and across periods internalizing the dependence of the debt Laffer curve on the debt choice. In fact, for all values of $B_1^*(\gamma)$ that are exposed to nontrivial risk of default (those corresponding to $\gamma \geq 0.5$), $B_1^*(\gamma)$ coincides with the maximum point of the corresponding debt Laffer curve (see Figure A.6 of the Appendix). Hence, the optimal debt yields the maximum resources to the government that it can procure given its inability to commit to repay. Setting debt higher is suboptimal because default risk reduces bond prices sharply, resulting in a lower amount of resources, and setting it lower is also suboptimal, because then default risk is low and extra borrowing generates more resources since bond prices fall little.

5 Extensions

This Section summarizes results of four important extensions of the model. First, a political bias case in which the social welfare function assigns weighs to agents that deviate from the fraction of L and H types observed in the economy. Second, an economy in which risk-neutral foreign investors can buy government debt. Third, a case in which proportional distortionary taxes on consumption are used as an alternative tool for redistributive policy. Fourth, a case in which agents have access to a second asset as vehicle for saving.

5.1 Biased Welfare Weights

Assume now that the weights of the government's payoff function differ from the utilitarian weights γ and $1-\gamma$. This can be viewed as a situation in which, for political reasons, the government's welfare weights are biased in favor of one group of agents. The government's welfare weights on L- and H-type households are denoted ω and $(1-\omega)$ respectively, and we refer to ω as the government's political bias.

The government's default decision at $t = 1$ is determined by the following optimization problem:

$$\max_{d \in \{0,1\}} \left\{ W_1^{d=0}(B_1, g_1, \gamma, \omega), W_1^{d=1}(g_1) \right\}, \tag{18}$$

where $W_1^{d=0}(B_1, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ denote the government's payoffs in the cases of no-default and default respectively. Using the government budget constraints to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma, \omega) = \omega u(y - g_1 + b_1^L - B_1) + (1-\omega)u(y - g_1 + b_1^H - B_1) \tag{19}$$

and

$$W_1^{d=1}(g_1) = u(y(1-\phi(g_1)) - g_1). \tag{20}$$

We can follow a similar approach as before to characterize the optimal default decision by comparing the allocations it supports with the first-best allocations. The parameter ϵ is used again to represent the dispersion of hypothetical decentralized consumption allocations under repayment: $c^L(\epsilon) = y - g_1 - \epsilon$ and $c^H(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon$. Under default the consumption allocations are again $c^L = c^H = y(1-\phi(g_1)) - g_1$. Recall that under repayment, the dispersion of consumption across agents increases with ϵ, and under default there is zero consumption dispersion. The repayment government payoff can now be rewritten as:

$$W^{d=0}(\epsilon, g_1, \gamma, \omega) = \omega u(y - g_1 + \epsilon) + (1-\omega)u\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon\right).$$

The socially efficient planner chooses its optimal consumption dispersion ϵ^{SP} as the value of ϵ that maximizes the above expression. Since as of $t = 1$ the only instrument the government can use to manage consumption dispersion relative to what the decentralized allocations support is the default decision, it will repay only if doing so allows it to get closer to ϵ^{SP} than by defaulting.

The planner's optimality condition is now:

$$\frac{u'(c_1^H)}{u'(c_1^L)} = \frac{u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y - g_1 - \epsilon^{SP}\right)} = \left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right). \tag{21}$$

This condition implies that optimal consumption dispersion for the planner is zero only if $\omega = \gamma$. For $\omega > \gamma$ the planner likes consumption dispersion to favor L types so that $c_1^L > c_1^H$, and the opposite holds for $\omega < \gamma$.

The key difference with political bias v. the model with a utilitarian government is that the former can support equilibria with debt subject to default risk even without default costs. Assuming $\phi(g_1) = 0$, there are two possible scenarios depending on the relative size of γ and ω. First, if $\omega \geq \gamma$, the planner again always chooses default as in the setup of Section 2. This is because for any decentralized consumption dispersion $\epsilon > 0$, the consumption allocations feature $c^H > c^L$, while the planner's optimal consumption dispersion requires $c^H \leq c^L$, and hence ϵ^{SP} cannot be implemented. Default brings the planner the closest it can get to the payoff associated with ϵ^{SP} and hence it is always chosen. In the second scenario $\omega < \gamma$ (i.e. the political bias assigns more (less) weight to H (L) types than the fraction of each type of agents that actually exists). In this case, the model can support equilibria with debt even without default costs. In particular, there is a threshold consumption dispersion $\hat{\epsilon}$ such that default is optimal for $\epsilon \geq \hat{\epsilon}$, where $\hat{\epsilon}$ is the value of ϵ at which $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ intersect. For $\epsilon < \hat{\epsilon}$, repayment is preferable because $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W_1^{d=0}(g_1)$. Thus, without default costs, equilibria for which repayment is optimal require two conditions: (a) that the government's political bias favors bond holders ($\omega < \gamma$), *and* (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Figure 6 illustrates the main quantitative predictions of the model with political bias. The scenario with $\omega = \gamma$, shown in blue corresponds to the utilitarian case of Section 4, and the other two scenarios correspond to high and low values of ω ($\omega_L = 0.25$ and $\omega_H = 0.45$ respectively).[24]

Figure 6: Equilibrium of the Model with Political Bias for different values of ω

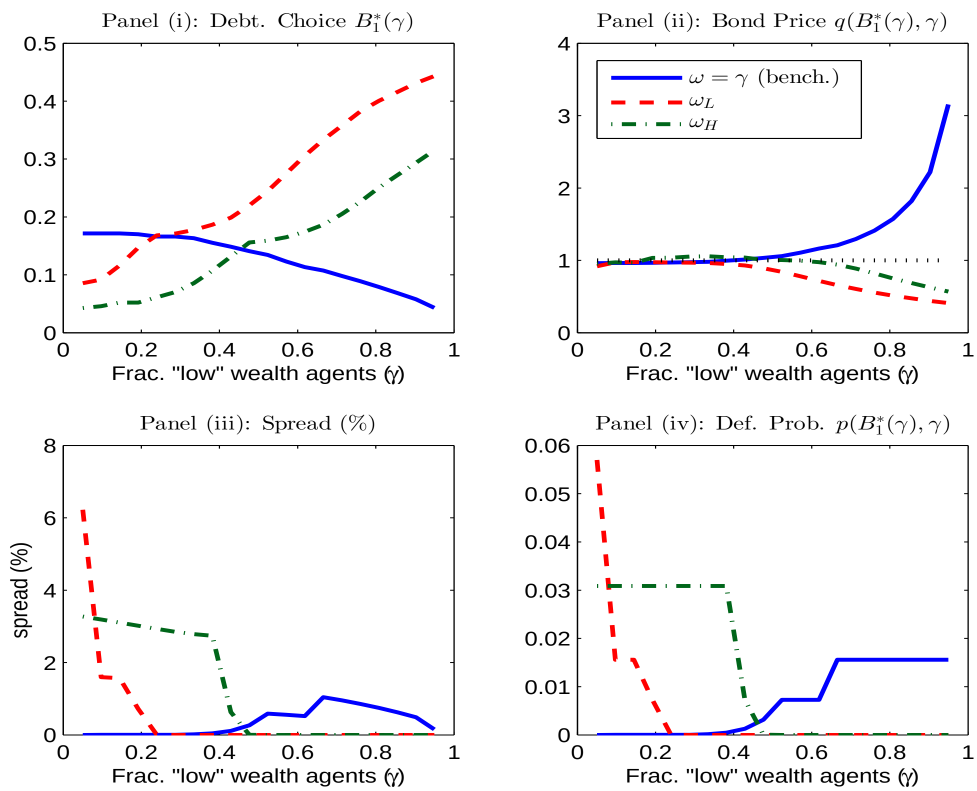


Figure 6 shows that the optimal debt level is increasing in γ. This is because the incentives to default grow weaker and the repayment zone widens as γ increases for a fixed value of ω. Moreover, the demand composition effect of higher γ is still present, so along with the lower default incentives we still have the increasing per capita demand for bonds of H types. These

[24]Note that along the blue curve of the utilitarian case both ω and γ effectively vary together because they are always equal to each other, while in the other two plots ω is fixed and γ varies. For this reason, the line corresponding to the ω_L case intersects the benchmark solution when $\gamma = 0.25$, and the one for ω_H intersects the benchmark when $\gamma = 0.45$.

two effects combined drive the increase in the optimal debt choice of the government. It is also interesting to note that in the ω_L and ω_H cases the equilibrium exists for all values of γ (even those that are lower than ω). Without default costs each curve would be truncated exactly where γ equals either ω_L or ω_H, but since these simulations retain the default costs used in the utilitarian case, there can still be equilibria with debt for lower values of γ (as explained earlier).

In this model with political bias, the government is still aiming to optimize debt by focusing on the resources it can reallocate across periods and agents, which are still determined by the debt Laffer curve, and internalizing the response of bond prices to debt choices.[25] This relationship, however, behaves very differently than in the benchmark model, because now *higher* optimal debt is carried at decreasing default probabilities, which leads the planner internalizing the price response to choose higher debt, whereas in the benchmark model *lower* optimal debt was carried at increasing equilibrium default probabilities, which led the planner internalizing the price response to choose lower debt.

In the empirically relevant range of γ, and for values of ω lower than that range (since $\omega_L = 0.25$ and $\omega_H = 0.45$, while the relevant range of γ is $[0.55 - 0.85]$), this model can sustain significantly higher debt ratios than the model with utilitarian payoff, and those ratios are close to the observed European median. At the lower end of that range of γ, a government with ω_H chooses a debt ratio of about 25 percent, while a government with ω_L chooses a debt ratio of about 35 percent.

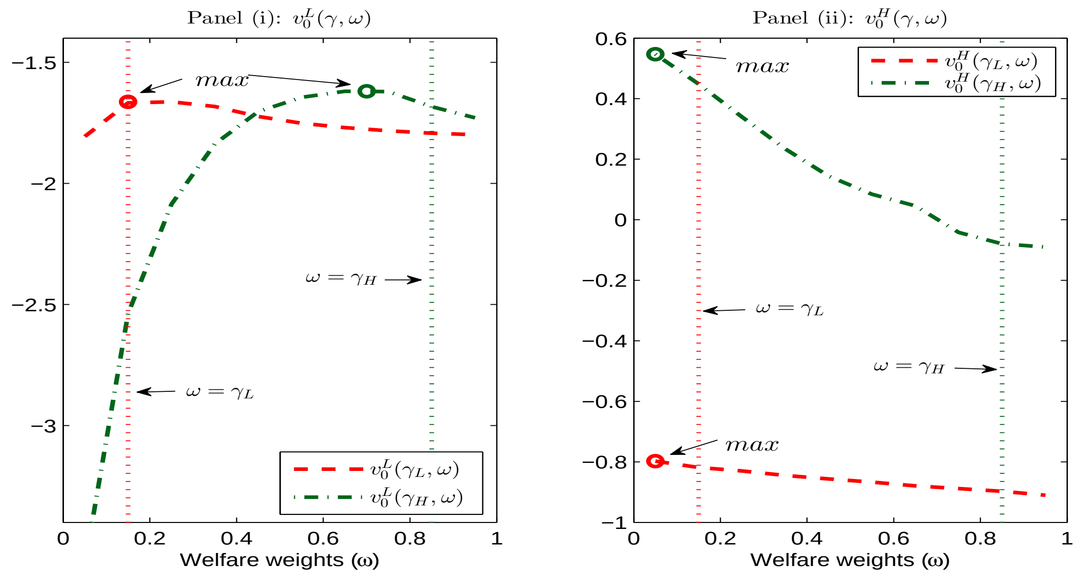
The behavior of equilibrium bond prices (panel (ii)) with either $\omega_L = 0.25$ or $\omega_H = 0.45$ differs markedly from the utilitarian case. In particular, the prices no longer display an increasing, convex shape, instead they are (for most values of γ) a decreasing function of γ. This occurs because the higher supply of bonds that the government finds optimal to provide offsets the demand composition effect that increases individual demand for bonds as γ rises. At low values of γ the government chooses lower debt levels (panel (i)) in part because the default probability is higher (panel (iv)), which also results in higher spreads (panel (iii)). But as γ rises and repayment incentives strengthen (because ω becomes relatively smaller than γ), the probability of default falls to zero, the spreads vanish, and debt levels increase. The price remains relatively flat because, again, the higher debt supply offsets the demand composition effect.

The political bias extension yields an additional interesting result: For a sufficiently concentrated distribution of bond holdings (high γ), $L-$type agents prefer that the government weighs the bond holders more than a utilitarian government (i.e, there are values of γ and ω for which, comparing equilibrium payoffs under a government with political bias v. a utilitarian

[25]When choosing B_1, the government takes into account that higher debt increases disposable income for L-type agents in the initial period but it also implies higher taxes in the second period (as long as default is not optimal). Thus, the government is willing to take on more debt when ω is lower.

government, $v^L(B_1, \omega, \gamma) > v^L(B_1, \gamma)$). To illustrate this result, Figure 7 plots the equilibrium payoffs in the political-bias model for the two types of agents as ω varies for two values of γ ($\gamma_L = 0.15$ and $\gamma_H = 0.85$). The payoffs for the $L-$ and $H-$ types are in Panels (i) and (ii), respectively. The vertical lines identify the payoffs that would be attained with a utilitarian government (which by construction coincide with those under political bias when ω=γ).

Figure 7: Welfare as a function of Political Bias for different values of γ



Panel (ii) shows that the payoff of H-types is monotonically decreasing in ω, because H types always prefer the higher debt levels attained by low ω governments, since it enhances their ability to smooth consumption at a lower risk of default. In contrast, Panel (i) shows that the payoff of L-types is non-monotonic in ω, and has a well-defined maximum. In the $\gamma = \gamma_L$ case, the maximum point is at $\omega = \gamma_L$, which corresponds to the equilibrium under the utilitarian government, but when $\gamma = \gamma_H$ the maximum point is at about $\omega = 0.75$, which is smaller than γ_H. Thus, in this case, ownership of public debt is sufficiently concentrated for the agents that do not hold it to prefer a government that chooses B_1 weighting the welfare of bond holders by more than the utilitarian government. This occurs because with γ_H the utilitarian government has strong incentives to default, and thus the equilibrium supports low debt, but $L-$type agents would be better off if the government could sustain more debt, which the government with $\omega = 0.75$ can do because it weighs the welfare of bond holders more, and thus has weaker default incentives. The $L-$ type agents desire more debt because they are liquidity-constrained (i.e. $\mu^L > 0$) and higher debt improves the smoothing of taxation and thus makes this constraint less tight.

The above results yield an important political economy implication: Under a majority voting electoral system, in which candidates are represented by values of ω, it can be the case that majorities of either L or H types elect governments with political bias $\omega < \gamma$. This possibility is captured in Figure 7. When the actual distribution of bond holdings is given by γ_H, the majority of voters are L types, and thus it follows from Panel (i) that the government represented by the ω at the maximum point (around 0.75) is elected. In this case, agents that do not hold government bonds vote for a government that favors bond holders (i.e. L types are weighed at 0.75 instead of 0.85 in the government's payoff function). When the distribution of bond holdings is given by γ_L, the majority of voters are H types and the electoral outcome is determined in Panel (ii). Since the payoff of H types is decreasing in ω, they elect the government at the lower bound of ω. Hence, under both γ_L and γ_H, a candidate with political bias beats the utilitarian candidate. This result is not general, however, because we cannot rule out the possibility that there could be a $\gamma \geq 0.5$ such that the maximum point of the L-types payoff is where $\omega = \gamma$, and hence the utilitarian government is elected under majority voting.

5.2 International Investors

While a large fraction of sovereign debt in Europe is in hands of domestic households (for the countries in Table 1, 60 percent is the median and 75 percent the average), the fraction in hands of foreign investors is not negligible. For this reason, we extend the benchmark model to incorporate foreign investors and move from a closed to an open economy. In particular, we assume that there is a pool of international investors modeled in the same way as in the Eaton-Gersovitz class of external default models: risk-neutral lenders with an opportunity cost of funds equal to an exogenous, world-determined real interest rate $\bar{r}$. As is common practice, we assume that these bonds are pari passu contracts, which rules out the possibility for the government to discriminate among borrowers when choosing to default. This also maintains the symmetry with the baseline model, in which the government was not allowed to default on a particular set of domestic households.

