Monitoring Synchronization of Regional Recessions: A Markov-Switching Network Approach*

[JOB MARKET PAPER]

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Abstract

This paper develops a new framework for monitoring changes in the degree of synchronization between many stochastic processes subject to regime changes. In the proposed methodology the estimated time-varying dependence relations among the hidden Markov processes governing the system can be interpreted as a dynamic weighted network. Bayesian estimates of an empirical application to the synchronization of business cycle phases in U.S. states suggest that national recessions can be anticipated by an index that accounts for the global synchronization between states, confirming its predictive ability with real-time exercises. Moreover, the way in which an upcoming national recession could simultaneously affect each of its smaller economies at the state level can be dynamically evaluated.

Keywords: Regional Business Cycle, Markov-Switching, Network Analysis.

JEL Classification: E32, R12, C32, C45.

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1 Introduction

The interest in identifying changes in business cycles synchronization started to markedly increase since the implementation of the European Monetary Union, due to more synchronized countries are expected to face smaller costs of joining the Union than those countries with relatively less synchronized cycles, Camacho et al. (2006). In other words, analyzing synchronization changes is crucial for policy makers in order to determine the countries, regions or even sectors of an economy that could be more sensitive to global policies or aggregate economic shocks and the others that could remain less affected by them.

Since the seminal work by Hamilton (1989) in which U.S. business cycle phases are characterized by using a Markov-switching (MS) model, a broad range of extensions related to this approach have been developed due to its great success. In particular, multivariate MS models have become a useful tool in analyzing synchronization between the business cycle phases of different countries (Smith and Summers, 2005 and Camacho and Perez-Quiros, 2006) or regions (Owyang et al., 2005 and Hamilton and Owyang, 2010). Although all these studies provide a picture about how much some business cycles are in sync for a given period of time, they are not able to endogenously capture potential changes in the degree of synchronization. This is because in order to preserve parsimony in the model, a still non explored question that could help to unveil this feature has remained unnoticed: What is the dynamic relationship between the unobserved state variables governing a multivariate MS setting?

The approaches followed in the literature traditionally assume a fixed over time dependence relation between such state variables that can be divided into two categories. There are studies in which such relation is just a priori assumed based on the econometrician's judgment. Apart from the general Markovian specification, that involves the estimation of the full transition probability matrix,[1] multivariate MS models are usually analyzed under three different types of relationships between the unobserved state variables governing

[1] This approach presents computational difficulties as the model increases in the number of series, states or lags.

each time series, see Hamilton and Lin (1996) and Anas et al. (2007). The first one refers to the case in which all series follow a common regime dynamics, Krolzig (1997). Second, the use of totally independent Markov chains, which is the most followed approach, Smith and Summers (2005) and Chauvet and Senyuz (2008). Third, the dynamics in one state variable precede those of other state variables, Hamilton and Perez-Quiros (1996) and Cakmakli et al. (2011).

Other studies focus, on the one hand, in obtaining a degree of synchronization between MS processes for a given sample period, providing *average* dependence relationship estimates, as in the case of Artis et al. (2004) who compute cross-correlations between the smoothed state probabilities after estimating several univariate models. On the other hand, some works focus on testing synchronization by relying on the extreme cases of independence and perfect synchronization. Some examples are Harding and Pagan (2006), who propose tests of the hypotheses that cycles are either unsynchronized or perfectly synchronized under complications caused by serial correlation and heteroscedasticity in cycle states, and Pesaran and Timmermann (2009) who test independence between discrete multicategory variables based on canonical correlations. One intuitive approach followed by Camacho and Perez-Quiros (2006), Bengoechea et al. (2006), and Leiva-Leon (2011), is based on modeling the data generating process as a linear combination between the two polar cases, claiming that in most of real situations, the true MS multivariate dynamics should be somewhere in between them. Although all these previous approaches can be used to study how much synchronized are a set of MS processes for a given period of time, they are not able to identify possible synchronization changes.

This paper proposes a novel framework for monitoring changes in the degree of aggregate synchronization between many stochastic processes that are subject to regime changes. Specifically, it computes regime inferences from a multivariate Markov-switching model and simultaneously obtains a measure of the time-varying synchronization between the unobserved state variables governing each process. Such measure is endogenously estimated as a weighted average between the dependent and independent polar cases by making inference on the regimes of high and low synchronization without requiring *a posteriori* computations as in the case of the concordance in Harding and Pagan (2006). One advantage of this approach in comparison to the conventional dynamic correlation measure, is that pairwise synchronizations, which are estimated through Bayesian methods, can be easily converted into desynchronization measures to be combined with dynamic multidimensional scaling, and network analysis in order to provide assessments regarding to possible changes in the clustering patterns that could experiment a system of time series, the key components leading the system, and even to make inference about its future behavior.

The proposed framework is used as a tool for monitoring the time-varying synchronization among U.S. states business cycle phases in order to assess, first, up to which extend the interconnectedness between smaller economies can be helpful to anticipate national recessions? and second, how an upcoming national recession could simultaneously affect each of its smaller economies at the state level?

In a network where U.S. states are interpreted as nodes and the strength of the links between nodes is given by the degree of business cycle synchronization between two states, an index of global prominence or centrality of U.S. states is computed. It shows a markedly high tendency to increase some months before national recessions take place, keeping high values during the whole contractionary episode and even some months after it ends, then returning to a stable level that seems to prevail during the rest of the expansion period. This agrees with the premise that when the global heterogeneity between U.S. states business cycles starts to decrease, meaning that their phases start to closely move in the same way, the economy becomes more fragile and recessions tend to take place. The reliability of this index to anticipate recessions is confirmed with real time exercises by reestimating it with the available information up to some period before national recessions start.

The pairwise interdependence among states and national business cycles is assessed relying on a network analysis. The clustering coefficient and closeness centrality measures are used to make time-varying assessments of the comovement among economic phases, reporting high values when economies are following the same pattern (recessions or expansions), and dropping when their phases are independent of each other by following idiosyncratic behaviors. Since the degree of interdependence between states can be monitored month-to-month, it is possible to evaluate how much state $"A"$ could be affected by economic shocks hitting state $"B"$ or the nation as a whole.

Additionally, the study provides three noteworthy features that deserve attention. First, the dynamic cohesiveness between U.S. states reports abrupt changes, specifically, before the mid 90's it was cyclically changing between high and low levels, but after that time it has stabilized, remaining at a high cohesiveness level.

Second, there is a set of economies with the highest centrality is composed by the most central state, North Carolina, followed by Missouri, Wisconsin, Tennessee, and Alabama. Interestingly, these five states are also geographically linked and their output composition is very close to the one of the U.S. nation, constituting a region of the U.S. business cycle core in terms of synchronization. This core is located in the intersection among three of the eight BEA regions, Southeast, Great Lakes and Plains, and can be interpreted as the leading dynamics of the national economy, which is useful for anticipating contractionary episodes.

Third, apart from dynamic estimates of the interdependence degree, this framework also provides the possibility of computing stationary estimates. They are obtained with the ergodic or time-independent probabilities associated to the latent variable measuring synchronization. This stationary results report that states in the core roughly coincide with the ones showing high concordance with the national business cycle found in Owyang et al. (2005), that were obtained under a univariate approach. Moreover, U.S. states can be grouped into three clusters, a highly, discreetly and lowly in sync with the national business cycle, result that agrees with the one in Hamilton and Owyang (2010).

The paper is structured as follows. Section 2 provides the Markov-switching synchronization modelling approach, the filtering algorithm and the procedure of the Bayesian parameter estimation. Section 3 analyzes the business cycle phases synchronization in U.S. states relying on network analysis, providing multidimensional scaling, clustering coefficient, and closeness centrality estimates. Section 4 concludes.

2 Modelling Markov-Switching Synchronization

In this section, it is proposed an algorithm to monitor changes in the degree of pairwise synchronization between stochastic processes that are subject to regime changes. Let $y_{i,t}$ be a time series modeled as a function of a latent variable, $S_{i,t}$, that indicates the regime at which it is, an idiosyncratic component $\epsilon_{i,t}$, and a set of parameters, θ_i, to be estimated. Accordingly, for $i = a, b$,

$$y_{a,t} = f(S_{a,t}, \epsilon_{a,t}, \theta_a) \quad (1)$$

$$y_{b,t} = f(S_{b,t}, \epsilon_{b,t}, \theta_b), \quad (2)$$

the goal will be to make an assessment on their synchronization for each period of time, that is,

$$\delta_t^{ab} = sync(S_{a,t}, S_{b,t}) = \Pr(S_{a,t} = S_{b,t}). \quad (3)$$

Specifically, this paper will focus on the time-varying sync between the two unobserved state variables governing a bivariate Markov-switching model. In order to mainly focus on modelling this dynamic dependence relation and to avoid complex notation, it is considered the following parsimonious and very tractable bivariate two-state Markov-switching specification:

$$\begin{bmatrix} y_{a,t} \\ y_{b,t} \end{bmatrix} = \begin{bmatrix} \mu_{a,0} + \mu_{a,1} S_{a,t} \\ \mu_{b,0} + \mu_{b,1} S_{b,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \right), \quad (4)$$

The results obtained in this section can be straightforwardly applied to an extended specification that could includes more lags in the dynamics or even Markov-switching variance-covariance matrix. The state variable $S_{k,t}$ indicates if y_{kt} is in regime 0 with a mean equal to $\mu_{k,0}$, where $S_{k,t} = 0$, or if y_{kt} is in regime 1 with a mean equal to $\mu_{k,1}$, where $S_{k,t} = 1$, for $k = a, b$. Moreover $S_{a,t}$ and $S_{b,t}$ evolve according to irreducible two-state Markov chains, whose transition probabilities are given by

$$\Pr(S_{k,t} = j | S_{k,t-1} = i) = p_{k,ij}, \text{ for } i, j = 0, 1 \text{ and } k = a, b. \quad (5)$$

In order to characterize the dynamics of $y_t = [y_{a,t}, y_{b,t}]'$, the information contained in $S_{a,t}$ and $S_{b,t}$ can be summarized in the state variable, $S_{ab,t}$, it will account for the possible combinations that the vector, $\mu_{S_{ab,t}} = \left[\mu_{a,0} + \mu_{a,1} S_{a,t},\ \mu_{b,0} + \mu_{b,1} S_{b,t}\right]'$, could take trough the different regimes. It is defined as:

$$S_{ab,t} = \begin{cases} 1, \text{ If } S_{a,t} = 0, S_{b,t} = 0 \\ 2, \text{ If } S_{a,t} = 0, S_{b,t} = 1 \\ 3, \text{ If } S_{a,t} = 1, S_{b,t} = 0 \\ 4, \text{ If } S_{a,t} = 1, S_{b,t} = 1 \end{cases} . \tag{6}$$

In contrast to the previous ways of modelling $\Pr(S_{ab,t} = j_{ab})$ in the literature, where some exogenous prior specific relationship between $S_{a,t}$ and $S_{b,t}$ is assumed, this paper proposes to model $\Pr(S_{ab,t} = j_{ab})$ based on the individual dynamics of $S_{a,t}$ and $S_{b,t}$, and simultaneously accounting for the dynamic degree of dependence between each other, which is endogenously estimated.

Although the degree of dependence between $S_{a,t}$ and $S_{b,t}$ is unknown, the two opposite extreme cases of dependence relationships are known. That is, on the one hand, if $S_{a,t}$ and $S_{b,t}$ are fully independent, then $\Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b)$. On the other hand, if $S_{a,t}$ and $S_{b,t}$ are totally dependent, in the sense that they are fully synchronized, then $y_{a,t}$ and $y_{b,t}$ are driven by the same state variable, S_t, i.e. $S_{a,t} = S_{b,t} = S_t$, remaining in this case $\Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(S_t = j)$. In empirical applications, the true dependence degree should be located between these two extreme possibilities.

In order to make inference on the type of dependence between $S_{a,t}$ and $S_{b,t}$, a new unobserved state variable will be defined as:

$$V_t = \begin{cases} 0 \text{ If } S_{a,t} \text{ and } S_{b,t} \text{ are fully independent} \\ 1 \text{ If } S_{a,t} \text{ and } S_{b,t} \text{ are totally dependent} \end{cases} , \tag{7}$$

In order to maintain the nonlinear nature of the framework, it will also evolve according to an irreducible two-state Markov chain whose transition probabilities are given by

$$\Pr(V_t = j_v | V_{t-1} = i_v) = p_{v,kl}, \quad \text{for } i_v, j_v = 0, 1 \tag{8}$$

Model in Equation (4) hence remains fully characterized by the state variable S_t^*, which collects information regarding to joint dynamics, individual dynamics and their dependence relationship over time simultaneously. In this way $S^*_{ab,t}$ is defined as:[2]

$$S^*_{ab,t} = \begin{cases} 1, \text{ If } V_t = 0, S_{a,t} = 0, S_{b,t} = 0 \\ 2, \text{ If } V_t = 0, S_{a,t} = 0, S_{b,t} = 1 \\ 3, \text{ If } V_t = 0, S_{a,t} = 1, S_{b,t} = 0 \\ 4, \text{ If } V_t = 0, S_{a,t} = 1, S_{b,t} = 1 \\ 5, \text{ If } V_t = 1, S_{a,t} = 0, S_{b,t} = 0 \\ 6, \text{ If } V_t = 1, S_{a,t} = 0, S_{b,t} = 1 \\ 7, \text{ If } V_t = 1, S_{a,t} = 1, S_{b,t} = 0 \\ 8, \text{ If } V_t = 1, S_{a,t} = 1, S_{b,t} = 1 \end{cases}, \tag{9}$$

Inference on the possible states of $S^*_{ab,t}$, i.e. $\Pr(S^*_{ab,t} = j^*_{ab})$ for $j^*_{ab} = 1, \ldots, 8$, can be done by computing

$$\Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v) \Pr(V_t = j_v) \tag{10}$$

where $\Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v)$ indicates the inference on the dynamics of $S_{ab,t}$ conditional on total independence, $V_t = 0$, or conditional on full dependence, $V_t = 1$. In the former case the joint probability of $S^*_{ab,t}$ will be

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t} = j_b, V_t = 0) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 0) \Pr(V_t = 0) \\ &= \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b) \Pr(V_t = 0), \end{aligned} \tag{11}$$

and in the latter case, it will be

$$\begin{aligned} \Pr(S_{a,t} &= j_a, S_{b,t} = j_b, V_t = 1) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 1) \Pr(V_t = 1) \\ &= \Pr(S_t = j) \Pr(V_t = 1), \end{aligned} \tag{12}$$

therefore probabilities of the state variable $S_{ab,t}$ in Equation (6) after accounting for synchronization, can be easily computed as

$$\Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(V_t = 1) \Pr(S_t = j) + (1 - \Pr(V_t = 1)) \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b), \tag{13}$$

[2] States 6 and 7 in Equation (9) are truncated to zero by construction, since the two state variables can not be in different states if they are perfectly synchronized, i.e. $\Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 1) = 0$ for any $j_a \neq j_b$.

which indicates that joint dynamics of $S_{a,t}$ and $S_{b,t}$ are characterized by a linear combination between the extreme dependent case and the extreme independent case, where the weights assigned to each of them are endogenously determined by their sync degree

$$\delta_t^{ab} = \Pr(V_t = 1). \tag{14}$$

2.1 Filtering Algorithm

Following the line of Hamilton's (1994) algorithm, I propose an extension to estimate the model described in Equations (4) and (13). The algorithm is composed by two unified steps, in the first one the goal is the computation of the likelihoods, while in the second one the goal is to compute prediction and updating probabilities.

