Human Capital Formation, Inequality, and Competition for Jobs

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Abstract

This paper develops a model of human capital accumulation and competition for jobs in the presence of instrumental concerns for relative ranking. Firms operating with different technologies rank individuals in the human capital dimension, and hire the best available individual(s). As a result, individuals care about their relative ranking in the population because it affects the wage they receive in equilibrium. More inequality in the distribution of endowments negatively affects aggregate efficiency in human capital formation as it weakens competition for jobs. However, we find that the opposite is true for wage inequality, namely, more wage inequality generates incentives for competition, and as a result agents exert more effort and accumulate more human capital in equilibrium.

Keywords: Human Capital, Inequality, Competition, Relative Ranking.

JEL Classification Numbers: J24, J31, O15, D33.

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1. Introduction

This paper develops a model of human capital accumulation and competition for jobs where there are strategic interactions between heterogeneous agents that compete for job positions with different wages. We argue that higher inequality of the endowments necessary to accumulate human capital negatively affects aggregate efficiency in human capital formation. This effect is beyond the standard Jensen’s inequality channel because inequality also affects individuals’ incentives to accumulate human capital when they confront competition for jobs from their close peers. Intuitively, as the mass of “close” competitors for job positions with different compensations increases, the incentives to differentiate from each other, by exerting higher effort and accumulating more human capital, also increases. However, wage inequality (i.e. inequality in the returns to human capital accumulation) has the opposite effect, fostering competition for job positions between individuals, and by doing so increasing aggregate efficiency in human capital formation. In equilibrium, individuals’ optimal choices depend on both, the distribution of endowments complementary to effort in the accumulation of human capital (the distribution of opportunities), and on wages (the distribution of returns). However, as we will show, changes in the level of inequality in these distributions have opposite effects on individuals’ choices and on aggregate efficiency in human capital formation.

On the demand side of the labor market, we assume that firms operating with different technologies rank individuals in the human capital dimension, and hire the best available candidate due to complementarities between human capital and technologies in the production process. On the supply side, in choosing the optimal level of investment in the accumulation of human capital, individuals take two effects into account. The first effect is the usual direct marginal benefit and cost from allocating one extra unit of time and effort to the accumulation of human capital (as in a standard model á-la-Becker (1964)). The second effect is the benefit coming from an extra unit of effort invested in the accumulation of human capital on the relative position of the individual in the distribution of human capital with a corresponding wage increase. As a result of this last effect, there is a so-called “rat-race” where individuals try to out-compete other individuals for the best available jobs. Although this rat-race might create excessive competition for jobs, the level of investment is socially optimal because individuals receive the full returns of the
investment in human capital accumulation. That is, we assume that there is a perfectly competitive labor market where human capital is remunerated according to its marginal product.

Our model assumes an individual’s concerns for relative ranking are instrumental. That is, individuals care about their relative position in the distribution of human capital not because they derive utility from relative ranking *per se*, but because their relative position determines the wage they will receive in equilibrium.

The literature on how inequality affects human capital formation has focused mostly on the role of credit market imperfections, wherein relatively poor individuals face financial constraints to finance the accumulation of human capital, as they cannot use future earnings as collateral for the loans necessary to pay the costs associated with accumulating human capital. Furthermore, if there are decreasing returns to human capital accumulation, it is precisely these individuals (the poor) who have the largest returns to resource investments in education. As a result, a redistribution of resources from rich to poor individuals would increase aggregate efficiency in the accumulation of human capital because of the reallocation of resources towards more profitable investments. This theoretical idea has been extensively developed in the literature since the work of Galor and Zeira (1993) and Banerjee and Newman (1993). Other developments have been proposed by De Gregorio (1996) and Bénabou (1996, 2000).¹ Empirical evidence has been found in favor of the hypothesis that inequality affects human capital accumulation in the presence of credit constraints (see Flug et al., 1998 and De Gregorio, 1996). In a recent paper Mejia and St-Pierre (2007) show that inequality in the endowments of complementary factors to the schooling process affect aggregate efficiency in the accumulation of human capital, without relying on credit market imperfections. The argument in that paper is that there are crucial complementary factors to the schooling process that are non-purchasable when the time comes for making investment decisions in education (i.e. parental schooling level, pre and post natal care, etc.). Because there are decreasing returns to time investment in human capital accumulation, and time investment in education is complementary to these factors, more inequality negatively affects aggregate human capital.

Our main contribution in this paper is to provide a rationale for a new, perhaps com-

¹See Aghion et al. (1999) for review of the literature.
plementary, channel through which the inequalities of endowments, and returns, affect incentives for human capital accumulation. The explanation we offer does not rely on credit market imperfections for inequality to affect human capital formation. Another important difference with the existing literature is that the model we propose here includes strategic interactions between individuals. That is, an individual’s return from the accumulation of human capital depends not only on his own choices and the production technologies, but also on the entire distribution of endowments and returns. In other words, we argue that in deciding the optimal investment in human capital formation, there are strategic interactions between individuals. In this respect our model is also related to existing works on human capital externalities, and on peer effects in education. While most of the empirical literature on peer effects has focused on the effect of average education of peers on different measures of educational attainment of each student in a given class (that is, on linear-in-means peer effects), Weingarth and Hoxby in a recent paper find that the structure of peer effects is highly non linear. That is, students benefit differently from the inclusion of a new student in the class, depending on their relative position in the class and the relative position of the entering student. In particular, students benefit significantly more from the inclusion in their class of new students that are similar to them (see Weingarth and Hoxby, 2007). Human capital externalities have also been modeled in the literature as an average mean effect, that is, it is average human capital in the economy that affects each individual’s marginal productivity in production (Lucas, 1988). In much of the literature on peer effects and on human capital externalities, individuals benefit from being close to more educated students or colleagues because of the close collaboration in the classroom or in the workplace. In this paper there are two novel things relative to the literature just described on peer effects and human capital externalities. First, individuals are affected differently from an entering student in their cohort depending on their relative position and the relative position of the entering student. In particular, an individual is affected more by the choices made by those individuals close to her in the distribution than by the choices of individuals who are very different (as was shown empirically in the recent paper by Weingarth and Hoxby, 2007). And, second, we argue that individuals are affected by other individuals not because of close collaboration in the classroom or workplace but because they are competing with each other for the best available jobs. While our model does not rule out important effects due to collaboration and cooperation, we do propose an-
other important way through which peer effects or human capital externalities might work: competition for jobs (or other relevant prizes associated with relative position). Thus, our paper has important implications (predictions) for the empirical literature on peer effects and human capital externalities. Namely, we argue that in measuring human capital externalities or peer effects one should not only account for the mean human capital in the population but, also, for the higher moments of the distribution of education. In particular, human capital externalities (due to competition) should be larger in societies with less inequality of opportunity, more inequality of returns and, in those parts of the distribution of endowments with a greater mass of individuals.

