When Brute Force Fails: An Economic Formalization *

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Abstract

This paper develops a model where a government simultaneously confronts heterogeneous drug trafficking organizations (DTOs). If the government allocates its resources according to a violence ranking between those organizations, then the DTOs' problem is strategic as their utility depends on the violence decision of others. We elicit two situations. In the first one the government induces a separating equilibrium with two different strategies, while in the second it induces a pooling equilibrium. In the first situation, the optimal allocation of resources results in a situation where the resources allocated to each DTO are somehow proportional to their levels of violence. In the second one, by means of making an announcement that it will concentrate all of its resources in the most violent DTO, it induces a pooling equilibrium in which all DTOs decide on an inefficiently low level of violence (from the DTO's perspective, of course), thus formalizing Mark Kleiman's dynamic concentration theory.

Keywords: Deterrence, optimal enforcement, violence.

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1 Introduction

In all countries in which drug trafficking or drug producing takes place, drug related violence is a central public policy problem. In those countries, Drug Trafficking Organizations (DTOs) have become the *de facto* rulers in vast areas within their territories. Nevertheless, Mexico has been lately the country with the most acute and visible problem of drug related violence. In the six years of Felipe Calderón's administration, more than 60,000 drug related killings occurred. During that administration, the Mexican Federal Government declared an open war against DTOs. The fact that the sharp increase of drug related homicides coincides approximately with the beginning of the war has generated a wide controversy regarding whether or not such increase in violence is related to the federal government's strategy of directly confronting DTOs.

On the one hand, authors like Guerrero (2010) have argued that most of the escalation of violence results from the federal government's strategy. For instance, the dismantling of DTOs has been driven by a beheading strategy. Several DTO's leaders have been neutralized to date, either being killed or captured by the government. However, this led to new surges of violence in the areas where their criminal work took place. Guerrero explains this fact as ensuing from tensions within the DTO to decide who will occupy the vacant position, and from other DTOs taking advantage of the momentary void of power in order to expand their business. The beheading strategy has thus generated a spread of violence throughout the Mexican territory, leading to several harmful side effects. For instance, many areas of Mexico have witnessed increasing rates of other profitable forms of violence such as extortion and kidnappings. The overall result has been a sharp increase of violent murder rates. Rios (2012) supports Guerrero's position and argues that this interaction between the beheading strategy and the new surges of violence, generated by the temporary void of power, generates a "self-reinforcing violent equilibrium".

On the other hand, several authors have positions contrary to those of Guerrero. For instance, Calderón et al. (2012) argue that the aforementioned beheading strategy results in a temporary rise of violence levels but in lower levels of violence in the long run, while Castillo, Mejía and Restrepo (2012) argue that violence in Mexico has also been affected by the success of the Colombian war against drug trafficking. Chabat (2010) and Astoraga (2010) argue that the previous Mexican authorities' "tolerance" regarding drug trafficking created a violence-enabling environment.

As long as drugs remain illegal and criminal groups control illegal drug market rents, the question regarding how a government could deter drug related violence efficiently is one of its most important public policy problems. Mark Kleiman (2011) has argued that a government may accomplish this goal by creating disincentives for the decision of DTOs to use violence to fulfill their objectives. That DTOs find it less profitable to be more violent is the way to do this. This may be accomplished by violence-targeted enforcement. Kleiman proposes a way in which this method could be developed. He argues that if a government creates a violence-related metric applied to all DTOs over a period of time, then it can announce that it will target the DTO ranked first in that metric. He argues that this strategy would result in all DTOs pooling into an equilibrium characterized by low levels of violence.

In this paper we develop a model in which the government and the DTOs interact and analyze the public policy problem regarding how a government confronting several DTOs simultaneously should allocate its enforcement resources among them. We assume that violence is the means by which DTOs acquire income in an illegal market. We make the simplifying but realistic assumptions that while the objective of the government is to minimize the aggregate level of violence, the objective of the DTOs is to maximize profits.

We model this interaction for three different cases. In all of them the government plays first by announcing how its allocation of resources takes place and then the DTOs decide their optimal levels of violence. Moreover, in all of them DTOs behave as neoclassical firms, e.g. as profit maximizing firms that face decreasing marginal returns from exercising violence and increasing costs. The costs DTOs face depend on the opportunity costs of violence and the costs of attracting the government's defense resources. The costs of attracting the government's attention is what differs in the three cases. In the first two cases the government's announcement and its corresponding allocation results in a separating equilibrium while in the third case it results in a pooling equilibrium characterized by low levels of violence.

In the first case, the government announces a constant marginal cost of violence that it would allocate to the DTOs' exercise of violence. Hence, each DTO will add such cost into their violence decision-making. As a result, there is an overall decrease in violence. Naturally, the constant marginal cost of violence announced by the government is determined by its budget constraint. From the government's budget constraint we may outline the trade-off it faces. On the one hand, making a higher announcement raises its costs because it has to allocate more resources on every unit of violence exercised by the DTOs. On the other hand, the announcement generates an overall decrease of DTOs' violence.

We consider the model previously summarized to be relevant because it may be seen as a theoretical approximation of the criminal justice system in most countries. This model shows that whenever the optimal cost has been reached, the only way of obtaining lower levels of violence is the straightforward way, e.g. increasing enforcement resources. Nevertheless, drug trafficking enforcement resources are huge in many countries and so are drug related violence levels in them. This situation calls for alternative deterrence strategies. This fact motivates the development of the other two cases.

In the next case, the government announces an allocation of resources according to a ranking of DTOs based on their use of violence. In this situation, the choice of violence by each DTO becomes necessarily strategic, because each DTO must anticipate the violence level of others in making its optimal violence decision. Therefore a negative concern for *violence status* may be induced by an allocation of resources according to a ranking of violence. Here we model *violence status* as an ordinal rank in the distribution of violence in a simultaneous move game. In order to solve this game, a symmetric Nash equilibrium is assumed, where all DTOs decrease their violence level simultaneously[1]. Then the existence of such equilibrium is formally demonstrated.

In the case where the government has complete information, treating violence status strategically allows us to derive an optimal allocation of scarce enforcement resources across different DTOs. This follows from the fact that when the government announces its allocation of resources, it may anticipate all DTOs' moves. As in the case in which the government faces incomplete information, the problem of the government is as if it had complete information and the DTOs' types are obtained within the expected order statistics.

In this version of the model the government faces diminishing marginal returns in terms of violence reduction, e.g. each additional unit of enforcement resources has a lower return in terms of violence reduction. We explicitly show how the government's optimal allocation of resources depends on its diminishing marginal returns in terms of violence reduction. Due to the fact that in this model DTOs decide to lower their violence levels because of the negative violence concern the ranking induces, in the optimal allocation, the amount of resources assigned to each position of the ranking should have the same marginal reduction of the *aggregate* level of violence. This is the reason behind the counterintuitive result that whenever the government faces faster diminishing marginal returns it should concentrate more resources in the most violent positions of the ranking. This result follows from the fact that even when having faster marginal returns, in terms of decreasing the level of violence in that particular position, it is optimal to assign a greater amount to the first positions of the ranking in order to ensure that the marginal return in terms of the reduction of the *aggregate* violence is constant across all ranking positions.

