Industry Dynamics, Investment and Business Cycles*

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Abstract

Dispersion in marginal products within narrowly defined industries is ubiquitous in modern economies, and it is often interpreted as a symptom of inefficiencies in production. When there are non-convexities in firm level technologies and firms operate under uncertainty, dispersion in marginal products can arise as the outcome of an efficient allocation. I analyze a fully fledged model of industry dynamics, where entry, exit and indivisible and irreversible technology investment are real options. The competitive allocation is inefficient because i) firms compete monopolistically and ii) dispersion in marginal products induces a gap between the social and private value of a firm. I characterize and compute the efficient allocation. Only a small fraction of the allocative inefficiency is directly reflected in dispersion in marginal products. For a calibrated economy to the US manufacturing sector, I find that productivity can be increased by 11 percent when implementing the efficient allocation with state contingent subsidies and taxes. Shifts in equilibrium exit, entry and technology investment account for two-thirds of the gains. Only a third of the gains are explained by reallocation of inputs across incumbents that reduces dispersion in marginal products. [JEL Codes: E32,L11,E23].

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1 Introduction

Dispersion in marginal products within narrowly defined industries is a stylized fact of modern economies[1]. There are many reasons for which marginal productivity of inputs may differ across firms. Some of the most extensively analyzed mechanisms in the literature are size dependent policies [2], subsidies or taxes for particular firms[3] and market incompleteness (i.e. financial frictions [4]). These mechanisms can explain a large portion of the documented dispersion. They also imply that if this dispersion is eliminated, efficiency can be improved. In this paper, I argue that dispersion in marginal products may arise as the outcome of an efficient allocation. Hence, some of the observed pattern in the data need not be detrimental for productivity or welfare. Such understanding of the origin of the observed dispersion, is important for the design of productivity enhancing policies.

In the economy that I study, irreversibilities and indivisibilities in investment, when firms operate under uncertainty, generate dispersion in marginal products. Consider the following example. Suppose we observe two firms that have access to the same set of indivisible production technologies and different marginal products: an incumbent operating at high capacity and an entrant operating at a lower scale. In models without indivisbility and irreversibility in technology choice, such disparity can be interpreted

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[1]For cross country evidence refer to Asker et al. (2013). For evidence for Korea, refer to Midrigan and Xu (2009). Also, Hsieh and Klenow (2009) provide evidence for the US, India and China. For evidence in Latin America, see Buso et al. (2013).

[2]Barstelman et al. (2013) document and study the impact of distorsions that are correlated with the size of firms.

[3]Restuccia and Rogerson (2008) analyze a broad range of policy distorsions.

[4]See Buera and Shin (2011), Moll (2013), Midrigan and Xu (2009) and the extensive literature thereafter.

as an inefficiency. Suppose, however, that the incumbent had entered the market at an earlier time during a boom, and current market conditions have worsened since. Therefore, the entrant finds it optimal to wait to scale up its capacity until market conditions improve, while the incumbent does not exit (or scales down) because its option value of remaining in the market is positive. Hence, a gap between the marginal products of capital for these two otherwise identical firms, is consistent with optimal investment strategies for both firms. Furthermore, in this environment, the planners' allocation would also generate dispersion in marginal products.

Indivisibilities and irreversibilities in technology investment are known to impact the shadow value of inputs across firms. They have the potential of generating efficient dispersion in marginal products, and hence the natural starting point for normative analysis. While in certain economies, dispersion in marginal product can be a symptom of inefficiencies in production, they do not necessarily reflect other inefficiencies associated with dynamic decisions of firms. In this paper, I show that shifts in the patterns of entry, exit and investment have a larger contribution to productivity gains than those associated to drops in marginal product dispersion. This finding is consistent with micro empirical analysis that documents substantial productivity improvements associated to shifts in the patterns of firm churning.

I study an economy where firms invest in alternative technologies and decide when to enter and exit the market. These decisions are modeled as real options. A technology is a productivity level and an associated minimum capacity in terms of capital. More productive technologies have a higher minimum capacity associated to them. At the moment of entry, each investor is assigned a blueprint (a technique to produce a good), the quality of which varies over a continuum of types. One could interpret a blueprint as a product and a technology as a process. The same product can be generated with alternative processes. The productivity of a firm depends on the combination of its blueprint and the technology it operates, plus an exogenous aggregate shock. Investment in technology is indivisible, because only a finite set of technologies (and associated minimum capacities) is available. It is also irreversible, in the sense that disinvestment in technology entails the firm liquidation and obtaining a new blueprint. When the options to enter, to improve technology or to exit the market are exercised, a one time fixed cost is incurred. Finally, firms compete monopolistically.

Due to the real option feature of the model there are states of the world where firms hold a particular technology while being constrained by its minimum capacity (holding excess capacity). The marginal product of capital for a constrained firm is lower than that of a comparable unconstrained firm. The equilibrium observed dispersion in

marginal products and the physical productivity of firms operating in the market depend on their investment decisions in technology, entry and exit patterns. Hence, physical productivity at the firm level and dispersion in marginal products, are endogenously related.

In this economy, the equilibrium allocation is inefficient. On the one hand, monopolistic competition generates a gap between the social and private of the firm, equal to the constant markup charged by the firms in the decentralized market. On the other hand, non-convexities in firm production, which in our environment induce dispersion in marginal products, generates an additional endogenous state dependent gap. Dispersion in marginal products are associated to disparate capital labor ratios across firms (static impact) and differences in investment and disinvestment strategies between the efficient and decentralized allocations (dynamic impact). The shift in incentives to entry, exit and technology investment affects the dynamics of aggregate productivity, often permanently. Assessing whether efficiency losses stem from lack of equalization in capital labor ratios(static inefficiency) or from inefficient investment and disinvestment decisions (dynamic inefficiency) is a novel contribution of this paper.

The social value of a firm type (blueprint and technology) includes the private value of the firm and an adjustment term. This term dictates that when the marginal product of capital of a firm is relatively low with respect to some moment of the marginal products in the market, the planner has an additional incentive, vis a vis the firm, to liquidate it. By doing so, the planner can free up labor from a firm with excess capacity and reallocate it to less constrained active firms. In the decentralized allocation, firms that hold excess capacity display low marginal product of capital but have a positive option value of remaining in the market. The planner can undo some of the effect of the irreversibility by liquidating a few of these firms, and replacing them with firms running at lower capacity (higher marginal product of capital), which can later on be expanded. Because this process is costly, the planner does not liquidate all firms with excess capacity. The gap between the private and social value of a firm varies endogenously with the conditions in the market. During downturns, it is more likely that firms hold excess capacity and hence, the decentralized allocation will be further away from the efficient one. It is worth to point out that if technologies are such that marginal products are equalized in equilibrium, the gap between decentralized and centralized allocations disappears.

The study of optimality in models with aggregate uncertainty, heterogenous firms and irreversibilities and indivisibilities is challenging. I show that if cost of adjustments are presented in the form of sunk costs and there is a continuum of firm types, there is a centralized problem whose allocation can be decentralized as a market allocation, i.e. the second welfare theorem holds. To bring decentralized and efficient allocations together, the optimal policy entails a transfer scheme similar to a state dependent Pigouvian tax/subsidy. As a by-product of the equivalence result, I can sidestep the standard approximation methods of Krusell and Smith (1998) to solve economies with heterogeneity and aggregate shocks. I solve the planner's allocation and use the decentralization result to compute the equilibrium of this economy.

For the economy calibrated to the US manufacturing sector, the planner's allocation dictates higher equilibrium investment, and a shift in output production towards larger, more productive firms. Improvements in aggregate productivity are 11% under the optimal policy. Suppose that instead of characterizing the efficient allocation, the economist assumes all dispersion in marginal products is associated to inefficiencies. He will compute gains from full elimination of dispersion in marginal products of 76%. The efficiency improvement would be widely overestimated.

In the model, efficiency gains from the implementation of the optimal policy are accounted mostly by a change in firms entry, exit and investment patterns. Only a third of the gains in productivity are explained by reallocation of labor and capital across incumbent firms. The employment distribution varies slightly between the decentralized and planner's allocation.

Suppose that one would like to compare alternative economies for which we observe some statistic of marginal product dispersion. If dispersion were a sign of inefficiency, the economy with higher dispersion should display lower aggregate productivity. Suppose that these economies differ in the process characterizing the aggregate shock. In the model, it is possible for these economies to have similar dispersion marginal products and substantial differences in aggregate productivity. At one extreme, when the volatility of the aggregate productivity process is low, the economy approximates a stationary one. There is exit and entry in equilibrium as well as upgrades in technology. However, because the size of the aggregate shock is small, the main determinant of investment decisions is the firm's idiosyncratic productivity (as it will be in an economy with no shocks). The mechanism discussed in the example at the beginning becomes irrelevant. At the other extreme, when the volatility of the process is very high, incumbent firms find it more valuable to wait and not upgrade. Hence, in equilibrium upgrades in technology are delayed. Exit rates increase so that firms holding capital away from the level that they would have chosen in the current period are selected out of the market whenever a bad shock hits the economy. The mechanism described above vanishes again. While both economies display low dispersion in marginal products, the one with higher volatility is on average less productive than the one with lower volatility. Hence, the link between aggregate productivity and dispersion in marginal products depends on features of the macroeconomy and the patterns of firms entry, exit and investment.

The rest of the paper is organized as follows. The following section reviews the literature, then we introduce the model. Finally we calibrate the model to the USA economy and analyze the implications of the model for cross sectional dispersion in productivity, aggregate productivity, industry dynamic and optimal industrial policy.

1.1 Literature Review

Models of industry equilibrium with complete markets (for example Hopenhayn (1992)) typically display aggregation. Hence, there is very little effect of heterogeneity in equilibrium allocations. As marginal product and capital labor ratios are equalized, the model boils down to one of a representative firm with average productivity. When the relationship between productivity, size (employment or assets) and output is non-monotonic, heterogeneity matters in a non-trivial way. Marginal products may differ in excess of productivity differences as is typically the case in incomplete market models.

Some efforts departing from this one to one map are those by Lee and Mukoyama (2008), Clementi and Palazzo (2010) and Khan and Thomas (2008) (in incomplete markets). Lee and Mukoyama (2008) provide evidence of differential entry and exit behavior along the business cycle and propose a model to quantitatively explain those facts. They analyze the effect of fluctuations in fixed production costs and labor adjustment costs on the industry dynamic in a model with no capital. Clementi and Palazzo (2010) analyze the propagation of aggregate shocks due to entry and exit of firms when firms are allowed to accumulate capital. Khan and Thomas (2008) study the effect of irreversibilities and collateral constraints in equilibrium allocations in an economy without exit and entry. They find that both frictions reinforce each other in slowing down reallocation. In open economies, Alessandria and Choi (2007) and Bilbiie et al. (2012) studied the effect of exit, entry and export behavior on aggregate fluctuations too.

One of the main drivers for the introduction of industry dynamic into standard RBC models seems to be the observed heterogeneity in firms behavior as of capital accumulation, employment (Moscarini and Postel-Vinay (2012), Davis et al. (1998)), and entry and exit (Barstelman et al. (2009)) . If reallocation is costless, economies with heterogeneous firms will be unidentifiable from an economy in which all firms hold the average productivity in the market (See Melitz (2003) for example). The result follows form the equalization of the shadow price of capital as in the Gorman (1953) tradition (in that case the shadow price of wealth). Departures from the standard frictionless model are necessary to reconcile the theory with the patterns observed in the data. In the model of the paper, I introduce indivisibilities and irreversibilities in the technologies operated by firms. The average productivity is not sufficient to predict the response of the economy to the shock. Marginal products may differ for a given period of time (static inefficiency) but capacity choices are dynamically optimal, so there will be no room for reallocation. The result resembles earlier intuitions drawn in Caballero and Hammour (1998) when analyzing factor specificity.

First generation models of irreversibility analyzed the impact of lumpy investment at the micro level on aggregate investment dynamic. Exploiting a partial equilibrium setup, they found substantial contribution of lumpiness to aggregate investment fluctuations (Caballero and Hammour (1998),Caballero et al. (1995) ,Abel and Eberly (1997), Dums and Dunne (1998)). In the second generation, the seminal paper of Veracierto (2002) and later Thomas (2002) argued that although in partial equilibrium their impact on investment dynamic is substantial, those effects fade away in general equilibrium. In both cases, a costless adjustment economy cannot be told apart from one with irreversibilities. Later work by Khan and Thomas (2003) found additional support for this hypothesis.

Such results suggest that irreversibilities can be dispensed with when analyzing aggregate fluctuations. Recent work by Bachmann et al. (2006) show that irreversibilities play an important role in explaining the conditional heteroscedasticity in aggregate investment behavior observed in the data. On a different line, Khan and Thomas (2008) show that quantitatively the interaction of irreversibilities with financial frictions may explain large drops in aggregate efficiency and slow recoveries. As exposit by Caballero (1999) irreversibilities might have important consequences for the aggregate behavior of the economy when interacted with market incompleteness or informational asymmetries.

This paper shows that even in a complete market set up, there is a relevant mechanism through which irreversibility affects aggregate efficiency. When interacted with aggregate uncertainty in a fully fledged industry dynamic model, they generate a disparity between investment decisions of entrants and incumbents in the market. The vintage of the firm becomes relevant in explaining their investment behavior. The most salient difference between this paper and previous work by Veracierto (2002), is that he abstracts from the entry and exit problem while it is determined endogenously in this paper[5]. Also, the nature of noncovexities in production is different from the one exploited in Veracierto (2002): while there partial irreversibility is allowed, here there is full irreversibility and invisibilities in technology adoption.

The mechanism that the model study relates to the literature on irreversible investment as started by Dixit and Pyndick (1994), Abel and Eberly (1997), Mariotti et al. (2006) and Caballero and Engel (1999). However, the set up of the model is closer to the work of Melitz and Guironi (2007). Distinctively, I allow firms to endogenously determine efficiency through investments in distinct technologies. I take a radical stand as of the idiosyncratic productivities of the firms, they are constant. I assume that the log productivity is drawn from an exponential distribution, so that the model can be interpreted as the limiting case of a model in which firms idiosyncratic productivity is stochastic and follows a Brownian Motion (See Luttmer (2010) for an example). The mechanism of the model does not vanish when idiosyncratic productivity is al-

[5]As can be seen from table 4 in Veracierto (2002), when there is full irreversibility, the change in the exogenous death rate has considerable effect on investment dispersion across production units.

lowed to change in time. It can rather be reinforced, as negative idiosyncratic shocks may render previous investment decisions statically inefficient. Assuming idiosyncratic risk away allows me to separate the impact of technological restrictions versus market incompleteness.