2. Concerns for relative ranking in the economics literature

Since the seminal work of Thorstein Veblen (1899), *A Theory of the Leisure Class*, several economists have argued that concerns for status (or the relative position in some relevant dimension(s)) have important economic consequences. A central discussion in the literature that deals with concerns for relative ranking has to do with how we should understand such concerns, that is, whether they are direct or instrumental. In the first case people have concerns for status because they obtain utility from having high status in its own sake. In the second case, people care about status because status directly affects the goods and services that individuals ultimately consume (Postlewaite, 1998). While the strongest argument for incorporating direct concerns for relative position is an evolutionary one, the case for not incorporating direct concerns for status in the utility function is that economic models that incorporate them typically allow for very diverse behavior, there are almost no restrictions on equilibrium behavior and, as a result, the models loose predictive power.

Most of the contributions that have emphasized the importance of concerns for relative ranking have focus on conspicuous consumption. The idea is the following: because wealth is unobservable, the consumption of conspicuous goods serves as a signal of non observable ability. Furthermore, if there are complementary interactions between individuals (for instance, at the household level between men and women, or at the workplace between

\[2\text{The reader is refered to Bastani (2007) for a thorough review of the literature on concerns for relative ranking.}\]

\[3\text{See Postlewaite (1998).}\]
employees and employers) conspicuous consumption might be welfare enhancing even when the costs of conspicuous consumption\(^4\) are taken into account as they allow for a better (more efficient) matching (among others, see Cole et al., 1992 and 1995, Bagwell and Bernheim, 1996, and Rege, 2000). While concerns for status might generate excessive competition, this does not mean that excessive competition is inefficient (as has been argued by Frank, 1999 and others). In fact, when status can be purchased in a competitive market, the cost of acquiring status is simply a transfer payment that adds to the seller’s wealth. For instance, Becker and Murphy (2000, ch. 4) show that competition for mates is fully efficient if the value that someone brings to the marriage is fully priced. In the same book, Becker and Werning take Frank’s (1999) example of wearing high heels and argue that “the demand for high heels is efficient, even when such shoes cause foot and back damage, if the marriage, or other, markets that match men and women compensates women fully for the utility gain to their husbands or other companions from their wearing high heels. This behavior is efficient even when it lowers the relative attractiveness of other women, including women who also wear high heels.” (see Becker and Murphy, 2000, ch. 8). In fact, when women decide to wear high heels they trade off the cost of wearing high heels for the utility gain they obtain from getting better husbands. Thus, wearing high heels can be understood as a Nash equilibrium in the presence of competition between women for better partners.

Only a few contributions in the economics literature on human capital and labor markets have incorporated concerns for relative ranking. Moen (1999) studies the incentives to invest in human capital in a model with labor market frictions and unemployment. In his model, an unemployed worker’s chances of getting a job depends on his human capital relative to that of other unemployed workers because firms prefer to hire the most productive applicant due to rent sharing between them and the workers. Relative ranking affects the job finding rate and, as a result, there is a “rat race” between unemployed individuals competing for job positions. Because wages are assumed to be determined by rent sharing between firms and workers (that is, the gains from education will not fully accrue to the workers in the form of higher wages) excessive competition might lead to inefficient overinvestment in human capital.

\(^4\)Conspicuous consumption (or "Veblen effects") exists when consumers are willing to pay a higher price for a functionally equivalent good (see Bagwell and Bernheim, 1996).
The most related contribution to this paper is a recent paper by Hopkins and Kornienko (2006). They study the effects of inequality in a tournament model where individuals compete for different rewards. Individuals, given their resources, make a simultaneous investment and output decision and then each individual is rewarded according to her relative position. The authors also emphasize the differential effect of inequality of resources and of inequality of rewards on individual equilibrium choices. However, their main focus of interest is on how changes in inequality of resources and rewards affect welfare for different segments of the population. In particular, they find that more inequality of resources lowers utility for agents in the middle and upper parts of the distribution, whereas an increase in the inequality of resources leads to lower utility for the relatively poor agents in society.\(^5\)

3. A simple illustration: The 2 agents - 2 firms model.

This section presents a simple model with only two firms and two agents that captures some of the main results (but not all) that will be presented in the next section of the paper.

3.1. Firms

Let us assume that there are two firms, \(l\) and \(h\), that produce a single homogeneous good, \(q_j\), using a production function that combines technology and human capital as follows:

\[
q_j = a_j \cdot h_j,
\]

where: \(a_j\) is the technology used by firm \(j = \{l, h\}\). Assume, without loss of generality, that \(a_h > a_l\). \(h_j\) is the human capital of the individual hired by firm \(j\). Furthermore, we assume that each firm hires only one individual.\(^6\)

Firms pay their workers their marginal product per unit of human capital employed in production. That is, firm \(l\) pays the worker it hires \(w_l = a_l\) per unit of human capital

\(^5\) Galí and Fernandez (1999) also develop a tournament model of competition for places at college but their main interest was to compare the efficiency of two different mechanisms in allocating rewards: markets vs. tournaments.

\(^6\) One can also think about one firm that has two available job positions, each operating with a different technology.
and firm $h$ pays the worker it hires $w_h = a_h$ per unit of human capital employed in the production process.

In this framework job positions differ in their payments because different firms operate with different technologies. The assumption that the production technology is linear in human capital greatly simplifies the analysis and also allows us to isolate the standard effect of inequality in the distribution of human capital on aggregate production efficiency that works through Jensen’s inequality (see Mejía and St-Pierre, 2007). That is, if the amount of output produced is a concave function of human capital then a more unequal distribution of this factor of production across individuals would reduce aggregate production efficiency.