[1]From this fact it follows that we model violence status as in Frank (1985).

Nevertheless, up to this point we have only dealt with a separating equilibrium. However, we show that under the assumption that the government has enough resources to deter any DTO when concentrating all of its resources on it, if the government announces that it will concentrate all its resources in the most violent DTO, this results in a pooling[2] equilibrium in which all DTOs decide the same level of violence. This level of violence turns out to be lower than the level of violence of the least violent DTO in the previous model. Therefore, all DTOs decide on an inefficiently low level of violence (from their point of view). The inefficiency arises from the fact that they would all be better off if deciding the level of violence of the least violent DTO in the absence of the interaction with the government. The previous result supports the theory developed by Mark Kleiman (2011), known as *dynamic concentration* theory, in which the best allocation of resources under a dynamic time frame work is achieved by applying this strategy. Moreover, we show this strategy is optimal even on a one-shot game.

The rest of the paper is organized as follows. Section 2 introduces a benchmark model in which the government and DTOs do not interact and therefore the amount of violence of DTOs is the same as in the case where the government does not confront DTOs. Section 3 introduces a model in which the government and the DTOs interact directly through the government's announcement of a constant marginal cost of violence across all DTOs. Section 4 develops the model in which DTOs interact according to a violence ranking. Section 5 presents a version of the model where the government announces it will concentrate all enforcement efforts on the most violent DTO, and shows how Mark Kleiman's theory holds even in a static setting. Section 6 concludes.

2 The Benchmark Case

In the benchmark case presented in this section, we assume that there are no strategic interactions between the government and DTOs[3]. We assume that DTOs behave as neoclassical firms, e.g. as profit maximizing firms that face decreasing marginal returns from exercising violence and increasing costs. In this case, the DTO's optimization condition, when choosing the optimal level of violence, is the classical result of marginal benefits equal to marginal

[2]The fact that we allow pooling equilibrium is what motivated us to model violence status as in Frank (1985) rather than as in Hopkins and Kornienko (2004)

[3]The following scenarios result in this situation: the government does not confront DTOs, it assigns the same amount of resources to all of them or it assigns its resources directly to DTOs regardless of their violence levels

costs.

We assume that all DTOs are equal in all relevant dimensions except in the degree of efficiency with which they are able to generate income using violence. This efficiencies allow us to recreate situations such as the one in which DTOs, for instance, have to confront each other over the control of drug trafficking routes, from which they derive income out of drug trafficking activities. The efficiency with which they generate income with violence will be denoted by A.

The DTOs' profits function can be broken down into two elements. If we denote the violence decision of DTOs by a, the first element, $AP(a)$, is a conventional income function that depends only on the violence level chosen by each DTO. The second, $-a$, defines the opportunity costs of violence. We assume that DTOs face constant diminishing marginal income returns from violence (e.g., we assume that $P'(a) > 0$ and $P''(a) < 0$). Therefore we assume that $P(a) = a^{\beta}$ where β is a constant that measures diminishing returns and $\beta \in [0, 1]$.

DTO i's problem is:

$$\underset{a_i}{\text{Max}}\, A_i a_i^{\beta} - a_i \tag{1}$$

with the first order condition being:

$$a(A_i) = (\beta A_i)^{\frac{1}{1-\beta}} \tag{2}$$

The previous first order condition will be our reference for further results. We are going to compare the level of violence in (2) with the ones obtained under several strategic interactions between the government and DTOs.

In the next section a model in which there is a direct interaction between the government and DTOs but no interaction between DTOs is developed.

3 When Does *Brute Force* Fail?

Consider the problem of a government who has to simultaneously confront several DTOs. Since the government has limited resources, its problem is to choose the share of enforcement resources it will allocate to each DTO. In order to do so, it will assign a cost to every unit of violence exercised by DTOs. A priori, however, we allow the government to assign a different cost to different DTOs, but we then show that assigning the same cost to all DTOs is its optimal way to choose those costs. The objective of the government is to minimize the aggregate level of violence.

We assume that the government has complete information regarding DTOs and that it plays first by choosing (and announcing) the cost for every unit of violence exerted by each DTO. Complete information implies the government knows all DTOs' efficiency to generate profits from exerting violence. Therefore it may anticipate the DTOs' move when choosing the optimal cost of violence it will impose to each DTO. This fact allows it to know in advance the share of enforcement resources that will be allocated to each DTO.

After the government makes its announcement, each DTO decides its optimal amount of violence taking into account that they receive direct profits from exerting violence but face two costs of doing so: first they face the opportunity costs of violence; second, the higher the level of violence they choose the higher the share of enforcement resources they will have. For the case at hand, consider that the government confronts $N \in \mathbb{N}^+$ DTOs. Naturally, every DTO knows its degree of efficiency. Since we assume that there is no interaction between DTOs, the other DTOs' degree of efficiency does not affect each others decisions.

The timing of the model is as follows. Nature plays first, giving each DTO a type, A_i. Then the government announces the share of its resources it will allocate on each DTO by means of announcing a cost to every unit of violence of each DTO. Finally, DTOs decide their level of violence, denoted by a, in order to maximize their profits.

We now introduce the agents involved in the game in more detail. We do this in the same order in which they appear when solving the model by backward induction.

Consider the problem of a DTO who must decide its optimal level of violence. We assume that DTOs, as in the benchmark case, behave as neoclassical firms. Nevertheless, in this problem, DTOs must deal with another cost, namely that of attracting more government attention (e.g., enforcement resources) when being more violent.

Assume that the same profits function may be applied to all DTOs. This function has the same form of the one applied in the benchmark case, but with an additional term that captures the extra cost of attracting the government's attention when they generate more violence.

Assume further that the government faces diminishing marginal returns, in terms of violence reduction, in the amount of enforcement resources assigned to each DTO. To capture this fact we establish that the cost announced to each DTO has also diminishing marginal returns. Therefore, if the government announces a constant cost c_i to every unit of violence exerted by DTO i, the cost DTO i faces is $-c_i^{\alpha} a_i$. Here α is a constant which reflects the government's diminishing marginal returns. Naturally we assume $\alpha \in (0,1]$. Therefore, in the case at hand, the government assigns to each DTO an amount of resources equal to $c_i a_i$ but, due to the diminishing returns it faces, the DTOs costs are only $-c_i^\alpha a_i$. Hence, the profit maximization problem of DTOs may be stated as:

$$\underset{a_i}{\text{Max}}\, A_i a_i^\beta - a_i - c_i^\alpha a_i \tag{3}$$

With the objective function in (3), the optimal violence level for a DTO with degree of efficiency to make violence profitable A_i is given by:

$$a(A_i) = \left(\frac{\beta A_i}{1 + c_i^\alpha}\right)^{\frac{1}{1-\beta}} \tag{4}$$

Now, the government's budget constraint, when anticipating the DTOs move, is given by:

$$\sum_i c_i \left(\frac{\beta A_i}{1 + c_i^\alpha}\right)^{\frac{1}{1-\beta}} \leq z \tag{5}$$

and since the government has complete information, its minimization problem is:

$$\underset{c_j}{\text{Min}} \sum_i \left(\frac{\beta A_i}{1 + c_i^\alpha}\right)^{\frac{1}{1-\beta}} \quad s.t. \quad \sum_i c_i \left(\frac{\beta A_i}{1 + c_i^\alpha}\right)^{\frac{1}{1-\beta}} \leq z \tag{6}$$

The government's first order condition is:

$$(\beta A_j)^{\frac{1}{1-\beta}} \left[\frac{d}{dc_j}\left(\frac{1}{1 + c_j^\alpha}\right)^{\frac{1}{1-\beta}} - c_j \lambda \frac{d}{dc_j}\left(\frac{1}{1 + c_j^\alpha}\right)^{\frac{1}{1-\beta}} - \lambda \left(\frac{1}{1 + c_j^\alpha}\right)^{\frac{1}{1-\beta}}\right] = 0 \tag{7}$$

where λ is the Lagrange multiplier associated with the budget constraint.

