Balancing the Power to Appoint Officers*

Salvador Barberà (MOVE, UAB and Barcelona GSE) and Danilo Coelho (IPEA)†

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Abstract

Rules of k names are frequently used methods to appoint individuals to office. They are two-stage procedures where a first set of agents, the proposers, select k individuals from an initial set of candidates, and then another agent, the chooser, appoints one among those k in the list. In practice, the list of k names is often arrived at by letting each of the proposers screen the proposed candidates by voting for v of them and then choose those k with the highest support. We then speak of v-rules of k names. Our main purpose in this paper is to study how different choices of the parameters v and k affect the balance of power between the proposers and the choosers. From a positive point of view, we analyze a strategic game where the proposers interact to determine what list of candidates to submit. From a normative point of view, we study the performance of different rules in expected terms, under different informational assumptions. The choice of v and k is then analyzed from the perspectives of efficiency, fairness and compromise.

Key Words: Voting rules; Constitutional Design; Strong Nash equilibrium; Rule of k Names. **JEL classification:** D02, D71, D72.

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†salvador.barbera@uab.cat and danilo.coelho@ipea.gov.br. The authors gratefully acknowledge support from the Spanish Ministry of Science and Innovation through grant "Consolidated Group-C" ECO2008-04756 and FEDER, from the Generalitat de Catalunya, Departament d'Universitats, Recerca i Societat de la Informació through the Distinció per a la Promoció de la Recerca Universitària and grant SGR2009-0419. Salvador Barbarà acknowledges financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2011-0075). We thank Miguel Ballester, Alexandre Carvalho, Anke Gerber, Matthew Jackson, David Jimenez and Carmelo Rodríguez-Álvarez for useful comments.

1 Introduction

Appointing people to office is one of the main ways how the powerful exert their influence in society. But the ability of any authority to appoint officers is often limited by the existence of other “de iure” or “de facto” powers. Even the President of the United States has to submit his proposals for cabinet members, for supreme court judges and for many other appointments to the approval of the legislators.

In this paper we study a class of methods that allow several agents to share the power to appoint. We call them rules of k names, and they work as follows. The set of deciders is divided into two groups: the proposers and the chooser. Proposers consider the set of all candidates to a position and screen k of them. Then, the chooser picks the appointee out of these k names. Indeed, rules of k names can vary, depending on the composition of the set of proposers, on the value of k, and also on the voting procedure adopted to form a list of k candidates.

Here we focus on a specific family of rules of k names that are used in many practical cases. This family adopts the following procedure to form the list of k names: each proposer submits a list of v candidates, and then the k most voted candidates get into the list. Though one can think of other methods to select the k names, the ones we consider are simple and frequently used. We call these procedures the v-rules of k names.

Rules of that form have been used in the past and are still very much used in the present. They seem particularly fitted to provide a balance of decision power between parties that have an interest to control who will be the rulers of certain institutions, like government bodies of the judiciary, public universities or local churches. These are examples of institutions that governments are interested to influence from the outside, but whose members would rather have under internal control. Historically, rules of k names were used within the Roman Church since the early middle ages, when secular rulers tried to control the appointment of bishops, while the clergy would rather decide on its leaders. And similar rules are still used to share the power between Rome and the local congregations. At present, the members of bodies that control the administration of justice are elected under rules of k names in many countries (Chile, Brazil, Colombia etc), thus allowing the legal professionals to propose their rulers, but also letting governments, or parliament, have a say on which internal candidates will eventually be appointed. Different countries (Mexico, Brazil, Turkey etc) have also adopted rules of k names to select their university rectors, with the government choosing from a short list generated within the university.

The purpose of this paper is to compare the ability of different v-rules of k names to balance the power between the proposers and the chooser. From a positive point of view, we analyze a strategic game where the proposers interact to determine what list of candidates to submit. From a normative point of view, we study the performance of different rules in expected terms, under different informational assumptions. The choice of v and k is then analyzed from the perspectives of efficiency, fairness and compromise.

Notice that a great variety of methods that are used in practice do differ on the values of both k, the size of the list, and v, the number of candidates that each proposer can vote for. There are cases where in order to participate in the choice of k candidates, each voter is allowed to submit k names. The rule used to elect Irish bishops or prosecutor-general in most of Brazilian states are of this sort, with $k = v = 3$. Yet, in most cases we know, each proposer is asked to submit a vote for v candidates, with v less than k. This is the case, for example, when choosing public university rectors in Brazil ($k = 3, v = 1$), members of Chile's courts of justice ($k = 3, v = 2$) or Chile's Supreme Court ($k = 5, v = 3$).

It is clear that the size k of the list to be submitted has an important effect on the distribution of power between the proposers and the chooser. In the extreme case where the proposers have to submit the whole list of candidates, all power goes to the chooser. In the opposite extreme case where $k = 1$, it is now the chooser who has no room left, and all the power stays in the hands of the proposers. In that case, however, there is still room for analysis, to determine what candidate would arise, depending on v and on the preferences of proposers. Our focus will be directed to those intermediate cases where both sides have some influence in the result. Specifically, we shall be interested in the kind of ex ante evaluation that a designer could make of different v-rules of k names, in terms of the expected utility that the chooser and the proposer might derive from each possible choice of the parameters k and v, under different scenarios/utility profiles. Armed with that kind of comparative assessment of different rules, a designer could eventually propose the use of a specific one in order to arbitrate between both parties, balance their power or distribute it according to different criteria.

In order to compute the expected utility that the proposers or the chooser would derive from the adoption of any specific v-rule of k names, a designer would have to gather information, or else to make assumptions regarding a number of relevant issues. Here are the ingredients required for such a calculation. First of all, the designer needs to know the distribution of utility profiles under which the rules to be compared are expected to eventually operate. Then, she needs to determine the outcome of the vote at each of the utility profiles. This outcome, in fact, will depend on the type of information available to agents at the time where their vote takes place, and the strategic behavior that they adopt, given that information. Our main calculations are made under the assumption that agents are fully informed about their preferences and those of all others at the time of vote, and that proposers play a strong Nash equilibrium of the ensuing strategic game where they decide what set of v candidates to vote for. An important part of our analytical effort goes to analyze the game that proposers play among themselves to determine the list of k names they present the chooser with. Because these games may have several strong Nash equilibria, or none, we concentrate attention to sets of environments where a unique strong Nash equilibrium is assured to exist, thus avoiding the need to make further predictions about the proposer's behavior in case of multiplicity or non-existence. For our restricted, yet non-trivial sets of environments, we characterize the equilibrium outcomes at each profile, and calculate the expected utility of the proposers and the chooser for each value of v and k. Of course, our calculations are made for specific distributions and environments, but we expect the reader to appreciate that the general method we propose to compare different rules could be extended to more complex situations.

Before we proceed, let us comment on some related papers. Unfortunately there does not seem to be a body of literature specifically devoted to study appointment rules with their checks and balances. Of course there exist many voting rules that can be adapted to this specific purpose, but we feel that it may be useful to focus on those that are especially fit for appointments. Our preceding papers on the subject (Barberà and Coelho, 2006 and Barberà and Coelho, 2010) provided an initial analysis of rules of k names and of possible ways to screen candidates. Our main results in Barberà and Coelho (2010) did focus on majoritarian rules.[1] Here we extend the analysis to the much wider class of v-rules of k names, of which only the case $v = k$ is majoritarian. In addition, we provide a fresh start

[1] We say that a screening rule is majoritarian if and only if for every set with k candidates there exists an action such that every strict majority coalition of proposers can impose the choice of this set provided that all of its members choose this action.

toward the normative evaluation of these rules. To mention some related work, Holzman and Moulin (2013) and Alon et al. (2011) concentrate on what they call nomination rules, leading to the choice of a fixed number of candidates where the candidates are also the voters. Even if different from our analysis, these papers show how being specific on the nature of the choice to be made can help in focusing on new axioms and new questions. We would also like to mention some sequential methods where different agents play different roles, as voters or vetoers, like Mueller's voting by veto (see Mueller,1978), Moulin's successive elimination procedures (Moulin, 1982) or Stevens, Brams and Merril's final-offer arbitrage procedures (Brams and Merrill, 1986 and Stevens, 1966). All of them are multi-stage procedures that also demand a game theoretic and a normative analysis, though in fact they are all different from each other and of v-rules of k names. What we can certainly say is that v-rules of k names are among the most widely used methods in that general vein.

As for the normative analysis in terms of expected utility, the papers closest to ours are those that study the design of egalitarian and utilitarian voting schemes. See for instance, Rae's (1969), Curtis (1972), Badger (1972), Coelho (2004) and Barberà and Jackson (2004). However, this literature focuses on the case where a society faces dichotomous choices. On the game theoretical side, the most related literature is the one characterizing the set of strong Nash equilibrium outcomes of voting games. See Barberà and Coelho (2010), Ertemel, Kutlu and Sanver (2010), Sertel and Sanver (2004), Polborn and Messner (2007), Moulin (1982) and Gardner (1977).

The paper is organized as follows. In the next section we introduce notation and definitions. In Section 3 we study the case where there is only one proposer. In Section 4 we analyze the case with several proposers, show some of the difficulties involved and concentrate on the specific but significant case where the proposers' interests are polarized. Conclusions follow in Section 5, and proofs appear in the Appendix.

2 The setup

In this section we formally define rules of k names and the games they induce. We observe that, in addition to other structural features, like the number of proposers, the number of candidates and the size k of proposed candidates, a full specification of a rule of k

names also requires to define the screening rules by which the proposers decide what names go into the list. In principle, this method could remain unspecified, or be rather complicated. But in actual practice simple and well specified screening rules are usually set. Basically, proposers are allowed to vote for a number v of candidates, and then the k most voted ones are selected (with a tie break if needed). These votes will typically be cast as the result of strategic calculations that may involve the cooperative coordination among players.

Let C be the finite set of candidates and $\mathbf{c}$ be its cardinality. For any $h < \mathbf{c}$, C_h denotes the family of subsets of C with cardinality h. Let $\mathbf{N} = \{1, ..., n\}$ be the finite set of proposers. The set of agents is $\mathbf{A} = \mathbf{N} \cup \{chooser\}$, where chooser is interpreted as an individual not in $\mathbf{N}$. Let W be the set of all strict orders[2]on C. Elements in W are denoted by $\succ_i, \succ_j$,....

Societies, or preferences profiles, are elements of W^{n+1}, denoted as $(\succ_1, \succ_2, ..., \succ_n, \succ_c)$. The first n components are interpreted to be the preferences of the proposers and the last component stands for the preferences of the chooser.

In addition to stand for the preferences of agents, orders of the set of candidates will also be used to break ties among alternatives, as we shall be later. Given an order $\succ$, and any subset of candidates $B \subset C$, we denote by $\alpha(B, \succ)$ the best candidate in B according to $\succ$.

We now define v-rules of k names, where $k \leq \mathbf{c}$ is the size of the set of candidates that the proposers must submit and $v \leq k$ is the number of candidates that each proposer can support.