Since foreign lenders are the marginal investor of sovereign debt, in this model the price of the bond is given by $q(B_1, \gamma) = \frac{1-p(B_1,\gamma)}{(1+\bar{r})}$ where $p(B_1, \gamma)$ is the default probability. More precisely, $p(B_1, \gamma) = E_{g_1}[d(B_1, \gamma, g_1)]$. While the arbitrage condition is functionally identical to the one of the Eaton-Gersovitz models, they embody different mechanisms. The two are similar in indicating that, because of risk neutrality, risk premia are equal to default probabilities. But there is a critical difference in how these probabilities are determined. In Eaton-Gersovitz models they follow from the values of continuation v. default of a representative agent, while in our model they are determined by comparing those values for a utilitarian social welfare function, which in turn depend on the dispersion of individual payoffs of default v. repayment (and on the welfare weights). Hence, concentration of debt ownership affects default probabilities via changes in the relative magnitudes of individual payoffs of default v. repayment.

We let the position of foreign investors be denoted B_t^f, which defines also the economy's net foreign asset position. We assume that a fraction ϕ^f of the initial stock of debt is in their hands.[26] That is $B_0^f = \phi^f B_0$. A fraction $(1-\phi^f)$ is distributed among domestic households according to γ. We denote the domestic demand by $B_t^d = \gamma b_t^L + (1-\gamma) b_t^H$. The debt market clearing condition is

$$B_t^d + B_t^f = B_t \tag{22}$$

We do not restrict the value of $[\gamma b_1^L + (1-\gamma) b_1^H]$ to be less than or equal to B_1, so B_1^f could be positive or negative. When $B_1^f > 0$ the country is a net external borrower, because the bonds issued by the government are less than the domestic demand for them, and when $B_1^f < 0$ the country is a net external saver.

The problems of the agents and the government remain identical to those described in Section 3. Of course, agents understand that there is a new pricing equation and that market-clearing conditions incorporate the foreign demand. We solve the model numerically using the same parameter values of the benchmark model. We set $\phi^f = 0.25$ to match the average fraction of foreign debt observed in our sample of European countries and $\overline{r}$ to 2 percent to match the average real interest rate in Germany in the 2000-2007 period.

Figure 8 shows how the planner's welfare gain of default varies with γ and B_1 for different levels of government expenditures ($g_1 = \underline{g}_1$ and $g_1 = \overline{g}_1$). The no-equilibrium region, which exists for the same reasons as before, is shown in dark blue. In line with the characteristics of default incentives of the benchmark model, within the region where the equilibrium is well defined, for a given γ, the planner's value of default increases monotonically with the level of debt B_1. However, we observe that, contrary to the benchmark case, in the economy with foreign lenders, conditional on B_1, the welfare gain of default has an inverted-U shape in the γ dimension. That is, for a given B_1 the value of α decreases with γ for low γ, reaches a minimum point and then increases with γ. This also determines a bell-shaped "No Equilibrium Zone." The intuition for this result is simple and derives from the decision rules of domestic agents (see Figure A.12 of the Appendix for the corresponding plot). For a given level of B_1, when γ is below ($\gamma < 0.25$ for $B_1 = B_M$ for example), the country is on average a foreign borrower (i.e. $B^f > 0$). That implies that a default generates a direct increase in domestic resources equal to

[26] This assumption is made only to approximate the quantitative predictions of the model with the data but is not crucial for any of the results presented below.

the forgone debt payments to foreign lenders. In this region, both $L-$type and $H-$type agents are at the borrowing limit. As γ increases, the country becomes a net saver in foreign markets. Increases in γ are associated with an increasing portion of domestic debt in hands of $H-$types. This reduces the benefit of a default on foreign lenders. However, as in the model without foreign lenders, domestic consumption dispersion increases. For mid-range γ's the first effect dominates the second and repayment is the preferred option. As γ increases even further the dispersion in domestic consumption increases to points where default is again the optimal alternative for the government. In this region, the main driver of domestic default is redistribution among domestic agents as in our benchmark economy.

Figure 8: Planner's welfare gain of default $\overline{\alpha}(B_1, g_1, \gamma)$

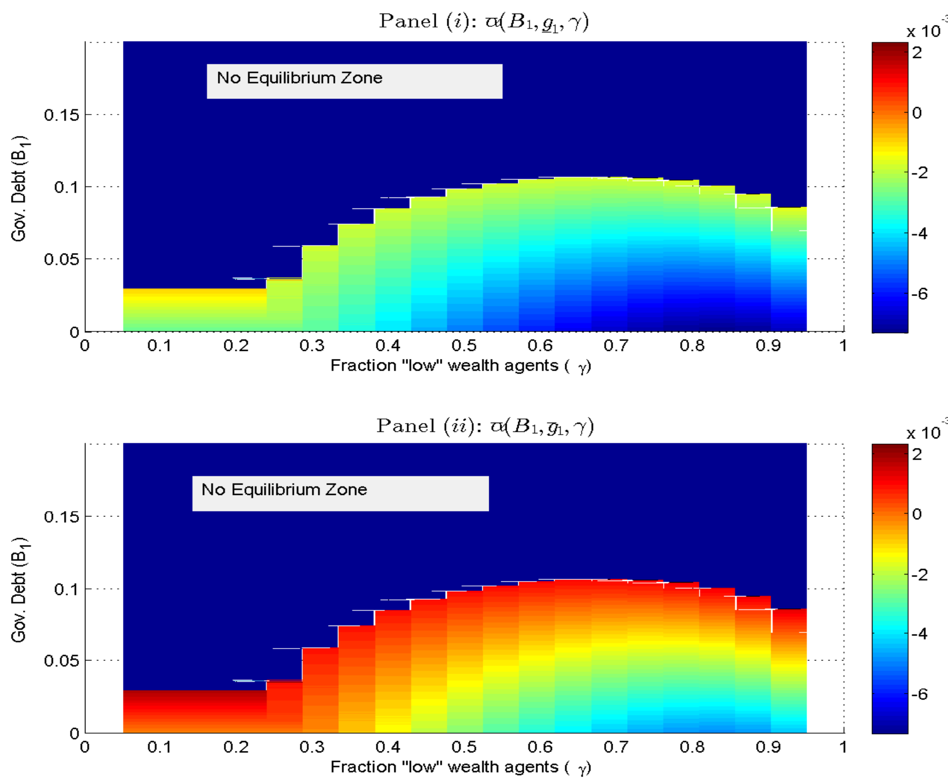
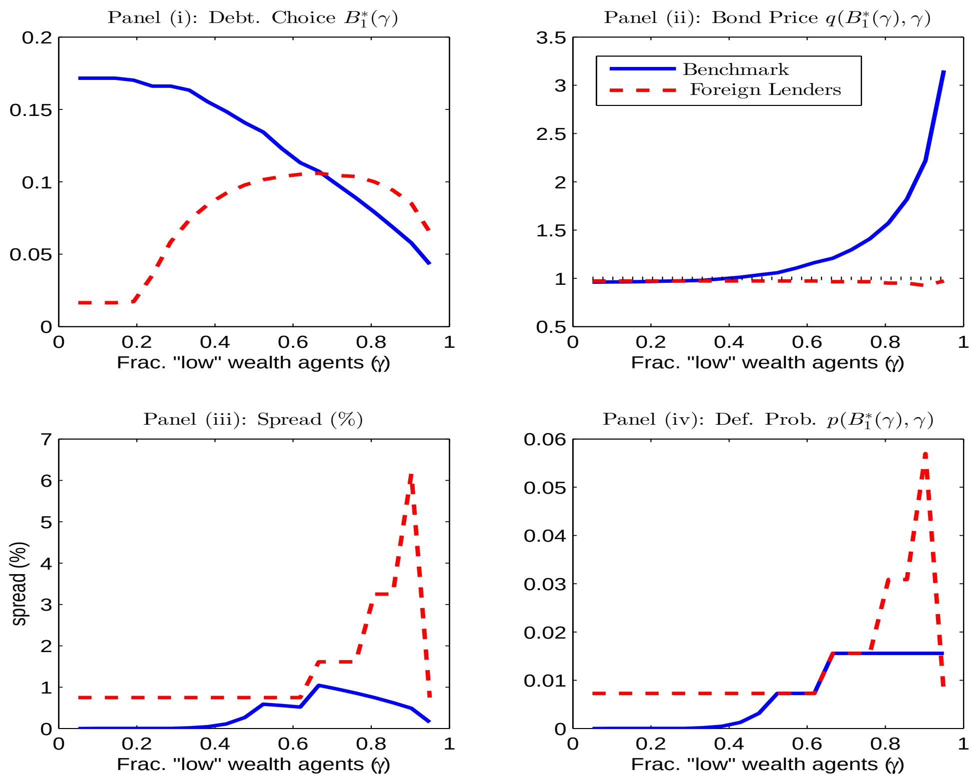


Figure 9 shows the comparison of the equilibrium functions in the economy with foreign lenders v. the benchmark economy. As we described before, the introduction of foreign lenders and the possibility of a "foreign" default constraint debt values and result in lower debt levels than in the benchmark, for values of γ lower than 0.75 (see Panel (i)). This is also reflected in higher default probabilities and spreads in the economy with foreign lenders than in the benchmark (see Panel (iii) and (iv)). By construction, the upper bound on the price of the economy with foreign lenders is $(1+r)^{-1}$, so the the distributive effect that negative real interest rates have in the benchmark economy dissipate (see Panel (*ii*)). This induces the government to take on more risk and redistribute via debt issuance in the economy with foreign lenders than in the benchmark.

Figure 9: Equilibrium Domestic and Foreign Debt Holdings



5.3 Redistributive Taxation (Partial Default)

In the benchmark model, issuing debt in the first period and defaulting in the second are the only tools the government can use to reduce consumption dispersion. We now examine how the model's predictions change by adding an alternative tool for redistribution. In particular, we introduce a proportional consumption tax that is invariant across periods and realizations of g_1.[27]

[27]Using this tax as opposed to other taxes simplifies the solution of the model, because aggregate consumption (as a function of y and g_t) is pinned down by the resource constraint, and thus known before solving the agents' problem. As a result, the algorithm does not need to iterate on the level of transfers to close the government budget constraint, while introducing a wealth tax would require it. An income tax would require not only iterating on aggregate transfers but also extending the model to consider endogenous labor supply, because the

With consumption taxes, the cost of purchasing goods in the left-hand-side of the agents' budget constraints becomes $(1+\tau^c)c_t^i$ for $i = L, H$ and $t = 0, 1$. Since aggregate consumption in each period is still given by $y - g_t$, the government budget constraints simplify to the following expressions. At date 0, the constraint is:

$$\tau_0 + \tau^c y = g_0(1 + \tau^c) + B_0 - q_0 B_1. \tag{23}$$

In the second period, under repayment, the constraint is:

$$\tau_1^{d_1=0} + \tau^c y = g_1(1 + \tau^c) + B_1. \tag{24}$$

In the second period, under default the budget constraint is:

$$\tau_1^{d_1=1} + \tau^c y(1 - \phi(g_1)) = g_1(1 + \tau^c). \tag{25}$$

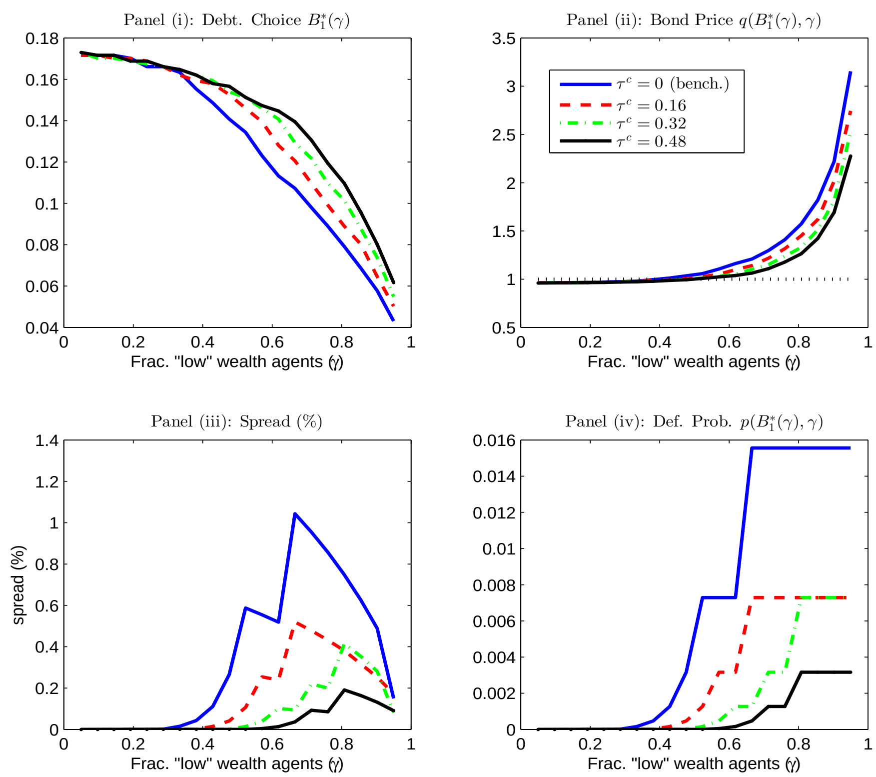
Using the budget constraints of agents and the government, we can proceed as with the benchmark model and derive expressions for the difference in consumption of the two agents in the two periods and default scenarios: $c_0^H - c_0^L = \frac{1}{1-\gamma}\left[\frac{B_0 - q(B_1,\gamma)B_1}{(1+\tau^c)}\right]$, $c_1^{H,d=0} - c_1^{L,d=0} = \frac{1}{1-\gamma}\frac{B_1}{(1+\tau^c)}$ and $c_1^{H,d=1} - c_1^{L,d=1} = 0$. These expressions are similar to those presented in Section 4, except that the terms that include public debt in the date-0 difference and the date-1 difference under repayment are divided by $(1 + \tau^c)$. Consumption taxes reduce consumption dispersion in both instances. Intuitively, the consumption tax plays a role akin to inflation in reducing the real value of public debt. Hence, this tax always reduces consumption dispersion in the first period (as long as there is a primary deficit), and also always reduces dispersion in the second period under repayment. In fact, assuming repayment, this tax is a better mechanism for redistribution because debt can only reduce dispersion at date 0 at the expense of increasing it a date 1. Moreover, the tax's ability to redistribute is not hampered by the debt Laffer curve that hampers the ability to redistribute with debt because of default risk. The tax acts in fact as a de-facto partial default in both periods. As a result, if we allow it to be chosen optimally, letting τ^c go to infinity would be optimal because it completely removes consumption dispersion in all periods at no cost. The purpose of this analysis, however, is to see how the existence of an alternative redistribution tool affects the results we have obtained for debt and default risk, rather than focus on the optimal use of the consumption tax.[28]

benchmark model assumes no income heterogeneity, which makes proportional income tax close to a lump-sum tax.

[28]The consumption tax as modeled lacks important trade-offs that would make it suboptimal to set it infinitely large. In particular, it is non-distortionary, because there is no labor supply choice, and because, since the tax

We solve the model numerically using the parameters of the benchmark calibration. For the value of the consumption tax, we use $\tau^c = 0.16$ which is the value estimated for the Eurozone in Mendoza, Tesar, and Zhang (2014). We also solve the model for $\tau^c = 0.32$ and $\tau^c = 0.48$, which are two and three times larger that the data proxy.[29] Figure 10 presents the comparison of the equilibrium of the benchmark economy with the three economies with taxes. This figure shows that the main result of the paper (the ability to support the existence of public debt with positive default risk) is robust to incorporating distributive taxes.

Figure 10: Comparison Equilibrium Benchmark vs Proportional Consumption Taxes



As consumption taxes increase, default incentives grow weaker (see lower default probabilities in Panel (iv)), which is natural because the tax reduces consumption dispersion, and hence reduces the need for the government to use default to lower dispersion. As a result, the government can sustain higher debt levels(see Panel (iv)), although the effect is not very large (debt

is constant, it does not distort savings decisions. The tax is also assumed to be known with perfect foresight, whereas uncertainty about inflation would have an effect similar to a partial risk of default that would affect demand for bonds and bond prices.