In the above equation, the first term in parenthesis shows the marginal benefits to the government of assigning an extra unit to the cost assigned to DTO j. This benefits arise from the fact that DTO j lowers its violence level due to that as a result of the higher cost it confronts. The second term shows the opportunity costs of the enforcement resources assigned to that particular DTO. The third term shows the cost faced by the government due to the fact that when raising a particular DTO's marginal costs it has a higher expenditure on each unit of violence exercised by that DTO.

Notice that the term $(\beta A_j)^{\frac{1}{1-\beta}}$ is irrelevant in the first order condition. Therefore, the optimal constant marginal cost to be announced by the government to each DTO does not depend on A_j. Hence, the marginal costs that the government announces to the DTOs must be equal among them in the optimum.

We denote by c the marginal cost announced by the government to all DTOs. Its budget constraint, when anticipating the DTOs move, maybe written as:

$$c \sum_i \left(\frac{\beta A_i}{1 + c^\alpha} \right)^{\frac{1}{1-\beta}} \leq z \tag{8}$$

Recall that the objective of the government is to minimize the aggregate level of violence. Therefore, at this point its objective is to find the maximum marginal cost it may announce such that its budget constraint is satisfied.

In the above equation we can also easily analyze the trade-off the government faces when deciding the optimal marginal cost to announce. The first term in the left hand side of the above equation shows the costs associated with applying a marginal cost to the DTOs. It reflects the fact that increasing the constant marginal costs raises its overall costs because it has a higher expenditure for every unit of violence. On the other hand, the second term in the left hand side shows the benefits associated with the reduction of violence due to the marginal cost. It reflects the fact that increasing the constant marginal costs results in an overall reduction of the aggregate violence of DTOs, thus resulting in a decrease in defense expenditure.

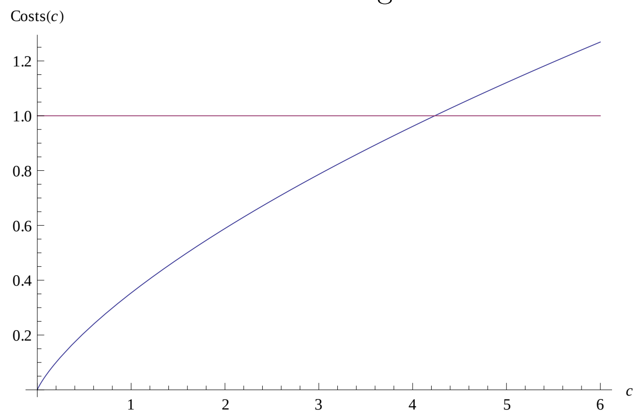
Whenever $\alpha + \beta < 1$ [4], we have a maximum marginal cost of violence that satisfies the government's budget constraint and it does not results in zero violence.

Figure 1 shows the overall costs of the government as a function of the marginal cost announced. From it we may observe that whenever the government has reached the maximum marginal cost it cannot reduce violence further with the same resources.

We consider the model previously exposed relevant because it may be seen as a theoretical approximation of the criminal justice system in most countries. This is due to the fact that we may understand the cost announced and assigned by the government as the penalties imposed by the criminal justice system. This model shows that whenever the optimal cost has been reached, the only way of obtaining lower levels of violence is the straightforward way, e.g. increasing enforcement resources. Nevertheless, drug trafficking enforcement resources

[4] The minimization problem of the government is not well defined whenever $\alpha + \beta \geq 1$. For that reason we are going to concentrate in the case where $\alpha + \beta < 1$. In such case, only a high violence equilibrium may be achieved. This follows from the fact that there is a maximum cost the government may assign in order to satisfy its budget constraint. Therefore, the appliance of a cost higher than the one obtained, *brute force*, doesn't result in the budget constraint being satisfied.

Figure 1: Evolution of the overall costs the government faces as a function of c.



are huge in many countries and so are the levels of violence associated with that activity in them. This fact motivates the development of alternative deterrence strategies, as the ones we expose next.

4 Deterring with *Status*

We now develop an asymmetric game in which the government and DTOs interact. The asymmetry follows from the fact that the government knows the DTOs' degree of efficiency to make violence profitable while each DTO ignores the other DTOs' degree. The objectives of the government and DTOs are the same as in the previous case.

In this case the government plays first announcing how the allocation of defense resources is going to take place according to a ranking of violence of DTOs. Therefore each DTO must anticipate the levels of violence of the other DTOs when deciding its level of violence. Since we assume that the government faces complete information when making its resource allocation decision it may anticipate the DTOs move.

After the government makes its announcement, the DTOs decide their optimal amount of violence taking into account that they receive direct benefits from the exercise of it but face the cost of attracting the government's attention. In this model we consider a continuum of DTOs whose degree of efficiency is distributed on a finite support.

The timing of the model is as follows. Nature plays first, giving each DTO a type (an efficiency with which they produce profits with violence, A_i). Then the government announces how the allocation of enforcement resources, z, among DTOs with the objective of minimizing the aggregate level of violence. Finally DTOs decide their level of violence, again denoted with a.

We now introduce the agents involved in the game in more detail. Again, this is done in the same order in which they appear when solving the model by backward induction.

4.1 The Drug Trafficking Organization's Problem

Consider the problem of a DTO that must decide its optimal level of violence. As in the previous cases, assume that DTOs behave as neoclassical firms. Nevertheless, in this problem DTOs must deal with the cost of attracting government defense resources in a different way than in the previous model. We are going to model this cost under the assumption that DTOs are affected by their relative position in a violence ranking, since the government announces how its resources will be allocated based on a violence rankings across DTOs.

We shall assume a continuum set of DTOs, identical in all respects except in their degree of efficiency, as shown in their technology to produce income. Each DTO is given a technology, A, which is private information and is an independent draw from a common distribution. This is described by a cumulative distribution function $\mathcal{F}(A)$ which is twice continuously differentiable with a strictly positive density over the support $[A_{min}, A_{max}]$ with $A_{min} > 0$.