STEP 1: For the moment it is assumed that model's parameters are known and collected in the vector $\theta = (\mu_{a,0}, \mu_{a,1}, \mu_{b,0}, \mu_{b,1}, \sigma_a^2, \sigma_b^2, \sigma_{ab}, p_{a,00}, p_{a,11}, p_{b,00}, p_{b,11}, p_{00}, p_{11}, p_{v,00}, p_{v,11})'$, in the next section the Bayesian procedure to estimate θ will be clarified. By using the prediction probabilities[3] $\Pr(S_{k,t} = j_k|\psi_{t-1};\theta)$ for $k = a, b$, $\Pr(V_t = j_v|\psi_{t-1};\theta)$ and $\Pr(S_t = j|\psi_{t-1};\theta)$, the joint probability corresponding to the state variable that fully characterizes the model dynamics, $S^*_{ab,t}$, can be obtained relying on Equations (11) and (12), that is

$$\begin{aligned}
\Pr(S^*_{ab,t} &= j^*_{ab}|\psi_{t-1};\theta) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v|\psi_{t-1};\theta) \\
&= \Pr(S_{a,t} = j_a, S_{b,t} = j_b|V_t = j_v, \psi_{t-1};\theta)\Pr(V_t = j_v|\psi_{t-1};\theta), \qquad (15)
\end{aligned}$$

thereafter the density of y_t given that it is on regime S_t^*, and the prediction probabilities of the realizations of S_t^*, are used to compute the joint likelihood

$$\begin{aligned}
f(y_t, S^*_{ab,t} &= j^*_{ab}|\psi_{t-1};\theta) = f(y_t|S^*_{ab,t} = j^*_{ab}, \psi_{t-1};\theta)\Pr(S^*_{ab,t} = j^*_{ab}|\psi_{t-1};\theta) \\
&= f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v|\psi_{t-1};\theta), \qquad (16)
\end{aligned}$$

then summing across the respective terms, specific likelihoods corresponding individual

[3] The steady state or ergodic probabilities can be used as starting values of the filter.

processes are computed

$$
\begin{aligned}
f_a(y_{a,t}, S_{a,t} &= j_a|\psi_{t-1};\theta) = \sum_{j_b=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, V_t=j_v|\psi_{t-1};\theta) \quad (17)\\
f_b(y_{b,t}, S_{b,t} &= j_b|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_v=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, V_t=j_v|\psi_{t-1};\theta) \quad (18)\\
f_{ab}(y_t, S_t &= j|\psi_{t-1};\theta) = \sum_{j_a=j_b} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, V_t=j_v|\psi_{t-1};\theta) \quad (19)\\
f(y_t, V_t &= j_v|\psi_{t-1};\theta) = \sum_{j_a=0}^{1}\sum_{j_b=0}^{1} f(y_t, S_{a,t}=j_a, S_{b,t}=j_b, V_t=j_v|\psi_{t-1};\theta). \quad (20)
\end{aligned}
$$

STEP 2: Once y_t is observed at the end of time t, the prediction probabilities $\Pr(S_{k,t} = j_k|\psi_{t-1};\theta)$ for $k = a, b$, $\Pr(V_t = j_v|\psi_{t-1};\theta)$ and $\Pr(S_t = j|\psi_{t-1};\theta)$ can be updated as follows

$$
\begin{aligned}
\Pr(S_{a,t} &= j_a|\psi_t;\theta) = \frac{f_a(y_{a,t}, S_{a,t}=j_a|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (21)\\
\Pr(S_{b,t} &= j_b|\psi_t;\theta) = \frac{f_b(y_{b,t}, S_{b,t}=j_b|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (22)\\
\Pr(S_t &= j|\psi_t;\theta) = \frac{f(y_t, S_t=j|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (23)\\
\Pr(V_t &= l|\psi_t;\theta) = \frac{f(y_t, V_t=l|\psi_{t-1};\theta)}{f(y_t|\psi_{t-1};\theta)} \quad (24)
\end{aligned}
$$

Where $\psi_t = \{\psi_{t-1}, y_t\}$, and the unconditional likelihood function is given by

$$
f(y_t|\psi_{t-1};\theta) = \sum\nolimits_{j^*=1}^{8} f(y_t, S^*_{ab,t} = j^*_{ab}|\psi_{t-1};\theta). \quad (25)
$$

Forecasts of the updated probabilities in Equations (21)-(24) are done by using the corresponding transition probabilities in the vector θ, that is $p_{a,ij}, p_{b,ij}, p_{ij}, p_{v,ij}$ for $S_{a,t}$, $S_{b,t}$, S_t, V_t respectively,

$$
\begin{aligned}
\Pr(S_{k,t+1} &= j_k|\psi_t;\theta) = \sum_{i_k=0}^{1} \Pr(S_{k,t+1}=j_k, S_{k,t}=i_k|\psi_t;\theta)\\
&= \sum_{i_k=0}^{1} \Pr(S_{k,t+1}=j_k|S_{k,t}=i_k)\Pr(S_{k,t}=i_k|\psi_t;\theta), \text{ for } k=a,b \quad (26)
\end{aligned}
$$

$$\begin{aligned}\Pr(V_{t+1} &= j_v|\psi_t;\theta) = \sum_{i=0}^{1}\Pr(V_{t+1}=j_v, V_t=i_v|\psi_t;\theta)\\ &= \sum_{i=0}^{1}\Pr(V_{t+1}=j_v|V_t=i_v)\Pr(V_t=i_v|\psi_t;\theta) \qquad (27)\\ \Pr(S_{t+1} &= j|\psi_t;\theta) = \sum_{i=0}^{1}\Pr(S_{t+1}=j, S_t=i|\psi_t;\theta)\\ &= \sum_{i=0}^{1}\Pr(S_{t+1}=j|S_t=i)\Pr(S_t=i|\psi_t;\theta). \qquad (28)\end{aligned}$$

Finally the above forecasted probabilities are used to predict inferences on the realizations of $S^*_{ab,t+1}$, again relying on Equations (11) and (12)

$$\begin{aligned}\Pr(S^*_{ab,t+1} &= j^*_{ab}|\psi_t;\theta) = \Pr(S_{a,t+1}=j_a, S_{b,t+1}=j_b, V_{t+1}=j_v|\psi_t;\theta)\\ &= \Pr(S_{a,t}=j_a, S_{b,t}=j_b|V_t=j_v,\psi_t;\theta)\Pr(V_t=j_v|\psi_t;\theta), \qquad (29)\end{aligned}$$

By iterating these two steps for $t = 1, 2, \ldots, T$, the algorithm simultaneously provides inferences on the joint dynamics and individual dynamics along with their time-varying degree of dependence of the model in Equations (4) and (13).

2.2 Bayesian Parameter Estimation

The approach to estimate θ will be relied on a bivariate extended version of the multi-move Gibbs-sampling procedure implemented by Kim and Nelson (1998) for Bayesian estimation of univariate Markov-switching models.[4] In this setting both the parameters of the model θ and the Markov-switching variables $\tilde{S}_{k,T} = \{S_{k,t}\}_1^T$ for $k = a, b$, $\tilde{S}_T = \{S_t\}_1^T$ and $\tilde{V}_T = \{V_t\}_1^T$ are treated as random variables given the data in $\tilde{y}_T = \{y_t\}_1^T$. The purpose of this Markov chain Monte Carlo simulation method is to approximate the joint and marginal distributions of these random variables by sampling from conditional distributions.

[4] The motivation for the use of Bayesian methods relies on the fact that as the number of possible states increase, the likelihood function could be characterized by many local maxima and there could be strong convergence problems in performing maximum likelihood estimation, Boldin (1996).

2.2.1 Priors

For the mean and variance parameters in vector θ, the Independent Normal-Wishart prior distribution is used

$$p(\mu, \Sigma^{-1}) = p(\mu)p(\Sigma^{-1}), \tag{30}$$

where

$$\begin{aligned} \mu &\sim N(\underline{\mu}, \underline{V}_\mu) \\ \Sigma^{-1} &\sim W(\underline{S}^{-1}, \underline{v}) \end{aligned}$$

for the transition probabilities $p_{a,00}, p_{a,11}$ from $S_{a,t}$, $p_{b,00}, p_{b,11}$ from $S_{b,t}$, p_{00}, p_{11} from S_t and $p_{v,00}, p_{v,11}$ from V_t, Beta distributions will be used as conjugate priors

$$p_{k,00} \sim Be(u_{k,11}, u_{k,10}),\ p_{k,11} \sim Be(u_{k,00}, u_{k,01}), \text{ for } k = a, b \tag{31}$$

$$p_{00} \sim Be(u_{11}, u_{10}),\ p_{11} \sim Be(u_{00}, u_{01}) \tag{32}$$

$$p_{v,00} \sim Be(u_{v,11}, u_{v,10}),\ p_{v,11} \sim Be(u_{v,00}, u_{v,01}) \tag{33}$$

2.2.2 Drawing $\tilde{S}_{a,T}$,$\tilde{S}_{b,T}$,$\tilde{S}_T$ and $\tilde{V}_T$ given θ and $\tilde{y}_T$

Following the result in Equation (13), in order to make inference on the bivariate dynamics of the model (4) driven by $\tilde{S}_{ab,T} = \{S_{ab,t}\}_1^T$ and described in (6), it is just needed to make inference on the dynamics of the single state variables $\tilde{S}_{a,T}$, $\tilde{S}_{b,T}$, $\tilde{S}_T$ and $\tilde{V}_T$, this can be done following the results in Kim and Nelson (1998) by first computing draws from the conditional distributions

$$g(\tilde{S}_{k,T}|\theta, \tilde{y}_T) = g(S_{k,T}|\tilde{y}_T) \prod_{t=1}^{T} g(S_{k,t}|S_{k,t+1}, \tilde{y}_t), \text{ for } k = a, b \tag{34}$$

$$g(\tilde{S}_T|\theta, \tilde{y}_T) = g(S_T|\tilde{y}_T) \prod_{t=1}^{T} g(S_t|S_{t+1}, \tilde{y}_t) \tag{35}$$

$$g(\tilde{V}_T|\theta, \tilde{y}_T) = g(V_T|\tilde{y}_T) \prod_{t=1}^{T} g(V_t|V_{t+1}, \tilde{y}_t). \tag{36}$$

In order to obtain the two terms in the right hand side of Equation (34)-(35) the following two steps can be employed:

Step 1: The first term can be obtained by running the filtering algorithm developed in Section 2.1, to compute $g(\tilde{S}_{k,t}|\tilde{y}_t)$ for $k = a, b$, $g(\tilde{S}_t|\tilde{y}_t)$ and $g(\tilde{V}_{k,t}|\tilde{y}_t)$ for $t = 1, 2, \ldots, T$, saving them and taking the elements for which $t = T$.

Step 2: The product in the second term can be obtained for $t = T-1, T-2, \ldots, 1$, by following the result:

$$\begin{aligned} g(S_t|\tilde{y}_t, S_{t+1}) &= \frac{g(S_t, S_{t+1}|\tilde{y}_t)}{g(S_{t+1}|\tilde{y}_t)} \\ &\propto g(S_{t+1}|S_t)g(S_t|\tilde{y}_t), \end{aligned} \tag{37}$$

where $g(S_{t+1}|S_t)$ corresponds to the transition probabilities of S_t and $g(S_t|\tilde{y}_t)$ were saved in Step 1.

Then, it is possible to compute

$$\Pr[S_t = 1|S_{t+1}, \tilde{y}_t] = \frac{g(S_{t+1}|S_t = 1)g(S_t = 1|\tilde{y}_t)}{\sum_{j=0}^{1} g(S_{t+1}|S_t = j)g(S_t = j|\tilde{y}_t)}, \tag{38}$$

and generate a random number from a $U[0, 1]$. If that number is less than or equal to $\Pr[S_t = 1|S_{t+1}, \tilde{y}_t]$, then $S_t = 1$, otherwise $S_t = 0$. The same procedure applies for $S_{a,t}$, $S_{b,t}$ and V_t, and by using Equation (13) inference of $\tilde{S}_{ab,T}$ can be done.

2.2.3 Drawing $p_{a,00}$,$p_{a,11}$,$p_{b,00}$,$p_{b,11}$, p_{00},p_{11},$p_{v,00}$,$p_{v,11}$ given $\tilde{S}_{a,T}$,$\tilde{S}_{a,T}$,$\tilde{S}_T$ and $\tilde{V}_T$

Conditional on $\tilde{S}_{k,T}$ for $k = a, b$, $\tilde{S}_T$ and $\tilde{V}_T$, the transition probabilities are independent on the data set and the model's parameters, hence the likelihood function of p_{00}, $p_{11}$is given by:

$$L(p_{00}, p_{11}|\tilde{S}_T) = p_{00}^{n_{00}}(1 - p_{00}^{n_{01}})p_{11}^{n_{11}}(1 - p_{11}^{n_{10}}), \tag{39}$$

where n_{ij} refers to the transitions from state i to j, accounted for in $\tilde{S}_T$.

Combining the prior distribution in Equation (32) with the likelihood, the posterior distribution is given by

$$p(p_{00}, p_{11}|\tilde{S}_T) \propto p_{00}^{u_{00}+n_{00}-1}(1 - p_{00})^{u_{01}+n_{01}-1}p_{11}^{u_{11}+n_{11}-1}(1 - p_{11})^{u_{10}+n_{10}-1} \tag{40}$$

which indicates that draws of the transition probabilities will be taken from

$$p_{00}|\tilde{S}_T \sim Be(u_{00} + n_{00}, u_{01} + n_{01}),\ \ p_{11}|\tilde{S}_T \sim Be(u_{11} + n_{11}, u_{10} + n_{10}) \tag{41}$$

2.2.4 Drawing $\mu_{0,a}$,$\mu_{1,a}$,$\mu_{0,b}$,$\mu_{1,b}$ given σ_a^2,σ_b^2,σ_{ab},$\tilde{S}_{a,T}$,$\tilde{S}_{b,T}$,$\tilde{S}_T$,$\tilde{V}_T$ and $\tilde{y}_T$

The model in Equation (4) can be compactly expressed as

$$\begin{aligned}
\begin{bmatrix} y_{a,t} \\ y_{b,t} \end{bmatrix} &= \begin{bmatrix} 1 & S_{a,t} & 0 & 0 \\ 0 & 0 & 1 & S_{b,t} \end{bmatrix} \begin{bmatrix} \mu_{a,0} \\ \mu_{a,1} \\ \mu_{b,0} \\ \mu_{b,1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \right) \\
y_t &= \bar{S}_t \mu + \xi_t, \quad \xi_t \sim N(\mathbf{0}, \Sigma),
\end{aligned} \tag{42}$$

stacking as:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}, \ \bar{S} = \begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \\ \vdots \\ \bar{S}_T \end{bmatrix}, \text{ and } \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_T \end{bmatrix},$$

the model in Equation (42) remains written as a normal linear regression model with an error covariance matrix of a particular form:

$$y = S\mu + \xi, \quad \xi \sim N(\mathbf{0}, I \otimes \Sigma) \tag{43}$$

Conditional on the covariance matrix parameters, state variables and the data, by using the corresponding likelihood function, the conditional posterior distribution $p(\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \Sigma^{-1}, \tilde{y}_T)$ takes the form

$$\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \Sigma^{-1}, \tilde{y}_T \sim N(\overline{\mu}, \overline{V}_\mu), \tag{44}$$

where

$$\begin{aligned}
\overline{V}_\mu &= \left(\underline{V}_\mu^{-1} + \sum_{t=1}^{T} \bar{S}_t' \Sigma^{-1} \bar{S}_t \right)^{-1} \\
\overline{\mu} &= \overline{V}_\mu \left(\underline{V}_\mu^{-1} \underline{\mu} + \sum_{t=1}^{T} \bar{S}_t' \Sigma^{-1} y_t \right).
\end{aligned}$$

After drawing $\mu = \begin{bmatrix} \mu_{a,0} & \mu_{a,1} & \mu_{b,0} & \mu_{b,1} \end{bmatrix}'$ from the above multivariate distribution, if the generated value of $\mu_{a,1}$ or $\mu_{b,1}$ is less than or equal to 0, that draw is discarded, otherwise it is saved, this is in order to ensure that $\mu_{a,1} > 0$ and $\mu_{b,1} > 0$.

2.2.5 Drawing σ_a^2,σ_b^2,σ_{ab} given $\mu_{0,a}$,$\mu_{1,a}$,$\mu_{0,b}$,$\mu_{1,b}$,$\tilde{S}_{a,T}$,$\tilde{S}_{b,T}$,$\tilde{S}_T$,$\tilde{V}_T$ and $\tilde{y}_T$

Conditional on the mean parameters, state variables and the data, by using the corresponding likelihood function, the conditional posterior distribution $p(\Sigma^{-1}|\tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \mu, \tilde{y}_T)$ takes the form

$$\Sigma^{-1}|\tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \mu, \tilde{y}_T \sim W(\overline{S}^{-1}, \overline{\upsilon}), \tag{45}$$

where

$$\begin{aligned} \overline{\upsilon} &= T + \underline{\upsilon} \\ \overline{S} &= \underline{S} + \sum_{t=1}^{T} \left(y_t - \bar{S}_t\mu\right)\left(y_t - \bar{S}_t\mu\right)', \end{aligned}$$

after Σ^{-1} is generated the elements is Σ are recovered.

3 Business Cycle Phases Synchronization in U.S. States

The global financial crisis has stimulated the interest on the sources and propagation of contractionary episodes, calling to take a more careful look at the disaggregation of the business cycle in order to study the mechanisms underlying economic fluctuations. Two recent works have shown interesting features about the business cycle phases in U.S. states, Owyang et al. (2005), and the propagation of regional recessions in the same country, Hamilton and Owyang (2010). On the one hand, the former study that follows a univariate approach, finds that U.S. states differ significantly in the timing of switches between regimes of expansions and recessions, indicating large differences in the extent to which state business cycle phases are in concord with those of the aggregate economy. On the other hand, the later work which follows a multivariate approach, focuses on clustering the states sharing similar business cycle characteristics finding that differences across states appear to be a matter of timing and they can be grouped into three clusters with some of them entering in recession or recovering before others.