[29]In the tax rate estimates in Mendoza, Tesar, and Zhang (2014), the highest consumption tax is observed in Finland at 24 percent.

increases by at most 5 percentage points for $\gamma = 0.6$ and a change in the consumption tax from 0 to 48 percent). This effect is negligible when default incentives are weak to start with, which occurs at low values of γ. The properties of the price function observed in the benchmark case (i.e. increasing in γ for a given level of default risk) are sustained in the economy with consumption taxes (see Panel (*ii*)). In line with the reduction in default probabilities, the model with consumption taxes also displays lower spreads than the benchmark (see Panel (*iii*)).

The findings in this Section suggest that the existence of other tools for redistribution contributes to support higher debt levels at lower default frequencies. The results are also interesting as an illustration of what happens if we combine de-jure outright default with partial, de-facto default (the latter commonly takes place via inflation, but as noted earlier the consumption tax in this setup plays the same role as inflation in reducing the real debt burden).

5.4 A Second Asset

We review now the implications of adding a risk-free asset that agents can use as a vehicle of savings, in addition to public debt. In particular, agents have access to a non-stochastic production technology $y_t^i = z(k_t^i)^\theta$ with $0 < \theta < 1$, where y_t^i is total output and k_t^i is capital for agent of type i in period t, respectively.[30] The initial aggregate level of capital is denoted by K_0. L-type agents are now endowed with b_0^L and k_0^L units of public debt and capital respectively, while$H$-type agents have endowments given by $b_0^H = \frac{B_0 - \gamma b_0^L}{(1-\gamma)}$ and $k_0^H = \frac{K_0 - \gamma k_0^L}{(1-\gamma)}$. Capital depreciates at rate δ. At period 0, agents choose how much of their savings they want to allocate to public bonds $b_1^i \geq 0$ and capital $k_1^i \geq 0$.

The full characterization of the equilibrium and the detailed quantitative results are discussed in Section A.10 of the Appendix. Here we focus only on the main results. Intuitively, the existence of the second asset implies that agents arbitrage returns across public debt and risk-free capital, taking into account the risk of default on public debt. In particular, the optimality conditions for $H-$ type agents imply:

$$\frac{q_0}{\left[\theta z(k_1^i)^{\theta-1} + 1 - \delta\right]^{-1}} = \frac{E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1)\right]}{E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1) + u'(c_1^{i,d=1})d_1\right]}. \tag{26}$$

Thus, the default spread of this model (i.e. the gap between the yield on government bonds and the marginal productivity of capital) is determined by default risk, weighted by marginal utility in each state of the world, since agents are risk-averse. From this perspective, the second asset

[30]The curvature in production allows us to obtain a well-defined portfolio choice of bonds and capital for private agents.

has a similar effect as introducing the foreign investor, because it introduces an opportunity cost of funds, but with the difference that in this cost is now endogenous and falls as capital investment rises.

Adding the second asset also affects the government's debt and default choices. In particular, the government now has to consider the initial distributions of both bonds and capital across agents, and the effects of its choices on the individual capital and bond decisions $\{b_1^i, k_1^i\}$, which depend on B_1 and default risk. Default is more costly, because via its effects on capital allocations it can cause higher consumption dispersion than before.

To make this case as close as possible to the benchmark model for the quantitative analysis, we assume $k_0^i = K_0$ (i.e. all initial heterogeneity is in initial bond holdings). Still, this results in heterogeneous bond and capital holdings for the second period. We set z to normalize GDP to 1, $\theta = 0.33$ and$\delta = 0.10$ (standard values), and set K_0 so the capital to output ratio is equal to 2. All other parameters are the same as in the benchmark.

Agents in this setup have the option to switch from debt to capital as default risk rises, which affects adversely the capacity of the government to issue debt. On the other hand, the weaker default incentives because of the additional adverse consumption dispersion effects enhance the ability to issue debt. In line with these arguments, we found that for low enough γ, so that default is a zero-probability event, both models have nearly identical results. As γ rises up to $\gamma < 0.6$, the model with two assets sustains higher optimal debt than the benchmark (by at most 4 percentage points), but at higher values of γ the opposite is true (debt in the bechnmark is at most 2 percentage points higher). Thus, optimal debt levels do not differ by wide margins, even tough the effects of adding the second asset are noticeable.

The arbitrage condition connecting bond prices and capital returns also has interesting quantitative implications for spreads and bond prices. First, since reallocation of savings from debt to capital reduces the marginal product of capital, the default risk spread rises as γ rises, and this happens even tough the probability of default, which rises with γ, is actually lower than in the benchmark. Second, bond prices rise at a much lower rate as γ rises than in the benchmark, and in fact are never higher than 1 even at the highest values of γ considered (i.e. bond yields are never negative). In this dimension, the quantitative results are very different in the setup with two assets, although they are qualitatively similar.

6 Conclusions

This paper proposes a framework in which domestic sovereign default and public debt subject to default risk emerge as an equilibrium outcome. In contrast with standard models of sovereign default on external debt, this model highlights the role of the domestic distribution of public debt ownership and the distributional effects of default across domestic agents in shaping the government's default incentives. These are features common to both the historical episodes of outright domestic default documented by Reinhart and Rogoff (2011) and the ongoing European debt crisis.

In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Distributional incentives alone cannot support equilibria with debt, because default is always optimal in the second period, and hence the debt market cannot function in the first period. We also showed that equilibria with debt exposed to default risk can exist if we introduce either exogenous default costs or government preferences biased in favor of bond holders.

The main finding of the paper (i.e. that distributional incentives to default tempered by default costs can support equilibria with debt exposed to default risk) is robust to other three important extensions of the model: Adding foreign investors who can hold a portion of the debt issued by the government, introducing consumption taxes as an alternative tool for redistributing resources across agents, and adding a second asset as a vehicle for saving.

This paper is a first attempt at developing a blueprint for research into models of domestic sovereign default driven by distributional incentives, and their interaction with agent heterogeneity and incomplete insurance markets. It has two main limitations, both byproducts of the two-period life horizon: First, it takes as given a distribution of bond ownership across agents. Second, it does not capture endogenous costs of default that would result from losing the benefits of public debt as a vehicle for self-insurance and liquidity-provision (see Aiyagari and McGrattan (1998))). In further work D'Erasmo and Mendoza (2014)) we are aiming to develop a framework that takes both issues into account.

References

Aghion, P. and P. Bolton, 1990. "Government Domestic Debt and the Risk of Default: A Political-Economic Model of the Strategic Role of Debt." *In Public Debt Management: Theory and History, edited by Rudiger Dornbusch and Mario Draghi. New York: Cambridge University Press*: 31544.

Aguiar, M. and M. Amador, 2013. "Fiscal Policy in Debt Constrained Economies." *NBER Working Papers 17457*.

Aguiar, M. and M. Amador, 2014. "Sovereign Debt." *Handbook of International Economics*, 4: 647–87.

Aguiar, M. and G. Gopinath, 2006. "Defaultable debt, interest rates and the current account." *Journal of International Economics*, 69(1): 64–83.

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath, 2013. "Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises." *NBER Working Papers 19516*.

Aiyagari, R. and E. McGrattan, 1998. "The optimum quantity of debt." *Journal of Monetary Economics*, 42: 447–469.

Aiyagari, R., A. Marcet, T. Sargent, and J. Seppala, 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy*, 110(6): 1220–1254.

Alesina, A. and G. Tabellini, 1990. "A Positive Theory of Fiscal Deficits and Government Debt." *The Review of Economic Studies*, 57: 403–414.

Amador, M., 2003. "A Political Economy Model of Sovereign Debt Repayment." *Stanford University mimeo*.

Andreasen, E., G. Sandleris, and A. Van der Ghote, 2011. "The Political Economy of Sovereign Defaults." *Universidad Torcuato Di Tella, Business School Working Paper*.

Arellano, C., 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review*, 98(3): 690–712.

Azzimonti, M., E. de Francisco, and V. Quadrini, 2014. "Financial Globalization, Inequality, and the Rising Public Debt." *American Economic Review*, 104(8): 2267–2302.

Barro, R., 1979. "On the determination of the public debt." *Journal of Political Economy*, 87(5): 940–971.

Basu, S., 2009. "Sovereign debt and domestic economic fragility." *Manuscript, Massachusetts Institute of Technology*.

Bocola, L., 2014. "The Pass-Through of Sovereign Risk." *Manuscript, University of Pennsylvania*.

Boz, E., P. D'Erasmo, and B. Durdu, 2014. "Sovereign Risk and Bank Balance Sheets: The Role of Macroprudential Policies." *Manuscript.*

Broner, F. and J. Ventura, 2011. "Globalization and Risk Sharing." *Review of Economic Studies*, 78(1): 49–82.

Broner, F., A. Martin, and J. Ventura, 2010. "Sovereign Risk and Secondary Markets." *American Economic Review*, 100(4): 1523–55.

Brutti, 2011. "Sovereign defaults and liquidity crises." *Journal of International Economics*, 84 (1): 6572.

Calvo, G., 1988. "Servicing the Public Debt: The Role of Expectations." *American Economic Review*, 78(4): 647–61.

Cuadra, G., J. Sanchez, and H. Sapriza, 2010. "Fiscal Policy and Default Risk in Emerging Markets." *Review of Economic Dynamics*, 13(2): 452–469.

Davies, J., S. Sandstrm, A. Shorrocks, and E. Wolff, 2009. "The Level and Distribution of Global Household Wealth." *NBER Working Paper 15508.*

Davies, J., R. Lluberas, and A. Shorrocks, 2012. "Credit Suisse Global Wealth Databook." *Zurich: Credit Suisse Reseach Institute.*

D'Erasmo, P., 2011. "Government Reputation and Debt Repayment in Emerging Economies." *mimeo.*

D'Erasmo, P. and E. Mendoza, 2014. "Optimal Domestic Sovereign Default." *Manuscript, University of Pennsylvania.*

Di Casola, P. and S. Sichlimiris, 2014. "Domestic and External Sovereign Debt." *Working Paper, Stockholm School of Economics.*

Dias, D., C. Richmond, and M. Wright, 2012. "In for a Penny, In for a 100 Billion Pounds: Quantifying the Welfare Benefits from Debt Relief." *mimeo.*

Dixit, A. and J. Londregan, 2000. "Political Power and the Credibility of Government Debt." *Journal of Economic Theory*, 94: 80105.

Dovis, A., M. Golosov, and A. Shourideh, 2014. "Sovereign Debt vs Redistributive Taxes: Financing Recoveries in Unequal and Uncommitted Economies." *mimeo.*

Eaton, J. and M. Gersovitz, 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies*, 48(2): 289–309.

Ferriere, 2014. "Sovereign Default, Inequality, and Progressive Taxation." *mimeo.*

Floden, M., 2001. "The effectiveness of government debt and transfers as insurance." *Journal of Monetary Economics*, 48: 81–108.

Gennaioli, N., A. Martin, and S. Rossi, 2014. "Sovereign default, domestic banks and financial institutions." *Journal of Finance*.

Golosov, M. and T. Sargent, 2012. "Taxation, redistribution, and debt with aggregate shocks." *working paper Princeton University*.

Guembel, A. and O. Sussman, 2009. "Sovereign Debt without Default Penalties." *Review of Economic Studies*, 76: 1297–1320.

Hall, G. and T. Sargent, 2014. "Fiscal Discrimination in Three Wars." *Journal of Monetary Economics*, 61: 148–166.

Hatchondo, J.C., L. Martinez, and H. Sapriza, 2009. "Heterogeneous Borrowers in Quantitative Models of Sovereign Default." *International Economic Review*, 50: 12951.

Heathcote, J., 2005. "Fiscal Policy with Heterogeneous Agents." *Review of Economic Studies*, 72: 161–188.

Jeon, K. and Z. Kabukcuoglu, 2014. "Income Inequality and Sovereign Default." *working paper, University of Pittsburgh*.

Lorenzoni, G. and I. Werning, 2013. "Slow moving debt crises." *NBER Working Paper No. w19228*.

Mendoza, E.G. and V.Z. Yue, 2012. "A General Equilibrium Model of Sovereign Default and Business Cycles." *Quarterly Journal of Economics*, 127(2): 889–946.

Mendoza, E.G., L. Tesar, and J. Zhang, 2014. "Saving europe?: The unpleasant arithmetic of fiscal austerity in integrated economies." *National Bureau of Economic Research, No. w20200*.

Mengus, E., 2014. "Honoring Sovereign Debt or Bailing Out Domestic Residents? A Theory of Internal Cost of Default." *WP Banque de France 480*.

Panizza, U., F. Sturzenegger, and J. Zettelmeyer, 2009. "The Economics and Law of Sovereign Debt and Default." *Journal of Economic Literature*, 47(3): 651–98.

Perez, D., 2015. "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity." *Manuscript*.

Pitchford and Wright, 2012. "Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment." *Review of Economic Studies*, 79(2): 812–837.

Pouzo, D. and I. Presno, 2014. "Optimal Taxation with Endogenous Default under Incomplete Markets." *U.C. Berkeley, mimeo*.

Reinhart, C. and K. Rogoff, 2011. "The Forgotten History of Domestic Debt." *Economic Journal*, 121, Issue 552: 319–350.

Sosa-Padilla, C., 2012. "Sovereign Defaults and Banking Crises." *Manuscript*.

Tabellini, G., 1991. "The Politics of Intergenerational Redistribution." *Journal of Political Economy*, 99: 33557.

Tauchen, G., 1986. "Finite state Markov-chain approximation to univariate and vector autoregressions." *Economics Letters*, 20: 177–81.

Tomz and Wright, 2012. "Empirical research on sovereign debt and default." *Working Paper Series WP-2012-06, Federal Reserve Bank of Chicago*.

Vasishtha, 2010. "Domestic versus external borrowing and fiscal policy in emerging markets." *Review of International Economics*, 18(5): 1058–1074.

Yue, V., 2010. "Sovereign Default and Debt Renegotiation." *Journal of International Economics*, 80 (2): 176–187.

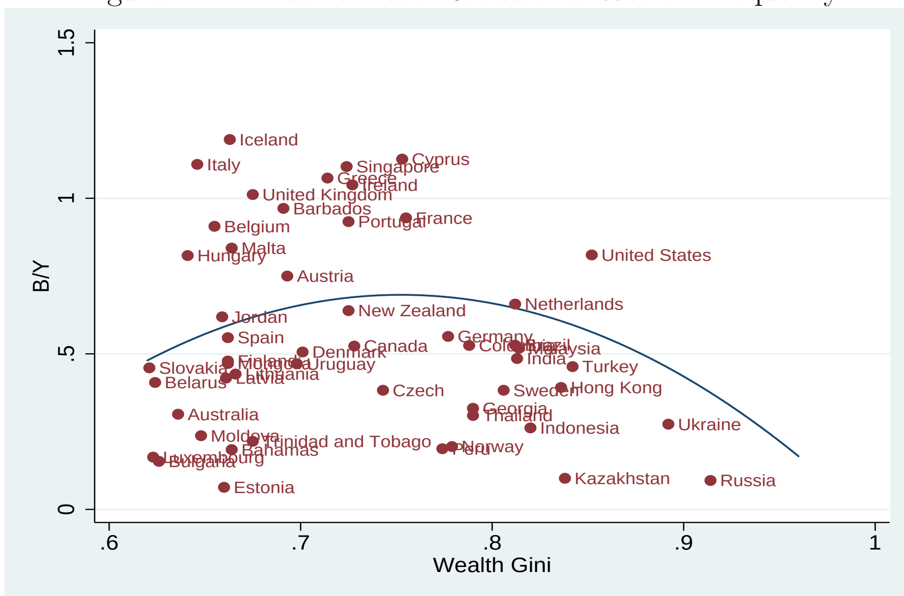
Appendix
Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default

This Appendix includes additional material that provides background for some of the arguments made in the paper. Section A.1 provides an empirical analysis of the relationship between domestic debt and inequality. Section A.2 presents a deterministic version of our model without default risk and log preferences. Section A.3 extends this simplified model to incorporate aggregate risk. Section A.4 derives analytically the equilibrium price function for this model and describes its determinants. Section A.5 provides the details of the calibration exercise presented in the paper. Section A.6 displays a set of additional figures of the benchmark model. Section A.7 contains a sensitivity analysis of the benchmark model. Sections A.8 and A.9 introduce additional figures for the extension of the model with biased welfare weights and foreign investors, respectively. Finally, Section A.10 describes the extension of the model with two assets in detail and presents a set of figures that complement those in the main body of the paper.