We follow the methodology presented in Frank (1985) regarding the construction of status rankings. Suppose violence is distributed among DTOs with density function $r(a)$ and that a_0 is the smallest violence level all DTOs. Then, a DTO with violence level $a = a_i$ would be ranked as:

$$R(a_i) = \int_{a_0}^{a_i} r(a)da \tag{9}$$

Therefore, $R(a_i)$ is the relative position of a DTO with violence level a_i in the violence ranking. Notice that $R(a)$ is a number between 0 and 1, indicating the percentile ranking of a in the population of a values. In this model, we assume that the government's allocation of defense resources is a function of this ranking. We define this distribution henceforth as $g(R(a))$. Therefore, if we define the government's defense resources as constant z, we may establish the costs of attracting the government's attention as $g(R(a))z$.

Assuming further that their income and costs function behave as in the previous models, the benefits function to be applied to all DTOs is:

$$\Pi(a) = AP(a) - a - g(R(a))z \tag{10}$$

We assume that $P(.)$ is nonnegative, strictly increasing, strictly quasiconcave and twice differentiable. Therefore, each DTO maximizes its benefits function, as stated above.

In the context of this game we seek a *symmetric equilibrium*. Such an equilibrium will be a Nash equilibrium in which all DTOs will use the same strategy, $a(A)$, mapping from the degree of efficiency to the level of violence.

Assume for the timebeing that $a(A)$ is increasing and differentiable. If we assume that all DTOs apply such an equilibrium strategy, then the probability that a DTO i with violence technology A_i and violence choice a_i will have a higher violence level than an arbitrarily chosen DTO j is $R(a_i) = Pr(a_i > a(A_j)) = Pr(a^{-1}(a_i) > A_j) = \mathcal{F}(a^{-1}(a_i))$. Hence we may restate the maximization problem of the DTOs as:

$$\underset{a_i}{\text{Max}}\, A_i P(a_i) - a_i - g(\mathcal{F}(a^{-1}(a_i)))z \tag{11}$$

Assuming that the problem is well defined and that its maximum is characterized by its first order condition, then the unique separating equilibrium (if it exists) must satisfy the following differential equation:

$$\dot{a}\left[a^{-1}(a_i)\right] = \frac{\dot{g}(\mathcal{F}(a^{-1}(a_i)))f(a^{-1}(a_i))z}{A_i P_a - 1} \tag{12}$$

In the above equation $\dot{a} = \frac{\partial a}{\partial A}$, $\dot{g} = \frac{\partial g}{\partial A}$ and $f(a^{-1}(a_i))$ is the density distribution of the technology. Henceforth assume that DTOs follow the equilibrium path explicitly given by the above equation. Therefore, replacing $a^{-1}(a_i) = A_i$ and noticing that $\dot{g}(A) = \frac{dg}{dA} = \dot{g}(\mathcal{F}(a^{-1}(a_i)))f(a^{-1}(a_i))$, we may rewrite the above expression as:

$$\dot{a} = \frac{\dot{g}(A)z}{A_i P_a - 1} \tag{13}$$

Note that, in this problem, the following relation holds under the symmetric Nash equilibrium: $\Pi_A > 0$. This means that the DTO with the lowest A, i.e. the one with lowest degree of efficiency, is the one that will reveal its type in equilibrium.

4.2 The Government's Problem

Now consider the problem of a government that must decide how to allocate its resources in an optimal manner in order to minimize the aggregate level of violence. In this case, finding the optimal scheme for resource allocation across DTOs is the same as determining the share of resources that must be allocated to each agent. This follows from the fact that the government has complete information and the DTOs follow a symmetric Nash equilibrium. Therefore, in this case we can write the government's objective function as:

$$\underset{g(A_i)}{\text{Min}} \int_{A_{min}}^{A_{max}} a(A_i, g(A_i)) f(A_i) \, \mathrm{d}A_i \tag{14}$$

Naturally, the constraint that the government faces is:

$$\int_{A_{min}}^{A_{max}} g(A_i) \, \mathrm{d}A_i = 1 \tag{15}$$

Notice also that since the government expects the DTO with the lowest degree of efficiency to reveal its type, the optimal allocation of resources for that particular DTO is $g(A_{min}) = 0$. With the above equations we may derive the following first order condition:

$$\frac{\dot{a}}{\dot{g}} f(A_i) = \lambda \tag{16}$$

The above equation says that the government faces diminishing marginal returns when concentrating resources in a particular DTO. Therefore its optimal condition is to have the same marginal benefits when concentrating resources among all DTOs. In other words, the last unit of defense resources allocated to each DTO must have the same return in terms of decreasing violence. Recall that when we developed the DTOs' problem we assumed a symmetric Nash equilibrium. Having $\lambda > 0$ is a *sufficient* condition to obtain a separating equilibrium.

In order to solve the problem explicitly we must solve the system of differential equations formed with the government's optimality condition as shown in equation (15) and the DTO's equilibrium path (equation (12)). We are going to develop the following example to explicitly show how this process takes place.

4.2.1 Example

Let us develop the simple example in which: $A_{max} = 10$, $A_{min} = 1$, $P(a) = \sqrt{a}$ and the DTOs' distribution of A is uniform along the support $[A_{min}, A_{max}]$. Therefore, by means of inserting the equilibrium path followed by DTOs in the government's optimality condition and solving the resulting equation for $a(A_i)$ we obtain:

$$a(A_i) = \frac{A_i^2}{4\left(1 + \frac{z}{\lambda}\right)^2} + C \tag{17}$$

In the above equation C is an integration constant that must be solved later on in order to have $a(A_{min}) = \frac{A_{min}^2}{4}$.

Inserting the above equation in the government's optimality condition and solving for $g(A_i)$ we obtain:

$$g(A_i) = \lambda \frac{A_i^2 - A_{min}^2}{4\left(1 + \frac{z}{\lambda}\right)^2} \tag{18}$$

Recall that λ is an integration constant that allows us to satisfy the government's resource constraint. Notice that since $\frac{\lambda}{4\left(1+\frac{z}{\lambda}\right)^2}$ is a constant there is a unique allocation of resources regardless of the amount z. Also, notice that since the DTO with the lowest degree of efficiency reveals its type, the aforementioned optimal allocation results in a marginal cost to the amount of violence higher than the one done by the least violent DTO.

In Figure 2 we show the optimal level of violence for DTOs with different technologies. In Figure 3 we show the government's optimal allocation of resources.

Figure 2: Evolution of the level of violence as z increases.

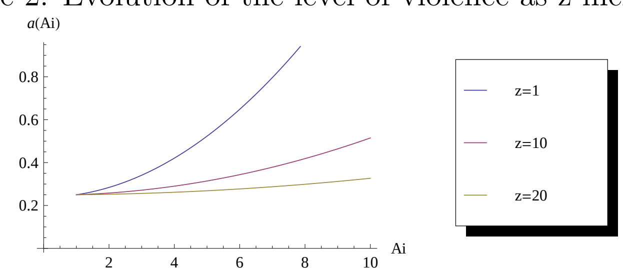
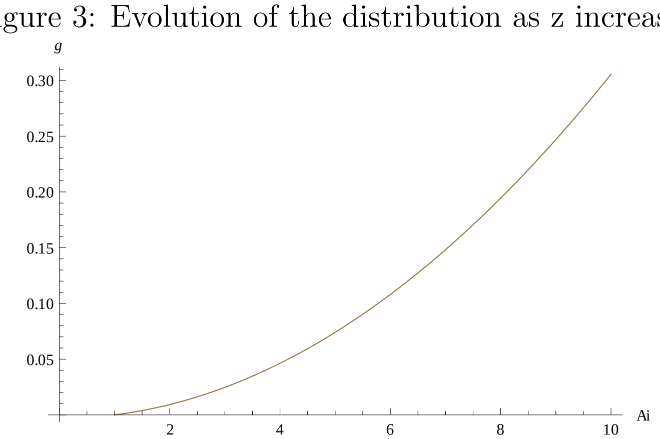


Figure 3: Evolution of the distribution as z increases.