On the theoretical side, I contribute to the work initiated by Lucas and Prescott (1971). They showed that a competitive equilibrium can be decentralized as an industry equilibrium in which the planner maximizes overall surplus in the economy by allocating labor across firms. I show that when firms compete monopolistically and there are irreversibilities in investment and indivisibilities in capital allocations, there is a pseudo planner problem whose equilibrium allocation coincides with the decentralized solution as long as state contingent subsidies and taxes are available. The equivalence result follows closely the result described in Jones and Manuelli (1990) to study policy questions in convex economies with growth. A recent paper that also studies the characteristics of the constrained optima allocation in models with industry dynamic and wedges in marginal products is Fattal Jaef and Hopenhayn (July 2012). There, wedges are calibrated to match the characteristics of the allocation of employment across firms. As in this paper, they restrict the planner to face the same distortions that the firms in the economy face. They find that while the competitive allocation generates the efficient allocation of resources across a given set of technologies, it fails to generate the efficient level of entry and exit, and hence the measure of active firms. The model analyzed here departs from their environment in several ways. First, the allocation of technologies run by the firm is endogenous, so even in the absence of imperfect competition, the allocation of employment need not efficient in the competitive equilibrium as outlined in the previous section. Second, this paper studies an economy with imperfect competition. Finally, this paper characterizes the optimal industrial policy. This has been done for models of international trade under oligopolistic competition in prices and quantities (Eaton and Grossman (1986)). For a model of industry dynamic without capital accumulation Lee and Mukoyama (2008) study the impact of alternative policies on labor regulations. However, their policies are ad hoc in the sense that there is no notion of efficiency associated to them. Lee and Mukoyama (2008) study the impact of taxes to output and inputs in production over aggregate TFP, for both i.i.d. policies and policies correlated with the productivity of the firms. Guner et al. (2008) study policies that target the size of the establishment, which in turn is correlated with their idiosyncratic productivity, and find substantial role in shaping aggregate productivity. Distinctively, this paper characterizes the optimal policy in an environment in which the efficient allocation does not dictate equalization of marginal products across all firms in the economy.

2 Model

This is an infinite horizon economy with time indexed by t. There is a final good which agents use for consumption and capital accumulation. It is produced by means of a continuum of intermediate goods. Intermediate goods are produced by combining capital and labor. Each intermediate good is perfectly differentiated and each firm producing it faces a constant elasticity demand. Final goods are traded competitively while there is monopolistic competition in intermediate goods. The technology for production of intermediate goods is endogenously chosen, and each one is associated to a minimum running capacity in terms of capital goods.

There is aggregate uncertainty in the economy. The exogenous shock is denoted s_t that takes two values, i.e. $\{g, b\}$, $g > b$ associated to the "good" and "bad" state, respectively. The transition probabilities are given by the matrix $\mathbf{P} \equiv \begin{bmatrix} \gamma_g & 1-\gamma_g \\ 1-\gamma_b & \gamma_b \end{bmatrix}$ where $P(s_{t+1} = g/s_t = g) = \gamma_g$ and $P(s_{t+1} = b/s_t = b) = \gamma_b$.

2.1 Households

The representative household derives utility from consumption of the final good C_t.

The household is endowed with a unit of labor that for simplicity is supplied inelastically to the firms. She receives a wage w_t for the services. She can also accumulate capital K_t, priced in terms of the final good (the numeraire) and rent it at price r_t to the firms. The aggregate stock depreciates at rate $\widehat{\delta}$.The household can buy shares of two different mutual funds that entitled it to the dividends generated by the firms operating in the economy. The first mutual fund consist of all the firms running with low minimum capacity technology, and the second is build with all firms using a technology with higher minimum capacity. After dividends are paid, assets can be traded.

Her problem reads

$$Max \ _{C_t, n_t^L, n_t^H, K_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right] \tag{1}$$

subject to

$$C_t + K_{t+1} - (1 - \widehat{\delta}) K_t + \sum_{j=L,H} P_t^j n_t^j = w_t + r_t K_t + \sum_{j=L,H} \left(d_t^j + P_t^j \right) n_{t-1}^j \tag{2}$$

$$X_{t+1} = \Gamma_c(X_t) \tag{3}$$

where P_t^j is the price of shares n_t^j of a mutual fund of firms of technology with minimum capacity $j = L, H$ at period t+1, which pay dividends d_{t+1}^j and can be sold tomorrow at price P_{t+1}^j. In computing the return to the share holdings, the agent needs to forecast the law of motion of the distribution of firms in the market for each possible realization for the exogenous aggregate shock, s_t. The aggregate state $X_t = \left(s_t, v_t^L, v_t^H, K_t \right)$ includes the exogenous shock, s_t ; the distribution of firms per technology, v_t^j for $j = L, H$;and the available aggregate stock of capital. To save on notation I denote $v_t^j(z) \equiv v_t^j([0, z))$ the measure of firms with productivity at most z and technology j. The subjective law of motion for the representative consumer is denoted by Γ_c. U fulfills the standard assumptions of concavity, monotonicity and differentiability. $\beta \in (0, 1)$ is the discount factor. The optimality conditions of the problem are standard. Dynamic optimality yields

$$U'(C_t)P_t^k = \beta E_t \left[U'(C_{t+1}) \left(d_{t+1}^k + P_{t+1}^k\right)\right] \tag{4}$$

For a standard CES specification $U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$ one can rewrite the price of shares as the present discounted value of all future dividends of firms that are active in period $t+1$with technology j, adjusted by the corresponding pricing kernel

$$P_t^j = E_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{C_\tau^{-\theta}}{C_t^{-\theta}} d_\tau^j \tag{5}$$

2.2 Final Goods Sector

There is a a representative competitive firm with a CES technology that produces final goods Y_t out of intermediate inputs y_{it}. The firm maximizes profits as

$$Max_{y_{it}}\ Y_t - \int p_{it} y_{it}\ di$$

subject to

$$Y_t \leq \left(\int y_{it}^{\rho} di\right)^{\frac{1}{\rho}}$$

where p_{it} is the cost of good y_{it}. It is assumed $\rho \in (0,1)$ so that goods are substitutes in production.

The corresponding input demand for each variety i emerges from the FOC of the problem, i.e.

$$Y_t^{1-\rho} y_{it}^{\rho-1} = p_{it}$$

2.3 Intermediate Goods

2.3.1 Capital and Labor Allocation

To produce each differentiated good, firms use capital and labor which are available at cost w_t and r_t, respectively, in units of the composite good. The technology is Cobb-Douglas,

$$y_t \leq s_t z \psi^j l_t^{1-\alpha} k_t^{\alpha}$$

There are two alternative technologies associated to a minimum capacity and a productivity shifter, $\left\{k_j, \psi^j\right\}$ for $j = L, H$. For simplicity we assume, $\psi^L = 1$ and $\psi^H > 1$. The capital choice sets are $[k_L, \infty)$ and $[k_H, \infty)$ for each technology, respectively. We interpret this indivisibility as the construction of a plant, or the set up of machinery which entails a particular capacity. The adoption of technology is costly. The problem of adoption, entry and exit into the market will be analyzed later. In this section, I study the allocation of capital and labor only.

Define x_t as the vector of idiosyncratic state variables to the firm, i.e. $x_t = \left(z, \psi^j\right)$. Let X_t be defined as before and define Γ_f as the law of motion for the aggregate state as perceived by any arbitrary firm; i.e. $X_{t+1} = \Gamma_f(X_t)$. The problem of a firm producing an intermediate good i in any period t is

$$\pi(x_t, X_t) = Max_{p_t, l_t, k_t} \; (p_t y_t - w_t l_t - r_t k_t)$$

subject to

$$y_t \leq s_t z \psi^j l_t^{1-\alpha} k_t^{\alpha}$$

$$\left(\frac{Y(X_t)}{y_t}\right)^{1-\rho} = p_t$$

$$k_t = [k_j, \infty)$$

Firms are assumed to be entirely equity owned. Because the elasticity of the demand is constant, the optimal price set by a firm is a constant markup over marginal cost. In particular,

$$p_t = \frac{(r_t - \lambda_t)^{\alpha} w_t^{1-\alpha}}{\rho\alpha^{\alpha}(1-\alpha)^{1-\alpha} s_t z}$$

where $\lambda_t \geq 0$ is the lagrange multiplier associated to the feasible set for capital. If the minimum capacity requirement is not binding then, $\lambda_t = 0$, otherwise $\lambda_t > 0$ and the markup for this firm is lower than otherwise.

From the FOC of the firms we can compute the labor and capital demand as follows

$$l_t = (s_t z \psi^j)^{\frac{\rho}{1-\rho}} \left[\left(\frac{1-\alpha}{w_t} \right)^{\frac{1}{\rho}-\alpha} \left(\frac{\alpha}{r_t - \lambda_t} \right)^{\alpha} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t) \tag{6}$$

$$k_t = \max \left\{ k_j, \left[s_t z \psi^j \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)} \left(\frac{\alpha}{r_t} \right)^{\frac{1}{\rho}-(1-\alpha)} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t) \right\} \tag{7}$$

The higher the relative efficiency in production the higher the demand of labor when intermediate goods are substitutes in production. Labor and capital demands are non-creasing in their costs, and they are increasing in the demand level as summarized by $Y(X_t)$.

Importantly, capital labor ratios need not be equal across all firms in the economy as the shadow value of capital depends on whether firms are constrained or not

$$\frac{k_t}{l_t} = \frac{w_t}{r_t - \lambda_t} \frac{\alpha}{1-\alpha}$$

If the minimum capacity requirement is binding, the firm adjusts its resource allocation through the flexible factor, in this case labor. However, the last condition indicates that constrained firms' labor demand does not increase enough to equalize the firms capital labor ratios across all firms [6]. This disparity is at the heart of the dynamics studied in

[6] In models where firms can be financially constrained, the capital labor ratios of constrained firms is usually lower than that of unconstrained firms. Constrained firms hold lower capital than they would if unconstrained, while in our model, constrained firms hold more capital.

this paper. In a static model with complete markets, disparate capital labor ratios are a sign of inefficiencies in the allocation. In the current set up however, these gaps might be consistent with a constrained efficient allocation as described later in the paper.

Define $Z^l = \int (\frac{\psi_i^j z_i}{\left(r_t(X_t)-\lambda_t^i\right)^{\alpha}})^{\frac{\rho}{1-\rho}} di$ and $Z^k = \int (\frac{\psi_i^j z_i}{\left(r_t(X_t)-\lambda_t^i\right)^{\frac{1-(1-\alpha)\rho}{\rho}}})^{\frac{\rho}{1-\rho}} di$, both statistics of productivity adjusted by the shadow value of capital across firms in the economy. Labor and capital demand are proportional to these statistics

$$l(x_t, X_t) = \frac{1}{Z^l} (\frac{\psi^j z}{(r_t(X_t) - \lambda_t)^{\alpha}})^{\frac{\rho}{1-\rho}} \quad (8)$$

$$k(x_t, X_t) = \frac{K_t}{Z^k} (\frac{\psi^j z}{(r_t(X_t) - \lambda_t)^{\frac{1-(1-\alpha)\rho}{\rho}}})^{\frac{\rho}{1-\rho}} \quad (9)$$

If no firm is constrained, shadow values of capital equalize across firms, and capital and labor demand are only a function of the relative productivity of the firms versus the average in the economy. When some firms are capacity constrained, the allocation of labor and capital is adjusted so that constrained firms can indeed retain more capital and labor inputs than if they were unconstrained.

2.3.2 Exit and Upgrade

Firms are exogenously liquidated with probability δ, getting a scrap value of Π_e^f.They can select out voluntarily, getting a scrap value of Π_e, net of exit costs. Without loss of generality, assume $\Pi_e^f = 0$, $\Pi_e > 0$, so that the option to exit is meaningful.For simplicity, I assume the latter is constant along the cycle and across sizes, but the model can accommodate richer structures in which the value depends on the technology operated by the firm and potentially different across states. This is depicted in the quantitative section.

A firm using a high minimum capacity technology may choose to operate or exit in the current period. If it operates it will get profits according to

$$\pi(x_t, X_t) = (1-\rho)Y_t^{1-\rho}\left[\frac{s_t K_t^{\alpha}}{\left(Z^k\right)^{\alpha}\left(Z^l\right)^{1-\alpha}}\right]^{\rho}\left(\frac{z\psi^j}{MPK_t^{\alpha}}\right)^{\frac{\rho}{1-\rho}}$$

Profits depend on the aggregate demand, a measure of productivity in the economy summarized by $\left(Z^k\right)^{\alpha}\left(Z^l\right)^{1-\alpha}$ and the productivity of the firm, adjusted for the value of its marginal product of capital. Next period it may be liquidated with probability δ or continue operating. If it continues, it can exercise the option to exit irrespective of which state of the world s_t is realized. To save notation, let $x_t^j = z\psi^j$ for $j = L, H$.The value of the firm W_t follows

$$W_t(x_t^H, X_t) = Max\left\{\Pi_e, \pi(x_t^H, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})W_{t+1}(x_t^H, X_{t+1})\right)\right\}$$

subject to

$$X_{t+1} = \Gamma_f(X_t)$$

where $\widetilde{\beta}_{t+1}(X_t, X_{t+1}) \equiv \beta\left(1-\delta\right)\frac{U'(C(X_{t+1}))}{U'(C(X_t))}$ is the stochastic discount factor of the household adjusted for the probability of survival of the firm, $\widetilde{\beta}_{t+1}$ to save notation.

On the continuation region, when the option to exit is not exercised, the value of the firms is the present discounted value of all future expected profits. We call it $\widetilde{W}_t$ and it reads

$$\widetilde{W}_t(x_t^H, X_t) = \pi(x_t^H, X_t) + E_t\left(\widetilde{\beta}_{t+1}W_{t+1}(x_t^H, X_{t+1})\right)$$

Let the function $z^e(\psi^H, X_t)$ determine the threshold for exit of high capacity firms when the aggregate state of the economy is X_t.

