Dual Employment Protection Legislation and the Size Distribution of Firms

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PRELIMINARY[1]

Abstract:

We develop a theoretical model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers (dual employment protection legislation, DEPL). We characterize the equilibrium labor composition that firms with different productivity choose over their life cycle, and we study the effect of DEPL on the distribution of firms' size and productivity. The results indicate that DEPL play similar role as a tax to big firms and a subsidy to small firms (size-dependent-policies) by distorting firm selection as well as the allocation of resources across firms, and thus generating a decline in the level of TFP. Consistent with the evidence documented in this paper, in spite of having similar labor productivity by firms' size-classes, countries with stricter DEPL that incentives or extend the use of temporary contracts have relatively smaller firms (that concentrate a higher fraction of employment), and lower aggregate productivity. In this sense the model gives new insights into the sources of the considerable differences in the firm-size distributions across countries.

Keywords: Economic development, firm dynamics, idiosyncratic uncertainty, plant heterogeneity, policy distortions, productivity differences, resource misallocation, size distribution of firms.

JEL Classification: D21, D24, D92, E23, J41, L11, O1, O40.

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1 Introduction

In this paper, we develop a new theory in which dual employment protection legislation (DEPL) have non trivial effects on the size distribution of firms and aggregate total factor productivity (TFP) by distorting firm selection and the resource allocation among firms. Our theory is motivated by three observations: 1) there exist evidence showing that differences in the size distribution of firms across countries is important to account for differences in aggregate productivity across countries; 2) there is evidence documenting that strict DEPL tended to be associated with a higher share of temporary employment across countries and lower productivity; and 3) we present data indicating that countries with particularly high fraction of temporary employment have a relatively large share of employment allocated in small firms.

We develop a theory of firm dynamics with search frictions and asymmetric firing costs of permanent and temporary workers in order to study the effect of dual employment protection legislation on the size distribution of firms and aggregate TFP. In the model economy, from a firm's perspective, DEPL puts in place two different workers. Temporary workers that remain matched with a firm for a short period of time (they have an exogenous separation rate equal to one) and are dismissed at zero firing cost; and permanent workers that remain matched with a firm until it decides to fire them (they have an exogenous separation rate equal to zero), but have high firing costs. In this framework firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. From one hand, firms that expect their productivity to growth and therefore survive and last longer in the market have incentives to hire more permanent workers. From the other hand, firms with bad prospects on the evolution of their productivity have incentives to hire more temporary workers since they keep reducing production and employment over time and exit the industry relatively soon. In this context, an increase in the firing costs of permanent workers distorts the optimal employment composition that firms choose over their life-cycle penalizing relatively more to firms with high productivity growth. Hence, an increase in firing costs of permanent workers reduces the mass of businesses hiring permanent workers. In addition there are general equilibrium effects that reinforces this result. First, on the intensive margin, an increase in the firing costs of permanent workers reduces profits of high productivity growth firms inducing to less vacancy posting and reducing the labor market tightness. In turn, this increases the probability that a firm matches with a worker making low productivity firms to expand. Second, an increase in the firing costs of permanent workers subsidizes firms with low productivity growth by reducing the costs of filling up temporary jobs which distorts the exit decision of firms. As a result, low productivity growth firms last longer in the market and the age of shutdown of high productivity growth firms decreases. Adding up, larger firing costs for permanent contracts shifts employment from high productivity growth firms, which contract and last shorter in the market, to firms with low productivity growth, which expand and last longer in the market.

Our paper is closely related to the large literature on labor market regulations and firm dynamics.[2,3] Most papers studying the effects of separation taxes have focused on the analysis of unemployment and worker turnover. However, our contribution is to show that DEPL has nontrivial effects on the size distribution of firms and TFP. In addition, the literature has emphasized positive effects of temporary contracts since they provide firms considerable flexibility in the hiring and firing process.[4] However, another contribution of our paper is to show that there are also negative effects of higher flexibility. Our model shows a mechanism through which higher flexibility in the hiring and firing process distorts the equilibrium selection of firms and the allocation of resources among firms.

[2] Regarding the literature focusing on differences in labor market regulations and their outcomes across countries, see for example Blanchard and Summers (1986), Blanchard and Wolfers (2000), Layard, Nickell, and Jackman (1991), Ljungqvist and Sargent (1998), Machin and Manning (1999), and Freeman (2007).

[3] Regarding the literature focusing on separation taxes, see for example Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993), Millard and Mortensen (1997), Güell (2000), among others. Regarding the extensive literature studying the effects of temporary contracts, see for example Bentolila and Saint Paul (1992), Cabrales and Hopenhayn (1997), Blanchard and Landier (2002), Nagypal (2002), Aguiregabiria and Alonso-Borrego (2004), Alonso-Borrego et al. (2005), Veracierto (2007), and Alvarez and Veracierto (2012).

[4] For instance, Aguirregabiria and Alonso-Borrego (2009) develops a dynamic structural models of labor demand to analyze longitudinal Spanish firm-level data during the period 1982-1993 (before and after the reform), and their results indicate an important positive effects on total employment and job turnover, and small effects on labor productivity and the value of firms. Within the papers studying the effects of temporary contracts the paper by Alonso-Borrego et al. (2005) and Alvarez and Veracierto (2012) are probably the most closely related to our paper since, in contrast with the other papers, they consider firm dynamics in a general equilibrium model. However, Alonso-Borrego et al. (2005) find that temporary contracts increase productivity. In the paper by Alvarez and Veracierto (2012) the main argument is that the presence of temporary contracts provides an employment buffer that firms can use to adjust to their idiosyncratic shocks without having to incur firing costs.

Of course, there is an empirical literature analyzing employment protection legislation and showing that DEPL is widely used across countries and it has potential effects on employment (and unemployment), worker turnover and productivity.[5,6] Regarding employment protection legislation Autor et al. (2007) and Bassanini et al. (2008) provide empirical evidence showing that strict employment protection legislation has a depressing impact on productivity because it reduces the level of risk that firms are ready to endure in experimenting with new technologies or because there is less threat of layoff in response to poor work performance. With regards to the effect of DEPL on productivity, Boeri and Garibaldi (2007), Sanchez and Toharia (2000), and Alonso-Borrego (2010) find a negative relationship between the share of temporary workers and firms' labour productivity. Dolado and Stucchi (2008) suggest that workers on temporary contracts may be motivated to exert low effort levels because of the high probability of being fired at the end of their contracts. They attribute one-third of the fall of TFP in Spanish manufacturing firms during the period 2001–2005 to the disincentive effects of the low conversion rates on temporary workers' effort.[7] Our paper provides a new mechanism to interpret the negative effect of DEPL on TFP documented in these papers.

There is an important recent macroeconomic literature analyzing the sources of resource misallocation among production units. For instance, Erosa and Hidalgo (2008) and Buera, Kaboski and Shin (2011) focus on financial market imperfections as a source of misallocation. Hsieh and Klenow (2009) study the impact of misallocation across establishments in explaining productivity in manufacturing in China and India. Furthermore, they recover the underlying distortions from observed allocations and, as well as Bertelsman, Haltiwanger and Scarpetta (2008), follow Restuccia and Rogerson (2008) and model

[5] Belgium, Denmark, Germany, Greece, Italy, Netherlands, Portugal, Spain and Sweden have introduced or intensified the use of temporary contracts since the mid 80's (European Commision, 2010).

[6] Some papers use longitudinal data of countries exploiting the differences in severance pay across countries. For instance, Lazear (1990), Addison and Grosso (1996), Burgess, Knetter and Michelacci (2000), Heckman and Pagés (2004), Abraham and Houseman (1993, 1994), Bover, García-Perea and Portugal (2000), among others. While a second line of research has exploited data before and after specific reforms in the labor market using a differences-in-differences approach. See for example Kugler (2004), Hunt (2000), and Bentolila and Saint-Paul (1992), among others. For the effects on employment and worker turnover see, the surveys by Dolado, García-Serrano and Jimeno (2002) and by Bentolilla, Dolado and Jimeno (2008) on the Spanish experience and the extensive literature cited therein.

[7] Dolado, Ortigueira and Stucchi (2012) propose a model in which both temporary workers' effort and firms' temp-to-perm conversion rates decrease when the gap in firing costs between permanent and temporary workers increases. In addition, they test the implications of the model using as natural experiments some labour market reforms entailing substantial changes in firing costs gap and they find that reforms leading to a lower gap enhanced conversion rates and increased firms' TFP.

distortions as firm or plant-specific. Moreover, a key insight in Restuccia and Rogerson (2008) is that idiosyncratic distortions are more important (have the potential to do much more damage) when they are positively correlated with firm productivity (establishments with low TFP receive a subsidy and establishments with high TFP are taxed). In our model, firing costs to permanent workers act as a subsidy to small firms (which are intensive in the use of temporary workers) and a tax to big firms (which are intensive in the use of permanent workers). In this sense, our paper is related to Restuccia and Rogerson (2008). In addition, Guner, Ventura and Xu (2008) consider policies that directly target the size of the establishment (size-dependent policies) such as a tax on establishments with more than given number of employees. When a general configuration of these policies are restricted to achieve a given reduction in average establishment size, they find a substantial reduction in aggregate output per worker. In line with this idea, our paper shows that DEPL plays similar role as a size-dependent policy, penalizing more to firms that use permanent employees (big firms).

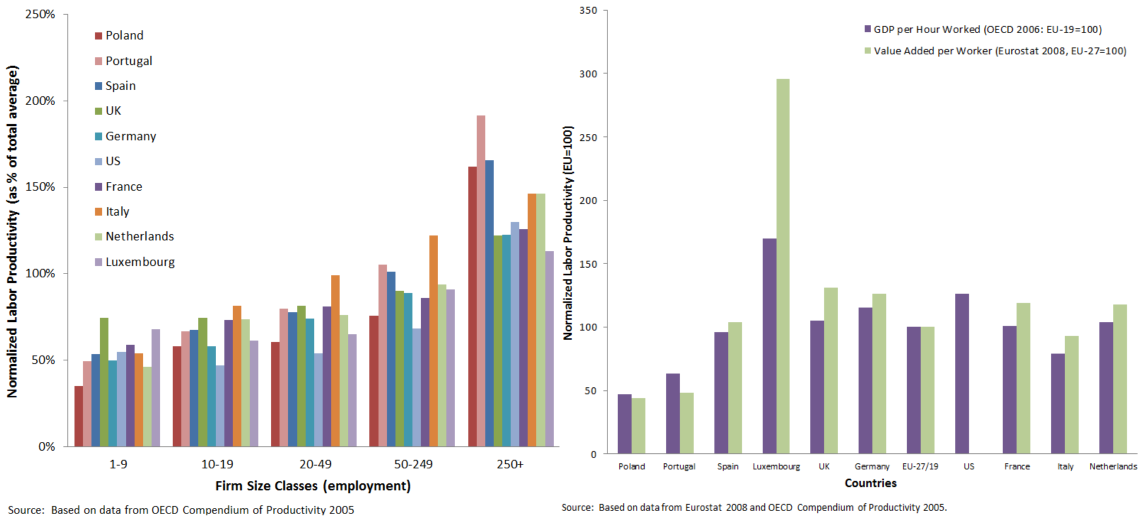
To summarize, we provide evidence showing that countries with dual employment protection legislation that incentives or extend the use of temporary contracts have relatively more employment concentrated in small firms and thus have lower productivity. Motivated by the evidence, we next develop a theoretical model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers and characterize the equilibrium labor composition that firms with different productivity growth rate choose over their life cycle. In this framework we analyze the impact of higher firing costs of permanent workers in the distribution of firms' size and productivity.

1.1 Empirical evidence

In order to motivate our theory, in this section we bring along evidence showing that it is important to analyze the size distribution of firms to understand differences in productivity across countries. We also show that, across countries, DEPL tends to be associated with a higher share of temporary employment, and that countries with particularly high fraction of temporary employment have a relatively large share of small firms as well as a high share of employment allocated in small firms and lower aggregate productivity.

Taking data from the OECD, Compendium of Productivity (2005), in the panel to the left of Figure 1 we present the normalized firms's labor productivity by size-classes (employment bins) in the manufacturing sector for US and many European countries.[8] The panel to the right of Figure 1 presents the aggregate labor productivity for the same group of countries.[9] Despite countries differ significantly in their aggregate labor productivity, across countries firms display similar labor productivity levels within the same size-classes. The biggest enterprise size class has the highest productivity. This is a common pattern in Europe since for the majority of countries, about 75 % (taking into account other European economies not presented here), productivity increases monotonically with size class.

Figure 1: Normalized Productivity (manufacturing sector), 2005.



How can significant differences in aggregate labor productivity across countries be reconciled with the fact that firms display similar labor productivity levels within the same size-classes across countries? The data exposed in Figure 2 and Table 1 suggests that one possible reason is that the distribution of firm sizes is different across counties. In Figure 2, panel A presents the cumulative fraction of firms for different employment levels comparing three economies, US, UK and Spain; and Table 1 shows the size distribution of firms and the labor productivity. It is clear that in Europe there is a higher concentration

[8] The normalised labour productivity is calculated as value added per worker in a given size class as a percentage of the average labour productivity across all size classes (see OECD, Compendium of Productivity 2005).

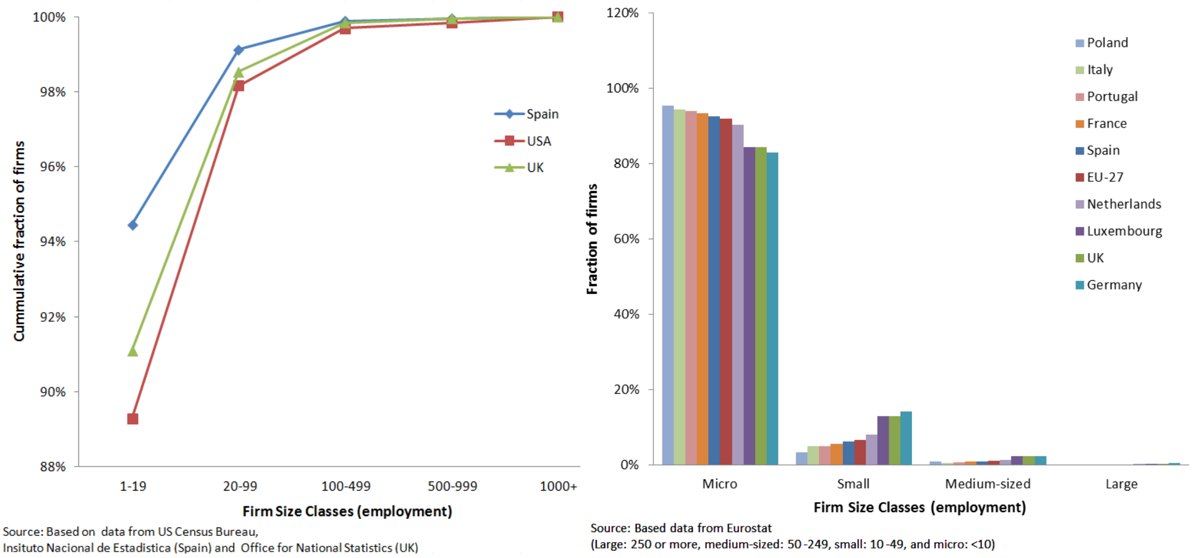
[9] The Labor productivity in the panel to the right of Figure 1 is Gross Domestic Product at constant prices and using PPP's, divided by either total employment or total hours worked.

of firms in low size levels, measured by employment, than in US. While 0.12% of firms in Spain and 0.16% of firms in UK have more than 500 employees, in US a fraction of 0.31% of firms belong to that employment size-class. Furthermore, the fraction of firms with 100 to 499 and 20 to 99 employees is 0.75% and 4.67% in Spain, 1.31% and 7.44% in UK, and 1.52% and 8.88% in US, respectively. In addition, Spain and UK have a higher fraction of firms in the lowest employment level, 94.46% and 91.10% in the 1 to 9 workers bin, respectively, than US (with 89.29%).

