Strategic spending in voting competitions with social networks.

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Abstract I propose a model of strategic spending on vote buying competitions (political campaigns and lobbying) where voters are influenced by the opinion of their neighbors on a social network. I find that resources are targeted toward individuals with a high eigenvector centrality, which contrasts with previous models that predict that spending should be targeted toward individuals with a higher probability of being pivotal for the vote. I then test both hypothesis by using data on campaign contributions by Political Action Committees and data on the network of cosponsorship of bills in the US. House of Representatives. I find both network influence and pivotality are significant predictors of campaign contributions, which suggests network influence is important for some, but not all, votes.

Keywords: Network games, strategic spending, Colonel Blotto games, counteractive lobbying, Bonacich centrality.

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1 Introduction

When people are deciding how to vote or which product to buy, they discuss their decision with people in their social environment. Studying the pattern of social relationships is important to understand how individuals are influenced directly and indirectly by the opinion of others. This paper studies strategic spending when voters influence each others' opinion. Current models of strategic spending do not take these effects into account.

I propose a model where two *persuaders* strategically assign resources across voters based on their position on a social network and the likely impact of their vote on the final outcome. My model is tractable and allows a rich structure of influence between individuals. For example, I allow for influence to be asymmetric and I put no restriction on the number of connections in the network.

Previous papers on strategic spending in voting competitions have found that resources should be targeted toward voters who have a higher probability of casting a pivotal vote.[1] In contrast, I find that when network effects are strong, persuaders target their resources toward voters who have a high eigenvector centrality, a measure of influence in the network. This measure is frequently used in the sociology literature[2] and is the basis for Google's PageRank, the algorithm to rank websites.

The shift away from pivotal voters is surprising because, with or without the network, these voters have the highest marginal impact on the outcome of the election. Under perfect targeting, spending resources to change a vote that isn't pivotal is a waste of resources. The shift in spending patterns occurs because the network prevents resources from being targeted in an effective way. As the network effects become very strong, it becomes impossible to persuade voters in isolation. Persuaders react by moving resources away from pivotal voters and focusing on influential voters.

To test the model I combine data on campaign contributions by Political Action Com-

[1] See Shubik and Weber (1981); Snyder (1989); Lever (2010).

[2] See Wasserman and Faust (1994); Bonacich (1987); Bonacich and Lloyd (2001).

mittees (PACS) with data on cosponsorships networks in the US. House of Representatives. Since I can observe each legislator repeatedly across electoral cycles, I can control for unobservable legislator characteristics that might bias my result. The effect of the network is identified by measuring how year-to-year changes in network influence predict the changes in campaign contributions.

I find that both pivotality and network influence are significant predictors of campaign contributions, even after controlling for several confounds. I find that increasing network influence by one standard deviation increases the campaign contributions of the average legislator by 26,000 US. dollars, which is 6% of the contributions. ($p = 0.03$) Increasing pivotality by one standard deviation increases the contributions by 39,000 dollars, or 9% of the average contributions. ($p = 0.00$)

My paper brings together two strands of research: vote buying competitions and social networks. In the research on political competitions there is a literature on counter-active lobbying[3] and on strategic spending in presidential elections,[4] but these papers do not allow for voters to influence one other. In the social networks literature there has been much work on identifying the influential members of a network, but very little work has been done on how this information would be used in a strategic competition. There exists a vast number of measures of network influence but in my model only eigenvector centrality matters.[5] [6]

The only previous papers on political competitions with network effects are Galeotti and Mattozzi (2011) and Gröenert (2009). Galeotti and Matozzi build a model of information disclosure when voters inform themselves through a social network. Their work focuses on the amount of information revealed when political parties have an incentive to hide their platforms. They also study how the network alters which candidates run for office.

[3] Austen-Smith and Wright (1994, 1996).

[4] Merolla, Munger and Tofias (2005).

[5] For references on the many measures used to measure influence and centrality on networks, see Jackson (2008); Wasserman and Faust (1994).