Definition 1 *Given any n-tuple $(B_1, ..., B_n)$ belonging to C_v^n, of sets of size v, the score of a candidate $x \in C$ at $(B_1, ..., B_n)$ is the number of B_i's containing x, $s(x, B_1, ..., B_n) = \#\{B_i | x \in B_i\}$. A set T is most voted in $(B_1, ..., B_n)$ if for all $x \in T$ and $y \in C \backslash T$, $s(x, B_1, ..., B_n) \geq s(y, B_1, ..., B_n)$. A $v-$**screening rule of** k **names** is a function $g : C_v^n \to C_k$ that selects a set T of k most voted candidates for each n-tuple of sets of size v.*

Definition 2 *A $v-$**rule of** k **names** is a function $f : C_v^n \times W \to C$ defined so that*

[2]Transitive: For all $x, y, z \in A : (x \succ y$ and $y \succ z)$ implies that $x \succ z$.

Asymmetric: For all $x, y \in A : x \succ y$ implies that $\neg(y \succ x)$.

Irreflexive: For all $x \in A, \neg(x \succ x)$.

Complete: For all $x, y \in A : x \neq y$ implies that ($y \succ x$ or $x \succ y$).

$f(B_1, ..., B_n, \succ) = \alpha(g((B_1, ..., B_n)), \succ)$, *for some v-screening rule of k names g.*

Notice that our definition of a set of most voted candidates allows for some candidates in T and some outside of T to get the same number of votes. This is because, in our setting, there may be cases where several candidates get the same number h of votes, the set of those getting more than h votes is smaller than k and the set of those getting at least h votes is larger than k. In these cases, the screening rule must "break the tie" between these candidates who just got h votes, and select enough of them to complete a set of size k. From now on, we will assume that our "tie breaking rules" are given by an order of candidates, and that this order is either fixed, or coincides with the preferences of some predetermined agent[3].

An important part of our work will consist in analyzing the type of strategic interactions that may arise among the proposers, as a function of their preferences and those of the chooser. We model these interactions as a normal form game with complete information, and concentrate our analysis on the study of its strong Nash equilibria.

Definition 3 *(Barberà and Coelho, 2010) Given* $k \in \{1, 2, ..., \mathbf{c}\}$ *and* $v \in \{1, 2, ..., k\}$, *a v-screening rule for k names* $g : C_v^n \rightarrow C_k$ *and a preference profile* $(\succ_1, \succ_2, ..., \succ_n, \succ_c) \in W^{n+1}$, *the* ***Constrained Chooser Game*** *is the simultaneous game with complete information where each player* $i \in \mathbf{N}$ *chooses a strategy* $B_i \in C_v$. *Given* $(B_1, ..., B_n) \in C_v^n$, $g((B_1, ..., B_n))$ *is the chosen list with k names and the winning candidate is* $\alpha(g((B_1, ..., B_n)), \succ_{chooser})$.

In the Constrained Chooser Game, the chooser's strategy set is restricted to a single element. In that sense, we could say that she is not an active player. Specifically, we take it that the chooser will simply select that candidate that is best for her among those that he will be presented with. Thus, the chooser's preferences determine the game's outcome function, and will have an impact on the equilibrium play of the proposers. But, in the spirit of subgame perfection, and given the sequential form of our rules, we exclude the possibility that the chooser may select a candidate that is not her best in the list she is presented with.

[3]For example, when $k = 3$ and $C = (x, y, z, w)$, if x,y receive 3 votes and the remaining two candidates z, w receive the same number of votes (say 2, one or none), the proposed set will be formed by x, y and the highest ranked among z and w according to the tie breaking rule.

We choose to analyze the set of strong Nash equilibria of this game. This is consistent with the idea that proposers have complete information about their preferences and those of the chooser, and that they must find ways to cooperate among themselves, in order to come up with a favorable list.

Definition 4 *Given* $k \in \{1, 2, ..., \mathbf{c}\}$ *and* $v \in \{1, 2, ..., k\}$*, a* v*-screening rule for* k *names* $g : C_v^n \longrightarrow C_k$ *and a preference profile* $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{n+1}$*, a joint strategy* $(B_1, ..., B_n) \in C_v^n$ *is a* ***pure strong Nash equilibrium of the Constrained Chooser Game*** *if and only if, given any coalition* $G \subseteq N$*, there is no* $(B'_1, ..., B'_n) \in C_v^n$ *with* $B'_{,j} = B_{,j}$ *for every* $j \in \mathbf{N} \backslash G$ *such that* $\alpha(g((B'_1, ..., B'_n)), \succ_{chooser}) \succ_i \alpha(g((B_1, ..., B_n)), \succ_{chooser})$ *for each* $i \in G$*.*

3 The case of one proposer

A designer is interested in determining the expected utility that a proposer and a chooser would derive from using any given rule within our class. Since there is only one proposer, only the value of k matters, and v does not play a role.

In order to carry out the ex ante calculations, the designer has to attribute utilities on candidates to each of the agents. We'll assume here that the designer takes the (negative of the) ranking of the different candidates as a proxy for the utility they derive from their election. Formally, $u_i(y) = -r_i$ where $r_i = 1 + \#\{z \in C | z \succ_i y\}$.

Under these simple specifications, each preference order is associated to one utility function, and from now on we use the terms preference profile and utility profile interchangeably.

We also assume that, once a utility profile is realized, both agents are informed about it, and the proposer uses this information to vote strategically, knowing that the chooser can only choose her best candidate in the received list. It is easy, in that case, for the designer to characterize the equilibrium behavior at each realized profile. Notice that a candidate can only be the chooser's best in a list with k names if it is among the chooser's best $\mathbf{c} - k + 1$-top candidates. Therefore, the equilibrium outcome is the best candidate for the proposer out of the $\mathbf{c} - k + 1$-top candidates of the chooser. This outcome can be obtained by not presenting the chooser with any of the candidates that would be better for her.

If the designer is endowed with a distribution indicating the probability that each profile of preferences is realized, then she is ready to compute the expected utility from choosing each possible value of k. For the purpose of illustration, we proceed under the assumption that the agents' preferences over the set of candidates are the result of independent random draws from a uniform distribution over the domain of all strict preferences.

Under the scenario where agents know each other's preferences at the time of vote, the expected utilities that the designer attributes to the proposer and to the chooser can be computed as their utility for the equilibrium outcome resulting at each profile times the probability that the profile obtains. *In what follows, and for simplicity, x will stand for the equillibrium outcome at each profile.*

Under the scenario where agents know each other's preferences at the time of vote, the expected utilities that the designer attributes to the proposer and to the chooser are:

$$E(u_p(x)|\mathbf{c}, k) = -\frac{(\mathbf{c}+1)}{(\mathbf{c}-k+2)} \qquad (1)$$

$$E(u_c(x)|\mathbf{c}, k) = -\frac{(\mathbf{c}-k+2)}{2} \qquad (2)$$

From expressions (1) and (2), notice that the proposer's expected utility is strictly decreasing with k, while the chooser's expected utility is strictly increasing with k. Thus, when $k = 1$ the chooser's expected utility reaches its minimum and $E(u_c(x)|\mathbf{c}, k = 1) = -\frac{\mathbf{c}+1}{2}$, while proposer's reaches its maximum, $E(u_c(x)|\mathbf{c}, k = 1) = -1$.

The expected utilities have these functional forms because the random variable r_p ($r_p \equiv 1+\#\{y \in C | y \succ_p x\}$) has the same distribution as that of the smallest element of a random sample with size $s = \mathbf{c} - k + 1$ drawn without replacement from a uniformly distributed population $D = \{1, 2, \ldots, \mathbf{c}\}$ and the random variable r_c ($r_c \equiv 1 + \#\{y \in C | y \succ_c x\}$) has the same distribution as that of a discrete random variable uniformly distributed over $\{1, 2, \ldots, \mathbf{c} - k + 1\}$.

Now, given these expected utility values, the designer can select a k satisfying any desirable criteria. We have considered three possible criteria for selection:

-egalitarianism: choose a k that minimizes, $|E(u_p(x)|\mathbf{c}, k) - E(u_c(x)|\mathbf{c}, k)|$,the difference between the expected utility of the proposer and that of the chooser, and thus equalizes them whenever possible with an integer value of k.

-utilitarianism: choose a k that maximizes, $E(u_p(x)|\mathbf{c}, k) + E(u_c(x)|\mathbf{c}, k)$, the sum of the proposer and the chooser's expected utilities.

-Nash bargaining: choose a k that maximizes, $(E(u_p(x)|\mathbf{c},k)-d)(E(u_c(x)|\mathbf{c},k)-d)$, the product of the proposer and the chooser's expected utilities where d, the status quo expected utility for each of the players, is equal to $-\frac{\mathbf{c}+1}{2}$. The status quo expected utility can be interpreted as the one that the agents would obtain if the winning candidate was chosen at random with uniform probability, rather than through any bargaining process.

Interestingly, these three criteria lead to the selection of the same values for k, in our case. The reason is simple: the combination of expected utilities for the proposer and for the chooser that we get as k changes constitute a symmetric set. Since the egalitarian and the utilitarian solutions satisfy Nash's axiom of symmetry, and our bargaining problem is symmetric, they both coincide with Nash's solution in this nice case. In fact, they may lead to the choice of one or at most two rules, defined by consecutive values of k, depending on the number of candidates. At any rate, we can always say that the chosen value must be greater or equal than half the number of candidates. That fact may be a bit disturbing, since in real life we observe the use of small values of k. But this is due to the specificity of the one proposer case, where the proposer gets a large advantage, that can only be compensated by a larger k. In the next section, we will see that these k values become smaller as the polarization among the proposers increases.

The following proposition expresses our preceding remarks more formally and with additional detail.

Proposition 1 *The egalitarian, utilitarian and Nash bargaining choice of k coincide in the one proposer case. If $z=\mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}$ is an integer, the two values $\{z-1,z\}$ are selected. Otherwise, the selected value is unique and equal to $\lfloor z\rfloor$[4]. If, in addition, $z=\mathbf{c}-\sqrt{2\mathbf{c}+2}+2$, then full equalization of expected utilities is achieved at $k=z$. The value of the optimal k's according to the egalitarian, utilitarian and Nash bargaining criteria is always greater or equal than $\frac{\mathbf{c}+1}{2}$.*[5]

Before we leave this one proposer case, let us just compare it with a different scenario. This is the case where at the time of vote agents would know their own preferences, but still remain completely ignorant about the characteristics of others. To simplify the analysis, suppose that the proposer selects a strategy assuming that the chooser's

[4] $\lfloor z\rfloor$ is the largest integer not greater than z.