Because technologies are complementary to human capital in the production process, the firm operating with the most advanced technology would like to hire the individual with the highest human capital available in the labor market. That is, we assume that firms rank individuals in the human capital dimension and that they make job offers to the individual with the highest human capital available in the job market. We will also assume that there is a perfectly assortative matching and that there are no search costs.

### 3.2. Individuals

There are two individuals with endowments of the complementary factors to the schooling process equal to $\theta_p$ and $\theta_r$, respectively. $\theta_i$ can be thought of as a combination of all factors that complement individual’s effort in the educational process, such as parental education, school and teacher quality, etc. (see Mejia and St-Pierre, 2007). Without loss of generality assume that $\theta_r \geq \theta_p$. That is, individual $r$ (the rich individual) has a larger (or equal) endowment of the complementary factors than individual $p$ (the poor individual).

Individuals accumulate human capital combining effort and the complementary factors to the schooling process (from now on, complementary factors) according to the following human capital production function:

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7This assumption also implies that the distribution of wages is independent of the distribution of human capital in the economy, which greatly simplifies the analysis and allows us to isolate changes in the distribution of returns to human capital accumulation from changes in the distribution of endowments.

8With any production function where human capital and technology are complements in production the analysis that follows would go through.
\[ h = h(e, \theta), \]

where \( e \) stands for effort and \( \theta \) for the endowment of the complementary factors.

Assumption A1: \( h_e \gg 0, h_{\theta} \gg 0, h_{ee} < 0, h_{e\theta} \leq 0, h_{ee} > 0, h(0, \theta) = 0 \).

According to A1, human capital is an increasing and strictly concave function of both effort and the complementary factors, and the marginal effect of effort on the accumulation of human capital is increasing in the complementary factors. In other words, human capital and the firm’s technology are complements in production. Also, effort is strictly necessary for the accumulation of human capital.

Each individual \( i \) maximizes a utility function that depends positively on consumption and negatively on effort. Furthermore we assume that the utility function is separable in the two arguments.\(^9\)

\[
\max_{\{e\}} U(c, e) = u(c) - v(e)
\]

Assumption A2: \( u'(c) > 0, u''(c) \leq 0, v'(e) > 0 \), and \( v''(e) > 0 \).

Consumption equals income which, in turn, is equal to the expected wage per unit of human capital times the amount of human capital accumulated by an individual. That is, consumption equals the expected wage times the amount of human capital that an individual brings to the market, \( E(w) * h \). Before going to the job market both individuals accumulate human capital and they know that the two firms will rank them in the human capital dimension. As a result, individual \( i \)'s perceived probability of being hired by the advanced technology firm (that is, her expected wage) is a function of her human capital, \( h_i \), and the human capital of individual \( j, h_j \), in the following way:

\[
E(w_i) = p(h_i, h_j) * w_h + (1 - p(h_i, h_j)) * w_l,
\]

where \( p(h_i, h_j) \) is the probability, as perceived by individual \( i \), of being hired by the firm that pays the high wage (that is, the firm operating with the advanced technology).

Assumption A3: \( p_{hi} > 0, p_{hj} < 0, p_{h_i,h_i} < 0 \).

\(^9\)This is perfectly equivalent to a situation where consumption and leisure are the only arguments in the utility function and where leisure time is sacrificed when time is devoted to the accumulation of human capital.
A3 says that the probability of being hired by the advanced technology firm for individual $i$ increases as her human capital increases and decreases with the human capital of individual $j$. Furthermore, this probability is strictly concave on $h_i$.

Assuming, again, without loss of generality, that $u(c) = c$, individual $i$ takes individual $j$'s effort as given and chooses her own effort to maximize utility.\(^{10}\) Individual $i$'s problem is:

$$\max_{\{e_i\}} E(w_i)h(e_i, \theta_i) - v(e_i). \quad (5)$$

The first order condition of individual $i$'s problem is:\(^{11}\)

$$\frac{\partial p(h_i, h_j)}{\partial h_i} \frac{\partial h_i(e_i, \theta_i)}{\partial e_i} (w_h - w_i)h(\hat{e}_i, \theta_i) + E(w_i)h_e(\hat{e}_i, \theta_i) - v'(\hat{e}_i) = 0. \quad (6)$$

The second and third term on the left hand side of equation 6 are the standard terms: the marginal benefits and costs from exerting one extra unit of effort in the accumulation of human capital in a standard model of human capital accumulation (see Becker, 1964, and Ben-Porath, 1967). The first term captures how an extra unit of time and effort allocated in the accumulation of human capital affects the probability of being hired by the firm that pays the high wage (that is, the firm operating with the advanced technology). When firms operate with different technologies and pay different wages individuals have an extra incentive to invest time and effort in the accumulation of human capital to increase the probability of being hired in the best available job.\(^{12}\)

3.3. Labor market equilibrium and comparative statics results

A Nash equilibrium of the game of competition for jobs is a pair of strategies $\{e_r, e_p\}$ that satisfy the first order conditions for both agents, $r$ and $p$, in equation 6. These two first order conditions describe the reaction function (the choice of effort) of each agent to every possible choice of effort by the other agent.

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\(^{10}\)In other words we assume that both agents make human capital investment decisions simultaneously.

\(^{11}\)Assumptions A1 through A3 guarantee that the maximization problem in equation 5 has a unique and interior solution and that the first order condition 6 is sufficient.

\(^{12}\)Standard models of human capital accumulation do not incorporate this effect because they implicitly assume that all available jobs operate with the same technology. As a result, there is no incentive for competition between applicants as the wage rate per unit of human capital is the same in all available jobs.
Before proceeding it is worth specifying a benchmark case where individuals do not take into account the effect of effort on the probability of being hired by the firm operating with the high technology (the first term in equation 6). In the benchmark case individuals either take as given the probability of being hired by the firm operating with the advanced technology, or, alternatively, take as given the expected wage. The important point of setting up the benchmark case is that individuals are not able to affect the probability of being hired by the advanced technology firm by exerting more effort. In this case, the first order condition is:

\[ E(w_i)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0, \quad (7) \]

where \( E(w_i) \) is taken as given by individual \( i \).