Panel B of Figure 2 presents a similar comparison among many other European countries. For instance, France, Italy and Spain have a higher fraction of micro and small firms and a lower fraction of medium and large firms than the average among European Countries. In contrast, Germany, UK and Netherlands have a lower fraction of micro and small firms and a higher fraction of medium and large firms than the average among European Countries.

Table 2 shows that the same pattern occurs when comparing the fraction of total employment concentrated in different firm size classes. For instance, in Spain small firms concentrate a bigger fraction of total employment than the average small firms in the EU. In Spain the joint employment of micro, small and medium firms represents 82.2%, versus 73.9% in the EU. In contrast, small firms in the UK employ a lower fraction of total employment with respect to the average small firms in the EU (62.87% versus 73.9%). The opposite evidence is found when we look at the fraction of employment at big firms. In Spain and the UK big firms concentrates 17.8% and 37.2% of total employment, respectively.

Figure 2: Fraction of firm by size-classes (manufacturing sector), 2008.



Panel A Panel B

Table 1: Firms size distribution and productivity.

Country	Employment-Size Classes (% of firms)				Productivity (EU-27=1)
	Micro	Small	Medium	Large	Value added per worker
Poland	95.5	3.3	1.0	0.2	0.44
Italy	94.3	5.1	0.5	0.1	0.93
Portugal	94.0	5.1	0.7	0.1	0.48
France	93.3	5.6	0.9	0.2	1.19
Spain	93.1	6.0	0.8	0.1	1.04
Netherlands	90.4	8.0	1.4	0.3	1.18
Luxembourg	84.0	12.9	2.4	0.3	2.96
UK	89.3	8.8	2.5	0.4	1.31
Germany	83.0	14.1	2.4	0.5	1.26
EU-27	92.0	6.7	1.1	0.2	1.00

Source: Taken from Eurostat 2009

Large: 250 or more, medium-sized: 50-249, small: 10-49, and micro: <10

Table 2: Employment by firms sizes.

Country	Employment-Size Classes (% of firms)			
	Micro	Small	Medium	Large
Poland	.	.	.	.
Italy	61.3	15.8	9.1	13.9
Portugal	57.7	17.1	11.9	13.3
France	35.9	17.7	13.8	32.6
Spain	49.6	20.6	12.0	17.8
Netherlands	41.1	17.8	13.9	27.2
Luxembourg	24.7	22.3	21.9	21.1
UK	35.3	14.8	12.7	37.2
Germany	32.6	18.2	16.2	33.0
EU-27	43.8	16.5	13.5	26.1

Source: Taken from Eurostat 2009

Large: 250 or more, medium-sized: 50-249, small: 10-49, and micro: <10

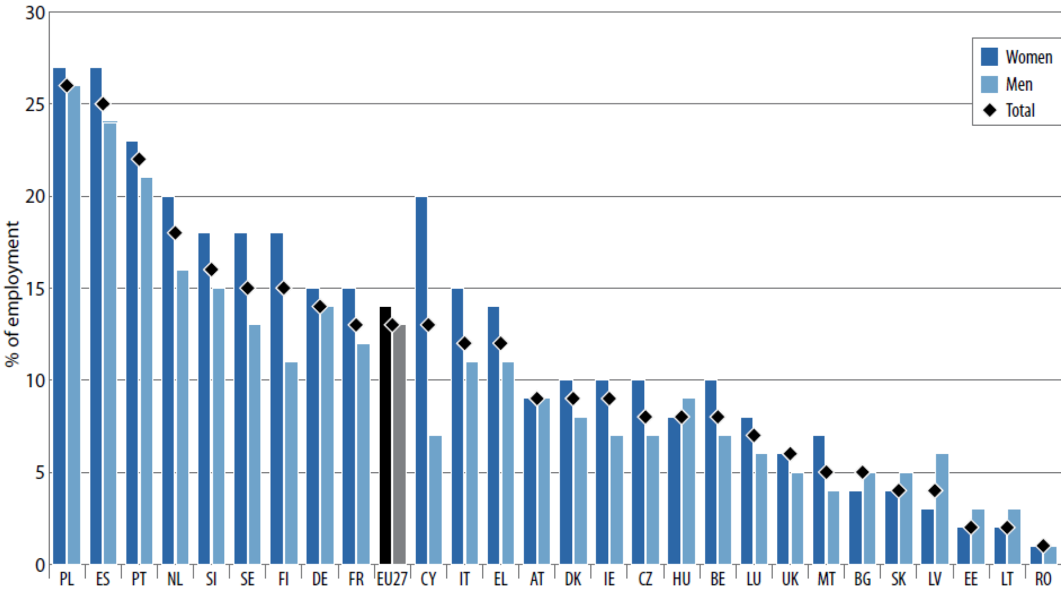
All this evidence suggests that is important to study the determinants of the size distribution of firms to interpret the differences in relative aggregate productivity. In line with this fact, there are other papers showing that in order to understand the cross country differences of economies' performance in many dimensions it is relevant to study the size distribution of firms. For instance, Navaretti, et al (2011), in the second EFIGE policy report, find that European countries perform very differently in terms of their trade competitiveness (exports and global production strategies) because the within-country distribution of firms characteristics (size, innovative capacity and productivity) differ significantly across these economies.[10]

As mentioned before, the growing literature studying the sources of resource misallocation among production units focuses on many reasons to interpret the differences in the size distribution of firms across countries. We next show additional evidence indicating that countries with particularly high fraction of temporary contracts in their labor force have a relatively large share of small firms and lower productivity, suggesting that labor market regulations that stimulate firms to extend the use of temporary may play an important role in shaping the size distribution of firms.

[10] For instance, they find that among European Union countries trade performance differ significantly where Germany is by far the most export oriented, with a share of exports to GDP of 39.9 percent, followed by Italy (23.4 percent), France (21.3 percent), the United Kingdom(17.2 percent) and Spain (16.7 percent). Then they show that German firms tend to be larger and Italian firms smaller than the EU average in all sectors. Furthermore, using firm level microdata collected in 2008 they conduct a counterfactual exercise suggesting that if the industrial structure (in terms of firm size and sectors) of countries such as Italy and Spain were to converge to the structure of Germany, the value of Italian and Spanish total exports would rise considerably, by 37 percent and 24 percent respectively.

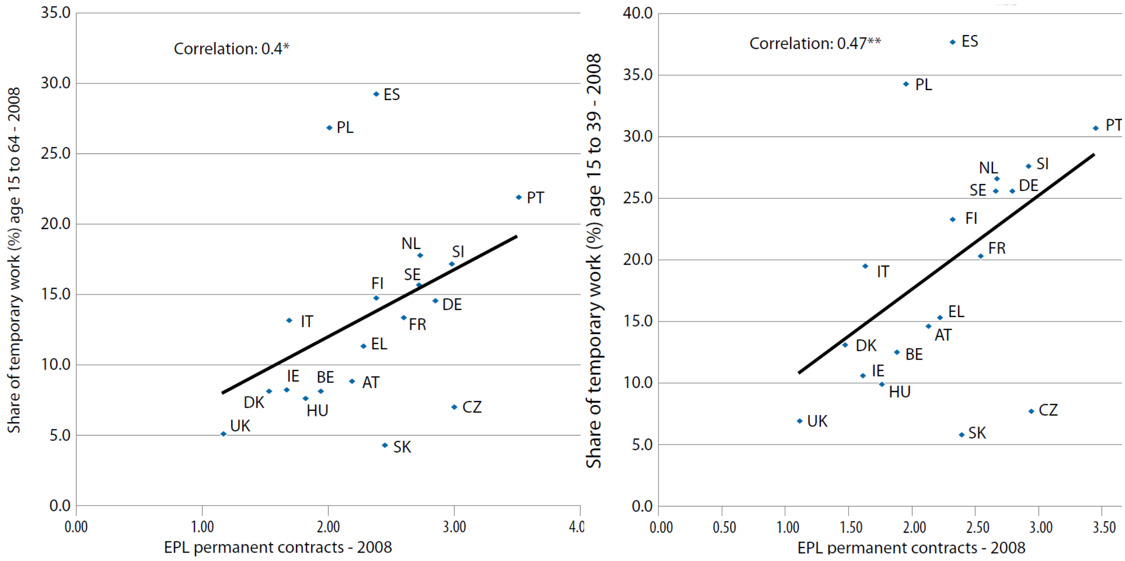
Fixed-term contracts are relatively common in some countries such as Poland and Spain, where 25% or more of employees have such a contract, and Portugal with more than 20% in 2009 (see Figure 3). In contrast, temporary contracts accounts for less than 15% of employees in Germany, and just a bit more than 5% of employees in Luxembourg and UK. Following Boeri (2010b), the 2010 Eurostat Report on Employment in Europe (Figure 4) shows that stricter employment protection legislation (EPL) for permanent contracts tended to be associated with a higher share of temporary employment across EU countries (firms substitute permanent workers for temporary workers to avoid higher firing costs).

Figure 3: Fixed-term employment for Member States by gender, 2009.



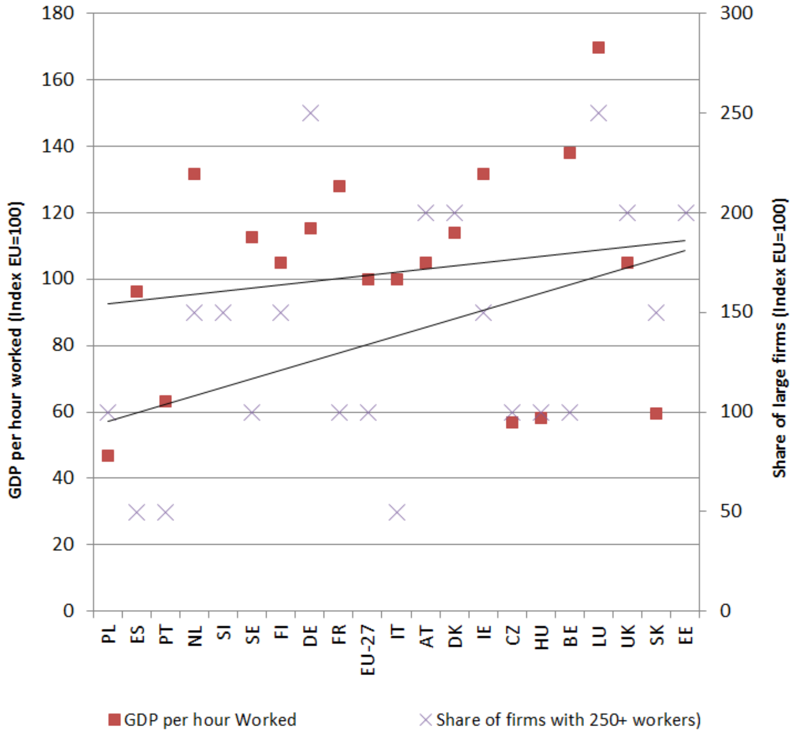
Source: Taken from Eurostat 2009

Figure 4: EPL on permanent contracts and temporary work.



In the horizontal axis of Figure 5 I have ranked countries according to the share of temporary work in decreasing order and the vertical axis contains the share of big firms (indicating an important aspect of the shape of the size distribution of firms) and the GDP per hour worked (an indicator of productivity). It provides preliminary evidence showing that there is a negative relationship between the share of temporary workers and the fraction of big firms as well as with the aggregate productivity across countries. The same results are obtained when using value added per worker and the share of employment in big firms, respectively.

Figure 5: Temporary work and TFP.



Source: Based on data from Eurostat and OECD 2008.

The previous unconditional data analysis is in line with formal empirical evidence on the relationship between DEPL (and temporary work) and the level of TFP documented in other papers, and it represents preliminary evidence about the relationship between temporary work and the size distribution of firms. In addition, it is common knowledge that labor market regulations clearly distinguish the first three countries (in particular in Spain and Portugal) from the rest, and that dual employment protection legislations in those countries have been at the center of the economic debate. Regarding formal empirical evidence, in addition to the literature discussed in the introduction, it is relevant the paper by Dolado, Ortigueira and Stucchi (2012) since it is well known that Spain represents a key case study due to the widespread use of temporary work and the relative strictness of DEPL. Dolado, Ortigueira and Stucchi (2012) argue that since the early nineties Spain has been the EU country with the highest proportion of temporary workers and, in parallel, it has suffered from a drastic productivity slowdown since the mid-1990s. Using a panel of Spanish manufacturing firms they estimate that up to 20% of the slowdown of TFP growth in Spanish manufacturing firms could be explained by the reduction in conversion rates that dual employment protection legislation generates. It is also well known that the lower TPF level that Spain has relative to other European countries is connected with the lower fraction of large firms (or lower share of employees in big firms).[11]

All in all, the unconditional analysis documented in the current paper suggests that countries with labor market regulations that disincentive the relative use of permanent contracts have a higher fraction of temporary work, and a higher fraction of temporary work is associated with a lower fraction of large firms and lower productivity. In addition, there is well documented microevidence for the Spanish economy that points to the dual employment protection legislation as a key determinant for the lower productivity. In the current paper we explore a new specific mechanism linking changes in the strictness of the dual employment protection legislation to aggregate productivity by means of distortions on firm selection and the allocation of resources across firms.

2 Model

2.1 Key features of the model

In this section we describe the main ingredients of the model, introduce some notation, and explain the induced optimal behavior by agents and the main trade-offs before going to the model more in detail. We develop a theory of firm dynamics with search frictions and asymmetric firing costs of permanent and temporary workers. Let's now explain each of these components.

First, by firm dynamics we mean the following. There is a continuum of potential entrant firms that upon entry have the same initial productivity and draw a rate of productivity growth, g, from a distribution with cdf $G(g)$. Firms live at most $\hat{J}$ periods, they produce homogeneous goods that are sold in a competitive market and for production they need to employ workers.