A.1 Empirical Analysis Debt and Inequality

Further and more systematic empirical analysis of the relationship between wealth inequality and public debt shows a statistically significant, bell-shaped relationship: Debt is increasing in wealth inequality when inequality is low, until the Gini coefficient reaches about 0.75, and then becomes decreasing in wealth inequality. This finding follows from estimating a cross-country panel regression using the international database of Gini coefficients for wealth produced by Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012), together with data on public debt-output ratios and control variables (i.e. the size of the government, proxied by the ratio of government expenditures to GDP, and country fixed effects). The results of panel regressions of the debt ratio on the Gini coefficient, the square of this coefficient, and the controls (using the observations for 2000 and 2012) are shown in Table A.3, and Figure A.11 shows a plot of the 2012 data and the fitted regression curve. The regressions only explain about a tenth of the cross-country variations on debt ratios, but the linear and quadratic effects of the wealth Gini coefficient are statistically significant and opposite in sign.

Figure A.11: Public Debt Ratios and Wealth Inequality



Note: Author's calculations based on WDI, Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012). Fitted line from results presented in Table A.3

Table A.3: Panel Regressions of Public Debt Ratios and Wealth Inequality

	Panel Country Fixed Effect	
	Dep. Var: Debt to GDP	
Wealth Gini	17.65**	18.11**
s.e.	7.06	7.22
Wealth Gini^2	-11.80**	-12.03**
s.e.	4.76	4.85
G/Y	-	-0.010
s.e.	-	0.012
avg. country FE	-5.92**	-5.95**
s.e.	2.60	2.64
years	2000-2012	2000-2012
R^2	0.118	0.136
# obs.	139	139

Note: Author's calculations based on WDI, Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012). Debt to GDP refers to Central Government Debt as share of GDP (source WDI). Wealth Gini as reported for each country for year 2000 and 2012 by Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012). G/Y corresponds to government final consumption as a share of GDP (source WDI). Coefficients and standard errors reported are derived from a panel linear regression with Country Fixed Effects. The sample contains 87 countries with data for years 2000 and 2012. If debt to GDP or government expenditure ratio is not available for year 2000 or 2012 data for year 1999 or 2011 is used.

A.2 Deterministic Model without Default Risk and Log Utility

This part of the Appendix derives solutions for a version of the model in which low-wealth (L) types do not hold any bonds and high-wealth (H) types buy all the debt. We cover first the fully deterministic case, without any shocks to income or government policies, and no default risk, but government expenditures may be deterministically different across periods. Government wants to use debt to relocate consumption across agents and across periods optimally given a utilitarian welfare function. Ruling out default on initial outstanding debt, the planner trades off the desire to use debt to smooth taxation for L types (reduce date-0 taxes by issuing debt) against the cost of the postponement of consumption this induces on H types who save to buy the debt. In what follows, we work with cases where $b_1^L = 0$ which might not hold depending on the parameters of the model; however, it is a relevant benchmark since this holds for our baseline parameterization and all the extensions of the model we present.[31] This assumption together with log utility provides closed form solutions. The goal is to illustrate the mechanisms that are driving the model when default risk and stochastic government purchases are taken out. Later in the Appendix we derive some results for the model with stochastic government purchases, and make some inferences for the case with default risk.

A.2.1 Households

A fraction γ of agents are L types, and $1-\gamma$ are H types. Preferences are:

$$\ln(c_0^i) + \beta \ln(c_1^i) \qquad for \quad i = L, H \tag{A.1}$$

If $b_1^L = 0$, budget constraints are:

$$\begin{aligned} c_0^L &= y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - q b_1^H && \text{(A.2)} \\ c_1^L &= y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H && \text{(A.3)} \end{aligned}$$

Since L types do not save they can only consume what their budget constraints allow. This is important because altering taxes affects disposable income, which will in turn affect the optimal debt choice of the government. For H types, the Euler equation is:

$$q = \beta \frac{c_0^H}{c_1^H} \tag{A.4}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, it must be the case that they are credit constrained (i.e. they would want to hold negative assets). That is, at the equilibrium price of debt their Euler equation for bonds would satisfy:

$$q > \beta \frac{c_0^L}{c_1^L} \tag{A.5}$$

[31] The $L-$ type agents will be credit constrained (i.e. $b_1^L = 0$) the lower b_0^L, B_0 and B_1.

A.2.2 Government

The government budget constraints are:

$$\tau_0 = g_0 + B_0 - qB_1 \tag{A.6}$$
$$\tau_1 = g_1 + B_1 \tag{A.7}$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma(\ln(c_0^L) + \beta \ln(c_1^L)) + (1-\gamma)(\ln(c_0^H) + \beta \ln(c_1^H)) \tag{A.8}$$

A.2.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $\gamma b_1^L + (1-\gamma) b_1^H = B_1$. By construction the same condition is assumed to hold for the initial conditions b_0^H and B_0. Since $b_0^L = 0$, this implies that the initial wealth of H-types is given by $b_0^H = B_0/(1-\gamma)$.

Rewriting the Euler equation of H types using the budget constraint, the government budget constraints and the bond market-clearing conditions when $b_1^L = 0$ yields:

$$q = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q\left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \tag{A.9}$$

Hence, the equilibrium price of bonds for a given government supply is:

$$q(B_1) = \beta \frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta) B_1} \tag{A.10}$$

Note that this price is not restricted to be lower than 1 (i.e. $q(B_1) > 1$ which implies a negative real rate of return on government debt can be an equilibrium outcome). In particular, as γ rises the per capita bond demand of H-types increases and this puts upward pressure on bond prices, and even more so if the government finds it optimal to offer less debt than the initial debt, as we showed numerically and explain further below. As $\gamma \to 1$, the limit of the equilibrium price goes to $q(B_1) = \frac{\beta}{1+\beta}\frac{B_0}{B_1}$ even tough market-clearing requires the demand of the infinitesimal small H type to rise to infinity.

After some simplification, the derivative of this price is given by:

$$q'(B_1) = \frac{-q(B_1)\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)}{\left[y - g_1 + \left(\frac{\gamma}{1-\gamma}\right)(1+\beta) B_1\right]} \tag{A.11}$$

which at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$ (notice $c_1^H > 0$ implies that the denominator of this expression must be positive).

Consider now what happens to this equilibrium as the fraction of L-types vanishes. As $\gamma \to 0$, the economy converges to a case where there is only an H type representative agent, and the price is simply $q(B_1) = \beta \frac{y-g_0}{y-g_1}$, which is in fact independent of B_1 and reduces to β if government purchases are stationary. Trivially, in this case the planner solves the same problem as the representative agent and the equilibrium is efficient. Also, for an exogenously given B_0 and stationary g, the competitive equilibrium is stationary at this consumption level:

$$c^H = y - g + \left(\frac{\gamma}{1-\gamma}\right) \frac{B_0}{1+\beta} \tag{A.12}$$

and the optimal debt is:

$$B_1 = \frac{B_0}{1+\beta} \tag{A.13}$$

Hence, in this case consumption and disposable income each period are fully stationary, yet the optimal debt policy is always to reduce the initial debt by the fraction $1/(1+\beta)$. This is only because of the two-period nature of the model. With an infinite horizon, the same bond price would imply that an equilibrium with stationary consumption and an optimal policy that is simply $B_1 = B_0$. It also follows trivially that carrying no initial debt to start with would be first-best, using lump-sum taxation to pay for g.

A.2.4 Optimal debt choice

The government's optimal choice of B_1 in the first period solves this maximization problem:

$$\max_{B_1} \left\{ \begin{array}{c} \gamma \left[\ln\left(y - g_0 - B_0 + q(B_1)B_1\right) + \beta \ln\left(y - g_1 - B_1\right)\right] \\ +(1-\gamma)\left[\ln\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q(B_1)\left(\frac{\gamma}{1-\gamma}\right) B_1\right) + \beta \ln(y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1)\right] \end{array} \right\} \tag{A.14}$$

where $q(B_1) = \beta \frac{y-g_0+\left(\frac{\gamma}{1-\gamma}\right)B_0}{y-g_1+\left(\frac{\gamma}{1-\gamma}\right)(1+\beta)B_1}$.

The first-order condition is:

$$\begin{aligned} &\gamma \left[u'(c_0^L)\left[q'(B_1)B_1 + q(B_1)\right] - \beta u'(c_1^L)\right] \\ &+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right)\left[-u'(c_0^H)\left[q'(B_1)B_1 + q(B_1)\right] + \beta u'(c_1^H)\right] = 0 \end{aligned} \tag{A.15}$$

Using the Euler equation of the H types and simplifying:

$$u'(c_0^L) + \left[\frac{u'(c_0^L)q(B_1) - \beta u'(c_1^L)}{q'(B_1)B_1}\right] = u'(c_0^H) \tag{A.16}$$

This expression is important, because it defines a wedge between equating the two agents' marginal utility of consumption that the planner finds optimal to maintain, given that the only instrument that it has to reallocate consumption across agents is the debt. Notice that, since as noted earlier for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation implies that at the equilibrium price: $u'(c_0^L)q(B_1) - \beta u'(c_1^L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L) > u'(c_0^H)$ or $c_0^H > c_0^L$, and notice that from the budget constraints this also implies $B_0 - q(B_1)B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$.

Comparison with no-debt equilibrium: Notice that since $B_0 - q(B_1)B_1 > 0$, the planner is allocating less utility to L type agents than those agents would attain without any debt. Without debt, and a tax policy $\tau_t = g_t$, all agents consume $y - g_t$ every period, but with debt L-types consume less each period given that $B_1 > 0$ and $B_0 - q(B_1)B_1 > 0$. Compared with these allocations, when the planner finds optimal to choose $B_1 > 0$ is because he is trading off the pain of imposing higher taxes in both periods, which hurts L types, against the benefit the H types get of having the ability to smooth using government bonds. Also, $B_0 - q(B_1)B_1 > 0$ highlights that there is a nontrivial role to the value of B_0, because if B_0 were zero B_1 would need to be negative which is not possible by construction. Hence, the model only has a sensible solution if there is already enough outstanding debt (and wealth owned by H type agents) that gives the government room to be able to improve the H type's ability to smooth across the two periods, which they desire to do more the higher is B_0.

Comparison with sub-optimal debt equilibrium: By choosing positive debt, the government provides tax smoothing for L types. Given B_0 and the fact that $B_0 - q(B_1)B_1 > 0$, positive debt allows to lower date-0 taxes, which increases consumption of L types (since $c_0^L = y - g_0 - B_0 + q(B_1)B_1$). The same policy lowers the consumption of H types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)(B_0 - q(B_1)B_1)$). Hence, debt serves to redistribute consumption across the two agents within the period. This also changes inter-temporal consumption allocations, with the debt reducing L types consumption in the second period and increasing H types consumption. Hence, with commitment to repay B_0, the debt will be chosen optimally to trade off these social costs and benefits of issuing debt to reallocate consumption atemporally across agents and intertemporally.

It is also useful to notice that the demand elasticity of bonds is given by $\eta \equiv q(B_1)/\left(q'(B_1)B_1\right)$, so the marginal utility wedge can be expressed as $\eta\left[u'(c_0^L) - \frac{\beta u'(c_1^L)}{q(B_1)}\right]$ and the planner's optimality condition reduces to:

$$1 + \eta\left[1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)}\right] = \frac{u'(c_0^H)}{u'(c_0^L)} \tag{A.17}$$

Hence, the planner's marginal utility wedge is the product of the demand elasticity of bonds and the L-type agents shadow value of being credit constrained (the difference $1 - \frac{\beta u'(c_1^L)}{q(B_1)u'(c_0^L)} > 0$, which can be interpreted as an effective real interest rate faced by L-type agents that is higher than the return on bonds). The planner wants to use positive debt to support an optimal wedge in marginal utilities only when the demand for bonds is elastic AND L-type agents are constrained.

A.3 Extension to Include Government Expenditure Shocks

Now consider the same model but government expenditures are stochastic. In particular, realizations of government purchases in the second period are given by the set $[g_1^1 < g_1^2 < ... < g_1^M]$ with transition probabilities denoted by $\pi(g_1^i|g_0)$ for $i = 1, ..., M$ with $\sum_{i=1}^{M} \pi(g_1^i|g_0) = 1$.

A.3.1 Households

Preferences are now:

$$\ln(c_0^i) + \beta \left(\sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^i) \right) \qquad for \quad i = L, H \tag{A.18}$$

Budget constraints are unchanged:

$$c_0^L = y - \tau_0, \qquad c_0^H = y - \tau_0 + b_0^H - q b_1^H \tag{A.19}$$
$$c_1^L = y - \tau_1, \qquad c_1^H = y - \tau_1 + b_1^H \tag{A.20}$$

We still analyze cases where L types do not save, so they only consume what their budget constraints allow. For H types, the Euler equation becomes:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \left(\frac{c_0^H}{c_1^H} \right) \tag{A.21}$$

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, their Euler equation for bonds must satisfy:

$$q > \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \left(\frac{c_0^L}{c_1^L} \right) \tag{A.22}$$

A.3.2 Government

The government budget constraints are unchanged:

$$\tau_0 = g_0 + B_0 - q B_1$$
$$\tau_1 = g_1 + B_1$$

The initial debt $B_0 \geq 0$ is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

$$\gamma \left(\ln(c_0^L) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^L) \right) + (1-\gamma) \left(\ln(c_0^H) + \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \ln(c_1^H) \right) \tag{A.23}$$

A.3.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt B_1 is given by a price q that satisfies the market-clearing condition of the bond market: $b_1^H = B_1/(1-\gamma)$. When $b_1^L = 0$ the equilibrium price can be derived from the H-types Euler equation.

We can solve the model in the same steps as before. First, rewrite the Euler equation of H types using their budget constraints, the government budget constraints and the market-clearing conditions:

$$q = \beta \sum_{i=1}^{M} \pi(g_1^i|g_0) \left[\frac{y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 - q \left(\frac{\gamma}{1-\gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1} \right] \tag{A.24}$$

From here, we can solve again for the equilibrium price at a given supply of bonds:

$$q(B_1) = \beta \frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1}\right)}{1 + \left(\frac{\gamma}{1-\gamma}\right) \beta B_1 \left(\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1 + \left(\frac{\gamma}{1-\gamma}\right) B_1}\right)} \tag{A.25}$$

As $\gamma \to 0$ we converge again to the world where there is only an H type representative agent, but now the pricing formula reduces to the standard formula for the pricing of a non-state-contingent asset $q(B_1) = \beta \left(\sum_{i=1}^{M} \pi(g_1^i|g_0) \frac{y-g_0}{y-g_1}\right)$. As $\gamma \to 1$ the equilibrium degenerates again into a situation where market clearing requires the demand of the infinitesimal small H type to rise to infinity.

The derivative of the price at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$. To show this, define $\Pi(B_1) \equiv \sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)}{y - g_1^i + \left(\frac{\gamma}{1-\gamma}\right) B_1}$ which yields $\Pi'(B_1) = -\sum_{i=1}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y - g_1^i + \left(\frac{\gamma}{1-\gamma}\right) B_1\right)^2} < 0$. Then taking the derivative $q'(B_1)$ and simplifying we get:

$$q'(B_1) = \frac{\beta \left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0\right) \left[\Pi'(B_1) - \beta \left(\frac{\gamma}{1-\gamma}\right) (\Pi(B_1))^2\right]}{\left(1 + \beta \left(\frac{\gamma}{1-\gamma}\right) B_1 \Pi(B_1)\right)^2} \tag{A.26}$$

Since $\Pi'(B_1) < 0$ and positive c_0^H implies $y - g_0 + \left(\frac{\gamma}{1-\gamma}\right) B_0 > 0$, it follows that $q'(B_1) < 0$.

We can also gain some insight into the implicit risk premium (the ratio $q(B_1)/\beta$)) and the related question of why the asset price can exceed 1 in this setup. Recall that in fact the latter was already possible without uncertainty when γ is large enough, because of the demand composition effect (higher γ implies by market clearing that the fewer H type agents need to demand more bonds per capita, so the bond price is increasing in γ and can exceed 1). The issue now is that, as numerical experiments show, an increase in the variance of g_1 also results in higher bond prices, and higher than in the absence of uncertainty, and again for γ large enough we get both $q(B_1) > 1$ and $q(B_1)/\beta > 1$. The reason bond prices increase with the variability of government purchases is precautionary savings. Government bonds are the only vehicle of saving, and the incentive for this gets stronger the larger the variability of g_1. Hence, the price of bonds is *higher* in this stochastic model than in the analogous deterministic model because of precautionary demand for bonds, which adds to the effect of demand composition (i.e. the price is higher with uncertainty than without at a given γ).

