Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy

(First draft)

A. Kerem Coşar, Nezih Guner and James Tybout
Pennsylvania State U.;
ICREA, Universitat Autònoma de Barcelona and Barcelona GSE;
and Pennsylvania State U. and NBER

January 22, 2010

1Without implicating them, we wish to thank John Haltiwanger, Marc Muendler, Nina Pavcnik and Andres Rodriguez-Clare for helpful discussions. This research was supported by the National Science Foundation (Grant No.SES-0617888) and the Excellence Project of the Bank of Spain. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation or the Bank of Spain.
Abstract

As Latin American countries have become more open, their job turnover rates have risen, their informal sectors have become larger, and their wage distributions have become less equal. We develop a dynamic general equilibrium trade model that explains these phenomena. The model combines standard search frictions in labor markets with heterogeneous firms that experience ongoing productivity shocks. Each period, firms decide whether to exit or continue producing. Those firms that remain active choose their export volumes and adjust their employment levels through vacancy postings or layoffs.

Openness matters in our model because it makes profits more sensitive to productivity shocks, as Rodrik (1997) argued. Thus when trade barriers are low, firms drawing negative shocks shed labor relatively rapidly (and perhaps exit), while firms drawing positive shocks acquire new workers relatively rapidly. Further, since openness decreases the rents of the former and increases the rents of the latter, it spreads the wage distribution. After fitting this model to Colombian micro data on establishments and households, we isolate the effects of trade frictions on labor market outcomes using counter-factual simulations. Preliminary results suggest that the mechanisms highlighted by our model can be important.
1 Introduction

In developing countries, globalization is often blamed for exacerbating wage inequality, reducing job security, and increasing the size of the informal sector. This has been particularly true in Latin America, where many countries that pursued trade liberalization programs also experienced greater wage dispersion, heightened job turnover, and/or informality.¹

But the extent to which trade liberalization is to blame remains an open question. Labor market outcomes reflect many factors besides foreign competition, and reduced-form regressions have not convincingly isolated causal relationships. To better isolate the effects of openness on developing countries’ workers, we formulate a dynamic structural model of trade with labor market frictions. Then we fit our model to plant-level panel data and household survey data from Colombia—a country that in many respects typifies Latin American experiences.

The mechanism that links openness and labor market outcomes in our model is related to one posited by Rodrik (1997). He argued that openness increases the elasticity of demand for goods, and thus makes firms’ labor demand functions more sensitive to idiosyncratic productivity shocks. Workers’ job security and bargaining power are therefore compromised as trade barriers come down. In our model, although the elasticity of demand for goods is parametrically fixed at home and abroad, openness likewise increases the effects of idiosyncratic productivity shocks on vacancy postings and lay-offs. This occurs mainly because openness shifts the size distribution of producers in favor of large firms, and with production concen-

¹ "Between the mid-1980s and the beginning of the 1990s, countries in Latin America began trade liberalization programs, with reductions of at least 15 percentage points in the average tariff rate, which fell from an average of 48.9 percent in the prereform years to 10.7 percent in 1999." (Inter-American Development Bank, 2004, p. 137). Haltiwanger et al (2004) document the association between job turnover and openness in Latin America. Goldberg and Pavcnik (2007) survey the evidence linking openness to wage inequality and informality in Latin America and other developing regions.
trated in fewer establishments, idiosyncratic shocks have a bigger influence on job turnover. The greater volatility in firm-specific labor demand induces associated changes in the equilibrium turnover rate, the wage distribution, and the rate of self-employment/informality.

Our formulation also shares some features with a number of recent trade models that describe the effects of openness on labor markets (Helpman and Itskhoki, 2007; Helpman, et al, 2008; Egger and Kreickemeier, 2007; Amiti and Davis, 2008; Davis and Harrigan, 2008; Felbermayr et al, 2008). In particular, it embodies Melitz’s (2003) basic insight that openness compounds the advantage of relatively efficient firms by creating new exporting opportunities for them, while it compounds the problems of relatively inefficient firms by intensifying the competitive pressures they face. However we depart from this literature in two ways. First, we assume that firms experience ongoing, idiosyncratic productivity shocks (as in Hopenhayn, 1992, and Hopenhayn and Rogerson, 1993), and they respond by adjusting their vacancy postings, lay-offs and exit decisions. (as in Bertola and Caballero, 1994, and Bertola and Garibaldi, 2001). Second, we fit our model to micro data and use it to perform numerical experiments that quantify the effects of openness under alternative assumptions. Simulations at plausible parameter values show that these effects can be important.

2Several less-related linkages between openness and labor market outcomes have been modeled in the recent trade literature. One strand of this literature emphasizes the changes in skill-premia and/or unemployment rates that result from trade-induced changes in the relative demand for different types of labor (e.g., Albrecht and Vroman, 2002; Yeaple, 2005; Davidson et al, 2006). Another characterizes the adjustments in wages, unemployment and labor turnover patterns that derive from trade-induced changes in sectoral output prices (e.g., Kambourov, 2006; Artuc, Chaudhuri and McClaren, 2008). And finally, some studies have focussed on cross-country differences in the flexibility of labor markets as a source of comparative advantage (Davidson et al, 1999; Cunat and Melitz, 2007; Helpman and Itskhoki, 2008).

While we do not pretend to capture all of the channels through which openness can affect labor market outcomes, our focus on firm-level entry, exit and idiosyncratic productivity shocks is supported by existing empirical evidence on the sources of job turnover and wage heterogeneity. Studies of job creation and job destruction invariably find that most reallocation is due to idiosyncratic (rather than industry-wide) adjustments (Davis et al, 1998; Roberts, 1996; Inter-American Development Bank, 2004). “This is true even in Latin America’s highly volatile macro environment” where producer entry and exit alone account for 30-40 percent of job creation and destruction (Inter-American Development Bank 2004, chapter 2). Further, as Goldberg and Pavcnik (2007) note, if openness has had a significant effect on job flows, it has mainly been through intra-sectoral effects: "Most studies of trade liberalization in developing countries find little evidence in support of [trade-induced labor] reallocation across sectors.” Finally, while cross-worker differences in wages are obviously partly due to differences in worker characteristics, much is attributable to labor market frictions and firm heterogeneity.4

2 Environment: The Closed Economy

For expositional clarity, we first develop our model for the case of a closed economy. This formulation extends Bertola and Cabellero (1994) and Bertola and Garibaldi (2001) to a general equilibrium setting with fully articulated product markets, arbitrary (stationary) Markov processes for productivity shocks and endogenous firm entry and exit. Once we have characterized the interactions between the labor markets, product markets, and productivity shocks

4Studying data from France and the United States, Abowd, et al (1999) and Abowd, et al (2002) show that roughly half of the cross-worker variation in compensation in French workers is due to employer effects. The only study of employer-employee data in developing countries we are aware of is Menezes-Filho and Muendler (2007). This paper does not report results on sources of wage variation.
in this setting, it is straightforward to generalize the analysis to an open economy and allow for intra-sectoral trade.

There are two types of output in our economy—services and industrial goods. The former are non-traded while the latter are tradable, subject to transport costs. Services are supplied by firms and, less efficiently, by unemployed workers engaged in home production. Regardless of their source, services are produced with labor alone, homogeneous across suppliers, and sold in competitive product markets. Firms that supply services use a common constant returns technology, and face no hiring or firing costs.

Industrial goods cannot be home-produced. They must be supplied by firms, which pay a sunk start-up cost to initiate production of a single variety of output. Each firm produces its output using labor alone, and competes in a monopolistically competitive product market. Unlike service sector firms, suppliers of industrial goods are subject to ongoing idiosyncratic productivity shocks, and they must create costly vacancies in order to attract new workers. The shocks they face can equally well be thought of as affecting the relative appeal of their products.

Producer dynamics in the industrial sector resemble those in Hopenhayn and Rogerson (1993) in that firms react to their productivity shocks by optimally hiring, firing or exiting. Also, new firms enter whenever their expected future profit stream exceeds the entry costs they face. However, unlike Hopenhayn and Rogerson (1993), we assume hiring in the industrial sector is subject to search frictions captured by a standard matching function. Labor market frictions generates rents that are bargained between worker and firms, and firms end up paying different wages depending on their current productivity and labor force as well as whether
they are hiring or firing workers. Further, workers make forward-looking decisions concerning which sector to work in and what job offers to accept.

Each worker decides whether to participate in the industrial labor market at the beginning of each period. Those who are already employed in the industrial sector can continue with their current job unless their employer lays them off or shuts down entirely. (They can also quit in order to move to the service sector or to search for other industrial sector jobs, although in equilibrium none find it optimal to do so.) Those not yet employed in the industrial sector can forego certain employment with a service sector firm in order to search for a higher-wage industrial sector job, but they risk remaining unemployed if they fail to match with an industrial sector producer.\(^5\) Those who end up unemployed subsist during the current period by using a relatively inefficient technology to home-produce services.

### 2.1 Production Technologies

All service-sector firms exploit a common constant-returns technology to produce the homogenous good. So with an appropriate choice of output units, we may write their combined supply of services as

\[
S = L_S,
\]

where \(L_S\) is labor hired in services.

In the industrial sector, output of producers with productivity level \(z\) is given by

\[
q(z, l) = zl^\alpha,
\]

where \(l\) is the labor input and \(\alpha > 0\). Productivity is firm-specific, independent across firms,

\(^5\)The notion that workers trade job security in a low wage sector for the opportunity to search in a higher wage sector traces back at least to the Harris and Todaro (1970) model.
and serially correlated. Its evolution is characterized by the transition density $h(z'|z)$, which is common to all firms.

### 2.2 Preferences

Worker-consumers in the economy are homogenous and their measure is normalized to unity. Each has lifetime utility given by

$$U = \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t U(s^c_t, q^c_t),$$

where $r$ is the rate of time preference, $s^c_t$ is consumption of services, and $q^c_t$ is an index of differentiated good consumption. The momentary utility function $U$ takes the form

$$U(s^c, q^c) = \frac{(s^c)^{1-\gamma} (q^c)^\gamma}{(1-\gamma)^{1-\gamma}};$$

(2)

where $\gamma \in (0, 1)$ and our index of industrial goods consumption is

$$q^c = \left( \int_0^N q^c(n)^{\frac{\sigma-1}{\sigma}} dn \right)^{\frac{\sigma}{\sigma-1}}.$$

(3)

Here $N$ is our measure of differentiated varieties, $q^c_t(n)$ is consumption of variety $n$, and $\sigma > 1$ is the elasticity of substitution between varieties.