Second, regarding the presence of search frictions, just like in Mortensen and Pissarides (1994), we assume that firms and workers have to search for each other and there is a matching technology that relates the probability of workers and employers finding a

[11] See FEDEA (2009), and Pol Antras in NeG (2010), http://www.fedeablogs.net/economia/?p=7234.

counterpart in the labor market to the ratio of vacancies to unemployed members of the labor force, denoted by θ. Due to the linear homogeneity of the matching function, $m(\theta)$, job seekers meet firms at the rate $\theta m(\theta)$ which is increasing in θ. Following Felbermayr, Prat and Schmerer (2011), the cost of posting vacancies is proportional to a function c_v, so that recruiting x workers entails spending $c_v x$, where c_v is the cost of a vacancy divided by the probability of filling the vacancy, $c_v = c/m(\theta)$. Whereas marginal recruitment costs are increasing at the aggregate level because of congestion externalities, they are exogenous from a firm's point of view. In addition, we assume that there are unemployment benefits, denoted by b.

Third, regarding temporary and permanent workers and the asymmetric firing costs we assume that, temporary workers are employees that remain matched with a firm for a short period of time (they have an exogenous separation rate equal to one) and are dismissed at zero firing cost. Permanent workers are employees that remain matched with a firm until it decides to fire them (they have an exogenous separation rate equal to zero), but have high firing costs, τ_f. In addition we allow permanent workers to have exogenous higher productivity than temporary ones (this is not a fundamental assumption; the results remain the same if we assume that temporary and permanent workers have similar exogenous productivity).

With respect to the timing, after entry a firm observes its productivity growth rate, it decides the optimal age to exit, and decides how many vacancies to post. After matches have taken place, it decides how many workers to hire as temporary or permanent workers. Then firms and workers bargain over wages according to their particular labor contracts (permanent or transitory contracts). There is free entry, thus new firms are born (enter) every period.

In this framework firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. Furthermore, firms with the same productivity level may choose different optimal sizes and fraction of permanent and temporary workers. In order to provide a better interpretation on this optimal firms´ behavior, let's focus on an example in discrete time to analyze the optimal decisions of firms with high productivity growth rate and firms with low productivity growth rate. Let's assume that every firm begins its life with the same productivity level, according to which its optimal size is ten workers. In addition, let's assume that permanent workers are equally productive than temporary workers.

First, let's assume that for a firm with the highest productivity growth rate its optimal employment sequence is $\{10, 15, 20, 25, 30, 35\}$ and thus it lives for $\widehat{J} = 6$ periods. Therefore, if the firm only hires permanent workers then the total cost over the life cycle, net of wages, is $35(c_v + \tau_f)$, where $35c_v$ is the total search cost and $35\tau_f$ is the total firing cost incurred when the firm exit the market. Alternatively, in case the firm decides to hire only temporary workers the net total cost is $135c_v$. Notice that the total search costs in the first case is much lower than in the second case and it may compensate the higher firing costs the firm pay when hiring permanent workers. Therefore, firms with positive growth rate, which survive and last longer in the market, have incentives to hire permanent workers.

Focusing just on the first ten workers that the firm hires right after entry, it can be seen that under a permanent labor contract the costs over the life cycle is $10(c_v + \tau_f)$ versus $\widehat{J}10c_v$ if the firm employs ten workers under a temporary labor contract. Therefore, it is convenient for the firm to use permanent workers whenever $\tau_f < (\widehat{J} - 1)c_v$. If we now focus on the second period that the firm is active in the market, and assuming that it has hired ten permanent workers in the previous period, under the same reasoning it hires five additional permanent workers if $\tau_f < (J - 2)c_v$. Notice that the left hand side of the previous inequality is constant while the right hand side decreases as firm ages. Therefore, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. Moreover, firms optimally modify the fraction of permanent workers they hire over their life cycle.

Second, let's focus on a firm with negative productivity growth rate. This example shows that firms with bad prospects on the evolution of their productivity have incentives to hire more temporary workers since they keep reducing production and employment over time and exit the industry relatively soon. Let's assume that a firm's optimal employment path is a sequence of workers $\{10, 5, 2\}$ and exit after $J^* = 2$ periods. Following the same reasoning as in the previous example its easy to notice that; first, it is not optimal for this firm to employ only permanent workers (since some of them will be fired next period); and second, it may be optimal for the firm to employ some permanent workers, for instance 3 permanent workers and 7 temporary workers in the first period, 2 temporary workers in the second period, fire one permanent worker in the third period, and then exit. Therefore, even firms with negative productivity growth rate may find it optimal to hire permanent workers to save on search costs and modify the fraction of temporary and permanent workers along their life-cycle.

The model generates a cross sectional distribution of firms by size and employment composition (different share of temporary and permanent workers). Furthermore, this environment gives rise to rich industry dynamics as firms enter, exit, decide the composition of their labor force over their life cycle and bargain wages for each type of labor contract. Despite the difficulty of the issue we develop a simple model that is analytically tractable to show that high relative firing costs of permanent workers distort firm selection and the allocation of resources across firms. But the main mechanism and insights of the model apply to a more general and realistic model. In the conclusion we argue that, under the presence of search frictions, as long as it takes longer to high productivity firms to exit (they have longer expected life span) and thus it is more valuable for them to hire permanent workers, a more realistic Hopenhayn and Rogerson's (1992) style model capture similar behavior and yields the same results as in the model developed in current paper. We will discuss further the evidence supporting this assumption in the conclusion of the paper.

2.2 Value functions for firms and workers

In this section we describe the model more in detail. As mentioned before, since there is a continuum of ex-ante homogeneous firms that are ex-post heterogeneous, there is a cross sectional distribution of firms over the states. As there is free entry and exit and no aggregate uncertainty, by a law of large numbers all aggregate quantities and prices are constant over time. Therefore, when describing the value of firms, employed and unemployed workers we focus on the stationary equilibrium and thus value functions are not indexed by time.

2.2.1 Firms

Firms produce goods according to the production function $z(n^T + \gamma n^P)^\alpha$ where z is the initial productivity level that we assume equal to one, n^T and n^P are the number of temporary and permanent workers, respectively. The parameter α is the span of control parameter ($\alpha < 1$), and γ is the relative productivity of permanent workers. As mentioned before, firms sell their output in a competitive market, and the price is normalized to 1.

We normalize the firing cost of a temporary worker to zero while the firing cost of a permanent worker τ_f , measured in units of output, is positive. We also assume that firms incur the cost τ_t (a training cost) when hiring a permanent worker. We assume that firms pay the training cost when they hire permanent workers (this is without loss of generality because it is optimal to train permanent workers upon hiring them) while the firing cost is paid when firms fire permanent workers. In addition, firms have to pay a per period fixed cost of production, $c_f > 0$, that generates endogenous exit in the model.

Value of an incumbent firm

The state variables of a firm is given by the specific growth rate of productivity, g, and firm's age, a. The aggregate state variable is the labor market tightness, θ. Firms choose a production plan that maximizes discounted lifetime profits. A production plan for a firm with growth rate g and age a is described by the mass of temporary workers, $n^T(g, a)$, and for the mass of hired and fired permanent workers , $h(g, a)$ and $f(g, a)$, respectively, and a time of exit , $J(g)$. As described below, wages are determined through bargaining. In general, the wage that results from bargaining depends on firm characteristics (g and a). For the moment, let us postulate that the bargaining outcome can be summarized by the wage functions $w^T(g, a)$ and $w^P(g, a)$. Therefore, the value function of an incumbent firm is

$$V(g,a;\theta) = \underset{J(g,a),\{n^T(g,a),f(g,a),h(g,a)\}_a^{J(g,a)\leq \hat{J}}}{Max} \int_a^J \Big[e^{ga}(n^T(g,a)+\gamma n^P(g,a))^{\alpha}$$

$$- w^T(g,a)n^T(g,a) - w^P(g,a)n^P(g,a)$$

$$-\frac{c}{m(\theta)}\left(n^T(g,a)+h(g,a)\right) - h(g,a)\tau_t - f(g,a)\tau_f\Big] da$$

$$- n^P(g,J(g,a))\tau_f - c_f(J(g,a)-a)$$

$$\dot{n}^P = h(g,a) - f(g,a)$$
$$J(g,a) \leq \hat{J}$$
$$h(g,a), f(g,a), n^T(g,a), n^P(g,a) \geq 0$$
$$n^P(g,a) \text{ given.}$$

Thus, a firm with state (g,a) decides how many vacancies to post so as to hire the desired amount of workers $\left(n^T(g,a)+h(g,a)\right)$, incurring a proportional cost of posting vacancies $c_v = c/m(\theta)$ per unit of vacancy posted. It has to pay wages, as well as firing costs and the per period fixed production cost. When the firm dies, at age $J(g,a)$, it has to pay the firing costs to all permanent workers being fired. The first constraint is the law of motion for permanent workers, and the other are just feasibility constraints.

As explained before, when deciding on $n^T(g,a)$, $f(g,a)$, and $h(g,a)$ firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. Firms may find it optimal to hire permanent workers and pay the firing cost τ_f for two reasons. First, to economize on matching costs. Second, permanent workers may be more productive than temporary workers ($\gamma \geq 1$,) where the case $\gamma - 1$ represents the human capital increase due to on the job training paid by the firm.

Entry decision

There is free entry of firms who are ex-ante identical in terms of productivity growth rate and start with the same initial unitary productivity level. In order to enter the industry firms must pay a sunk entry cost, $C_e \geq 0$. After paying C_e they get a draw of productivity growth rate g from the distribution $G(g)$. After observing g they pay the fixed production cost, c_f, post vacancies, pay training costs to the new permanent workers, and bargain over wages with matched workers. Therefore, the value of the expected future discounted profits of a new firm is

$$V_e(\theta) = \int_g V(g;\theta)dG(g). \tag{1}$$

Since the function $V(g)$ is increasing in g, the optimal exit decision involves a threshold value g^e such that a firm survive if $g \geq g^e$.

2.2.2 Workers

Here we present the value function for employes and unemployed workers. The appendix 4.1 contains details on the derivations of these equations. The value function of a worker employed in a firm in state (g, a) with a labor contract $s = \{T, P\}$, where s indicates temporary or permanent contract, is given by

$$\rho W(g,a,s) = w^s(g,a) + \delta(g,a,s)\left[U - W(g,a,s)\right] + \left[1 - \delta(g,a,s)\right]\dot{W}(g,a,s),$$

where $\delta(g, a, s)$ denotes the probability that the firm decides to terminate the labor contract. As explained before temporary contracts are terminated with probability 1. The value function of an unemployed worker is

$$\rho U = b + \theta m(\theta)\int \left[W(g,a,s) - U\right] d\mu(g,a,s), \tag{2}$$

where $\mu(g, a, s)$ denotes the conditional probability of being matched with a firm in state (g, a) offering a labor contract of type s. Of course, this probability is an equilibrium object that will be formally defined later on and represents the distribution of job offers.

2.2.3 Equilibrium wages

The total surplus generated by a match between workers and firms is split between them. The worker's surplus is equal to the difference between the value of being employed $W(g, a, s)$ by a firm with age a, rate of productivity growth g, under contract s, and the value of being unemployed U. Firm's surplus is simply equal to the marginal increase in the firm's value $\partial V(g,a)/\partial n^P$ (notice that it depends on the labor contract, $s=\{T,P\}$) since each employee is treated as the marginal worker. Following Felbermayr et al (2011) we assume that the outcome of bargaining over the division of the total surplus from the match satisfies the following "surplus-splitting" rule for the case of temporary workers

$$(1-\beta)[W(g,a,T)-U]=\beta\partial V(g,a;\theta)/\partial n^T(g,a)=c_v,$$

and for the case of permanent workers

$$\begin{aligned}(1-\beta)[W(g,a,P)-U] &= \beta\frac{\partial[V(g,a;\theta)-(0-\tau_f n^P(g,a))]}{\partial n^P(g,a)}\\ &= \beta(\lambda(g,a)+\tau_f),\end{aligned}$$

where $\beta\in[0,1]$ denotes the bargaining power of the worker, and $\lambda(g,a)$ is a co-state variable representing the shadow value of a permanent worker (which will be defined formally later on). Applying the envelope theorem for each type of labor contract we get the solutions. Given that we set up the problem of the firm in continuous time, temporary workers obtain the value of being unemployed (the unemployment benefit) for every value of the bargaining power parameter, this is $w^T(g,a)=b\ \forall\beta$, as there is no stock value of a temporary worker to the firm. In the case of permanent worker labor contract, as the threat point of the firm is $\tau_f n^P$, the wage w^P will be higher that the unemployment benefit except in the case in which the bargaining power of the firm is one ($\beta=0$).

Let's solve for the permanent worker's wage. We know from the value function of a permanent worker that

$$W(g,a,P)=\frac{w^P(g,a)+\delta(g,a,P)U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)},$$

by plugging this equation in the bargaining problem we get

$$\begin{aligned}(1-\beta)[\frac{w^P(g,a)+\delta(g,a,P)U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)}-U] &= \beta(\lambda(g,a)+\tau_f)\\ \frac{w^P(g,a)-\rho U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)} &= \frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f),\end{aligned}$$

thus we have

$$w^P(g,a)=\frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)(\rho+\delta(g,a,P))-(1-\delta(g,a,P))\dot{W}(g,a,P)+\rho U.$$

From the value function of an unemployed worker we have

$$\rho U = b + \theta m(\theta) \left[\int \underbrace{(W(g,a,T) - U)}_{=0} d\mu(g,a,T) + \int \underbrace{(W(g,a,P) - U)}_{=\frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)} d\mu(g,a,P) \right].$$

where the term inside the first integral is equal to zero since we have shown that $(1-\beta)[W(g,a,T)-U] = 0$ in the bargaining problem of temporary workers, and the term inside the second integral is $\frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)$ since $(1-\beta)[W(g,a,P)-U] = \beta(\lambda(g,a)+\tau_f)$ in the bargaining problem of permanent workers. Therefore we have that

$$\rho U = b + \theta m(\theta)\frac{\beta}{1-\beta}\int [\lambda(g,a)+\tau_f] d\mu(g,a,P).$$

By plugging this equation in the wage equation we have found previously we get the following expression for the wage of permanent workers

$$\begin{aligned} w^P(g,a,P) &= \frac{\beta}{1-\beta}\left[(\lambda(g,a)+\tau_f)(\rho+\delta(g,a,P)) + \theta m(\theta)\int [\lambda(g,a)+\tau_f] d\mu(g,a,P)\right] \\ &\quad -(1-\delta)\dot{W}(g,a,P) + b, \end{aligned}$$

where remember that $\delta(g,a,P)$ and $\dot{W}(g,a,P)$ are equilibrium objects. Given that these variables are not easy to compute since now on we will follow the analysis for the case in which the firm has all the bargaining power, $\beta = 0$, therefore, $w^T = w^P = b$.