A.3.4 Optimal debt choice

The government's optimal choice of B_1 solves again a standard maximization problem:

$$\max_{B_1}\left\{\begin{array}{c}\gamma\left[\ln\left(y-g_0-B_0+q(B_1)B_1\right)+\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)\ln\left(y-g_1-B_1\right)\right]\\ +(1-\gamma)\left[\ln\left(y-g_0+\left(\frac{\gamma}{1-\gamma}\right)B_0-q(B_1)\left(\frac{\gamma}{1-\gamma}\right)B_1\right)\right.\\ \left.+\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)\ln(y-g_1+\left(\frac{\gamma}{1-\gamma}\right)B_1)\right]\end{array}\right\} \tag{A.27}$$

where $q(B_1)$ is given by the expression solved for in the competitive equilibrium.

The first-order condition is:

$$\gamma\left[u'(c_0^L)\left[q'(B_1)B_1+q(B_1)\right]-\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)u'(c_1^L)\right] \tag{A.28}$$

$$+(1-\gamma)\left(\frac{\gamma}{1-\gamma}\right)\left[-u'(c_0^H)\left[q'(B_1)B_1+q(B_1)\right]+\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)u'(c_1^H)\right]=0$$

Using the stochastic Euler equation of the H types and simplifying:

$$u'(c_0^L)\left[q'(B_1)B_1+q(B_1)\right]-\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)u'(c_1^L)=u'(c_0^H)q'(B_1)B_1 \tag{A.29}$$

$$u'(c_0^L)+\left[\frac{u'(c_0^L)q(B_1)-\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)u'(c_1^L)}{q'(B_1)B_1}\right]=u'(c_0^H) \tag{A.30}$$

This last expression, compared with the similar expression of the planner without uncertainty, implies that in the planner's view, the government expenditure shocks only matter to the extent that they affect the shadow price of the binding credit constraint of the L types. As before, since for L types to find it optimal to hold zero assets it must be that they are "credit constrained," their Euler equation would imply that at the equilibrium price: $u'(c_0^L)q(B_1)-\beta\sum_{i=1}^{M}\pi(g_1^i|g_0)u'(c_1^L)>0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c_0^L)>u'(c_0^H)$ or $c_0^H>c_0^L$, and notice that from the budget constraints this implies again $B_0-q(B_1)B_1>0$, which implies

$B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial B_0 for any $q(B_1) \geq 1$, and also for "sufficiently high" $q(B_1)$. Thus the optimal debt choice again has an incentive to be lower than the initial debt.

A.4 Pricing Function: Stochastic Model with Default

We can also make an inference about what the pricing function looks like in the model with default risk, because with default we have a similar Euler equation, except that the summation that defines the term $\Pi(B_1)$ above will exclude all the states of g_1 for which the government chooses to default on a given B_1 (and also at a given value of γ). That is, the term in question becomes $\Pi^D(B_1) \equiv \sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)}{y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1} \leq \Pi(B_1)$, and the pricing function with default risk is:

$$q^D(B_1) = \beta\frac{\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right)\Pi^D(B_1)}{1+\left(\frac{\gamma}{1-\gamma}\right)\beta B_1\Pi^D(B_1)} \leq q(B_1) \tag{A.31}$$

Moreover, it follows from the previous analysis that this pricing function is also decreasing in B_1 ($q^{D\prime}(B_1) < 0$), and $\Pi^{D\prime}(B_1) = -\sum_{\{i:d(B_1,g_1^i,\gamma)=0\}}^{M} \frac{\pi(g_1^i|g_0)\left(\frac{\gamma}{1-\gamma}\right)}{\left(y-g_1^i+\left(\frac{\gamma}{1-\gamma}\right)B_1\right)^2}$is negative but such that $\Pi'(B_1) \leq \Pi^{D\prime}(B_1) < 0$. Also, it is clear from the above pricing functions that if the probability of default is small, so that are only a few values of i for which $d(B_1, g_1^i, \gamma) = 1$ and/or the associated probability $\pi(g_1^i|g_0)$ is very low, the default pricing function will be very similar to the no-default pricing function.

If we define the default risk spread as $S(B_1, \gamma) \equiv [1/q^D(B_1, \gamma)] - [1/q(B_1, \gamma)]$, where γ has been added as an argument of the price functions because those prices also depend on variations in inequality, the spread reduces to the following expression:

$$S(B_1, \gamma) = \left(\frac{1}{\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right)}\right)\left[\frac{1}{\Pi^D(B_1,\gamma)} - \frac{1}{\Pi(B_1,\gamma)}\right]$$

Clearly since $\Pi^D(B_1, \gamma) \leq \Pi(B_1, \gamma)$ the spread is non-negative, and it is strictly positive if there is default at equilibrium. The spread is increasing in B_1, because as the debt rises default is chosen optimally in more of the possible realizations of g_1^i and hence $\Pi^D(B_1, \gamma)$ falls further below $\Pi(B_1, \gamma)$, so that the gap between the reciprocals of these two terms widens. Note also that the spread is a multiple of the gap between these reciprocals, with the multiple given by $1/\beta\left(y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0\right)$. As a result, the total date-0 resources available for consumption of the H-types ($y - g_0 + \left(\frac{\gamma}{1-\gamma}\right)B_0$) have a first-order *negative* effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. The are also second order effects, because the equilibrium allocation of B_1 also depends on that income measure, and thus $\Pi^D(B_1, \gamma)$ and $\Pi(B_1, \gamma)$ vary with it as well, but these are not considered here.

In terms of the effect of changes in γ on S, notice that there are two effects. First, there is a negative effect because higher γ means that, for a given B_0, the resources available for date-0 consumption of H types increase, since fewer H type agents need to demand enough initial bonds to clear the bond market, which means that per-capita each of the H types hold more date-0 bonds and have more bond income. Second, there is a positive effect because rising γ weakens default incentives as the welfare of the wealthy is valued more, and hence default is optimally chosen in more states, which increases $\left[1/\Pi^D(B_1,\gamma) - 1/\Pi(B_1,\gamma)\right]$. Thus, in principle the response of the spread to increases in inequality is ambiguous. The weaker the response of the default probability to changes in γ, however, the more likely it is that the first effect will dominate and the spreads will be a decreasing function of inequality.

A.5 Calibration of the Benchmark Model

The model is calibrated to annual frequency, and most of the parameter values are set so as to approximate some of the model's predicted moments to those observed in the European data. The preference parameters are set to standard values: $\beta = 0.96$, $\sigma = 1$. We also assume for simplicity that L types start with zero wealth, $b_0^L = 0$.[32] We assume that

$$\ln(g_1) \sim N\left((1-\rho_g)\ln(\mu_g) + \rho_g \ln(g_0), \frac{\sigma_e^2}{(1-\rho_g^2)}\right).$$ [33]

The parameters of this process are pinned down by estimating an AR(1) model with 1995-2012 data of the government expenditures-GDP ratio (in logs) for France, Germany, Greece, Ireland, Italy, Spain and Portugal. Given the parameter estimates for each country, we set μ_g, ρ_g and σ_e to the corresponding cross-country average. This results in the following values $\mu_g = 0.1812$, $\rho_g = 0.8802$ and $\sigma_e = 0.017$. Given these moments, we set $g_0 = \mu_g$ and use Tauchen (1986) quadrature method with 45 nodes in $G_1 \equiv \{\underline{g}_1, \ldots, \overline{g}_1\}$ to generate the realizations and transition probabilities of the Markov process that drives expectations about g_1.[34]

We abstain from setting a calibrated value for γ and instead show results for $\gamma \in [0,1]$. Note, however, that data for the United States and Europe suggest that the empirically relevant range for γ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of γ is useful we use $\gamma = 0.7$, which is the mid point of the plausible range.[35]

[32] $\sigma = 1$ and $b_0^L = 0$ are also useful because, as we show in the Appendix, under these assumptions we can obtain closed-form solutions and establish some results analytically.

[33] This specification allows us to control the correlation between g_0 and g_1 via ρ_g, the mean of the shock via μ_g and the variance of the unpredicted portion via σ_e^2. Note that if $g_0 = \mu_g$, $\ln(g_1) \sim N(\ln(\mu_g), \frac{\sigma_g^2}{(1-\rho_e^2)})$.

[34] As it is standard, we assume that g_1 is an equally spaced grid with $\log(\underline{g}_1)$ and $\log(\overline{g}_1)$ located -/+ 3 standard deviations from $\log(\mu_g)$ respectively.

[35] In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds but 50.4% have retirement accounts (which are very likely to own government bonds). These figures would suggest values of γ ranging from 50% to 88%. In Europe, comparable statistics are not available for several countries, but recent studies show that the wealth distribution is highly concentrated and that the wealth Gini coefficient ranges between 0.55 and 0.85 depending on the country and the year of the study (see

Average income y is calibrated such that the model's aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households' aggregate endowment must equal GDP net of fixed capital investment and net exports, since the latter two are not explicitly modeled. The average for the period 1970-2012 for the same set of countries used to estimate the g_1 process implies $y = 0.7883$. Note also that under this calibration of y and the Markov process of g_1, the gap $y - g_1$ is always positive, even for $g_1 = \overline{g}_1$, which in turn guarantees $c_1^H > 0$ in all repayment states.

We set the value of B_0 to match the median value of government debt for Euro-zone countries reported in Table 1 of the paper. As we describe in the next section, even though the optimal B_1 (close to 15 percent of GDP for $\gamma \in [0.55, 0.85]$) is higher than those observed in the sovereign debt literature, in this two-period model the consumption smoothing mechanism induces a reduction in the optimal B_1 relative to the initial condition B_0 even in a deterministic version of the model with stationary government purchases. Under these assumptions, the optimal debt choice is decreasing in γ and has an upper bound of $B_1 = B_0/(1+\beta)$ as $\gamma \to 0$.[36] In addition, when default risk is introduced debt needs to be below the level that would lead the government to default in the second period with probability 1, and above the level at which either c_0^L or c_1^H become non-positive, otherwise there is no equilibrium.

The functional form of the default cost function is the following: $\phi(g_1) = \phi_0 + (\overline{g}_1 - g_1)/y$, where $\overline{g}_1$ is calibrated to represent an "unusually large" realization of g_1 set equal to the largest realization in the Markov process of government expenditures, which is in turn set equal to 3 standard deviations from the mean (in the process in logs).[37] We calibrate ϕ_0 to match an estimate of the observed frequency of domestic defaults. According to Reinhart and Rogoff (2011), historically, domestic defaults are about 1/4 as frequent as external defaults (68 domestic v. 250 external in data since 1750). Since the probability of an external default has been estimated in the range of 3 to 5% (see for example Arellano (2008)), we estimate the probability of a domestic default at about 1%. The model is close to this default frequency on average when solved over the empirically relevant range of γ's ($\gamma \in [0.55, 0.85]$) if we set $\phi_0 = 0.004$. Note, however, that this calibration of ϕ_0 to target the default probability and the calibration of B_0 to the target described early needs to be done jointly by repeatedly solving the model until both targets are well approximated.

Davies, Sandstrm, Shorrocks, and Wolff (2009)). In our model, if $b_0^L = 0$, the Gini coefficient of wealth is equal to γ.

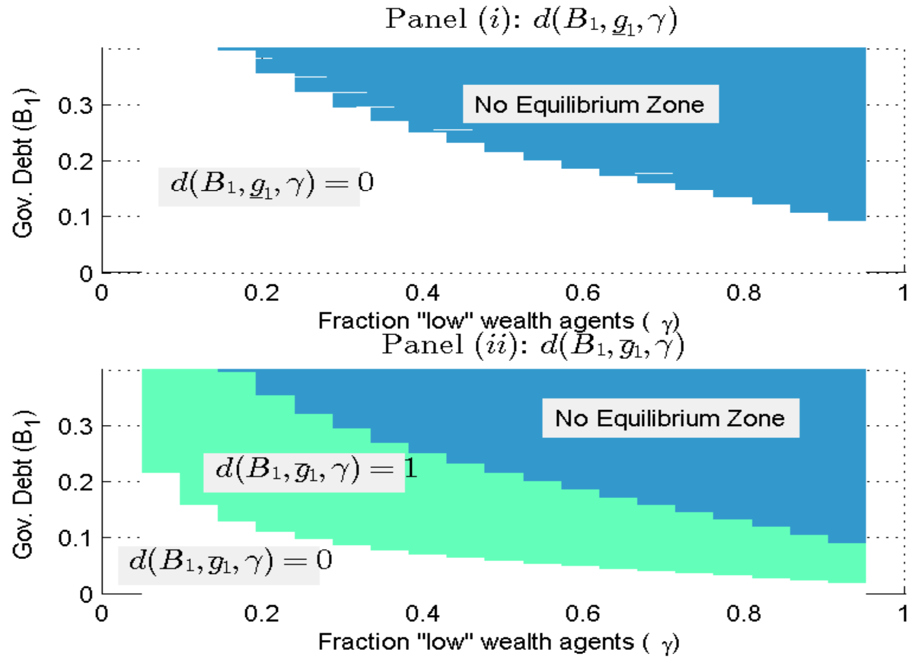
[36] This occurs because as $\gamma \to 0$, the model in deterministic form collapses to a representative agent economy inhabited by H types where the optimal debt choice yields stationary consumption, $q_0 = 1/\beta$, and $B_1 = B_0/(1+\beta)$. In contrast, an infinite horizon, stationary economy yields $B_1 = B_0$ (see Appendix for details).

[37] This cost function shares a key feature of the default cost functions widely used in the external default literature to align default incentives so as to support higher debt ratios and trigger default during recessions (see Arellano (2008) and Mendoza and Yue (2012)): The default cost is an *increasing* function of disposable income $(y - g_1)$. In addition, this formulation ensures that households' consumption during a default never goes above a given threshold.

A.6 Additional Figures: Benchmark Model

Figure A.12 shows two panels with the optimal default decision for different values of g_1. The plots separate the regions where the government chooses to repay ($d(B_1, g_1, \gamma) = 0$ shown in white), where it chooses to default ($d(B_1, g_1, \gamma) = 1$ in green) and where the equilibrium does not exist (in blue).

Figure A.12: Government default decision $d(B_1, g_1, \gamma)$



The repayment region ($d(B_1, g_1, \gamma) = 0$) corresponds to the region with $\overline{\alpha}(B_1, g_1, \gamma) < 0$. Hence, the government defaults at higher B_1 the lower a given γ, or at higher γ the lower a given B_1. Moreover, the two plots show that when $g_1 = \overline{g}_1$ the government defaults for combinations of γ and B_1 for which it repays when $g_1 = \underline{g}_1$. Thus, default occurs over a wider set of (B_1, γ) pairs at higher levels of government expenditures, and thus it is also more likely to occur.

We examine further the behavior of the default decision by computing the threshold value of γ such that the government is indifferent between defaulting and repaying in period for a given (B_1, g_1). These indifference thresholds ($\hat{\gamma}(B_1, g_1)$) are plotted in Figure A.13 against debt levels ranging from 0 to 0.4 for three values of government expenditures $\{\underline{g}_1, \mu_g, \overline{g}_1\}$. For any given (B_1, g_1), the government chooses to default if $\gamma \geq \hat{\gamma}$.