The price of services is our numeraire, and given our representation of preferences above, the exact price index for the composite good $q^c$ is

$$P = \left( \int_0^N p(n)^{1-\sigma} dn \right)^{\frac{1}{1-\sigma}},$$

(4)

where $p(n)$ is the price of variety $n$.\(^6\)

---

\(^6\)Since all domestic firms in the same $(z, l)$ state charge the same price, $P$ can be restated as $N \int p(z, l)^{1-\sigma} f(z, l)dz dl^{1/(1-\sigma)}$, where $f(z, l)$ is the density of firms over states.
Letting $I_i$ be the income of worker $i$ and disallowing savings, the period-by-period budget constraint is

$$I_i = s_i^c + P q_i^c.$$  

Utility maximization implies that consumer $i$ spends a fraction $\gamma$ of her income on the composite industrial good and her demand for variety $n$ is

$$q_i^c(n) = \frac{\gamma I_i}{P} \left( \frac{p(n)}{P} \right)^{-\sigma} = D_i p(n)^{-\sigma}, \quad (5)$$

where $D_i = \gamma I_i P^{\sigma-1}$. Finally, since worker-consumers are risk neutral, consumer $i$ enjoys momentary indirect utility

$$W_i = I_i P^{-\gamma}. \quad (6)$$

### 2.3 Labor Markets and the Matching Technology

The service sector labor market is frictionless so, given that the price of services is unity, the service sector wage is $w_s = 1$. Search frictions make things more complicated in the industrial sector. Each period the number of new matches between unemployed workers and vacancy posting firms is given by

$$M(V, L_u) = \frac{V L_u}{(V^\theta + L_u^\theta)^{1/\theta}},$$

where $L_u$ is the measure of unemployed workers searching in industrial sector and $V$ is the measure of vacancies in industry.\(^7\) Consequently, industrial firms fill each vacancy they post with probability

$$\phi^f(V, L_u) = \frac{M(V, L_u)}{V} = \frac{L_u}{(V^\theta + L_u^\theta)^{1/\theta}},$$

\(^7\)The functional form of the matching function follows den Haan et al. (2000). It is constant returns to scale, and increasing in both arguments. In contrast to the standard Cobb-Douglas form, it has no scale parameter and the implied matching rates are bounded between zero and one.
while unemployed workers searching for industrial jobs find matches with probability

\[ \phi^w(V, L_u) = \frac{M(V, L_u)}{L_u} = \frac{V}{(V^\theta + L_u^\theta)^{1/\theta}}. \]

Each period, after firms learn their current productivity, they decide whether to post vacancies and hire workers or shrink by firing some of their workers. After hiring and firing take place, firms pay wages that are determined by bargaining between workers and firms. Workers not fortunate enough to be retained—because their employers either contracted or exited—choose whether to look for work in the industrial sector or the service sector. Workers who spent the previous period producing service goods also decide where to seek work. Those who choose the service sector are employed with certainty at the wage \( w_s \). Those who choose the industrial sector must match with a vacancy-posting producer before then begin work. If they succeed in doing so, their wages reflect the rents associated with their match (details are provided below). The probabilities of these different events shape workers’ sectorial choices, as well as firms’ employment policies. We start by describing the latter.

### 2.4 Incumbent Firm’s Problem

Each industrial firm produces a unique variety and thus determines its own price and revenue by choosing an employment level. The demand function (5) and the production function (1) imply that any producer with productivity \( z \) who chooses employment level \( l \) will earn revenue

\[
    r(z, l) = D_d^{\frac{1}{\theta}}(zl^\alpha)^{\frac{\alpha-1}{\alpha}},
\]

where \( D_d = \int_0^1 D_i di \), and \( D_i \) depends upon the aggregate price level and the \( i^{th} \) worker’s income as discussed in section 2.2.
When choosing employment levels, firms weigh this revenue stream against wage costs, the effects of changes in $l$ on the the continuation value, and hiring costs. To characterize the latter, let the cost of posting $v$ vacancies for a firm of size $l$ be

$$C_h(l, v) = \left( \frac{c_h}{\lambda_1} \right) \left( \frac{v}{l^{\lambda_2}} \right)^{\lambda_1},$$

where $c_h$ and $\lambda_1 > 1$ are positive parameters. The parameter $\lambda_2 \in [0, 1]$ determines the strength of scale economies in hiring. If $\lambda_2 = 0$, there are no economies of scale and the cost of posting $v$ vacancies is the same for all firms. On the other hand if $\lambda_2 = 1$, the cost of a given employment growth is the same for all firms, and hence a given number of vacancies cost less for larger firms.

Firms are large in the sense that cross-firm variation in realized arrival rates is ignorable. (That is, all firms fill the same fraction $\phi^f$ of their posted vacancies.) It follows that expansion from $l$ to $l'$ simply requires the posting of $v = \frac{\ell - l}{\phi^f}$ vacancies, and we can write the cost of expanding from $l$ to $l'$ workers as

$$C_h(l, l') = \left( \frac{c_h}{\lambda_1} \right) \left( \phi^f \right)^{-\lambda_1} \left( \frac{l' - l}{l^{\lambda_2}} \right)^{\lambda_1}.$$

Clearly, when labor markets are slack, hiring is less costly because each vacancy is relatively likely to be filled.

Finally, each firm bargains with its workers individually and continuously, ensuring, as in Stole and Zwiebel (1996) and Cahuc and Wasner (2000), that all workers at a given firm are paid the same wage at a given point in time.

---

8 This specification generalizes Nilsena et al (2007), who set $\lambda_2 = 1 - 1/\lambda_1$. As discussed in Bertola and Caballero (1994) “convexity is necessary to obtain a well-defined vacancy-posting equilibrium when productivity is heterogeneous across firms, as firms with high productivity and low employment levels would want to post infinitely many vacancies for arbitrarily short intervals of time if such policies were not made prohibitively costly.”
To derive firms’ optimal employment policies, we first specify the sequencing of events within each period (Figure 1). An incumbent firm enters the current period with the productivity $z$ and work force $l$, which were determined at the end of the previous period. Immediately the firm decides whether to stay in business or to exit. If it stays, it proceeds to an interim stage in which it observes its current-period productivity realization $z'$. Then, taking stock of its updated state, $(z', l)$, the relevant wage schedules, and the sector-wide worker arrival rate, $\phi'$, it chooses its current period work force, $l'$. The firm can decide to hire ($l' > l$) or fire ($l' \leq l$) workers. If it hires workers, they are immediately available to produce output in the current period. Finally, revenues accrue and wages and other costs are paid at the end of the period.

Given the presence of search frictions, workers at hiring firms generate rents, and as we will detail shortly, these are bargained over to determine wages. However, since firms can shed
workers costlessly, the marginal worker at a firing firm creates no rents and has no bargaining power. Hence expanding and contracting firms face different wage schedules, and current operating profits depend upon both \( l \) and \( l' \). More precisely, defining \( w_h(z', l') \) to be the wage function faced by a hiring firm and \( w_f(z', l') \) to be the wage function faced by a firing firm, profits are

\[
\pi(z', l, l') = \begin{cases} r(z', l') - w_h(z', l')l' - c_f & \text{if } l' > l \\
r(z', l') - w_f(z', l')l' - c_f & \text{otherwise.} \end{cases} \tag{8}
\]

where \( c_f \), the per-period fixed cost of operation, is common to all firms.

Using (8), the beginning-of-period value of a firm in state \((z, l)\) is

\[
\mathcal{V}(z, l) = \max \left\{ 0, \frac{1}{1 + r} E_{z'|z} \max \left[ \pi(z', l, l') - C(l, l') + \mathcal{V}(z', l') \right] \right\}, \tag{9}
\]

where the max of the term in square brackets is the value of the firm in the interim state (after it has realized its productivity shock), and

\[
C(l, l') = \begin{cases} C_h(l, l'), & \text{if } l' > l, \\
0, & \text{otherwise.} \end{cases}
\]

The solution to (9) implies an employment policy function,

\[
l' = L(z', l), \tag{10}
\]

an indicator function that distinguishes hiring firms from others,

\[
I^h(z', l) = \begin{cases} 1, & \text{if } L(z', l) > l, \\
0, & \text{otherwise.} \end{cases} \tag{11}
\]

and an indicator function that characterizes firm’s continuation/exit policy,

\[
I^c(z, l) = \begin{cases} 1, & \text{if } \mathcal{V}(z, l) > 0 \\
0, & \text{otherwise.} \end{cases} \tag{12}
\]
2.5 Entry

In the steady state, a constant (endogenous) fraction $\mu_{exit}$ of firms exits the industry. These firms are replaced by an equal number of entrants, who find it optimal to pay a sunk entry cost of $c_e$ and create new firms. Upon creating their firms, these entrants acquire $l_e > 0$ workers and learn their initial productivity, which is drawn from the density function $f_e(z)$ with support $[\underline{z}, \bar{z}]$. (The search costs for the initial $l_e$ workers are included in $c_e$.) Thereafter entrants behave exactly like incumbent firms in the interim stage (see figure 1), with their interim state given by $(z, l_e)$. So by the time they begin producing, most new entrants have adjusted their workforce (subject to search costs) in accordance with their initial productivity. Free entry implies that

$$V_e = \int_{\underline{z}}^{\bar{z}} V(z, l_e) f_e(z) dz \leq c_e,$$  \hspace{1cm} (13)

which holds with equality if there is a positive mass of entrants, $M$.

2.6 Worker’s Problem

Figure 2 presents the intra-period timing of events for workers. Consider first a worker who is employed by an industrial firm in state $(z, l)$ at the beginning of the current period. This worker learns immediately whether her firm will continue operating. If her firms exits, she joins the pool of industrial job seekers (enters state $u$) in the interim stage. Otherwise, she enters the interim stage as an employee of the same firm that she worked for in the previous period. (No one voluntarily quits because, in equilibrium, firms always pay their workers at least their reservation wage.) Her firm then realizes its new productivity level $z'$ and enters the interim state $(z', l)$. At this point her firm decides whether to hire or fire workers. In the
former case it expands its workforce to $l' > l$, she earns $w_h(z', l')$, and she is positioned to start the next period in state $(z', l')$. In the latter case, she either loses her job and reverts to state $u$ or she retains her job, earns $w_f(z', l')$, and starts next period in state $(z', l')$. All workers at contracting firms are equally likely to be laid off, so each loses her job with probability $p_f = (l - l')/l$.