[6] Eigenvector centrality is closely related to the inter-centrality measure found in the model of Ballester, Calvó-Armengol and Zenou (2006). In their model, inter-centrality identifies the members of a crime network that should be targeted for removal.

Their work puts much less emphasis on the structure of the network. Gröenert studies the problem of a single lobbyist who wishes to persuade legislators that follow a simple behavioral voting rule: they vote in favor of a proposal if the fraction of their neighbors favoring the proposal exceeds an idiosyncratic threshold. She finds that the optimal spending strategy for lobbying on threshold networks consists of successively targeting the legislators with the most connections, but she also proves this strategy cannot be guaranteed to be optimal for networks that are not threshold networks.

The rest of the paper is structured as follows: Section 2 sets up the model; Section 3 solves the model; Section 4 tests the model with data on campaign contributions by lobbies in the US. House; and Section 5 concludes. I present two extensions in appendices. For most of the paper I assume persuaders have a fixed amount of resources, but in Appendix C I solve the model when persuaders have to raise their resources at a cost. Appendix D extends the model for competitions in proportional representation systems, where persuaders maximize their share of votes.

2 The Model

2.1 The players.

There is a finite number N of voters that select between two options, A and B. These options can be two candidates in the case of a general election, or the option to pass a bill vs. upholding the status quo in the case of a legislature. A subscript i denotes voter i. All voters have to chose A or B, so turnout is not an issue.

Each voter has an opinion $v_i \in (0,1)$ of the relative value of A vs. B. A larger v_i is more favorable to A. These opinions are a summary statistic of the relevant information required to chose between A and B. For example, v_i could capture the the difference in the candidates' ability to deal with a financial crisis, the perception on which candidate is more determined to carry out difficult reforms or differences in charisma.

In addition to his opinion on the value of a candidate, each voter has an ideological preference $\theta_i \in (0,1)$ for one candidate or the other. A larger θ_i means a voter has larger intrinsic preference for A, but his final decision will depend both on his opinion v_i and his preference θ_i.

Games of strategic spending frequently only have equilibria in complicated mixed strategies.[7] Characterizing these equilibria is hard. The added complexity of network influence would make the model intractable. To avoid the problem I assume that voters maximize the following stochastic utility function.

$$U(\textit{voting for } A) = \frac{v_i + \theta_i}{2} - \eta_i$$

$$U(\textit{voting for } B) = 0$$

Where η_i is distributed Uniform[0, 1] and drawn independently across voters. The stochastic element η_i represents uncertainty about the elements that determine a vote. This variable need not be random from the point of view of the voter, it only matters that it's unknown by the persuaders. Because of η_i, increasing the opinion v_i only increases the probability that voter i choses A over B. Voter i choses A if ${}^{(v_i+\theta_i)}/_2$ is greater than η_i. Let p_i denote the probability voter i choses A: ${}^{(v_i+\theta_i)}/_2$. Voter i choses B with probability $(1 - p_i)$. To simplify the proofs, I assume each individual has a strictly positive probability of choosing each option, although the probability a voter swings his vote can be arbitrarily small.

There are two persuaders, one associated with each option A and B. The persuaders have to decide how to spend resources over voters. They can be thought of as political parties or competing lobbies. Interpreting A and B as political parties is straightforward: the parties have to convince voters to chose them and whoever gets a majority wins. To interpret them as lobbies, A and B are assumed to be spending to influence the vote over

[7] The most well known reference are the Colonel Blotto games. In the standard game, Colonel Blotto and his opponent must assign a limited resources to N different battlefields. Whoever assigns more resources to a field wins the battle, and whoever wins most battles wins the war. See Roberson (2006) for a great reference.

a bill in Congress. One lobby wants the bill to pass and the other wants it to fail. A and B target their resources over different legislators to convince them to vote in their preferred direction. Without loss I assume A wants the bill to pass while B prefers the status quo.