From figure 2, notice that a *symmetric Nash* equilibrium indeed exists, i.e. DTOs follow an increasing equilibrium path. Also note that, as z increases, the violence levels change;

nevertheless, they keep the same trend. In figure 3, notice that there is a unique optimal allocation for every amount of resources possible and that under this allocation, the government assigns a higher amount of resources to DTOs with higher degree of efficiency.

Up until now, we have shown the optimal allocation for the government when it can concentrate resources in each DTO without having decreasing returns in it. In the previous model this would be the case in which $\alpha = 1$. In the next section we are going to elicit the case in which the government faces diminishing marginal returns when concentrating resources, i.e. the case in which $\alpha \in (0,1)$. We show that when this is the case, its optimal allocation results in a higher concentration of resources in those DTOs with higher degree of efficiency.

4.3 Deterring with *Status* while Facing Diminishing Returns

We now develop the same problem as in the previous section with the additional feature that the government faces diminishing marginal returns, in terms of violence reduction, when concentrating resources on attacking a given DTO. As in the first model, the government's diminishing marginal returns may be understood as the fact that each additional unit of resources results in a lower reduction in violence by the DTOs. For the DTOs, the problem changes only in the costs derived of attracting the governments attention. Those costs in this problem take the form $(g(R(a))z)^{\alpha}$. The parameter $\alpha \in (0,1)$ captures the degree of the diminishing marginal returns the government faces. As usual, we solve this problem by backward induction.

Following the same steps as in the previous model we find the equilibrium path followed by DTOs to be:

$$\dot{a} = \frac{\alpha g(A)^{\alpha-1}\dot{g}(A)z}{A_i P_a - 1} \tag{19}$$

Define $g(A)^{\alpha} = \gamma(A)$. Therefore we may rewrite the equilibrium path as:

$$\dot{a} = \frac{\dot{\gamma}(A)z}{A_i P_a - 1} \tag{20}$$

We assume that the government's problem is the same as the one stated in the previous section. Assume further that its budget constraint has the form:

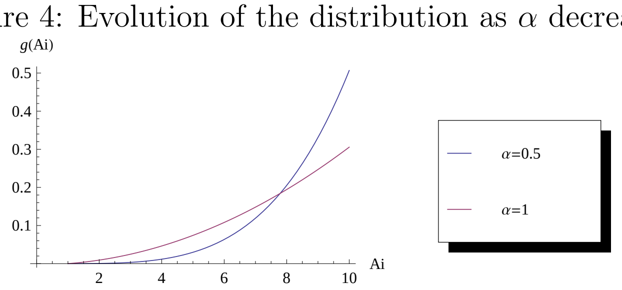
$$\int_{A_{min}}^{A_{max}} \gamma(A_i)\,\mathrm{d}A_i = C \tag{21}$$

where C is a constant such that the original budget constraint of the government, $\int g(A_i)dA = 1$, is satisfied. Therefore this problem is exactly the same to the one solved before replacing $\gamma(\cdot)$ with $g(\cdot)$. Therefore, the optimal allocation of the government in this problem, mathematically, is simply the same as the one obtained in the previous problem raised to the power of $\frac{1}{\alpha}$.

In this problem we assume that the government faces diminishing marginal returns, i.e. $\alpha \in (0, 1)$. Therefore we may conclude that, in this case, in the optimal allocation more resources are concentrated in the most violent ranking positions than when facing constant marginal returns. Due to the fact that in this model DTOs lower their violence levels because of the negative violence concern the ranking induces, in the optimal allocation the amount of resources assigned to each position of the ranking should have the same marginal reduction on the *aggregate* level of violence. This is the reason behind the counterintuitive result that whenever the government faces faster diminishing marginal returns it should concentrate more resources in the most violent positions of the ranking. In other words, the result follows from the fact that the marginal effect of assigning more enforcement resources on the most violent DTOs does not only depend on how much violence is reduced on that particular position of the ranking. It also depends on how the other positions change their optimal violence decision in response to a larger assignment of enforcement resources on high positions of the ranking. This explains why despite having diminishing marginal returns, the government assigns a higher share of resources on the higher positions of the ranking of violence.

The following graph shows, given the same parameters as in the example of the previous section, the optimal allocation of the government under several values for α.

Figure 4: Evolution of the distribution as α decreases.



Notice how, as α decreases, the optimal allocation of resources results in a higher concentration of enforcement expenditure on the more violent DTOs.

We have only dealt so far with the problem of the government allocating its resources when confronting a continuum of DTOs. In the appendix we show that deterring with status is also possible when confronting a discrete number of DTOs.

In the next section we analyze the pooling equilibrium that arises when the government announces it will concentrate its resources in the most violent DTO.

5 Mark Kleiman's Dynamic Concentration Theory

In this section we delve into what would happen, under the framework we have followed until now, if the government applies the violence deterring strategy proposed by Mark Kleiman (2011). The following text exposes the deterrence strategy underlined by Kleiman's *dynamic concentration theory*:

> Mexico's different problem calls for a different strategy: creating disincentives for violence at the level of the largest trafficking organizations. Those six organizations vary in their use of violence; total violence would shrink if market shares changed in favor of the currently least violent groups or if any group reduced its violence level. Announcing and carrying out a strategy of violence-targeted enforcement could achieve both ends. The Mexican government could craft and announce a set of violence-related metrics to be applied to each organization over a period of weeks or months. Such a scoring system could consider a group's total number of killings, the distribution of its targets (among other dealers, enforcement agents, ordinary citizens, journalists, community leaders, and elected officials), its use or threat of terrorism, and its nonfatal shootings and kidnappings. Mexican officials have no difficulty attributing each killing to a specific trafficking organization, in part because the organizations boast of their violence rather than trying to hide it. At the end of the scoring period, or once it became clear that one organization ranked first, the police would designate the most violent organization for destruction. That might not require the arrest of the kingpins, as long as the targeted organization came under sufficiently heavy enforcement pressure to make it uncompetitive. The points of maximum vulnerability for the Mexican trafficking organizations might not even be within Mexico. U.S. law enforcement agencies believe that for every major domestic distribution organization in the United States, they can identify one or more

> of the six dominant Mexican trafficking organizations as the primary source or sources. If the U.S. Drug Enforcement Administration were to announce that its domestic target-selection process would give high priority to distributors supplied by Mexico's designated "most violent organization," the result would likely be a scramble to find new sources. Removing an organization would not reduce total smuggling capacity; someone would pick up the slack. But the leaders of the targeted trafficking group would, if the program were successful, find themselves out of business. The result might be the replacement of more violent trafficking activity by less violent trafficking activity. Less happily, it could lead to a temporary upsurge in violence due to the disruption of existing processes and relationships. But in either case, if the destruction of the first designated target was followed by an announcement that a new target selection process was under way using the same scoring system, there would be great pressure for each of the remaining trafficking groups to reduce its violence level to escape becoming the next target. The process could continue until none of the remaining groups was notably more violent than the rest (Kleiman, 2011, p. 100)

In this section we are going to show first why, when dealing with strategies such as the one proposed by Kleiman, we should drop the separating equilibrium assumption. Then we build the pooling equilibrium under the assumption that even the most violent DTO has incentives to mimic the least violent one. Finally, we show under which conditions is the aforementioned assumption possible and discuss what happens if it is not.