Workers in state $u$ are searching for industrial jobs. They are hired by entering and expanding firms that post vacancies. If they are matched with a firm, they receive the same wage as those who were already employed by the firm. If they are not matched, they remain unemployed in the current period, and support themselves by home-producing $b \in [0, 1)$ units of the service good. At the start of the next period, they can choose to work in the service sector (enter state $s$) or look for a job in the industrial sector (remain in state $u$). Likewise, workers who start the current period in the service sector choose between continuing to work at the service wage $w_s = 1$ and entering the pool of industrial job-seekers. As these workers have the option to choose either labor market, they are said to be in state $o$.

We now specify the value functions for the workers in the interim stage. Going to the service sector generates an end-of-period income of 1 and returns a worker to the $o$ state at the beginning of next period. Accordingly, the interim value of this choice is

$$J^s = \frac{1}{1 + r} (1 + J^o), \quad (14)$$

Searching in the industrial sector exposes workers to the risk of spending the period unemployed, supporting themselves by home-producing $b$ units of the service good. But it also opens the possibility of landing in a high-value job. Since the probability of finding a match
is $\phi^w$, the interim value of searching for an industrial job is

$$J^u = \frac{1}{1 + r} \left[ \phi^w E J_e^h + (1 - \phi^w)(b + J^o) \right], \tag{15}$$

where $E J_e^h$ is the expected value of matching with a hiring firm.

The value of the sectorial choice is $J^o = \max\{J^s, J^u\}$, and since services and industrial goods are both consumed in equilibrium, workers must be equally attracted to both types of production, i.e.,

$$J^o = J^s = J^u. \tag{16}$$

Combined with (14), this condition implies that $J^o$, $J^s$, and $J^u$ are all equal to $1/r$.

The expected value of matching with an industrial job, $E J_e^h$, depends on the distribution of hiring firms and the value of the jobs they offer. For workers who match with a hiring firm
in the interim state \((z', l)\), the interim period value is given by

\[
J^c_h(z', l) = \frac{1}{1 + r} \left[ w_h(z', l') + J^c(z', l') \right],
\]

(17)

where \(l' = L(z', l)\) and \(J^c(z', l')\) is the value of being employed at an industrial firm in state \((z', l')\) at the start of the next period. Accordingly, the expected value of a match for a worker as perceived at the interim stage is

\[
EJ^c_h = \int_{z'} \int_{l} J^c_h(z', l) g(z', l) dldz',
\]

(18)

where \(g(z', l)\) is the density of vacancies across hiring firms

\[
g(z', l) = \frac{v(z', l) \tilde{f}(z', l)}{\int_{z'} \int_{l} v(z', l) \tilde{f}(z', l) dldz'}.
\]

(19)

Here \(v(z', l) = \mathcal{I}^h(z', l) \cdot [L(z', l) - l] / \phi^f\) gives the number of vacancies posted by a firm in interim state \((z', l)\), and \(\tilde{f}(z', l)\) is the interim stage unconditional density of firms over \((z', l)\).

It remains to specify the value of starting the period matched with an industrial firm, \(J^c(z, l)\), which appears in (17) above. The value of being at a firm that exits immediately is simply the value of being unemployed, \(J^u\). This is also the value of being at a firing firm, since workers at these firms are indifferent between being fired and retained. Hence \(J^c(z, l)\) can be written as

\[
J^c(z, l) = \mathcal{I}^c(z, l) E_{z'|z} \left\{ \mathcal{I}^h(z', l) J^c_h(z', l) + \left[ 1 - \mathcal{I}^h(z', l) \right] J^u \right\} + \left[ 1 - \mathcal{I}^c(z, l) \right] J^u
\]

(20)

### 2.7 Wage Schedules

It remains to characterize the wage schedules. After hiring firms have posted their vacancies and matching has taken place, the labor market closes. Firms then bargain with their workers
simultaneously and on a one-to-one basis, treating each worker as the marginal one. At this point vacancy posting costs are already sunk and workers who walk away from the bargaining table cannot be replaced in the current period. Similarly, if an agreement between firm and the worker is not reached, the worker remains unemployed in the current period. These timing assumptions create rents to be split between the firm and the worker.

As detailed in Appendix 1, it follows that the wage schedule for expanding/replenishing firms with an end-of-period state \( (z', l') \) is given by

\[
w_h(z', l') = (1 - \beta) r \frac{b + J_0}{1 + r} + \Gamma(\alpha, \beta, \sigma) D_1^\#(z')^{\frac{\sigma - 1}{\sigma}} (l')^{-[\# + (1 - \alpha)]},
\]

where \( r \frac{b + J_0}{1 + r} \) is the flow value of unemployment for a worker who is bargaining with a firm at the end of the period, \( \beta \in [0, 1] \) measures the bargaining power of the firm, and \( \Gamma(\alpha, \beta, \sigma) = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)} \) is a constant.

The marginal worker at a contracting firm generates no rents, so the firing wage just matches her reservation value (see Appendix 1):

\[
w_f(z', l') = r J^u - [J^c(z', l') - J^u]. \tag{21}
\]

Note that \( w_f(z', l') \) varies across firms, since those workers who continue with a firing firm may enjoy higher wages next period. This option to continue has positive value (captured by the bracketed term), so firing firms may pay their workers less than the value of being unemployed.

3 Closed Economy Equilibrium

Five basic conditions characterize our equilibrium. First, the distribution of firms over \((z, l)\) states reproduces itself each period through the Markov processes on \(z\), the policy functions
(including hiring, firing, entry and exit), and the productivity draws that firms receive upon entry. Second, supply matches demand for services and for each differentiated good, where supplies are determined by employment and productivity levels in each type of good. Third, the flow of workers into unemployment matches the flow of workers out of unemployment—that is, the Beveridge condition holds. Fourth, aggregate income matches aggregate expenditure. And finally, workers optimally choose the sector in which they are working or seeking work. Appendix 2 provides a formal definition of equilibrium.

4 Allowing for International Trade

We now allow consumers to import foreign industrial goods and industrial firms to export some of their output. In addition to generating the efficiency and welfare effects demonstrated by Melitz (2003), these modifications shift patterns of job creation and destruction across firms in different \((z, l)\) states, and thereby affect aggregate job turnover rates, wage distributions, and unemployment rates.

In keeping with our focus on developing countries, we assume that the home country is too small to affect foreign income levels. Also, to limit the complexity of our model and speed numerical solution, we assume that the set of available foreign varieties and their foreign currency-denominated f.o.b. prices are exogenously determined. However, we allow the domestic currency price of imported goods to be endogenously determined as the nominal exchange rate adjusts to establish balanced trade.

There are several types of trade frictions in our model. First, as in Melitz (2003), there are fixed costs of exporting. This allows the model to replicate the well-known fact that exporters tend to be larger and more efficient than other firms. Second, and also as in Melitz (2003),
there are iceberg transport costs. This allows the model to capture the fact that exporters typically sell most of their output at home, even when foreign markets are large. It also provides us with a basis for "globalization" experiments in which the costs of international commerce decline. Third, in order to examine unilateral trade policy reforms, we allow for ad-valorem tariffs on imports.

4.1 Import demand

To characterize import demand, assume that consumers have access to \( N^* \) foreign varieties, and denote the f.o.b. foreign-currency prices of these varieties by \( p^*(n) \), \( n \in [0, N^*] \). Also, let \( k \) be the price of foreign currency, \( (\tau_c - 1) \) be the iceberg transport cost per unit shipped and \( (\tau_m - 1) \) be the ad valorem tariff rate on imports. Then, assuming that imported varieties enter consumers’ utility functions symmetrically with domestic varieties, total domestic demand for imported variety \( n \) can be written as:

\[
q_c^e(n) = \frac{\gamma I}{\bar{P}} \left( \frac{\tau_m \tau_c k p^*(n)}{\bar{P}} \right)^{-\sigma},
\]

where \( \bar{P} = [P^{1-\sigma} + (P^*)^{1-\sigma}]^{1/(1-\sigma)} \) is the exact price index for all \( N + N^* \) industrial goods available to domestic consumers, and \( P^* = \tau_m \tau_c k \left[ \int_0^{N^*} p^*(n)^{1-\sigma} \, dn \right]^{1/(1-\sigma)} \) is the exact price index for imported varieties.

The term \( \left[ \int_0^{N^*} p^*(n)^{1-\sigma} \, dn \right] \) is an exogenous constant, so we normalize it to unity by choice of foreign currency units. This allows us to write domestic spending on imported varieties as:

\[
E^* = \int_0^{N^*} (\tau_m \tau_c k) \cdot q_c^e(n) \, dn = D \cdot [\tau_m \tau_c k]^{1-\sigma},
\]

where \( D = \gamma I P^{\sigma-1} \) is an index of aggregate domestic demand for industrial goods, as before.
It follows that domestic demand for foreign currency (expressed in domestic currency) is
\[ R^* = \frac{E^*}{\tau_m} = \frac{D[\tau_m \tau_c k]^1 - \sigma}{\tau_m} = D^{\tau_c - \sigma} [\tau_c k]^{1 - \sigma}, \]
and tariff revenues collected by the home country government amount to \( T = R^*(\tau_m - 1) \).

There are no public goods in our model, so we assume all tariff revenues are returned to worker/consumers in the form of lump-sum transfers.