Each persuader has a fixed amount of resources to spend: R_A, R_B. These finite resources could be money for advertising budgets, money for campaign contributions or time spent campaigning. I denote by (a_i, b_i) the percentage of resources that persuader A and persuader B spend on voter i; therefore $(a_i R_A, b_i R_B)$ are the amounts in units of resources. In Appendix C, I solve the model when persuaders have to chose the level of R_A and R_B at a cost.

I assume that A and B only care about winning the election, but not by how many votes.[8] Let $\bar{N}$ be the minimum amount of votes that A needs to win and let $\delta_i \in \{0, 1\}$ represent the final decision of voter i. Persuader A wants to maximize $\pi(p_1, \ldots, p_N) = Prob(\sum \delta_i \geqslant \bar{N})$ and persuader B wants to maximize $1 - \pi$, which is the same as minimizing π. Because of the stochastic element of the votes, A and B are never guaranteed to win; instead they seek to maximize their probability of winning. Since some bills need a qualified majority of votes to pass, I solve the model for any supermajority rule.

2.2 The timing of the game.

The game moves in several stages. For tractability I separate strategic spending and network influence into different stages of the game. Inside each stage there are periods which repeat similar actions. The timing of the game is as follows.
Let v_i^t represent the opinion of voter i at period t.

- **The initial stage:** (Period 0) Voters begin with an initial opinion v_i^0 and a known preference θ_i. The network is fixed and known.

- **The persuasion stage:** (Period 1) Persuaders simultaneously spend resources to influence v_i^0, which changes to v_i^1. (Section 2.3.)

[8]Appendix D solves a model where A and B wish to maximize the percentage of votes they receive. This is important for political systems with proportional representation where the number of seats in congress depends on the share of the vote. The results are qualitatively similar.

- **The network stage:** (Periods 2 through T) After the persuaders spend all their budget, voters update v_i^t through the social network. (Section 2.4.)

- **Final stage.** The η_i's are realized. Voters pick A with probability p_i^T.

2.3 The persuasion stage.

During the persuasion stage, persuaders simultaneously spend resources to influence opinions. I assume persuaders can change opinions through the following contest success function. (See Figure 1.)

$$v_i^1(0,0) = v_i^0$$

$$v_i^1(a_i R_A, b_i R_B) = \frac{v_i^0 (a_i R_A)^\gamma}{v_i^0 (a_i R_A)^\gamma + (1 - v_i^0)(b_i R_B)^\gamma};\ \gamma > 0$$

This functional form has the following desirable characteristics.

- **Smoothness and monotonicity**: The contest success function takes values in $[0,1]$ and varies smoothly with the amount of resources each persuader spends. If $b_i > 0$, v_i^1 monotonically tends to 1 as $a_i R_A \to \infty$. Conversely, if $a_i > 0$, v_i^1 monotonically tends to 0 as $b_i R_B \to \infty$.

- **Symmetry in spending**: If $a_i R_A = b_i R_B$ we have $v_i^1 = v_i^0$.

- **A scale-free property**: If both persuaders scale the amount they are spending on voter i by any positive factor, the opinion v_i^1 is left unaffected: $v_i^1(\lambda a, \lambda b) = v_i^1(a,b); \forall \lambda > 0$. This happens because the contest success function only depends on the ratio of resources spent on each voter: $(a_i R_A)/(b_i R_B)$.

- **Decreasing marginal persuadability:** As A and B scale their resources up, the marginal persuadability of each voter decreases. This is crucial to get equilibria in pure strategies.

$$\frac{\partial v^1}{\partial a}(\lambda a, \lambda b) < \frac{\partial v_i^1}{\partial a}(a,b); \forall \lambda > 1$$