Although these previous studies provide insights about the synchronization among different business cycles during a given period, they are not able to capture changes in the degree of synchronization between the phases of economic fluctuations, that can be potentially caused by economic unions, policy changes, aggregate recessionary shocks, etc. Analyzing synchronization changes is crucial for policy makers in order to determine at every period of time, which states could be more sensitive to specific policy or recessionary shocks and which others would remain less affected.

The framework developed in Section 2 is applied for monitoring monthly changes in the degree of sync among economic phases of U.S. states, and also between states and the national business cycle. The state coincident indexes constructed in Crone (2002) and provided by the Federal Reserve Bank of Philadelphia, are used as indicators of economic activity at state level. Alaska and Hawaii are excluded, as in Hamilton and Owyang (2010), and The Chicago Fed National Activity Index (CFNAI) is used as monthly measure of the aggregate U.S. business cycle, the sample period is 1979:8 to 2012:3.

3.1 Bivariate Synchronization

The analysis that was performed for 48 U.S. states required to model each of the $C_2^{48} = 1,128$ pairwise comparisons, where in each of them 12,000 draws were generated from the posterior with an initial burn of 2,000 draws that were discarded.[5] In order to assess the performance of the proposed MS synchronization model, two selected examples are analyzed.[6]

The first one focuses on the case of two states which presents high and similar percentage of national GDP, that is, New York with 7.68% and Texas with 7.95%. Table 1 shows the posterior means and medians for the model parameters along with their corresponding standard deviation. All means and medians of parameters are similar and also statistically significant. It is worth to highlight the estimates of the transition probabilities associated to the state variable that measures synchronization, V_t. For the *New York vs. Texas* case, the probability of going from a regime of high sync to another regime of high sync is almost equal to the probability of going from a regime of low sync to another regime of low sync, since both are about 0.95. These estimates are corroborated in Chart A of

[5] Due to the high technology nowadays available, such high number of computations is not a problem anymore.

[6] The results for the other cases are available upon request to the author.

Figure 1, where the probabilities of recession for New York and Texas along with their synchronization are plotted. As can be seen during the first half of the sample, both states were lowly synchronized, however since 1994 they started to experiment high sync levels.

The second example focuses on analyzing two states of different GDP share by selecting the polar case, that is, the state with the highest share, California with 13.34%, along with the state with the lowest share, Vermont with 0.18%. Table 2 presents the Bayesian estimation results of the model parameters. Unlike to the previous case, in the *California vs. Vermont* model, the probability of remaining in a high sync regime (0.97) is higher than the probability of remaining in a low sync regime (0.93). This can be observed in Chart B of Figure 1, since with the exception of the 1987-1994 period, both states have experimented high levels of synchronization among their business cycle phases.

These two examples, apart from documenting the existence of abrupt changes in business cycle sync phases between U.S. states, show that their synchronization is independent on their GDP share, since the proposed approach only focuses on making a time-varying assessment of the comovement among economic phases, reporting high values when both are following the same pattern (recessions or expansions), and dropping when their phases are independent of each other by following idiosyncratic behaviors.

The analysis regarding to the cooncordance between states and national recessions is performed in Owyang et al. (2005) by following the line in Harding and Pagan (2006), where the degree to which two business cycles are in sync is calculated as the percentage of time the two economies were in the same regime. However, such approach provides an *average* synchronization measure without the possibility of making inference on the potential changes that it could experiment trough time. The approach in this paper is used to analyze this issue. Figure 2 shows the dynamic sync degree between states and national business cycle, showing a high heterogeneity across states. On the one hand, some of them show an almost constant and low sync degree, e.g. Louisiana, Oklahoma and Wyoming, while others an almost constant and high sync degree, e.g. Alabama, Minnesota, Nebraska, South Carolina, Tennessee and Vermont. On the other hand, the rest of states in general experiment abrupt sync changes, that usually take place before or after national recessions, as in the case of Massachusetts and North Dakota respectively for the 1990's recession, and North Carolina for 2007's recession, these features agree with Hamilton and Owyang (2010) who states that differences across states appear to be a matter of timing.

By means of the performed analysis, it can be evaluated how much state *"A"* could be affected by an economic shock that is hitting state *"B"*, or which states could be more sensitive when a period of national recession is coming, since the degree of interdependence between states and national economic activity can be monitored month-to-month. However, an aggregate dynamic representation of the results presented so far seems needed in order to deal with the question: How an upcoming national recession could simultaneously affect each of its smaller economies at the state level?

As suggested by Timm (2002) and Camacho et al. (2006), multidimensional scaling methods are a helpful tool to identify cyclical affiliations between economies. Although, traditionally studies focus in providing a map of just one general picture of the business cycle similarities for a given sample period, for the present case a dynamic approach is needed.

3.2 Dynamic Multidimensional Scaling

This methodology seeks to find a low dimensional coordinate system to represent n-dimensional objects and create a map of lower dimension (k) which gives approximate distances among them. The dimensional coordinates of the projection of any two objects, a and b, are computed by minimizing a stress function which measure the squared sum of divergences between the true distances or the measure of desynchronization, $\gamma_t^{ab} = 1 - \delta_t^{ab}$, and the approximate distances, $\tilde{\gamma}_t^{ab}$, among these objects. Moreover given that this representation can be obtained for each $t = 1, \ldots, T$, it is applied a dynamic version, in which the evolution of states synchronization is presented by a discrete-time sequence of graph snapshots. In order to preserve the "mental map" between snapshots so that the transition between frames in the animation can be easily interpreted, the movements between time steps are constrained by adding a temporal penalty to the stress function of a static graph layout method, Xu et al. (2011), so that the minimization problem is:

$$\min_{\tilde{\gamma}_t^{ab}} = \frac{\sum_{a=1}^{n}\sum_{b=1}^{n}(\gamma_t^{ab} - \tilde{\gamma}_t^{ab})^2}{\sum_{a,b}(\gamma_t^{ab})^2} + \beta \sum_{a=1}^{n} \tilde{\gamma}_{t|t-1}^{a}, \tag{46}$$

where β can be chosen as the average standard deviations of the series in order to adjust the movements according to the variability of the data and with

$$\tilde{\gamma}_t^{ab} = (||z_{a,t} - z_{b,t}||^2)^{1/2} = \left[\sum_{i=1}^{k}(z_{ai,t} - z_{bi,t})^2\right]^{1/2} \tag{47}$$

$$\tilde{\gamma}_{t|t-1}^{a} = (||z_{a,t} - z_{a,t-1}||^2)^{1/2} = \left[\sum_{i=1}^{k}(z_{ai,t} - z_{ai,t-1})^2\right]^{1/2}. \tag{48}$$

where $z_{a,t}$ and $z_{b,t}$ are the k-dimensional projection of the objects a and b, and $z_{ai,t}$ and $z_{bi,t}$ each of their corresponding elements at time t.

Figure 3 plots the U.S. states synchronization map for the first month of the 1990's recession, showing that for this period of time U.S. states could be grouped into three clusters, coinciding with the number of clusters found in Hamilton and Owyang (2010). Specifically, there is a bunch of states that were more affected by that recession, being highly in sync with the national business cycle, some examples are Nevada, Ohio, Michigan, Florida, etc., and two separate groups that experimented low synchronization with respect to the national activity and moreover with respect to each other, in one of them are located states like Texas, Oklahoma, Wyoming, etc., while in the other asynchronized group are found states like Rhode Island, New Jersey, Maine, etc.[7]

In Figure 4 and Figure 5, the same exercise performed in the previous figure is applied for the 2001's and 2007's recessions respectively, showing that for both episodes U.S. states can be grouped only into two clusters, which can be interpreted as the "core" and the "periphery". The "core" that is highly in sync with the national cycle contains the great part of the states, while the "periphery" just few of them, some examples are North Dakota, Wyoming, Oklahoma, Montana, etc.

[7]The full animated representation can be found at `https://sites.google.com/site/daniloleivaleon/media`

3.3 Clustering Coefficient

The intuition behind the proposed framework for monitoring synchronization in Section 2, relies on the fact that if $\delta_t^{ab} \approx 1$, then it is likely that states a and b are sharing the same business cycle dynamics, or in other words, economic shocks affecting to a could also be affecting to b creating a link of dependence between them. By means of this, U.S. states, denoted by h_i for $i = 1, \dots, n$, and collected in $H = \{h_i\}_1^n$, can be interpreted as nodes, interacting in the weighted network g_t, with the relationship between each pair of nodes h_a and h_b at t, given by the strength of their link $\delta_t^{ab} \in [0, 1]$ and collected in Θ_t.

The previous graphs mapping states business cycle similarities for the beginning of the last three recessions showed two different scenarios. In the first one, 1990's recession, states tend to be less synchronized between them and also with respect to the National business cycle than the other two, 2001's and 2007's recessions. These changes in interconnectedness can be monitored by using the clustering coefficient for each of the $n = 48$ states weighted by the degree of synchronization, $CL_{i,t}$, in order to compute the clustering coefficient for the whole network at time t by averaging them, see Watts and Strogatz (1998):

$$CL_t = \frac{1}{n} \sum_{i=1}^{n} CL_{i,t}, \tag{49}$$

where CL_t would measure the statistical level of cohesiveness between the business cycle phases of U.S. states for every period of time.

In Figure 6 is plotted the dynamic average clustering coefficient, not surprisingly it presents low values during the 1990's recession, but high values during the 2001's and 2007's recessions. Moreover, that series shows that in the mid 90's there was a change in the cohesiveness between states since before that date it was cyclically changing from high to low values, but after that, it remained almost stable in high values.

In order to assess if these features can be evaluated in real time for monitoring purposes, the framework was reestimated two times with data up to one month before the beginning of the last two recessions.[8] The top and bottom charts in Figure 7 plot the index of states cohesiveness for the 2007's and 2001's recessions respectively, showing a very similar

[8] The same analysis was not performed for the 1990's, an the previous recessions due to the small size of the sample.

performance as in the estimation with the full data set, corroborating the reliability of the U.S. states cohesiveness index.

Another fundamental issue regarding to the business cycle synchronization literature is the existence of a well defined core or atractor, which is usually represented by a leading economy or a weighted average between a set of economies representing such core, that could help to anticipate the movements in the aggregate business cycle. Camacho et al. (2006) develop a specific procedure, based on simulations of the similarities among pairs of economies, in order to assess if the points (economies) plotted in the business cycle map are randomly distributed or there is any kind of atractor that keep them together. This paper tries to explore up to which extend the interconnectedness between smaller economies can be helpful to anticipate national recessions?

The Markov-Switching Synchronization Network (MSYN) will be used to determine the connectedness between nodes (states) and therefore the "key" nodes (states) composing the U.S. business cycle core or leading economies, that will help to anticipate global recessions. In order to have a glimpse about the form that the MSYN would take during contractionary episodes, Figure 8 plots the corresponding graph for the first month of the last three recessions. Given that the MSYN is a weighted network, in order to make possible the graphical representation, the link between a and b is plotted if $\delta_t^{ab} > 0.5$, otherwise there is no link between them. It is important to note that although the U.S. business cycle is not included in the network analysis, just the states, all the three plots show close relation with the ones in Figures 3, 4 and 5, respectively, where the national cycle is included, corroborating the grouping pattern.

By looking at a disaggregated perspective, this paper has shown that U.S. states present a high dynamic heterogeneity among their business cycle phases, due to while some states are entering in recession, the others may or may not follow the same pattern. This study takes advantage of this fact departing from the hypothesis that when this heterogeneity starts to decrease, meaning that all states start to converge to the same phase, *something bad* is going to happen, agreeing with the traditionally view that economies tend to show higher converge during periods of recessions than during periods of expansion.

In order to find the core dynamics leading the national economy, it is required to look at the prominence of each node (state) in the present synchronization network, for this purpose the concept used to answer these kind of questions is the one of centrality. There are several measures regarding to the centrality of a node in a network, but given that desynchronization measures, γ_t^{ab}, under the present framework are interpreted as distances, providing information about the farness among U.S. states business cycles, the most appropriate measure for this context is the one of closeness centrality.

3.4 Closeness Centrality

The farness of a given node is defined as the sum of its distances to all other nodes, where the distance between two nodes i and j is given by the length of the shortest path between them, denoted by $d(i,j)$. In order to compute $d(i,j)$, the Dijkstra's (1959) algorithm is applied since it solves the shortest path problem on a weighted graph $G_t = (H, \Theta_t)$ for the case in which all edges weights are nonnegative. For example, in a set $H' = \{a, b, c\}$ where $\delta^{ab} = 0.5$, $\delta^{ac} = 0.9$ and $\delta^{bc} = 0.2$, the shortest path between a and c will be 0.7, since $\delta^{ab} + \delta^{bc} < \delta^{ac}$, hence notice that $d(a,c)$ does not necessarily have to be equal to δ^{ac}.

The closeness between two nodes is defined as the inverse of the farness. Thus, the more central is a node, the lower is its total distance to all other nodes. Closeness can be regarded as a measure of how fast it will take to spread information, risk, economic shocks, etc., from node i to all other nodes sequentially.[9] The total distance from node i to all other nodes in the network g, i.e. the farness of i at time t, is given by $\sum_{j\neq i|t} d_t(i,j)$, being its reciprocal the closeness centrality of node i:

$$C_t(i) = \frac{1}{\sum_{j\neq i|t} d_t(i,j)}. \tag{50}$$

This measure of centrality has a natural analogue at the aggregate network level. Let i^* be the node that attains the highest closeness centrality across all nodes and let $C_t(i^*)$ be this centrality at time t, then the closeness centrality of the network is given by

$$C_t = \sum_{i=1|t}^{k} [C_t(i^*) - C_t(i)]. \tag{51}$$

[9] For an overview regarding to definitions in network analysis, see Goyal (2007).

This measure is calculated for every period of time with the information available in Θ_t for the whole sample, i.e. since 1979:8 until 2012:3, plotting in Figure 9 the dynamics of the closeness centrality for the MSYN involving U.S. states, showing a clear and easy to interpret pattern. The centrality shows a markedly high tendency to increase some months before national recessions take place, keeping high values during the whole contractionary episode and even some months after it ends, then returning to an stable level that seems to prevail in general during the rest of the expansion period. High centrality levels reported after recessions have ended could be in part attributed to the so-called "third phase" in the business cycle of some U.S. states, since the pattern associates stable growth with low centrality, while recessions and recoveries with high centrality.

The most useful feature of the centrality index, $C_{c,t}$, is the capacity to anticipate national recessions, since some periods before a contractionary episode begin, it starts to increase. In order evaluate this predictive power the index was reestimated two times with data up to one month before the beginning of the last two recessions.[10] The top and bottom charts in Figure 10 plot the centrality index for the 2007's and 2001's recessions respectively, corroborating its reliability to anticipate U.S. recessions.

Additionally, by accounting for the centrality of each node (state) trough time using Equation (50), the states with the highest centrality at each month during the sample period can be interpreted as the atractor. The results show that there is a set of five "main" states which includes, the main atractor, North Carolina with the 43.9% of times being the state with the highest centrality, followed by Missouri with 22.7%, Wisconsin with 21.4%, Tennessee with 11.2%, and Alabama with a 0.8%, the rest of states never experienced the highest centrality at any period. These states are highlighted in Figure 11, showing that, with the exception of Wisconsin, they are also geographically linked, composing a region of the U.S. business cycle sync core.

[10] The same analysis was not performed for the 1990's, an the previous recessions due to the small size of the sample.

3.5 Stationary Synchronization

Apart from dynamic estimates of the interdependence degree, this framework also provides the possibility of computing stationary estimates, obtained with the ergodic or time-independent probabilities of V_t, as in Hamilton (1994), that can be interpreted as an *average sync* during the period under study. Table 3 reports the matrix containing the stationary synchronization among U.S. states, Θ, in which the shaded cells refer to pairs of states with the highest sync degree ($\delta \geq 0.9$). The values in Θ range from 0.1, which is the *Delaware vs. Louisiana* case, until 0.95, some examples are *Kentucky vs. Tennessee*, *North Carolina vs. South Carolina*, and *Tennessee vs. Wisconsin*, which no coincidentally contain states that belong to the core previously specified. This information can be helpful, for example, to assess which states would be more affected if there is an oil shock hitting Texas, since according to Table 3, the answer will be Colorado, followed by Oklahoma.