2.3 Stationary Equilibrium

Before defining a stationary equilibrium, it is convenient to introduce some additional notation and define the law of motion describing equilibrium aggregates. In steady state there will be a constant influx of new firms. To characterize the equilibrium distribution of firms we denote by M the mass of new entrants and we define the following indicator function

$$I(g,a) = \begin{cases} 1 & \text{if } a = J(g) \\ 0 & \text{otherwise.} \end{cases}, \tag{3}$$

and the mass of firms of age a and productivity growth g, $X(g,a)$, which satisfies

$$\begin{aligned} \frac{\partial X(g,a)}{\partial a} &= -I(g,a)X(g,a) \\ X(g,0) &= M\frac{dG(g)}{1-G(g^e)} \text{ if } g \geq g^e \end{aligned}$$

The above law of motion states that the mass of age 0 firms with productivity growth g is given by the mass of entrants times the fraction of businesses that draw productivity growth g among those businesses drawing $g \geq g^e$. As firms aged, the mass of businesses with productivity g stays constant until the optimal exit time $J(g)$. At this age, the mass of businesses with productivity g decreases by 100%.

The probability measure of firms with productivity growth rate $\widetilde{g}$ and age $\widetilde{a}$, hiring workers with contract type $s \in T, P$, are defined as

$$\begin{aligned}
\mu(\widetilde{g}, \widetilde{a}, P) &= \frac{X(\widetilde{g}, \widetilde{a})h(\widetilde{g}, \widetilde{a})}{\int_{g \geq g^e} \int_0^{J(g)} X(g,a)h(g,a)dgda}, \\
\mu(\widetilde{g}, \widetilde{a}, T) &= \frac{X(\widetilde{g}, \widetilde{a})n^T(\widetilde{g}, \widetilde{a})}{\int_{g \geq g^e} \int_0^{J(g)} X(a,g)n^T(a,g)dadg}.
\end{aligned}$$

In addition, the probability that a permanent worker in a business with state (g, a) is fired satisfies

$$\delta(a, g, P) = \begin{cases} 1 & \text{if } a = J(g) \\ \frac{f(g,a)}{n^P(g,a)} & \text{if } a < J(g). \end{cases} \tag{4}$$

A *stationary equilibrium* is a list of firm decisions rules on temporary and permanent employment $n^T(g, a)$, hiring $h(g, a)$ and firing $f(g, a)$, age of exit $J(g)$, entry threshold g^e, value functions for firms, permanent and temporary employed workers and unemployed workers $(V((g, a; \theta)), W(g, a, P), W(g, a, T), U)$, wage functions $w^T(g, a)$ and $w^P(g, a)$, probability measure for new hires $\mu(g, a, s)$, a mass of entrants M, unemployed N^U and employed N^E, and labor market tightness θ such that:

i) Prices $w^T(g, a) = w^T(g, a) = b$ are given by Nash bargaining (under the assumption $\beta = 0$).

ii) Given $w^T(g, a)$ and $w^T(g, a)$, the firms' production plans $J(g)$, $n^T(g, a)$, $f(g, a)$, $h(g, a))$ are optimal.

iii) Free entry condition is satisfied $C_e = V_e(\theta) = \int_g \max\{0, V(g; \theta)dG(g)\}$.

iv) Laws of motions of different cohorts and entrants as described above,

$$\begin{aligned}
\frac{\partial X(g,a)}{\partial a} &= -I(g,a)X(g,a), \\
X(g,0) &= M\frac{dG(g)}{1 - G(g^e)} \text{ if } g \geq g^e.
\end{aligned}$$

v) The mass of unemployed individuals finding jobs should be equal to the mass of vacancies filled by firms

$$N^U \theta m(\theta) = \int_{g \geq g^e} \int_0^{J(g)} X(g,a) \left[h(g,a) + n^T(g,a) \right] dgda. \tag{5}$$

vi) The mass of employed individuals satisfies

$$N^E = 1 - N^U = \int_{g \geq g^e} \int_0^{J(g)} X(g,a) \left[n^T(g,a) + n^P(g,a) \right] dgda. \tag{6}$$

vii) The mass of workers finding jobs should be equal to the mass of workers being fired (so that unemployment is constant) due to exits and downsizing:

$$N^U \theta m(\theta) = \int_{g \geq g^e} \int_0^{J(g)} X(g,a) \left[f(g,a) + n^T(g,a) \right] dgda + \int_{g \geq g^e} n^p(g, J(g)) X(g, J(g)) dg. \tag{7}$$

viii) The mass of entrant firms, M, is consistent with the equilibrium labor market tightness, θ, according to $1 = N^E(M,\theta) + N^U(M,\theta)$.

Proposition 1 *Equilibrium Existence and Uniqueness:* There exist a unique pair (θ^*, M^*) that satisfies the equilibrium definition

Proof Since the newborn firm value, $V(g,a;\theta)$, is monotone decreasing and continuous in θ, thus the expected present discounted value at entry, $V_e(\theta)$, is also monotone decreasing and continuous in θ. Given a value for C_e such that $V_e(\theta = 0) > Ce > V_e(\theta = 1)$, by the intermediate value theorem there exist a unique value θ^* such that $\mathrm{V}_e(\theta^*) - Ce = 0$. By linear homogeneity of $\widehat{X}(\cdot)$, $\widehat{N}^E(\cdot)$ and $\widehat{Vac}(\cdot)$ in M (since policy functions h, f, n^T are invariant in M) then employment and vacancies are continuous and increasing in M and unemployment is decreasing in M, there exist a unique value M^* such that $\theta^* = \frac{M^* \widehat{Vac}(g,a;\theta^*)}{1 - M^* \widehat{N}^E(g,a;\theta^*)}$[12].■

[12] For more details see the solution algorithm in the appendix.

2.4 Solving and characterizing the model for $\beta = 0$

As it was shown previously, the wage for temporary workers does not depend on the bargaining power of the firm and is always equal to the unemployment benefit. On the other hand, the wage for permanent workers depends not only on the bargaining power of the firm but also on equilibrium objects such as the fraction of firms searching for permanent workers, and the optimal firing probability a firm decides on their permanent workers. As it can be noticed in the following solutions, dealing with such complicated expressions would make the analysis less clear and increases significantly the difficulty to focus in the main mechanisms and the economic intuition of the model. Therefore, in this section, and for the rest of the paper, we solve and characterize the solutions of the model for the case in which firms have all the bargaining power. We leave the discussion of the general case for the last sections.

The value of a new born firm with productivity growth rate g is given by

$$V(g;\theta) \equiv \underset{J(g),\{n^T(g,a),f(g,a),h(g,a)\}_{a=0}^{J(g,a)\leq \hat{J}}}{Max} \int_0^J \Big[e^{ga}(n^T(\cdot)+\gamma n^P(\cdot))^\alpha - (w^T n^T(\cdot) + w^P n^P(\cdot)) - \frac{c}{m(\theta)}[n^T(\cdot)+h(\cdot)] - h(\cdot)\tau_t - f(\cdot)\tau_f\Big] da - n^P(\cdot,J)\tau_f - c_f J$$

$$\dot{n}^P = h(g,a) - f(g,a)$$
$$J(g,a) \leq \hat{J}$$
$$h(g,a), f(g,a), n^T(g,a), n^P(g,a) \geq 0$$
$$n^P(g,0) = 0.$$

Under the assumption that firms have all the bargaining power both wages, for temporary and permanent workers, are equal to the unemployment benefit, $w^T = w^P = b$. Therefore, the Hamiltonian associated to the above optimal control problem is

$$H(g,n^P(\cdot),n^T(\cdot),h(\cdot),f(\cdot),\lambda(\cdot)) \equiv \big[e^{ga}(n^T(\cdot)+\gamma n^P(\cdot))^\alpha - b\left(n^T(\cdot)+n^P(\cdot)\right) - c_v\left(n^T(\cdot)+h(\cdot)\right) - h(\cdot)\tau_t - f(\cdot)\tau_f\big] - \tau_f n^P(\cdot,J,\cdot) - c_f J + \lambda(g,J,n^p)\left[h(\cdot)-f(\cdot)\right]$$

The Pontryagan's Maximum Principle implies that the necessary and sufficient conditions for an optimal solution to the above problem are given by the control equations (to save space, we do not put the states of the control variables, except for λ):

$$\frac{\partial H}{\partial n^T} = e^{gt}\alpha(n^T + \gamma n^P)^{\alpha-1} - b - c_v \leq 0 \text{ with } = \text{ if } n^T > 0, \tag{8}$$

$$\frac{\partial H}{\partial h} = -c_v - \tau_t + \lambda(g,a) \leq 0 \text{ with } = \text{ if } h > 0, \tag{9}$$

$$\frac{\partial H}{\partial f} = -\tau_f - \lambda(g,a) \leq 0 \text{ with } = \text{ if } f > 0, \tag{10}$$

the multiplier equation

$$\frac{\partial H}{\partial n^P} = e^{gt}\gamma\alpha(n^T + \gamma n^P)^{\alpha-1} - b = -\dot{\lambda}(g,a) \tag{11}$$

and the state equation

$$\frac{\partial H}{\partial \lambda} = \dot{n}^P \Rightarrow \dot{n} = h - f, \tag{12}$$

and the transversality conditions

$$\begin{aligned} 0 &\leq e^{gJ}(n^T(g,J) + \gamma n^P(g,J))^{\alpha} - b\left[n^P(g,J) + n^T(g,J)\right] \\ &\quad -c_v\left[n^T(g,J) + h(g,J)\right] - \tau_t h(g,J) - \tau_f f(g,J) - c_f \end{aligned} \tag{13}$$

$$\text{with } = \text{ if } J < \hat{J},$$

$$\lambda(g,J) = -\tau_f \tag{14}$$

The optimality conditions can easily be interpreted. Equation (8) states that when firms hire temporary workers, they equate the marginal product of temporary workers to the cost of hiring temporary workers (the wage rate plus the vacancy cost). Firms that do not hire temporary workers exhibit a marginal product of temporary workers below their hiring cost, thereby equation (8) holding with inequality. The co-state variable λ represents the shadow value of a permanent worker. Firms hiring permanent workers, equate the marginal value of permanent workers to the sum of recruiting and training costs (see (9)). The shadow value of a permanent worker decreases over time ($\dot{\lambda} < 0$). Intuitively, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. The transversality condition (14) states that at the end of the match, the value of a permanent worker is equal to $-\tau_f$. The firing decision (10) ensures that the value of a permanent worker cannot decrease below $-\tau_f$. Equation (11) and $\dot{\lambda} < 0$ imply that the marginal product of permanent workers is above the wage (b) paid to permanent workers. Firms exit at the maximum possible age ($\hat{J}$) when profits are positive at the end of the life cycle. Otherwise, firms exit when profits become equal to zero (see equation (13)). As we shall see, firms exit at $J < \hat{J}$ only if $g < 0$.

2.4.1 Characterizing firms' decisions.

In this section we show how the model works by providing a characterization of the optimal composition of the labor force for each firm's type as well as the optimal age at exit. Besides, we show that there is a particular productivity growth rate $g^* < 0$ below which firms only use temporary labor contracts over their life cycle and above which firms also employ permanent workers and exit at age $\widehat{J}$. In addition we analyze the behavior of firms with zero productivity growth rate (that just hire permanent workers). Once we show how the model works, in the following sections we analyze the general equilibrium effects that changes in the labor market regulations have on the size distribution of firms and aggregate productivity.

We have assumed that the production function is of the form

$$f(n^T + \gamma^P) = (n^T + \gamma n^P)^\alpha. \tag{15}$$

Let's make the following two assumptions:

$$\textit{Assumption 1.} \qquad (1-\alpha)\left[\frac{\alpha}{b+c_v}\right]^{\frac{\alpha}{1-\alpha}} > c_f$$

$$\textit{Assumption 2.} \qquad \hat{J} > \frac{c_v + \tau_t + \tau_f}{(b+c_v)\gamma - b}$$

Assumption 1 ensures that firms are profitable at age 0 so that the model economy features a non-trivial equilibrium with production. In characterizing the behavior of firms, we find it convenient to partition firms in two groups depending on whether they ever hire permanent workers or not. The first group is comprised by firms whose rate of growth g is higher than the threshold value g^*:

$$g^* \equiv (1-\alpha)\frac{(b+c_v)\gamma - b}{c_v + \tau_t + \tau_f} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0 \tag{16}$$

where $g^* < 0$ follows from Assumption 1. On the other hand, firms with low productivity growth ($g < g^*$) do not hire permanent workers.

Using the fact that permanent workers are only hired if their marginal product is above the wage rate (b), (11) implies that the shadow value of permanent workers decreases with the age of the firm,

$$\gamma e^{gt}\alpha(n^T + \gamma n^P)^{\alpha-1} - b = -\dot{\lambda} \geq 0 \Rightarrow \dot{\lambda} \leq 0. \tag{17}$$

The declining value of permanent workers implies that if a firm does not hire permanent workers at age 0, it will not do it at a later age ($n(g,0)^P = 0 \Rightarrow n(g,a)^P = 0$ for all a). Whether a firm finds it profitable to hire a permanent worker at age 0, depends on its expected lifetime at birth. Intuitively, if the expected lifetime is long enough, the firm can recoup the fixed cost of hiring a permanent worker. The expected life of a firm at birth is (weakly) increasing in its rate of productivity growth (g). Assumption 2 implies that firms with positive productivity growth have incentives to hire permanent workers at age 0 ($n^P(g,0)) > 0$).

Case I: Firms with low productivity growth ($g < g^*$). As it was explained in previous sections, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. In particular, there are firms with such a low productivity growth rate that find it optimal to exit relatively soon, thus they do not hire permanent workers at all. This is the case we analyze here. We start by solving the optimization problem under the assumption that $n^P(g,0) = 0$. We then find restrictions in the parameter space so that the optimal solution has this property.

The optimal amount of temporary workers follows from (8):

$$n^T(g,a) = \left[\frac{\alpha e^{ga}}{b + c_v}\right]^{\frac{1}{1-\alpha}}. \tag{18}$$

Operating profits at age a satisfy:

$$\begin{aligned} \pi(g,a) &= e^{gt} f(n^T(g,a)) - (b + c_v) n^T(g,a) - c_f, & (19) \\ &= (1-\alpha) e^{\frac{ga}{1-\alpha}} \left[\frac{\alpha}{b + c_v}\right]^{\frac{\alpha}{1-\alpha}} - c_f. & (20) \end{aligned}$$

Using (20) it follows that firms make positive profits at age 0 ($\pi(g,0) > 0$) if Assumption 1 holds.