**Proposition 1:** Effort and human capital accumulation are higher when individuals compete for job positions than in the benchmark case where there is no competition.

*Proof:* If \( p_{e_i} = \frac{\partial p(h_i, h_j)}{\partial h_i} \frac{\partial h_i(e_i, \theta_i)}{\partial e_i} > 0 \), that is, if the probability of individual \( i \) being hired by the firm operating with the advanced technology increases as her human capital increases (i.e. as his effort increases), then \( p_{e_i}(w_h - w_l)h(\hat{e}_i, \theta_i) > 0 \) and, using equation 6, \( (p_h w_h + (1 - p_h) w_l)h_{e_i}(e_i^*, \theta_i) - v'(\hat{e}_i) < 0 \). However, in the benchmark case, \( (p_h w_h + (1 - p_h) w_l)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0 \) and so it must be that \( \hat{e}_i > e_i^* \) if the function \( (p(h_i, h_j) * w_h + (1 - p(h_i, h_j)))h(e_i, \theta_i) - v(e_i) \) is strictly concave in \( e_i \), as it is by assumptions A1 through A3.

Intuitively, when there is competition for available job positions individuals will exert more effort because they have an extra incentive to accumulate human capital beyond the standard marginal benefit (second term in equation 6). This extra incentive is the marginal increase in the probability of being hired by the firm operating with the advanced technology that results form an extra unit of time and effort allocated to the accumulation of human capital.

**Proposition 2:** Higher inequality in the distribution of the complementary factors decreases average human capital. The decrease in average human capital as inequality increases is larger when individuals compete for jobs than in the benchmark case.

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**Proof (sketch):** Define the average endowment of the complementary factors as \( \overline{\theta} \), and let \( \theta_r = \overline{\theta} + \delta \), and \( \theta_p = \overline{\theta} - \delta \). The parameter \( \delta \) captures inequality in distribution of the complementary factors. With this definition, the larger is \( \delta \), the larger is inequality in the distribution of the complementary factors. From A3 and the assumption that \( h_{\text{eff}} > 0 \) from A1, \( \left| \frac{\partial p(h_p, h_r)}{\partial \delta} \right| > \left| \frac{\partial p(h_r, h_p)}{\partial \delta} \right| < 0 \). That is, when inequality increases, the probability perceived by the poor individual of being hired by the firm with the advanced technology decreases more than the same probability perceived by the relatively rich individual increases. This result follows directly from Jensen’s inequality after noticing that \( p_{c_i e_i} < 0 \).

Because the probability of being hired by the advanced technology firm is a strictly concave function of effort, and the endowment of the complementary factors and effort are complements in the accumulation of human capital, higher inequality in the distribution of endowments reduces competition for available jobs.

**Proposition 3:** As the difference between wages in the two available job positions \( (w_h - w_l) \) increases, average human capital in the economy increases. That is, a larger difference in wages (i.e. the technologies employed by the two firms) increases the incentives to exert more effort in the accumulation of human capital.

**Proof:** The results follows directly from the first order condition (equation 6) by noticing that the larger is \( w_h - w_l \), the larger is the return from exerting effort that is associated with the increase in the probability of being hired by the advanced technology firm.

Notice that inequality of endowments and inequality of returns (wages) affect differently the incentives to compete for available job positions. While more inequality of endowments disincentives competition, more wage inequality does the opposite.\(^{13}\)

In order to have some sense of the magnitude of the effect of inequality in the endowments of the complementary factors, and of inequality in returns, on effort and human capital accumulation, Figure 1 (a) and (b) present the results obtained from the calibration of the model presented above.\(^{14}\) Effort and hence human capital accumulation are higher

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\(^{13}\)This result is in line with that obtained by Hopkins and Kornienko (2006).

\(^{14}\)We use the following functional forms for the calibration of the model: \( h(e_i, \theta_i) = e_i^\alpha \theta_i^{1-\alpha} \), with \( 0 < \alpha < 1 \), \( p(h_i, h_j) = \frac{h_i}{h_i + h_j} \), and \( e_i = \frac{\theta_i}{2} \). Note that these functional forms satisfy assumptions A1
when individuals compete for job positions than in the benchmark case (Proposition 1). This is seen in Figure 1a by comparing the two lines for any given level of endowment inequality. Note also from this figure that as inequality increases average human capital in the economy decreases, but in the case of competition for jobs it decreases faster (Proposition 2). Figure 1b shows how average human capital changes as the difference between wages in the two available job positions increases for a given level of endowment inequality. As the wage difference becomes larger, individuals have a higher incentive to compete for the high paying job position and thus exert more effort and accumulate more human capital.

![Figure 1: Average HK vs. Endowment Inequality (a) and vs. Wage Inequality (b).](image)

through A3. We set $\alpha = 3/4$, but the results presented in Figure 1 (a) and (b) are robust to variations of this parameter between 0 and 1.

For the benchmark case we take the probability of being hired by the advanced technology firm to be the probability that would obtain if the two agents had engaged in a contest for the high paying position. Note that individuals take as given the probability that results in equilibrium but cannot affect it by exerting more effort.
4. The General Model

4.1. Firms

Suppose that there is a continuum of firms indexed by \( j \) that produce a homogeneous good according to the following production function:

\[ q_j = a_j \times h_j, \]

where, as in equation 1, \( a_j \) is the technology used by firm \( j \) and \( h_j \) is the human capital of the individual hired by firm \( j \). Assume that each firm hires only one individual. Furthermore, assume that \( a_j \sim H(a) \).\(^{16}\) There is perfect competition in the labor market so firms remunerate human capital according to its marginal product. That is, the wage rate paid by firm \( j \) is equal to \( a_j \). Wages, therefore, are distributed according to \( H(a) \).

4.2. Individuals

There is a continuum of individuals indexed by \( i \). As in the two agents two firms model, each individual has a given endowment of the factors that complement time and effort in the educational process, \( \theta_i \). The endowment of the complementary factors is distributed in the population according to \( G(\theta) \), with support in \([a, b] \). Human capital is accumulated (produced) using individual’s effort and the complementary factors, according to \( h(e, \theta) \). The human capital production function satisfies A1 above.