5.1 The Pooling Equilibrium under the new Government's Strategy

We are now going to show why a separating equilibrium under Kleiman's strategy is not feasible. Assume that the government's defense resources are high enough so that if they are concentrated in a particular DTO, the benefits that DTO derives from violence are zero. In other words, they are sufficient to drive that DTO out of business. Therefore, we are assuming that for any DTO i if i is the target, then $\Pi_i = 0 \ \forall \ a_i$.

Assume further that we expect the resulting equilibria to satisfy the following condition: no matter what equilibrium strategy is followed by DTOs, whenever $A_i > A_j$ then $\Pi_i\left(a(A_i)\right) > \Pi_i\left(a(A_j)\right)$. This condition is sufficient to obtain the desired relationship between the violence ranking and the DTO's efficiency in using violence to produce profits.

Suppose the government applies Kleiman's strategy. Assume that DTOs follow a separating equilibrium as stated before: All DTOs follow an increasing strategy $a(A_i)$ mapping from the efficiency to the levels of violence. Then any increasing separating equilibrium strategy does not satisfies the aforementioned condition. This is due to the fact that the benefits of the DTO with highest degree of efficiency are zero, due to the assumption that whenever the government targets a DTO its benefits are zero. Therefore, the benefits of the other DTOs, with lower efficiency, are strictly higher than its.

Hence, when dealing with Kleiman's strategy one cannot obtain a separating equilibrium while satisfying the aforementioned condition. Therefore, we are going to drop the separating equilibrium condition and thus build a pooling equilibrium.

5.2 The Pooling Equilibrium

In order to build the pooling equilibrium, again assume that the government confronts $N \in \mathbb{N}^+$ DTOs and they behave as neoclassical firms with income function $P(.) = \sqrt{.}$. Each DTO has an efficiency that is private information. Again, this information is an independent draw from a common distribution described by the cumulative distribution function $\mathcal{F}(A)$. Assume that if the government follows Kleiman's strategy and it observes all DTOs with the same level of violence, it randomly picks one and deters it.

For now, assume that even a DTO with the highest possible degree of efficiency, i.e. one with $A_i = A_{max}$, finds it optimal to mimic an eventual DTO with the lowest possible degree of efficiency, i.e. $A_j = A_{min}$. In the next section, we are going to elicit the conditions that make it so.

Suppose that initially all DTOs decide to mimic the lowest possible efficiency. Therefore they all exercise the level of violence a DTO with that efficiency would do in the absence of the interaction with the government. As shown in the benchmark case, this level of violence would be given by $a^{bc}_{amin} = A^2_{min}/4$.

The strategy resulting in all DTOs deciding the aforementioned level of violence is not an equilibrium strategy. This follows from the fact that if there is a DTO with the least possible efficiency, it has incentives to lower its level of violence in order to secure its profits. This is due to the fact that if all other DTOs decide $a(A_i) = a^{bc}_{min}$ and it decides that level of violence, then its profits are:

$$\Pi(A_{min})^{pool} = \frac{N-1}{N}(A_i\sqrt{a(A_{min})} - a(A_{min})) \tag{22}$$

In the above equation $A_i\sqrt{a(A_{min})} - a(A_{min})$ is the actual profits it would obtain if it is not randomly picked by the government and $\frac{N-1}{N}$ is the probability of not being picked by the government. If it infinitesimally lowers its level of violence it would secure its profits and they would be:

$$\Pi(A_{min})^{lower} = A_i\sqrt{a^{bc}_{min}} - a^{bc}_{min} \tag{23}$$

These profits are clearly higher than the ones it would obtained if it did not lower its level of violence. Since the other DTOs anticipate this fact, they must keep lowering their violence level together with the least violent DTO. The question at hand is when is the eventual least violent DTO going to stop decreasing its violence.

In order to know where the least violent DTO is going to stop, we must analyze the trade-off it faces. On the one hand, when lowering its level of violence, it secures its benefits. Actually, since its benefits go from $\Pi(A_{min})^{pool}$ to $\Pi(A_{min})^{lower}$ when infinitesimally lowering its violence, we may say that the difference between the aforementioned quantities is its marginal benefit of reducing its violence decision. On the other hand, whenever it reduces its violence decision, it obtains lower benefits. Therefore we may say that the marginal costs of reducing its violence decision are equivalent to its first order condition in the absence of interactions with the government. This simply follows from the fact that it is lowering its violence decision in order to completely avoid the government.

In summary, if we denote the marginal benefits and the marginal costs of the least violent DTO of decreasing its violence decision with MB and MC, respectively, this quantities would be:

$$MB = \Pi(A_{min})^{lower} - \Pi(A_{min})^{pool} = \frac{1}{N}(A_{min}\sqrt{a_{min}} - a_{min}) \tag{24}$$

$$MC = \frac{d\Pi}{da_{min}} = \frac{A_{min}}{2 * \sqrt{a_{min}}} - 1 \tag{25}$$

The optimal violence decision for the least violent DTO, or equivalently the place where the race stops, is at the violence level in which the marginal benefits from being less violent are equal to its associated marginal cost:

$$\frac{1}{N}(A_{min}\sqrt{a_{min}} - a_{min}) = \frac{A_{min}}{2 * \sqrt{a_{min}}} - 1 \tag{26}$$

Notice that the aforementioned condition has a single crossing point on the interval

$0 < a < a^{bc}_{amin}$. This follows from the fact that $MB(a^{bc}_{amin}) > MC(a^{bc}_{amin})$, $\lim_{a\to 0^+} MC(x) > \lim_{a\to 0^+} MB(a)$ and that on the aforementioned interval $\frac{dMB}{da} > 0$ and $\frac{dMC}{da} < 0$.

If we denote the level of violence where the above condition is satisfied with $a(A_{min}) = a_p$, the strategy $a(A_i) = a_p$ is an equilibrium strategy. This follows from the fact that not even the least violent DTO has incentives to deviate to a lower violence level and no DTO has incentives to deviate to a higher violence level (it would obtain zero profits). The fact that no DTO has incentives to deviate to a lower level of violence makes the aforementioned strategy an *undercut-proof equilibrium*[5].

Nevertheless this equilibrium is inefficient for the DTOs. This follows from the fact that each DTO would be better off if the they all exercise the level of violence in the absence of the race.

Notice that in the pooling equilibrium's associated level of violence is higher when the number of DTOs increases. This result follows from the fact that the marginal benefits of decreasing violence levels are lower whenever there is a higher number of DTOs.

In order to build this pooling equilibrium we made the assumption that even for the most violent DTO it is optimal to mimic the least violent DTO. We are going to elicit next the condition that makes this assumption true and what happens if it is not.