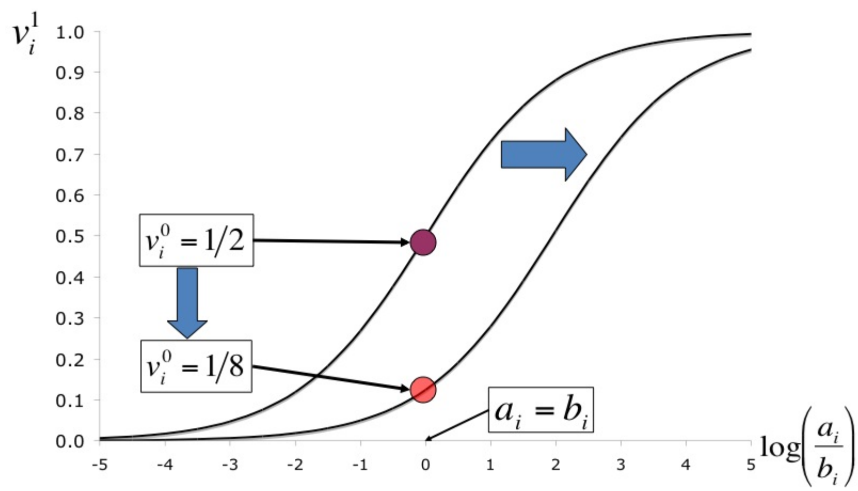


Figure 1: The contest success function depends on the ratio of resources and is "S shaped" in log units. The picture shows two potential functions with a different v_i^0 parameter. The picture assumes $R_A = R_B$.

Contest success functions have been used in the economics and political science literature to study strategic spending in tournaments, arms races and political competitions.[9] Skaperdas provides axiomatizations for this and other contest success functions.[10] Shubik and Weber use this contest success function to solve a smooth Colonel Blotto game.[11] Snyder uses a slightly more general function that does not depend on the ratio of resources but has all the other characteristics above.[12] His results are qualitatively similar to mine but the ratio formula gives convenient analytical solutions that depend on percentages of resources. My model is different from all these previous models in that I allow influence through a network.

The parameter γ determines the impact of resources on opinions. For a large γ, a small difference in the level of spending between A and B dramatically swings opinions in one direction or the other. As γ tends to infinity, the game becomes a standard Colonel Blotto game. To guarantee the existence of a pure strategy equilibria, γ has to be small.

[9]See Hirshleifer (1991); Skaperdas (1992) and Siegel (2009, 2010).
[10]Skaperdas (1996).
[11]Shubik and Weber (1981).
[12]Snyder (1989).

2.4 The network stage.

After persuaders have spent all their budget, voters update their opinion by taking a weighted average of the opinion their neighbors on a social network. A network is a row-stochastic matrix M which summarizes all the information on how much voters influence each other and who listens to whom. The network is exogenous and common knowledge by the persuaders. Each non-negative entry M_{ij} represents the weight voter i puts on voter j's opinion; while M_{ii} represents the weight he puts on his previous opinion. The interpretation is that each voter has a unit of attention that he divides between the opinions of his neighbors and his own. Voters can have asymmetric weights on each other's opinion, i.e. M_{ij} can be different than M_{ji}. It can even be that voter i influences j but j does not influence i.

Every period, voters update their opinion using the weights of the network. Let $\mathbf{v^t}$ be the vector of opinions at time t. This vector evolves according to:

$$\mathbf{v^{t+1}} = M\mathbf{v^t} = M^t\mathbf{v^1}$$

This implies that opinions evolve according to a linear transition system, which is very useful to solve the model because, in general, it is very hard to keep track of the evolution of opinions in models with complex network structures. This is even more true for models of strategic spending in voting competitions because it's hard to calculate how opinions change the probability of winning. By assuming a linear updating process I can apply powerful tools from linear algebra and markov-chain theory to study the problem.

There are two ways to interpret the updating process. It can be interpreted as a model of information processing or as a model of social preferences. In the information interpretation, there is a common value v that captures the true difference in the quality between A and B. All voters agree that more quality is better, but voters disagree on the value of v because they have different signals, or opinions, about it. Voters therefore try to update their assessment of v using the opinion of their neighbors on the network. As people update their opinion, information gets disseminated through society. Voters update their opinion many times using their neighbors' opinions to incorporate the new information that propagates through the network.