Individuals derive utility from consumption and disutility from effort according to:

\[ U(c, e) = u(c) - v(e). \quad (8) \]

The utility function in equation 8 satisfies A2.

4.3. Matching between firms and workers in the labor market

\(^{16}\)We assume that the CDF \( H(.) \) is strictly increasing and continuous.
Following the approach of Hopkins and Kornienko (2004), if we let $F(h)$ be the distribution of human capital across individuals, individual $i$’s ranking in the distribution of human capital will be given by

$$\gamma F(h(e, \theta)) + (1 - \gamma) F^-(h(e, \theta)),$$

(9)

where $F^-(h) = \lim_{h \to h} F(h)$ is the mass of individuals with human capital strictly less than $h^{17}$, and $\gamma \in [0, 1)$ is a parameter that captures the decrease in the payoff from “ties”.$^{18}$

We will assume that in hiring workers firms rank individuals and, because technologies are complementary to human capital in the production process, the firm with the most advanced technology would like to hire the individual with the highest human capital available in the market, the firm ranked second would like to hire the individual with the highest human capital available in the market (the individual who ranks second in the distribution of human capital), and so on and so forth. That is, there is a perfectly assortative matching between firms and individuals.

Recalling that $H(a)$ denotes the distribution of technologies across firms and that $w_j = a_j$, then individual $i$’s ranking in the distribution of human capital coincides with his ranking in the distribution of wages in the economy. That is:

$$\gamma F(h(e, \theta)) + (1 - \gamma) F^-(h(e, \theta)) = H(w_i) \Rightarrow$$

(10)

$$R [\gamma F(h(e, \theta)) + (1 - \gamma) F^-(h(e, \theta))] = w_i,$$

(11)

where $w_i$ is the wage rate per unit of human capital that individual $i$ receives and $R = H^{-1}$ is the inverse function (the quantile function) of the CDF of $a$.\footnote{A simpler definiton of rank would be just having $F(h)$ (as in Frank, 1985). The problem with this definition is that if all agents accumulate the same level of human capital, $h$, then, because $F(h) = 1$, all agents would have the highest ranking, and since there is a continuum of individuals, each having zero weight, an individual that increases her investment in human capital just above $h$ would see no increase in her ranking (see Hopkins and Kornienko, 2004).}

\footnote{If all agents were to choose a level of human capital equal to $h$, then they would have ranking $\gamma$ whereas if one individual chooses a level of human capital slightly greater than $h$ her ranking would be 1 ($> \gamma$) (see Hopkins and Kornienko, 2004).}

\footnote{Recall that we have assumed before that the CDF $H(a)$ is strictly increasing and continuous, so it has an inverse (quantile) function.}

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4.4. Individuals’ optimization problem

Individuals take as given other individual’s effort and choose their own effort, \( e \), to maximize \( U(c,e) \). Assuming, without loss of generality, that \( u(c) = c \), the objective of agent with endowment \( \theta \in [a,b] \) is to solve the following problem:

\[
\max_{e \in [0,d]} R \left[ \gamma F(h(e, \theta) + (1 - \gamma) F^-(h(e, \theta)) \right] h(e, \theta) - v(e),
\]

where \( R \left[ \gamma F(h(e, \theta) + (1 - \gamma) F^-(h(e, \theta)) \right] h(e, \theta) \) (\( = w * h \)) is the level of income (and consumption) that an agent with an endowment \( \theta \) will attain.

**Assumption A4:**

(i) \( v(\cdot) \) is differentiable, increasing and strictly convex (from A2),
(ii) \( h(\cdot, \cdot) \) is differentiable, strictly increasing and concave in both arguments (\( h_{e\theta} > 0, \ h_e >> 0, h_{\theta} >> 0, \ h_{ee} < 0, \ h_{\theta\theta} < 0 \)). Also, \( h(0, \theta) = 0 \) for all \( \theta \in [a,b] \),
(iii) \( e(a) = e_a > 0 \).
(iv) \( R(\cdot) \) is differentiable, increasing and concave, and
(v) As a CDF, \( F(\cdot) \) inherits of the following property: \( F(\cdot) \) is non-decreasing and continuous from the right.

4.5. Labor market equilibrium

A symmetric Nash equilibrium solution is a mapping \( e : [a, b] \rightarrow [c, d] \) that assigns a choice of effort \( e(\theta) \) for any possible endowment level \( \theta \), where \( e(\theta) \) is chosen so to maximize (12). At this point, the solution need not be a function. Let \( h^{eq}(\theta) \equiv h(e(\theta), \theta) \) be the equilibrium human capital mapping. However, if the solution \( e(\theta) \) is a function, then the next proposition shows that the solution satisfies several useful properties.

**Proposition 3:** If the solution \( e(\cdot) \) exists and is a function then we have the following properties:
(i) \( h^{eq}(\cdot) \) is strictly increasing , (ii) \( e(\cdot) \) is continuous, and (iii) \( e(\cdot) \) is differentiable.

**Proof:** see the Appendix.
**Proposition 4:** Under A4, a solution function $e(\cdot)$ to the maximization problem stated in equation 12 exists, is unique, and is characterized by the following differential equation (first order condition), which forms a symmetric Nash equilibrium of the game of competition for jobs:

$$e'(\theta) = -\left[R'(G(\theta))g(\theta)\frac{h(e, \theta)}{R(G(\theta))h_e - e'} + \frac{h_\theta}{h_e}\right]$$

(13)

Proof: see the Appendix.

4.6. Comparative static results and simulations

Having found the equilibrium allocation of time and effort for each agent we are now interested in studying how inequality in the distribution of endowments affects aggregate efficiency in human capital formation. Also, as we will see below, interesting results are obtained regarding how different agents might be better off in more unequal societies whereas some others might be worse off. Recall from the 2 agents - 2 firms model presented in the previous section that more inequality of endowments is associated with a lower level of average human capital in the economy. In order to do the comparative statics results in the general model we use numerical methods to solve the differential equation 20 and then carry out the simulation of a mean-preserving spread in the distribution of endowments of the complementary factors.\(^{20}\)

In order to solve numerically equation 20 we use the same functional forms that we used in the 2 agents - 2 firms model (see footnote 14) and we assume, without loss of generality, that $e(a) = 1$, that is, the agent with the lowest endowment of the complementary factors exerts a level of effort equal to 1. We assume that $H(a) \sim U(0, 1)$, that is, technologies (and therefore wages) are distributed according to a standard uniform. Also, we assume that $G(\theta) \sim U(1 - \varepsilon, 2 + \varepsilon)$ and in doing the mean preserving spread in the distribution of endowments we will increase the parameter $\varepsilon$.