The same procedure can be followed to compute stationary *state vs. national* synchronizations, that are reported in Table 4. Not surprisingly the states in the core show high values, that is, Missouri (0.84), Wisconsin (0.87), Alabama (0.92), Tennessee (0.93), and the main atractor, the highest value, i.e. North Carolina (0.94). Interestingly, these states also show high cooncordance with the National business cycle according to the results in Owyang et al. (2005), obtained under a univariate approach. Moreover, a whole stationary sync picture is obtained and the resulting map is reported in Figure 12, showing that for 1979:8-2012:3, U.S. states can be grouped into three clusters, a highly, discreetly and lowly in sync with the national business cycle. This results also nests the one in Hamilton and Owyang (2010), where it is found three clusters with individual states in a given cluster sharing certain business cycle characteristics.

4 Conclusions

This paper proposes a novel framework for monitoring changes in the degree of aggregate synchronization between many stochastic processes that are subject to regime changes. Specifically, it computes regime inferences from a multivariate Markov-switching model and simultaneously estimate a measure of the time-varying synchronization between the unobserved state variables governing each process involved in the system.

The proposed framework is used as a tool for monitoring the time-varying synchronization among U.S. states business cycles in order to assess, first, up to which extend the interconnectedness between smaller economies can be helpful to anticipate national recessions? and second, how a national recession that is coming soon is going to affect each of its smaller economies at the state level?

The results suggest that national recessions can be anticipated by an index that accounts for the aggregate synchronization of states in an interdependent environment, confirming its reliability with real time exercises, and that it can be evaluated how much state "A" could be affected by economic shocks hitting state "B" or the nation as a whole, since the degree of interdependence between states can be monitored month-to-month.

Additionally, by taking into account the centrality of each state in terms of synchronization, it is obtained the leading economies in the U.S. business cycle, composed by the main atractor, North Carolina, followed by Missouri, Wisconsin, Tennessee, and Alabama. Interestingly this set of five states are also geographically linked, providing a region of the core leading the national economy that helps to anticipate contractionary episodes, and that is located in the intersection among three of the eight BEA regions, Southeast, Great Lakes and Plains.

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Table 1. Bivariate MS Synchronization Model for New York and Texas

Parameter	Mean	Std. Dev.	Median
$\mu_{ny,0}$	-0.0934	0.0131	-0.0944
$\mu_{ny,1}$	0.2480	0.0126	0.2485
σ_{ny}^{2}	0.0073	0.0005	0.0072
$p_{ny,11}$	0.9837	0.0073	0.9847
$p_{ny,00}$	0.9312	0.0263	0.9346
$\mu_{tx,0}$	-0.0601	0.0102	-0.0601
$\mu_{tx,1}$	0.1691	0.0107	0.1692
σ_{tx}^{2}	0.0064	0.0004	0.0064
$p_{tx,11}$	0.9839	0.0070	0.9852
$p_{tx,00}$	0.9358	0.0251	0.9390
$\sigma_{ny,tx}$	0.0013	0.0004	0.0013
p_{11}	0.9808	0.0079	0.9819
p_{00}	0.9292	0.0264	0.9323
$p_{V,11}$	0.9501	0.0481	0.9647
$p_{V,00}$	0.9480	0.0360	0.9565

Note: The selected example presents the case of two states with high and similar U.S. GDP share, New York with 7.68%, and Texas with 7.95%. The results for the other cases are available upon request to the author.

Table 2. Bivariate MS Synchronization Model for California and Vermont

Parameter	Mean	Std. Dev.	Median
$\mu_{ca,0}$	-0.0211	0.0100	-0.0215
$\mu_{ca,1}$	0.1674	0.0103	0.1673
σ_{ca}^{2}	0.0068	0.0005	0.0068
$p_{ca,11}$	0.9773	0.0090	0.9785
$p_{ca,00}$	0.9450	0.0205	0.9472
$\mu_{vt,0}$	-0.0523	0.0136	-0.0523
$\mu_{vt,1}$	0.2012	0.0141	0.2010
σ_{vt}^{2}	0.0127	0.0010	0.0127
$p_{vt,11}$	0.9764	0.0093	0.9774
$p_{vt,00}$	0.9413	0.0217	0.9432
$\sigma_{ca,vt}$	0.0028	0.0005	0.0027
p_{11}	0.9772	0.0092	0.9784
p_{00}	0.9432	0.0210	0.9458
$p_{V,11}$	0.9742	0.0242	0.9817
$p_{V,00}$	0.9362	0.0417	0.9461

Note: The selected example presents the case of the states with the highest and the lowest U.S. GDP share, i.e. California with 13.34% and Vermont with 0.18%. The results for the other cases are available upon request to the author.

Table 3. Stationary MS Synchronization among U.S. States Business Cycle Phases

	AL	AZ	AR	CA	CO	CT	DE	FL	GA	ID	IL	IN	IA	KS	KY	LA
AL	1	0.84	0.92	0.64	0.54	0.73	0.64	0.62	0.89	0.78	0.84	0.83	0.81	0.93	0.90	0.23
AZ	0.84	1	0.75	0.84	0.55	0.85	0.82	0.92	0.94	0.73	0.73	0.65	0.44	0.75	0.66	0.25
AR	0.92	0.75	1	0.60	0.56	0.56	0.66	0.71	0.78	0.80	0.83	0.74	0.86	0.87	0.89	0.15
CA	0.64	0.84	0.60	1	0.52	0.78	0.74	0.82	0.83	0.61	0.76	0.61	0.45	0.73	0.62	0.15
CO	0.54	0.55	0.56	0.52	1	0.57	0.27	0.45	0.59	0.69	0.58	0.36	0.66	0.61	0.51	0.53
CT	0.73	0.85	0.56	0.78	0.57	1	0.89	0.68	0.80	0.36	0.66	0.49	0.53	0.62	0.58	0.14
DE	0.64	0.82	0.66	0.74	0.27	0.89	1	0.85	0.93	0.48	0.75	0.58	0.47	0.60	0.62	0.10
FL	0.62	0.92	0.71	0.82	0.45	0.68	0.85	1	0.91	0.59	0.73	0.48	0.38	0.53	0.52	0.22
GA	0.89	0.94	0.78	0.83	0.59	0.80	0.93	0.91	1	0.61	0.81	0.56	0.60	0.73	0.70	0.14
ID	0.78	0.73	0.80	0.61	0.69	0.36	0.48	0.59	0.61	1	0.62	0.64	0.86	0.87	0.66	0.33
IL	0.84	0.73	0.83	0.76	0.58	0.66	0.75	0.73	0.81	0.62	1	0.67	0.89	0.77	0.85	0.16
IN	0.83	0.65	0.74	0.61	0.36	0.49	0.58	0.48	0.56	0.64	0.67	1	0.74	0.84	0.89	0.29
IA	0.81	0.44	0.86	0.45	0.66	0.53	0.47	0.38	0.60	0.86	0.89	0.74	1	0.88	0.92	0.24
KS	0.93	0.75	0.87	0.73	0.61	0.62	0.60	0.53	0.73	0.87	0.77	0.84	0.88	1	0.92	0.27
KY	0.90	0.66	0.89	0.62	0.51	0.58	0.62	0.52	0.70	0.66	0.85	0.89	0.92	0.92	1	0.22
LA	0.23	0.25	0.15	0.15	0.53	0.14	0.10	0.22	0.14	0.33	0.16	0.29	0.24	0.27	0.22	1
ME	0.68	0.88	0.69	0.76	0.39	0.92	0.91	0.74	0.89	0.62	0.74	0.62	0.56	0.68	0.66	0.21
MD	0.63	0.90	0.67	0.84	0.46	0.77	0.91	0.73	0.83	0.64	0.63	0.52	0.49	0.53	0.52	0.17
MA	0.53	0.70	0.55	0.64	0.43	0.92	0.80	0.59	0.83	0.19	0.65	0.39	0.50	0.49	0.48	0.13
MI	0.86	0.65	0.79	0.67	0.33	0.48	0.65	0.60	0.62	0.54	0.66	0.95	0.62	0.85	0.89	0.23
MN	0.80	0.83	0.82	0.73	0.56	0.51	0.68	0.69	0.83	0.84	0.87	0.84	0.88	0.88	0.88	0.28
MS	0.86	0.51	0.84	0.49	0.37	0.43	0.44	0.40	0.62	0.77	0.74	0.81	0.80	0.82	0.87	0.23
MO	0.91	0.89	0.92	0.69	0.55	0.70	0.89	0.85	0.93	0.72	0.92	0.76	0.79	0.91	0.93	0.13
MT	0.60	0.35	0.41	0.24	0.51	0.23	0.17	0.25	0.33	0.85	0.31	0.41	0.53	0.49	0.46	0.31
NE	0.85	0.62	0.78	0.49	0.60	0.40	0.43	0.43	0.66	0.87	0.88	0.60	0.89	0.89	0.84	0.26
NV	0.80	0.93	0.66	0.85	0.48	0.41	0.72	0.91	0.84	0.75	0.67	0.73	0.65	0.73	0.67	0.24
NH	0.71	0.92	0.69	0.73	0.59	0.94	0.87	0.81	0.87	0.50	0.64	0.36	0.58	0.62	0.46	0.16
NJ	0.66	0.93	0.67	0.85	0.50	0.94	0.91	0.90	0.87	0.55	0.61	0.52	0.52	0.64	0.54	0.18
NM	0.72	0.84	0.62	0.57	0.60	0.37	0.27	0.78	0.79	0.85	0.70	0.66	0.54	0.85	0.63	0.35
NY	0.70	0.73	0.49	0.88	0.57	0.92	0.89	0.74	0.80	0.33	0.68	0.63	0.53	0.71	0.61	0.19
NC	0.92	0.91	0.88	0.82	0.62	0.76	0.92	0.78	0.95	0.65	0.84	0.82	0.62	0.91	0.90	0.17
ND	0.32	0.23	0.22	0.16	0.43	0.26	0.14	0.20	0.14	0.42	0.26	0.33	0.36	0.35	0.33	0.30
OH	0.89	0.67	0.78	0.77	0.32	0.45	0.68	0.53	0.76	0.60	0.79	0.94	0.68	0.86	0.85	0.30
OK	0.31	0.26	0.22	0.23	0.74	0.29	0.18	0.25	0.28	0.27	0.34	0.18	0.45	0.61	0.26	0.77
OR	0.89	0.71	0.90	0.62	0.47	0.41	0.74	0.50	0.63	0.73	0.91	0.88	0.91	0.79	0.91	0.27
PA	0.87	0.78	0.64	0.79	0.42	0.73	0.80	0.73	0.81	0.59	0.88	0.84	0.77	0.83	0.89	0.26
RI	0.53	0.81	0.41	0.48	0.17	0.63	0.84	0.62	0.67	0.56	0.37	0.45	0.30	0.45	0.34	0.18
SC	0.93	0.91	0.86	0.79	0.42	0.74	0.87	0.77	0.94	0.66	0.86	0.79	0.70	0.83	0.85	0.21
SD	0.89	0.63	0.89	0.57	0.55	0.44	0.35	0.43	0.47	0.85	0.73	0.73	0.90	0.87	0.85	0.27
TN	0.94	0.88	0.90	0.75	0.49	0.66	0.67	0.67	0.90	0.69	0.84	0.93	0.84	0.91	0.95	0.22
TX	0.49	0.41	0.39	0.42	0.92	0.41	0.28	0.43	0.50	0.35	0.63	0.30	0.56	0.59	0.47	0.72
UT	0.69	0.85	0.73	0.61	0.89	0.60	0.31	0.47	0.63	0.78	0.82	0.34	0.78	0.86	0.77	0.30
VT	0.74	0.90	0.58	0.71	0.48	0.91	0.84	0.79	0.89	0.53	0.68	0.64	0.49	0.74	0.52	0.22
VA	0.81	0.93	0.76	0.79	0.53	0.86	0.95	0.88	0.95	0.63	0.83	0.57	0.56	0.68	0.62	0.15
WA	0.93	0.85	0.91	0.60	0.60	0.65	0.73	0.53	0.89	0.76	0.91	0.83	0.86	0.88	0.91	0.18
WV	0.45	0.39	0.46	0.27	0.22	0.22	0.19	0.47	0.22	0.38	0.77	0.44	0.71	0.67	0.56	0.39
WI	0.92	0.78	0.87	0.69	0.59	0.60	0.68	0.58	0.81	0.79	0.84	0.89	0.91	0.93	0.92	0.27
WY	0.27	0.26	0.16	0.17	0.30	0.15	0.14	0.24	0.16	0.25	0.16	0.25	0.27	0.27	0.25	0.82

Table 3 (cont.) Stationary MS Synchronization among U.S. States Business Cycle Phases

	ME	MD	MA	MI	MN	MS	MO	MT	NE	NV	NH	NJ	NM	NY	NC	ND
AL	0.68	0.63	0.53	0.86	0.80	0.86	0.91	0.60	0.85	0.80	0.71	0.66	0.72	0.70	0.92	0.32
AZ	0.88	0.90	0.70	0.65	0.83	0.51	0.89	0.35	0.62	0.93	0.92	0.93	0.84	0.73	0.91	0.23
AR	0.69	0.67	0.55	0.79	0.82	0.84	0.92	0.41	0.78	0.66	0.69	0.67	0.62	0.49	0.88	0.22
CA	0.76	0.84	0.64	0.67	0.73	0.49	0.69	0.24	0.49	0.85	0.73	0.85	0.57	0.88	0.82	0.16
CO	0.39	0.46	0.43	0.33	0.56	0.37	0.55	0.51	0.60	0.48	0.59	0.50	0.60	0.57	0.62	0.43
CT	0.92	0.77	0.92	0.48	0.51	0.43	0.70	0.23	0.40	0.41	0.94	0.94	0.37	0.92	0.76	0.26
DE	0.91	0.91	0.80	0.65	0.68	0.44	0.89	0.17	0.43	0.72	0.87	0.91	0.27	0.89	0.92	0.14
FL	0.74	0.73	0.59	0.60	0.69	0.40	0.85	0.25	0.43	0.91	0.81	0.90	0.78	0.74	0.78	0.20
GA	0.89	0.83	0.83	0.62	0.83	0.62	0.93	0.33	0.66	0.84	0.87	0.87	0.79	0.80	0.95	0.14
ID	0.62	0.64	0.19	0.54	0.84	0.77	0.72	0.85	0.87	0.75	0.50	0.55	0.85	0.33	0.65	0.42
IL	0.74	0.63	0.65	0.66	0.87	0.74	0.92	0.31	0.88	0.67	0.64	0.61	0.70	0.68	0.84	0.26
IN	0.62	0.52	0.39	0.95	0.84	0.81	0.76	0.41	0.60	0.73	0.36	0.52	0.66	0.63	0.82	0.33
IA	0.56	0.49	0.50	0.62	0.88	0.80	0.79	0.53	0.89	0.65	0.58	0.52	0.54	0.53	0.62	0.36
KS	0.68	0.53	0.49	0.85	0.88	0.82	0.91	0.49	0.89	0.73	0.62	0.64	0.85	0.71	0.91	0.35
KY	0.66	0.52	0.48	0.89	0.88	0.87	0.93	0.46	0.84	0.67	0.46	0.54	0.63	0.61	0.90	0.33
LA	0.21	0.17	0.13	0.23	0.28	0.23	0.13	0.31	0.26	0.24	0.16	0.18	0.35	0.19	0.17	0.30
ME	1	0.92	0.90	0.69	0.69	0.54	0.80	0.43	0.61	0.75	0.90	0.93	0.56	0.87	0.86	0.18
MD	0.92	1	0.76	0.59	0.75	0.37	0.82	0.31	0.54	0.66	0.79	0.92	0.37	0.82	0.75	0.20
MA	0.90	0.76	1	0.50	0.42	0.35	0.53	0.17	0.32	0.30	0.93	0.91	0.23	0.88	0.74	0.17
MI	0.69	0.59	0.50	1	0.87	0.81	0.86	0.31	0.54	0.61	0.46	0.60	0.67	0.69	0.88	0.32
MN	0.69	0.75	0.42	0.87	1	0.72	0.89	0.48	0.87	0.85	0.62	0.58	0.72	0.66	0.86	0.42
MS	0.54	0.37	0.35	0.81	0.72	1	0.84	0.57	0.75	0.78	0.41	0.33	0.71	0.52	0.86	0.27
MO	0.80	0.82	0.53	0.86	0.89	0.84	1	0.47	0.88	0.84	0.72	0.75	0.79	0.69	0.94	0.31
MT	0.43	0.31	0.17	0.31	0.48	0.57	0.47	1	0.62	0.40	0.23	0.32	0.67	0.17	0.40	0.34
NE	0.61	0.54	0.32	0.54	0.87	0.75	0.88	0.62	1	0.77	0.44	0.47	0.81	0.42	0.74	0.39
NV	0.75	0.66	0.30	0.61	0.85	0.78	0.84	0.40	0.77	1	0.67	0.72	0.91	0.46	0.78	0.23
NH	0.90	0.79	0.93	0.46	0.62	0.41	0.72	0.23	0.44	0.67	1	0.94	0.53	0.84	0.82	0.15
NJ	0.93	0.92	0.91	0.60	0.58	0.33	0.75	0.32	0.47	0.72	0.94	1	0.59	0.93	0.85	0.15
NM	0.56	0.37	0.23	0.67	0.72	0.71	0.79	0.67	0.81	0.91	0.53	0.59	1	0.37	0.81	0.29
NY	0.87	0.82	0.88	0.69	0.66	0.52	0.69	0.17	0.42	0.46	0.84	0.93	0.37	1	0.83	0.27
NC	0.86	0.75	0.74	0.88	0.86	0.86	0.94	0.40	0.74	0.78	0.82	0.85	0.81	0.83	1	0.30
ND	0.18	0.20	0.17	0.32	0.42	0.27	0.31	0.34	0.39	0.23	0.15	0.15	0.29	0.27	0.30	1
OH	0.62	0.52	0.45	0.95	0.89	0.82	0.89	0.35	0.74	0.68	0.55	0.57	0.63	0.64	0.91	0.39
OK	0.24	0.21	0.25	0.19	0.35	0.27	0.27	0.24	0.45	0.21	0.25	0.22	0.26	0.26	0.31	0.43
OR	0.74	0.65	0.44	0.85	0.89	0.85	0.77	0.42	0.79	0.60	0.48	0.59	0.56	0.63	0.79	0.34
PA	0.80	0.78	0.64	0.81	0.88	0.79	0.91	0.38	0.82	0.75	0.67	0.77	0.78	0.89	0.87	0.42
RI	0.89	0.58	0.64	0.54	0.45	0.39	0.45	0.30	0.40	0.48	0.83	0.80	0.37	0.54	0.51	0.18
SC	0.80	0.84	0.73	0.87	0.86	0.82	0.92	0.43	0.72	0.82	0.79	0.75	0.64	0.81	0.95	0.26
SD	0.64	0.54	0.38	0.68	0.88	0.77	0.75	0.76	0.90	0.60	0.45	0.41	0.71	0.54	0.75	0.38
TN	0.74	0.57	0.51	0.94	0.90	0.87	0.94	0.40	0.79	0.80	0.68	0.70	0.69	0.79	0.94	0.30
TX	0.29	0.28	0.45	0.26	0.55	0.36	0.50	0.21	0.54	0.27	0.48	0.34	0.59	0.50	0.48	0.48
UT	0.47	0.51	0.41	0.34	0.69	0.67	0.74	0.48	0.67	0.76	0.61	0.58	0.88	0.63	0.70	0.35
VT	0.91	0.79	0.87	0.75	0.70	0.52	0.81	0.29	0.58	0.69	0.91	0.94	0.55	0.83	0.89	0.24
VA	0.92	0.94	0.83	0.73	0.83	0.54	0.91	0.35	0.58	0.77	0.81	0.93	0.47	0.84	0.93	0.18
WA	0.67	0.76	0.63	0.80	0.86	0.83	0.91	0.50	0.83	0.84	0.65	0.65	0.63	0.57	0.89	0.25
WV	0.38	0.25	0.19	0.47	0.74	0.72	0.61	0.34	0.67	0.37	0.29	0.29	0.41	0.31	0.37	0.53
WI	0.73	0.46	0.52	0.90	0.92	0.87	0.93	0.43	0.91	0.77	0.60	0.55	0.82	0.74	0.93	0.44
WY	0.22	0.20	0.14	0.22	0.30	0.27	0.16	0.25	0.27	0.25	0.15	0.20	0.37	0.22	0.20	0.31