Proposition 1 $\widetilde{W}_t(x_t^H, X_t)$ *is monotonic increasing in idiosyncratic productivity,* z*. Hence, the optimal exit strategy of the firm is a trigger strategy such that if* $z < z^e(\psi_H, X_t)$ *the firm exits the market.*

The value of the high capacity firms is

$$
\begin{aligned}
W_t(x_t^H, X_t) &= \widetilde{W}_t(x_t^H, X_t) \qquad \text{if } z \geq z^e(\psi^H, X_t) \\
W_t(x_t^H, X_t) &= \Pi_e \qquad\qquad \text{o/w}
\end{aligned}
\tag{10}
$$

Next, we move to the problem of firms currently holding a low minimum capacity requirement technology. After observing the aggregate state, they may decide to exit the market, to upgrade capacity or to operate at the current one. The cost of upgrade in technology is I_H units of the composite good that should be paid in the period of upgrade, after the aggregate shock is realized.

$$
W_t(x_t^L, X_t) = Max\{\Pi_e, W_t(x_t^H, X_t) - I^H, \widetilde{W}_t(x_t^L, X_t)\} \tag{11}
$$

subject to

$$
X_{t+1} = \Gamma_f(X_t)
$$

Their continuation value is

$$
\widetilde{W}_t(x_t^L, X_t) \ = \pi(x_t^L, X_t) + E_t\left(\widetilde{\beta}_{t+1} W_{t+1}(x_t^L, X_{t+1})\right)
$$

Proposition 2 *$\widetilde{W}_t(x_t^L, X_t)$ is monotonic increasing in idiosyncratic productivity, z. The optimal exit and upgrade strategy for the firm is such that if $z < z^e(\psi^L, X_t)$, the firm exits the market; if $z \geq z^u(\psi^H, X_t)$ the firm upgrades technology; if $z^e(\psi^L, X_t) \leq z < z^u(\psi^H, X_t)$ the firm holds a low minimum capacity requirement technology.*

The value of a firm with the low minimum capacity technology is

$$
\begin{aligned}
W_t(x_t^L, X_t) &= W_t(x_t^H, X_t) - I_H \quad \text{if } z > z^u(\psi^H, X_t) \\
W_t(x_t^L, X_t) &= \widetilde{W}_t(x_t^L, X_t) \qquad\quad \text{if } z \in \left[z^e(\psi^L, X_t), z^u(\psi^H, X_t)\right] \\
W_t(x_t^L, X_t) &= \Pi_e \qquad\qquad\qquad\quad \text{o/w}
\end{aligned}
\tag{12}
$$

2.3.3 Entry

A fraction δM_t of the total mass of firms operating in the market M_t, are forced out of the market each period. There is a continuum of firms ready to enter the market at any period t. They observe their productivity before investing I_L units to buy a low minimum capacity technology. Their productivity z is drawn from an exogenous distribution $G(z)$ with finite support $[\underline{z}, \overline{z}]$. For the problem to be well defined we need to assume $I_L \geq \Pi_e$. Otherwise, entrepreneurs could create resources just by entering and exiting immediately from the market. After entry, they may choose to upgrade technology immediately at cost I_H.

The mass of entrants M_t^{ent} is determined by the free entry condition,

$$I_L \geq \int_{z^e(\psi^L, X_t)} W(z, \psi^L, X_t) dG(z) \tag{13}$$

with equality if $M_t^{ent} > 0$.

It is worth noting that the equilibrium distribution of firms across productivity and technologies, which is used in the computation of the expected value of the firms (summarized in X_t), is indeed endogenously determined by the choice of exit and upgrade thresholds of firms in the market. Entrants correctly anticipate their future expected profits, so that pre-entry expected profit equalize the post entry value.

3 Aggregates

Let the measure of firms operating in the market $M_t = v_t^L(z^u(\psi^L, X_t)) + v_t^H(\overline{z})$ and define a scaled measure $\widehat{v}_t^j = \frac{v_t^j}{M_t}$. Replacing capital and labor demands in the aggregate production function, we obtain

$$Y(X_t) = TFP_t K_t^{\alpha}$$

where

$$TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(Z^l\right)^{\frac{1-\rho}{\rho}} \left(\frac{Z^l}{Z^k}\right)^{\alpha} \tag{TFP}$$

In other words, aggregate efficiency is determined by the realization of the exogenous shock, the measure of firms operating in the market (as usual in models of monopolistic competition), and a moment of the productivity of the firms operating in the market. If there are no firms capacity constrained, $\frac{Z^l}{Z^k} = r$, and the model boils down to the canonical firm dynamic one where $TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(\sum_j \int (\psi_i^j z_i)^{\frac{\rho}{1-\rho}} d\widehat{v}_t^j(z_i)\right)^{\frac{1-\rho}{\rho}}$. Also, as alpha goes to zero, disparity in marginal products becomes irrelevant for aggregate productivity, because the share of the factor for which the minimum constraint may bind becomes negligible. In general none of those is the case. It is important to note also that there might be multiple allocations (distributions across technologies) that yield the same TFP_t conditional on the aggregate state and the measure of operating firms.

Before moving to the definition of the equilibrium, let me close the model description by computing the dividends received by the household. They correspond to the sum of the profits of operating firms, plus the scrap value of the liquidated ones minus the costs of entry and upgrade.

$$\begin{aligned} d_t^L(X_t) &= \int \pi(z, \psi^L, X_t)\ dv_t^L(z) + \Pi_e M_t^{eL}(X_{t-1}, X_t) - I_L M_t^{ent}(X_t) \\ d_t^H(X_t) &= \int \pi(z, \psi^H, X_t)\ dv_t^H(z) + \Pi_e M_t^{eH}(X_{t-1}, X_t) - \\ &\quad -I_H \left[M_t^u\ (X_t) + M_t^{ent}(X_t) \frac{1 - G(z_t^u(\psi^H))}{1 - G(z_t^e(\psi^L))} \right] \end{aligned}$$

where $M_t^{ej}(X_{t-1}, X_t)$ the measure of exits for firms running technology j, $M_t^u\ (X_t)$ is the measure of incumbent upgrades in state X_t; $M_t^{ent}(X_t)$ the corresponding measure of entrants, and $M_t^{ent}(X_t)\frac{1-G(z_t^u(\psi^H))}{1-G(z_t^e(\psi^L))}$ is the measure of entrants that upgrade immediately. The measure of exits is zero if $z^e(\psi^j, X_t) \leq z^e(\psi^j, X_{t-1})$ and positive otherwise, i.e. $M_t^{ej}(X_{t-1}, X_t) = (1-\delta)\, v_{t-1}^j(z^e(\psi^j, X_t))$ if $z^e(\psi^j, X_t) > z^e(\psi^j, X_{t-1})$. Also, $M_t^u\ (X_t) = (1-\delta)\left[v_{t-1}^L(z^u(\psi^H, X_{t-1})) - v_{t-1}^L(z^u(\psi^H, X_t))\right]$ whenever $z^u(\psi^H, X_{t-1}) > z^u(\psi^H, X_t)$ and zero otherwise.

4 Equilibrium

4.1 Competitive Equilibrium

Definition 1 *A competitive equilibrium is a system of thresholds* $\left\{z^e(\psi^j, X_t), z^u(X_t)\right\}_{t=0}^{\infty}$*, distribution of firms* $\left\{v_t^L(z), v_t^H(z)\right\}_{t=0}^{\infty}$*, a law of motion for the dynamic of the distributions of firms,* Γ*, entrants* $\{M_t^{ent}\}_{t=0}^{\infty}$ *with productivities drawn from* $G(z)$*, and consumption, aggregate capital and share holdings functions ,*$\left\{C(X_t), K_{t+1}(X_t), n^H(X_t), n^L(X_t)\right\}_{t=0}^{\infty}$ *such that given prices* $\left\{r(X_t), w(X_t), P^L(X_t), P^H(X_t)\right\}_{t=0}^{\infty}$ *, the cost structure* $\Upsilon_c = \left[\Pi^e, I^H, I^L\right]$*, the initial stock of capital in the economy* K_0*, share holdings,* $n_0^H = n_0^L = 1$*, and the exogenous law of motion for aggregate shocks* s_t *as characterized by* $\mathbf{P}$*,*

i) The representative consumer maximizes utility (as in (1)*)*

ii) Firms in the intermediate goods sector maximize their value as described by (10) and (12) given their residual demand and productivity z*.*

iii) Firms in the final good sector maximize profits.

iv) $W(z^e(\psi^L, X_t), \psi^L, X_t) \leq I_L$ *with equality if* $M_t^{ent} > 0$

v) $M_t = M_t^{ent} + (1-\delta)\, M_{t-1} - \left(M_t^{eL} + M_t^{eH}\right)$ *where* $M_t = v_t^L(z^u(\psi^H, X_t)) + v_t^H(\overline{z})$*.*

vi) Markets clear

(a) $\int l(x_t, X_t)dv_t^L(z) + \int l(x_t, X_t)dv_t^H(z) = 1$

(b) $\int k(x_t, X_t)dv_t^L(z) + \int k(x_t, X_t)dv_t^H(z) = K_t$

(c) $n_t^j = 1, j = L, H$

(d) $C_t + K_{t+1} - (1-\widehat{\delta})K_t + I_L M_t^{ent} + I_H \left[M_t^u + M_t^{ent} \frac{1-G(z_t^u(\psi^H))}{1-G(z_t^e(\psi^L))} \right] = Y_t + \Pi_e \left(M_t^{eL} + M_t^{eH} \right)$

vii) *Consistency for the law of motion of the aggregate state:* $\Gamma = \Gamma_f = \Gamma_c$.

In the following section I prove that such equilibrium exist. Before moving into the details of that proof, let me describe the features of the stationary equilibrium.

4.2 Stationary Equilibrium

Law of Motion for the distribution of firms For notational convenience I redefine any function $f(a, X_t)$ as $f_t(a)$. Let $\Lambda : C^v \times C^v \times \{s_g, s_b\} \to C^v \times C^v$ be the equilibrium law of motion for the distribution of firms of low and high minimum capacity technologies. The law of motion is characterized by

$$v_t^L(z) = (1-\delta)\, v_{t-1}^L(z) - M_t^{eL} + M_t^{ent} \frac{G(z) - G(z_t^e(\psi^L))}{1-G(z_t^e(\psi^L))} \qquad z_t^u(\psi^H) > z > z_t^e(\psi^L)$$
$$v_t^L(z) = 0 \qquad \text{o/w}$$

In other words, the measure of firms running the low minimum capacity technology equals the measure of firms from the previous period with productivity larger than the current exit threshold, net of exogenous liquidations, plus the measure of entrants with productivity up to the upgrade threshold.

The dynamic for the distribution of high minimum capacity technology is

$$v_t^H(z) = (1-\delta)\, v_{t-1}^H(z) - M_t^{eH} \qquad z_t^u(\psi^H) > z > z_t^e(\psi^H)$$
$$v_t^H(z) = (1-\delta)\ v_{t-1}^H(z) + M_t^{ent} \frac{G(z) - G(z_t^u(\psi^H))}{1-G(z_t^e(\psi^L))} \qquad \overline{z} > z > z_t^u(\psi^H)$$

whenever $z^u_{t-1}(\psi^H) \leq z^u_t(\psi^H)$.Otherwise

$$
\begin{array}{ll}
v^H_t(z) = (1-\delta)\ v^H_{t-1}(z) - M^{eH}_t & z^u_t(\psi^H) > z >\ z^e_t(\psi^H) \\
v^H_t(z) = (1-\delta)\, v^H_{t-1}(z) + M^u_t(z) + M^{ent}_t \frac{G(z)-G(z^u_t)}{1-G(z^e_t(\psi^L))} & z^u_{t-1}(\psi^H) > z >\ z^u_t(\psi^H) \\
v^H_t(z) = (1-\delta)\, v^H_{t-1}(z) + M^u_t(z^u_{t-1}(\psi^H)) + M^{ent}_t \frac{G(z)-G(z^u_t)}{1-G(z^e_t(\psi^L))} & \overline{z} > z > z^u_{t-1}(\psi^H)
\end{array}
$$

The measure of firms running at high minimum capacity equals the measure of firms that survived from the previous period, minus exits, plus entrants with productivity larger than the current upgrade threshold. If the current threshold is above the previous one, this measure also includes firms that upgraded technologies under the previous threshold rule and decide to remain in the market under the current exit rule.

Lemma 1 *The measure of firms per technology v^j_t,belongs to the space of bounded and continuous measures on $[0, \overline{z}]$,i.e. C^v.*

Properties of the allocation

Proposition 3 *The optimal allocation is such that*

1. *Exit thresholds for firms running the low minimum capacity technology are higher than for firms running the high minimum capacity one, i.e. $z^e(\psi_L, X_t) > z^e(\psi_H, X_t)$ if neither firm is constrained by the minimum capacity.*

2. *Exit thresholds are increasing in the cost of capital, i.e. $\frac{\partial z^e(\psi^j, X_t)}{\partial r_t} \geq 0$.*

3. *The upgrade threshold across technology is higher than the exit threshold for high minimum capacity firms, i.e. $z^u(\psi^H, X_t) > z^e(\psi_H, X_t)$.*

4. *The measure of entrants is procyclical.*

The first result indicates that firms running the low minimum capacity technology find optimal to exit before firms of the same idiosyncratic productivity running the high capacity technology . The second, that increases in the cost of capital, increase the likelihood of voluntary exit as equilibrium profits drop. The third result is important as it assures that costs are such that there is no upgrade in technology and immediate exit. Finally, the levels of entry are procyclical as they are in the data.

4.3 Industry Equilibrium (PP)

To prove existence of the competitive equilibrium I define a centralized problem, whose solution coincides with the decentralized allocation under certain conditions. In particular, I define a pseudo-planner problem, with the same technological restrictions of the firms in the economy, and compensate it through taxes/subsidies to entry, exit, and upgrade in technology to generate the decentralized allocation. Because there is monopolistic competition in the intermediate good sector, the allocation will in general not be surplus maximizing. Furthermore, differently from Fattal Jaef and Hopenhayn (July 2012), the allocation of employment across firms need not be efficient after controlling for the number of firms operating in the market.