Figures 2 and 3 present the results of the numerical solutions of the general model for different values of the relative importance of effort in the accumulation of human capital, $\alpha$ (see footnote 14). In each Figure, Panel (a) shows how much effort is exerted in equilibrium

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\(^{20}\)We use a 4(5) imbedded pair Runge-Kutta Scheme called the Dormand-Prince 4(5) (explicit) scheme (see Ascher and Petzold, 1998, ch. 4). We than Lydia Boroughs for kindly helping us with this methodology.
by each agent in the economy. Note that the distribution of effort depends crucially on how important is effort relative to the endowment of the complementary factors in determining the accumulation of human capital. As effort becomes relatively less important (that is, as the endowments become more important) relatively poor agents have to exert more effort in order to compensate for their low endowment.\textsuperscript{21} Panel (b) in each of the figure shows that human capital, in equilibrium, is a strictly increasing function of the endowment (see point i. in Proposition 3). Panel (c) in each figure shows the result of the simulation of a mean preserving spread in the distribution of endowments (an increase in the parameter $\varepsilon$ in the distribution of endowments $G(\theta)$). In the two cases more inequality in the distribution of endowments (higher $\varepsilon$) is associated with lower aggregate efficiency in human capital formation (as measured by average human capital in the population). In other words, the proposed model does not predict a trade-off between equality and aggregate efficiency in human capital formation.

5. Extensions

5.1. Human capital accumulation in meritocratic vs. elitist societies

A natural extension of the framework presented before is to consider how human capital accumulation changes when the firm’s ranking of individuals does not fully depend on their human capital, but also on other factors that are beyond the individual’s control. In order to do so, let us assume that the firm’s ranking of individual is now a (linear) combination of the individuals’ ranking in the human capital dimension and the individuals’ ranking in the endowments dimension in the following way

$$\phi \left[ \gamma F(h(e, \theta) + (1 - \gamma)F^{-}(h(e, \theta)) \right] + (1 - \phi) \left[ \gamma G(\theta) + (1 - \gamma)G^{-}(\theta) \right], \quad (14)$$

where $\phi \in [0, 1]$ is a parameter that measures how meritocratic the firms’ ranking is. If $\phi = 1$ we are back in the general model in the previous section. However, if $0 < \phi < 1$, the firms rank individuals both in the human capital dimension and in the endowment dimension. One can think of $\phi$ as a measure of how social connections and other types of

\textsuperscript{21}From this observation we conjecture that relatively poor agents are worse off in economies where the complementary factors are relatively more important in determining human capital formation. This is a result that we will explore more formally in a future version of the paper.
influences generally associated with higher wealth might improve the ranking of some, most probably the relatively rich, individuals and make it easier for them to get the best available jobs. The important point is that the second part of the ranking (in the endowments ($\theta$) dimension) is beyond the individuals’ control.

The new solution to the maximization problem in equation 12 using the ranking by firms in equation 14 is:

$$e'(\theta) = -[\phi R'(G(\theta))g(\theta) \frac{h(e, \theta)}{R(G(\theta))h_e} - \frac{h_e}{h_e}] + \frac{h_\theta}{h_e}, \quad (15)$$

which is in fact very similar to that in equation 20 except for the inclusion of the parameter $\phi$.

We now compare the levels of effort and average human capital in the economy for a value $\phi < 1$ with those obtained in the previous section where $\phi$ was assumed to be equal to 1. Figure 4, 5, and 6 compare equilibrium effort, equilibrium human capital, and the relationship between inequality of endowments and aggregate efficiency in human capital formation for meritocratic societies (Panel a in each Figure) with those obtained in non meritocratic societies (Panel b in each Figure). Notice that both effort and human capital accumulation are lower for all individuals in non meritocratic societies. Even those individuals with higher endowments, and, in the case of non meritocratic societies, better social connections, exert less effort in equilibrium and accumulate less human capital. This is because for $\phi < 1$, the incentives to compete for job positions is weakened for all individuals as a portion of the firms’ ranking of individuals is beyond the individuals’ control.

5.2. Concluding remarks

to be written...
Appendix

Proof of Proposition 3.

STEP 1: we show that $h^{eq}(\cdot)$ is non decreasing.

(i) Let $\theta_1, \theta_2 \in [a, b]$ such that $\theta_2 > \theta_1$. We show that $h^{eq}(\theta_2) \geq h^{eq}(\theta_1)$. The statement is trivial if either $e(\theta_1) = 0$ or $e(\theta_2) = d$. Suppose that $d > e(\theta_2)$ and $0 < e(\theta_1)$. Notice that the range of $h(\cdot, \theta)$ is an interval $[0, h(d, \theta)]$ where $h(d, \theta)$ is strictly increasing in $\theta$. Since $h(d, \theta_2) > h(d, \theta_1)$ then there exists $\hat{e} \in [0, h(d, \theta_2)]$ such that $h(h, \theta_2) = h(e(\theta_1), \theta_1).

Therefore, $R(F(h(\hat{e}, \theta_2)))h(\hat{e}, \theta_2) = R(F(h(e(\theta_1), \theta_1)))h(e(\theta_1), \theta_1).

Now we show that $e(\theta_2) \geq \hat{e}$ so that $h^{eq}(\theta_2) \geq h^{eq}(\theta_1)$. By contradiction, suppose that there exists $\bar{e} < \hat{e}$ so that $R(F(h(\bar{e}, \theta_2)))h(\bar{e}, \theta_2) - v(\bar{e}) > R(F(h(\hat{e}, \theta_2)))h(\hat{e}, \theta_2) - v(\hat{e})$, or that:

$$R(F(h(\bar{e}, \theta_2)))h(\bar{e}, \theta_2) - R(F(h(\hat{e}, \theta_2)))h(\hat{e}, \theta_2) + (v(\bar{e}) - v(\hat{e})) > 0 \tag{16}$$

Let $e' < e(\theta_1)$ such that $h(\bar{e}, \theta_2) = h(e', \theta_1)$. Because $h_{e\theta} > 0$, it follows that $(e(\theta_1) - e') > (\hat{e} - \bar{e})$. (This last line may need more details if we keep it this way).