The firm shuts down if $\pi(J) = 0$ for some $J < \hat{J}$. Solving for J we obtain the age of exit as a function of productivity growth (g):

$$J(g) \equiv \frac{(1-\alpha)}{g} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b + c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right]. \tag{21}$$

The value of a permanent worker at age J is equal to $-\tau_f$ (see equation (14)). Using equations (8) and (11), we can obtain an expression for $\lambda(g,a)$:

$$\lambda(g,a) = \lambda(g,J) - \int_a^J \dot{\lambda} dj \tag{22}$$
$$= -\tau_f + [(b+c_v)\gamma - b](J-a) \tag{23}$$

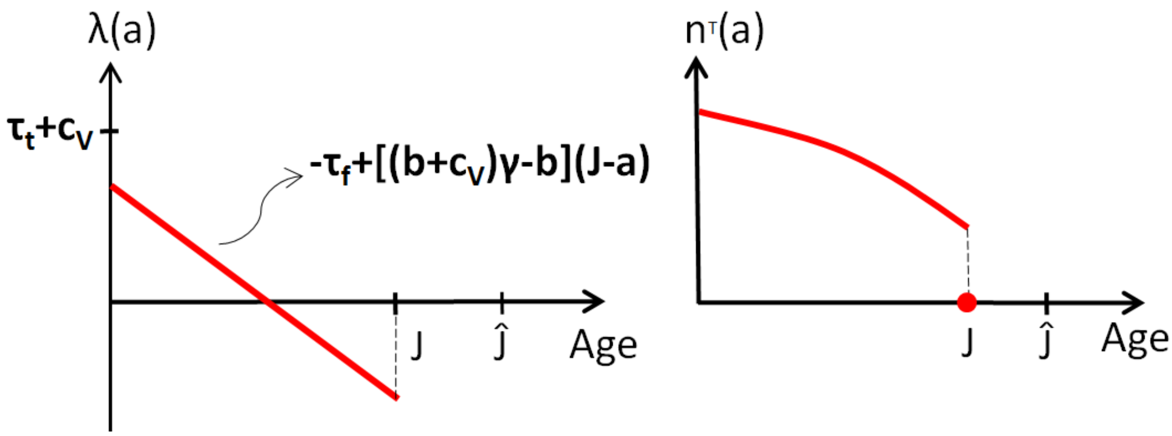
Note that when $\lambda(g,0) < c_v + \tau_t$ (9) implies that $h(g,0) = 0$ so that the initial assumption of $n^P(g,0) = 0$ holds true. Using (23) it follows that $\lambda(g,0) < c_v + \tau_t$ holds if

$$J < \frac{c_v + \tau_t + \tau_f}{(b+c_v)\gamma - b} \tag{24}$$

Note that the condition in equation (24) is fairly easy to interpret when $\gamma = 1$. In this case, firms do not hire permanent workers at age 0 when the the optimal exit age is such that $J < \frac{c_v+\tau_t+\tau_f}{c_v}$, which implies that the cost of hiring one temporary worker for J periods (c_vJ) is lower than the cost of hiring one permanent worker ($c_v + \tau_t + \tau_f$). Obviously, this condition will be violated when the optimal exit age J is large enough.

Similar interpretation follows when $\gamma > 1$. Firms do not hire permanent workers at age 0 when the cost of hiring one temporary worker for J periods ($J(b+c_v)$) is lower than the cost of hiring one effective permanent worker for J periods ($\frac{c_v+\tau_t+\tau_f+Jb}{\gamma}$). Figure 6 presents the evolution of the shadow value of a permanent worker as well as the dynamics of employment for temporary workers. Since the optimal path value for hiring, firing and employment for permanent workers is zero, we do not plot them.

Figure 6: Dynamics of $\lambda(g,a)$ and $n^T(g,a)$ for $g < g^*$.



Combining (21) and (24) we find that firms do not hire permanent workers when the rate of productivity growth (g) is such that:

$$\frac{(1-\alpha)}{g} ln \left[\frac{c_f}{1-\alpha} \left(\frac{b+c_v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \right] < \frac{c_v + \tau_t + \tau_f}{(b+c_v)\gamma - b} \tag{25}$$

Rewriting the last expression and using the fact that $g < 0$ we obtain[13]

$$g < g^* \equiv (1-\alpha) \frac{(b+c_v)\gamma - b}{c_v + \tau_t + \tau_f} ln \left[\frac{c_f}{1-\alpha} \left(\frac{b+c_v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \right] \tag{26}$$

Hence, we have shown that firms with $g < g^*$ do not hire permanent workers. Moreover, Assumption 2 and equation (21 imply that firms with productivity g^* exit before the terminal period since $J(g^*) < \hat{J}$ and the optimal age at exit $J(g)$ increases with the rate of productivity growth.

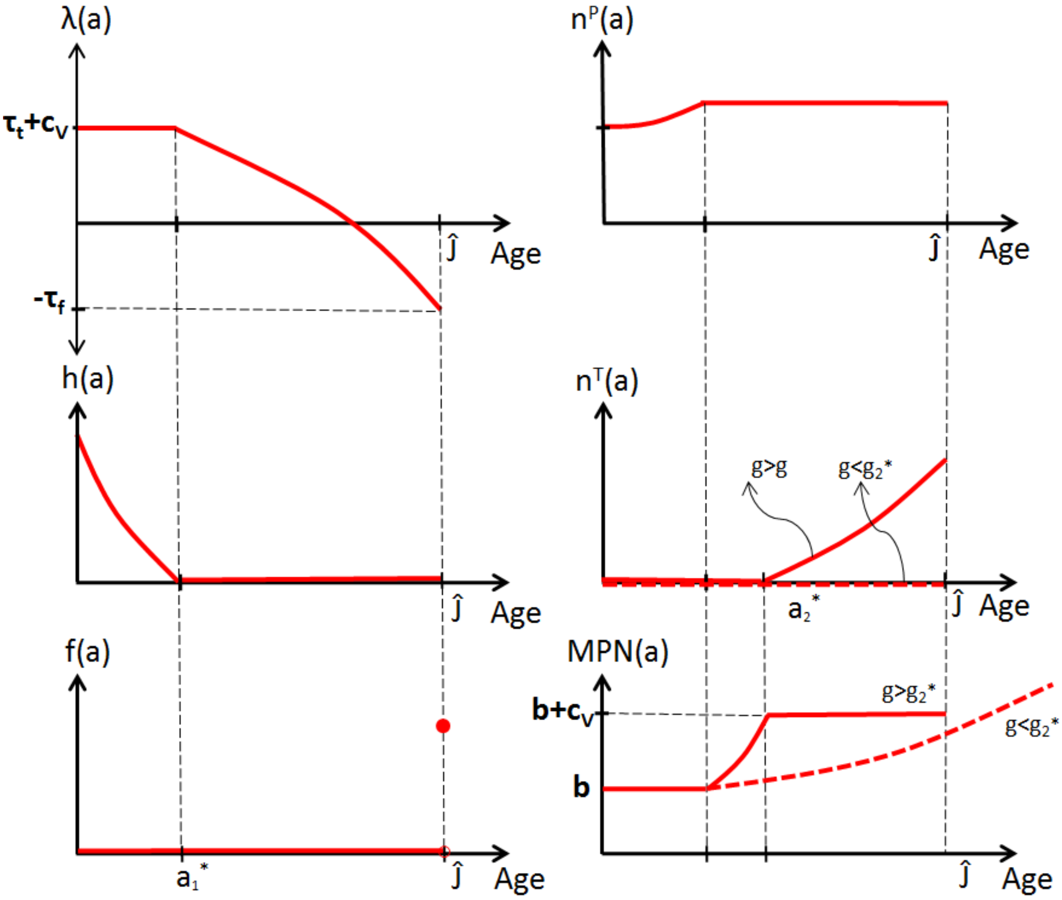
Case II: Firms with high productivity growth $(g > g^*)$**.** In this case we consider firms whose productivity growth is sufficiently high enough, even though it could be negative, to make them profitable to hire permanent workers at age 0 ($n^P(g, 0) > 0$). These are firms that last longer in the marker than the ones in Case I. These firms can further be partitioned in two groups: Those who only hire permanent workers at age 0 and those who hire permanent workers for a period of time after birth of the firm. The second group of firms are those with $g > 0$.

Case II-A: $g > 0$. Assumption 1 ensures that $\pi(g, 0) > 0$ and $g > 0$ implies that $\pi(g, a) > 0$ for all a. As a result, the optimal exit time satisfies $J = \hat{J}$. Note that equation (9) implies that firms hire permanent workers at age a if $\lambda(g, a) = c_v + \tau_t$. Now, since $\lambda(g, a)$ is a continuous function and $\lambda(g, J) = -\tau_f$ we know that there exist some $a_1^* < J$ such that firms do not hire permanent workers for all $a \geq a_1^*$. Intuitively, it is not optimal to incur the fixed costs of hiring and training permanent workers when the age of firms is sufficiently close to the age at which they exit the industry. In figure 7 it can be noticed that firms hire permanent workers until age a_1^*, and since then on the employment of permanent workers remains constant until the firm exit the industry. At that point in

[13] Assumptions A2 implies that firms with $g > 0$ will find it optimal to hire permanent workers at age 0.

time the firing policy for permanent workers becomes positive.

Figure 7: Dynamics of main variables for $g > 0$.



Now, notice that (9) implies

$$h(g,a) > 0 \Rightarrow \lambda(g,a) = c_v + \tau_t \Rightarrow \dot{\lambda} = 0. \tag{27}$$

Substituting $\dot{\lambda} = 0$ in (11), we obtain that the marginal product of permanent workers at age a is equal to b. Then, the first order condition on temporary workers (8) holds with strict inequality so that $n^T(g,a) = 0$ for all $a \leq a_1^*$. Setting $\dot{\lambda} = 0$ and $n^T(g,a) = 0$ in (11), we obtain

$$n^P(g,a) = \begin{cases} \left[\frac{\alpha}{b}\gamma^\alpha e^{ga}\right]^{\frac{1}{1-\alpha}} & \text{for } a \in [0, a_1^*], \\ n^{P*} = \left[\frac{\alpha}{b}\gamma^\alpha e^{ga_1^*}\right]^{\frac{1}{1-\alpha}} & \text{for } a \in [a_1^*, J]. \end{cases} \tag{28}$$

For $a \in [a_1^*, J]$ we have $h(g,a) = 0$ so that (9) holds with inequality. After age $a > a_1^*$, the marginal product of labor rises at a rate g until it reaches the value of $c_v + b$. Let's denote by a_2^* the age at which the marginal product of labor becomes equal to $b + c_v$. At this age, it becomes profitable to hire temporary workers and the FOC with respect to temporary workers hold with equality. The threshold age a_2^* is obtained from

$$a_2^* \equiv \frac{1}{g} ln \left[\frac{c_v + b}{\alpha} \left(\gamma n^{p*} \right)^{1-\alpha} \right]. \tag{29}$$

Defining the threshold growth rate of productivity

$$g_2^* \equiv \frac{1}{\hat{J}} ln \left[\frac{c_v + b}{\alpha} \left(\gamma n^{p*} \right)^{1-\alpha} \right], \tag{30}$$

it follows that firms with productivity growth $g < g_2^*$ do not hire temporary workers ($n^T(g,a) = 0$ for all a) since $a_2^* > \hat{J}$ so that they do not reach age a_2^*. Firms with $g > g_2^*$ live beyond age a_2^* and hire temporary workers during the period $a \in [a_2^*, \hat{J}]$. In Figure 8, the last two panels on the right, the optimal path for the hirings of temporary workers, $n^T(a)$, and the marginal product of labor are indicated by the dashed lines (case in which $g < g_2^*$), while the solid lines indicate the case for firms with $g > g_2^*$. For the case of firms with $g > g_2^*$, the optimal amount of temporary workers $n^T(g,a)$ is obtained by solving (8) with equality and setting $n^P(g,a) = n^{P*}$.

We close the characterization of the firm problem for firms with $g > 0$, by showing how to obtain the threshold age a_1^* (the values of a_2^* and n^{P*} are expressed in terms of a_1^*). To this end, we use

$$-\tau_f = \lambda(g, J) = \lambda(g, 0) + \int_J^0 \dot{\lambda} da \tag{31}$$

If the firm does not hire temporary workers during its life cycle ($g < g_2^*$ so that $a_2^* > \hat{J}$), a_1^* is obtained by solving the following equation

$$\begin{aligned} -\tau_f &= c_v + \tau_t + \int_0^{a_1^*} 0 dt + \int_{a_1^*}^{J} (b - e^{gt} \alpha\gamma \left(\gamma n^{P*} \right)^{\alpha-1}) dt & (32) \\ &= c_v + \tau_t + b(J - a_1^*) - \alpha\gamma \left(\gamma n^{P*} \right)^{\alpha-1} \frac{1}{g} \left(e^{gJ} - e^{g a_1^*} \right), & (33) \end{aligned}$$

where we have used (11), (31), and the fact that $\lambda(g,a)$ is constant while $h(g,a) > 0$.

If the firm hires temporary workers at the end of its life cycle ($g > g_2^*$ so that $a_2^* < \hat{J}$),

a_1^* is obtained by solving the following equation

$$-\tau_f = c_v + \tau_t + \int_0^{a_1^*} 0dt + \int_{a_1^*}^{a_2^*} (b - e^{gt}\alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1})dt \tag{34}$$

$$+ \int_{a_2^*}^{\hat{J}} (b - e^{gt}\alpha\gamma \left(\gamma n^{P*} + n^T(g,t)\right)^{\alpha-1})dt \tag{35}$$

$$= c_v + \tau_t + b(a_2^* - a_1^*) - \alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1} \frac{1}{g} \left(e^{ga_2^*} - e^{ga_1^*}\right) \tag{36}$$

$$+b(\hat{J} - a_2^*) - [\gamma(b + c_v) - b] (\hat{J} - a_2^*). \tag{37}$$

where we have used (8), (11), (31), and the fact that $\lambda(g, a)$ is constant while $h(g, a) > 0$.