Before going into the core of this section, let's define some notation. The relevant aggregate state space from the point of view of the planner is $\Xi_t = \left\{s_t, v_{t-1}^L, v_{t-1}^H, K_t\right\}$,i.e. the realization of the shock, and the distribution of firms across technologies and stock of capital carried over from the previous period. The cost structure is given by tuples $\Upsilon^p(\Xi_t) = \left[I^{L*}, I^{H*}, \Pi^{e*}(\psi^L), \Pi^{e*}(\psi^H), T\right]$. Each element of the vector depends on the aggregate state, but it has been dropped for notational convenience. The first element is the cost associated to entry of new firms, the second is that associated to upgrades, Π^{e*} is the scrap value of firms exiting the market for each type of technology. Each of the four elements described before equalizes the cost/scrap value from the decentralized allocation plus a wedge, i.e. $\Upsilon^p(\Xi_t) = \Upsilon(1 + \boldsymbol{\tau}(\Xi_t))$ where $\boldsymbol{\tau}(\Xi_t)$ is the vector of subsidies/taxes . To assure that the goods associated to those wedges are not lost, the total value of the wedges as well as any difference generated in total output, is transferred back lump sum, $T(\Xi_t) = \Upsilon\boldsymbol{\tau}(\Xi_t) + Y_t - Y_t^*$. The corresponding productivity thresholds are $z^{e*}(\psi^L, \Xi_t), z^{e*}(\psi^H, , \Xi_t), z^{u*}(\psi^H, \Xi_t)$ for exit of low and high minimum capacity firms and upgrades. The level of entry is $M^{ent}(\Xi_t)$, the measure of upgrades is $M^u(\Xi_t)$ and the measure of exits per technology are, $M^{eL}(\Xi_t), M^{eH}(\Xi_t)$.

Define the problem of the planner as follows

$$V(\Xi_t) = \underset{C_t,\ K_{t+1},\ Y_t,\ z_t^{e*}(\psi^L), z_t^{e*}(\psi^H),\ z_t^{u*}(\psi^H), M_t^{ent}, l_t^i, k_t^i}{Max} U(C_t) + \beta EV(\Xi_{t+1}) \tag{PP}$$

subject to

$$\begin{aligned} & C_t + I_t^{L*} M_t^{ent} + I_t^{H*}\left[M_t^u + M_t^{ent}\frac{1-G(z_t^{u*}(\psi^H))}{1-G(z_t^{e*}(\psi^L))}\right] \\ & + K_{t+1} - K_t(1-\widehat{\delta}) \\ \leq\ & Y_t^* + T_t + \Pi_t^{e*}(\psi^L)M_t^{eL} + \Pi_t^{e*}(\psi^H)M_t^{eH} \end{aligned}$$

$$s_t\left(\int\left(\psi^L z_i l_i^{1-\alpha}k_i^{\alpha}\right)^{\rho} dv_t^L(z_i) + \int\left(\psi^H z_i l_i^{1-\alpha}k_i^{\alpha}\right)^{\rho} dv_t^H(z_i)\right)^{\frac{1}{\rho}} = Y_t^*$$

$$\int l_i di = 1, \text{ and } \int k_i di = K_t$$

$$k_i \geq k_L \text{ if } \psi_i = \psi_L, \text{ and } k_i \geq k_H \text{ if } \psi_i = \psi_H$$

$$\left(v_t^L, v_t^H\right) = \Lambda\left(v_{t-1}^L, v_{t-1}^H\right)$$

Hence, given the distribution of firms in the market, the realization of the aggregate shock and the available stock of capital, the planner chooses the allocation of firms across technologies to maximize utility.

Theorem 1 *a) For a given transfer scheme Υ^p, the solution to the centralized problem exists and it is unique.*

b) There exist a cost structure $\{\Upsilon^p(\Xi_t)\}_{t=0}^{\infty}$ such that the allocation of firms that solves the planner problem (PP) coincides with the competitive allocation.

For expositional purposes the full proof can be found in the Appendix. Heuristically it goes as follows. For part a) note that the problem would be a standard concave problem if there were no sunk costs to technology adoption and no minimum capacity constraint that may bind in equilibrium. The presence of a continuum of heterogenous firms mitigates potential non-convexities as in Mas-Colell (1977). The continuum of firms works as the divisible commodity necessary to convexify the aggregate feasible set. For part b), the proof has two steps. Analogous to Jones and Manuelli (1990), first we define an operator on the transfers, $\Omega\left(T(\Xi_t)\right)$ and prove that it has a fixed point. At the fixed point, the feasibility constraint of the planner and competitive equilibrium are the same. Second, we need to define prices and a cost structure such that the optimality conditions hold in both cases. The price of capital and salaries are defined such that the optimal consumption and capital accumulation paths for the representative consumer coincide with those predicted by (PP). To assure that the allocation of firms across technologies and the measure of active firms coincide, I use the linearity of the optimality conditions of the (PP) for the allocation of firms, as well as in the indifference conditions for the firms in the decentralized problem. I show that one can define a unique set of subsidies/taxes, $\widehat{\boldsymbol{\tau}}\left(\Xi_t\right)$ such that the thresholds of the decentralized problem satisfy the optimality necessary conditions of (PP). I show that the transfer generated by $\widehat{\boldsymbol{\tau}}\left(\Xi_t\right), T(\widehat{\boldsymbol{\tau}}\left(\Xi_t\right))$ is a fixed point of Ω. Hence, the equivalence is proven. Note that if the equilibrium was pareto optimal, then $\widehat{\boldsymbol{\tau}}\left(\Xi_t\right)$ should be equal to zero across all states.

Corollary 1 *The solution to the competitive equilibrium exists*[7]

[7] We cannot say much about the determinacy of the competitive equilibrium. Let me illustrate an example with no aggregate uncertainty. Suppose that the household would like to consume more in the current period and less in the following one. It implies that the marginal utility of consumption today is higher and the marginal utility tomorrow lower than in the stationary equilibrium. If this is the case, the intertemporal Euler equation of the household would not hold. However, higher demand today for final goods implies higher demand for all intermediate goods, which triggers entry, raising labor demand and wages. The productivity cutoff for exit may raise, as profits are now lower than before on average. The shift in the cutoff implies that the average productivity in the market goes

4.3.1 Analysis

Before moving to the quantitative results, it is useful to illustrate how the optimality conditions for technology selection and input allocation differ between the centralized and decentralized allocation. Away, from the fixed point of the transfer operator Ω, the social and private value of investment in technology will be different. Hence the measure of firms holding high and low minimum capacity technology need not be efficient. Employment and capital are proportional to relative productivity of the firm versus the rest in the economy. Therefore, once the technology selection margin is distorted, so is the capital and labor allocation.

The optimal path for aggregate capital in the PP problem is dictated by

$$U'(C_t) = E_t\left[U'(C_{t+1})\,\beta\left(\lambda^k_{t+1} + 1 - \widehat{\delta}\right)\right]$$

If allocations are the same in the decentralized and centralized allocations, then aggregate TFP (as shown in the aggregation section of the paper) will be the same, and the marginal product of capital too as

$$\lambda^k_{t+1} \equiv \frac{\partial Y_{t+1}}{\partial K_{t+1}} = \alpha s_{t+1} TFP_{t+1} K^{\alpha-1}_{t+1}$$

This condition equalizes the optimal accumulation policy for the household, if $r_t = \lambda^k_t$.

The allocation of employment and capital across firms in PP and in the decentralized allocation are dictated by

$$(Y_t)^{1-\rho}(1-\alpha)\frac{y^{\rho}_{it}}{l_t} = \lambda^l_t \qquad\qquad (Y_t)^{1-\rho}(1-\alpha)\frac{y^{\rho}_{it}}{l_t} = \frac{w_t}{\rho}$$

up, and the average markup drops. If the overall effect induces lower equilibrium profits, the price of shares P^k_t can drop bringing back the Euler equation to hold which yields the indeterminacy. If entry costs were denominated in terms of labor cost instead of the composite good, higher demand may not induce entry, as the costs of entry raises with the number of firms operating in the market.

$$Y_t^{1-\rho}\alpha\frac{y_{it}^{\rho}}{k_t} = \lambda_t^k - \widehat{\lambda}_{it} \qquad\qquad Y_t^{1-\rho}\alpha\frac{y_{it}^{\rho}}{k_t} = \frac{r_t - \lambda_{it}}{\rho}$$

Hence, the shadow value of labor in the centralized allocation equals the wage rate in the decentralized allocation adjusted by the elasticity of substitution in intermediate inputs. This is the well known gap introduced by the monopolistic competition assumption. If the allocation across technologies is the same, the shadow value of labor across allocation is proportional to each other. An analogous condition is satisfied by the capital allocation across firms when the firm is not capacity constrained. When the firm is constrained the shadow value of capital is adjusted by a factor $\widehat{\lambda}_{it}$ in the planners allocation. In the decentralized allocation, the adjustment factor is $\frac{\lambda_{it}}{\rho}$.

In terms of the dynamic investment decisions, it is important to highlight the differences between the private and social value of a firm type (blueprint plus technology). The exit condition for a firm operating technology j in the planners' problem reads

$$\Pi_t^{e*}(\frac{\partial M_{t-1}^{ej}}{\partial z_t^e}(1-\delta)) = \frac{\partial Y_t^*}{\partial z_t^{e*}}$$

where

$$\frac{\partial Y_t^*}{\partial z_t^{e*}} = s_t M_t^{\frac{1-\rho}{\rho}}\left(Z^l\right)^{\frac{1-\rho}{\rho}}\left(\frac{Z^l}{Z^k}\right)^{\alpha} K_t^{\alpha}\frac{\psi_i^j z_t^{e*}}{\left(\lambda_t^k - \widehat{\lambda}_{it}\right)^{\alpha}}\Theta$$

The last term, Θ is an efficiency adjustment factor, which for the firms equals

$$\Theta = 1-\rho$$

and for the planner equals

$$\Theta = \frac{1-\rho}{\rho} - \alpha[\frac{Z^l}{Z^k}\frac{1}{\left(\lambda_t^k - \widehat{\lambda}_{it}\right)^{\frac{1-\rho}{\rho}}} - 1]$$

When there is no dispersion in marginal products, $\frac{Z^l}{Z^k} = \left(\lambda_t^k\right)^{\frac{1-\rho}{\rho}}$ and therefore, $\Theta = \frac{1-\rho}{\rho}$.

Only the distortion introduced by the monopolistic competition remains. If marginal products are disparate across firms, the planner will have incentives to liquidate firms whose shadow value of capital is below the "average" in the market. This will allow him to free up labor towards the unconstrained firms.

If there are upgrades in equilibrium, the optimality condition in PP reads

$$I_t^{H*}\left(\frac{\partial M_t^u}{\partial z_t^u}+M_t^{ent}\frac{\partial \Pr_G(z>z_t^u)}{\partial z_t^u}\right)=\frac{\partial Y_t^*}{\partial z_t^{u*}}+E\left\{\widetilde{\beta}_{t+1}\frac{\partial M_{t+1}^u}{\partial z_t^u}\left(I_{t+1}^{H*}-\lambda_{t+1}^H\right)\right\}$$

where λ_{t+1}^H is the shadow value of a high capacity firm. Suppose that the level of entry in the decentralized allocation and the centralized allocation are the same, as well as the upgrade costs. The optimality condition in the decentralized allocation in the following period reads $W_{t+1}(\psi^j, z_{t+1}^u, X_{t+1}) = W_{t+1}(\psi^H, z_{t+1}^u, X_{t+1}) - I_{t+1}^H$. Hence, the difference in current period upgrade policy can originate in the gap between the social and private value of the firm as depicted for the exit condition. The planner optimality condition has also an additional term, $E\left[\widetilde{\beta}_{t+1}\frac{\partial M_{t+1}^u}{\partial z_t^u}\lambda_{t+1}^H\right]$ which accounts for the dynamic impact of current upgrading on the equilibrium measure of future upgrades.

The optimality condition for entry in the planner's problem is the same that the free entry condition imposed for the firms in the decentralized market. However, because the social and private value of a firm differ, exit and upgrade decisions do not coincide across allocations and neither will the level of entry.

5 Quantitative Analysis

In this section I assume there is a finite level (N) of minimum capacities/technologies, and there is no further investment in capacity conditional on a particular technology. I assume that there is a stock of capital ready to be used in any particular company. The stock is large enough so that any firm that decides to invest in capacity or enter the market can be supplied with the corresponding stock. The dynamic of the aggregate stock of capital will be pinned down by the consumption decisions of the planner, which in turn will pin down the dynamic of the measure of firms in the economy.

Production under each alternative technology is given by

$$y_t = s_t z k_j^{\alpha} l(x_t, X_t)^{1-\alpha} \text{ for } j = 1, ..., N$$

where $k_j < k_{j+1}$ for any j. A detailed explanation of the algorithm for computing the equilibrium is provided in Appendix A.

5.1 Calibration

The model is calibrated to the USA economy[8]. Although business cycle statistics are typically presented at quarterly frequency, industry dynamics statistics are only available on a yearly basis. Hence, the time unit of the model is a year. Some of the calibrated parameters are standard in the RBC literature. The persistence of expansions and recession periods were set to match the average duration of the phase of the business cycle in the USA. In particular, $\gamma_s = 1 - 1/t_s$ where t_s is the average length of a particular phase of the business cycle $s = g, b$. The average duration of an expansion was set to 3.175 years (or 12.7 quarters), and that of a recession to 1.425 years (or 5.7 quarters). The discount factor was set to match a steady state interest rate of 2%, $1 + r = \beta^{-1}$. Log utility was assumed.

The substitutability across intermediate goods in the final good aggregator was set to match returns to entrepreneurship (ρ shapes the curvature of the profit function). Atkenson and Kehoe (2005) set a value of 15% to the returns to entrepreneurship, whose analogous in the model is $1 - \rho$ ($\rho = 0.85$). The share of capital in value added is set to 1/3 as standard in the literature. The hazard rate for exogenous exit, δ was set to 5,5%. It corresponds to the mean exit rate reported in Lee and Mukoyama (2008)

[8]There are substantial differences in the firm size distribution of the USA versus other OECD countries (see Barstelman et al. (2009)). In particular, the right tail of the distribution is "fatter" in the USA than in other developed economies. Alternative calibrations can be accomodated.

based on statistics from the Annual Survey of Manufactures. Finally, the number of technologies is set arbitrarily to 4 and the lower bound of possible productivities equal to 0.01[9].