### 4.2 Export supply

It remains to characterize firms’ exporting decisions. Let \( D^* \) be the foreign market analog to \( D \), denominated in foreign currency.\(^9\) Then a firm in state \((z, l)\) that exports the fraction \( \eta \) of its output generates foreign sales revenues amounting to:
\[ r_x(z, l, \eta, k) = kD^* \left( \frac{\eta}{\tau_c} \right)^{\frac{\sigma - 1}{\sigma}}. \]

There are are no start-up costs or adjustment costs associated with exporting, so firms choose \( \eta \) each period to maximize their total current revenues, net of fixed exporting costs, \( c_x \). The associated revenue function is:
\[
\begin{align*}
    r(z, l, e) &= \max_{\eta \in [0, 1]} (r_d(z, l, \eta) + r_x(z, l, \eta, k) - c_x I^x(z, l)) \\
    &= \max \left\{ \left[ D^{\tau_c} (1 - \eta^o) \frac{\sigma - 1}{\sigma} + kD^* \left( \frac{\eta^o}{\tau_c} \right)^{\frac{\sigma - 1}{\sigma}} \right] (zl^o) \frac{\sigma - 1}{\sigma} - c_x I^x(z, l, k), \ D^{\tau_c} (zl^o) \frac{\sigma - 1}{\sigma} \right\},
\end{align*}
\]
where
\[ \eta^o = \frac{1}{\left( 1 + \frac{\tau_c - 1}{\kappa \sigma D^*} \right)} \]
is the optimal fraction of output to export, \textit{given} foreign market participation, \( r_d(z, l, \eta) \) is the revenue generated by selling \((1-\eta)zl^o\) units of output in the domestic market, and \( I^x(z, l, k) \) is

\(^9\)More precisely, \( D^* = \gamma (P^* \tau)^{-\sigma - 1} I^* \), where \( P^* \) is the exact price index for industrial goods available abroad and \( I^* \) is foreign income, both expressed in foreign currency. These objects are exogenous to the model, given our assumption that the home country is too small to influence foreign market aggregates.
an indicator function that takes a value of unity when $\eta > 0$. Whether the latter occurs simply depends upon $zl^\alpha$, since the gains from foreign market participation increase monotonically with production.

Embedded in our general equilibrium model, this standard revenue function delivers a number of desirable features. First, it implies that for any given $(z, l)$, the marginal revenue product of labor is larger when the economy is open. This is the underlying reason that productivity shocks induce larger adjustments in vacancy postings or firings when foreign markets are accessible. Second, since larger revenues at a given $(z, l)$ mean more surplus to bargain over, it is also the reason that the wage paid by a firm that exports in state $(z, l)$ is higher than what it is in the closed economy equilibrium. This result is consistent with the empirical finding that, controlling for employment, exporters pay higher wages (Bernard and Jensen, 1999). Third, combined with the fact that search frictions make marginal costs vary across firms with identical $z$ values, it explains why productive efficiency is a noisy predictor for exporting status.\(^\text{10}\) Fourth, re-interpreting $z$ shocks to be product appeal indices rather than productivity indices, it explains why exporters manage to be larger than non-exporters, even though they charge higher prices and pay higher wages.\(^\text{11}\)

Finally, and perhaps most interestingly, this expression implies that firms’ exporting status affects their total revenue for a given amount of labor and a given productivity level.\(^\text{12}\)

(Consider, for example, the change in revenue induced by a reduction in $\tau_c$ sufficiently large

\(^\text{10}\)This fact has attracted some attention recently. Hallak and Sivadasan (2008) explain it by assuming that (1) firms differ in terms of both their quality and their productivity efficiency, and (2) exporting requires that firms meet a minimum quality standard.

\(^\text{11}\)Kugler and Verhoogen (2008) note that this pattern could alternatively be due to complementarities in production between worker ability and product quality.
to cause a firm to begin exporting.) Thus, it provides a new interpretation for the common finding that measured productivity—i.e., deflated revenue per unit input bundle—is higher among exporters.\textsuperscript{12} The reason this result emerges is that labor market frictions prevent firms from freely adjusting their size as exporting opportunities come and go.

4.3 Open economy equilibrium

Once the closed economy revenue function (7) has been replaced with (22), and we have redefined our price and quantity indices appropriately, the analysis proceeds exactly as before. Total export revenues are

$$R_x = N \int_z \int_{l_e} \int_{z}^{\infty} r_x(z, l, k) I^x(z, l, k) f(z, l) dldz,$$

and since service goods are non-traded, balanced trade obtains when $R^* = R_x$. The exchange rate $k$ moves to ensure that this condition holds. Appendix 3 provides further details and confirms that if all other market clear, trade balance holds by Walras’ Law.

5 Quantitative Analysis

5.1 An Application to Colombia

To explore the implications of the small-country version of our model, we use a combination of econometric estimation and calibration techniques to fit it to Colombia. This country suits our purposes for several reasons. First, Colombia underwent a significant trade liberalization during the late 1980s and early 1990s, reducing its average nominal tariff rate from 21.5 percent to 11 percent (Table 1).\textsuperscript{13} Second, despite stable average unemployment rates, these

\textsuperscript{12}In support of this interpretation, De Loecker and Warzynski (2009) report evidence that mark-ups are higher among exporting firms.

trade reforms were associated with an increase in job turnover rates from 18.4 percent to 23 percent, an increase in informal self-employment from 17.8 to 20.7, and an increase of 0.34 in the ratio of wages at the 90th percentile to wages at the 10th percentile, controlling for observable worker characteristics (Table 1). These patterns are typical of Latin American experiences. Finally, although Colombia did experience some macro shocks during the period of interest, they were relatively mild. Thus the consequences of Colombia’s liberalization are relatively likely to come through in its data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>pre-liberalization</th>
<th>post-liberalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average nominal import tariff</td>
<td>21.50</td>
<td>11.30</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>18.43</td>
<td>22.95</td>
</tr>
<tr>
<td>Economy-wide unemployment rate</td>
<td>9.99</td>
<td>9.87</td>
</tr>
<tr>
<td>Informal self-employment rate</td>
<td>17.79</td>
<td>20.68</td>
</tr>
<tr>
<td>Log wage gap (90th versus 10th percentile)</td>
<td>1.43</td>
<td>1.51</td>
</tr>
</tbody>
</table>

5.2 The Revenue Function and Productivity Process

The job turnover and wage inequality documented in Table 1 help us to calibrate our model, as we will discuss shortly. But the parameters that characterize the revenue function and the productivity process can be estimated econometrically using Colombia’s annual industrial survey. Note that the revenue function (22) and CES preferences imply that log revenues (gross of fixed exporting costs) can be written as a function of employment, productivity and are for 1992-98, 1992-99, 1992-98, 1992-99, 1992-98 periods, respectively. The tariff data come from Attanasio, Goldberg and Pavcnik (2004), Table 1a. Job turnover figures are based on DANE’s annual industrial survey, which covers all manufacturing establishments with at least 10 workers. The log wage distribution is based on the residuals from a Mincerian regression of log wages on education, age, and sectoral and occupational dummies. The data set pools biennial household survey data from Colombia’s national statistical agency (DANE) for the period 1986-98. Coefficients on all variables are allowed to shift through time in order to exclude changing skill premiums as a source of dispersion. The informal self employment rate is constructed from the same data base. It is the fraction of the work force that is self-employed, non-professional, and informal (i.e., not paying social security).
an index of market-wide demand,
\[ d_H = \ln[D^{\frac{1}{\gamma}}(1 - \eta^o)^{\frac{\sigma-1}{\sigma}}], \] (24)

an index of the *percentage increase* in total demand associated with exporting,
\[ d_F = \ln[(k^\sigma D^*)^{\frac{1}{\gamma}} (\eta^o / \tau_c)^{\frac{\sigma-1}{\sigma}} e^{-d_H} + 1], \] (25)

and an indicator for whether firm \( i \) is an exporter, \( I^x_{it} \):
\[ \ln r_{it} = d_H + I^x_{it} d_F + \frac{\sigma - 1}{\sigma} \ln z_{it} + \alpha \frac{\sigma - 1}{\sigma} \ln l_{it} \] (26)

Further, assuming that \( \ln(z) \) follows an exogenous AR(1) process,
\[ \ln z_{it} = \rho \ln z_{it-1} + \epsilon_{it}, \] (27)

equation (26) can be restated as:
\[ \ln r_{it} = (d_H + I^x_{it} d_F) - \rho (d_H + I^x_{it-1} d_F) + \rho \ln r_{it-1} \]
\[ -\alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it-1} + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it} + \frac{\sigma - 1}{\sigma} \epsilon_{it}, \] (28)

If we could obtain consistent estimates of the coefficients that appear on the right-hand-side observable variables, these would allow us to infer consistent estimates of \( \rho, \alpha, \) and \( \sigma \). Also, the variance of the error term would allow us to infer \( \sigma_\epsilon \), the standard deviation of error terms in (27). However, selection bias and simultaneity bias prevent us from consistently estimating (28) with ordinary least squares. The former problem occurs because firms choose whether to exit the market partly on the basis of their \( \epsilon_{it} \) realizations, so the \( \epsilon_{it} \) realizations observed for active producers are not random draws from the unconditional distribution of \( \epsilon ' s \). The latter problem occurs because firms’ current exporting decisions and employment
levels are chosen after the current realization on $\epsilon$ is observed, so $\epsilon_{it}$ is correlated with both $T_{it}^x$ and $\ln l_{it}$. Appendix 4 develops a generalized method of moments (GMM) estimator related to Olley and Pakes' (1996) that deals with both problems.

Applying this estimator to the set of all Colombian manufacturing plants observed for at least three years during the pre-liberalization period 1982 and 1991, we obtain the results summarized in Table 1 below.\textsuperscript{14} Since $\sigma$ is not identified, we fixed this parameter at a value typical of the literature, $\sigma = 5$. All remaining parameters are estimated with considerable precision. It should be noted, however, that the estimates are somewhat sensitive to choice of the instrument set, and to the weights we used on different types of workers—managers, technicians, skilled laborers, unskilled workers, and apprentices—when constructing the number of "effective" workers.\textsuperscript{15}

\begin{table}[h]
\centering
\caption{Revenue function and productivity process \hfill (GMM estimates, given $\sigma = 5.0$)}
\begin{tabular}{lccc}
\hline
parameter & estimate & std. error & z-ratio \\
\hline
$\alpha$ & $0.592$ & $0.057$ & $10.41$ \\
$\rho$ & $0.848$ & $0.007$ & $118.73$ \\
$\sigma^2$ & $1.668$ & $0.042$ & $39.54$ \\
$d_H$ & $1.682$ & $0.047$ & $35.78$ \\
$d_F$ & $0.213$ & $0.004$ & $51.31$ \\
\hline
\end{tabular}
\end{table}

\subsection{5.3 Remaining Parameters}

Using our estimates for $d_F$ and $d_H$, equations (24), (25), and (23), and the fact that exporters earned 8.03% of their revenue from foreign sales during the base period, we can solve for $D$.