Case II-B: $g = 0$. Firms with $g = 0$ hire permanent workers at age 0 and then do not hire any additional worker (neither temporary nor permanent). Assumption 1 implies that these firms exit at the terminal period ($J = \hat{J}$). Equation (9) implies that $\lambda(g, 0) = c_v + \tau_t$. For $n^P(g, a) = n^P(g, 0)$ for all a, equation (11) requires that $\dot{\lambda} = \bar{\lambda}$ for some constant $\bar{\lambda} < 0$. We then have

$$\lambda(g, a) = \lambda(g, 0) + \int_0^a \dot{\lambda}(g, t)dt = \lambda(0) + \int_0^a \bar{\lambda}dt \tag{38}$$

$$= c_v + \tau_t + a\bar{\lambda} \tag{39}$$

Using the transversality condition $\lambda(g, \hat{J}) = -\tau_f$, we obtain $\bar{\lambda} = -\frac{c_v+\tau_t+\tau_f}{J}$. Equation (11) at time 0 implies that the marginal product of labor with respect to permanent workers is such that

$$\gamma^\alpha \alpha n^P(g, 0)^{\alpha-1} = b + \frac{c_v + \tau_t + \tau_f}{\hat{J}}. \tag{40}$$

Since firms do not hire temporary workers, the FOC with respect to temporary workers at time 0 (equation 8) implies that

$$\alpha n^P(g, 0)^{\alpha-1} < b + c_v. \tag{41}$$

Combining (40) and (41), it is optimal for firms with $g = 0$ to hire permanent workers at time 0 but not temporary workers when the following parameter restriction applies

$$\frac{1}{\gamma^\alpha} \left(b + \frac{c_v + \tau_t + \tau_f}{\hat{J}}\right) < b + c_v,$$

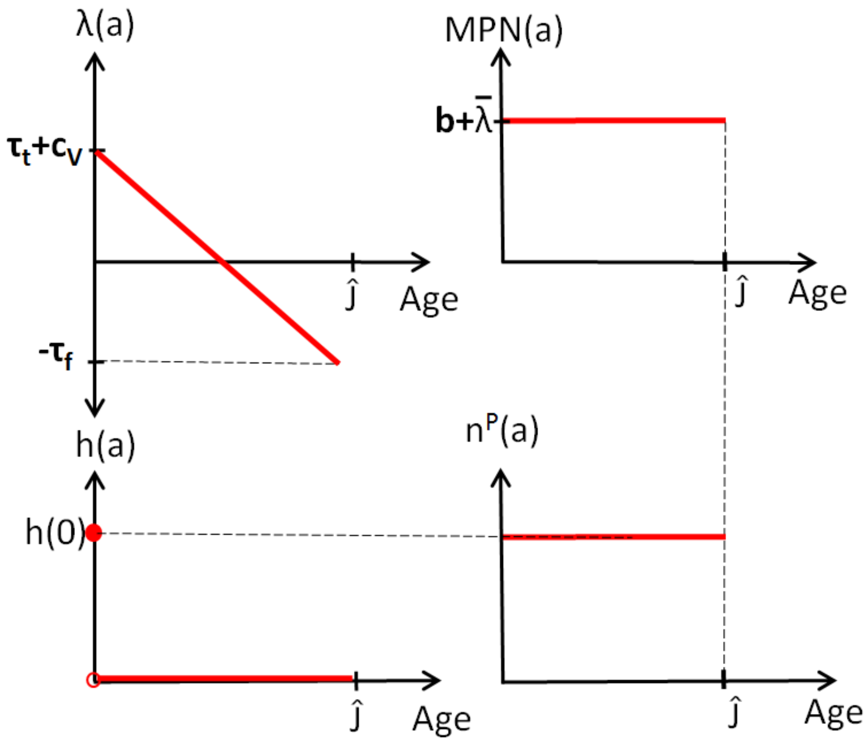
which holds true under Assumption 2.

Using (40) to solve for $n^P(g,0)$ we obtain

$$n^P(g,0) = \left(\frac{\gamma^\alpha \alpha J}{b\hat{J} + c_v + \tau_t + \tau_f} \right)^{\frac{1}{1-\alpha}}. \tag{42}$$

Figure 8 presents the optimal path for the shadow value of hiring permanent workers, the hirings and total employment of permanent workers, and its marginal productivity.

Figure 8: Dynamics of main variables for $g = 0$.



Case II-C: $g^* < g < 0$. Firms with productivity growth rate $g^* < g < 0$ hire permanent workers in period 0 and exit at age $J < \hat{J}$. From equation (9) we have that the shadow value of permanent workers is such that $\lambda_0 = c_v + \tau_t$. Permanent employment remains constant up to a period time in which the growth rate declines enough so as to induce the firm to begin firing permanent workers. Formally, $n^P(g,a) = n^P(g,0)$ for $t \in [0, a_f^*]$, for some $a_f^* > 0$. The shadow value of permanent workers decrease and may reach the value of $-\tau_f$ at age $a_f < J$ (see Figure 9). To put it differently, firms start firing workers before the age of exit and there is a period at the end of the life cycle in which the FOC (10) holds with equality ($f(g,a) > 0$ for $a \in [a_f, J]$). Formally, define $a_f \equiv min_a \lambda(a) = -\tau_f$. Note that a_f is the first age at which $\lambda(g,a) = -\tau_f$.

Firms start firing workers before exiting. For $a \geq a_f$,

$$f(g,a) > 0 \Rightarrow \lambda(g,a) = -\tau_f \Rightarrow \dot{\lambda} = 0. \tag{43}$$

Using (11) we have that permanent employment, before firings take place, is given by

$$n^P(g,a) = \left[\frac{\alpha\gamma^\alpha e^{ga}}{b}\right]^{\frac{1}{1-\alpha}} \text{ if } a \geq a_f \tag{44}$$

The optimal age to exit is obtained by solving for J the following equation

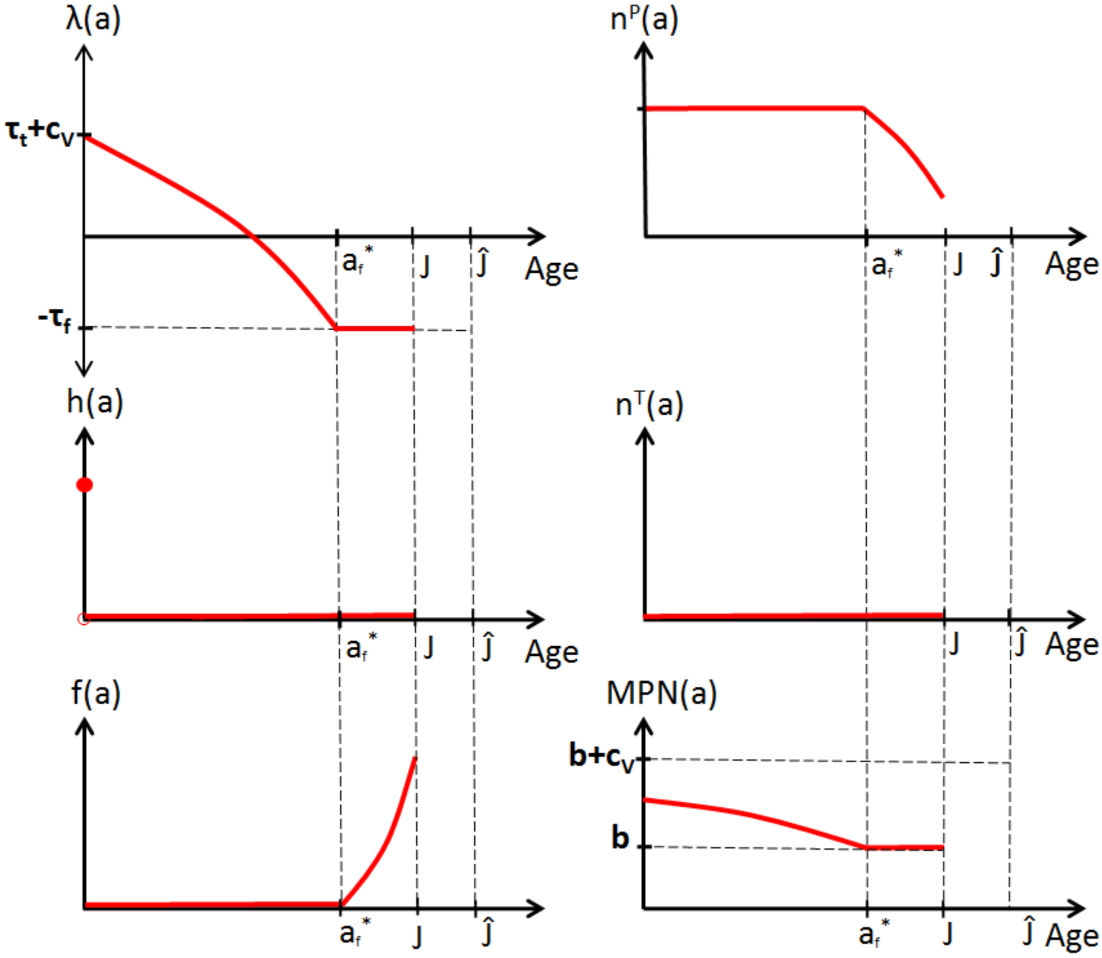
$$\pi(g,J) = e^{gJ}\left[\gamma n^P(g,J)\right]^\alpha - bn^P(g,J) - c_f = 0, \tag{45}$$

where $n^P(g,J)$ is obtained from (44). The value of a_f is obtained from

$$\begin{aligned} -\tau_f &= \lambda(g,a_f) = \lambda(g,0) + \int_0^{a_f} \dot{\lambda} dt & (46)\\ &= c_v + \tau_t + \int_0^{a_f} \left(b - e^{gt}\alpha\gamma\left[\gamma n^P(g,a_f)\right]^{\alpha-1}\right) dt & (47)\\ &= c_v + \tau_t + ba_f - \alpha\gamma\left[\gamma n^P(g,a_f)\right]^{\alpha-1}\frac{e^{ga_f}-1}{g}, & (48) \end{aligned}$$

where $n^P(g,a_f)$ is obtained from (44). Figure 9 shows the dynamics of all relevant variables. The shadow value of permanent workers is positive when the firm is born and then declines over time, as the marginal product of labor decreases, while the employment of permanent workers remains constant up to age a_f. At that point in time the firm starts firing permanent workers and, the marginal product of labor remains constant and the firm reduces its size up to the optimal age to exit, J.

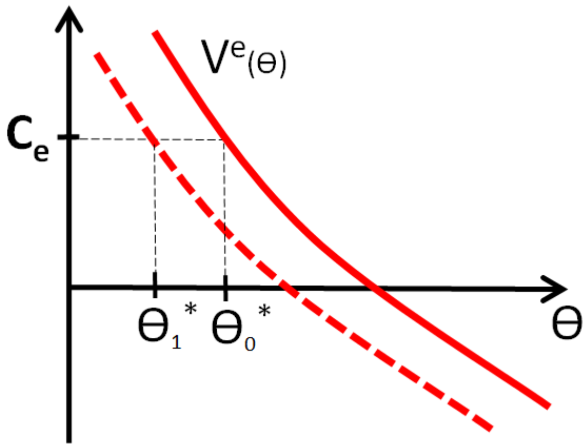
Figure 9: Dynamics of main variables for $g^* < g < 0$.



2.5 Analyzing the impact of labor market institutions

In this section we analyze and characterize the effect that an increase in the firing costs of permanent workers have on firm's selection and the employment of temporary and permanent workers of firms with different productivity levels affecting the equilibrium size distribution of firms and aggregate productivity. Formally, following the solution algorithm steps explained in the appendix 4.2, an increase in the firing cost of permanent workers, τ_f, generates a decline in the equilibrium labor market tightness (see Figure 10). This is, as τ_f increases, the expected future discounted profits of a new firm decreases (at the initial labor market tightness, θ_0^*). In Figure 10 this is reflected by a downward shift in the curve representing the expected value at entry. To match the entry cost, C_e, discounted profits need to be higher. Since the expected value at entry, $V_e(\theta)$, is strictly decreasing in θ, this implies that the equilibrium labor market tightness decreases.[14] We denote the new value for the labor market tightness as θ_1^*.

Figure 10: Effect on the equilibrium θ^*.



The intuition behind the previous result is the following. An increase in the firing costs to permanent workers reduces profits (in particular, penalizing more to firms with high productivity growth rate which concentrates a big fraction of employment), inducing to less vacancy posting. Therefore, the labor market tightness declines up to a point in which the value at entry matches again the cost of entry ($V_e(\theta_1^*) = C_e$). As a result, in the new situation there are less vacancies relative to the unemployed workers in the economy.

Furthermore, recall that firms with $g < g^*$ do not hire permanent workers, where

$$g^* \equiv (1-\alpha)\frac{(b+c_v)\gamma - b}{c_v + \tau_t + \tau_f} ln \left[\frac{c_f}{1-\alpha} \left(\frac{b+c_v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \right] < 0. \tag{49}$$

As mentioned before, note that Assumption 1 implies that $g^* < 0$ since $ln \left[\frac{c_f}{1-\alpha} \left(\frac{b+c_v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \right] < 0$. It then follows that an increase in τ_f has two effects on g^* that go in the same direction (which are analyzed in detail later on):

a) Partial equilibrium effect: $\frac{\partial g^*}{\partial \tau_f} > 0$. The threshold value of productivity growth at which firms start hiring permanent workers increase. Hence, an increase in firing costs reduces the mass of businesses hiring permanent workers.

[14] See the solution method in the appendix for further details.

b) General equilibrium effect I: An increase in τ_f reduces profits, inducing to less vacancy posting, and a rise in the unemployment to vacancy ratio. In turn, this increases the probability that a firm matches with a worker (reduces c_v for a fixed vacancy cost c, this is $c_v = c/m(\theta)$ decreases). Thus, we have that $\frac{\partial c_v}{\partial \tau_f} < 0$ and $\frac{\partial g^*}{\partial c_v} < 0$ so that the general equilibrium effect of an increase in τ_f is to increase g^*.

These two forces give incentives to high productivity firms to reduce their size. Thus, both effects act on the intensive margin. In addition to the previous effects there is another general equilibrium force that induces more distortions in the economy. This additional mechanism has an impact on the extensive margin:

c) General equilibrium effect II: Exit. In general equilibrium, an increase in τ_f subsidizes firms with low growth by reducing the costs of filling up temporary jobs. This subsidy distorts the exit decision of firms by encouraging the hiring of temporary workers. The age of shutdown of low growth firms increases (exit margin). On the other hand, the age of shutdown of high growth firms decreases (exit margin).[15]

In what follows, we show that as τ_f increases firms with low productivity growth rates $g < g^*$ expand, but still employ only temporary workers, and live longer. Firms with intermediate productivity growth rates, $g^* < g < 0$, contract and exit earlier. Firms with zero productivity growth contract, and finally the most productive firms, with $g > 0$, contract. As a result, higher firing costs for permanent workers penalizes relatively more to firms with high productivity growth and subsidizes firms with low productivity growth, shifting employment from the first ones, which contract and last shorter in the market, to the second ones, which expand and last longer in the market. A higher relative firing costs for permanent workers play similar role as a size-dependent-policy by distorting firm selection as well as the allocation of resources across firms implying changes in the size distribution of firms and resulting in a lower aggregate productivity.

We now analyze formally the effect of an increase in firing costs to permanent workers in detail. We organize the analysis by focusing on the effects on firms with different

[15] Our model economy assumes that all businesses start with the same productivity and Assumption 1 ensures that businesses are profitable at age 0. Now, if we assume that businesses also differ in terms of their initial productivity then some businesses might not be profitable at age 0. In this case, the hiring subsidy of temporary workers (induced by τ_f) will also distort the entry margin by encouraging entry of businesses with low initial productivity. This extension is left for future research.