The remaining parameters of the model were calibrated jointly to match moments of the firm size distribution, as well as features of the industry dynamic and the aggregate volatility of the economy. To calibrate them I simulate the model economy via Montecarlo: I run the optimal policies for 1000 periods (discard the first 200) over 100 alternative paths for a variety of parameter specifications. The list of parameters calibrated jointly is presented in Table 2

While some parameters have closer tights to certain moments, they are not independent of the remaining variables of the economy. Let me describe their roles briefly. First, the size of aggregate shocks measured by $s_g - s_b$ is closely related to the volatility of the cyclical component of log GDP. The target in the data corresponds to the standard deviation of the hp-filtered series of log GDP from 1930 to 2011, equal to 2.1%. Positive shocks take a value of 1.027 and negative shocks of 0.97 (shocks are assumed symmetric around one). The observed variation in aggregate output is not independent however of the cost structure of the economy, as the latter determines how much investment or exit is observed in equilibrium, which in turn affects aggregate output.

The set of capacities as well as the range for idiosyncratic productivities, are related to the levels of log employment produced by the model[10]. The upper bound on capacities was set to 4 while the upper bound on productivities was set to 4.25. The firms at the top of the employment have a level of employment slightly above 10000 employees, consistent with the data. The distribution of sizes in the economy inherits also some of the properties of the exogenous distribution of idiosyncratic productivity, $G(z)$. The distribution of entrants is calibrated such that the $log(z)$ is exponential with parameter

[9]The minimum effective productivity operating in the market is determined endogenously.

[10]The finite level of capacities model predicts that relative labor demands are described by $\frac{l_i}{l_j} = \frac{(z_i k_i^{\alpha})^{\frac{\rho}{1-\rho(1-\alpha)}}}{(z_i k_j^{\alpha})^{\frac{\rho}{1-\rho(1-\alpha)}}}$

$\zeta_G = 1.9$. In other words, $G(z)$ is Pareto with parameter ζ_G.

The generated firm size distribution is also related to the entry and upgrade costs per capacity, through the equilibrium allocations. To calibrate the cost structure, I assumed state independent costs for the pseudo planner problem. Once the allocations generated by the economy matched the targets for the US, I backed out the cost structure in the decentralized allocation. In other words, I computed the costs that would make the exit and upgrade threshold of the decentralized allocation coincide with the ones in the calibrated economy.

The total number of parameters for calibration is thirteen. The complete list of moments that were targeted to calibrate them are found in Table 3.The identified costs indicate slightly higher entry costs during expansions, fairly constant scrap values across states, but increasing in the capacity of the firms as expected. Upgrade costs are identified higher during expansions. In the ergodic distribution of the model, upgrades in capacity for incumbent firms average 2.8% of the total population of active firms, costs of upgrade should raise when incentives to upgrade increase to avoid shifts in the firm size distribution that will make it inconsistent with its fairly constant shape in the data. The establishment and employment shares are as reported by Lee and Mukoyama (2008), as well as the average exit and entry rates. Overall, the model predicts well the behavior of the establishment and employment distribution. The share of employment for firms at the top of the log employment distribution is slightly underpredicted. The model predicted share of establishments with less than 19 employees is below the observed number in the data. The firms at the top of the distribution reported by the BDS have 10.000 or more employees. They correspond to 6% of the total population of establishments in the economy. The model is conservative in this sense as the largest firm in the economy employs 10.829 employees.

In terms of firm entry and exit rates the model overpredicts exit rates by 0.7%, and underpredicts entry rates 0.6%. For the measure of firms to be stable in the ergodic distribution, these flows should be roughly the same, the model is calibrated to go half the way the difference in entry and exit rates reported in the data.I also targeted the percentage of firms with positive investment spikes as reported by Dums and Dunne (1998). A spike is defined as firm that reports an investment rate of 30% or higher in any given year. Given the capacity grid, any upgrade in capacity will be considered an investment spike, as well as any entry decision. The model produces a measure of spikes of about 1% higher than in the data once we account for investment of entrants. In the model, 40% of the measure of firms with investment spikes corresponds to incumbent firms. The contribution is rather small as for the calibrated aggregate shocks, investment thresholds move mildly. The introduction of firm specific shocks will increase fluctuations in the thresholds, potentially inducing more equilibrium investment for incumbents.

5.2 Results

5.2.1 Productivity

We first describe the predictions of the model for aggregate Total Factor Productivity (TFP). To express the results as close as those in the literature, note that when technology is Cobb[Douglas, total factor physical productivity ($TFPQ_i$) per firm is proportional to a geometric average of capital and labor productivity

$$TFPQ_i \triangleq MPK_i^{\alpha} MPL_i^{1-\alpha}$$

where the marginal product of capital and marginal product of labor are defined as $MPK_i = \rho\alpha\frac{y_i}{k_i}$ and $MPL_i = \rho(1-\alpha)\frac{y_i}{l_i}$respectively. Aggregating up, we obtain

$$TFP_t = \left(\sum_{j=L,H} \int (TFPQ_{z_i})^{\rho}\, dv_t^j(z_i) \right)^{\frac{1}{\rho}} \tag{14}$$

This expression is analogous to (TFP) presented in the aggregates section and is our baseline measure.

If there is no dispersion in marginal product across firms, aggregate total factor productivity simplifies to

$$TFP_t^{MC} = s_t \left[\sum_{j=L,H} \int (z_i)^{\frac{\rho}{1-\rho}} \, dv_t^j(z_i) \right]^{\frac{1-\rho}{\rho}} \tag{15}$$

Although in this case there are no losses in efficiency stemming from the technological friction, the presence of monopolistic competition might still affect productivity through the equilibrium number of operating firms in the market. We use this measure to test the properties of the baseline model against.

Table 4 shows the effect of irreversibilities and indivisibilities in production on computed aggregate TFP. All values are reported in log points. The first column reports the statistic described in (14) .The second column reports the same statistic for the optimal allocation of firms which is computed imposing the decentralized cost structure into the pseudo-planner problem absent of transfers. The first row reports aggregate productivity and the second row the standard deviation of the time series. The third row reports a measure of dispersion in computed TFPQ across firms. I report the coefficient of variation across economies.

Aggregate productivity under the optimal allocation is 11% higher than in the Baseline economy. While the optimal policy induces a drop in the coefficient of variation of TFPQ across firms, it induces higher volatility of productivity in the time series. From the definition of TFP one can see that the gains in efficiency in the constrained optima may stem from disparities in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. Further analysis on the sources of gains is included when describing the optimal policy.

To isolate the effect of irreversibilities and indivisibilities from the changes in the equilibrium measure of firms due to the monopolistic competition, I normalize the measure of active firms to one. Table 5 reports the statistics described in the previous table for the baseline economy, the optimal policy, and an economy in which marginal product of inputs in production equalizes across firms, i.e.(15). The allocation in which marginal products are equalized across firms yields the highest aggregate productivity and the lowest coefficient of variation for TFPQ. This is not surprising since the constrained optima cannot completely undo the impact of indivisibilities and irreversibilities on marginal product dispersion. The differences between them are large, while aggregate productivity almost double, the cross sectional dispersion drops to a third. Also time series productivity volatility raises even more when marginal products are equalized. Fluctuations in productivity in such economy stem from changes in the productivity of the marginal firm operating in the market. The irreversibilities and indivisibilities in the model induce lower adjustment, and less volatile aggregate productivity.

The measure of dispersion in TFPQ potentially hides distributional issues, i.e. the distortion generated by the irreversibility and the indivisibility is disparate across capacities/technologies. I compute the ratio of mean productivity per capacity in the model and under the assumption that firms equalize marginal products. An entry equal to 1 in Table 6 indicates the same mean productivity. The results suggest that the friction in the model generates firms with low capacity to held few resources (hence high marginal products), and productive firms running high capacity technologies, with too many resources compared to what they would held if marginal products were equalized. The friction in the model generates selection towards bigger more productivity firms. In the economy with equalization of marginal products, labor is shifted from the high capacity, low marginal productivity firms to low capacity higher marginal productivity ones. It is worth noting that the improvement in aggregate productivity induced for the optimal policy, is attained for a distribution of employment that resembles largely the one in the baseline economy.

5.2.2 Optimal Policy

As mentioned in the previous section efficiency gains may stem from improvements in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. For the calibrated economy, while total efficiency gains associated to the optimal policy are 11%, only a third of them stem from pure reallocation of resources. The rest, is induced by a larger measure of firms operating in the market in equilibrium: 17% more firms than in the baseline economy.

Accordingly, the industry dynamic is different. While entry and exit rates are lower under the optimal policy, the upgrade rate increases. Both combined indicate that there is a shift toward more productive larger firms. Upgrade rates of incumbent firms raises by 1% if compared to the baseline economy. Table 10 reports the firm dynamic. These patterns are consistent with the planner assigning a higher value to holding an additional large capacity firm than the private value of the firm in the decentralized equilibrium. The thresholds for upgrade and exit move accordingly. While in the baseline economy the exit thresholds are lower, the upgrade threshold are above the optimal levels as dictated by the efficient allocation. Average output per firm increases in the optimal allocation by 24.7% and average consumption increases 27%. The consumption equivalent compensation that would make an agent indifferent between living in the efficient or in the baseline economy should be 44% of the consumption in the baseline economy. Note that in this economy consumption equals output minus the good cost of entries and upgrades, plus the scrap value of the firms in the economy. Differences in the firm dynamic across allocations will be reflected in differences consumption equivalents even if the yield the same levels of output.

The optimal policy induces shifts in the contribution to output across firm sizes. It predicts a slightly larger share of output to be accounted for firms with more than 500 employees, as well as a larger contribution in employment. Capital however is allocated in the opposite direction, with a slightly higher share of the total used by the firms at the bottom of the distribution. This is not surprising since the marginal products at the bottom tend to be higher than those predicted by an economy with equalization of marginal products. Table 9 compares the predictions of the model and the optimal policy for the distribution of output, capital and employment.

One of the advantages of having the second welfare theorem to hold, is that we can study the characteristics of the optimal industrial policy, i.e. the cost structure that would induce a decentralized allocation that is efficient. Table 11 reports such cost structure and the one from the calibrated economy. The optimal policy dictates

subsidies to the cost of entry in recessions and higher entry costs during booms. Both policies combined induce less fluctuations in the measure of entrants to the market. Upgrade costs are subsidized across all aggregate states. Less costly upgrades induce shifts in the productivity distribution of the firms operating in the market to the right. Scrap values are identified lower than in the calibrated economy for all capacities except at the very bottom. Lower scrap values are consistent with lower exit rates predicted in by the optimal policy.

Note that I only describe differences across stationary equilibria. The exercises are silent as of the gains/losses that the economy may incur along the transition. Studying the path across equilibria is particularly challenging in economies like this one, where not only a statistic of the distribution needs to be carried along in the state space, but potentially full histories of a continuum of firms need to be considered. In the case where only two capacities are operated and there is no aggregate uncertainty the transition can be computed. In that case, the gains across stationary equilibrium are a lower bound to total gains whenever the transition occurs from an economy with a relatively low measure of active firms, to one with higher level of operating firms. For an increase in the measure of firms comparable to the one observed across steady states in the full model (17%), predicted transition gains are 60% larger than the steady state gains. Steady state gains in the simplified economy are 1%. This number is not readily comparable to the ones in the full economy because the cost structure and investment strategies do not map to each other. However, the exercise is useful to gain intuition. Gains are larger accounting for the transition because consumption convergence occurs from "above". By doing so, the planner avoids entering firms in the transition that will later on find themselves holding more capital that what they would need at the new steady state. In the transition the upgrade threshold jumps an overshoots the new steady state upgrade threshold. Any entrant that finds optimal to upgrade in the beginning of the transition will find optimal to do so all along it. Also, induced entry decreases the relative measure of firms that are holding more capacity that what they would have chosen if entering the market this period. Hence, if the measure of firms is increasing in the market, the effect of the irreversibility on firms holding high capacity in the initial steady state vanishes in the aggregate.

5.2.3 Volatility and Aggregate TFP

In this section I investigate how features of the business cycle impact the entry and exit behavior of firms as well as our measures of aggregate productivity. The spirit of the exercise is to understand how the level of uncertainty that firms face affects aggregate productivity and equilibrium dispersion in marginal products.

In particular, I focus on changes in the unconditional variance of the shock. Suppose the aggregate shock s_t follows an AR(1)

$$s_t = \phi s_{t-1} + e_s$$

where ϕ is the persistence of the shock and e_s an i.i.d. shock with mean zero and standard deviation σ_e. The unconditional volatility of the aggregate shock is

$$\sigma_s^2 = \frac{\sigma_e^2}{1 - \phi^2}$$

Hence, changes in unconditional volatility can be brought about by changes in the persistence or in the variance of the e_s shock. If the AR(1) process is approximated by a two state Markov chain, a la Rouwenhorst (1995), then

$$\left(\frac{s_g - s_b}{2}\right)^2 = \sigma_e^2$$

and

$$\gamma_g + \gamma_b - 1 = \phi$$

I first study whether changes in the persistence and the variance of e_s (for a given unconditional volatility) have different impact in entry and exit patterns as well as in aggregate efficiency. Second, I vary the unconditional variance by changing the variance of e_s only, and assess the implications for aggregate efficiency.

I assume that expansions are shorter than in the calibrated economy ($\gamma_g = .237$), about 1.1 years on average. I will call this Case G, for change in gamma. Alternatively, I set γ_g back to its calibration value, and increase $s_g - s_b$ to generate the same unconditional volatility. I will call this Case S, for change in the size of the shock.