[5] The proofs of the propositions are in the Appendix A.

preferences are the result of independent random draws from a uniform distribution over the domain of strict preferences. In that case, it is reasonable to assume that the proposer will always include their k best candidates in the list. Then, under the same distributional assumptions over preferences and preference profiles, the ex ante expected utilities for the proposer and for the chooser would be

$$E(u_p(x)|\mathbf{c}, k) = -\frac{(k+1)}{2} \qquad (3)$$

$$E(u_c(x)|\mathbf{c}, k) = -\frac{(\mathbf{c}+1)}{(k+1)} \qquad (4)$$

and the value of k that would simultaneously correspond to the egalitarian, utilitarian and Nash bargaining distribution of utilities would be equal to $\mathbf{c} - k^* + 1$ where k^* is the optimal k under the complete information scenario.

Let us now compare the results from the complete information case with those that obtain under ignorance at the time of vote. Take, as a reference point, the case where the proposer was allowed to nominate half of the candidates. Then the proposer has a first mover advantage in the complete information case, while that advantage goes to the second mover, the chooser, in the case of perfect ignorance. These advantages are symmetric, and the choices of k correspond to the need to compensate the weakest of the two players by allowing them to nominate more, or less than half of the candidates, depending on the scenario. Since these two scenarios are very extreme, we can interpret our result under the complete information as an upper bound for the optimal k, and the one under complete ignorance as a lower bound.

From now on, we abandon the scenario of complete ignorance, which was just introduced for comparative purposes, and we stick all along the paper to the realistic assumption that agents involved in the use of v-rules of k names are well informed about the characteristics of all players.

4 The case of several proposers

We now turn attention to the more general case, where several proposers must determine what list of k names they submit to the chooser, who then chooses one of them.

We have already seen in the preceding section that, as part of the search for the ex ante expected values of different v-rules of k names, the designer needs to assess what would be the list chosen by the proposers at each preference profile. The equilibrium choice of a list by a single informed proposer was easy to establish. But now, the presence of several proposers acting under a v-screening rule, introduces several new challenges. One needs to describe the game played by the proposers when deciding what candidates to support, and the solution concept that fits their expected behavior. We study the specific game where the strategies of the proposers are sets of v candidates, and the outcome function is the straightforward choice by the chooser of her best candidate among those in the list. We call it the Constrained Chooser Game, because we are implicitly assuming that the chooser will not act strategically, trying to reach agreements with any of the proposers. Given the sequential structure of the decision process, and in the spirit of subgame perfection, this chooser restricted game discards the possibility of the chooser committing to act in a non-maximizing way. As for the proposers, we assume that they can coordinate their actions, and thus study the outcomes associated with strong Nash equilibrium play. We have provided formal definitions for this game and for our equilibrium concept in Section 2.

We first present the reader with two examples that show how interesting, but also how complex and revealing the analysis of equilibrium can become. Then, we introduce a specific type of societies that capture some essential features of the interplay between several proposers whose interests are in conflict. These are what we call polarized proposer's societies: proposers come in two types, one majoritarian and the other minoritarian, whose preferences are exactly opposed. We are able to prove that in these societies the Constrained Chooser Game always has a unique strong Nash outcome. Moreover, we can provide an exact characterization of this equilibrium, and this allows us to reach our goal: for any given distribution of potential societies over which it must operate, we show how the designer can calculate the expected utility of the chooser, and the average utility of the proposers. Again, armed with this information and with some normative criterion, it is possible to arbitrate among different rules.

4.1 Two interesting voting situations

It is clear from the one proposer case that, in order to compute the expected value of a rule, one needs to predict the outcome of voting at any given profile. Finding the equilibria of the games generated under different v-rules of k names is thus a necessary step prior to the choice of those (v, k) values that identify those satisfying any desirable normative criterion. In our search for equilibria and their characterizations, we'll face a number of difficulties, that eventually lead us to concentrate on a simple model. But in order to give the reader a feeling of the interesting problems that arise, let us plunge directly in the following example.

There are five candidates $\{c1, c2, c3, c4, c5\}$ and eleven proposers. Each proposer is allowed to vote for one candidate ($v = 1$) and a list will be formed with the names of the three most voted candidates ($k = 3$), with ties being broken according to the order $c1 \succ c3 \succ c4 \succ c5 \succ c2$. The type (preferences) and the number of agents are given in the following table.

Preference Profile

1 type 1 proposer	7 type 2 proposers	3 type 3 proposers	Chooser
$c1$	$c3$	$c2$	$c1$
$c3$	$c2$	$c3$	$c2$
$c4$	$c5$	$c4$	$c3$
$c5$	$c1$	$c5$	$c4$
$c2$	$c4$	$c1$	$c5$

We shall argue, in what follows, that $c2$ can be the outcome induced from the strong Nash equilibrium play of the proposers when the chooser always picks his best candidate in the list.

Consider the following strategy profile that sustains $c2$ as a strong Nash equilibrium outcome: the seven type 2 proposers cast four votes for $c3$ and three votes for $c4$. The only one type 1 proposer casts a vote for $c1$, while the three type 3 proposers cast three votes for $c2$. Thus, the selected list is $\{c3, c2, c4\}$ and $c2$ is the winning candidate.

The argument behind this equilibrium is quite clear. Type 3's go ahead in support of $c2$, and then the type 2's have to prevent $c1$ from becoming the outcome by "wasting" their remaining votes in support of $c4$.

But there is another, maybe more interesting equilibrium. Notice that any coalition with at least three proposers can impose at least one candidate in the list, and that the chooser and the three proposers of type 3 prefer $c2$ to $c3$. In spite of this, candidate $c3$ can also be sustained as a strong Nash equilibrium outcome! To verify it, consider the following strategy profile: the seven type 2 proposers cast three votes for $c3$, two votes for $c4$, one vote for $c1$ and one vote for $c5$. Type 1 proposer casts a vote for $c1$, while the three type 3 proposers cast two votes for $c5$ and one for $c4$. So, $c3$,$c5$ and $c4$ will have three votes each, while $c1$ only two. Thus, the selected list is $\{c3, c5, c4\}$ and $c3$ is the winning candidate. The reader can check that no coalition of voters can profitably deviate.

Now, here is a intuition for this equilibrium, where the two proposers of type 2 cleverly distribute their votes in order to prevent the type 3's from being able to select $c2$, even if they all vote for it. Voters of type 2 ensure that candidate $c3$, their favorite, is among the proposed ones, by casting three votes in its favor. They also give enough support to candidate $c1$ so that, along with the vote of type 1, $c1$ is still not chosen, but would be as soon as candidates with two votes enter the list. Then, since $c1$ has two votes, proposers of type 3 cannot vote for their favorite, $c2$, because if they all spent their votes on $c2$, which would make $c2$ eligible, then some alternative with two votes would come in, and in this case it would be $c1$, which they hate but is the chooser's best. Given that they cannot get $c2$, they then concentrate, in alliance with type 2 people, in getting $c4$ and $c5$ into the list, both above their worse alternative $c1$, in order to at least get their second alternative.

Thus, the presence of the type 1 proposer voting for $c1$ leads types 2 and 3 into a sort of race: if one of them uses the most rewarding strategy in one of the two equilibria, the other must concede. If both used their most rewarding strategies, then $c1$, that they both hate, would come out!

In this example, we can observe several types of strategic behavior on the side of agents. The richness of the example also leads to the existence of several equilibria among which it is hard to choose. Multiplicity of equilibria adds to the difficulty of characterizing any of them. Hence, even if the steps to be taken toward any specific choice of optimal rules are quite clear, we cannot expect simple, general explicit solutions. This is why we shall eventually simplify the setting where we work.

Our second example is also clarifying. The choice of v and k has an impact on the balance between the satisfaction of the chooser and that of the proposers. But our following example shows that this impact is complex: without any further restrictions, the effects of k and v on the agents' payoffs are not monotonic.

There are four candidates $c2$,$c1$,$c3$ and $c4$, and three proposers. Each proposer votes for one candidate and the list has the names of the two most voted candidates ($v = 1, k = 2$), with a tie breaking rule when needed: $c3 \succ c4 \succ c1 \succ c2$.

Preference Profile

Proposer 1	Proposer 2	Proposer 3	Chooser
$c1$	$c2$	$c2$	$c1$
$c4$	$c3$	$c3$	$c2$
$c3$	$c4$	$c4$	$c3$
$c2$	$c1$	$c1$	$c4$

Candidate $c3$ is the unique strong Nash equilibrium outcome under $v = 1$ and $k = 2$. Here is an intuition for this result: notice that candidate $c2$ cannot be a strong equilibrium outcome, because as long as proposer 1 votes for $c1$, proposers 2 and 3 cannot get $c2$ to be the outcome, even if they can force $c2$ to be in the list. Short of that, proposers 2 and 3 coordinate their actions so that one of them votes for $c3$ and the other for $c4$. If 1 persists in voting for $c1$, this creates a tie between the three candidates that is solved in favor of $c3$ and $c4$, out of which the chooser selects $c3$. If 1 votes for $c3$ instead, the same outcome ensues. And all other actions by any combination for agents would lead some of them to outcomes that would be worse than $c3$ for some of them. Hence, $c3$ is the unique strong Nash equilibrium under $(v, k) = (1, 2)$. The case $(v, k) = (1, 1)$ is simple and lead to the election of $c2$.

The table below presents the set of strong Nash equilibrium for different values of v and k.

Set of strong Nash equilibrium outcomes

k=1	$v = 1$	$\{c2\}$
k=2	$v = 1$	$\{c3\}$
k=2	$v = 2$	$\{c2\}$
$k = 3$	$v = 1$	$\{c1\}$
$k = 3$	$v = 2$	$\{c2\}$

Notice that, with v fixed at 1, the chooser prefers $k = 1$ than when $k = 2$. This is quite surprising, since $k = 1$ means that the chooser has no power at all! Notice also that he prefers $k = 3$ to $k = 2$ when $v = 1$. On the other hand, for $k = 2$, the chooser prefers the higher value $v = 2$ to that of $v = 1$. However, for $k = 3$, he prefers the lower value

$v = 1$ to that of $v = 2$.

4.2 Polarized Proposers Model

The rich examples in the preceding section show that we cannot expect to develop a full, general analysis of the equilibrium outcomes when a diversity of proposers coexists. This also makes it hard to develop explicit formulas for the expected utilities of proposers and choosers would derive from different choices of k and v. Yet, our general reasoning can still be applied to analyze specific cases, however complex. Better than that, we can provide an explicit analytical development for the case of several proposers, for the special case where these are divided into two antagonistic groups. This model is able to take into account the tension that arises among the proposers, when drawing a list of k names, provided the tension is not too diffused, and concentrates between two groups in conflict. We call it the polarized proposer's case, and it is described by the following characteristics:

1. (Assumption 1). The set of proposers is partitioned into two groups G_1 and $G_2 = N \backslash G_1$,with sizes $\#G_1 = m > \#G_2 = n - m$.

2. (Assumption 2). All proposers in G_1 share the same preferences over the set of candidates.

3. (Assumption 3). All proposers in G_2 share the same preferences over the set of candidates, and they are the reverse of those of agents in G_1.