Figure A.13: Default Threshold $\hat{\gamma}(B_1, g_1)$

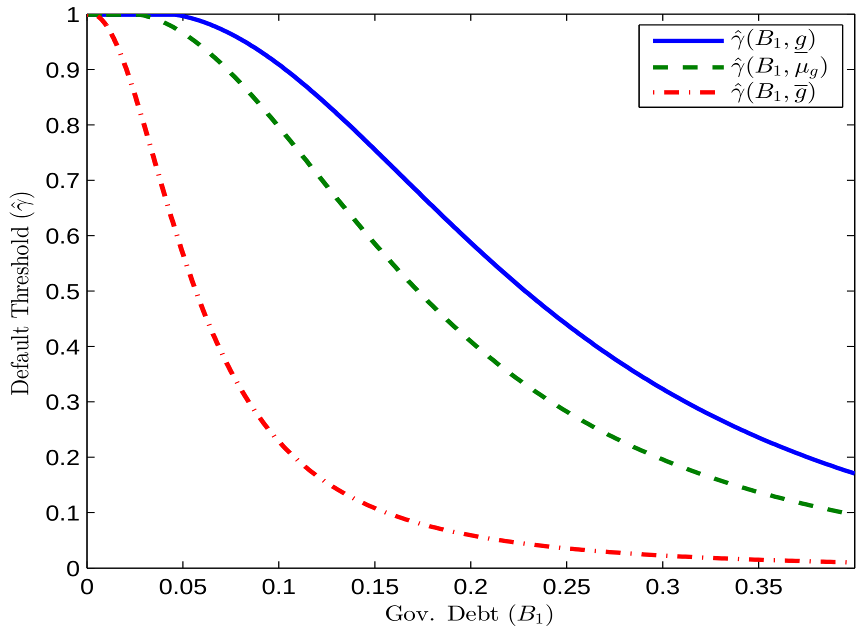
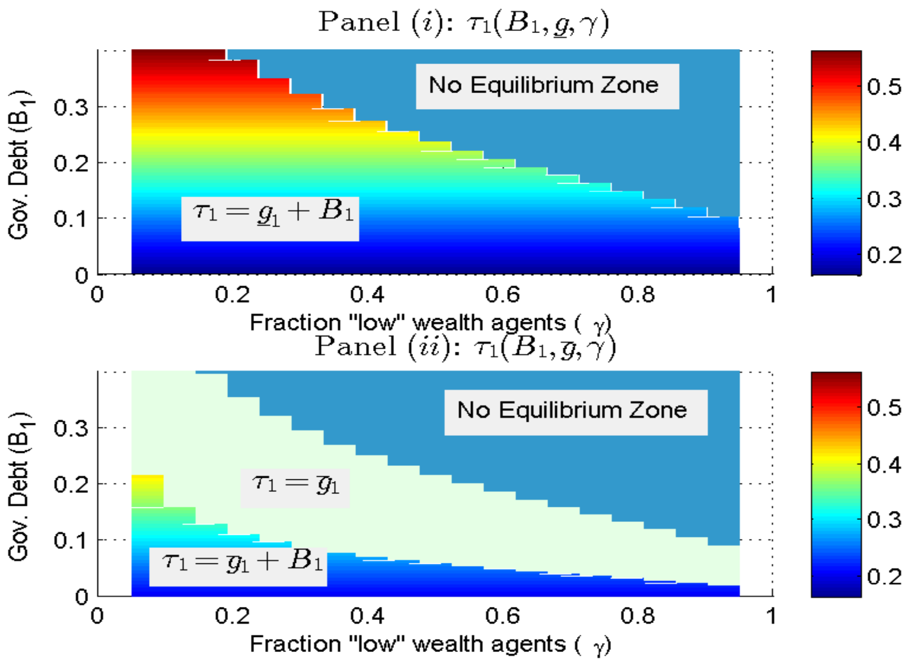


Figure A.13 shows that the default threshold is decreasing in B_1. Hence, the government tolerates higher debt ratios without defaulting only if wealth concentration is sufficiently low. Also, default thresholds are decreasing in g_1, because the government has stronger incentives to default when government expenditures are higher (i.e. the threshold curves shift inward).[38] This last feature of $\hat{\gamma}$ is very important to determine equilibria with debt subject to default risk. If, for a given value of B_1, γ is higher than the curve representing $\hat{\gamma}$ for the lowest realization in the Markov process of g_1 (which is also the value of $\underline{g_1}$), the government defaults for sure and, as explained earlier, there is no equilibrium. Alternatively, if for a given value of B_1, γ is lower than the curve representing $\hat{\gamma}$ for the highest realization of g_1 (which is the value of $\overline{g}_1$), the government repays for sure and debt would be issued effectively without default risk. Thus, for the model to support equilibria with debt subject to default risk, the optimal debt chosen by the government in the first period for a given γ must lie between these two extreme threshold curves. We show that this is the case later in this Section. Figure A.14 shows intensity plots of the equilibrium tax functions.

[38] $\hat{\gamma}$ approaches zero for B_1 sufficiently large, but in Figure A.13 B_1 reaches 0.40 only for exposition purposes.

Figure A.14: Equilibrium Tax Functions $\tau_1^{d(B_1,g_1,\gamma)}$



Putting together these plots with those of the default decision in Figure A.12 illustrates the model's distributional incentives to default from the perspective of tax policy. If the government defaults, the tax is $\tau_1 = g_1$, but if it repays the tax is $\tau_1 = g_1 + B_1$. Since all agents pay the same taxes but L types do not collect bond repayments, the lower taxes under default provide a distributional incentive to default that is larger when wealth is more concentrated. Figure A.14 shows that, for given g_1, the repayment scenarios with higher taxes are more likely when a large fraction of households hold debt (low γ) and thus benefit from a repayment, or when the debt is low so that the distributional incentives to default are weak. Moreover, equilibria with higher taxes are more likely to be observed at low than at high levels of government expenditures, because default is far likely with the latter.

Figure A.15 plots the bond demand decision rules (for given B_1) of the H types in the same layout as the bond prices (i.e. as functions of γ for three values of B_1).[39] This plot validates the intuition provided above about the properties of these agents' bond demand function. In particular, as γ increases, the demand for bonds of H types grows at an increasing rate, reflecting the combined effects of higher per-capita demand by a smaller fraction of H-type agents and a rising default risk premium. Thus, the convexity of these bond decision rules reflects the effects of wealth dispersion on demand composition and default risk explained earlier.

[39] We do not include the bond decision rules for L types because they are credit constrained (i.e. their Euler condition holds with inequality) and choose $b_1^L = b_0^L = 0$.

Figure A.15: Equilibrium HH bond decision rules b_1^i

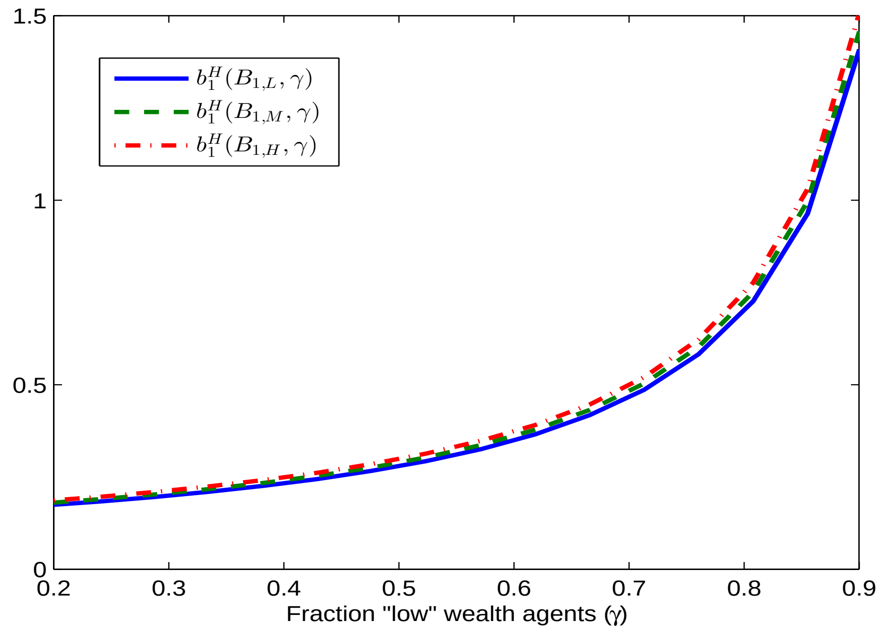
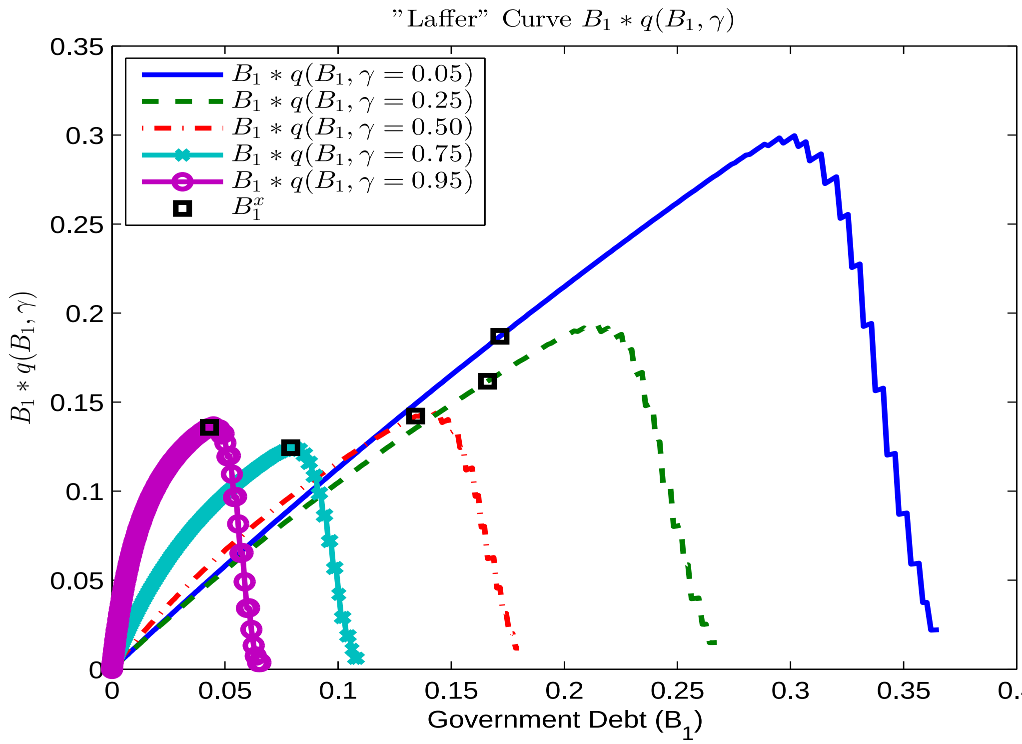


Figure A.16 shows the debt Laffer curves for five values of γ in the [0.05,0.95] range.[40]

[40]Each curve is truncated at values of B_1 in the horizontal axis that are either low enough for $c_0^L \leq 0$ or high enough for default to be chosen for all realizations of g_1, because as noted before in these cases there is no equilibrium.

Figure A.16: Debt Laffer Curve



When $\gamma \geq 0.50$, $B_1^*(\gamma)$ is located at the maximum of the corresponding Laffer curve. In these cases, setting debt higher than at the maximum of the Laffer curve is suboptimal because default risk reduces bond prices sharply, moving the government to the downward sloping region of the Laffer curve, and setting it lower is also suboptimal because then default risk is low and extra borrowing generates more resources since bond prices change little, leaving the government in the upward sloping region of the Laffer curve. Thus, if the optimal debt has a nontrivial probability of default, the government's debt choice exhausts its ability to raise resources by borrowing.

A.7 Sensitivity Analysis: Benchmark Model

This Section presents the results of a set of counterfactuals that shed more light on the workings of the model and also show some results for the case in which the social welfare function has biased weights that do not correspond to the fractions of L and H types in the economy. The sensitivity analysis studies how the main results are affected by changes in the initial debt B_0, initial government expenditures g_0, and the constant in the default cost function ϕ_0.

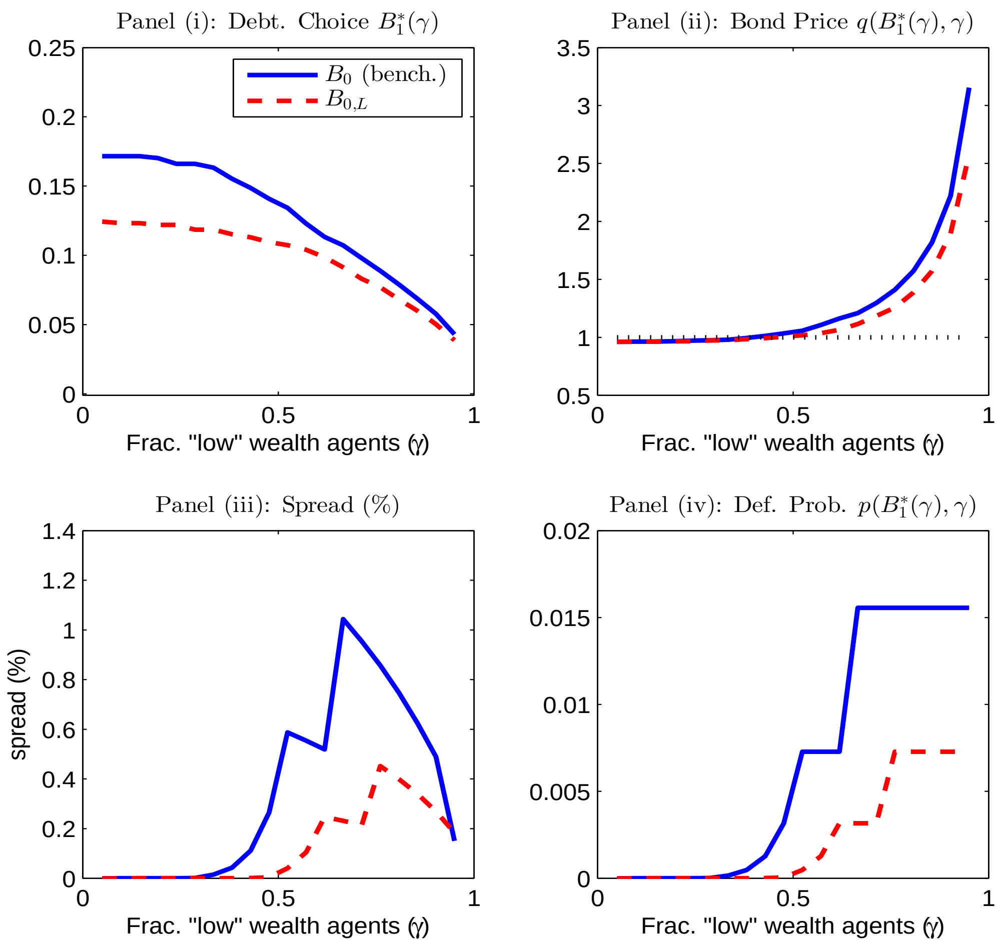
A.7.1 Lower Initial Debt Level B_0

Figure A.17 compares the optimal government debt and associated equilibrium bond prices, spreads and default probability under the original calibration with $B_0 = 0.35$ and a value that is 20% lower ($B_{0,L} = 0.28$).

Panel (i) shows that the optimal debt choice is always lower for the lower B_0, but the difference narrows as γ rises. This occurs because at low γ default risk plays a negligible role, and the demand composition effect implies lower per-capita demand for bonds from H type agents and a smaller fraction of credit-constrained L types, so the government wants to sell less debt. But once default risk becomes relevant, the optimal debt choice is about the same.

With the lower B_0, bond prices are uniformly lower albeit slightly (panel (ii)), spreads are increasing as γ rises for a wider range of γ and attain a lower maximum (panel (iii)), and the same is true for default probabilities (panel (iv)). Bond prices are lower with lower initial debt because for a given γ this implies lower initial wealth of H types (b_0^H), and in turn this requires a lower bond price to clear the market. This effect is stronger than two other effects that push bond prices in the opposite direction: First, the slightly lower debt B_1 that the government finds optimal to supply at lower levels of B_0. Second, the higher disposable income of households resulting from the lower date-0 taxes needed to repay lower levels of B_0, which increases demand for bonds.