\textsuperscript{14}The data are annual observations on all manufacturing firms with at least 10 workers. They were collected by Colombia’s National Statistics Department (DANE) and cleaned as described in Roberts (1996). Given that fixed capital and intermediate inputs do not appear in our model, we define revenue to be the value of output net of intermediate input and capital costs. Annual capital costs are 10 percent of the book value of firms’ capital stocks.

\textsuperscript{15}The weights used for reported estimates are based on cross-plant mean wage premiums for each type of employee, relative to unskilled workers. Weighting means (using plant size as weights) yields a larger $\alpha$ value, although it has little effect on $\rho$. 

24
and $k^\alpha D^*$ and $\tau$. The results appear in table 3 below, along with several other parameters values we fix using observable aggregates. First, the real borrowing rate in Colombia fluctuated around 15 percent between late 1980s and early 2000s, so we set $r$ to be 0.15 (Bond et al, 2008). Second, the share of tradables in total consumption expenditure in Colombia was about 40 percent in 2005 (Wold Bank, 2008, Table 11, p. 134).¹⁶ This allows us to set $\gamma$ to be 0.4.

We take several other parameters from the existing literature. Following den Haan, Ramey and Ramey (2000), we set the elasticity of the matching function, $\theta$, to be 1.27. As a benchmark we give equal bargaining power to firms and workers, i.e. $\beta = 0.5$, assume that entrants have start with the lowest possible employment level.¹⁷

<table>
<thead>
<tr>
<th>Table 3: Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(parameters that are set before simulations)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Par.</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.592</td>
<td>production function</td>
<td>GMM estimate (Table 2)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.848</td>
<td>persistance of $z$ process</td>
<td>GMM estimate (Table 2)</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>1.291</td>
<td>std. dev. of shocks to $z$</td>
<td>GMM estimate (Table 2)</td>
</tr>
<tr>
<td>$k^\alpha D^*$</td>
<td>47,978</td>
<td>foreign demand level</td>
<td>from GMM estimates (Table 2)</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>3.327</td>
<td>iceberg trade costs</td>
<td>from GMM estimates (Table 2)</td>
</tr>
<tr>
<td>$l_e$</td>
<td>1</td>
<td>initial size of entering firm</td>
<td>assumed (smallest possible)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>bargaining power</td>
<td>assumed (literature)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>5</td>
<td>elasticity of subs.</td>
<td>assumed (literature)</td>
</tr>
<tr>
<td>$r$</td>
<td>0.15</td>
<td>discount rate</td>
<td>Bond, et al (2008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4</td>
<td>share, $Q$ goods in utility</td>
<td>World Bank (2005)</td>
</tr>
</tbody>
</table>

Table 3 collects the parameters discussed thus far, and implies that 8 parameters remain to be determined: the fixed cost of operation $c_f$, the vacancy posting cost parameter $c_h$, the

¹⁶In order to find the expenditure share on tradables we sum the expenditure shares for food and nonalcoholic beverages, alcoholic beverages and tobacco, clothing and footwear. We also added half of furnishing, household equipment and maintenance and half of other consumption items as tradable. This leaves housing, water, electricity, health, transportation, communication, recreation and culture, education, restaurants and hotels as non-tradable items.

¹⁷Although newly created firms start at the lowest possible employment level, by the time they begin production they will have adjusted their initial labor force, and their observed employment will reflect their initial productivity.
fixed cost of exporting $c_x$, the value of home production $b$, the value of entry $c_e$, and the parameters of hiring cost function, $\lambda_1$ and $\lambda_2$. The value of entry is determined endogenously in the model to satisfy the free entry condition, as in Hopenhayn and Rogerson (1993). We estimate $c_f$, $c_h$, $c_x$, $b$, $\lambda_1$ and $\lambda_2$ by a method of moments. We select twelve targets that summarize key features of our model: the firm exit rate, the job turnover rate, the fraction of firms that export, the unemployment rate, the autocorrelation of firms employment levels, correlation between firms productivity and employment and the employment growth rates among expanding firms at the different quintiles of the size distribution.

While it is not possible to associate individual parameters with individual statistics, experiments do suggest that particular statistics play relatively key roles in identifying particular parameters. The quintile specific job growth rates and the aggregate labor turnover rate are responsive to the parameters of the vacancy cost function: $c_h$, $\lambda_1$ and $\lambda_2$ with cross-quintile differences governed by the scale economies parameter, $\lambda_2$. The unemployment rate is very responsive to the productivity of unemployed workers, $b$. And the rate of firm turnover is very responsive to the per-period fixed costs of operating a business, $c_x$.

The firm exit rate and the fraction of firms that exit are calculated from Colombian plant level data for the pre-liberalization period, 1978-91. The quintile-specific rates of job creation and the statistics $corr(l, l')$ and $corr(z, l)$ are based on the same data base, using the technology estimates in Table 2 to calculate $z$. The job turnover rate is calculated from Inter-American Development Bank Job Flows Data Set for 1978-1992 period. The unemployment rate is taken from Inter-American Development Bank (2004), and is based on DANE’s biennial household surveys.
Table 4 shows the data-based statistics and their model-based simulated counterparts. Note that we have (somewhat arbitrarily) divided the correlation statistics by 10 in order to keep all statistics a similar order of magnitude, and thereby prevent the correlation statistics from dominating the calibration exercise. Although we are using 6 parameters to try to match 12 statistics, the does a nice job of fitting the data overall.\textsuperscript{18} In particular, the model captures the contributions of firm entry/exit and intra-firm size adjustments to overall job turnover, the persistence in employment levels, the overall unemployment rate, and the general tendency for expanding firms to add workers at a more rapid rate when they are large.\textsuperscript{19}

<table>
<thead>
<tr>
<th>Industry-wide Statistics</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>exit rate</td>
<td>0.091</td>
<td>0.083</td>
</tr>
<tr>
<td>job turnover</td>
<td>0.211</td>
<td>0.196</td>
</tr>
<tr>
<td>export rate</td>
<td>0.120</td>
<td>0.128</td>
</tr>
<tr>
<td>Unempl.</td>
<td>0.086</td>
<td>0.107</td>
</tr>
<tr>
<td>$corr(l, l')/10$</td>
<td>0.095</td>
<td>0.084</td>
</tr>
<tr>
<td>$corr(z, l')/10$</td>
<td>0.059</td>
<td>0.068</td>
</tr>
<tr>
<td>$corr(z, l)/10$</td>
<td>0.057</td>
<td>0.077</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Employment Growth Rates, by Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>&lt;20th percentile</td>
</tr>
<tr>
<td>20th-40th percentile</td>
</tr>
<tr>
<td>40th-60th percentile</td>
</tr>
<tr>
<td>60th-80th percentile</td>
</tr>
<tr>
<td>&gt;80th percentile</td>
</tr>
</tbody>
</table>

*Figures to be updated*

Table 5 reports the parameter values associated with the calibration. Except for $\lambda_1$ and $\lambda_2$, parameters are measured in terms of the 1990 average annual wage for a service sector worker, taken from the annual household survey, which amounted to roughly $1,300 US (1977), or about $4,500 current US dollars. Accordingly, our model implies that the fixed costs of

\textsuperscript{18}The metric of fit we used was $\|X - Y\| / \|X\|$ where $X$ is the vector of data-based statistics and $Y$ is the vector of the model-based counterparts. At its minimized value, this metric was 0.095.

\textsuperscript{19}This feature of the Colombian data might seem at odds with the well-known finding that job turnover is higher among small firms. The apparent reason is that these statistics do not include turnover due to entry and exit, which occurs overwhelmingly among small firms. Once we include entry and exit in our statistics, both the data and our model replicate the standard finding.
operating a business amount to about $107,000, while the fixed costs of exporting are only about $16,000. Note also that those who end up doing home production take about an 80 percent wage cut relative to what they could have earned if they had committed to working for a service sector firm. Finally, the parameters of the vacancy cost function imply both short-run convexities ($\lambda_1 = 1.50$) and modest scale economies ($\lambda_2 = 0.18$). The latter, of course, is a reflection of the quintile-specific growth rates reported in Table 4 and the (mean-reverting) productivity process reported in Table 2. That is, if there were no scale effects, mean reversion would imply slightly smaller growth rates among the largest firms that are posting vacancies, and slightly higher growth rates among the smallest firms that are posting vacancies.

<table>
<thead>
<tr>
<th>Par.</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f$</td>
<td>23.77</td>
<td>fixed cost of oper.</td>
</tr>
<tr>
<td>$c_h$</td>
<td>4.18</td>
<td>posting cost scalar</td>
</tr>
<tr>
<td>$c_x$</td>
<td>3.66</td>
<td>fixed exporting cost</td>
</tr>
<tr>
<td>$b$</td>
<td>0.21</td>
<td>value of hhd. prod.</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1.52</td>
<td>convexity, $c(l, l')$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.18</td>
<td>scale effect, $c(l, l')$</td>
</tr>
</tbody>
</table>

* Figures to be updated

5.4 Simulated effects of openness: small country case

We are now prepared to examine the effects of trade reforms in our calibrated model. To do so we reduce the iceberg trade costs from 1.813 to 1.65. This reduction generates an increase in the fraction of firms that export from 12.2 percent to 21.2 percent, which is in line with Colombian liberalization experience.

Table 6 shows how key labor market statistics change with the trade reforms. Note first
that trade liberalization increases job turnover by 1.9 percentage points, which is about half of the increase in the data. Thus, while our model does not explain the entire 4.5 percentage point increase that Colombia experienced, it accounts for a significant fraction of the change. That is, increased sensitivity to productivity shocks among exporters appears to be a significant phenomenon.

Table 6: The Effects of Trade Reform – Labor Markets

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\tau_m = \tau_n = \text{Difference (model)}$</th>
<th>Difference (data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Firms that Export</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job Turnover</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment, or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informal self-employment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90-10 Wage Inequality</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In a sense, we have stacked the deck against finding large turnover effects. Our model filters all shocks to the prices of imported goods through a general price index, and thus does not allow idiosyncratic shocks to foreign suppliers’ prices to affect different Colombian firms differently. If we were to use a nested demand system in which different domestic producers compete with different foreign exporters, the effects of openness on job turnover would be magnified.\(^{20}\) And since all producers compete with imports while only 10 to 20 percent export, the effects could be dramatic.