4. (Assumption 4). The tie breaking rule coincides with at least one of the agents preferences over the set of candidates.[6]

As we shall see, these conditions guarantee that there will always exist a unique strong Nash equilibrium for the Constrained Chooser Game, thus allowing us to compare rules according to the expected utility of the chooser, and the average expected utility of the proposers. Before we plunge into the analysis of that case, we provide some general results that are useful to partially characterize equilibria even under more general conditions.

Notice that any v-screening rule of k names endows each group of proposers with some power to determine what candidates are to be included in the list submitted to the

[6] Without this assumption, the existence result may not hold. See an example in Appendix B.

chooser. The following definitions and results apply for any given v-rule of k names and any set X of candidates.[7]

Definition 5 *Let $q_k^v(X)$ be the minimum integer $\widehat{q}$ such that, for any coalition G of voters with size at least as large as $\widehat{q}$, agents in G can vote in such a way that all elements in X are included in the list, for any vote of the proposers in N/G. That is, $q_k^v(X)$ is computed in such a way that any coalition of that size or larger can always guarantee itself the inclusion of X in the list, if its members coordinate their votes.*

Remark 1 *The values of $q_k^v(\cdot)$ evolve monotonically with those of k and v. For any C and $v < v' < k < k' < c$, we have that:*

1. $q_k^v(X) \geq q_k^{v'}(X)$ for any $X \in C_k$;

2. $q_{k'}^v(X') \geq q_k^v(X)$ for any $X \in C_k$ and $X' \in \{Y \in C_{k'} | X \subset Y\}$;

3. $q_k^{v'}(\{x\}) \geq q_k^v(\{x\})$ for any $x \in C$;

4. $q_k^v(\{x\}) \geq q_{k'}^v(\{x\})$ for any $x \in C$.

Remark1 tell us that $q_k^v(\cdot)$ is increasing in k and decreasing in v, while $q_1^v(\cdot)$ is decreasing in k and increasing in v. Thus, an increase in k or a decrease in v alters the distribution of power among the proposers in the following ways: (1) it affects non-positively the cardinality of the set of possible coalitions of players that are able to impose all the names in the list and (2) it affects non-negatively the cardinality of the set of possible coalitions of players that that are able to impose at least one name in the list. Notice that (1) implies that some strong Nash equilibrium outcomes under (k, v) may not be a strong Nash equilibrium under $(k, v-1)$ or under $(k+1, v)$. Notice also that (2) implies that the chooser's best candidate in the list may become an equilibrium outcome under $(k, v-1)$ or under $(k+1, v)$ in spite of not being an equilibrium outcome under (k, v).

These q_k^v values may differ (but not too much) for sets of the same size, depending on the names of the alternatives that they include, because the tie breaking rule treats candidates asymmetrically. Hence, we may also define some absolute bounds that work

[7] Notice that definitions 5 and 6 are closely linked to that of effectivity functions studied by, among others, Peleg (1984), Abdou and Keiding (1991) and Sertel and Sanver (2004). These concepts of effectivity refer to the ability of agents to ensure an outcome, under the given rule.

for any set. In particular, we'll use those bounds that apply for singletons or for sets of size k, since they are the ones that will help in characterizing equilibria.

Definition 6 *Let* $q_1^v \equiv Max_{y \in \mathbf{C}}\{q_k^v(\{y\})\}$ *and* $q_k^v \equiv Max_{Y \in \mathbf{C}_k}\{q_k^v(Y)\}$.[8]

We are now ready to provide a necessary condition that must be satisfied by any strong Nash equilibrium outcomes for the Constrained Chooser Game, whatever the preferences of agents might be. In addition to its intrinsic interest, the result will be later used in our analysis of the polarized proposers' case.

Proposition 2 *If candidate x is a strong Nash equilibrium outcome of the Constrained Chooser Game, then it satisfies the following four conditions*

C1: It is among the chooser's $(c-k+1)$*-top candidates.*

C2: If $y \neq x$ *is among chooser's* $(c-k+1)$*-top candidates then* $\#\{i \in N | y \succ_i x\} < q_k^v(Y)$ *for any* $Y \in C_k$ *such that y is the chooser's best candidate in Y.*

C3: If y is the chooser's best candidate then $\#\{i \in N | y \succ_i x\} < q_k^v(\{y\})$.

C4: If y is the chooser's best candidate and also ranked above than x by the tie breaking criterion then $\#\{i \in N | x \succ_i y\} \geq q_k^v$.[9]

In the general case where all preferences are allowed, no conclusive statement can be reached regarding the gains for the chooser, as shown by our second example, in Section 4. The difficulty to make definite statements under a universal domain of preferences is compounded the possibility that, when changing parameters, one of them may guarantee existence of equilibria but not the other. In spite of these added difficulties, we can prove the following result that holds for the universal domain of preferences:

Proposition 3 *If the chooser 1-top-candidate is a strong Nash equilibrium outcome of the Constrained Chooser Game under v́-rule for ḱ names then it is also a strong Nash*

[8] We can actually compute these bounds explicitly, as follows: $q_k^v = \lceil \frac{kn}{(k+v)} \rceil + \mathcal{I}(\lfloor \frac{v \lceil \frac{kn}{(k+v)} \rceil}{k} \rfloor \leq n - \lceil \frac{kn}{(k+v)} \rceil)$ and $q_1^v = \lceil \frac{vn}{(k+v)} \rceil + \mathcal{I}(\frac{vn}{(k+v)} = \lceil \frac{vn}{(k+v)} \rceil)$, where $\mathcal{I}$ denotes the indicator function that takes value 1 if the expression in brackts is true, and 0 otherwise.

[9] In Appendix B, we show how Proposition 2 can be useful to locate an equilibrium outcome. We also give an example where the set of strong Nash equilibrium outcomes is empty.

equilibrium outcome of the Constrained Chooser Game under any v-rule for k names whenever $v \leq v'$ and $k' \geq k$ provided that both screening rules have the same tie breaking criterion.

In the case of polarized proposers, we can establish the existence of a unique strong Nash equilibrium and to provide a characterization.

Proposition 4 *Consider the Polarized Proposers Model and any v-rule of k names. A strong Nash equilibrium outcome of the Constrained Chooser Game always exists and it is unique. In addition:*
1) Suppose that the tie breaking criterion coincides with the majoritarian group's preferences over the set of candidates.
If $m \geq q_k^v > n - m$ then the strong Nash equilibrium outcome is the best candidate of the majoritarian group out of chooser's $(c-k+1)$-top candidates;
If $q_k^v > m \geq q_1^v > n - m$ then the strong Nash equilibrium outcome is the chooser's best candidate out of the majoritarian group's k-top candidates;
If $q_k^v > m > n - m \geq q_1^v$ then the equilibrium outcome is the chooser's best candidate.

2) Suppose that the tie breaking criterion coincides with the chooser's preferences over the set of candidates or with the minoritarian group's preferences over the set of candidates.
If $m \geq q_k^v$ then the strong Nash equilibrium outcome is the best candidate of the majoritarian group out of chooser's $(c-k+1)$-top candidates;
If $q_k^v > m$ then the strong Nash equilibrium outcome is the chooser's best candidate.[10]

The following three corollaries apply to the case where our v-rules of k names are used in societies where proposers are polarized. They follow from Proposition 4 and Remark 1.

Corollary 1 *The chooser cannot be worse off under v'-rule for k names than under $\widetilde{v}$-rule for k names whenever $\widetilde{v} > v'$.*

[10] A simple and interesting case of polarized societies arises when all proposers share the same preferences, i.e., $G_1 = N$ and $G_2 = \emptyset$. In that case, equilibria look essentially the same as when there is only one proposer. That is, the strong Nash equilibrium outcome is the proposers' best candidate out of chooser' s $(\mathbf{c} - k + 1)$-top candidates.

Corollary 2 *The chooser cannot be worse off under v-rule for k' names than under v-rule for $\widetilde{k}$ names whenever $k' > \widetilde{k}$.*

Corollary 3 *The chooser cannot be worse off under a more polarized set of proposers (small m) than under a less polarized set of proposers (big m).*

As shown by Corollaries 1 and 2 above, in the case of the polarized proposer's model, the chooser will always weakly prefer a smaller v and a larger k. As for the proposers, and given their polarization, some of them will gain and some will loose from any given parameter change.

Moreover, an increase in k has an additional effect: it increases the number of candidates available for the chooser. Thus, regarding chooser's payoff, a change in k tends to have a higher impact than a change in v.

4.3 The ex-ante analysis of different rules

In that section we present the calculations that a designer could make to determine the expected utility for the proposers and of the choosers under different v-rules of k names, in societies with polarized proposers. We are able to produce explicit computations under appropriate assumptions, which provide us with insights regarding the trade-offs between the choices of v and k, and the impact of these choices upon the agents. We can also calculate what rules could distribute expected utility in the most egalitarian way, between the proposers and the chooser.

Before engaging in these computations, let us remark that each of our modeling decisions in what follows could be altered without changing the essence of our exercise. Regarding the specification of possible worlds, it is not hard to extend it to cases where the preferences of agents are still based on the ranking of the outcomes but exhibit different degrees of risk aversion. As for the informational assumptions, one could also study easily the polar case where, once a profile is realized, each agent is only informed about her own preference, but remains ignorant about those of the rest. In that case, it becomes natural to assume that agents will behave sincerely, rather than strategically, and the computations carry over in a similar manner. Finally, it is clear that one could still resort, like we did for the one proposer case, to alternative evaluation criteria, like weighted utilitarianism, or any sort of distributional criterion regarding power, other than egalitarianism. But again, our main message here is that the use of some method over another may be discussed in expected utility terms and that, in our case, it is even possible to get a feeling for the trade-offs involved in the choice of any pair (v, k) over any other (v', k') through explicit numerical computations.

As in the one proposer case, here we assume that the preference of the majoritarian group of proposers and that of the chooser are drawn independently from a uniform distribution, and this generates the distribution over polarized profiles. Again, we assume that the utilities of agents assign to each candidate the negative of its rank. We denote by $r_p \equiv \frac{m}{n} r_{G_1} + \frac{n-m}{n} r_{G_2}$ and $u_p(x) \equiv \frac{m}{n} u_{G_1}(x) + \frac{n-m}{n} u_{G_2}(x)$ the average utility of an outcome for the proposers, given x the outcome, $r_{G_1} \equiv 1 + \#\{y \in C | y \succ_{G_1} x\}$ and $r_{G_2} \equiv 1 + \#\{y \in C | y \succ_{G_2} x\}$.