5.3 Optimal Mimicking

In this section we analyze the mimicking decision of DTOs. In order to do so, we must first analyze optimal violence decision when being the DTO mimicked by the DTOs that have a higher degree of efficiency. The reason behind why this is the first step when analyzing the mimicking decision is that when the DTOs decide which efficiency to mimic they must anticipate the fact that a DTO with that degree will lower its level of violence in an attempt to secure its payments.

In this problem, the probability with which DTO j is the *k-th* DTO with highest efficiency type will be denoted by $P_k(A_j)$. That probability is obtained with the following formula:

$$P_k(A_j) = \frac{N!}{(k-1)!(N-k)!}\mathcal{F}(A_j)^{N-k}(1-\mathcal{F}(A_j))^{k-1}f(A_j) \tag{27}$$

In the above equation $\frac{N!}{(k-1)!(N-k)!}$ is the number of possibilities in which a particular DTO results being the *k-th* DTO with the highest degree of efficiency and $\mathcal{F}(A_i)^{N-k}(1-$

[5]In an undercut-proof equilibrium each DTO maximizes its profits while ensuring that its level of violence is low enough such that any other DTO finds it profitless to set a lower level of violence in order to secure its payments

$\mathcal{F}(A_i))^{k-1}f(A_i)$ is the probability that each of the aforementioned possibilities occur.

Therefore the expected profits of a DTO with efficiency A_j, given that all DTOs are mimicking its efficiency, are:

$$\Pi(A_j)^{pool} = \left(\sum_{k=2} P_k(A_j)\frac{k-1}{k}\right)(A_j\sqrt{a(A_j)} - a(A_j)) \tag{28}$$

The above equation shows that its benefits, when being mimicked, are the summation of the probability its the $k-th$ DTO with the highest degree of efficiency (e.g., there are k DTOs with the same level of violence) times the probability of not being randomly picked by the government ($\frac{k-1}{k}$), times the benefits it would obtain when not being picked $(A_j\sqrt{a(A_j)} - a(A_j))$.

Following the same procedure as in the previous case, its marginal benefits from lowering its violence decision are then:

$$MB = \left(\sum_{k=2} P_k(A_j)\frac{1}{k}\right)(A_j\sqrt{a(A_j)} - a(A_j)) \tag{29}$$

Therefore, every efficiency has a unique optimal violence decision when being mimicked, obtained by solving the marginal costs and benefits equality. This is a result that is obtained following the same procedure that was developed in the previous section.

When a DTO decides which efficiency to mimic, it must take into account two things. The first one is the level of violence of that efficiency when being mimicked. This follows from the fact that it is going to obtain profits with that level of violence. The second one is the probability of not being picked by the government. This is due to the fact that whenever it decides to mimic a DTO with a lower efficiency, its probability of not being picked by the government is higher.

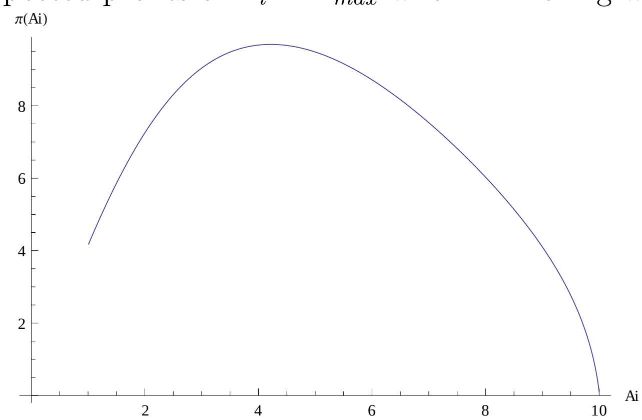
Hence, when mimicking there is a trade-off between the benefits lost whenever doing it in a low efficiency type and the increasing probability of not being picked by the government when doing so. If the least possible efficiency type is high enough, then the benefits of mimicking it are higher than its costs. This results in being optimal to mimic the lowest efficiency for all DTOs. Nevertheless, if the opposite is true, then an intermediate efficiency exists such that it is optimal to mimic it for all the DTOs with an efficiency higher than that intermediate efficiency.

We are going to develop two examples in order to illustrate the previous results:

5.3.1 Example 1

Let us develop the simple example in which: $A_{max} = 10$, $A_{min} = 1$, $P(a) = \sqrt{a}$ and the DTOs' distribution of A is uniform along the support $[A_{min}, A_{max}]$. In this case notice that the lowest possible degree of efficiency is 10 times lower than the highest possible one. So we expect an eventual DTO with degree of efficiency $A_i = A_{max}$ to mimic an intermediate degree of efficiency. In fact, figure 6 shows the expected profits when mimicking every possible efficiency.

Figure 5: Expected profits of $A_i = A_{max}$ when mimicking with $A_{min} = 1$.



Therefore, in this case, the optimal degree of efficiency to mimic by the DTOs with $A \geq 4.1$ is $A = 4.1$. As for the DTOs with an efficiency lower than the aforementioned one, they must follow a separating equilibrium between them.

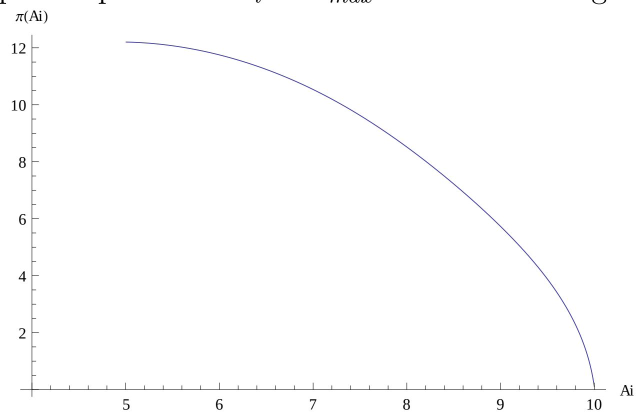
Even though, in this case, some DTOs mimic a DTO that is not the least violent one, notice that the equilibrium obtained is again inefficient for the DTOs who are mimicking. Notice that they would all be better off if they all mimicked the violence benchmark level of the efficiency that they are mimicking.

5.3.2 Example 2

Let us develop the simple example in which: $A_{max} = 10$, $A_{min} = 5$, $P(a) = \sqrt{a}$ and the DTOs' distribution of A is uniform along the support $[A_{min}, A_{max}]$. In this case notice that the highest possible efficiency is twice the lowest possible one. So we expect an eventual DTO with degree of efficiency to attack $A_i = A_{max}$ to mimic the least violent DTO. In fact, figure 6 shows the expected profits of such DTO when mimicking:

Therefore, in this case it's optimal for all DTOs is to mimic the least violent DTO.

Figure 6: Expected profits of $A_i = A_{max}$ when mimicking with $A_{min} = 5$.



This model supports the theory of *dynamic concentration* developed by Kleinman (2011). It shows how the announcement of the government to concentrate all of its resources in the most violent DTO lowers the level of violence of all DTOs to an inefficiently low level of violence. The inefficiency follows from the fact that all DTO would be better off if they all mimic the violence level of the DTO with the lowest degree of efficiency in the absence of the race. Naturally, an inefficiently low level of violence of the DTOs is a desirable equilibrium for the government.