Note next that openness also causes more wage inequality in our model. This is not a skill-premium effect; rather it reflects the fact that exporters who experience positive productivity shocks enjoy relatively large rents when trade costs are low. Thus expanding exporters pay larger wage premiums to their workers than they would have in a more closed economy. The left-hand tail of the wage distribution is also stretched by openness, but for a more

\(^{20}\)Atkeson and Burstein (forthcoming) provide an example of this type demand specification.
subtle reason. Large and productive firms get occasional negative shocks, so they shed labor. However, the option value of remaining in such a firm is higher in an open environment because if the firm experiences a positive shock, it will rehire at higher wages. So workers who stay accept lower wages. Unlike with our turnover results, the model over-predicts by a modest amount the extra wage dispersion caused by Colombia’s trade liberalization.

Despite the higher job turnover and greater wage inequality that our model predicts, it does not suggest that aggregate unemployment rates should respond much to openness. If one equates self-employment in our model with unemployment in the official statistics, this is consonant with Colombian (and Latin American) experiences. On the other hand, if one equates self-employment with working alone in the informal sector, our model under-predicts responses to globalization in this respect (refer to Table 1).

Finally, it is interesting to examine the welfare effects of opening. Aggregating across consumers, we find that Colombia’s tariff reductions increased welfare by 1.5 percent. But
different types of workers fared differently. Figure 1 depicts the change in the value of being a worker at firms located throughout \((z, l)\) space, going from \(\tau = 1.65\) to \(\tau = 1.813\). Not surprisingly, workers fortunate enough to be employed by high productivity firms do better in the relatively open environment. This is particularly true at the largest productive firms, which have the most rents to share with their workers.

6 Summary

In Latin America and elsewhere, globalization has been associated with less job security, more wage inequality, and more informality. We have formulated a dynamic structural model that explains these association as a consequence of heightened firm responsiveness to idiosyncratic productivity shocks, and we have shown that this mechanism alone could account for a substantial fraction of the heightened volatility and wage dispersion in Colombia.

In addition to providing a lens through which to interpret recently-observed changes in Latin American labor markets, our paper makes several methodological contributions. First, we have generalized the representation of labor markets developed by Bertola and Caballero (1994) to an open economy setting with fully articulated product markets, multiple sectors, and arbitrary Markov processes for productivity shocks. Second, we have demonstrated how to quantify some welfare and distributional effects of openness postulated by Rodrik (1999).\textsuperscript{21} Finally, we have developed a means to characterize plant-level productivity processes that does not require us to observe a measure of physical output, matches a large set of stylized facts, and is robust with respect to simultaneity bias and selection bias.

\textsuperscript{21}Previous attempts to examine Rodrik’s conjecture empirically have amounted to tests for structural shifts in the elasticity of demand for labor, pre- versus post-globalization.
Appendix 1: The Wage Functions

Hiring Wages

In order to characterize wages in hiring firms, we first determine the total surplus for a firm and a worker that are matched in the end-of-period state \((z', l')\). At the time of bargaining, the surplus that the marginal worker generates for a firm is given by

\[
\Pi^{\text{firm}}(z', l') = \frac{1}{1 + r} \left[ \frac{\partial \pi(z', l')}{\partial l'} + \frac{\partial \psi(z', l')}{\partial l'} \right].
\]

Note that at the time of bargaining, the vacancy posting and matching process are over and the costs of vacancy postings are sunk. As a result, if the bargaining fails, the firm is simply left with less workers. The surplus that a marginal worker generates consists of two parts: the current increase in the firms’ profits, i.e. marginal revenue product net of wages, and the increment to the value of being in state \((z', l')\) at the start of the next period. If the firm does not exit next period, i.e. if \(\psi(z', l') > 0\), the marginal worker will have a positive. As exit and firing are costless, if the firm exits or fires workers, his expected marginal value from its current marginal hire will be zero.

Similarly, the surplus for the marginal worker who is matched by a hiring firm in the end-of-period state \((z', l')\) is

\[
\Pi^{\text{work}}(z', l') = \frac{1}{1 + r} \left[ w_h(z', l') + J^e(z', l') \right] - \frac{b + J^o}{1 + r},
\]

where the worker enjoys \(w_h(z', l')\) in the current period, and starts next period in a firm with the beginning-of-period state \((z', l')\). Since at the time of bargaining the vacancy posting and matching process are over, if the bargaining fails, the worker is unemployed this period and starts next period in state \(o\).
The worker and firm split the total surplus by Nash bargaining where the bargaining power of the firm is given by $\beta$, i.e.

$$\beta \Pi^{\text{firm}}(z', l') = (1 - \beta) \Pi^{\text{wkr}}(z', l')$$

Wages are thus determined as a solution to the following equation

$$\beta \left[ \frac{\partial \pi(z', l')}{\partial l'} + \frac{\partial \nu(z', l')}{\partial l'} \right] = (1 - \beta) \left[ w_h(z', l') + J^e(z', l') - (b + J^o) \right]. \quad (29)$$

Adding and subtracting $(1 - \beta) \frac{b + J^o}{1 + r}$ on the right hand side of (29) gives

$$\beta \left[ \frac{\partial \pi(z', l')}{\partial l'} + \frac{\partial \nu(z', l')}{\partial l'} \right] = (1 - \beta) \left\{ w_h(z', l') - r \left( \frac{b + J^o}{1 + r} \right) + \left[ J^e(z', l') - \frac{b + J^o}{1 + r} \right] \right\} \quad \text{(30)}$$

where $r \left( \frac{b + J^o}{1 + r} \right)$ is the flow value of being unemployed and $(J^e(z', l') - \frac{b + J^o}{1 + r})$ is the expected continuation value of employment at a $(z', l')$-type firm net of the continuation value of unemployment. Weighted by $(1 - \beta)$, this latter term cancels with $\beta \frac{\partial \nu(z', l')}{\partial l'}$, which appears on the left hand side of equation (29), since the worker gets the fraction $1 - \beta$ of any future rents from the match while the firm gets $\beta$. Thus equation (30) becomes

$$\frac{\partial w_h(z', l')}{\partial l'} \beta' + w_h(z', l') - \beta \frac{\partial r(z', l')}{\partial l'} - (1 - \beta) r \left( \frac{b + J^o}{1 + r} \right) = 0,$$

which is the same as Bertola and Garibaldi (2001)’s equation (10).

Using

$$\frac{\partial r(z', l')}{\partial l'} = \frac{\alpha - 1}{\sigma} \frac{1}{D^\frac{1}{\sigma}} (z')^{\frac{\sigma - 1}{\sigma}} (l')^{\sigma \alpha (\frac{\sigma - 1}{\sigma}) - 1},$$

33
the wage schedule for expanding firms is given by

\[ w_h(z', l') = (1 - \beta) r \left( \frac{b + J^o}{1 + r} \right) + \Gamma(\alpha, \beta, \sigma) \frac{\sigma + 1}{\sigma} (l')^{-[\frac{\sigma + 1}{\sigma} - \frac{\sigma}{\sigma} - 1]}, \]

where \( \Gamma(\alpha, \beta, \sigma) \) is a function of the parameters of the problem

\[ \Gamma(\alpha, \beta, \sigma) = \frac{\alpha \beta (\sigma - 1)}{\sigma (1 - \beta) + \alpha \beta (\sigma - 1)}. \]

**Firing Wages**

To derive the firing wage schedule, we begin by writing the value of employment at a firing firm in the interim stage as

\[ J^f(z', l) = \frac{1}{1 + r} \left[ p_f(z', l)(1 + r)J^u + (1 - p_f(z', l)) \left( w_f(z', l') + J^e(z', l') \right) \right], \]

where \( l' = L(z', l) \). This expression reflects the possibility of losing one’s job, \( p_f(z', l) \), which we assume occurs at firing firms with probability

\[ p_f(z', l) = \frac{l - L(z', l)}{l}. \]

It also reflects the fact that workers who are not fired are paid just enough to retain them. Next we note that, since workers are indifferent between staying and leaving

\[ w_f(z', l') + J^e(z', l') = (1 + r)J^u, \]

and the wage schedule faced by firing firms can be written as

\[ w_f(z', l') = rJ^u - [J^e(z', l') - J^u]. \]

Note that as a hiring firm increases its employment level toward the point at which \( \Pi^{firm}(z', l') = 0 \), the hiring wage approaches \( w_f(z', l') \) by (29).
Appendix 2: Steady State Equilibrium

A steady state equilibrium consists of a measure of differentiated goods $N$, an exact price index for composite good $P$, an aggregate quantity index for composite good $Q$, aggregate income $I$, a measure of workforce in services $L_s$, a measure of unemployed workers in differentiated goods sector $L_u$, unemployment rate in differentiated goods sector $\mu_u$, job finding rate $\phi_u$, vacancy filling rate $\phi_f$, the exit rate $\mu_x$, the measure of entrants $M$; the value functions and associated policy functions $V(z, l), L(z, l), T^h(z, l), T^c(z-1, l), J^o, J^u, J^s, J^e$; the wage schedules $w_h(z, l)$ and $w_f(z, l)$, and end-of-the period and interim distributions $f(z, l)$ and $\tilde{f}(z, l)$ such that

1. **Steady State Distributions:** Because of the transitions that occur within a period, we have to distinguish the distributions at different points in time. Let $f(z, l)$ and $\tilde{f}(z, l)$ be the stationary probability distributions over $(z, l)$ at the end and interim states, respectively. In equilibrium, these distributions reproduce themselves through the Markov processes on $z$, the policy functions and the productivity draws upon entry.

   The interim distribution is defined as
   
   $$\tilde{f}(z, l) = \begin{cases} 
   \int h(z | \tilde{z}) f(\tilde{z}, l) I^c(\tilde{z}, l) d\tilde{z} & \text{if } l \neq l_e \\
   f_e(z) & \text{if } l = l_e 
   \end{cases}.$$ 

   In turn, the end-of-the period distribution is
   
   $$f(z, l') = \int \tilde{f}(z, \tilde{l}) I_{(L(z, \tilde{l}), l)},$$

   where $I_{(L(z, \tilde{l}), l)}$ is an indicator function with $I_{(L(z, \tilde{l}), l)} = 1$ if $L(z, \tilde{l}) = l$.