Our next two propositions may be a bit tedious, but we still include them in order to show that one may compute exact values for expected utilities in our model, and use them to determine the egalitarian values (v, k). They refer to the case of polarized proposers when the majoritarian group's preferences are used to break ties. Under these conditions, we can state

Proposition 5 *For any v-rule of k names, the agents' expected utilities are given by the following expressions:*

1. If $m \geq q_k^v > n - m$:

   $E(u_p(x)|c, k, v) = -\frac{m}{n} \frac{(\mathbf{c}+1)}{(\mathbf{c}-k+2)} - \frac{n-m}{n} \frac{(\mathbf{c}+1)(\mathbf{c}-k+1)}{(\mathbf{c}-k+2)}$

   $E(u_c(x)|\mathbf{c}, k, v) = -\frac{(\mathbf{c}-k+2)}{2}$

2. If $q_k^v > m \geq q_1^v > n - m$:

   $E(u_p(x)|\mathbf{c}, k, v) = -\frac{m}{n} \frac{(k+1)}{2} - \frac{n-m}{n} \frac{(2\mathbf{c}-k+1)}{2}$

   $E(u_c(x)|\mathbf{c}, k, v) = -\frac{(\mathbf{c}+1)}{(k+1)}$

3. If $q_k^v > m > n - m \geq q_1^v$:

   $E(u_p(x)|\mathbf{c}, k, v) = -\frac{(\mathbf{c}+1)}{2}$

$E(u_c(x)|\mathbf{c}, k, v) = -1$

Notice that the different cases in Proposition 5 arise because, in view of the size of the majorities, and the power assigned by the choice of v and k to the majority and the minority, equilibria will be differently characterized as shown by Proposition 4.

We can now use the results from this proposition to identify the parameters that would lead to a most egalitarian distribution of expected utilities in our polarized societies.

Specifically, we'll look for those pairs (v, k) that minimize the difference between the expected utility of the chooser and that of the proposers, which is itself and average of the utilities of proposers from both polar groups.

Definition 7 *A pair (k, v), such that $k \in \{1, ..., \mathbf{c}\}$, $v \in \{1, ..., k\}$ and $v \leq k$, is an egalitarian solution if $|E(u_p(x)|\mathbf{c}, k, v) - E(u_c(x)|\mathbf{c}, k, v)| \leq |E(u_p(x)|\mathbf{c}, k', v') - E(u_c(x)|\mathbf{c}, k', v')|$ for every $k' \in \{1, ..., \mathbf{c}\}$ and $v' \in \{1, ..., k'\}$. We denote by S_e the set of all values of (k,v) that are egalitarian solutions.*

Remark 2 *Notice that, ceteris paribus, the egalitarian value of k for polarized societies is non increasing in m. This is due to Corollary 3.*

Proposition 6 *For any v-rule of k names, the set S_e of pairs of parameters (v, k) defining the most egalitarian distribution of expected utilities has the following characteristics:*
$S_e \subseteq S_1 \cup S_2$.
where
$S_1 = \{(k, v) \in \{\lfloor \tau_1 \rfloor, \lceil \tau_1 \rceil\} \times \{1, ..., \lceil \tau_1 \rceil\} | m \geq q_k^v\}$
$S_2 = \{(k, v) \in \{\lfloor \tau_2 \rfloor, \lceil \tau_2 \rceil\} \times \{1, ..., \lceil \tau_2 \rceil\} | q_k^v > m \geq q_1^v > n - m\}$
$\tau_1 = \frac{m}{n}\left(\frac{n}{m} + (\mathbf{c}+1) - \sqrt{\frac{n}{m}(2 - \frac{n}{m}) + (2\mathbf{c}+1) + \mathbf{c}^2(\frac{n}{m} - 1)^2}\right);$
$\tau_2 = \frac{\frac{m}{n}}{2\frac{m}{n} - 1}\left((\mathbf{c}-1) - \mathbf{c}\frac{n}{m} + \sqrt{\frac{n}{m}(2 - \frac{n}{m}) + (2\mathbf{c}+1) + \mathbf{c}^2(\frac{n}{m} - 1)^2}\right).$[11]

These values arise from minimizing the expressions resulting from comparing the expected values for the chooser and the average proposer, as expressed in Proposition 5. For each inequality in Proposition 5, we obtain first the values of k that minimize differences between the agents' expected utilities, then for each value of k we find the values of v's that would be compatible with its corresponding inequality. In fact, we can ignore inequality

[11] Notice S_2 can be empty. But S_1 is never empty since $\{(\lfloor \tau_1 \rfloor, \lfloor \tau_1 \rfloor), (\lceil \tau_1 \rceil, \lceil \tau_1 \rceil)\} \subseteq S_1$.

3 in Proposition 5 since it is dominated by the other inequalities (since if inequality 3 holds, the chooser would have all the power).

Armed with these explicit calculations, one can proceed to analyze specific cases. Without going into much detail, the following example identifies the rules that guarantee an egalitarian distribution of power.[12]

Example 1 *Let* $\mathbf{c} = 10, n = 7,\ m = 5$. *Applying Proposition 6, we have that* $\tau_1 = 4.4633$, $\tau_2 = 1.919$. *Hence,* $S_1 = \{(4,4),(4,3),(5,5),(5,4)\}$, $S_2 = \{(5,3)(1,1),(2,1))\}$ *and the egalitarian solution is* $S_e = \{(2,1)\}$. *Now, consider* $m = 6$. *Again, applying Proposition 6, we have that* $S_e = \{(6,6),(6,5),(6,4),(6,3),(6,2)\}$. *Notice that as the size of the majority increases from 5 to 6, the egalitarian* k *increases from 2 to 6. In the homogeneous proposers' case* ($m = n$)*, the set of egalitarian solutions would be* $S_e = \{(7,7),(7,6),(7,5),(7,4),(7,3),(7,2),(7,1)\}$.

These examples suggest that, while the choice of k is rather stringent, the values of v that allow societies to reach an egalitarian distribution of power, given k, are not so precisely determined. This lack of full uniqueness is not surprising, nor bothersome, we believe, given that we are working in a setting with integer parameters. However, it suggests yet a final possible exercise, that we briefly discuss now.

Indeed, there are cases whether the constitutional detail provided by a planner stops short of fully specifying both parameters k and v. In particular, if k is exogenously fixed, one can still inquire what could be a power equalizing choice for v.

Here is an interesting example where this kind of problem arose in practice.

According to the Brazilian Constitution, one-third of the members of the Superior Court of Justice shall be chosen in equal parts among lawyers and members of the Public Prosecution. When there is position vacant assigned to be occupied by a lawyer, the constitution states the National Lawyer Association must propose six candidates to the court (so, $\mathbf{c} = 6$). Upon receiving the set of candidates, the court shall organize a list of three names and send it to the President of the Republic, who selects one of the listed names.

However, the constitution does not determine what screening rules should be used to screen the six initial names and then the three out of them. If we consider the list of

[12] These calculations are elementary and available from the authors.

six candidates as given, and concentrate on the choice of screening rule to determine the three to be sent to the President, it turns out that the Superior Court will be facing the question we just mentioned in the abstract. Given $k = 3$, what v should be used by the court to select the three names?

It turns out that, in fact, a specific screening rule was decided upon and is now established in the bylaws of Superior Court.

On what grounds was this rule chosen? We cannot tell. But, can we rationalize the proposed rules through our analysis?

Since there are 33 ministers in the Superior Court of Justice, when there is a position vacant, the number of proposers of the three names is 32. Therefore, we have the following parameters $n = 32$, $\mathbf{c} = 6$ and $k = 3$.

If we suppose that the assumptions of the Polarized Proposers Model hold and that $m = 24$, the size of the majority group, what would be the value of v that minimizes the absolute difference between the president's expected utility and the average of the ministers' expected utilities? It would be equal to three, that is, $v = k = 3$, which is a majoritarian method. In fact, $v = k = 3$ would be a egalitarian solution for any size of the majority group (m). The rule actually chosen by the ministers was in fact a cumbersome sequential method, which however boils down, in terms of the implied equilibrium, to using the majoritarian v rule with $v = k = 3$, precisely the value that would correspond to our calculations. Of course, we are not claiming that this was the reasoning underlying the choice of the screening rule that appears in the bylaws. But the example at least shows what kind of reasoning they could have adopted, and the use of our approach in selecting not only first best solutions, but also to perform second best analysis.

5 Concluding Remarks

Rules that contemplate several stages of choice are widely used. Some people are in charge of screening, then others choose among those candidates that were not screened out. We have concentrated in the case with only one chooser, because it is actually used in many cases, and also for simplicity, but hope to keep deepening our understanding of the advantages of each of the many forms in which societies divide their decision tasks.

In fact, as mentioned at the end of our introduction, the very idea to divide the tasks may arise from very diverse reasons. The one we have concentrated upon is to divide the decision power. This is in line with Arrowian tradition, where the interests of agents are taken as given, and the rules are methods to mitigate conflicts. But there is at least a second fundamental reason to subdivide decisions, this one based on common values, more in line of Condorcet Jury Theorem. This reason is to assign each agent to the partial decision that she is better informed about. When candidates can be judged on a multidimensional scale, different decision-makers in a team may contribute to a final choice by screening out candidates based on the dimension that they are better fit to judge. In this context, rules of k names can be seen as methods to make proper use of expert advise.

Even within our present framework, we are aware that our normative analysis can be enriched by endowing agents with more complex preferences, considering a wider range of distributions over preference profiles, relaxing the full information assumption and/or considering alternative equilibrium concepts under maybe different specifications of the game they interact within. But our purpose here was to open a line of work, to provide guidelines for a normative analysis of these widely used rules, and to exhibit the richness and the difficulties involved in following a similar program under alternative assumptions: hence our choice of relatively simple specifications, for utilities, probabilities and equilibria.

Finally, let us re-emphasize that, even if widely used, v-rules of k names are only one class among many others through people are eventually appointed. Given the power that comes attached with the possibility to appoint people to offices, we hope that these, along with other rules, can be systematically scrutinized and compared. We would like to think of our work as part of this potential stream of research

References

[1] Alon, N., Fisher, F., Procaccia, A. and Tennenholtz, M. (2011) Sum of us: strategyproof selection from the selectors, in: Proc. 13th Conference on Theoretical Aspects of Rationality and Knowledge pp. 101-110.

[2] Barberà, S., Sonnenschein, H. and Zhou, L. (1991) Voting by committees. Econometrica 59:595–609.

[3] Barberà, S., Bossert, W., Pattanaik, P. (2004) Ranking sets of objects. In: Barberà, S., Hammond, P. and Seidl, C.(Eds.) Handbook of Utility Theory, Volume II Extensions. Kluwer Academic Publishers 893–977

[4] Badger, W.W. (1972) Political individualism, positional preferences and optimal decision-rules. In: Niemi, R.G., Weisberg, H.F. (eds.) Probability Methods for Collective Decision Making. Merril Publishing, Columbus, Ohio.

[5] Barberà, S., Jackson, M. (2004) Choosing how to choose: Self-stable majority rules and constitutions. Quarterly Journal of Economics 119 (3).

[6] Barberà, S., Coelho, D. (2010) On the rule of k names. Games and Economic Behavior 70:44-61.

[7] Barberà, S., Coelho, D. (2008) How to choose a non-controversial list with k names. Social Choice and Welfare 31:79-96.

[8] Bernheim, D., Peleg, B. and Whinston, M (1987) Coalition-proof Nash equilibria I. Concepts. Journal of Economic Theory 42:1–12.