Table 3 (cont.) Stationary MS Synchronization among U.S. States Business Cycle Phases

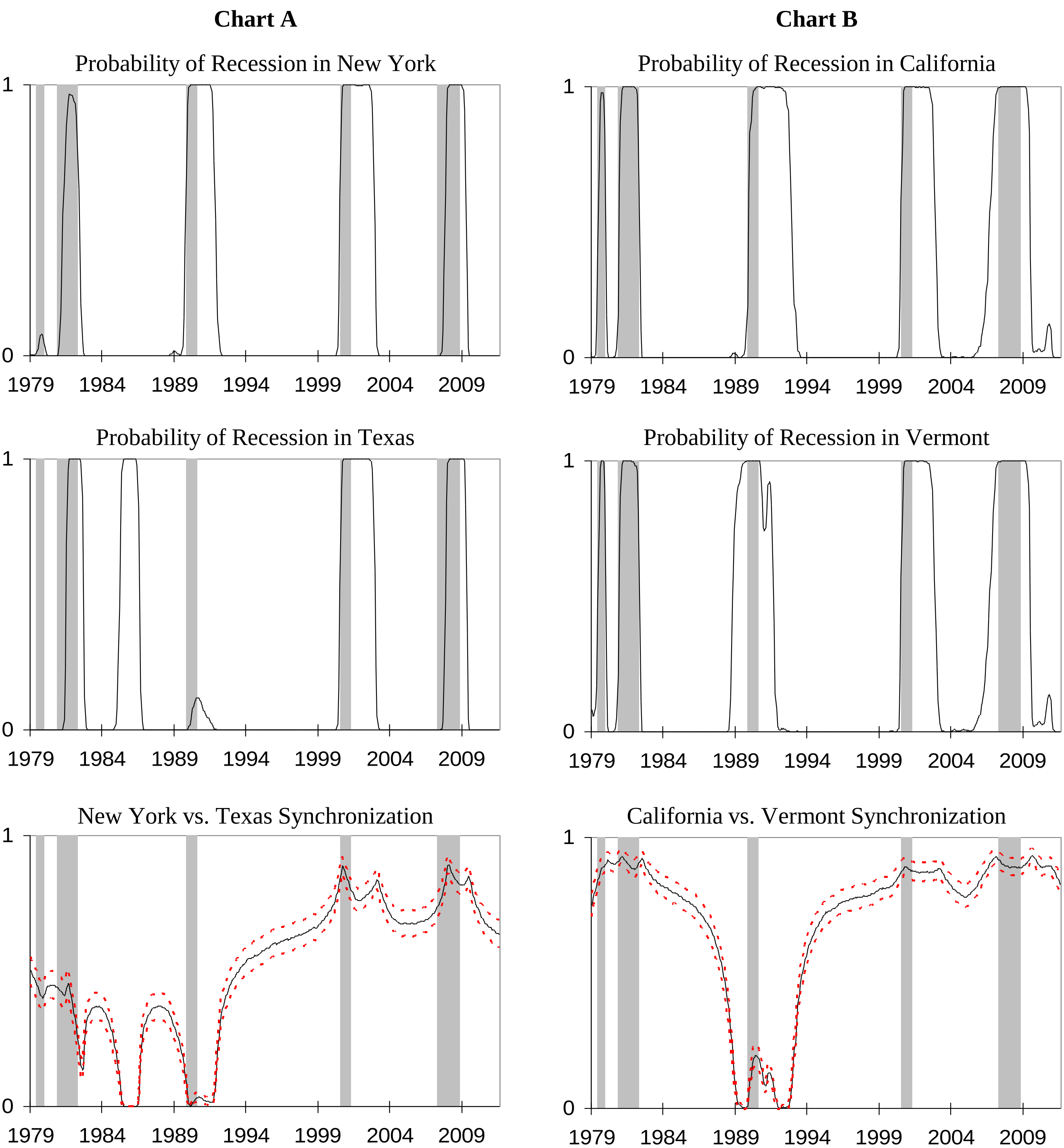
	OH	OK	OR	PA	RI	SC	SD	TN	TX	UT	VT	VA	WA	WV	WI	WY
AL	0.89	0.31	0.89	0.87	0.53	0.93	0.89	0.94	0.49	0.69	0.74	0.81	0.93	0.45	0.92	0.27
AZ	0.67	0.26	0.71	0.78	0.81	0.91	0.63	0.88	0.41	0.85	0.90	0.93	0.85	0.39	0.78	0.26
AR	0.78	0.22	0.90	0.64	0.41	0.86	0.89	0.90	0.39	0.73	0.58	0.76	0.91	0.46	0.87	0.16
CA	0.77	0.23	0.62	0.79	0.48	0.79	0.57	0.75	0.42	0.61	0.71	0.79	0.60	0.27	0.69	0.17
CO	0.32	0.74	0.47	0.42	0.17	0.42	0.55	0.49	0.92	0.89	0.48	0.53	0.60	0.22	0.59	0.30
CT	0.45	0.29	0.41	0.73	0.63	0.74	0.44	0.66	0.41	0.60	0.91	0.86	0.65	0.22	0.60	0.15
DE	0.68	0.18	0.74	0.80	0.84	0.87	0.35	0.67	0.28	0.31	0.84	0.95	0.73	0.19	0.68	0.14
FL	0.53	0.25	0.50	0.73	0.62	0.77	0.43	0.67	0.43	0.47	0.79	0.88	0.53	0.47	0.58	0.24
GA	0.76	0.28	0.63	0.81	0.67	0.94	0.47	0.90	0.50	0.63	0.89	0.95	0.89	0.22	0.81	0.16
ID	0.60	0.27	0.73	0.59	0.56	0.66	0.85	0.69	0.35	0.78	0.53	0.63	0.76	0.38	0.79	0.25
IL	0.79	0.34	0.91	0.88	0.37	0.86	0.73	0.84	0.63	0.82	0.68	0.83	0.91	0.77	0.84	0.16
IN	0.94	0.18	0.88	0.84	0.45	0.79	0.73	0.93	0.30	0.34	0.64	0.57	0.83	0.44	0.89	0.25
IA	0.68	0.45	0.91	0.77	0.30	0.70	0.90	0.84	0.56	0.78	0.49	0.56	0.86	0.71	0.91	0.27
KS	0.86	0.61	0.79	0.83	0.45	0.83	0.87	0.91	0.59	0.86	0.74	0.68	0.88	0.67	0.93	0.27
KY	0.85	0.26	0.91	0.89	0.34	0.85	0.85	0.95	0.47	0.77	0.52	0.62	0.91	0.56	0.92	0.25
LA	0.30	0.77	0.27	0.26	0.18	0.21	0.27	0.22	0.72	0.30	0.22	0.15	0.18	0.39	0.27	0.82
ME	0.62	0.24	0.74	0.80	0.89	0.80	0.64	0.74	0.29	0.47	0.91	0.92	0.67	0.38	0.73	0.22
MD	0.52	0.21	0.65	0.78	0.58	0.84	0.54	0.57	0.28	0.51	0.79	0.94	0.76	0.25	0.46	0.20
MA	0.45	0.25	0.44	0.64	0.64	0.73	0.38	0.51	0.45	0.41	0.87	0.83	0.63	0.19	0.52	0.14
MI	0.95	0.19	0.85	0.81	0.54	0.87	0.68	0.94	0.26	0.34	0.75	0.73	0.80	0.47	0.90	0.22
MN	0.89	0.35	0.89	0.88	0.45	0.86	0.88	0.90	0.55	0.69	0.70	0.83	0.86	0.74	0.92	0.30
MS	0.82	0.27	0.85	0.79	0.39	0.82	0.77	0.87	0.36	0.67	0.52	0.54	0.83	0.72	0.87	0.27
MO	0.89	0.27	0.77	0.91	0.45	0.92	0.75	0.94	0.50	0.74	0.81	0.91	0.91	0.61	0.93	0.16
MT	0.35	0.24	0.42	0.38	0.30	0.43	0.76	0.40	0.21	0.48	0.29	0.35	0.50	0.34	0.43	0.25
NE	0.74	0.45	0.79	0.82	0.40	0.72	0.90	0.79	0.54	0.67	0.58	0.58	0.83	0.67	0.91	0.27
NV	0.68	0.21	0.60	0.75	0.48	0.82	0.60	0.80	0.27	0.76	0.69	0.77	0.84	0.37	0.77	0.25
NH	0.55	0.25	0.48	0.67	0.83	0.79	0.45	0.68	0.48	0.61	0.91	0.81	0.65	0.29	0.60	0.15
NJ	0.57	0.22	0.59	0.77	0.80	0.75	0.41	0.70	0.34	0.58	0.94	0.93	0.65	0.29	0.55	0.20
NM	0.63	0.26	0.56	0.78	0.37	0.64	0.71	0.69	0.59	0.88	0.55	0.47	0.63	0.41	0.82	0.37
NY	0.64	0.26	0.63	0.89	0.54	0.81	0.54	0.79	0.50	0.63	0.83	0.84	0.57	0.31	0.74	0.22
NC	0.91	0.31	0.79	0.87	0.51	0.95	0.75	0.94	0.48	0.70	0.89	0.93	0.89	0.37	0.93	0.20
ND	0.39	0.43	0.34	0.42	0.18	0.26	0.38	0.30	0.48	0.35	0.24	0.18	0.25	0.53	0.44	0.31
OH	1	0.17	0.86	0.84	0.45	0.84	0.76	0.93	0.29	0.36	0.75	0.74	0.82	0.62	0.92	0.26
OK	0.17	1	0.22	0.33	0.16	0.24	0.35	0.22	0.83	0.42	0.27	0.22	0.31	0.48	0.22	0.81
OR	0.86	0.22	1	0.76	0.43	0.86	0.89	0.87	0.30	0.59	0.66	0.73	0.89	0.44	0.89	0.23
PA	0.84	0.33	0.76	1	0.63	0.86	0.70	0.92	0.52	0.62	0.84	0.86	0.82	0.71	0.92	0.30
RI	0.45	0.16	0.43	0.63	1	0.65	0.49	0.48	0.17	0.19	0.84	0.69	0.41	0.32	0.51	0.18
SC	0.84	0.24	0.86	0.86	0.65	1	0.76	0.94	0.41	0.54	0.76	0.94	0.90	0.41	0.90	0.24
SD	0.76	0.35	0.89	0.70	0.49	0.76	1	0.82	0.42	0.64	0.46	0.51	0.84	0.55	0.89	0.26
TN	0.93	0.22	0.87	0.92	0.48	0.94	0.82	1	0.40	0.73	0.74	0.81	0.91	0.52	0.95	0.27
TX	0.29	0.83	0.30	0.52	0.17	0.41	0.42	0.40	1	0.76	0.35	0.33	0.36	0.34	0.59	0.61
UT	0.36	0.42	0.59	0.62	0.19	0.54	0.64	0.73	0.76	1	0.60	0.51	0.85	0.51	0.87	0.33
VT	0.75	0.27	0.66	0.84	0.84	0.76	0.46	0.74	0.35	0.60	1	0.88	0.59	0.35	0.76	0.18
VA	0.74	0.22	0.73	0.86	0.69	0.94	0.51	0.81	0.33	0.51	0.88	1	0.80	0.21	0.77	0.16
WA	0.82	0.31	0.89	0.82	0.41	0.90	0.84	0.91	0.36	0.85	0.59	0.80	1	0.36	0.92	0.20
WV	0.62	0.48	0.44	0.71	0.32	0.41	0.55	0.52	0.34	0.51	0.35	0.21	0.36	1	0.77	0.47
WI	0.92	0.22	0.89	0.92	0.51	0.90	0.89	0.95	0.59	0.87	0.76	0.77	0.92	0.77	1	0.31
WY	0.26	0.81	0.23	0.30	0.18	0.24	0.26	0.27	0.61	0.33	0.18	0.16	0.20	0.47	0.31	1

Table 4. Stationary MS Synchronization between State and U.S. business cycle phases

State	Sync	State	Sync	State	Sync	State	Sync
Alabama	0.92	Iowa	0.64	Nebraska	0.61	Rhode Island	0.58
Arizona	0.79	Kansas	0.85	Nevada	0.75	S. Carolina	0.92
Arkansas	0.71	Kentucky	0.84	N. Hampshire	0.60	S. Dakota	0.71
California	0.87	Louisiana	0.24	New Jersey	0.78	Tennessee	0.93
Colorado	0.41	Maine	0.80	New Mexico	0.75	Texas	0.38
Connecticut	0.66	Maryland	0.48	New York	0.77	Utah	0.72
Delaware	0.80	Massachusetts	0.44	N. Carolina	0.94	Vermont	0.85
Florida	0.68	Michigan	0.90	N. Dakota	0.32	Virginia	0.90
Georgia	0.80	Minnesota	0.90	Ohio	0.92	Washington	0.75
Idaho	0.62	Mississippi	0.85	Oklahoma	0.22	W. Virginia	0.55
Illinois	0.86	Missouri	0.84	Oregon	0.74	Wisconsin	0.87
Indiana	0.92	Montana	0.39	Pennsylvania	0.88	Wyoming	0.28