Since $\hat{e} < e(\theta_1)$, we have that $0 < (v(\hat{e}) - v(\bar{e})) < v(e(\theta_1)) - v(e')$ by convexity of $v(\cdot)$. Using this last observation along with (16) above we get:

$$R(F(h(e', \theta_1)))h(e', \theta_1) - R(F(h(e(\theta_1), \theta_1)))h(e(\theta_1), \theta_2) + v(e(\theta_1)) - v(e') > 0 \tag{17}$$

This is a contradiction with the fact that $e(\theta_1)$ is maximizing (12). Therefore, $e(\theta_2) \geq \hat{e}$.

STEP 2 $h^{eq}(\cdot)$ is strictly increasing

By contradiction, suppose that it is not. Then there exists $\theta_0 < \theta_1$ with $\bar{h} = h^{eq}(\theta_0) = h^{eq}(\theta_1)$. Because $h^{eq}(\cdot)$ is non-decreasing, $h^{eq}(\theta) = \bar{h}$ for all $\theta \in [\theta_0, \theta_1]$. That is, there is a mass point in the distribution of human capital at $\bar{h}$ and therefore $F(\bar{h}) > F^{-}(\bar{h})$. It follows that $R((\gamma F(\bar{h}) + (1 - \gamma)F^{-}(\bar{h})) < R(F(\bar{h})) \leq R(\gamma F(\bar{h} + \epsilon) + (1 - \gamma)F^{-}(\bar{h} + \epsilon))$ for all $\epsilon > 0$. 

20
Notice that $e(\theta_1) < d$. If this was not true then we would have $h^{eq}(\theta_0) \equiv h(e(\theta_0), \theta_0) \leq h(d, \theta_0) < h(d, \theta_1) = \bar{h}$, a contradiction. Since $h(\cdot, \theta)$ and $v(\cdot)$ are both continuous in $e$, then for any $d > \delta_n > 0$, we have $\lim_{\delta \to 0} v(e(\theta_1) + \delta) = v(e(\theta_1))$ while $\lim_{\delta \to 0} h(e(\theta_1) + \delta, \theta_1) = \bar{h}$. From the preceding paragraph, however, $R(\gamma F(h(e(\theta_1) + \delta, \theta_1)) + (1 - \gamma) F^-(h(e(\theta_1) + \delta, \theta_1)))) > R(\gamma F(\bar{h}) + (1 - \gamma) F^-(\bar{h})))$ for any $\delta > 0$.

Therefore, there exists a small enough $\bar{\delta} > 0$ such that for any $\delta < \bar{\delta}$,

$$R(\gamma F(h(e(\theta_1) + \delta, \theta_1)) + (1 - \gamma) F^-(h(e(\theta_1) + \delta, \theta_1))))h(e(\theta_1) + \delta, \theta_1) - v(e(\theta_1) + \delta) > R(\gamma F(\bar{h}) + (1 - \gamma) F^-(\bar{h})))\bar{h} - v(e(\theta_1))$$

Thus, an individual with an endowment $\theta_1$ could increase her utility by choosing a slightly higher level of effort $e(\theta_1) + \delta$, which leads to a contradiction.

**STEP 3** We show that $e(\cdot)$ is continuous.

By contradiction, suppose not, so there is a jump in the equilibrium solution at some endowment level of the complementary factors $\hat{\theta} \in (a, b)$ so that $\lim_{\theta \to \hat{\theta}} e(\theta) = \hat{e} \neq e(\hat{\theta})$. Notice that $h^{eq}(\cdot)$ being strictly increasing implies the continuity of $R(\gamma F(h^{eq}(\cdot) + (1 - \gamma) F^-(h^{eq}(\cdot))))$, that is

$$\lim_{\theta \to \hat{\theta}} R(\gamma F(h^{eq}(\theta) + (1 - \gamma) F^-(h^{eq}(\theta)))) = R(\gamma F(h^{eq}(\hat{\theta}) + (1 - \gamma) F^-(h^{eq}(\hat{\theta})))) = R(\gamma F(h(\hat{\theta}, \hat{\theta}) + (1 - \gamma) F^-(h(\hat{\theta}, \hat{\theta}))))$$

Since, $v(\cdot)$, $h(\cdot, \cdot)$ and $R(\gamma F(h^{eq}(\cdot) + (1 - \gamma) F^-(h^{eq}(\cdot))))$ are continuous at $(\hat{e}, \hat{\theta})$ then a standard argument applies.

**STEP 4** We show that $e(\cdot)$ is differentiable on $(a, b)$. From the previous steps, notice that $\gamma F(h(e, \theta) + (1 - \gamma) F^-(h(e, \theta))) = F(h(e, \theta))$ for all $\theta \in [a, b]$. That is, there are no mass points.

Let $\hat{\theta} = \theta + \delta$ for some $\delta$. We have,

$$R(F(h(e(\hat{\theta}), \theta))h(e(\theta), \theta) - v(e(\theta)) \geq R(F(h(e(\hat{\theta}), \theta))h(e(\hat{\theta}), \theta) - v(e(\hat{\theta}))$$

Similarly, we have,
\[ R(F(h(e(\hat{\theta}), \hat{\theta})))h(e(\hat{\theta}), \hat{\theta}) - v(e(\hat{\theta})) \geq R(F(h(e(\theta), \hat{\theta})))h(e(\theta), \hat{\theta}) - v(e(\theta)) \]

By the Mean Value theorem we have,

\[ R(F(h(e(\hat{\theta}), \theta)))h(e(\hat{\theta}), \theta) = R(F(h(e(\theta), \theta)))h(e(\theta), \theta) + \]

\[ (R'(F(h(e_1, \theta)))f(h(e_1, \theta)h_e(e_1, \theta))h(e_1, \theta) + R(F(h(e_1, \theta)))h_e(e_1, \theta))(e(\hat{\theta}) - e(\theta)) \quad \text{for some} \quad e_1 \in [0, d] \]

so that,

\[ (v(e(\hat{\theta})) - v(e(\theta))) - (R'(F(h(e_1, \theta)))f(h(e_1, \theta)h_e(e_1, \theta))h(e_1, \theta) \]

\[ + R(F(h(e_1, \theta)))h_e(e_1, \theta))(e(\hat{\theta}) - e(\theta)) \geq 0 \]