[9] Brams, S. J., and Merrill, S. (1986) Binding versus final-offer arbitration: A combination is best. Management Science 32:1346-1355.

[10] Coelho, D. (2005) Maximin choice of voting rules for committees. Econ. Gov. 6:159–175.

[11] Curtis, R. (1972) Decision rules collective values in constitutional choice. In: Niemi, R.G., Weisberg, H.F. (eds.) Probability Methods for Collective Decision Making. Merril Publishing, Columbus, Ohio.

[12] Ertemel, S., Kutlu, L. and Sanver, R. (2010) Voting games of resolute social choice correspondence. Draft

[13] Gardner, R. (1977) The Borda game. Public Choice 30(1):43-50.

[14] Gehrlein W. (1985) The Condorcet criterion and committee selection. Mathematical Social Science 10:199–209

[15] Holzman, J. and Moulin, H. (2013) Impartial nomination for a prize. Econometrica 81:173-196.

[16] Kaymak, B. and Sanver, M. R. (2003) Sets of alternatives as Condorcet winners. Social Choice and Welfare 20:477-494.

[17] Moulin, H. (1982) Voting with proportional veto power. Econometrica 50(1):145-62.

[18] Moulin, H. (1988) Axioms of Cooperative Decision Making. Cambridge University Press, Cambridge.

[19] Mueller, D. (1978) Voting by veto. Journal of Public Economics 10:57-75.

[20] Peleg, B. (1984) Game Theoretic Analysis of Voting in Committees. Econometric Society Monographs in Pure Theory, Cambridge University Press, Cambridge.

[21] Polborn, M. and Messner, M. (2007) Strong and coalition-proof political equilibria under plurality and runoff rule. International Journal of Game Theory 35:287-314.

[22] Rae, D. (1969) Decision rules and individual values in constitutional choice. American Political Science Review 63: 40–56.

[23] Ratliff T (2003) Some startling inconsistencies when electing committees. Social Choice Welfare 21:433–454.

[24] Sertel, M. R., and Sanver M. R. (2004) Strong equilibrium outcomes of voting games are the generalized Condorcet winners. Social Choice and Welfare 22:331-347.

[25] Stevens, C.M. (1966) Is compulsory arbitration compatible with bargaining. Industrial Relations 5:38-52.

Appendix A

Proof of Proposition 1. Proposition 1 is a direct consequence of lemmas 1-5. ■

Lemma 1 *Let* $\mathbf{c} - \sqrt{2\mathbf{c}+2} + 2$ *be an integer number. If* $k = \mathbf{c} + 2 - \sqrt{2\mathbf{c}+2}$ *then* $E(u_p(x)|\mathbf{c}, k) = E(u_c(x)|\mathbf{c}, k)$.

Proof of Lemma 1. First notice that for every k we have that:
$E(u_p(x)|\mathbf{c},k)E(u_c(x)|\mathbf{c},k) = \frac{\mathbf{c}+1}{2}$
Take any $k^* \in [1,\mathbf{c}]$ such that $E(u_p(x)|\mathbf{c},k^*) = E(u_c(x)|\mathbf{c},k^*)$. Thus,
$E(u_c(x)|\mathbf{c},k^*)^2 = \frac{\mathbf{c}+1}{2}$
$E(u_c(x)|\mathbf{c},k^*) = \frac{\mathbf{c}-k^*+2}{2} = \sqrt[2]{\frac{\mathbf{c}+1}{2}}$
Therefore, $k^* = \mathbf{c} + 2 - \sqrt[2]{2\mathbf{c}+2}$. ■

Lemma 2 *A $k \in \{1,...\mathbf{c}\}$ maximizes $E(u_p(x)|\mathbf{c},k) + E(u_c(x)|\mathbf{c},k)$ if and only if it minimizes $|E(u_p(x)|\mathbf{c},k) - E(u_c(x)|\mathbf{c},k))|$.*

Proof of Lemma 2. *First notice that for every k we have that:*
$E(u_p(x)|\mathbf{c},k)E(u_c(x)|\mathbf{c},k) = \frac{\mathbf{c}+1}{2}$
The equality above implies that
$(E(u_p(x)|\mathbf{c},k) + E(u_c(x)|\mathbf{c},k))^2 = E(u_p(x)|\mathbf{c},k)^2 + E(u_c(x)|\mathbf{c},k)^2 + (\mathbf{c}+1)$
The expression above implies that, given that $E(u_p(x)|\mathbf{c},k) + E(u_c(x)|\mathbf{c},k) < 0$, a $k \in \{1,...\mathbf{c}\}$ maximizes $E(u_p(x)|\mathbf{c},k)+E(u_c(x)|\mathbf{c},k)$ if and only if it minimizes $E(u_p(x)|\mathbf{c},k)^2 + E(u_c(x)|\mathbf{c},k)^2$.
Notice also that:
$(E(u_p(x)|\mathbf{c},k) - E(u_c(x)|\mathbf{c},k))^2 = E(u_p(x)|\mathbf{c},k)^2 + E(u_c(x)|\mathbf{c},k)^2 - (\mathbf{c}+1)$.
The expression above implies that a $k \in \{1,...\mathbf{c}\}$ maximizes $E(u_p(x)|\mathbf{c},k)^2+E(u_c(x)|\mathbf{c},k)^2$ if and only if it maximizes $(E(u_p(x)|\mathbf{c},k) - E(u_c(x)|\mathbf{c},k))^2$.
Therefore, a $k \in \{1,...\mathbf{c}\}$ maximizes $E(u_p(x)|\mathbf{c},k)+E(u_c(x)|\mathbf{c},k)$ if and only if minimizes $|E(u_p(x)|\mathbf{c},k) - E(u_c(x)|\mathbf{c},k))|$. ■

Lemma 3 *A $k \in \{1,...\mathbf{c}\}$ maximizes $E(u_p(x)|\mathbf{c},k) + E(u_c(r_c)|\mathbf{c},k)$ if and only if it also maximizes $(E(u_p(x)|\mathbf{c},k) - d)(E(u_c(x)|\mathbf{c},k) - d)$ where $d < 0$.*

Proof of Lemma 3. First notice that for every k we have that:
$E(u_p(x)|\mathbf{c},k)E(u_c(x)|\mathbf{c},k) = \frac{\mathbf{c}+1}{2}$.
Thus, $(E(u_p(x)|\mathbf{c},k) - d)(E(u_c(x)|\mathbf{c},k) - d) = \frac{\mathbf{c}+1}{2} + d^2 - d(E(u_p(x)|\mathbf{c},k) + E(u_c(x)|\mathbf{c},k))$
Given that $d < 0$, the expression above implies that k maximizes $E(u_p(x)|\mathbf{c},k)+E(u_c(x)|\mathbf{c},k)$ if and only if it maximizes $(E(u_p(x)|\mathbf{c},k) - d)(E(u_c(x)|\mathbf{c},k) - d)$. ■

Lemma 4 *Consider any $\mathbf{c}$:*
1) $E(u_p(x)|k,\mathbf{c}) + E(u_c(x)|k,\mathbf{c}) > E(u_p(x)|k-1,\mathbf{c}) + E(u_c(x)|k-1,\mathbf{c})$ for every $k <$

$\mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}$;

2) $E(u_p(x)|k,\mathbf{c})+E(u_c(x)|k,\mathbf{c}) = E(u_p(x)|k-1,\mathbf{c})+E(u_c(x)|k-1,\mathbf{c})$ *if* $k=\mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}$;

3) $E(u_p(x)|k,\mathbf{c})+E(u_c(x)|k,\mathbf{c}) < E(u_p(x)|k-1,\mathbf{c})+E(u_c(x)|k-1,\mathbf{c})$ *for every* $k > \mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}$.

Proof of Lemma 4. For every $k\in\{2,..\mathbf{c}\}$ we have the following equality: $E(u_p(x)|\mathbf{c},k)+E(u_c(x)|\mathbf{c},k)-(E(u_p(x)|\mathbf{c},k-1)+E(u_c(x)|\mathbf{c},k-1)) = \frac{\mathbf{c}+1}{(\mathbf{c}-k+2)(\mathbf{c}-k+3)}-\frac{1}{2}$. Notice $\frac{\mathbf{c}+1}{(\mathbf{c}-k+2)(\mathbf{c}-k+3)}$ is decreasing with k and $\frac{\mathbf{c}+1}{(\mathbf{c}-k+2)(\mathbf{c}-k+3)}=\frac{1}{2}$ when $k=\mathbf{c}-\frac{1}{2}\sqrt{8\mathbf{c}+9}+\frac{5}{2}$. Let $P(k)=\frac{\mathbf{c}+1}{(\mathbf{c}-k+2)(\mathbf{c}-k+3)}-\frac{1}{2}$ and $k^*=\mathbf{c}-\frac{1}{2}\sqrt{8\mathbf{c}+9}+\frac{5}{2}$. Thus, $P(k^*)=0$, $P(k)>0$ for any $k<k^*$ and $P(k)<0$ for any $k>k^*$. ■

Lemma 5 *If* $\mathbf{c}-\sqrt{2\mathbf{c}+2}+2$ *is an integer number then it is equal to* $\left\lfloor \mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}\right\rfloor$.

Proof of Lemma 5. Let $z=\mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}$ and $y=\mathbf{c}-\sqrt{2\mathbf{c}+2}+2$. Notice that $z-y=\sqrt{2\mathbf{c}+2}+\frac{1}{2}-\sqrt{2\mathbf{c}+2+\frac{1}{4}}$. Thus, $1>z-y>0$ for every $\mathbf{c}>0$. Therefore if $\mathbf{c}-\sqrt{2\mathbf{c}+2}+2$ is an integer number we have that $\left\lfloor \mathbf{c}+\frac{5}{2}-\sqrt{2\mathbf{c}+\frac{9}{4}}\right\rfloor = \mathbf{c}-\sqrt{2\mathbf{c}+2}+2$.
■

Proof of Proposition 2. Suppose that candidate x is the outcome of a strong Nash equilibrium of the Constrained Chooser Game. In any strong Nash equilibrium where x is the outcome, the screened set is such that x is the best candidate in this set according to the chooser's preferences. This implies that x is a chooser's $(\mathbf{c}-k+1)$-top candidate. To prove that Condition 2 is necessary take any candidate $y\neq x$ among those that are chooser's $(\mathbf{c}-k+1)$-top candidates and let Y be any list with k names where y is the chooser's best candidate in Y. Notice that y cannot be considered better than x by any coalition with at least $q_k^v(Y)$ candidates. Otherwise, this coalition could impose Y, preventing x from being elected. So, if y is a chooser's $(\mathbf{c}-k+1)$-top candidate, then $\#\{i\in N|y\succ_i x\}<q_k^v(Y)$ for any $Y\in C_k$ such that y is the chooser's best candidate in Y.