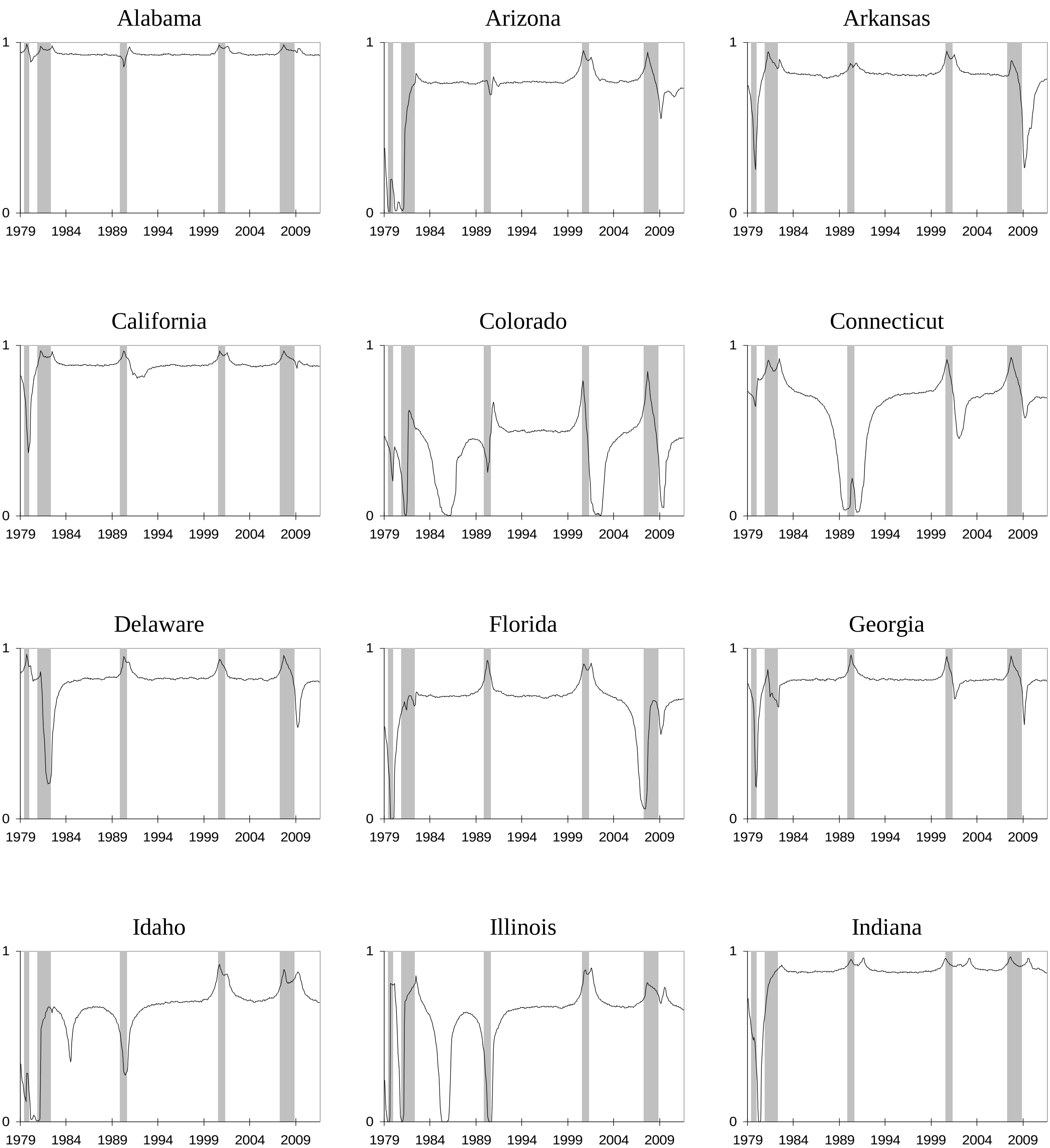
Note: The table reports the stationary degree of synchronization for 1979:8 to 2012:3. Those estimates correspond to the ergodic probability that the business cycle phases of the corresponding state and the one of the U.S. economic activity are totally dependent, i.e. $\Pr(V_t=1)$.

Figure 1. MS Synchronization between Selected States



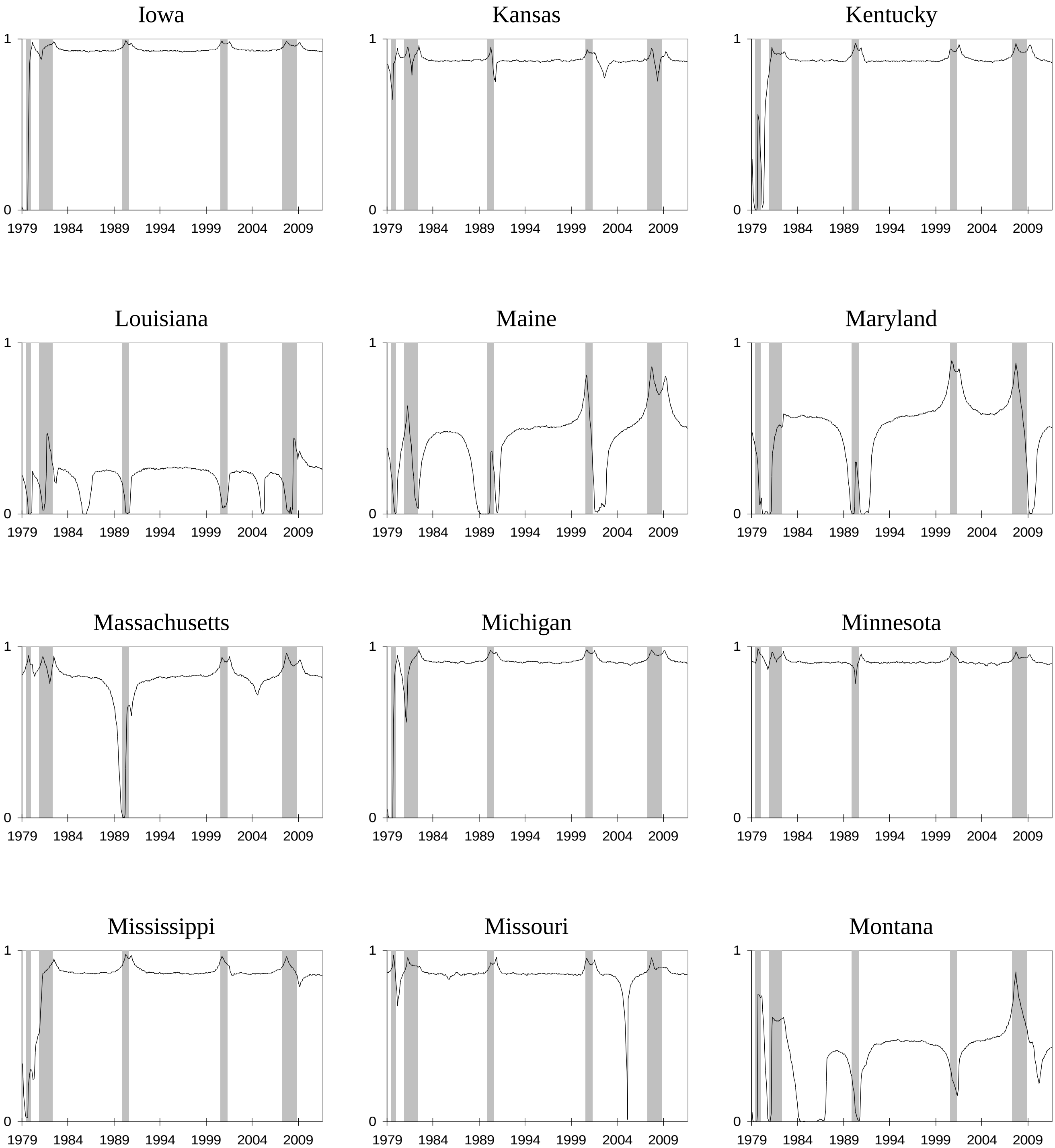
Note. The Chart A of the figure plots the probabilities of recession for New York and Texas and its time-varying synchronization. The Chart B of the figure plots the probabilities of recession for California and Vermont and its time-varying synchronization. Shaded areas correspond to NBER recessions. The full set of pairwise cases can be available upon request to the author.

Figure 2. MS Synchronization between States and U.S. Economic Activity



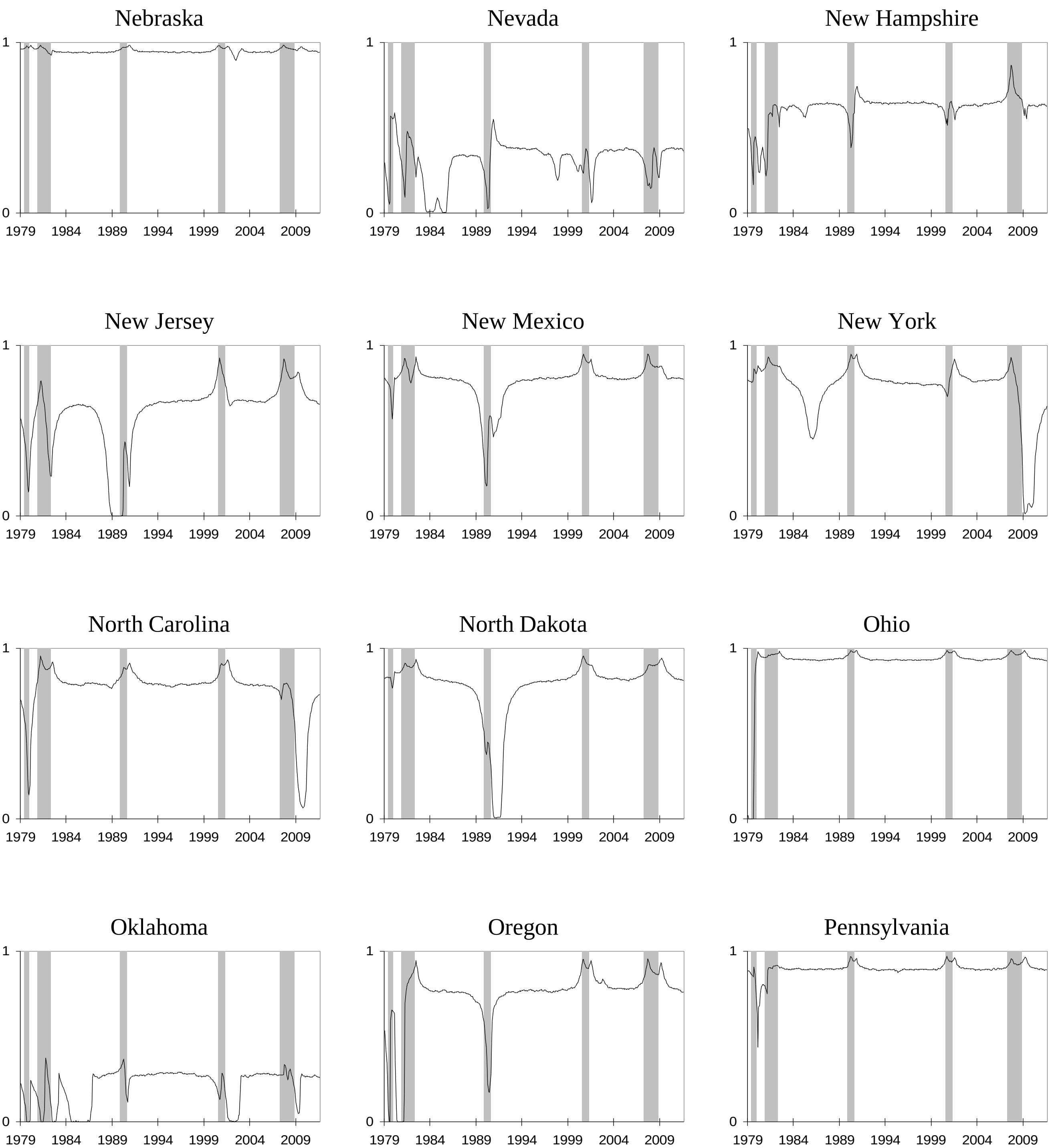
Note. Each chart plots the time-varying degree of synchronization between the U.S. business cycle phases and the ones of each State. Shaded areas correspond to NBER-referenced recessions.

Figure 2 (continued). MS Synchronization between States and U.S. Economic Activity



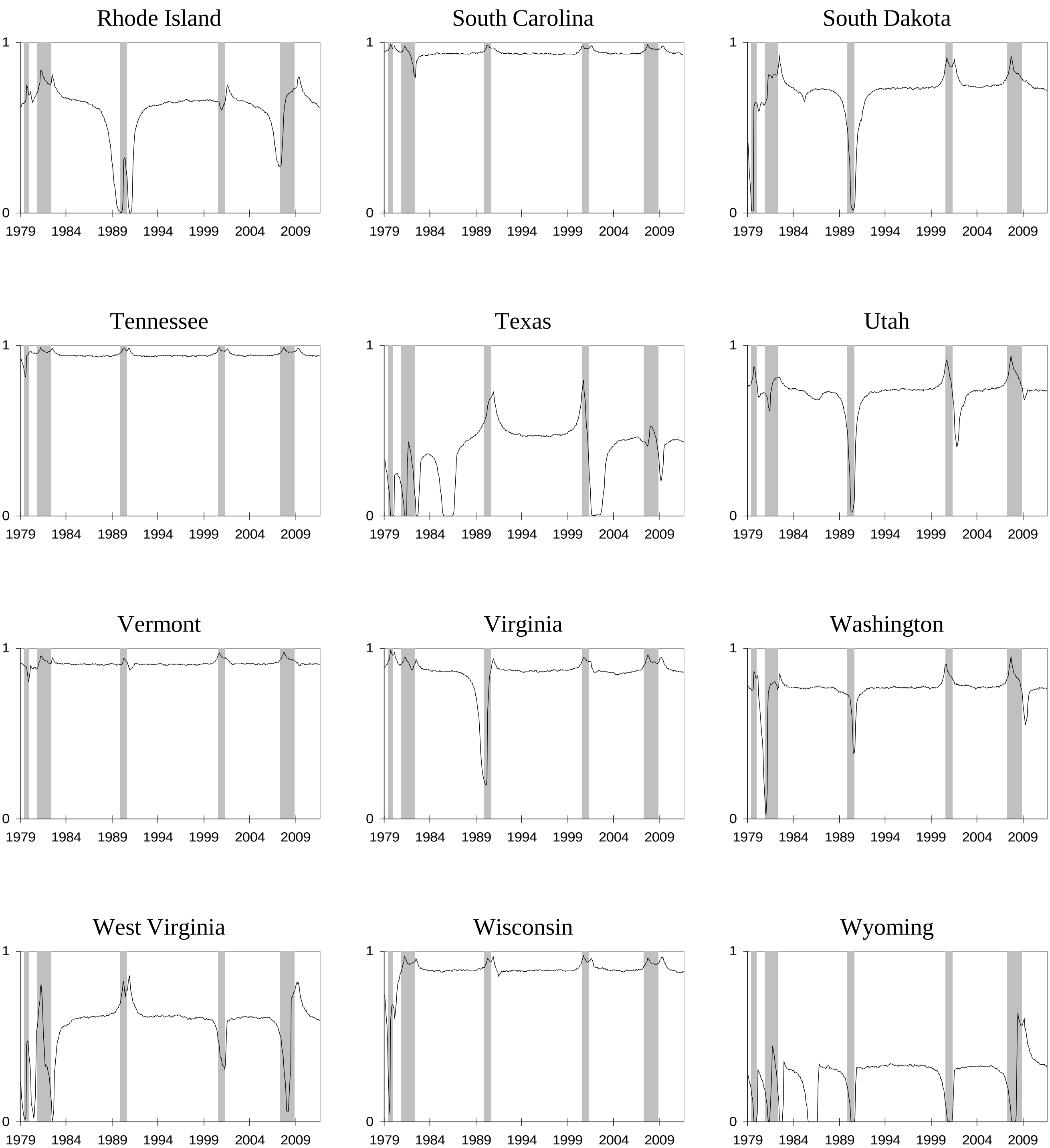
Note. Each chart plots the time-varying degree of synchronization between the U.S. business cycle phases and the ones of each State. Shaded areas correspond to NBER-referenced recessions.