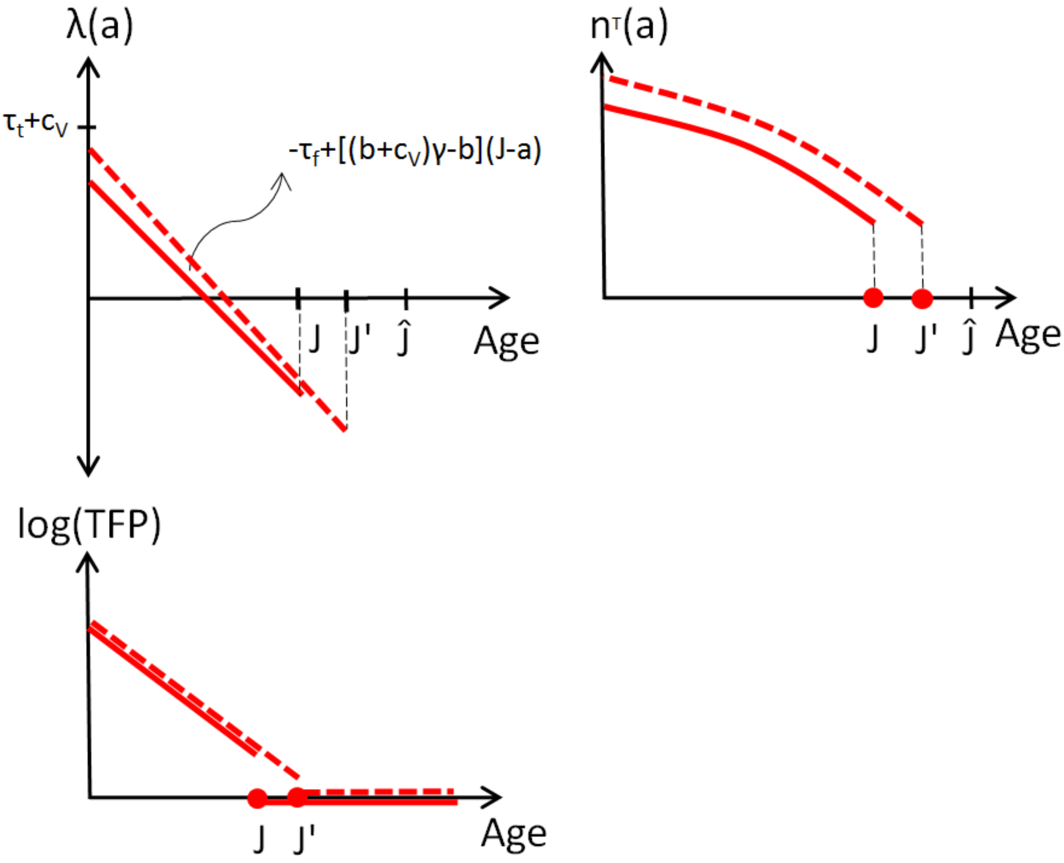
productivity growth rates. The derivations of all these cases follow the characterization of firms' optimal decisions made in the previous sections.

Case I. We first focus on firms with low productivity growth. When there is an increase in τ_f it can be seen from equation (23) that the slope of $\lambda(g,a)$ increases while change in the intersection with the vertical axis is ambiguous. Figure 11 documents the dynamics of the main relevant variables for the case of firms with $g < g^*$. From equation (21) we have that inefficient firms lasts longer in the market,

$$\frac{\partial J}{\partial \tau_f} = \frac{\alpha \frac{\partial c_v}{\partial \tau_f}}{(b + c_v)g} > 0.$$

In addition, from equation (18) it can be noticed that firms with productivity growth rates $g < g^*$ employ more temporary workers than before (see Figure 11). As these firms last longer in the market their total factor productivity declines further than before the increase in τ_f.

Figure 11: Dynamics of main variables for $g < g^*$.

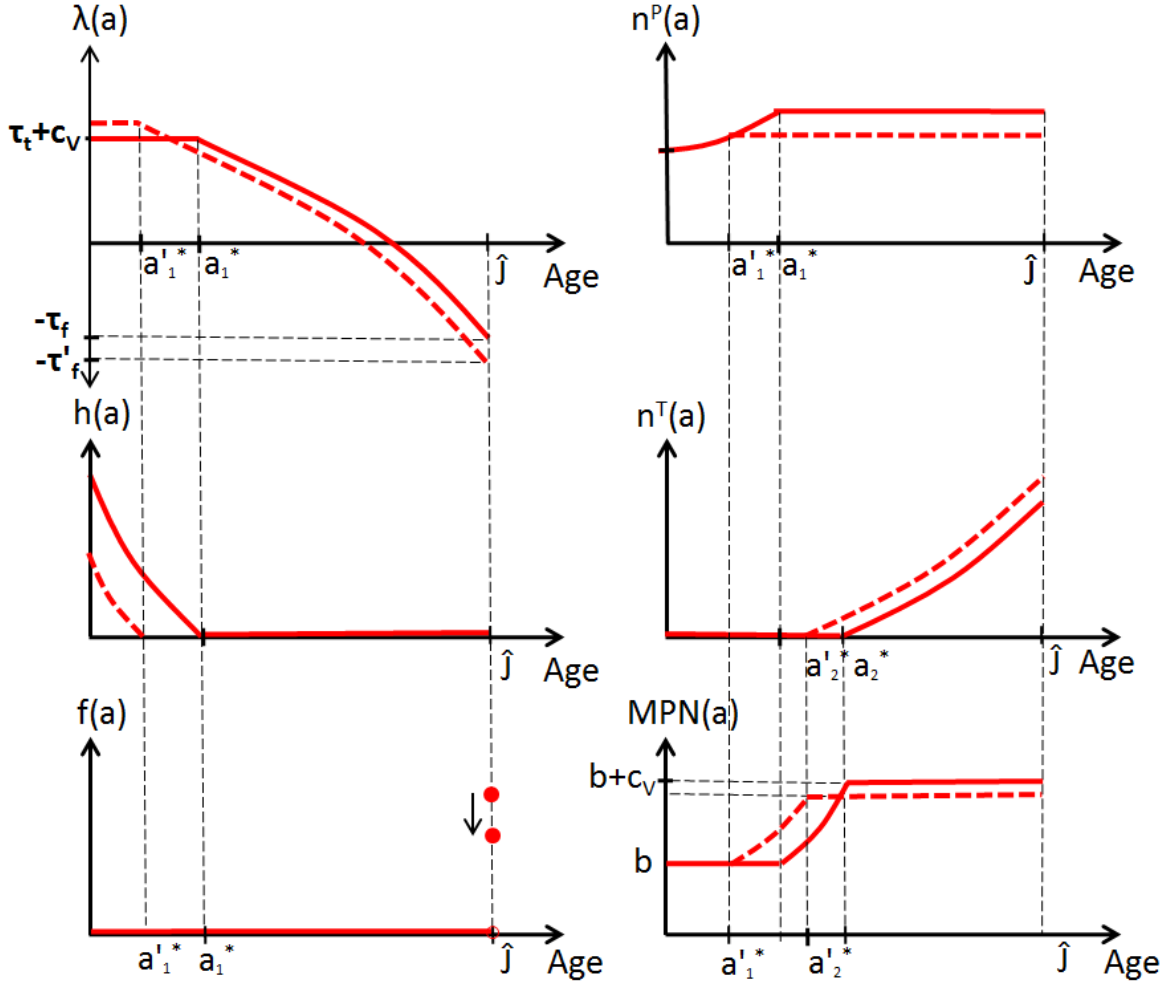


Case II-A. Figure 12 presents the comparative statics results for the case of firms with positive productivity growth rates. Since these firms make positive profits at the beginning of their life-cycle their duration in the market does not change ($\frac{\partial J}{\partial \tau_f} = 0$). An increase in τ_f generates a decline in the LHS of equation (33) and (34), which determines the threshold a_1^*. Therefore, the RHS has to decrease to restore the equality, and since $\frac{\partial n^P(g,a)}{\partial a_1^*} > 0$, thus a_1^* decreases (in Figure 12). The new age threshold is denoted by $a_1^{*\prime}$, and the new firing cost is denoted by τ_f' . Moreover, equations (28), (29), and (30) indicate that firms reduce the employment of permanent workers, $n^P(g,a)$ and $n^P(g,a_1^*)$ (which is reflected by a downward shift in the $n^P(g,a)-$curve in Figure 12), and they stop recruiting permanent workers sooner than before and start hiring temporary workers faster, as also a_2^* and g_2^* decline (notice that $\frac{\partial a_2^*}{\partial a_1^*} > 0$). In addition, as the following expression indicates,

$$n^T(g,a) = \left[\frac{\alpha e^{ga}}{b + c_v}\right]^{\frac{1}{1-\alpha}} - n^P(g,a_1^*),$$

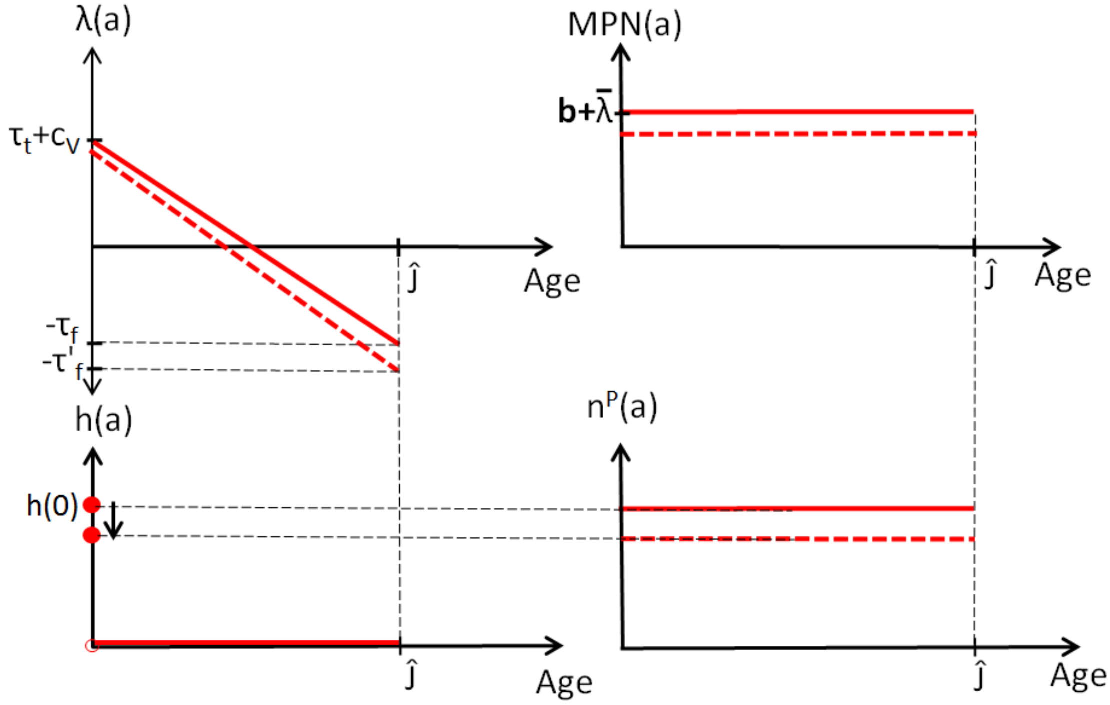
firms increase the employment of temporary workers. This is reflected by an upward shift in the $n^T(g,a)-$curve in Figure 12. As a result, the dynamic path for the marginal productivity of labor is lower than before the increase in the firing cost to permanent workers.

Figure 12: Dynamics of main variables for $g > 0$.



Case II-B. Firms with constant productivity stay active in the market up to period $\widehat{J}$. Since they optimally employ permanent workers to save on search costs they are penalized by the increase in the firing costs τ_f. Thus, they reduce the employment level of permanent workers. Formally, when τ_f increases, the lower labor market tightness induces lower search costs. Thus, equation (39) shows that the shadow value of permanent workers decreases for every point in time. From expression (42) it is clear that the level of permanent employment decreases (see Figure 13).

Figure 13: Dynamics of main variables for $g = 0$.

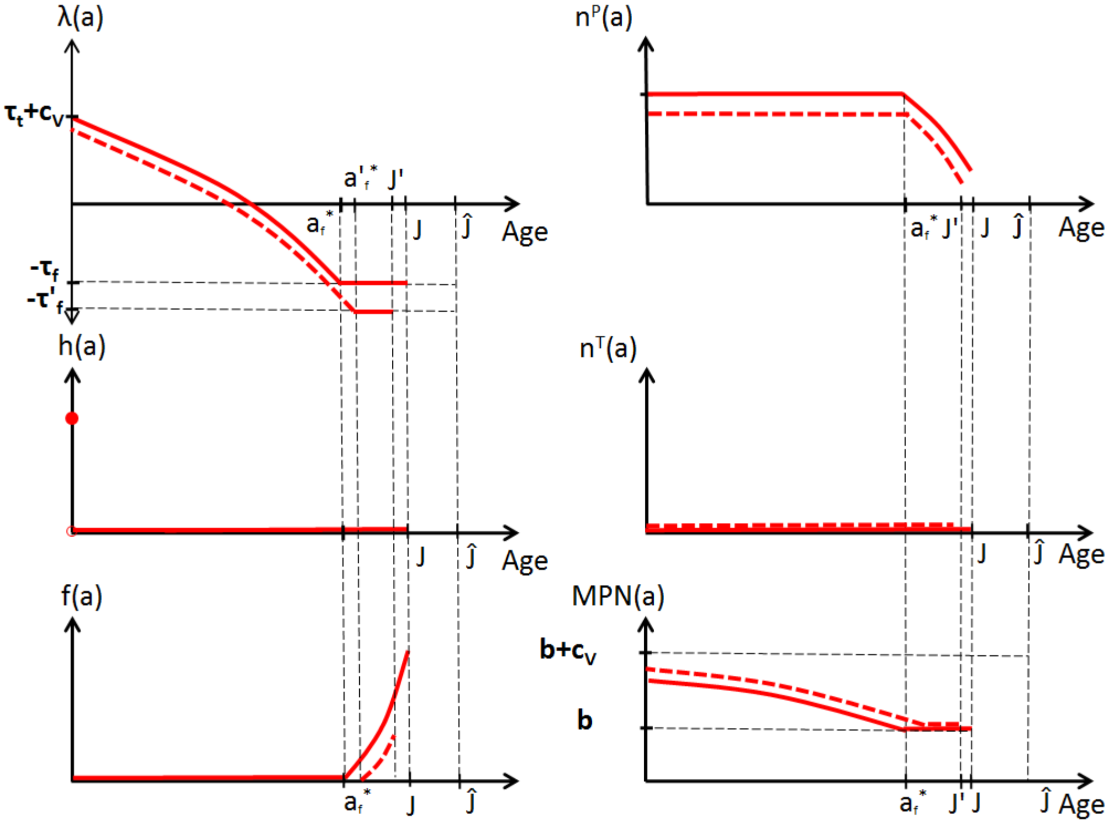


Case II-C. Firms with productivity growth $g^* < g < 0$ also employ permanent workers and thus are also directly affected by an increase in the firing costs to permanent workers. They hire fewer workers than before and exit the industry sooner than before the increase in τ_f. Formally, from equation (48), to match a higher τ_f the optimal age at which the firm starts to fire permanent workers (a_f^*) increases. Notice that, $n^P(g,a) = n^P(g,0)$ for $t \in [0, a_f^*]$. Since the permanent employment, before firings take place, is given by

$$n^P(g,a) = \left[\frac{\alpha\gamma^\alpha e^{ga}}{b}\right]^{\frac{1}{1-\alpha}} \text{ if } a \geq a_f,$$

the number of permanent workers a firm hires is a negative function of the age a_f^*. Therefore the number of permanent workers decreases. Since the shadow value of permanent workers is such that $\lambda_0 = c_v + \tau_t$, and then decreases over time, we have that the new path for $\lambda(g,a)$ is below the previous one. As profits are lower than before for these firms, the optimal age to exit decreases. Figure 14 shows the dynamics of all relevant variables for this case.