Now, to justify Condition 3, suppose, by contradiction, that it is not true that $\#\{i\in N|y\succ_i x\}\geq q_k^v(y)$. Let $S_1\equiv\{i\in N|y\succ_i x\}$, so $\#S_1\geq q_k^v(y)$. Then, the coalition of proposers in C_1 would be able to impose the inclusion of y in the list (since $\#S_1\geq q_k^v(y)$), and the chooser would select it instead of x. Hence, if y the chooser's best candidate, we have that $\#\{i \in N | y \succ_i x\} < q_k^v(y)$.

Finally, consider Condition 4. Let y be the chooser's best candidate, and assume that it is ranked above x by the tie breaking criterion. Suppose, by contradiction, that it is not true that $\#\{i \in N | x \succ_i y\} \geq q_k^v$. Hence, at any strategy profile that includes x in the selected list, the coalition $S_1 \equiv \{i \in N | y \succ_i x\}$ can find a profitable deviation to include y, becomes the winning candidate. Therefore, x cannot be a strong Nash equilibrium outcome. ■

Proof of Proposition 3. Let x be the chooser's 1-top candidate. First notice that given that x is a strong Nash equilibrium outcome under a v'-screening rule for k' names, it implies that any strategy profile where all proposers votes for x is a strong Nash equilibrium.

Take any strategy profile where all voters vote for x, and call by m'. Given that it is a strong Nash equilibrium, there is no coalition of voters that can make a profitable deviation. The voters that would wish to avoid the election of x are those that prefer another chooser's $(\mathbf{c} - k' + 1)$-top candidate to x (recall that only the chooser's $(\mathbf{c} - k' + 1)$-top candidates can be the chooser' best name among the candidates of a set with cardinality k').The only way to avoid the election of x would be to avoid the inclusion of x in the chosen list. Take any chooser's $(\mathbf{c} - k' + 1)$-top candidate and call it by y. If all the voters that prefer y to x deviate from m' by do not vote for x, x would continue to have enough votes to be one name of k listed names. Otherwise, the strategy profile where all the voters vote for x would no be a strong Nash equilibrium.

Now let us show that x is also a strong Nash equilibrium any v-screening rule for k names where $v \leq v'$ and $k' \geq k$. We need to show that there is a strategy profile that sustains x as strong Nash equilibrium outcome under v-screening rule for k names.

Take any strategy profile where all voters vote for x and call this strategy by m. So, x will be one of k listed name and it will be the elected candidate. We need to show that there is no coalition of voters that can make a profitable deviation under m. Given m' and m, notice that it is more difficult to make a profitable deviation under v-screening rule for k names than v'-screening rule for k' names. Because, under a v-screening rule for k names, any coalition of voters that would have incentive to avoid the election of x has less votes to distribute among the k candidates in order to avoid the inclusion of x in the list. Thus, given that there exists no coalition that can make a profitable deviation under $m\acute{}$, it implies that there exists no coalition that can make a profitable deviation under m. Therefore, x is a strong Nash equilibrium outcome under v-screening rule for k names. ■

Proof of Proposition 4.

1) Suppose that the tie breaking criterion coincides with the majoritarian group's preferences over the set of candidates.

1.1) Consider $m \geq q_k^v$.

Let x be the best alternative of the majoritarian group out of the chooser's $(\mathbf{c} - k + 1)$-top candidates. Since $m \geq q_k^v$, and by definition of q_k^v, there is a strategy profile that can be adopted by the majoritarian group that leads to the election of x, and the minoritarian group is unable to change it. Notice also that the majoritarian group will not have any incentive in changing this outcome. Therefore, there exists a strategy profile that sustains x as a strong Nash equilibrium outcome.

Now let us show that x is the unique strong Nash equilibrium outcome. Suppose, by contradiction that there is another strong Nash equilibrium outcome $y \neq x$. By Condition 2 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(X)$ where x is the chooser's best alternative in X. This is a contradiction since $\#\{i \in N | x \succ_i y\} > m > q_k^v > q_k^v(X)$.

1.2) Consider $q_k^v > m \geq q_1^v > n - m$.

Let x be the chooser's best alternative out of the majoritarian group's k-top candidates. Let X be the set of k-top candidates for the majoritarian group's. We first show that there exists a strategy profile that sustains x as an equilibrium outcome. Notice that $q_k^v > m \geq q_1^v > n - m$ implies that $m \geq q_k^v(X)$ and $n - m \geq q_k^v(\{x\})$. Consider the following strategy profile: the majoritarian group adopts a strategy profile that can allows it to impose the list X and the minoritarian group adopts a strategy profile that allows it to impose x in the list. In order to change the outcome, one of the groups could try to block the inclusion of x, but neither of them alone can do it. Notice also that only the majoritarian group would be able to include another candidate better than x in the list sent to the chooser. But this candidate would be worse than x for the majoritarian group. Therefore, there exists no coalition of proposers that has an incentive to deviate. Thus, we have proved that there exists a strategy profile that sustains x as an equilibrium outcome.

Now we shall prove that x is the unique strong Nash equilibrium outcome. By contradiction, suppose that there is another strong Nash equilibrium outcome $y \neq x$. By Condition 1 of Proposition 2, x is among the chooser's $(\mathbf{c} - k + 1)$-top candidates. By Condition 2 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(X)$. This is a contradiction since $\#\{i \in N | x \succ_i y\} > m \geq q_k^v(X)$.

1.3) Consider $q_k^v > m > n - m \geq q_1^v$.

Let x be the chooser's best candidate. First let us show that there exists a strategy profile that sustains x as an equilibrium outcome. Notice that $n - m \geq q_1^v$ implies that $m > n - m > q_v^v(\{x\})$. Consider the following strategy profile: every proposer casts a vote for x. Thus, x will be in the selected list and it will be elected. No group can take x out from the selected list by a unilateral deviation, since both have size larger than $q_v^v(\{x\})$. Since both group has the reverse preference profile of the other, they do not have incentive to jointly deviate from this strategy profile. Therefore, this strategy profile sustains x as an strong Nash equilibrium outcome.

Now let us prove that x is the unique strong Nash equilibrium outcome. By contradiction, suppose that there is another strong Nash equilibrium outcome $y \neq x$. By Condition 3 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(\{x\})$. This is a contradiction since $m > n - m \geq q_1^v(\{x\})$.

2) Suppose that the tie breaking criterion coincides with the chooser's preferences over the set of candidates.

2.1) Consider $m \geq q_k^v$.

Let x be the best alternative of the majoritarian group out of the chooser' s $(\mathbf{c} - k + 1)$-top candidates. Since $m \geq q_k^v$, and by definition of q_k^v, there is a strategy profile that can be adopted by the majoritarian group that leads to the election of x, and the minoritarian group is unable to change it. Notice also that the majoritarian group will not have any incentive in changing this outcome. Therefore, there exists a strategy profile that sustains x as a strong Nash equilibrium outcome.

Now let us show that x is the unique strong Nash equilibrium outcome. Suppose, by contradiction that there is another strong Nash equilibrium outcome $y \neq x$. By Condition 2 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(X)$ where x is the chooser's best alternative in X. This is a contradiction since $\#\{i \in N | x \succ_i y\} > m > q_k^v > q_k^v(X)$.

2.2) Consider $q_k^v > m$.

Let x be the chooser's best candidate. Notice that $q_k^v > m$ implies that $n - m \geq q_k^v(\{x\})$. Suppose the following strategy profile: every proposer cast a vote for x. Thus, x will be in the selected list and it will be elected. No group can take x from the selected list by a unilateral deviation, since both has size larger than $q_v^v(\{x\})$. Since both group have the reverse preference profile than the others, they do not have incentive to joint deviate from this strategy profile. Therefore, this strategy profile sustains x as strong Nash equilibrium outcome.

Now let us prove that x is the unique strong Nash equilibrium outcome. By contradiction, suppose that there exists another strong Nash equilibrium outcome $y \neq x$. By Condition 3 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(\{x\})$. This is a contradiction since $m > n - m \geq q_1^v(\{x\})$.

3) Suppose that the tie breaking criterion coincides with the minoritarian group's preferences over the set of candidates.

3.1) Consider $m \geq q_k^v$.

Let x be the best alternative of the majoritarian group out of the chooser' s $(\mathbf{c} - k + 1)$-top candidates. Since $m \geq q_k^v$, and by definition of q_k^v, there is a strategy profile that can be adopted by the majoritarian group that leads to the election of x, and the minoritarian group is unable to change it. Notice also that the majoritarian group will not have any incentive in changing this outcome. Therefore, there exists a strategy profile that sustains x as a strong Nash equilibrium outcome.

Now let us show that x is the unique strong Nash equilibrium outcome. Suppose, by contradiction that there is another strong Nash equilibrium outcome $y \neq x$. By Condition 2 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(X)$ where x is the chooser's best alternative in X. This is a contradiction since $\#\{i \in N | x \succ_i y\} > m > q_k^v > q_k^v(X)$.

3.2) Consider $q_k^v > m$.

Let x be the chooser's best candidate. Consider the following strategy profile: every proposer casts a vote for x. Thus, x will be in the selected list and it will be elected. Notice that the minoritarian group cannot take x out from the selected list by a unilateral deviation since $m \geq q_k^v(\{x\})$. If some y is better than x for the majoritarian group, then it will be ranked below than x by the tie breaking criterion. Thus, the majoritarian group cannot deviate in any way that simultaneously includes y and excludes x from the selected list. Since both groups have the reverse preferences, they do not have incentive to jointly deviate from this strategy profile. Therefore, this strategy profile sustains x as an strong Nash equilibrium outcome. Now let us prove that x is the unique strong Nash equilibrium outcome. By contradiction, suppose that there is $y \neq x$ that is also a strong Nash equilibrium outcome. Suppose that the minoritarian group of proposers prefers y to x. By Condition 3 of Proposition 2, we have that $\#\{i \in N | x \succ_i y\} < q_k^v(\{x\})$ which implies that $m < q_k^v(\{x\})$. This is a contradiction, since $m \geq q_k^v(\{x\})$. Suppose that the majoritarian group of proposers prefers y to x. Thus x is ranked above than y according by the tie breaking criterion. By Condition 4 of Proposition 2, we have that $\#\{i \in N | y \succ_i x\} \geq q_k^v$, which implies that $m \geq q_k^v$. This is a contradiction since $q_k^v > m$. Therefore, y cannot be strong Nash equilibrium outcome. ■

Proof of Proposition 5. Proposition 5 is a direct consequence of Proposition 4 and the assumption that agent's preferences are randomly drawn from a uniform distribution over the domain of preferences.