Figure 2 (continued). MS Synchronization between States and U.S. Economic Activity



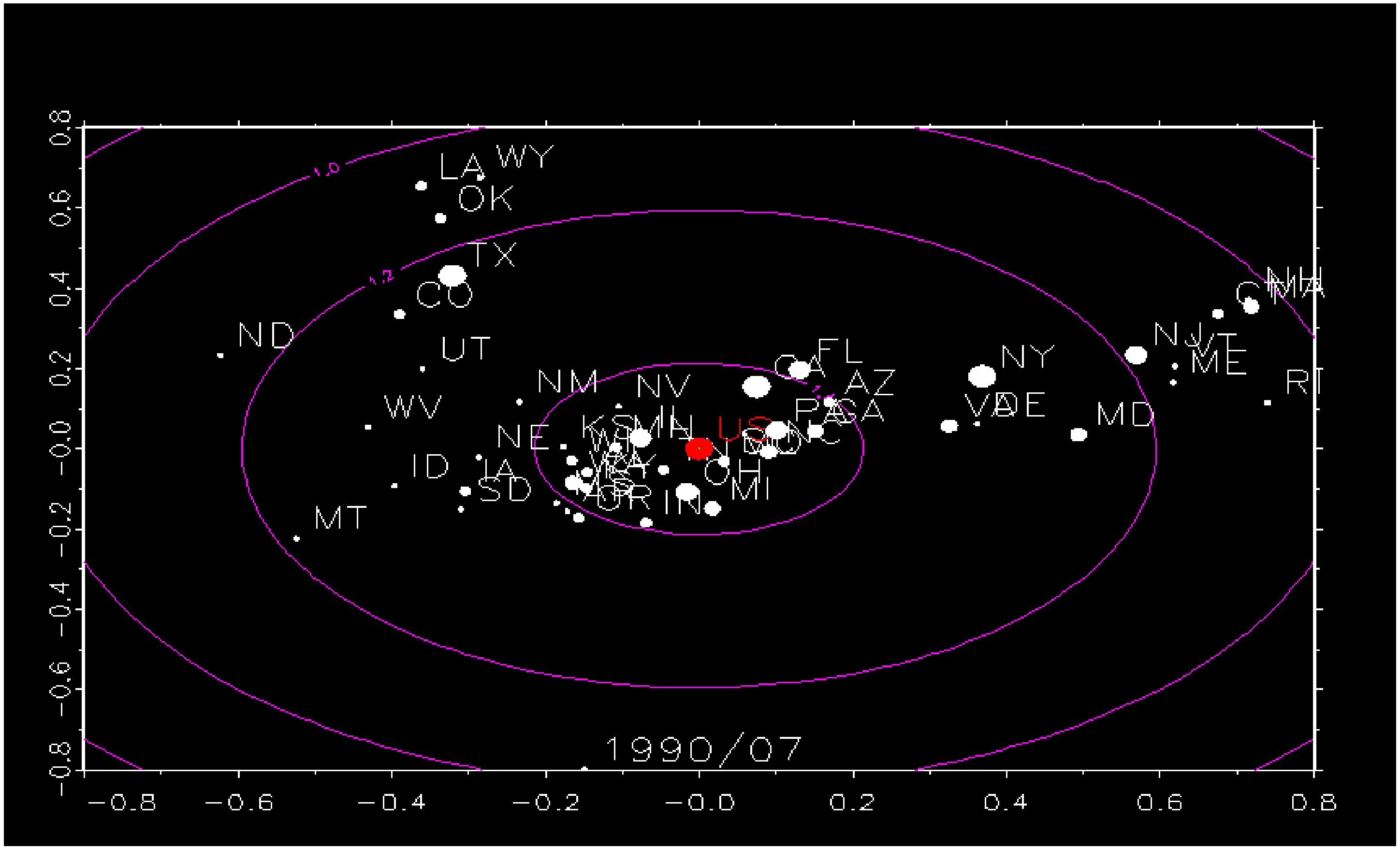
Note. Each chart plots the time-varying degree of synchronization between the U.S. business cycle phases and the ones of each State. Shaded areas correspond to NBER-referenced recessions.

Figure 2 (continued). MS Synchronization between States and U.S. Economic Activity



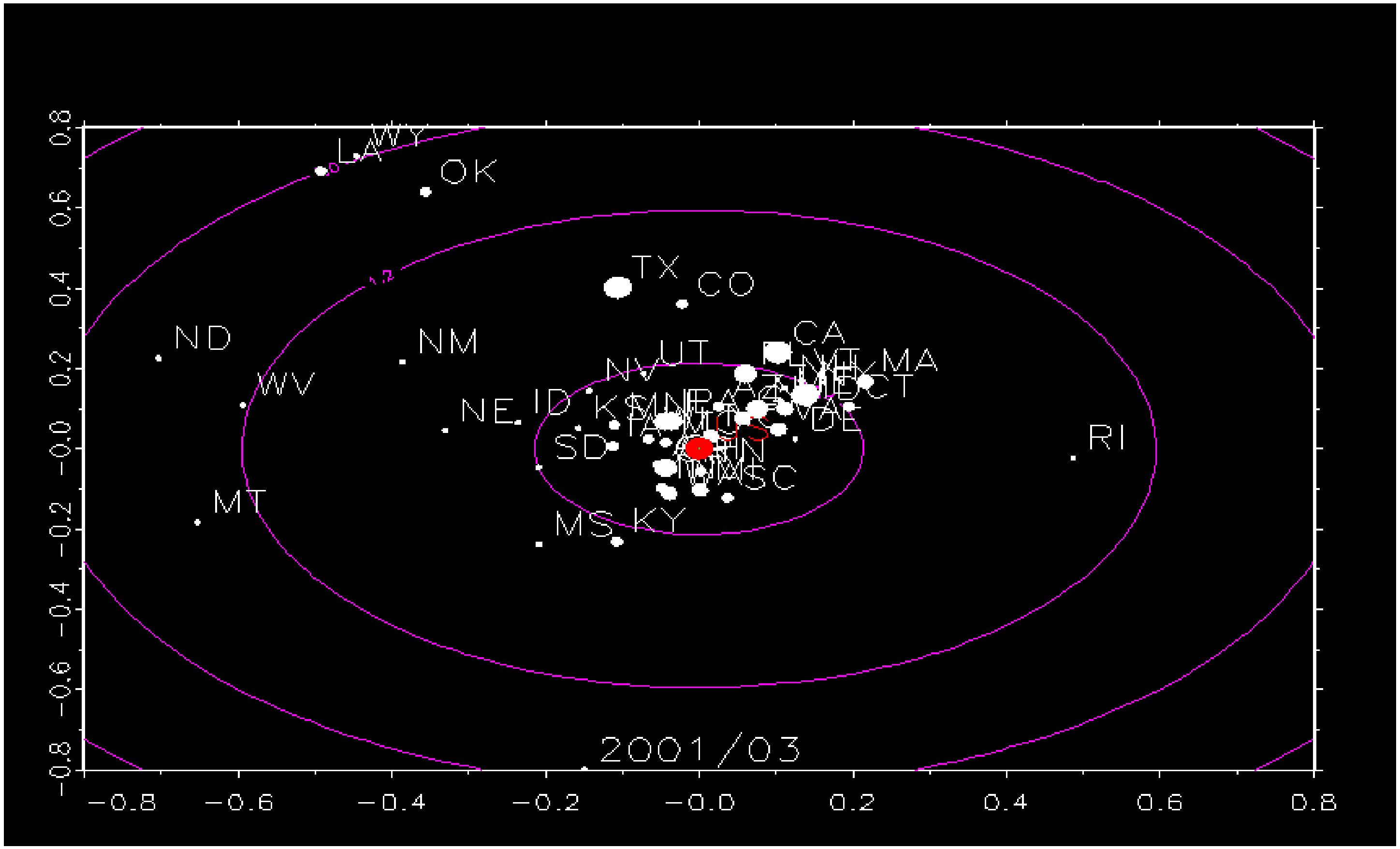
Note. Each chart plots the time-varying degree of synchronization between the U.S. business cycle phases and the ones of each State. Shaded areas correspond to NBER-referenced recessions.

Figure 3. U.S. States Synchronization Map for the 1990's recession



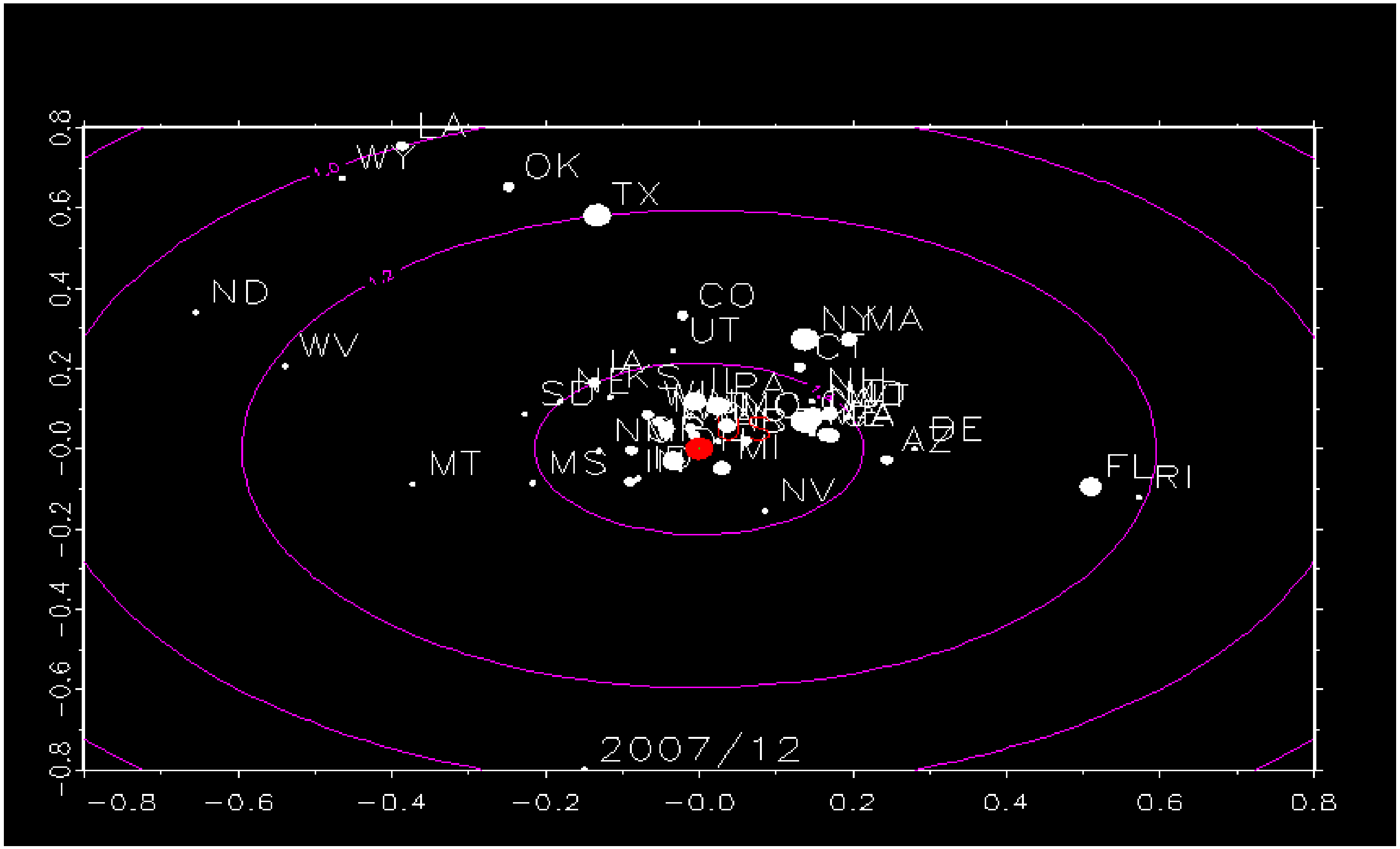
Note. The figure plots the multidimensional scaling map based on the Euclidean distance of the U.S. States business cycle characteristics for July 1990, which is the beginning of a recession. The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The size of the points makes reference to the GDP share of the corresponding state. The full animated version is available upon request to the author.

Figure 4. U.S. States Synchronization Map for the 2001's recession



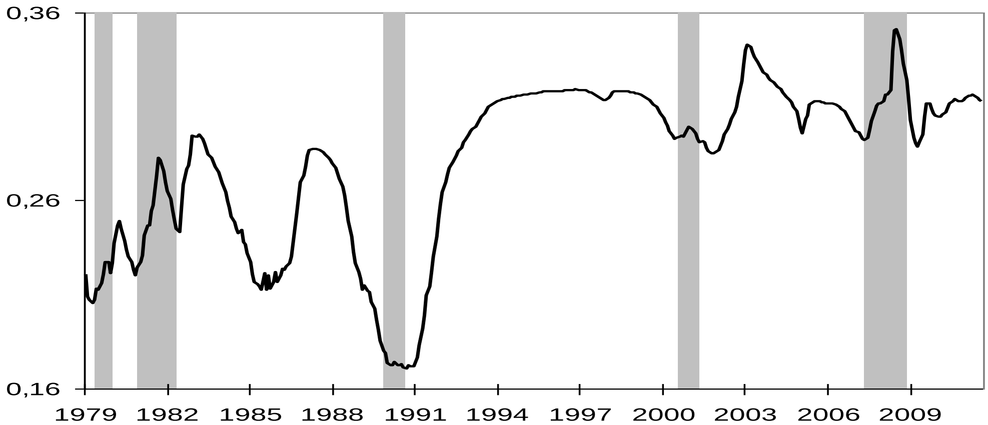
Note. The figure plots the multidimensional scaling map based on the Euclidean distance of the U.S. States business cycle characteristics for March 2001, which is the beginning of a recession. The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The size of the points makes reference to the GDP share of the corresponding state. The full animated version is available upon request to the author.

Figure 5. U.S. States Synchronization Map for the 2007's recession



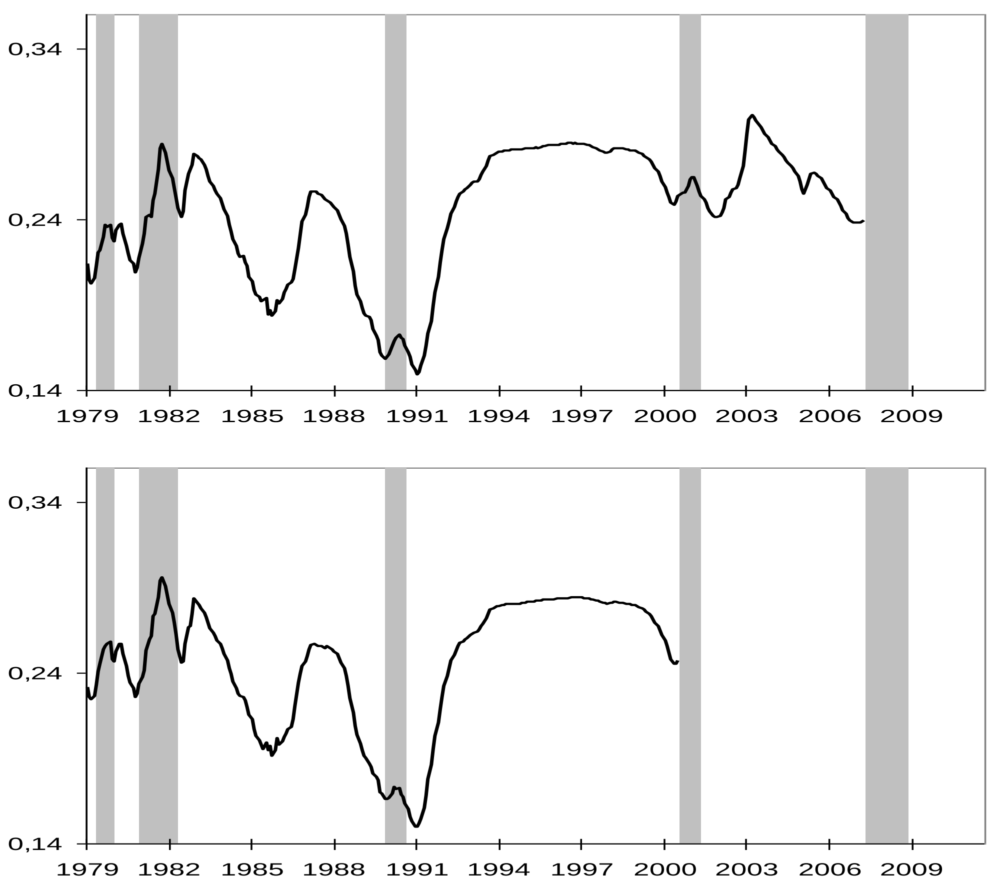
Note. The figure plots the multidimensional scaling map based on the Euclidean distance of the U.S. States business cycle characteristics for December 2007, which is the beginning of the so called "great recession". The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The size of the points makes reference to the GDP share of the corresponding state. The full animated version is available upon request to the author.