Table 7 reports the results. The first row reports aggregate TFP, the second its volatility. The third row reports the coefficient of variation of TFPQ across firms. The fourth, the ratio of aggregate TFP defined as (14)/(15) when the measure of firms is normalized to 1. The fourth row reports the implied volatility of output. The fifth and sixth columns report the cross sectional dispersion in productivity. As expected the predicted volatility of output is larger in the cases under study than under the calibrated model. In this particular example, the volatility of output is substantially higher when the size of shocks changes rather than when the persistence of the process does. On the one hand, lower persistence of the shock affects the discounting of future profits and hence the trade off between current and future consumption. While shocks are more frequent, firms are also less willing to respond to the aggregate fluctuations by investing or disinvesting On the other hand, the size of the shocks affects the actual payoffs of investment. Because the firms have an outside option given by their scrap value when exiting, increases in the size of the shock improve the payoffs of investment, inducing larger responses in output.

A feature to highlight is that the impact on aggregate TFP is not monotonous. While in Case G productivity raises about 10%, it drops one third in Case S. The cross sectional dispersion of TFPQ drops by similar magnitudes in both cases, yet aggregate efficiency is very different. The volatility of aggregate output raises substantially. In terms of allocations, the relative efficiency of these economies against their equal marginal products counterparts are fairly constant. Hence, much of the differences across economies stem from the equilibrium measure of firms in the market. The economy of Case G has 4 times more firms than the economy of Case S.

The underlying industry dynamic, i.e. patterns of entry, exit and investment, also differ. Table 8 depicts mean exit, entry and upgrade rates from montecarlo simulations. In both cases the increase in volatility induces higher upgrade rates. Although in Case S, upgrade rates augments almost 5 times with respect to the baseline, selection does not induce higher average productivity (in part because exit rates are also larger). In Case G instead, entry and exit rates drop with respect to the baseline, while upgrade increase and average productivity raises.

This example points out that different features of the underlying process of exogenous shocks, can produce substantially different responses of the economy even when the underlying measure of uncertainty (unconditional volatility) is the same. This is embedded in the non-convexities of the model. The disparity in the behavior of exit and entry rates as well as investment rates, may be a promising tool in identifying characteristics of the productivity process. A limitation however, is that the relationship between the industry dynamic and the nature of shock depends on the underlying friction in the economy.

Finally, I assess the impact of changes in the unconditional volatility of the shock from changes in the size of the shocks only. I simulate the economy for a grid of $s_g - s_b$ between 0.04 to 0.15 (equivalent to positive and negative shocks of sizes 0.02 and 0.07, respectively). The predicted relationship between the volatility of output (and hence the unconditional volatility of the aggregate shock) and the cross sectional dispersion in productivity is non-monotonic. Also, the relationship between dispersion in computed productivities at the firm level and aggregate productivity is not independent of aggregate uncertainty. Figure 3 displays a scatter plot of measures of dispersion and aggregate TFP under alternative shocks.

5.3 Sensitivity Analysis

I perform robustness check with respect to some of the parameters that characterize the size distribution of firms. In particular, the parameter ς_G that parametrizes the exponential distribution from which productivity draws for entrants are obtained. Second I compare the predictions of the calibrated Model to one in which the exogenous rate of exit is substantially lower.

I first set the parameter that characterizes the exponential distribution to 1.01. This number is not arbitrary as it correspond to the estimated parameter for the Pareto distribution that characterizes the firm size distribution in the data (See Axtell (2001)). The predicted distribution of establishment across log employment lies to the right of the calibrated one. Note that a lower parameter for an exponential distribution indicates a "fatter' tail. In other words, entrants in this alternative economy start too productive inducing selection at the bottom and a shift in the allocation towards larger firms.

As the parameter increases the average productivity of entrants gets lower. Entrants with lower productivity affect the average productivity in the market and the allocation of employment and capacity across productivities. Matching accurately the firm distribution by employment and establishment is important. The economy with $\varsigma_G = 1.2$ cannot match the employment distribution in the data. It generates a distribution highly skewed to the right.

I also test the predictions of the model when the exogenous exit rate drops to 1.1% per year. The equilibrium industry dynamic changes by construction generating lower entry and exit rates in equilibrium. The size distribution of firms gets skewed to the right, indicating reallocation towards high capacity more productive firms. The equilibrium number of firms operating in the market drops. Finally, the time of the transition to the stationary distribution of firms doubles. Although transitional dynamics is not the objective of this paper, this result indicates that the study of the impact of policies that changes the incentives to firm liquidation should account for longer or shorter transition paths.

6 Conclusion

This paper explores the implications of technological restrictions at the firm level for aggregate efficiency in production when there is aggregate uncertainty. I find that dispersion in marginal products among firms operating analogous technologies can be consistent with (constrained) efficient allocations. Furthermore, observed dispersion is not independent of other features of the economy, such as the business cycle or more broadly the degree of demand uncertainty that firms face. The paper highlights that dispersion in marginal products is an imperfect measure of the associated efficiency losses.

The paper contributes to the study of non-convex economies with heterogeneous agents by providing an equivalence result. The equivalence result paves the way for the study of optimal policy in richer environments. Although our equivalence result relies heavily in the existence of a continuum of firms and the presence of sunk costs, potentially other schemes of adjustment costs can be accommodated.

When the industry dynamic is incorporated in a general equilibrium framework, high aggregate productivity allocations are associated with relatively low dispersion in marginal products. But low aggregate productivity allocations can also be associated to low dispersion in marginal products and hence in measured productivity. Whether this is the case or not depends on the mechanism originating such dispersion. In this paper, I show that when uncertainty and indivisibilities and irreversibilities at the firm level are incorporated in a standard firm dynamic model, the model is consistent with those patterns.

Partial irreversibility and higher divisibility in capital allocations will lessen the model generated excess dispersion in marginal products, for a given volatility of the aggregate process. However, as long as the movements in investment thresholds are such that the measure of incumbents firms holding capital away from the one chosen by entrants with the same blueprint does not vanish, non-convexities at the micro level will induce dispersion in marginal products and computed productivity.

I have abstracted from idiosyncratic risk. If incorporated in the model, I expect higher induced dispersion in marginal products. Higher uncertainty at the firm level will move optimal investment thresholds at the firm level even more than in the economy with aggregate shocks only. Large regions of inaction for alternative realizations of the idiosyncratic productivity shock or demand shock, are consistent with sustained disparities in marginal products.

The relationship between uncertainty, investment, industry structure and disparities in marginal products across production units might be a promising line of research in the context of the study of cross country differences in aggregate TFP. In other words, are economies characterized by more instability (i.e. political instability that leads to uncertainty on tax schemes, or fluctuations in the terms of trade in economies with a highly concentrated production base) prone to higher and persistent disparities in marginal products? How does the industry structure and firm dynamics vary across these economies? Can those patterns help us identify features of the aggregate productivity process?

Finally, it is worth mentioning that the results presented in the paper correspond to the behavior of firm distributions in the long run. The properties of the transitions to the ergodic distributions remain to be studied. The presence of indivisibilities in technologies may slow down the transition, affecting not only the equilibrium technologies adopted but also the return to capital and the path of output and capital accumulation, as well as the implications for the design of optimal policy.

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7 Appendix (A)

7.1 Numerical Solution

Given a cost structure, Υ,the solution to the pseudo-planner problem is a set of functions $z^{e*}(k_j, \Xi_t; \Upsilon)$, $z^{u*}(k_j, \Xi_t; \Upsilon)$ and a measure of entrants M^{ent*} that solves the corresponding optimality conditions. The algorithm to solve the equilibrium allocations is

1. Assume an arbitrary cost structure for the planner $\Upsilon = \left[\Pi^e, \Pi^e, I^H, I^L, 0\right]$ (with no transfers, T).
2. Compute the dynamic of the joint distribution of capital and productivity for an arbitrary initial distribution v_0.
3. Approximate the value function of the planner
4. For a given optimal policy for the planner, run montecarlo simulations over the predicted distribution of $\{v_t\}_{t=1}^{T_M}$.
5. Calibration: The moments of $v = v_{T_M}$ for T_M large enough, are used to matched moments of firms dynamic in the data.
6. Use the calibrated cost structure of the planner Υ, and the optimality conditions delivered from the decentralized problem to compute the cost structure of the decentralized allocation $\Upsilon^c = [\Pi_e^c(k_j, \Xi_t), I_H^c(k_j, \Xi_t), I_L^c, 0]$.
7. Use the decentralized cost structure to solve for the optimal policy (planner's allocation).

7.1.1 Dynamic of the Distribution

We need first to construct the grid of capacity levels in the economy, Ψ^k and that of idiosyncratic productivities Ψ^z. The grid for capacities is equally spaced, and the grid of idiosyncratic productivities is log spaced. Points in the Ψ^z will be concentrated in the left tail.

Let J be the number of capacity levels. Define the grid if exit thresholds Ψ_j^e for $j = 1, ..., J$; and three grids for upgrade threshold grids Ψ_j^u for $j = 1, ..., J-1$ where Ψ_j^u indexes the grid of upgrade thresholds from capacity j to $j+1$. Finally, we need a grid for entry levels, Ψ^{ent}.

To generate the grids we do it jointly via the Smolyak algorithm.The algorithm constructs a sparse multidimensional grid.

The grid and transition matrix for the aggregate exogenous state s is constructed following Tauchen (1986).

For given Λ_0, I compute Λ_1 using the law of motion described in the body of the paper, for each of the points in the sparse grid.

7.1.2 Approximation of the Value Function

I implement standard value function iteration over the centralized problem.

To interpolate the value function, I use tensor products using the sparse grid as interpolation points.

I solve for the coefficients of the interpolating function given an initial guess of the value function, θ_0 and the cost structure of the model, Υ.

Then update the guess by optimizing numerically

$$
\begin{aligned}
V_1(v,s,M_t) &= \underset{\{z^x_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J,M^e_t}{Max} U\left(C_t(\{z^e_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J,M^e_t)\right) \\
&\quad +\beta[\Pr(s'=s_1/s)V_0\left(v',s_1,M_t(1-\delta)-M_t^{eL}-M_t^{eH}+M_t^{ent}\right) \\
+\Pr(s' &= s_2/s)V_0\left(v',s_2,M_t(1-\delta)-M_t^{eL}-M_t^{eH}+M_t^{ent}\right)]
\end{aligned}
$$

subject to

$$C_t + I^{L*}M_t^{ent} + I^{H*}M_t^{up} \leq Y_t + T_t + \Pi_j^{e*}M_j^e$$

$$v' = \Lambda(v,\{z^x_{jt}\}_{j=1}^J,\{z^u_{jt}\}_{j=1}^J, M_t^{ent}, M_t)$$

$$\left(\sum_{\Psi^k}\sum_{\Psi^z}\left(z_i l_i^{1-\alpha}k_j^{\alpha}\right)^{\rho}\eta^j(z_i)\right)^{\frac{1}{\rho}} = Y_t$$

$$\frac{v_t^j(z_i)-v_t^j(z_{i-1})}{z_i-z_{i-1}} = \eta^j(z_i)$$

$$\int l_i di = 1$$

Using the updated value function V_1 recompute θ.Iterate until convergence.

7.1.3 Montecarlo Simulations

From the calibrated transition probabilities of the aggregate shock, generate 100 paths of 1000 periods each and simulate the path of allocations given the optimal policy of the planner.

Compute statistics of interest characterizing the firm dynamics of the economy, i.e. entry rates, exit rates and investment rates per capacity, dispersion in productivity, etc.

7.1.4 Cost Structure in the Decentralized Allocation

The optimality conditions for the firms, as well as those of the centralized problem are linear in the adjustment costs. Hence, if we replace the allocation that solves the pseudo planner problem into the system of equations that solves the decentralized allocation, we can infer the cost structure that decentralizes the allocation.

At the centralized allocation, the optimality conditions from the decentralized problem would typically not hold. To bring the equilibrium about, we redefine the adjustment costs faced by firms as

$$\Pi_j^c(k_j, s_t, v_t) = \Pi_e(1 + \tau^e(k_j, s_t, v_t))$$

$$I_j^{Hc}(k_j, s_t, v_t) = I_H(1 + \tau^u(k_j, s_t, v_t))$$

$$I^{Lc}(s_t, v_t) = I_L(1 + \tau^{ent}(s_t, v_t))$$

and solve a system of nonlinear equations for the tax/subsidy scheme. The cost structure of the decentralized allocation is $\Upsilon^c = \left[\left\{\Pi_j^c(k_j, s_t, v_t)\right\}_{j=1}^{J}, \left\{I_j^{Hc}(k_j, s_t, v_t)\right\}_{j=1}^{J-1}, I^{Lc}(s_t, v_t), 0\right]$.