1) Consider $m \geq q_k$. By Proposition 4, the equilibrium outcome is the best alternative of the majoritarian group out of the chooser's $(\mathbf{c} - k + 1)$-top candidates. Thus, r_c has the same distribution as that of a discrete random variable uniformly distributed over $\{1, 2, \ldots, \mathbf{c} - k + 1\}$. r_{G1} has the same distribution as that of the smallest element of a random sample with size $s = \mathbf{c} - k + 1$ drawn without replacement from a uniformly distributed population $D = \{1, 2, \ldots, \mathbf{c}\}$. r_{G2} has the same distribution as that of the biggest element of a random sample with size $s = \mathbf{c} - k + 1$ drawn without replacement from a uniformly distributed population. Therefore, we have:

$$E(u_c(x)|\mathbf{c}, k, v) = -(\tfrac{\mathbf{c}-k+2}{2})$$

$$E(u_{G1}(x)|\mathbf{c}, k, v) = -\tfrac{(\mathbf{c}+1)}{(\mathbf{c}-k+2)}$$

$$E(u_{G2}(x)|\mathbf{c}, k, v) = -\tfrac{(\mathbf{c}+1)(\mathbf{c}-k+1)}{(\mathbf{c}-k+2)}$$

$$E(u_p(x)|\mathbf{c}, k, v) = -\tfrac{m}{n}\tfrac{(\mathbf{c}+1)}{(\mathbf{c}-k+2)} - \tfrac{n-m}{n}\tfrac{(\mathbf{c}+1)(\mathbf{c}-k+1)}{(\mathbf{c}-k+2)},$$

2) Consider $q_k^v > m \geq q_1^v > n - m$. By Proposition 4, the equilibrium outcome is the chooser's best alternative out of the majoritarian group's k-top candidates. Thus, r_c has

the same distribution as that of the smallest element of a random sample with size $s = k$ drawn without replacement from a uniformly distributed population $D = \{1, 2, \ldots, \mathbf{c}\}$. r_{G1} has the same distribution as that of a discrete random variable uniformly distributed over $\{1, 2, \ldots, k\}$. r_{G2} has the same distribution as that of a discrete random variable uniformly distributed over $\{\mathbf{c} - k + 1, \ldots, \mathbf{c}\}$. Therefore, we have:

$E(u_c(x)|\mathbf{c}, k, v) = -\frac{(\mathbf{c}+1)}{(k+1)}$

$E(u_{G1}(x)|\mathbf{c}, k, v) = -\frac{(k+1)}{2}$

$E(u_{G2}(x)|\mathbf{c}, k, v) = -\frac{(2\mathbf{c}-k+1)}{2}$

$E(u_p(x)|\mathbf{c}, k, v) = -\frac{m}{n}\frac{(k+1)}{2} - \frac{n-m}{n}\frac{(2\mathbf{c}-k+1)}{2}$

3) Consider $q_k^v > m > n - m \geq q_1^v$. By Proposition 4, the equilibrium outcome is the chooser's best candidate. Thus, r_c is a constant and it is equal to 1. r_{G1} and R_{G2} have the same distribution as that of a discrete random variable uniformly distributed over $\{1, 2, \ldots, \mathbf{c}\}$.

$E(u_c(x)|\mathbf{c}, k, v) = -1$

$E(u_p(x)|\mathbf{c}, k, v) = -\frac{(\mathbf{c}+1)}{2}$. ■

Proof of Proposition 6. Proposition 6 is a direct consequence of Proposition 5 and lemmas 6 and 7 below. ■

Lemma 6 *In the domain of all pairs (k, v) such that $m \geq q_k > n - m$, we have that:*
1) $E(u_p(x)|\mathbf{c}, k, v) > E(u_c(x)|\mathbf{c}, k, v)$ for every $k < \tau_1$;

2) $E(u_p(x)|\mathbf{c}, k, v) = E(u_c(x)|\mathbf{c}, k, v)$ if $k = \tau_1$ is an integer number;

3) $E(u_p(x)|\mathbf{c}, k, v) < E(u_c(x)|\mathbf{c}, k, v)$ for every $k > \tau_1$.

where $\tau_1 = \frac{m}{n}\left(\frac{n}{m} + (\mathbf{c}+1) - \sqrt{\frac{n}{m}(2 - \frac{n}{m}) + (2\mathbf{c}+1) + \mathbf{c}^2(\frac{n}{m} - 1)^2}\right)$

Proof of Lemma 6. Given that $m \geq q_k > n-m$, by Proposition 5, $E(u_p(x)|\mathbf{c}, k, v) = -\frac{m}{n}\frac{(\mathbf{c}+1)}{(\mathbf{c}-k+2)} - \frac{n-m}{n}\frac{(\mathbf{c}+1)(\mathbf{c}-k+1)}{(\mathbf{c}-k+2)}$, and $E(u_c(x)|\mathbf{c},k,v) = -(\frac{\mathbf{c}-k+2}{2})$.

Notice that:

$|E(u_p(x)|\mathbf{c},k,v) - E(u_c(x)|\mathbf{c},k,v)|$ is single dipped and reaches the minimum when $k = \tau_1$.When $k = \tau_1$, we have that: $|E(u_p(x)|\mathbf{c},k,v) - E(u_c(x)|\mathbf{c},k,v)| = 0$. ■

Lemma 7 *In the domain of all pairs (k,v) such that $q_k > m \geq q_1 > n-m$, we have that:*

1) $E(u_p(x)|\mathbf{c},k,v) > E(u_c(r_c)|\mathbf{c},k,v)$ for every $k < \tau_2$;

2) $E(u_p(x)|\mathbf{c},k,v) = E(u_c(x)|\mathbf{c},k,v)$ if $k = \tau_2$ is an integer number;

3) $E(u_p(x)|\mathbf{c},k,v) < E(u_c(x)|\mathbf{c},k,v)$ for every $k > \tau_2$.

where $\tau_2 = \frac{\frac{m}{n}}{2\frac{m}{n}-1}\left((\mathbf{c}-1) - \mathbf{c}\frac{n}{m} + \sqrt{\frac{n}{m}(2-\frac{n}{m}) + (2\mathbf{c}+1) + \mathbf{c}^2(\frac{n}{m}-1)^2}\right)$

Proof of Lemma 7. Given that $q_k > m \geq q_1 > n-m$, by Proposition 5, we have that $E(u_p(r_p)|\mathbf{c},k,v) = -\frac{m}{n}\frac{(k+1)}{2} - \frac{n-m}{n}\frac{(2\mathbf{c}-k+1)}{2}$

$E(u_c(x)|\mathbf{c},k,v) = -\frac{(\mathbf{c}+1)}{(k+1)}$.

Notice that:

$|E(u_p(r_p)|k,v,\mathbf{c}) - E(u_c(x)|k,v,\mathbf{c})|$ is single dipped and reaches the minimum when $k = \tau_2$. When $k = \tau_2$, we have that $|E(u_p(r_p)|k,v,\mathbf{c}) - E(u_c(x)|k,v,\mathbf{c})| = 0$. ■

Appendix B

Proposition 2 helps us to locate equilibria and provide a first step toward their characterization, when they exist! But knowing the necessary conditions alone is already of great help. We illustrate this point though an example.

Example 2 *Let* $\mathbf{A} = \{c1, c2, c3, c4, c5\}$ *and let* $\mathbf{N} = \{1, 2, 3\}$*. Suppose that each proposer votes for one candidate and the three most voted candidates form the list, with a tie breaking rule when needed:* $c2 \succ c1 \succ c5 \succ c4 \succ c3$*. The preferences of the chooser and the committee members are as follows:*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
$c5$	$c5$	$c5$	$c1$
$c4$	$c4$	$c4$	$c2$
$c3$	$c3$	$c2$	$c3$
$c1$	$c1$	$c1$	$c4$
$c2$	$c2$	$c3$	$c5$

Notice that, we have that $q_k^v(\{x\}) = 1$ *for any* $x \in A$ *and* $q_k^v(X) = 3$ *for any* $X \in A_k$*.*
The first step in describing the equilibrium outcomes is to identify those candidates that satisfy the three necessary conditions established in Proposition 2.
Inspecting the preference profile above, we have that:
1. *Condition 1:* $\{c1, c2, c3\}$*.*
2. *Condition 2:* $\{c1, c2, c3, c4, c5\}$*.*
3. *Condition 3:*$\{c1, c4, c5\}$*.*
4. *Condition 4:* $\{c1, c2, c4, c5\}$*.*

So, only candidate $c1$ *that satisfies all four conditions. Now we have to check whether there is a strategy profile that sustains candidate* $c1$ *as a strong Nash equilibrium candidate. The following strategy profile sustains* $c1$ *as a strong Nash equilibrium outcome: Proposer 1 votes for* $c1$*, Proposer 1 votes for* $c4$ *and Proposer 3 votes for* $c2$*.*

In the preceding example, the choice of candidates satisfying the necessary conditions could be in fact be sustained with an appropriate set of strong equilibrium strategies. But this need not be the case. In fact, there may be candidates that satisfy the necessary conditions and yet cannot be the outcome of any equilibrium. Worse of that, equilibria may not exist even if some candidates meet the necessary conditions, as shown by our next example. The example below also shows that without Assumption 4 the Polarized Proposers Model may not have a strong Nash equilibrium outcome.

Example 3 *Let* $C = \{c1, c2, c3, c4, c5, c6\}$ *and let* $\mathbf{N} = \{1, 2, 3\}$*. The proposers use the rule of 4 names,* $(k = 4, v = 1)$*, with the following tie breaking rule when needed:* $c6 \succ c5 \succ c4 \succ c3 \succ c2 \succ c1$*. The preferences of the chooser and the committee members are as follows:*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Proposer 4*	*Proposer 5*	*Chooser*
$c5$	$c5$	$c5$	$c5$	$c1$	$c1$
$c6$	$c6$	$c6$	$c6$	$c3$	$c2$
$c4$	$c4$	$c4$	$c4$	$c2$	$c3$
$c2$	$c2$	$c2$	$c2$	$c4$	$c4$
$c3$	$c3$	$c3$	$c3$	$c6$	$c5$
$c1$	$c1$	$c1$	$c1$	$c5$	$c6$

First, notice that $q_k^v(\{x\}) = 1$ *for any* $x \in \{c3, c4, c5, c6\}$, $q_k^v(\{x\}) = 2$ *for any* $x \in C \backslash \{c3, c4, c5, c6\}$ *and* $q_k^v(X) = 5$ *for any* $X \in C_k \backslash \{c3, c4, c5, c6\}$ *and* $q_k^v(\{c3, c4, c5, c6\}) = 4$ *.Notice that proposers 1, 2, 3, and 4 form the majoritarian group of proposers, so* $m = 4$*. Notice also that the tie breaking rule is equal to the reverse of the chooser's preference over the set of candidates. The first step in describing the equilibrium outcomes is to identify those candidates that satisfy the three necessary conditions established in Proposition 2.*
Inspecting the preference profile above, we have that:
1. *Condition 1:* $\{c1, c2, c3\}$.
2. *Condition 2:* $\{c2, c3, c4, c5, c6\}$.
3. *Condition 3:* $\{c1, c2, c3, c4, c5, c6\}$.
4. *Condition 4:* $\{c1, c2, c3, c4, c5, c6\}$.

So, only candidates $c2$ *and* $c3$ *satisfy all four conditions. However, there exists no strategy profile that can sustain them as a strong Nash equilibrium outcome of the Constrained Chooser Game.*