Figure 6. Cluster Coefficient



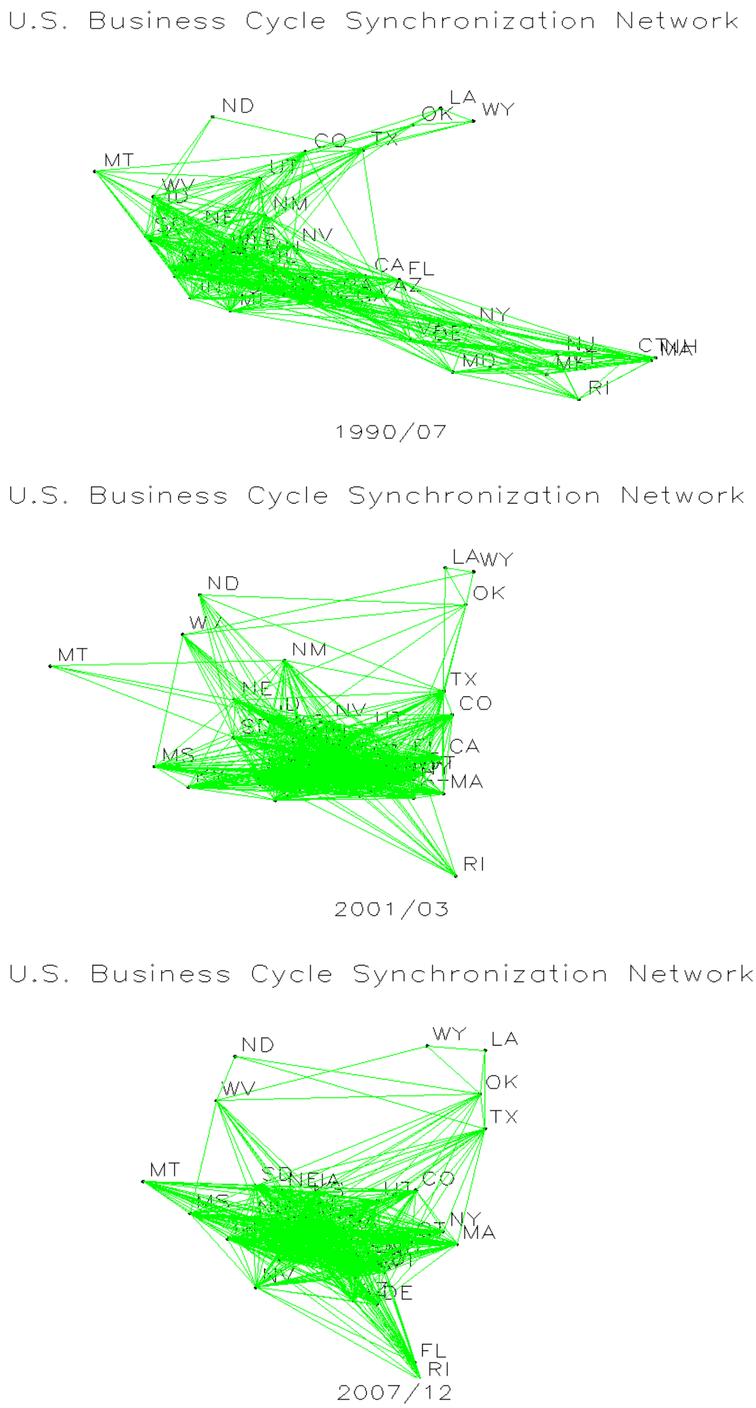
Note. The figure plots the cluster coefficient of the synchronization network composed by the U.S. State for each period of time. Shaded areas correspond to recessions as documented by the NBER.

Figure 7. Real Time Estimates of Closeness Centrality



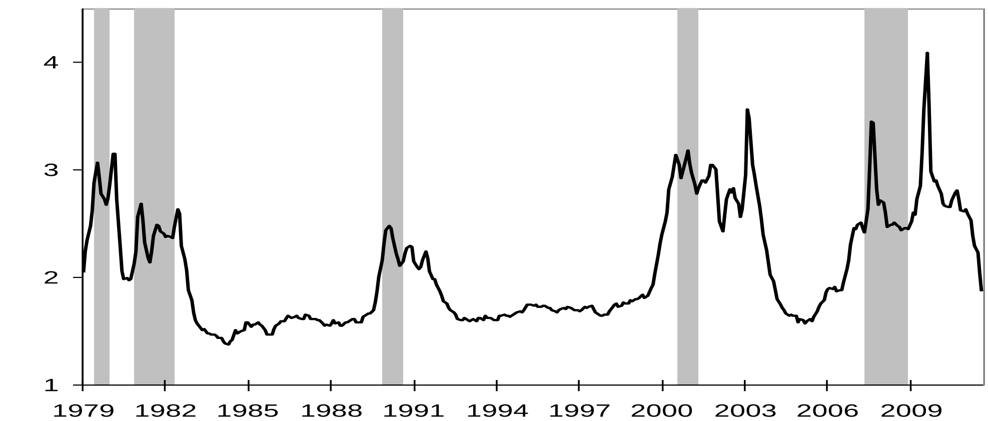
Note. The top chart plots the dynamic cluster coefficient with the data until 12/2007. The bottom chart plots the dynamic cluster coefficient with the data until 03/2001. Shaded areas correspond to recessions as documented by the NBER.

Figure 8. Synchronization Network for the Beginning of Recessions



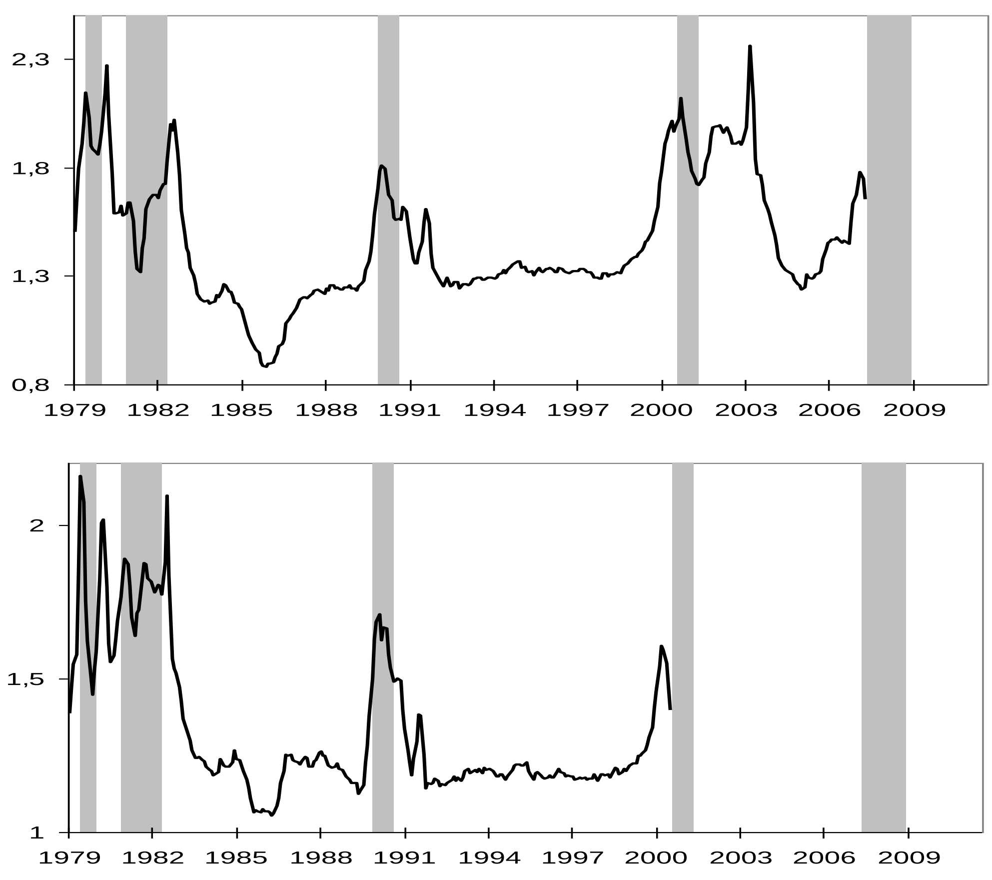
Note. The figure plots the interconnectedness in terms of synchronization between the business cycle phases of U.S. States. Each node represent a State, and the green each green line represent the link between two states, which take place only if $\Pr(V_t{=}1)>0{,}5$. The full animated version is available upon request to the author.

Figure 9. Closeness Centrality



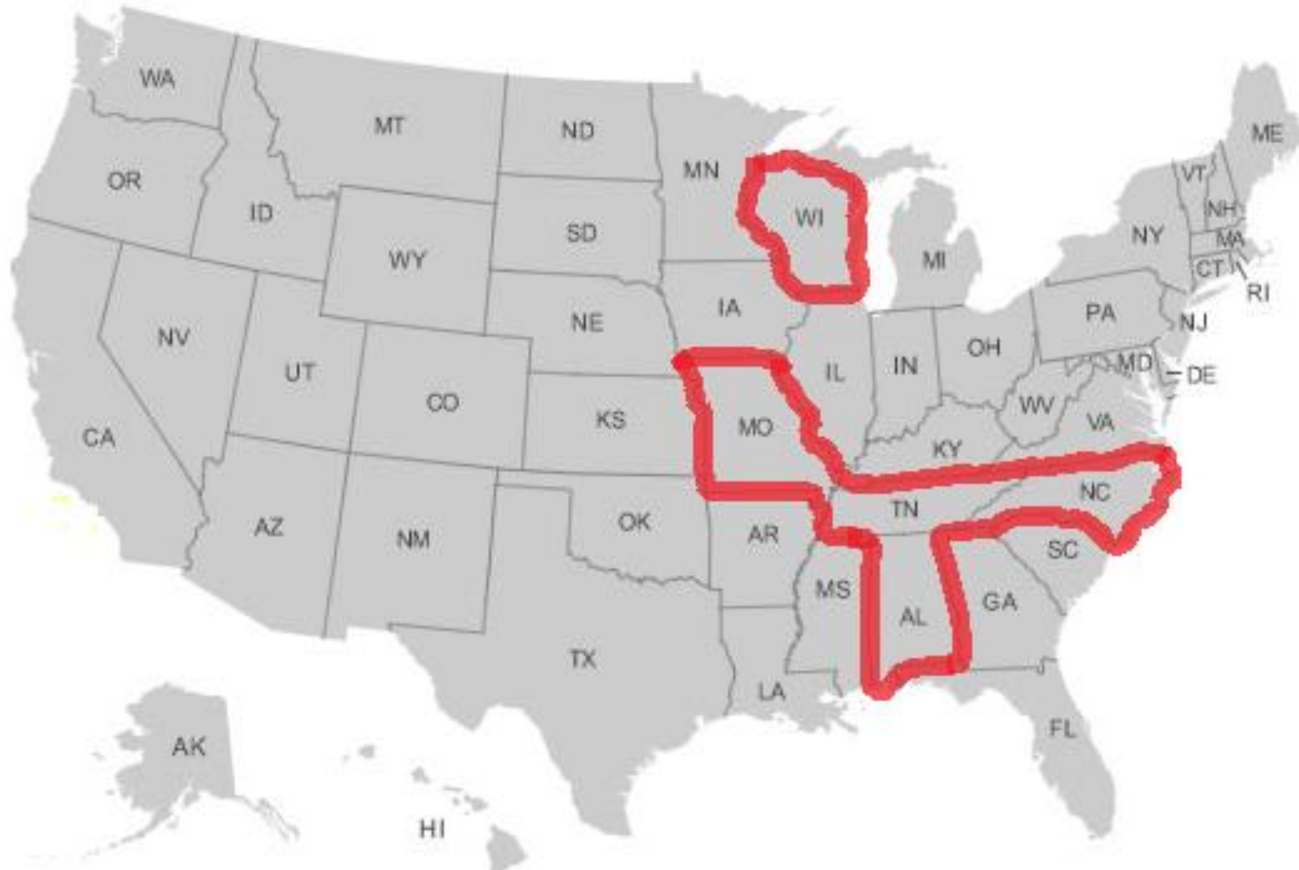
Note. The figure plots the closeness centrality of the synchronization network composed by the U.S. state for each period of time. Shaded areas correspond to recessions as documented by the NBER.

Figure 10. Real Time Estimates of Closeness Centrality



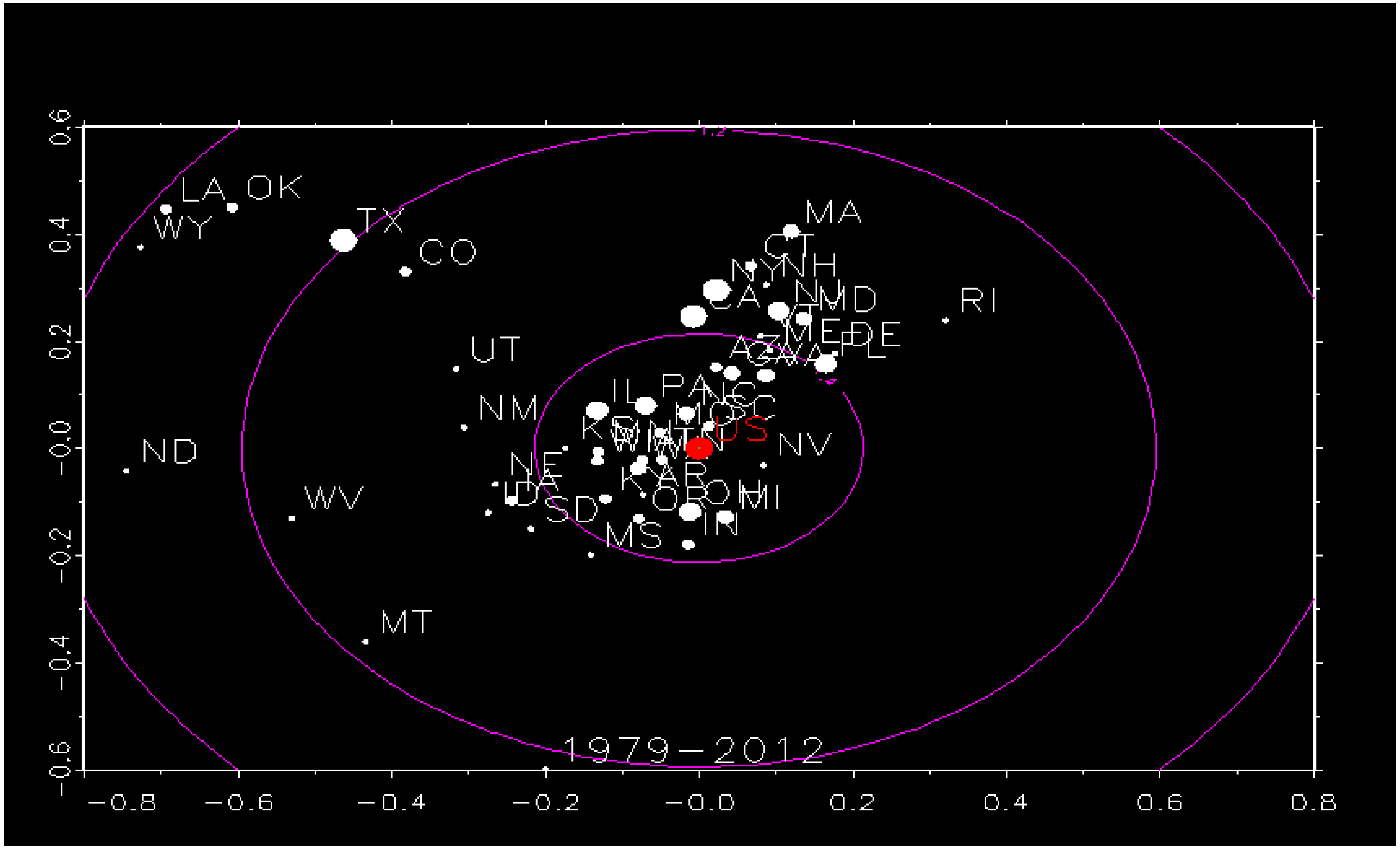
Note. The top chart plots the dynamic closeness centrality with the data until 12/2007. The bottom chart plots the dynamic closeness centrality with the data until 03/2001. Shaded areas correspond to recessions as documented by the NBER.

Figure 11. U.S. States with Highest Centrality during 1979:8 – 2012:3



Note. The states inside the marked area correspond to Alabama, Missouri, North Carolina and Tennessee, which are the states that have shown the highest closeness centrality in the Markov-switching synchronization network alternating among periods during 1979:8 until 2012:3.

Figure 12. U.S. States Stationary Synchronization Map



Note. The figure plots the multidimensional scaling map based on the stationary Euclidean distance of the U.S. States business cycle characteristics for the sample under study, August 1979-March 2010., The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The size of the points make reference to the GDP share of the corresponding state. The full animated version is available upon request to the author.