2. **Market Clearance in Sector $S$:** Demand for the $S$–sector comes from two sources: consumers spend a $(1 - \gamma)$ fraction of aggregate income $I$ on it, and firms demand it to
pay their fixed operation costs, labor adjustment and entry costs. Define the fraction of hiring firms as 
\[ \mu_h = \int_z \int_l l^h(z,l)\tilde{f}(z,l)dldz. \]
Average labor adjustment cost is given by
\[ \tilde{c} = \int_z \int_l C(l, L(z,l)l^h(z,l)\frac{\tilde{f}(z,l)}{\mu_h}dldz. \]

Market clearance condition in this sector is
\[ L_S + b\mu_u L_Q = (1 - \gamma)I + N(\tilde{c} + c_f) + Mc_e, \]
where \( L_S \) and \( L_Q \) are the size of the workforce in the two sectors, and \( \mu_u \) is the unemployment level within the \( Q \)-sector.

3. Labor Market: With a normalized measure of workers, the size of the workforce in the \( Q \)-sector is \( L_Q = 1 - L_S \). Total production employment in the differentiated good sector is given by
\[ E_Q = N \int_z \int_l f(z,l)dldz = (1 - \mu_u)L_Q. \]
The measure of unemployed workers is then
\[ L_u = L_Q - E_Q = \mu_u L_Q. \]
The equilibrium condition for the labor market in the \( Q \)-sector is that flows out of employment equal the flows into employment. Every period, a fraction \( k \) of workers in that sector are laid off due to exits and downsizing. The equilibrium flow condition is
\[ \mu_u L_Q\phi^w = (1 - \mu_u)L_Qk; \]
which yields the usual Beveridge curve
\[ \mu_u = \frac{k}{k + \phi^w}. \]
Aggregate number of vacancies in this economy is

\[ V = N \int \int v(z, l) \tilde{I}^h(z, l) \frac{\tilde{I}(z, l)}{\mu_h} dldz. \]

which, together with \( L_u \), determines matching probabilities \( \phi^f(V, L_u) \) and \( \phi^w(V, L_u) \) that firms and workers take as given.

4. **Firm turnover**: In equilibrium, there is a positive mass of entry \( M \) every period so that the free entry condition (13) holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,

\[ \mu_{exit} = \int \int \left[ 1 - \mathcal{I}^e(z, l) \right] \mathcal{F}(z, l) dldz, \]

and measure of exits equals that of entrants,

\[ M = \mu_{exit} N. \]

5. **Income and Market Clearance for the \( Q \)-sector**: The composite good \( Q \) and its price are given by:

\[ P = \left( N \int \int p(z, l)^{1-\sigma} f(z, l) dldz \right)^{\frac{1}{1-\sigma}}, \]

and

\[ Q = \left( N \int \int q(z, l) \frac{\sigma-1}{\sigma} f(z, l) dldz \right)^{\frac{\sigma}{\sigma-1}}. \]

Aggregating over the revenue functions (7) across producers, total revenues earned by the differentiated good sector is a fraction \( \gamma \) of total income in the economy:

\[ \gamma I = PQ = R. \]
By Walras’ Law, market clearance in the labor market and the $S$-sector implies the clearance of the $Q$-sector. We show that by writing aggregate income in the closed economy:

$$I = L_S + b\mu_uL_Q + W_Q + \Pi,$$

(31)

where $L_S$ is employment (and income earned) in the $S$-sector and $\mu_uL_Qb$ is the income earned by the unemployed through home production. Let $\tilde{T}^h(z, l)$ be an indicator function which equals one if a firm in state $(z, l)$ at the end of a period achieved this state by hiring in the interim. $\Pi$ is total profits net of entry, vacancy and firing costs,

$$\Pi = N \int_z \int_l \left[ \tilde{T}^h(z, l) \{ r(z, l) - w_h(z, l)l \} \right. \\
+ \left[ 1 - \tilde{T}^h(z, l) \right] \{ r(z, l) - w_f(z, l)l \} f(z, l)dldz \\
- N\tilde{c} - Nc_f - Mc_e$$

(32)

and $W_Q$ is the total wage bill in the $Q$-sector

$$W_Q = N \int_z \int_l \left\{ \tilde{T}^h(z, l)w_h(z, l) \cdot l + \left[ 1 - \tilde{T}^h(z, l) \right] w_f(z, l)l \right\} f(z, l)dldz.$$ 

(33)

Using (31), (32) and (33),

$$\gamma I = N \int_z \int_l \left[ \tilde{T}^h(z, l)r(z, l) + \left[ 1 - \tilde{T}^h(z, l) \right] r(z, l) \right] f(z, l)dldz.$$

Right-hand of that equation is total revenue $R$ earned by the differentiated good sector.

6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching a job in industrial sector.

$$J^o = J^s = J^u.$$ 

(34)
Appendix 3: Open Economy Equilibrium

Equilibrium conditions

The equilibrium definition in an open economy is similar to that in a closed economy with the addition of an export policy function \( T_{q>0}(z,l) \), the exchange rate \( \varepsilon_x \), the fraction of firms exporting \( \mu_x \) and the trade balance condition. Here we show that when all markets clear, trade balance condition follows from Walras’ Law.

By definition of income as before,

\[
I = L_s + b(uL_Q) + W_Q + \Pi + T_m
\]

\[
I = L_s + b(uL_Q) + W_Q + R_x + R_d - W_Q - Nc - Nc_f - Mc_e + T_m
\]

where \( Nc, Nc_f \) and \( Mc_e \) are aggregate hiring costs, overhead and entry costs respectively.

Market clearance for \( S \) sector is

\[
L_s + b(uL_Q) = (1 - \gamma) I + Nc + Nc_f + Mc_e
\]

which implies

\[
\gamma I = R_x + R_d + T_m.
\]

On the expenditure side, a fraction \( \gamma \) of income is spent on differentiated goods, foreign and domestic.

\[
\gamma I = E_m + E_d.
\]

By domestic market clearance, \( E_d = E_m \) which implies

\[
R_x + T_m = E_m.
\]
Payment to foreigners is given by \( R_m = \frac{E_m}{(1 + \tau_m)} \). Substituting \( E_m \) and cancelling tariff revenues \( T_m = R_m \tau_m \) leaves us with the trade balance condition:

\[ R_m = R_x. \]

**Price and quantity indices**

To re-state the price and quantity indices for the open economy case, let \( N \) now denote total varieties sold in each country, let \( N_D \) denote the number of varieties that each country produces domestically, and let \( \mu_x \) be the equilibrium fraction of firms in each country that export. (By symmetry, \( N = (1 + \mu_x)N_D. \) Then, the composite good \( Q \) is the following weighted average of foreign and domestic quantities:

\[ Q = N^\frac{\alpha}{\sigma - 1} \left( \frac{\mu_x}{1 + \mu_x} Q_F^{\frac{\sigma - 1}{\sigma}} + \frac{1}{1 + \mu_x} Q_D^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}, \]

where

\[ Q_F = \left( \int \int_1^{\infty} \left[ \frac{\eta^*}{\tau} z \right]^{\sigma - 1} \frac{\tau}{\mu_x} T_{\tau > 0}(z, l) f(z, l) dldz \right)^\frac{\sigma}{\sigma - 1}, \]

and

\[ Q_D = \left( \mu_x Q_{D,x}^{\frac{\sigma - 1}{\sigma}} + (1 - \mu_x)Q_{D,nx}^{\frac{\sigma - 1}{\sigma}} \right)^\frac{\sigma}{\sigma - 1}. \]

Also the domestic index \( Q_D \) is itself an aggregate across domestic sales of exporters given by \( Q_{D,x} \) and sales of domestic only producers represented by \( Q_{D,nx} \):

\[ Q_{D,x} = \left( \int \int_1^{\infty} [(1 - \eta^*) z^{l^\alpha}]^{\sigma - 1} \frac{\tau}{\mu_x} T^{x}(z, l) f(z, l) dldz \right)^\frac{\sigma}{\sigma - 1}, \]

and

\[ Q_{D,nx} = \left( \int \int_1^{\infty} (z^{l^\alpha})^{\sigma - 1} \left[ 1 - T^{x}(z, l) \right] \frac{f(z, l) dldz}{1 - \mu_x} \right)^\frac{\sigma}{\sigma - 1}. \]
The price index for $Q$ follows from a similar aggregation. The export indicator function $x(z, l)$ and the fraction of exporting firms $\mu_x$ are additions to the equilibrium definition in the open economy case. All conditions in the closed economy equilibrium are valid with the additional demand for the homogenous good resulting from fixed exporting costs $\mu_x N_D f_x$ and the modified aggregate profit function to account for export revenues and costs.
Appendix 4: Estimating the Revenue Function and Productivity Process

The Revenue Function

The equation we wish to estimate is:

\[ \ln r_{it} = \ln r_{it-1} + (d_H + I_{it}^x \cdot d_F) - \rho (d_H + I_{it-1}^x \cdot d_F) \]

\[ + \alpha \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it} - \alpha \rho \left( \frac{\sigma - 1}{\sigma} \right) \ln l_{it-1} + \left( \frac{\sigma - 1}{\sigma} \right) \epsilon_{it}. \]  

(A3.1)

But selection bias and simultaneity bias prevent us from consistently estimating this expression with ordinary least squares. The former problem occurs because firms choose whether to shut down partly on the basis of their \( \epsilon_{it} \) realizations, and the latter problem occurs because firms’ current exporting decisions (\( I_{it}^x \)) and employment levels (\( l_{it} \)) depend upon their current productivity levels.

Selection Bias and Identification

To deal with these problems, let \( I_{it}^c \) be an indicator variable that takes a value of 1 if the \( i^{th} \) firm continues to operate in period \( t \), and 0 otherwise. Then, defining \( \xi_{it} = \epsilon_{it} - E [\epsilon_{it}|I_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, I_{it-1}^x] \), the revenue function can be re-formulated as:

\[ \ln r_{it} = \rho \ln r_{it-1} + d_H (1 - \rho) + d_F (I_{it}^x - \rho \cdot I_{it-1}^x) + \alpha \frac{\sigma - 1}{\sigma} \ln \ell_{it} \]

\[ - \alpha \rho \frac{\sigma - 1}{\sigma} \ln \ell_{it-1} + \frac{\sigma - 1}{\sigma} E [\epsilon_{it}|I_{it}^c = 1, \ldots] + \frac{\sigma - 1}{\sigma} \xi_{it}, \]  

(A3.2)

where the error term \( \xi_{it} \) has zero mean and is orthogonal to \( \ln r_{it-1}, \ln \ell_{it-1}, I_{it-1}^x, \) and \( E [\epsilon_{it}|I_{it}^c = 1, \ldots] \). Also, although it is correlated with current exporting decisions (\( I_{it}^x \)), \( \xi_{it} \)
is orthogonal to \( E \left[ I_{it}^x | I_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, I_{it-1}^x \right] \). These implications of our model can be used as the basis for a generalized method of moments (GMM) estimator that identifies the parameters of equation (A3.1). And the efficiency of this estimator can be improved by exploiting the moment condition \( E \left( I_{it}^x (1 - e^{-d_F}) - x_{it} \right) = 0 \), where \( I_{it}^x (1 - e^{-d_F}) \) is the share of exports in total sales implied by our model and \( x_{it} \) is the observed ratio of export revenues to total sales (which we treat as a noisy measure of true export intensity).