Figure 14: Dynamics of main variables for $g^* < g < 0$.



Altogether, as mentioned before, an increase in τ_f leads to a reduction of the number of firms hiring permanent workers (extensive margin) and an increase in the number of firms hiring temporary workers. Moreover, among firms hiring permanent workers, an increase in τ_f reduces the number of permanent workers hired (intensive margin). The general equilibrium effect of an increase in firing costs τ_f implies that firms with $g < g^*$ hire more temporary workers due to the higher probability of filling a vacancy. Thus, an increase in τ_f shifts employment from permanent contracts to temporary contracts and from firms with high productivity growth rate to firms with low productivity growth rate. Total factor productivity decreases because the share of employment in highly productive firms decreases and firms spend less resources in training workers.

3 Quantitative Analysis: A calibrated discrete time model (THIS SECTION IS INCOMPLETE)

As long as it takes longer to high productivity firms to exit (they have longer expected life span) and there are search frictions, the main mechanisms, insights and results of the previous model will hold in a more realistic and richer Hopenhayn and Rogerson's (1992) style model. Moreover, there is empirical evidence suggesting that smaller firms have a lower probability of survival and younger firms have a higher probability of exiting.[16] Table 4 shows the pattern of exit rate. Conditional on size younger firms have lower survival rate, and conditional on age, bigger firms have higher survival rate.

Table 4: Firm exit rates by age and size.

	Firm Size (employees)											
Firm Age	**1 to 4**	**5 to 9**	**10 to 19**	**20 to 49**	**50 to 99**	**100 to 249**	**250 to 499**	**500 to 999**	**1000 to 2499**	**2500 to 4999**	**5000 to 9999**	**10000+**
1	37.4	9.3	7.4	6.6	7.0	9.2	8.8	14.9	5.9	0.0	0.0	66.7
2	28.6	7.4	5.3	4.4	4.2	6.6	7.7	7.9	3.1	13.3	20.0	28.6
3	25.8	6.6	4.5	3.8	4.3	3.9	4.0	4.4	0.7	7.1	0.0	0.0
4	23.8	5.6	4.5	3.2	2.5	2.3	1.4	2.9	1.9	7.8	8.7	3.9
5	22.4	5.2	3.8	3.4	3.0	3.6	4.4	8.9	3.7	1.9	2.6	20.0
6 to 10	19.1	4.1	2.9	2.6	2.1	2.0	3.5	3.3	10.2	7.9	2.9	1.4
11 to 15	16.5	3.2	2.3	2.3	2.6	2.2	3.6	4.3	6.8	3.2	9.6	6.9
16 to 20	14.9	2.8	2.0	1.9	1.7	1.8	2.3	1.8	3.2	2.6	4.6	4.5
21 to 25	13.5	2.6	1.9	1.9	2.2	2.1	3.6	2.8	4.5	4.4	4.2	2.2
26+	12.5	2.2	1.9	1.9	2.1	1.7	2.1	2.6	3.5	2.7	5.4	6.3
ALL	19.5	4.0	2.9	2.5	2.3	2.0	2.7	2.9	3.9	4.2	4.6	5.7

Source: US Census Bureau, Business Dynamics Statistics 2010.

Therefore, the data suggests that the mechanism of our paper is empirically relevant. In a more general model in which firms face mean reverting idiosyncratic productivity shocks, it takes longer for large firms to exit the industry. If persistence is very high, as documented in the data, firms with productivity shocks above the mean expects that high shocks today will be around for a long time, thus they are unlikely to exit the industry (they need to accumulate many negative shocks to abandon the industry). In contrast, for small firms (with low productivity level) few small negative productivity shocks make them exit relatively soon. In addition, if search frictions are present, it is more valuable for large firms with higher survival rate to hire permanent workers. Therefore, the insights of our paper translate into a more realistic Hopenhayn and Rogerson's (1992) style model with search frictions. In this section we develop such a model and calibrate it the US

[16] For further details see Sutton (1997), Caves (1998), Geroski (1998), Dunne, Roberts and Samuelson (1988, 1989a,b).

economy and perform quantitative analysis to evaluate how dual employment protection legislation (with coherent parameters for firing costs of permanent workers) account for differences aggregate productivity and size distributions of firms between Spain and US.

4 Conclusion and final remarks

Motivated by the fact that countries with strict employment protection legislation of permanent contracts have relatively smaller firms (that concentrate a higher fraction of total employment) and lower aggregate productivity, the current paper develops a model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers. We showed in a very stylized model that firing costs of permanent workers act as size-dependent-policies. Stricter DEPL distorts firm selection as well as the allocation of resources across firms.

5 Appendix

5.1 Derivation of value functions for employed and unemployed workers

Consider discrete time with a period length $\triangle$. Denote by p the probability of finding a job per unit of time so that in a period of length $\triangle$ the probability of finding a job is $\triangle p$. The discount rate per unit of time is ρ. The value of an unemployed worker is

$$\begin{aligned} U_t &= \triangle b + \triangle p e^{-\rho\triangle}\int W(g,t+\triangle,s)d\mu(g,t,s) \\ &\quad + (1-\triangle p)\, e^{-\rho\triangle}U_{t+\triangle} \\ U_t - e^{-\rho\triangle}U_{t+\triangle} &= \triangle b + \triangle p e^{-\rho\triangle}\int \left[W(g,t+\triangle,s) - U_{t+\triangle}\right] d\mu(g,t,s) \\ U_t - [1-\rho\triangle]U_{t+\triangle} &= \triangle b + \triangle p[1-\rho\triangle]\int \left[W(g,t+\triangle,s) - U_{t+\triangle}\right] d\mu(g,t,s), \end{aligned}$$

where the last row makes a Taylor expansion of the term $e^{-\rho\triangle}$ at $\triangle = 0$. Diving both sides of the equation by $\triangle$ and taking the limit as $\triangle \to 0$ gives

$$\dot{U} + \rho U = b + p\int \left[W(g,t,s) - U\right] d\mu(g,t,s), \tag{50}$$

and using stationarity of U we can set $\dot{U} = 0$ to obtain the value function of an unemployed worker

$$\rho U = b + \theta m(\theta)\int \left[W(g,t,s) - U\right] d\mu(g,t,s).$$

Similar algebra can be done to obtain the value function of a permanent worker,

$$\begin{aligned} W(g,a,P) &= \triangle_a w^P(g,a,P) + \left[1-\triangle_a\delta(g,a,P)\right] e^{-\rho\triangle_a}W(g,t+\triangle_a,P) \\ &\quad + \triangle_a\delta(g,a,P)e^{-\rho\triangle_a}U \end{aligned}$$

Rearranging terms,

$$\begin{aligned} W(g,a,P) - e^{-\rho\triangle_a}W(g,a+\triangle_a,P) &= \triangle_a w^P(g,a) \\ -\triangle_a\delta(g,a,n^P)e^{-\rho\triangle_a}W(g,a+\triangle_a,P) &+ \triangle_a\delta(g,a,P)e^{-\rho\triangle_a}U \end{aligned}$$

$$\begin{aligned} W(g,a,n^P) - [1-\rho\triangle]W(g,a+\triangle_a,P) &= \triangle_a w^P(g,a) \\ -\triangle_a\delta(g,a,P)e^{-\rho\triangle_a}W(g,a+\triangle_a,P) &+ \triangle_a\delta(g,a,P)e^{-\rho\triangle_a}U \end{aligned}$$

Diving both sides of the equation by $\triangle_a$ and taking the limit as $\triangle_a \to 0$ gives

$$\frac{dW}{da} + \rho W(g,a,P) = w^P(g,a,P) + \delta(g,a,P)\left[U - W(g,a,P)\right], \tag{51}$$

where $\frac{dW}{da} = \frac{\partial W}{\partial a} + \frac{\partial W}{\partial n^P}\frac{dn^p}{da}$, and $\frac{dn^p}{da} = \dot{n}^p$

5.2 Solution method

The algorithm to compute the equilibrium is as follows (see Figure 15):

1) Given an initial guess for labor market tightness, θ_0, obtain policy functions $n^T(g,a)$, $h(g,a)$, $f(g,a)$, and optimal age of exit $J(g)$ and entry threshold g^e.

2) Compute newborn firm's value functions $V(g;\theta)$ and the expected value at entry, $V_e(\theta) = \int_g \max\{0, V(g;\theta)dG(g)\}$, and

   – If $V_e(\theta) < C_e$ $\Rightarrow$guess a lower θ by bisection and repeat from point (1).

   – If $V_e(\theta) < C_e$ $\Rightarrow$guess a higher θ by bisection and repeat from point (1).

   When $V_e(\theta^*) \approx C_e$ $\Rightarrow$stop and go to next point.

3) Set the mass of entrants to one, $M = 1$, and use decision rules to compute the measure of firm of different age and growth rate, $\widehat{X}(g,a)$. Compute aggregate employment of permanent workers and aggregate vacancy postings when $M = 1$, denoted by $(\widehat{Vac})$:

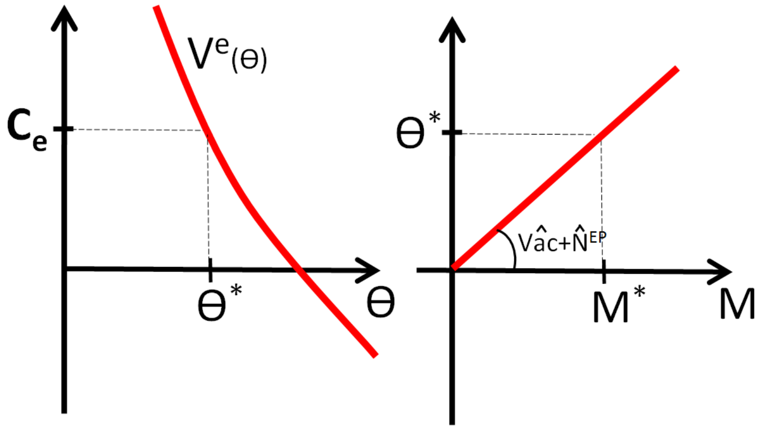
$$\begin{aligned}\widehat{N}^{EP}(g,a;\theta^*) &= \int_{g\geq g^e}\int_0^{J(g)} \widehat{X}(g,a)n^P(g,a)dadg \\ \widehat{Vac}(g,a;\theta^*) &= \frac{1}{m(\theta^*)}\int_{g\geq g^e}\int_0^{J(g)} \widehat{X}(g,a)\left[n^T(g,a)+h(g,a)\right]dadg.\end{aligned}$$

   Notice that the supra hat in all variables indicates when $M = 1$.

4) Use the linear homogeneity of $\widehat{X}(\cdot)$, $\widehat{N}^{EP}(\cdot)$ and $\widehat{Vac}(\cdot)$ in M to compute the equilibrium measure of entrants, M^*, consistent with the equilibrium labor market tightness, θ^*. This is, find M^* such that $\theta^* = \frac{M^*\widehat{Vac}(g,a;\theta^*)}{1-M^*\widehat{N}^{EP}(g,a;\theta^*)}$ where $1-M^*\widehat{N}^{EP}(g,a;\theta^*) =$

N^U.

Figure 15: Equilibrium θ^* and M^***.**



6 References

Alonso-Borrego, C. (2010): "Firm Behavior, Market Deregulation and Productivity in Spain", *Documentos de Trabajo*, Bank of Spain, N.^{o} 1035.

Alonso-Borrego, C., Fernández-Villaverde, J., E., Galdón-Sánchez (2005): "Evaluating Labor Market Reforms: A General Equilibrium Approach", *NBER Working Paper Series*, WP. 11519.

Autor, D., Kerr, W. and Kugler, A. (2007): "Does employment protection reduce productivity? Evidence from US States:, *Economic Journal*, 117, 189–217.

Bartelsman, E., J. Haltiwanger, and S.Scarpetta (2013): "Cross Country Differences in Productivity: The role of allocation and selection", forthcoming *American Economic Review*.

Boeri, T., and P. Garibaldi (2007): "Two Tier Reforms of Employment Protection Legislation. A Honeymoon Effect?", *The Economic Journal*, 117 (June), 357–385.

Caves, R. E (1998): "Industrial Organization and New Findings on the Turnover and Mobility of Firms", *Journal of Economic Literature*, 36 (4), 1947-1982.

Dolado, J., Ortigueira , S. and R. Stucchi (2012): "Does dual employment protection affect TFP? Evidence from Spanish manufacturing firms", *CEPR Discussion Papers,* DP8763.

Dolado, J., and R. Stucchi (2008): "Do Temporary Contracts Affect TFP? Evidence from Spanish Manufacturing Firms", *IZA Discussion Papers,* No. 3832.

Dunne, T., Roberts, M. and L. Samuelson (1988): "Patterns of Firms Entry and Exit in U.S. Manufacturing Industries", *RAND Journal of Economics*, 19, (4), 495-515.

Dunne, T., Roberts, M. and L. Samuelson (1989a): "The Growth and Failure of U.S. Manufacturing Plants", *Quarterly Journal of Economics*, 671-698.

Dunne, T., Roberts, M. and L. Samuelson (1989b): "Plant Turnover and Gross Employment Flows in the U.S. Manufacturing Sector", *Journal of Labor Economics*, 7, 48-71.

Erosa Andres and Ana Hidalgo (2008): "On Finance as a Theory of TFP, Cross-Industry Productivity Differences, and Economic Rents", *International Economic Review*, 49, (2), 437-473.

Felbermayr G., J. Prat, and H. J. Schmerer (2011): "Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity", *Journal of Economic Theory*, 146, 39-73.

FEDEA (2009): "Growth Agenda for Spain", (http://www.crisis09.es/agenda/Growth-agenda-for-Spain-Final.pdf).

Geroski, Paul A. (1998): "An Applied Econometrician's View of Large Company Performance", *Review of Industrial Organization,* 13, 271–94.

Hsieh, C. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India", *Quarterly Journal of Economics*, 1403-1448.

Hopenhayn H., and R. Rogerson (1993): "Job Turnover and Policy Evaluation: A General Equilibrium Analysis", *Journal of Political Economy,* 101, 915-938.

Mortensen, D. T., and C. A. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies*, 61, 397-415.

Navaretti, Bugamelli, Schivardi, Altomonte, Horgos and Maggioni (2011): "The global operations of European firms", *Bruegel Bluprint Series*, the second EFIGE policy report, Volume XII.

Pissarides, C. A. (2000): "Equilibrium Unemployment Theory", 2nd edition Cambridge MIT Press.

Sanchez, R., and L. Toharia (2000): "Temporary workers and productivity: the case of Spain", *Applied Economics*, 32, 583-591.

Sutton, J. (1997): "Gibrat's Legacy", *Journal of Economic Literature*, 35, 40-47