Similarly, using again the mean value theorem yields

\[ (v(e(\hat{\theta})) - v(e(\theta))) - \]

\[ (R'(F(h(e_2, \theta)))f(h(e_2, \theta)h_e(e_2, \theta))h(e_2, \theta) + R(F(h(e_2, \theta)))h_e(e_2, \theta))(e(\hat{\theta}) - e(\theta)) \leq 0 \quad \text{for some} \quad e_2 \in [0, d] \]

Combining these two last inequalities, we obtain:

\[ \frac{v(e(\hat{\theta})) - v(e(\theta))}{(R'(F(h(e_1, \theta)))f(h(e_1, \theta)h_e(e_1, \theta))h(e_1, \theta) + R(F(h(e_1, \theta)))h_e(e_1, \theta))\delta} \leq \frac{e(\hat{\theta}) - e(\theta)}{\delta} \leq \frac{v(e(\hat{\theta})) - v(e(\theta))}{(R'(F(h(e_2, \theta)))f(h(e_2, \theta)h_e(e_2, \theta))h(e_2, \theta) + R(F(h(e_2, \theta)))h_e(e_2, \theta))\delta} \]

By continuity, both the RHS and LHS of the expression converges to the same limit at \( \delta \) approaches 0 ensuring that \( \lim_{\delta \to 0} \frac{e(\hat{\theta}) - e(\theta)}{\delta} \) exists. By definition, this establishes that \( e(\cdot) \) is differentiable at \( \theta \).

END OF PROOF.

**Proof of Proposition 4**

**STEP 1**

Since both \( F(\cdot) \) and \( R(\cdot) \) are increasing, the fact that \( h(\cdot, \theta) \) is concave guarantees that the composite function \( R(F(h(\cdot, \theta))) \) is both quasiconcave and increasing in \( e \) for all \( \theta \in [a, b] \). Since \( h(\cdot, \theta) \) is, also, both quasiconcave and increasing in \( e \) for all \( \theta \in [a, b] \) then the product \( R(F(h(\cdot, \theta)))h(\cdot, \theta) \) is also quasiconcave in \( e \) for all \( \theta \in [a, b] \).
Furthermore, the objective function in 12 is therefore strictly quasiconcave because $v(\cdot)$ is strictly convex by assumption (A2).

**STEP 2**

From the previous step and since the functions $R(\cdot), F(\cdot), h(\cdot, \theta)$ and $v(\cdot)$ are continuously differentiable\(^{22}\) then the problem (12) can be characterized by the following first order condition:

$$
R'(F(h^{eq}(\theta))) f(h^{eq}(\theta)) h_e(e, \theta) + R(F(h^{eq}(\theta))) h_e(e, \theta) - v'(e) = 0.
$$

(18)

Alternatively, (18) can be rewritten as follows using the fact that $G((h^{eq})^{-1}(h(e, \theta))) \equiv F(h(\cdot, \theta)), G(\cdot) \equiv F(h(\cdot))$ and $g(\cdot) \equiv f(h^{eq}(\cdot))$.

$$
R'(G(\theta)) g(\theta) \frac{d(h^{eq})^{-1}(h^{eq}(\theta))}{dh} h_e(e, \theta) + R(G(\theta)) h_e(e, \theta) - v'(e) = 0
$$

(19)

**STEP 3** The above first order condition can be rewritten as the following differential equation using the fact that $1 = \frac{d(h^{eq})^{-1}(h^{eq}(\theta))}{dh}(h_e(e, \theta)e'(\theta) + h_{\theta}(e, \theta))$ and given the denominator does not vanish in the expression for $\psi(\cdot, \cdot)$ below.

$$
e'(\theta) = -[R'(G(\theta)) g(\theta)(1 - h_{\theta}(e, \theta))\psi(e, \theta)],
$$

(20)

where $\psi(e, \theta) = \frac{h(e, \theta)}{R(G(\theta)) h_e(e, \theta) - v'(e)}$

**STEP 4** In equilibrium $e^{eq}(a) = e_a > 0$.\(^{23}\) If not then the individual with the lowest endowment of the complementary factors ($a$) could improve her outcome by reducing her effort without affecting her ranking which is already at the lowest anyway. Moreover, given that $e^{eq}(a) = e_a > 0$, then $v'(e_a) > 0$ and therefore the denominator of $\psi(e, \theta)$ is always non zero making the differential equation (20) a complete characterization of the equilibrium.

**STEP 5**

\(^{22}\)The fact that $F(\cdot)$ is differentiable follows from the identity $G(\cdot) \equiv F(h^{eq}(\cdot))$ in which both $h^{eq}(\cdot)$ and $G(\cdot)$ are differentiable.

\(^{23}\)That is, we assume that there is a minimum guaranteed level of effort for all agents.
Lastly, notice that given $\psi(\cdot, \cdot)$ is continuous, such a differential equation with initial condition $e^{eq}(a) = e_a$ has a unique solution by virtue of the fundamental theorem of differential equations. Therefore, a unique equilibrium solution exists.

END OF PROOF.
References


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Figure 2: Simulation results ($\alpha = 3/4$).
Figure 3: Simulation results ($\alpha = 1/2$).
Figure 4: Equilibrium Effort in meritocratic societies ($\phi = 1$) (Panel a) vs Equilibrium Effort in non meritocratic societies ($\phi = 0.5$) (Panel b).

Figure 5: Equilibrium Human Capital in meritocratic societies ($\phi = 1$) (Panel a) vs Equilibrium Human Capital in non meritocratic societies ($\phi = 0.5$) (Panel b).
Figure 6: Inequality of endowments vs. Aggregate efficiency in HK formation in meritocratic societies ($\phi = 1$) (Panel a) vs non meritocratic societies ($\phi = 0.5$) (Panel b).