This estimation strategy requires that we calculate \( E \left[ I_{it}^x | I_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, I_{it-1}^x \right] \). To this end, recall that there is a threshold productivity level above which all firms with beginning-of-period employment level \( \ell_{it-1} \) will continue operating. Denoting this threshold productivity level \( g^*(\ell_{it-1}) \), the continuation condition is \( \ln z_{it} = \rho \ln z_{it-1} + \epsilon_{it} > g^*(\ell_{it-1}) \). Or, since \( \ln z_{it-1} = \frac{\sigma}{\sigma-1} \left[ \ln r_{it-1} - (d_H + T_{it-1}^x d_F) \right] - \alpha \ln l_{it-1} \) (by equation 26), continuation occurs when \( \frac{\epsilon_{it}}{\sigma} > g^*(\ell_{it-1}) - \rho \ln z_{it-1} \) \( \overset{def}{=} g(r_{it-1}, l_{it-1}, T_{it-1}^x) \), and the probability of continuation can be calculated as

\[
p_{it}^C = 1 - \Phi \left[ g(r_{it-1}, l_{it-1}, T_{it-1}^x) \right], \tag{A3.3}
\]

where \( \epsilon_{it} \sim N(0, \sigma^2) \) and \( \Phi() \) is the standard normal cumulative distribution. Treating \( g(\cdot) \) as a flexible function of its arguments, it follows that \( p_{it}^C \) values can be imputed from estimates of the probit function (A3.3), and the object of interest can be calculated using well-known properties of the normal distribution (e.g., Maddala, 1983):²³

---

²²Identification further requires that these conditional expectations be non-linear functions of their arguments and/or that they condition on additional arguments that do not appear in equation (A3.2). Note that the dependence of \( \ln \ell_{it} \) on \( \epsilon_{it} \) does not prevent us from obtaining consistent estimates of these parameters because the coefficient on \( \ln \ell_{it} \) can be inferred from the coefficients on \( \ln r_{it-1} \) and \( \ln r_{it-1} \).

²³When estimating this probit, we use a flexible (translog) functional form for \( g(r_{it-1}, l_{it-1}, l_{it-1}^x) \).
\[ E \left[ \epsilon_{it} \mid T_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^c \right] = \sigma_{\epsilon} \cdot M_{it}, \]

\[ \text{var} \left[ \epsilon_{it} \mid T_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^c \right] = \sigma_{\epsilon}^2 \cdot \left( 1 - M_{it} \left[ M_{it} - \Phi^{-1}(p_{it}^c) \right] \right), \]

where \( M_{it} = \frac{\phi(\Phi^{-1}(p_{it}^c))}{p_{it}^c} \) is the relevant Mills ratio and \( \phi() = \Phi'(\cdot) \).

Our estimation strategy also requires that we calculate \( E \left[ I_{it} x_{it}^j I_{it} = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x \right] \).

For this, note that equation (4) implies firms above some threshold productivity level will choose to export, given \((l_{it-1}, z_{it-1})\). Thus, once again exploiting the normality of \( \epsilon_{it} \), we can write

\[ E \left[ T_{it}^x \mid T_{it}^c = 1, \ln r_{it-1}, \ln \ell_{it-1}, T_{it-1}^x \right] = p_{it}^X = 1 - \Phi \left[ h(\ln r_{it-1}, \ln l_{it-1}, T_{it-1}^x) \right], \]  \hspace{1cm} (A3.4)

where \( p_{it}^X \) is the probability that firm \( i \) exports in period \( t \) and \( h(r_{it-1}, l_{it-1}, T_{it-1}^x) \) is a flexible function of its arguments.\(^{24} \)

Hence \( E \left[ T_{it}^x \mid T_{it}^c = 1, \ldots \right] \) can be calculated by estimating the probit (A3.4) and retrieving the imputed \( p_{it}^X \) values. Clearly, identification here comes from the non-linear form of the probit function.\(^{25} \)

### The Moment Conditions

To summarize, our GMM estimator is based on the moment conditions:

\[ E[\xi_{it} \ln r_{it-1}] = 0, \quad E[\xi_{it} \ln \ell_{it-1}] = 0, \quad E[\xi_{it} M_{it}] = 0, \quad E[\xi_{it} T_{it-1}^x] = 0, \]

\[ E[\xi_{it} p_{it}^X] = 0, \quad E[\xi_{it}] = 0, \quad E[\nu_{it}^x] = 0, \quad E[\nu_{it}^c] = 0. \]

\(^{24}\)It is interesting that lagged exports help predict current exports here, even though we have assumed away sunk entry costs. The reason is that, by (26), lagged exports help to explain lagged productivity.

\(^{25}\)Olley and Pakes (1996) develop a related strategy that posits a deterministic linkage between productivity shocks and investment levels. This allows them to get away from functional form as a basis for identification, but it is not an available option in the present setting.
where:

\[
\begin{align*}
\xi_{it} &= \frac{\sigma}{\sigma - 1} \left[ \ln r_{it} - d_H (1 - \rho) - d_F (I^x_{it} - \rho I^x_{it-1}) - \rho \ln r_{it-1} \right] + \alpha \rho \ln \ell_{it-1} - \alpha \ln \ell_{it} - \sigma \cdot M_{it}, \\
\nu^\ell_{it} &= \xi_{it}^2 - \sigma^2 \cdot (1 - M_{it} \left[ M_{it} - \Phi^{-1}(p_{it}) \right]), \\
\nu^{x^2}_{it} &= I^x_{it} (1 - e^{-d_X}) - x_{it}.
\end{align*}
\]

In principle, these conditions identify \( \rho, \alpha, \sigma^2, d_X, d_H, \frac{\sigma - 1}{\sigma} \). In practice, while \( \rho, \alpha, \sigma^2, d_X, \) and \( d_H \) can be estimated with some precision using this estimator, \( \frac{\sigma - 1}{\sigma} \) is poorly identified. We therefore fix \( \frac{\sigma - 1}{\sigma} \) at several alternative values taken from the existing literature, and generate corresponding sets of estimates for the remaining parameters. (Refer to Table 1 in the text.)

Our results proved not to be sensitive to the inclusion of time dummies in A1.1. Accordingly, since our theoretical model presumes that the macro environment is stable, we focus our attention on the case in which they are omitted.
Appendix 5: Numerical Solution Algorithm

In order to solve the model numerically, we discretize the productivity space using the method suggested by Tauchen (1986) and uniformly discretize the employment space between $[l_e, L]$. We make sure that the arbitrarily imposed upper bound $L$ is not binding by checking firms’ hiring policy functions. The following steps, where consecutively numbered items are loops nested in each other, describe the solution algorithm.

1. **Bisection over job filling probability $\phi_f$**: Take a lower bound $\underline{\phi}_f$ and an upper bound $\overline{\phi}_f$ in $[0, 1]$. Let $\phi_f = (\underline{\phi}_f + \overline{\phi}_f)/2$. When the nested loops are solved, we obtain a value for $EJ^*_h$, and we can calculate the implied $\phi_w$ using the definition of the matching function. Calculate the value of unemployment, $J^u$, by (15). Update either $\overline{\phi}_f$ or $\underline{\phi}_f$ depending on $J^u \leq J^o$. Iterate until workers’ indifference condition for sectoral choice holds approximately, that is, until $J^u$ is sufficiently close to $J^o$.

2. **Bisection over share of output exported $\eta$**: Taking $\phi_f$ as given, take a lower bound $\underline{\eta}$ and an upper bound $\overline{\eta}$ in $[0, 1]$. Let $\eta = (\underline{\eta} + \overline{\eta})/2$. Iterate until trade balance holds approximately.

3. **Iteration over firing wage schedule $w_f(z, l)$**: Start with some initial wage schedule $w^i_f(z, l)$. Taking $(\phi_f, \eta)$ as given, the nested loops solve for the value of employment in a firm, $J^o(z, l)$. Update the wage schedule $w^{i+1}_f(z, l)$ using expression (21), imposing the equilibrium condition $J^u = 1/r$. Iterate until $w^{i+1}_f(z, l)$ is sufficiently close to $w^i_f(z, l)$.

4. **Bisection over demand level $D_H$**: Start with lower and upper bounds $(D_H, \overline{D_H})$ and let
\[ D_H = \left( \frac{D_H + \overline{D}_H}{2} \right). \] Solve for firms' problem to get policy functions for exit, hiring and exporting. Find the value of entry, \( \mathcal{V}_e \), and update either \( D_H \) or \( \overline{D}_H \) depending on \( \mathcal{V}_e \leq c_e \). Iterate until free entry condition holds approximately such that \( \mathcal{V}_e \) is sufficiently close to the cost of entry, \( c_e \).

To find the value of matching with a hiring firm conditional on the event of matching occurring, \( EJ^e_h \), we first derive the steady state distribution \( f(z, l) \) as the fixed point of a contraction using the policy functions of the firm, the stochastic shocks \( h(z'|z) \) and productivity draws of the entrants. Using the vacancy posting policy of the firm, we then find the job offer distribution \( g(z, l) \) as in the text and obtain \( EJ^e_h \) by (18).

The above algorithm solves the model for a given set of exogenous parameter values, including the cost of entry \( c_e \). When we calibrate the model to obtain estimates of some of these parameters, we treat the set of data moments as one particular general equilibrium outcome. Thus in the modified algorithm employed for calibration, we use the empirical value of \( \eta \). We also take the value of \( D_H \) estimated in the first stage as given and set \( c_e \) such that free entry holds. This enables us to skip loops 2 and 4 in the calibration. When we do policy experiments by varying the parameters related to trade costs, the values of \( D_H \) and \( \eta \) change endogenously, so we use the complete algorithm to solve the model.
References