This model supports Kleiman's strategies and furthermore supports the prediction it makes, namely that DTOs would end in a pooling equilibrium. Nevertheless, Kleiman supports this as ensuing form the fact that, with such strategy, the government would make low violent drug trafficking more profitable than the more violent one. Notice that in this model this does not happen. Actually, in the pooling equilibrium DTOs with higher efficiency obtain higher profits than the other DTOs. Therefore, even though Kleiman's strategy and conclusions are the same as the ones obtained within this model, the arguments that support them differ and therefore gives this theory an additional credence. Moreover, Kleiman argues that the government must apply such strategy until DTOs end up having the same violence level. This model supports the fact that, even if they end up having the same level, the government can apply Kleiman's strategy and that they should optimally end up in the same level of violence even under a one-shot game.

6 Conclusions

In this paper we analyze the public policy problem regarding how a government should allocate scarce resources to fight organized crime with the objective of reducing violence. To do so, we develop a model in which the government confronts several criminal organizations. We do this under three different scenarios. In all of them we model this organizations as neoclassical firms as a way to reflect the fact that they receive direct benefits from crime itself.

In the first case we developed a model that aims to illustrate a simplified version of the criminal justice system in most countries. Therefore, in that model the government assigns a penalty, in the form of a constant marginal cost, to every unit of crime (e.g., violence) done by criminal organizations (DTOs). We show that under this model, whenever the maximum penalty has been assigned, the only way of increasing it is to obtain higher resources. Nevertheless, situations such as drug trafficking in which drug related crime is huge and so are the enforcement resources allocated to fight it calls for alternative deterring strategies.

The second case developed is one in which the government announces an allocation of resources according to a ranking of crime. Therefore, the choice of crime levels becomes necessarily strategic, because each criminal agent must anticipate the violence level of others in making its optimal violence decision. Therefore a negative concern for crime status may be induced by an allocation of resources according to a ranking. In that model a decrease in the overall amount of violence is accomplished. We discuss how the optimal allocation of resources results in the last unit of it being allocated to each organization having the same marginal return in terms of decreasing crime.

The third case developed is one in which the government applies Kleiman's *dynamic concentration* strategy (2011). This strategy argues that the government should announce that it will target the most violent criminal organization in a given period. Moreover, the government should announce it will concentrate all its resources on attacking that particular organization. Kleiman predicts that when applying this strategy during several periods, eventually all criminal organizations would end up having the same (low) level of violence. We show that this is possible even in a one-shot game. Moreover, we show that the pooling equilibrium that arises under this strategy is inefficient for the criminal organizations. This result gives an additional support to Kleiman's strategy, since nothing could be more desirable for a government than having criminal agents operating on an inefficiently low level of violence.

Therefore, our paper claims that if a government faces a situation in which *brute force* fails, as shown in the first case, it should apply Kleiman's *dynamic concentration* strategy because, even in the short run, it accomplishes the desired results.

7 References

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A Deterring with *Status*: Discrete Case

We now consider the game where the government confronts $N \in \mathbb{N}^+$ DTOs. In this case, the government deduces the DTOs' technologies with order statistics. This due to the fact that all of the degrees of efficiency are obtained with the same distribution and that is common knowledge.

In this problem, the probability of a DTO i to be the *k-th* DTO is:

$$P_k(A_i) = \frac{N!}{(k-1)!(N-k)!}\mathcal{F}(A_i)^{N-k}(1-\mathcal{F}(A_i))^{k-1}f(A_i) \tag{30}$$

In the above equation $\frac{N!}{(k-1)!(N-k)!}$ is the number of possibilities with which a particular DTO results being the *k-th* DTO with the highest degree of efficiency and $\mathcal{F}(A_i)^{N-k}(1-\mathcal{F}(A_i))^{k-1}f(A_i)$ is the probability that each of the aforementioned possibilities occur.

Denote with g_k the fraction of resources allocated in the k-th position of the violence ranking. Therefore, we may rewrite the maximization problem of the DTOs as:

$$\underset{a_i}{\text{Max}}\, A_i P(a_i) - a_i - \sum_k P_k g_k z \tag{31}$$

We assume that DTOs follow a separating equilibrium $a(A_i)$. Following a similar procedure to the one in the continuous problem, we find that the equilibrium path followed by DTOs is:

$$\dot{a} = \frac{\sum_k \frac{\partial P_k}{\partial A_i} g_k z}{A_i P_a - 1} \tag{32}$$

Notice that, since $\frac{d(1-\mathcal{F}(A_i))}{dA_i} < 0$, assigning any amount to the least violent DTO is inefficient (for the government in terms of decreasing violence). We may consider that assigning a positive amount to the least violent DTO results in an incentive to all DTOs to increase their level of violence. Hence we expect that under an optimal allocation of resources, no resources are assigned to the least violent DTO. On the other hand, the converse is true. Assigning any amount of resources to the most violent DTO results in an incentive to all DTOs to decrease their level of violence. All assignments to other positions result in an incentive to the more violent DTOs to increase their violence level and an incentive to the less violent DTOs to decrease their violence level. With this we conclude that any increasing array of the fraction of resources allocated to each position of the violence ranking results in an overall decrease of the violence levels. Nevertheless, in order to find the optimal allocation of resources we must solve a maximization problem that results from the solution to the previous equation.

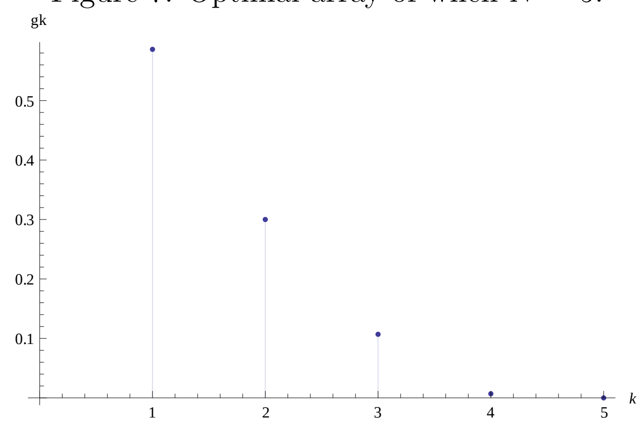
Unfortunately this problem does not always have a trivial solution. Nevertheless, whenever the distribution from which the degrees of efficiency are obtained is uniform, the problem has a solution.

Therefore we are going to develop the particular case where we have $\beta = \frac{1}{2}$, $A_{max} = 10$, $A_{min} = 1$, $N = 5$ and the DTOs' degree of efficiency follows a uniform distribution in the support $[A_{min}, A_{max}]$. Suppose $a(A_i)$ takes the form:

$$a(A_i) = \frac{A_i^2}{4(1+c)^2} \tag{33}$$

As in the continuous problem, the optimal allocation of resources does not depend on the amount of resources. Figure 5 shows the optimal allocation of resources, in terms of the fraction assigned to each position, for the problem at hand.

Figure 7: Optimal array of when $N = 5$.



As we expected, the optimal allocation results in high amounts of resources allocated to the most violent DTO and no resources to the least violent one.