Figure A.17: Changes in Initial Government Debt B_0

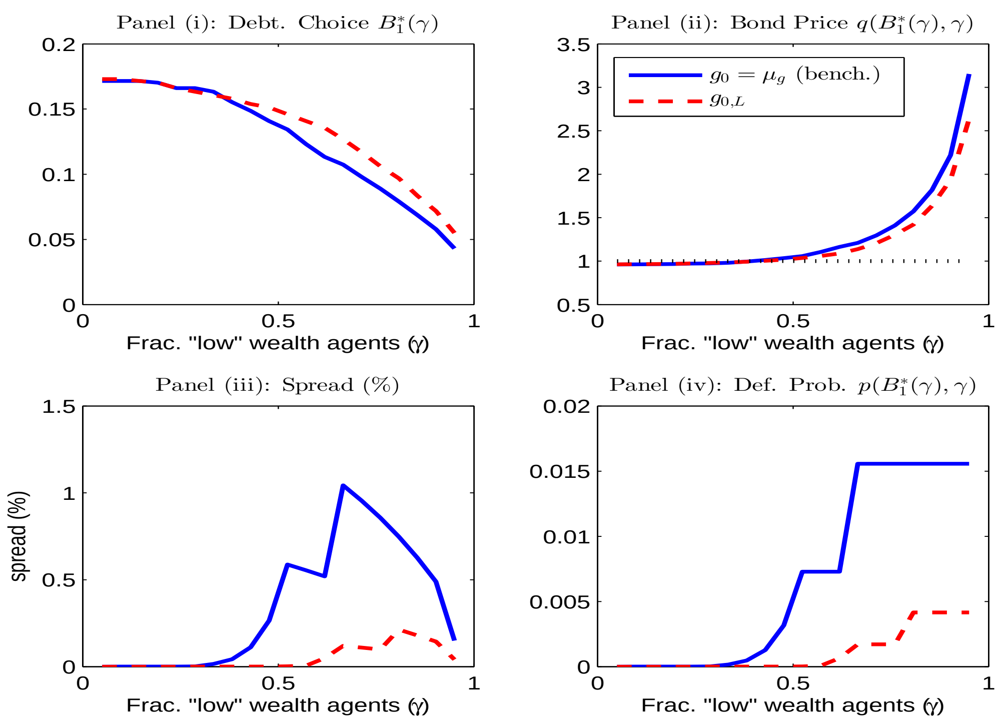


In terms of the implications for the empirically relevant range of γ in the European data, these results show that at the lower B_0 the model continues to predict debt ratios of about 8 to 12%, but now at spreads that are only half (40 v. 100 basis points) and at default probabilities below 1 percent instead of 1 to 1.5 percent. Moreover, these results also show that spreads and default probabilities can display richer patterns than the ones found in the initial calibration.

A.7.2 Lower Initial Government Expenditures g_0

Figure A.18 compares the model's equilibrium outcomes under the calibrated value of initial government expenditures, $g_0 = \mu_g = 0.181$, and a scenario in which g_0 is 1.5 standard deviations below the mean, $g_{0,L} = 0.171$.

Figure A.18: Changes in Initial Government Expenditures g_0



Like the reduction in B_0, the reduction in g_0 increases date-0 disposable income via lower date-0 taxes. They differ, however, in two key respects: First, changes in g_0 affect the expected level of government expenditures for $t = 1$, as reflected in changes in the transition probabilities which are conditional on g_0. Second, changes in g_0 do not affect the aggregate wealth of the economy and the initial bond holdings of H types.

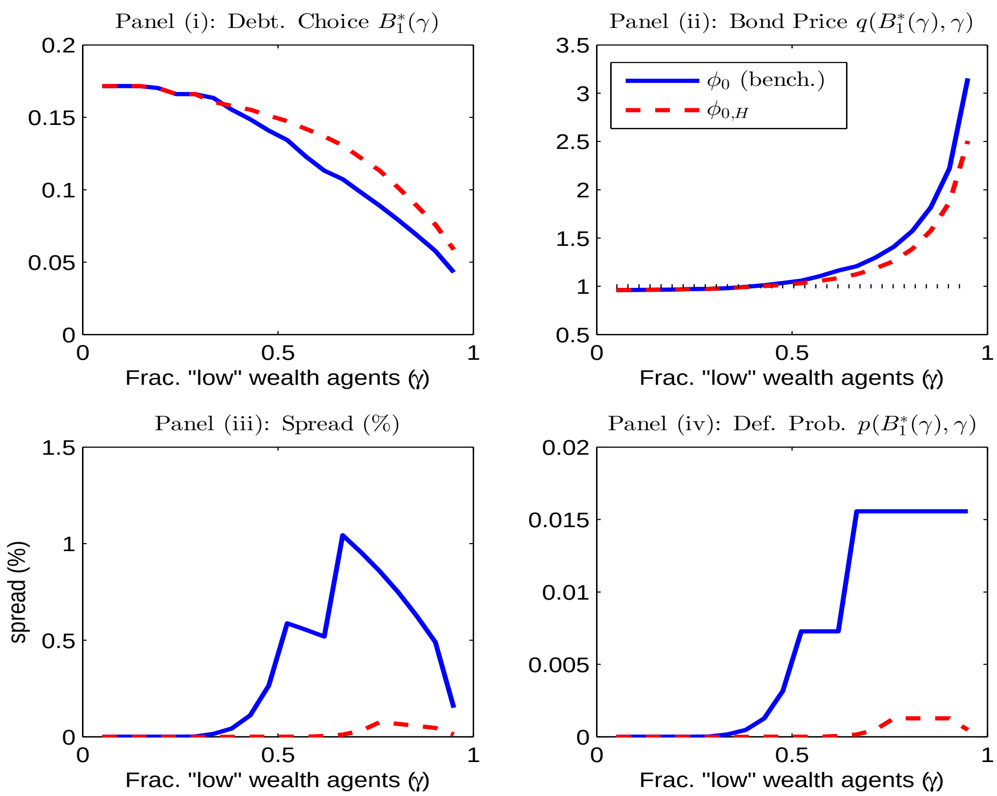
Panel (i) shows that the optimally debt is slightly higher with lower g_0, unless γ is below 0.35. This reflects the fact that the lower g_0 allows the government to issue more debt in the initial period, because the likelihood of hitting states with sufficiently high g_1 for optimal default to occur in the second period is lower. This also explains why in panels (ii)-(iv), despite the higher optimal debt with the lower g_0, bond prices, default probabilities and spreads are lower.

Moreover, optimal debt is about the same with lower g_0 as in the initial calibration for $\gamma \leq 0.35$ because at this low level of wealth concentration default risk is not an issue and the mechanism that we just described is irrelevant.

A.7.3 Fixed Cost of Default ϕ_0

Panels $(i) - (iv)$ in Figure A.19 compare the equilibrium outcomes for the initial calibration of the default cost parameter ($\phi_0 = 0.004$) with an scenario with a higher values ($\phi_{0,H} = 0.02$).

Figure A.19: Changes in Cost of Default ϕ_0

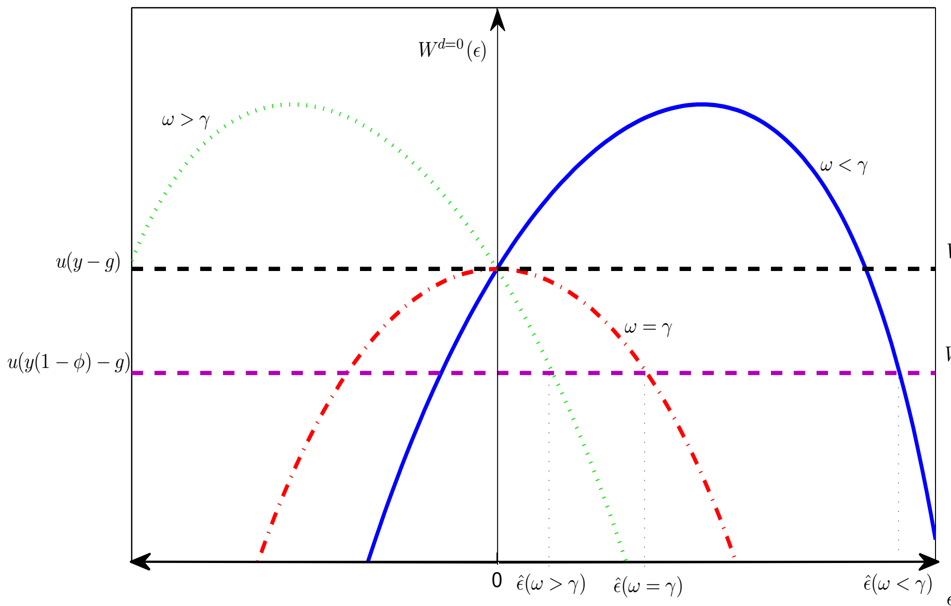


Qualitatively, the changes in the equilibrium are in the direction that would be expected. Higher default costs increase the optimal debt and reduce bond prices, spreads and default probabilities. Quantitatively, however, the changes in optimal debt and bond prices are small, while the reduction in spreads and default probabilities are significantly larger.

A.8 Additional Figures: Model with Biased Welfare Weights

Adding default costs to the political bias setup ($\phi(g_1) > 0$) makes it possible to support repayment equilibria even when $\omega \geq \gamma$. As Figure A.20 shows, with default costs there are threshold values of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega >=< \gamma$.

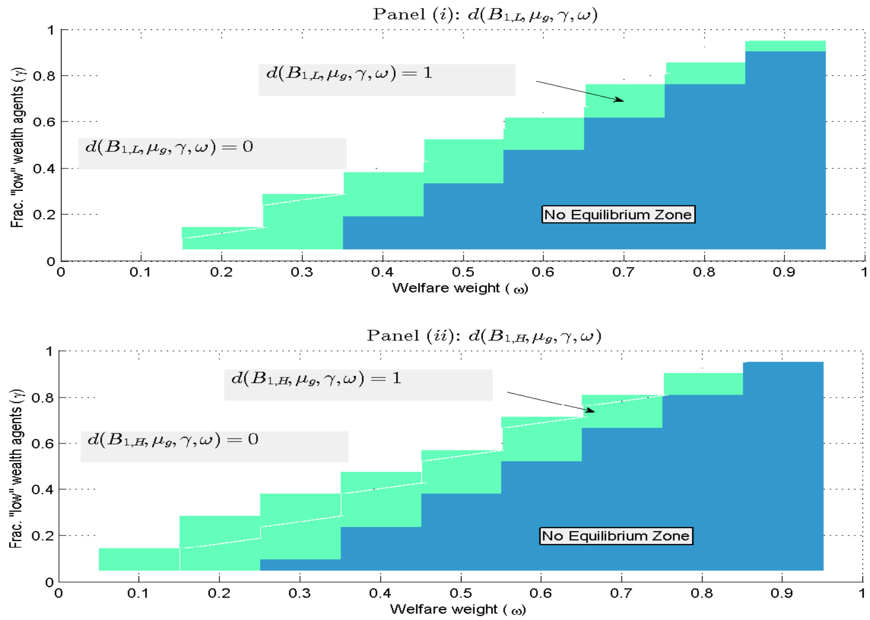
Figure A.20: Default decision with non-utilitarian planner when $\phi(g_1) > 0$



It is also evident in Figure A.20 that the range of values of ϵ for which repayment is chosen widens as γ rises relative to ω. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in ϵ, do not produce consumption dispersion larger than the value of $\hat{\epsilon}$ associated with a given (ω,γ) pair. Intuitively, the consumption of H-type agents must not exceed that of L-type agents by more than what $\hat{\epsilon}$ allows, because otherwise default is optimal.

Figure A.21 shows the default decision rule induced by the planner's welfare gains of default, again as a function of ω and γ for the same two values of B_1. The region in white corresponds to cases where $d(B_1, g_1, \gamma, \omega) = 0$, the green region corresponds to $d(B_1, g_1, \gamma, \omega) = 1$ and the blue region corresponds to cases in which there is no equilibrium.

Figure A.21: Default Decision Rule $d(B_1, g_1, \gamma, \omega)$



This Figure shows that when the ω is low enough, the government chooses default, and for a given ω the default region is larger the lower is γ. Taxes and prices for given values of B_1 and ω are linked to the default decision and γ as in the benchmark model and the intuition behind their behavior is straightforward.

A.9 Additional Figures: Model with Foreign Investors

Figure A.22 shows the bond decision rules for domestic and foreign investors as the value of γ and B_1 change.

Figure A.22: Decision Rule Domestic and Foreign Agents

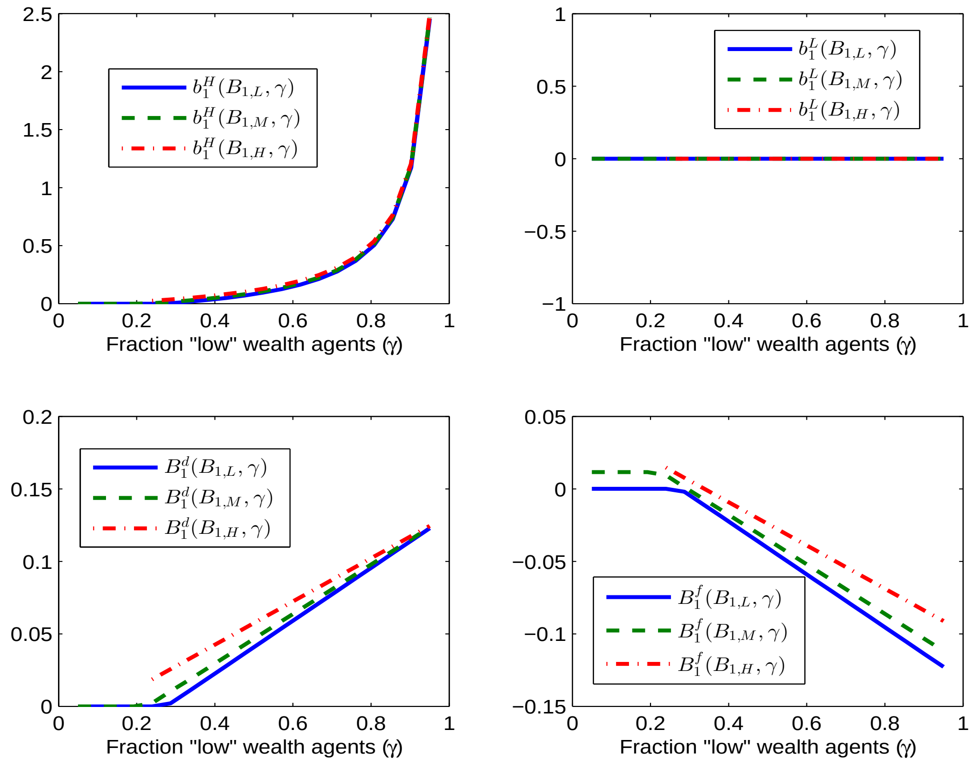
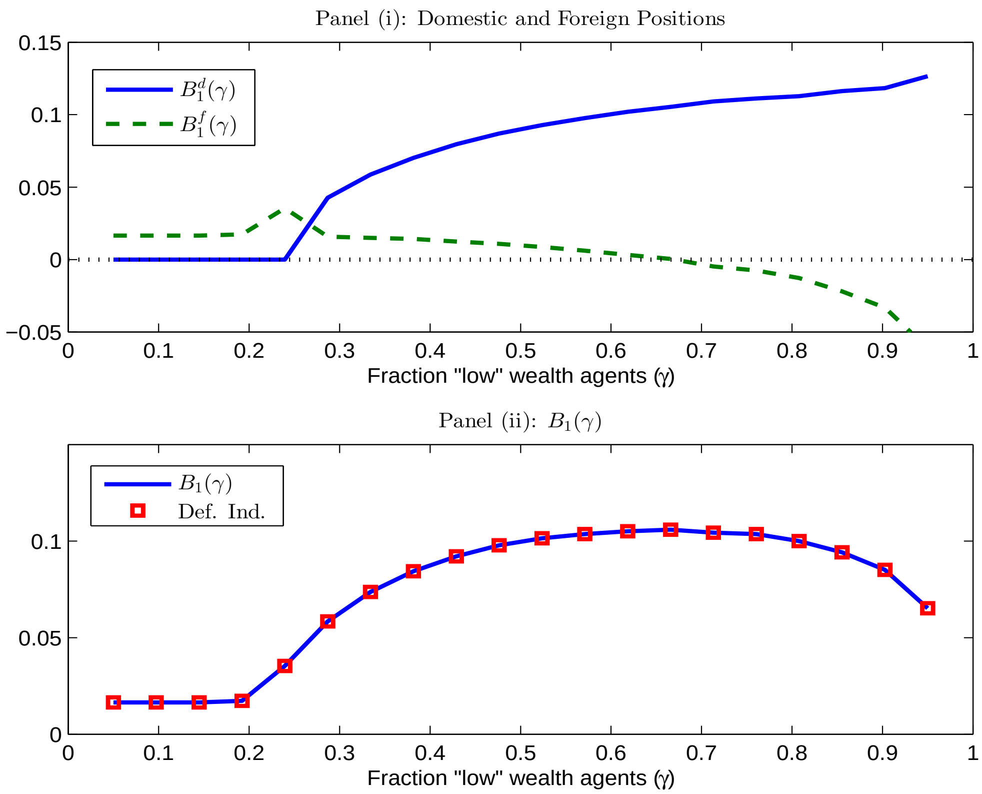


Figure A.23 presents the optimal domestic and foreign demand (Panel (i)) and the optimal debt issuance of the government (Panel (ii)). This Panel also marks with red squares the debt levels where there is positive risk of default and shows that the feature of equilibrium domestic default is also robust to the introduction of foreign lenders.