8 Appendix (B)

8.1 Results

Figure 1: Establishment Distribution

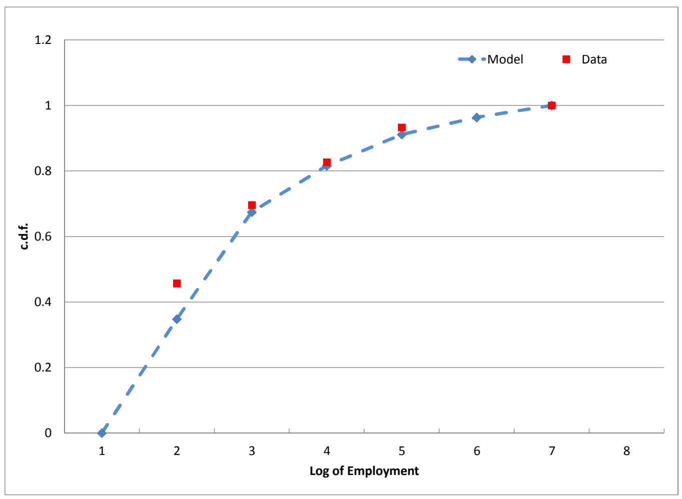
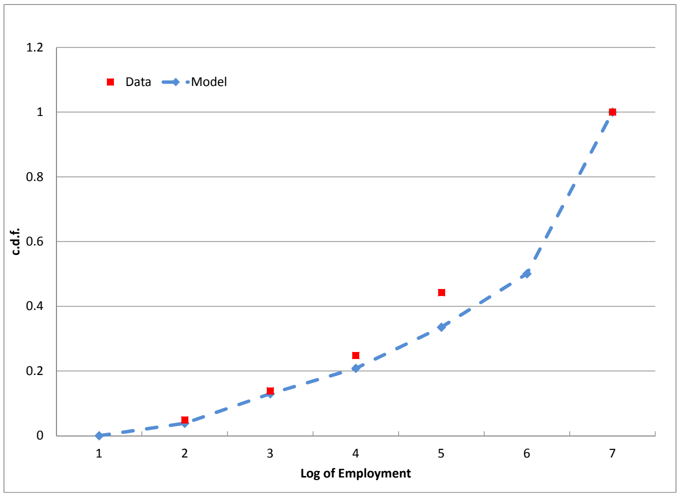


Figure 2: Employment Distribution



Parameter	**Target**	**Value**
γ_g	Persistence of Expansions	.685
γ_b	Persistence of Recessions	.298
β	Average Annual Interest Rate	.98
α	Share of Capital	33%
$\sigma(\rho)$	Returns to entrepreneurship	6.66 (0.85)
δ	Mean Exit Rate	0.055
θ	Intertemporal Elasticity of Substitution	1 (log utility)
$\underline{z}$	Lower Bound of Idyosincratic Productivity	0.01
N	Number of Technologies/Capacities	4

Table 1: Parametrization

Parameter	Definition	Value
$s_g - s_b$	Size of the Shocks (Symmetric)	$exp(0.0267) - exp(-0.0267)$
$\left[\underline{k}, \overline{k}\right]$	Range of Capacities	$[1, 4]$
$[\underline{z}, \overline{z}]$	Range for Idiosyncratic Productivity (Upper Bound)	$[0.01, 4.25]$
I^{L11}	Entry Costs	$\begin{bmatrix} 1.09 \\ 0.93 \end{bmatrix}$
I^H	Upgrade Costs	$\begin{bmatrix} 4.55 & 11.37 & 37.1 \\ 4.28 & 4.26 & 1.98 \end{bmatrix}$
Π^e	Scrap Values	$\begin{bmatrix} 0.85 & 2.47 & 9.27 & 9.1 \\ 0.86 & 2.46 & 9.2 & 9.13 \end{bmatrix}$
ς_G	Pareto Tail of the productivity distirbution at entry	1.9

Table 2: Jointly Calibrated Parameters

Moment	Data	Model	Moment	Data	Model
Emp. Share, 1-19	0.05	0.04	Estab. Share, 1-19	0.46	0.35
Emp. Share, 20-49	0.14	0.13	Estab. Share, 20-49	0.69	0.67
Emp. Share, 50-99	0.25	0.21	Estab., 50-99	0.83	0.82
Emp. Share, 100-249	0.44	0.36	Estab., 100-249	0.93	0.91
Entry Rate	6.9%	6.24%	Exit Rate	5.5%	6.23%
Investment Spikes[12]	8%	9.1%	Log Emp. (upper bound)	10000+	10829
Output Volatility	2.09%	2.1%			

Table 3: Targeted Moments

	Baseline	**Optimal Allocation**
Aggregate TFP	3.36	3.73
Standard Deviation TFP	7.9%	8.4%
Coefficient of Variation, TFPQ	3.01	2.66

Table 4: Productivity Statistics

	Baseline	**Optimal Allocation**	TFP^{mc}
Aggregate TFP	1.31	1.36	2.33
Standard Deviation TFP	2.6%	2.6%	3.3%
Coefficient of Variation, TFPQ	3.01	2.66	1.05

Table 5: Productivity Statistics: Normalized Measure

k	**Ratio mean** $TFPQ$
1	1.02
2	0.99
3	0.99
4	0.98

Table 6: Efficiency across capacities

	Baseline	**Case G**	**Case S**
	γ_g= **.685**	γ_g= **.237**	γ_g= **.685**
	$\mathbf{s}_g - \mathbf{s}_b$= **0.053**	$\mathbf{s}_g - \mathbf{s}_b$= **0.053**	$\mathbf{s}_g - \mathbf{s}_b$= **0.064**
TFP	3.36	3.72	2.19
Standard Deviation TFP	7.9%	9.1%	30.4%
Coefficient of Variation TFPQ	3.01	2.7	2.64
$\mathbf{TFP}_{M=1}/\mathbf{TFP}^{mc}$	0.56	0.58	0.56
Volatility of Output	2.1%	2.5%	8.6%

Table 7: Features of Aggregate Uncertainty

	Model	**Case G**	**Case S**
Entry Rate	6.24%	5.95%	20.4%
Exit Rate	6.23%	5.94%	12.6%
Upgrade Rate	9.1%	9.7%	45.1%

Table 8: Firm Dynamics

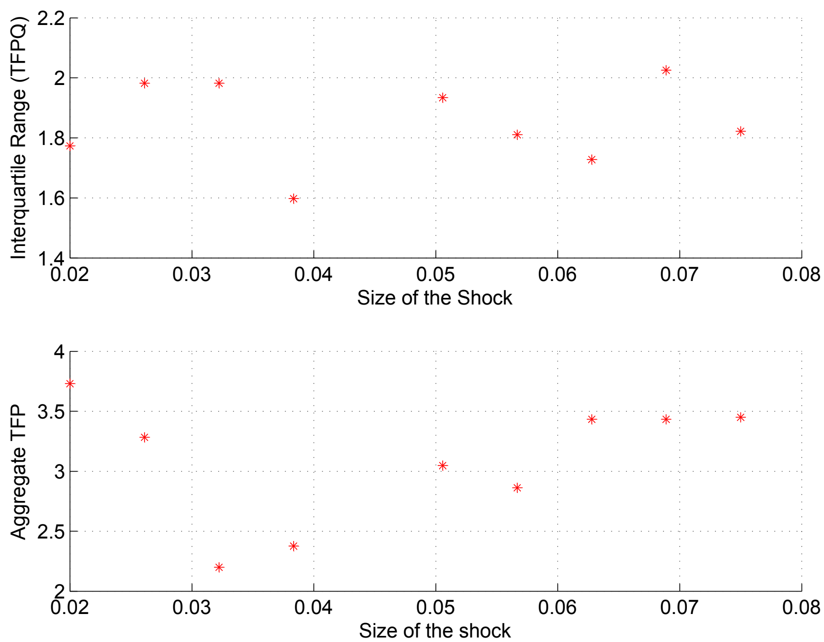


Figure 3: Dispersion in TFP, Aggregate TFP and the Cyclical Component of GDP

Employment	**0-49**	**50-149**	**150-499**	**500+**
Output Share	0.14	0.06	0.05	0.75
Opt. Policy	0.13	0.05	0.04	0.78
Capital Share	0.69	0.16	0.08	0.07
Opt. Policy	0.71	0.15	0.08	0.06
Employment Share	0.16	0.11	0.15	0.58
Opt. Policy	0.16	0.11	0.14	0.59

Table 9: Optimal Policy: Distributional Implications

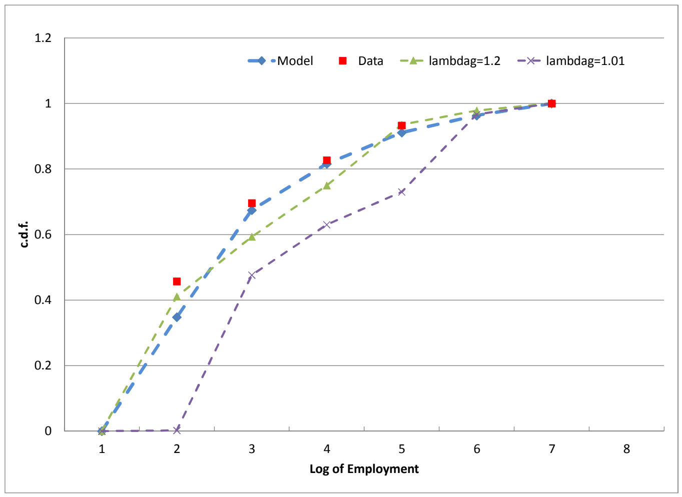
	Model	**Optimal Policy**
Entry Rate	6.24%	5.85%
Exit Rate	6.23%	5.84%
Upgrade Rate	9.1%	9.8%

Table 10: Optimal Policy: Firm Dynamics

	Good Times		**Bad Times**	
	Baseline	**Optimal Policy**	**Baseline**	**Optimal Policy**
I^L/Y	0.30	0.28	0.30	0.29
I^H/Y	$\begin{bmatrix} 0.87 & 3.28 & 3.22 \end{bmatrix}$	$\begin{bmatrix} 0.45 & 0.81 & 2.51 \end{bmatrix}$	$\begin{bmatrix} 0.87 & 3.25 & 3.23 \end{bmatrix}$	$\begin{bmatrix} 0.43 & 0.83 & 2.57 \end{bmatrix}$
Π^e/Y	$\begin{bmatrix} 0.39 & 1.61 & 4.02 & 13.11 \end{bmatrix}$	$\begin{bmatrix} 0.50 & 0.49 & 2.27 & 8.33 \end{bmatrix}$	$\begin{bmatrix} 0.33 & 1.51 & 1.51 & 0.70 \end{bmatrix}$	$\begin{bmatrix} -0.02 & 0.29 & 0.79 & 1.84 \end{bmatrix}$

Table 11: Tax/subsidy Structure in terms of output per worker

Figure 4: Establishment Distribution, Sensitivity Analysis



9 Appendix (C)

9.1 Features of the Solution

Proposition 4 *Continuation Values* $\widetilde{W}$ *are monotonic increasing in idiosyncratic productivity,* z *and the optimal investment/disinvestment strategy of the firm is a set of thresholds such that if* $z < z^e(\psi^J, X_t)$ *they exit the market, and if* $J = L$ *whenever* $z \geq z^u(\psi^H, X_t)$ *the firm upgrades capacity.*

Proof. First notice that $\pi(x_t, X_t)$ is bounded and continuous in z. (Replace the optimal factor demands in the profit function).

Second, let $W^*(\ x, X)$ be the unique fixed point to the operator T,

$$T(W(x, X_t)) = Max\left\{\Pi_e, \pi(x, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\}$$

We first show first that $W^*(\ x, X)$ is non-decreasing in z.

Let $C(Z)$ be the set of continuous bounded functions in z, and let $C'(Z)$ a closed subspace of non-decreasing functions. Take $W \in C(Z)$ and $z_1 < z_2$. then

$$\begin{aligned} T(W(z_1, \psi^j, X_t)) &= Max\left\{\Pi_e, \pi(z_1, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\ &\leq Max\left\{\Pi_e, \pi(z_2, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\ &= T(W(z_2, \psi^j, X_t)) \end{aligned}$$

so that $T(C'(Z)) \subseteq C'(Z)$. Hence by the Contraction Mapping Theorem $W^* \in C'(Z)$.

Now, we want to prove that for each $\left(\psi^j, X\right)$ the function $\widetilde{W}\ (z,\psi^j, X_t)$ is strictly increasing in z. Note that the expectation operator in the last term of the previous equation defined over the aggregate of the economy and independent of the productivity of the firm except through the function W^*. Take $z_1 < z_2$

$$\begin{aligned}\widetilde{W}(z_1,\psi^j,X_t) &= \pi(z_1,\psi^j,X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t,X_{t+1})\,(1-\delta)\,W^*(z_1,\psi^j,X_{t+1})\right) \\ &< \pi(z_2,\psi^j,X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t,X_{t+1})\,(1-\delta)\,W^*(z_2,\psi^j,X_{t+1})\right) \\ &= \widetilde{W}(z_2,\psi^j,X_t)\end{aligned}$$

which proves the claim.

Given the monotonicity of the continuation values, the optimality of the trigger strategy follows. Suppose not. Hence, there is a firm with productivity z, such that $z < z^e(\psi^J, X_t)$ and the firm does not exit the market. But the firm with productivity $z+\Delta < z^e(\psi^J, X_t)$ did, so $Max\left\{\Pi_e, \widetilde{W}(z+\Delta,\psi^j,X_t)\right\} = \Pi_e$. From the monotonicity of $\widetilde{W}$, it holds $\widetilde{W}(z+\Delta,\psi^j,X_t) > \widetilde{W}(z,\psi^j,X_t)$ so that $\Pi_e > \widetilde{W}(z,\psi^j,X_t)$ and hence remaining in the market cannot be optimal. Analogous argument hold for the upgrade thresholds. ■

Proposition 5 *The optimal allocation satisfies*

1. *If the minimum capacity constraint is not binding,* $z^e(\psi_L, X_t) > z^e(\psi_H, X_t)$
2. $\frac{\partial z^e(\psi^j, X_t)}{\partial r_t} \geq 0$
3. $z^u(\psi_H, X_t) > z^e(\psi_H, X_t)$
4. M^e *is procyclical*

Before proving the results note that the instantaneous profits of a firm are

$$\pi(x_t, X_t) = (1-\rho)Y_t^{1-\rho}\left[\frac{s_t K_t^{\alpha}}{(Z^k)^{\alpha}(Z^l)^{1-\alpha}}\right]^{\rho}\left(\frac{z\psi^j}{MPK_t^{\alpha}}\right)^{\frac{\rho}{1-\rho}}$$

1. **Proof.** Note first that the profit function $\pi(x_t, X_t)$ is monotonic in the firm idiosyncratic productivity and the technology shifter. We have proved that firms' continuation values are also monotonic. Hence $W(z, \psi_H, X_t) > W(z, \psi_L, X_t)$ for all z whenever the minimum capacity constraint is not binding. The scrap value of the firms is constant and independent of the technology of the firm. The optimality condition for the exit thresholds equalizes the value of the firm to the scrap value and therefore, $z^e(\psi_L, X_t) > z^e(\psi_H, X_t)$. ■

   Proof. The profit function is such that $\frac{\partial \pi(x_t, X_t)}{\partial r_t} < 0$. Following the same strategy than for the monotonicity in idiosyncratic productivity one can show that $W(z, \psi^j, X_t)$ is non increasing in the cost of capital and the continuation value $\widetilde{W}(z, \psi^j, X_t)$ is decreasing in r_t. As in the previous proof, the result follows from the optimality condition for the exit threshold. ■

   Proof. Given that upgrades in technology are costly and the scrap value at exit is independent of the technology operated by the firm. It cannot be optimal to upgrade and exit immediately. For this strategy, I_H units of goods are paid , while exiting while running the low minimum capacity technology yields the same scrap value and no associated cost. ■

   Proof. $\widetilde{W_t}(z, \psi_L, X_t)$ is increasing in the aggregate state of technology s. From the free entry condition the result follows. ■

The fact that the scrap value of the firm is independent of the cost capital and the idiosyncratic characteristics of the firm is critical to prove the previous results.