Figure A.23: Equilibrium Domestic and Foreign Debt Holdings



Panel (i) of Figure A.23 shows that domestic (foreign) demand is an increasing (decreasing) function of γ. Panel (ii) of Figure A.23 shows that the debt choice of the government is a concave function of γ. This is a direct result of default incentives. When γ is low the government has strong incentives to default of foreign lenders, so debt issuance remains low. As γ increases and domestic households increase their demand for sovereign debt, forces for a "foreign" default decrease and higher debt levels are attainable. As higher levels of γ, "domestic" default incentives constraint the government to increase the debt further and the optimal choice decreases.

A.10 Details for the Model with Two Assets

We now extend the baseline model by allowing domestic agents to save using a risk-free asset, in addition to government bonds. In particular, agents have access to a non-stochastic production technology $y_t^i = z(k_t^i)^\theta$ with $0 < \theta < 1$, where y_t^i is total output and k_t^i is capital for agent of type i in period t, respectively. The initial aggregate level of capital is denoted by K_0. As before, there is a fraction γ of $L-type$ agents that are endowed with b_0^L and k_0^L units of domestic sovereign debt and capital respectively. A fraction $(1-\gamma)$ of agents are of the $H-type$ and have endowments given by $b_0^H = \frac{B_0 - \gamma b_0^L}{(1-\gamma)}$ and $k_0^H = \frac{K_0 - \gamma k_0^L}{(1-\gamma)}$. We assume that capital depreciates at rate δ. At period 0, agents choose how much of their savings they want to allocate to public bonds $b_1^i \geq 0$ and capital $k_1^i \geq 0$.

The assumptions we made above to introduce the second asset, particularly the curvature of the production function, serve the purpose of supporting a well-defined portfolio choice for private agents (i.e. the fraction of wealth to allocate to bonds and capital), which is in turn useful to solve the government's problem.[41] This formulation is also helpful because it approximates the cases in which agents could buy a foreign risk-free asset or a domestic asset that is in zero net supply (and is also risk free). If, for example, agents could buy a foreign risk-free asset, we could obtain well defined portfolios by introducing adjustment costs.

The agents' budget constraint in the initial period is:

$$c_0^i + q_0 b_1^i + k_1^i = z(k_0^i)^\theta + k_0^i(1-\delta) + b_0^i - \tau_0 \;\text{ for } i = L, H. \tag{A.32}$$

The budget constraints in period 1 for the case of no-default and default are:

$$c_1^{i,d=0} = z(k_1^i)^\theta + k_1^i(1-\delta) + b_1^i - \tau_1 \;\text{ for } i = L, H. \tag{A.33}$$

$$c_1^{i,d=1} = (1-\phi(g_1))z(k_1^i)^\theta + k_1^i(1-\delta) - \tau_1 \;\text{ for } i = L, H. \tag{A.34}$$

The first-order condition with respect to b_1^i is

$$u'(c_0^i) \geq (\beta/q_0)E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1)\right], \; = \text{ if } \; b_1^i > 0.$$

As in the benchmark, the marginal benefit of an extra unit of domestic bond is positive only in those states where the government chooses to repay. Similarly, the first-order condition with respect to capital is given by

$$u'(c_0^i) \geq \beta E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1) + u'(c_1^{i,d=1})d_1\right]\left[\theta z(k_1^i)^{\theta-1} + 1 - \delta\right], \; = \text{ if } \; k_1^i > 0.$$

Since the asset is risk-free, the marginal benefit of an extra unit of capital is positive in all possible future states of the world. However, it will be more valuable in those states where the government chooses to default since the marginal utility is higher. These first-order conditions also show that the level of government debt B_1 (as well as the initial wealth dispersion γ) influence the individual asset demand through the default decision directly, and indirectly through the bond price.

When the borrowing constraint on bonds binds, this introduces a wedge between the agents' expected marginal product of capital and the expected return on the bond. However, if both decisions are interior (which is likely to happen for $H-$type agents), agents equalize the expected return across assets, which implies:

$$\frac{q_0}{\left[\theta z(k_1^i)^{\theta-1} + 1 - \delta\right]^{-1}} = \frac{E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1)\right]}{E_{g_1}\left[u'(c_1^{i,d=0})(1-d_1) + u'(c_1^{i,d=1})d_1\right]}. \tag{A.35}$$

This equation shows that when capital and bond decisions are interior, the wedge between

[41] This statement is of course conditional on parameters supporting the interesting case in which at least some agents do hold both assets at equilibrium.

the price of the bond and the marginal productivity of capital is driven uniquely by default risk (weighted by marginal utility in each state of the world). In the particular case in which in addition we are in a combination of B_1 and γ where there is no default risk, the optimal individual choice of capital is given by:

$$(1/q_0) = (\alpha z(k_1^i)^{\alpha-1} + 1 - \delta)$$
$$\Rightarrow k_1^i = \left[\frac{\alpha z q_0}{1 - q_0(1-\delta)}\right]^{\frac{1}{1-\alpha}}.$$

The demand for capital increases with the price of the bond, or falls with the interest rate, as in standard investment models.

In this environment, the government faces similar budget constraints to those presented in the benchmark model. However, when choosing the optimal level of debt and whether to default or not, it takes into account not only the distribution of bond holdings across agents but also the distribution of capital. In particular, at $t = 1$, the repayment value is given by:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(z(k_1^L)^\theta + (1-\delta)k_1^L - g_1 + b_1^L - B_1) + (1-\gamma)u(z(k_1^H)^\theta + (1-\delta)k_1^H - g_1 + b_1^H - B_1)$$

and the value of default is

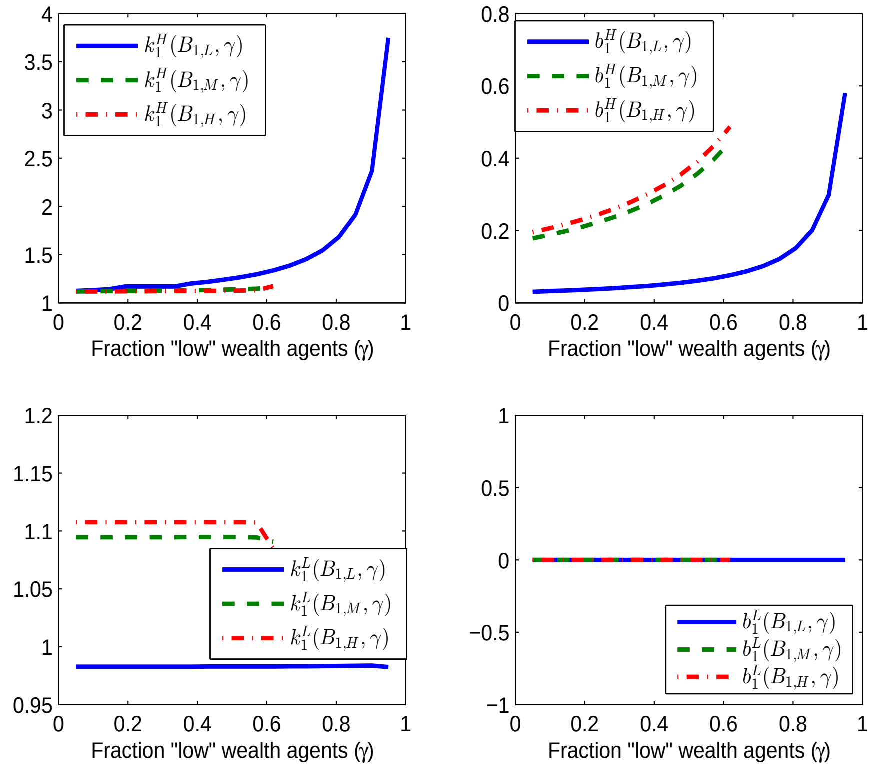
$$W_1^{d=1}(g_1, \gamma) = \gamma u(z(k_1^L)^\theta(1-\phi(g_1)) + (1-\delta)k_1^L - g_1) + (1-\gamma)u(z(k_1^H)^\theta(1-\phi(g_1)) + (1-\delta)k_1^H - g_1).$$

Note that in each case, the individual capital and bond decisions $\{b_1^i, k_1^i\}$ are a function of B_1 and γ. This implies that the variance of consumption, a key moment to determine the differential value of repayment v. default, is a function of the amount of debt the government has issued in period 0.

The model seems simple, but its solution needs to be obtained numerically. To make this case as close as possible to the benchmark model, we assume that $k_0^i = K_0$ (i.e. all initial heterogeneity is in initial bond holdings). Still, this results in heterogeneous bond and capital holdings in the second period. We set z to normalize GDP to 1, θ equal to 0.33 (a standard value), $\delta = 0.10$ and set K_0 so the capital to output ratio is equal to 2. All other parameters are set equal to those in the benchmark.

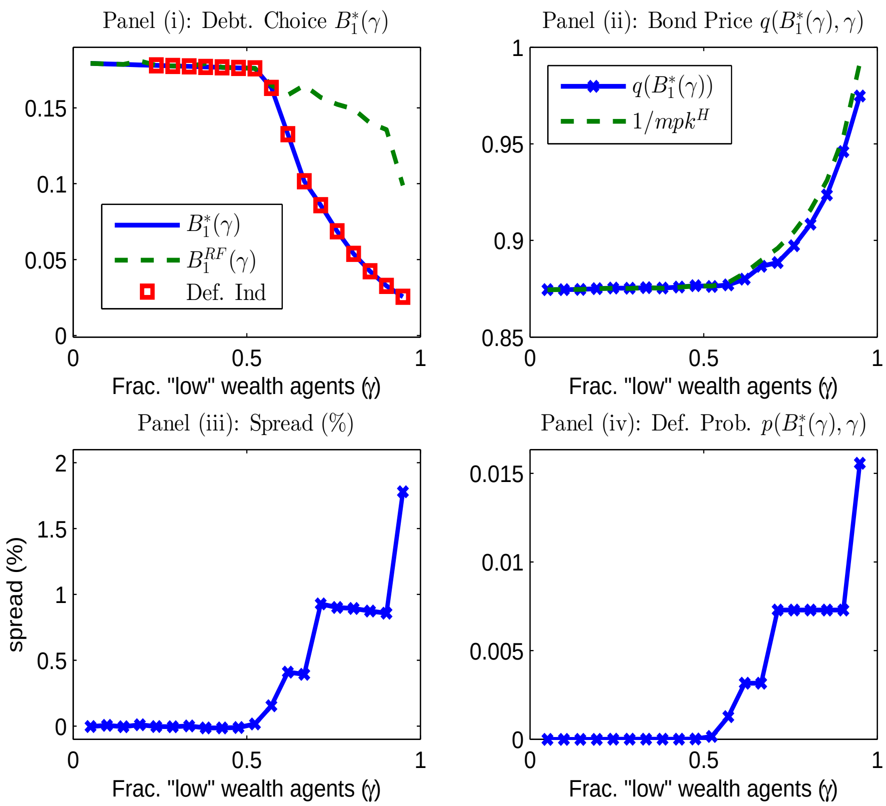
Figure A.24 presents the decision rules of the model. The top panels correspond to those for the $H-$type and the bottom panels to those for the $L - type$. They are truncated for combinations of B_1 and γ where the equilibrium does not exist because the government defaults for all possible realizations of g_1.

Figure A.24: Bond and Capital Decision Rules in Model with Two Assets



As in the benchmark model, $L-type$ agents are at the borrowing constraint for all values of B_1. Their capital choice is determined completely by the first-order condition for capital. As we get close to a region with default risk, their demand for capital decreases since the marginal benefit of an extra unit of savings decreases, because they expect to receive a transfer in the case of a default. On the other hand, $H-$type agents are never at the borrowing constraint, so their choice of capital is given by equation (A.35). As in the benchmark economy with one asset, while default risk is low ($B_1 = B_{1,L}$), the bond price increases with γ to induce the $H-$type to demand a decreasing fraction of their initial bond position (but still increasing in γ). This increase in bond prices drives the increasing demand for capital for the $H-$type when $B_1 = B_{1,L}$. As we move to higher levels of B_1, similar factors are at play but since default risk is not trivially small, bond prices do not rise as fast, inducing a relatively flat demand for capital with slightly increasing demand for domestic bonds for $H-$types with a decreasing demand for capital for $L-$types.

Figure A.25: Equilibrium Model with Two Assets



Note: mpk^H denotes the marginal product of capital for the $H-$type and is given by $\theta z(k^H)^{\theta-1} + 1 - \delta$. The spread is computed as $1/q(B_1,\gamma) - mpk^H$.

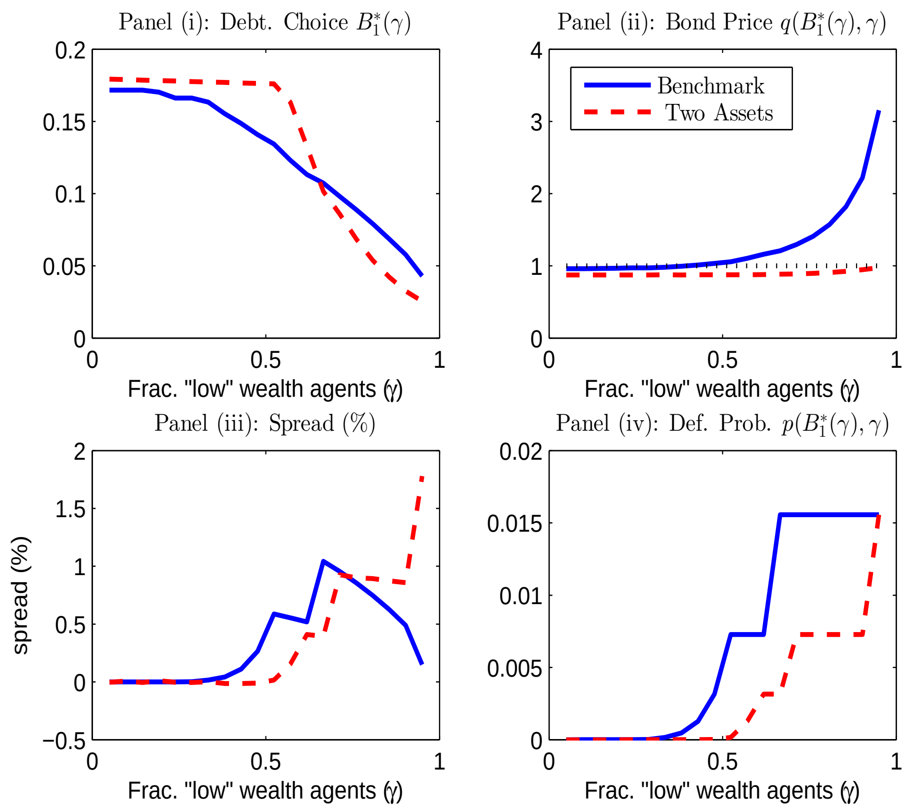
Figure A.25 presents the equilibrium functions as they vary with γ. As before, we present the optimal debt choice of the government, and the associated bond prices, spreads (measured against the marginal product of capital of the $H-$type) and the default probability.[42] This figure shows that the results of the benchmark model are robust to the introduction of an additional asset.

Panel (i) shows that optimal debt falls as γ increases and Panel (iv) that default risk is not zero for $\gamma > 0.20$ (non negligible for $\gamma > 0.50$). Importantly, the probability of a default event is increasing in the initial level of wealth inequality. As default risk increases, the wedge between the marginal product of capital and the equilibrium bond price increases driving up the spreads (see Panels (ii) and (iii)). Different from our baseline model, bond prices remain below 1, implying positive real interest payments for all values of γ.

Figure A.26 compares the equilibrium functions of the benchmark model with those of the economy with two assets.

[42] We measure spreads against the marginal product of capital of the $H-$type since the wedge is only driven by default risk. In the case of the $L-$type agents there is an additional component that arises from the multiplier on the non negativity constraint on domestic bonds.

Figure A.26: Comparison Equilibrium Benchmark vs Two Asset Model



The model with two assets sustains higher levels of debt, but as default risk increases the government chooses to reduce debt issuance faster in the economy with capital than in the benchmark. Even though we observe a lower default probability and spreads, the previous result derives from the fact that lower levels of debt are sustainable when agents have access to another asset. The cost of a default is not as high for the $H-$types and a default reduces consumption dispersion that increases faster with γ in the economy with capital than in the benchmark since initial dispersion is amplified via capital.