9.2 Existence and Uniqueness of the centralized allocation

Lemma 2 (AC) *The measure associated to the distribution of types is absolutely continuous(AC) with respect to the lebesque measure on the real line*

Proof. The claim follows from the absolute continuity of the exogenous distribution of types. We prove by induction.

By definition

$$v_1^L(z) = \left[\frac{G(z) - G(z^{e*}(1,X))}{1 - G(z^{e*}(1,X))}\right]$$

$$v_1^H(z) = \left[\frac{G(z) - G(z^{u*}(k,X))}{1 - G(z^{e*}(1,X))}\right]$$

Take a sequence of of intervals $(a_k, b_k)_{k=1}^K$ and let

$$\sum_{k=1}^{K} \left| v_1^j(b_k) - v_1^j(a_k) \right| \leq \varepsilon$$

Replacing by the definition

$$\sum_{k=1}^{K} \left| \frac{1}{1 - G(z_1^{eL})} \left(G(b_k) - G(a_k)\right) \right| \leq \varepsilon$$

Let $\widehat{\varepsilon} = \varepsilon \left[1 - G(z^{e*}(1,X))\right]$ .By AC of G, there exist $\widehat{\delta}$ such that

$$\sum_{k=1}^{K} |b_k - a_k| \leq \widehat{\delta}$$

Because ε was arbitrary, and $(a_k, b_k)_{k=1}^K$ too, v_1^j is absolutely continuous.

Suppose v_N^j is absolutely continuous. By definition,

$$v_{N+1}^L(z) = (1-\delta)\left[v_N^L(z) - v_N^L(\psi_L)\right]_{\chi_{z_t^e(\psi_L) > z_{t-1}^e(\psi_L)}} + \left[\frac{G(z) - G(z^{e*}(\psi_L))}{1 - G(z^{e*}(\psi_L))}\right] \quad z_t^{u*} > z > z^{e*}(\psi_L)$$

If $z_{t-1}^{u*} \leq z_t^{u*}$

$$\begin{aligned} v_{N+1}^H(z) &= (1-\delta)\ v_N^H(z) & z_t^{u*} > z > z_t^{e*}(\psi_H) \\ &= (1-\delta)\, v_N^H(z) + M_t^{ent} \frac{G(z) - G(z_t^{u*})}{1 - G(z_t^{e*}(\psi_L))} & 1 > z > z_t^{u*} \end{aligned}$$

If $z_{t-1}^{u*} > z_t^{u*}$

$$\begin{aligned}
v_{N+1}^{H}(z) &= (1-\delta)\ v_N^H(z) & z_t^{u*} > z > z_t^{e*}(\psi_H) \\
&= (1-\delta)\left[v_N^H(z) + v_N^L(z) - v_N^L(\ z_t^{u*})\right]\ + M_t^{ent}\frac{G(z)-G(z_t^{u*})}{1-G(z_t^{e*}(\psi_L))} & z_{t-1}^{u*} > z > z_t^{u*} \\
&= (1-\delta)\, v_N^H(z) + v_N^L(\ z_{t-1}^{u*}) - v_N^L(\ z_t^{u*}) + M_t^{ent}\frac{G(z)-G(z_t^{u*})}{1-G(z_t^{e*}(\psi_L))} & 1 > z > z_{t-1}^{u*}
\end{aligned}$$

therefore, the sum of absolutely continuous functions. Hence, v_{N+1}^{j} is absolutely continuous.

If v_t^j, is absolutely continuous then it is continuous. ■

Lemma 3 (M) *The feasible measure of firms in the market is bounded*

Proof. By definition, the total measure of firms in the market is $M_t = v_t^L + v_t^H$. Using the aggregation results, one could right the feasibility constraint of the economy as

$$\begin{aligned}
M_t^{\frac{1}{\rho}}\widetilde{Y}_t + T_t + \Pi_t^e M_t^{eL} + \Pi_t^e M_t^{eH} - I_t^L M_t^{ent} - I_t^H M_t^u &= C_t \\
M_t - (1-\delta)\, M_{t-1} - M_t^{ent} + \left(M_t^{eL} + M_t^{eH}\right) &= 0
\end{aligned}$$

where $\widetilde{Y}_t = Y_t M_t^{-\frac{1}{\rho}}$

A strategy to make the measure of firms grow without bound would be to never exit firms and enter as much as possible. Now, because entry is costly, optimality dictates that the marginal cost of an entrant equalizes the marginal return,

$$\frac{1}{\rho}\left((1-\delta)\, M_{t-1} + M_t^{ent}\right)^{\frac{1}{\rho}-1} y_t = I_t^L$$

which pins down a finite level of entry at each t. Replacing the entry level into the dynamic equation for the measure of firms we obtain

$$M_t = \left(\rho\frac{I_t^L}{\widetilde{Y}_t}\right)^{\frac{\rho}{1-\rho}}$$

which is bounded.

Alternatively, a strategy to make the measure of firms shrink without bound would be to never enter firms and exit as many as possible. Now, because exit is costly (in terms of foregone output), optimality dictates

$$\frac{1}{\rho}\left((1-\delta)M_{t-1}-M_t^e\right)^{\frac{1}{\rho}-1}y_t=\Pi_t^e$$

which pins down a finite level of entry at each t. Replacing the entry level into the dynamic equation for the measure of firms we obtain

$$M_t=\left(\rho\frac{\Pi_t^e}{\widetilde{Y}_t}\right)^{\frac{\rho}{1-\rho}-1}$$

which is bounded at a positive number. ■

Before moving to the next result define Θ as the set of bounded absolutely continuous functions from $[\underline{z},\overline{z}]\rightarrow R^{+}$. Hence, $\left(v_{t-1}^L,v_{t-1}^H\right)\in\Theta$. Let, $\overline{K}\subset R$ the feasible set for capital. Because there are decreasing returns to capital in the aggregate and there is no growth in the economy, it is without loss of generality to assume $\overline{K}$ is compact.

Lemma 4 (U) *$U:\Theta x\Theta x\overline{K}\rightarrow R$ is bounded and continuous*

Proof. Proof. As defined in the body of the paper U is CES with parameter θ. If $\theta<1$ then $U(C(v_{t-1}^L,v_{t-1}^H,v_t^L,v_t^H,K_t))$ is bounded below as $U(0)=0$. Now, potentially U is unbounded above. However, because the feasible measure of firms in the market is always bounded above (Lemma (M)), consumption is bounded and $U(.)$ too, along the relevant state space. $U(.)$ is continuous by assumption so the claim is proved. If instead $\theta\geq 1$, the return U is discontinuous at zero and potentially unbounded below. Because the feasible measure of firms is bounded away from zero, unboundedness below of U is also ruled out. ■

Claim 2 ***Proof.*** *If $\theta\geq 1, U$ can be unbounded below, but the feasible measure of firm* *is bounded below and consumption is bounded below hence U is bounded in the feasible set.* ∎

∎

Theorem 3 *a) The solution to the Planner problem exist and it is unique*

Proof. Let E_t be the expected value under the transition probabilities for the exogenous shock $\mathbf{P}$. We can write the planner's problem in terms of the operator $\digamma$ as

$$\digamma V(\Xi_t) = \underset{\left(v_t^L, v_t^H, K_{t+1}\right) \in \Gamma\left(v_{t-1}^L, v_{t-1}^H, K_t\right)}{Max} U\left(v_{t-1}^L, v_{t-1}^H, v_t^L, v_t^H, K_t\right) + \beta E_t\left[V(\Xi_{t+1})\right]$$

where $\left(v_t^L, v_t^H\right) \in \Theta$, The utility function is defined as $U : \Theta x \Theta x \overline{K} \to R$

Let $H(\Xi_t, \theta)$ be the set of functions (functional) $f : \Xi_t \to R$ that are homogenous of degree $(1-\theta)$, continuous except potentially at the origin if $\theta > 1$ and bounded in the norm

$$\|f\| = \sup_{\|\Xi_{t-1}\|=1, \Xi_{t-1} \in \Theta x \Theta x \overline{K}} \|f(\Xi_{t-1})\|$$

$\digamma : H(\Xi_t, \theta) \to H(\Xi_t, \theta)$.

From Lemma (U) we have that $U\left(v_{t-1}^L, v_{t-1}^H, v_t^L, v_t^H, K_t\right)$ maps a convex compact set[13] into a closed subset. Also,given the structure of the stochastic process for s_t, $\mathbf{P}$ has the Feller property.

Hence $\digamma$ is a contraction, with a unique fixed point in Ξ_t. ∎

9.3 Equivalence with the decentralized solution

To prove the equivalence between the centralized and decentralized solution define

$$\Omega(z^e, z^u, M^{ent}; \Xi_t) \equiv \tau \Pi^e M^e + \tau I^H M^u + \tau I^L M^{ent} + Y - Y^*$$

[13] Θ is a convex set as each convex combination of two AC functions is AC.

thresholds depend of the transfers, T_t

Lemma 5 $\Omega(z^e, z^u, M^{ent}; \Xi_t)$ *is continuous in the exit and upgrade thresholds as well as in the measure of entrants.*

Proof. Continuity in the measure of entrants is straightforward from the definition. Continuity in the thresholds follows from the definition of the measure of upgrades and exits in terms of the distribution of firms in the market and the absolute continuity of v_t^j that we proved in Lemma (AC), i.e. $M^{eJ} = v^J(z^e(\psi^J, \Xi_t))$ for $J = L, H$ and $M^u = \max\left\{v^L(z^u(\psi^H, \Xi_t)) - v^L(z^u(\psi^H, \Xi_{t-1})), 0\right\}$ ■

Lemma 6 *There exist a transfer scheme $T^*(\Xi_t)$ such that*

$$\Omega(z^e(\psi^L, T^*), z^e(\psi^H, T^*), z^u(\psi^H, T^*), M^{ent}(T^*); \Xi_t) = T^*$$

Proof. Lemma (M) shows that the measure of firms operating in the market is bounded. Hence, there exist B such that $\Omega(z^e, z^u, M^{ent}; \Xi_t) < B$[14]. The feasible measure of entrants is also bounded by Lemma (M). Let $\Phi \equiv [0, B]$, which is convex and compact by construction. The optimal thresholds are the maximizers of (PP). By the theorem of the maximum they are u.h.c. in $T^*(\Xi_t)$. Hence, Ω is an upper hemicontinuous convex valued correspondence and $\Omega \neq \varnothing$ for any $T \in \Phi$. Thus, Ω has a fixed point (Kakutani). ■

Note that there might be different combination of thresholds that generate the same transfer

Lemma 7 *If the allocation of firms in the decentralized and centralized problem are the same, there exist prices such that the dynamic of aggregate capital is the same across economies.*

[14] Output is bounded because the measure of firms is bounded and there are decreasing returns to capital in the economy.

Proof. The equivalence comes from setting $r_t \equiv \frac{\partial Y_{t+1}}{\partial K_{t+1}} = \alpha s_{t+1} TFP_{t+1} K_{t+1}^{\alpha-1}$, i.e. the marginal product of capital in the economy. If the allocation of firms is the same, then endogenous TFP is the same. If we replace the equilibrium prices of firm shares in the budget constraint of the representative household, we obtain the feasibility condition in terms of goods for the aggregate economy. The latter equals the budget constraint of the pseudo planner for $T = T^*$ the fixed point. Hence the allocations of capital are the same. ■

Theorem 4 *b) There exist a cost structure* $\{\Upsilon^p(\Xi_t)\}_{t=0}^{\infty}$ *such that the allocation of firms that solves the planner problem (PP) coincides with the competitive allocation.*

Proof. Define, $\Upsilon^p(\Xi_t) = \Upsilon^c \tau^*(\Xi_t)$ where $\tau^*(\Xi_t)$ generates $T^*(\Xi_t)$ (the fixed point of Ω)

When the PP is solved at $T = T^*(\Xi_t)$ the budget constraint reads

$$C_t + K_{t+1} - (1-\widehat{\delta})K_t + I_L M_t^{ent} + I_H \left[M_t^u + M_t^{ent} \frac{1 - G(z_t^u(\psi^H, T^*))}{1 - G(z_t^e(\psi^L, T^*))} \right] = Y_t + \Pi_e \left(M_t^{eL} + M_t^{eH} \right)$$

which is the market clearing condition in the decentralized allocation. Hence, for this cost structure the feasibility constraint of the planner coincides with that of the competitive equilibrium.

The dynamic optimality conditions for the firms in the decentralized conditions need to hold at $z_t^u(\psi^H, T^*)$.

I argue that there exist an industrial policy $\widehat{\tau}$ such that at the thresholds of the competitive equilibrium, the generated transfer T is a fixed point of $\Omega; T(\widehat{\tau}(\Xi_t)) = \Omega(T)$. Note that the pseudo-planner's optimality conditions in terms of the allocation of firms across technologies and entry levels (PPFOC) are linear in the cost of entry, upgrade and the scrap value. The indifference conditions for the firms in the decentralized problem are linear in the costs too. Define $\widehat{\tau}$ to satisfy (PPFOC) at zeta. The industrial policy is well defined because it solves a system of linear equations perfectly identified.

Suppose that $T(\widehat{\tau}(\Xi_t))$ is not a fixed point of Ω. The level of output generated in by the centralized allocation is the same as in the decentralized allocation because the thresholds and measure of entries are the same. The budget constraint would read

$$\begin{aligned} & C_t + K_{t+1} - (1-\widehat{\delta})K_t + I_L M_t^{ent} + I_H \left[M_t^u + M_t^{ent} \frac{1 - G(z_t^u(\psi^H, T))}{1 - G(z_t^e(\psi^L, T))} \right] \\ = \quad & Y_t + \Pi_e \left(M_t^{eL} + M_t^{eH} \right) + \widehat{\tau} \Pi^e M^e + \widehat{\tau} I^H M^u + \widehat{\tau} I^L M^{ent} \end{aligned}$$

which implies that the set of thresholds z^e, z^u, M^{ent} of the decentralized allocation violate the market clearing condition in the goods market. Therefore, the contradiction.

Finally, at the prices of capital and labor that we have chosen, the optimal investment and consumption of the representative consumer coincides with the allocation of the pseudo planner. ■