Updating opinions through a linear process is not usually the optimal bayesian way of processing information, but it can be justified as a simple heuristic for voters who are boundedly rational.[13] The optimal bayesian information processing can be very cumbersome to solve, while Golub and Jackson have shown that myopic linear updating provides a consistent estimate of the true value of v for networks with large numbers of individuals as long as the structure of the network has some reasonable assumptions. The required conditions ensure that the influence of any individual and of any finite group of individuals is not bounded away from zero.[14]

If we interpret the network as a model of social preferences, there is no true parameter v. Instead, voters have a social dimension of choice, v_i, and a private dimension, θ_i. The private dimension never changes, but the social dimension is positively influenced by the preferences of their neighbors on the network. The value of v_i captures the intensity of each voter's preference which determines the final choice probability. Voter's also incorporate the intensity of their neighbors' social preference into their choice.[15] Since different voters want to imitate different people, everybody continuously updates their social preferences to match their neighbors. The updating process assumes individuals are myopic in doing so.

Before moving on to solve the model, it is useful to define some network concepts. I refer to the voters as the nodes of the network and say there is a link from i to j if $M_{ij} > 0$. A network is *directed* if there can be a link from node i to node j without a link from j to i. A directed network is *path-connected* if for every pair of nodes i, j there is a directed path from i to j and a directed path back. That is, either i is connected to j or there exists a

[13] Linear updating is optimal if the value v and the signals v_i are jointly Normal, although the M_{ij} weights have to be adjusted after every period of updating.

[14] See Golub and Jackson (2010). For more on these issues, see DeMarzo, Vayanos and Zwiebel (2003) and Acemoglu, Ozdaglar and Parandeh-Gheibi (2010).

[15] This can be called altruism in the sense that the utility of a voter is a weighted average of his utility v_i and the expected utility of his neighbors.

sequence of nodes $\{k_1, \ldots, k_n\}$ such that $\{M_{i,k_1}, M_{k_1,k_2}, \ldots, M_{k_{n-1},k_n}, M_{k_n,j}\} > 0$.[16]

3 The equilibrium.

I solve for the unique pure strategy nash equilibrium of the model in three situations: without network influence ($T = 1$); with only one round of network updating ($T = 2$), which I refer to as *weak network effects*; and with an arbitrarily large number of rounds of network updating ($T \to \infty$), which I refer to as *strong network effects.* In each case, A solves

$$\max_{(a_1,\ldots,a_N)} \pi\big(p_1^T(a_1 R_A, b_1 R_B), \ldots, p_N^T(a_N R_A, b_N R_B)\big)$$

$$\text{s.t. } \sum a_1 = 1$$

While B solves

$$\min_{(b_1,\ldots,b_N)} \pi\big(p_1^T(a_1 R_A, b_1 R_B), \ldots, p_N^T(a_N R_A, b_N R_B)\big)$$

$$\text{s.t. } \sum b_1 = 1$$

Proposition 1 gives a general formulation for the unique pure strategy nash equilibrium. In the following subsections I use this to analyze how the equilibrium changes in the presence of weak and strong network effects.

Proposition 1 (Equilibrium strategies.)**.** *The strategies below constitute the unique pure strategy nash equilibrium, if one exists.*

$$a_i^* = b_i^* = \frac{(\partial\pi/\partial v_i^1) v_i^1(R_A, R_B)\big(1 - v_i^1(R_A, R_B)\big)}{\sum(\partial\pi/\partial v_j^1) v_j^1(R_A, R_B)\big(1 - v_j^1(R_A, R_B)\big)}$$

Proof. See Appendix A. □

[16] I also need to assume that the network is aperiodic. Aperiodicity is a technical condition that is verified if at least one voter places a positive weight on his previous opinion. See Jackson (2008) for more details on the definitions.

The proposition follows because at an equilibrium persuaders spend on each voter to equate the marginal benefit (${}^{\partial\pi}/_{\partial v_i^1}$) with the marginal cost ($v_i^1(1-v_i^1)$) of changing i's opinion. Since the probability of winning the election must add up to 1, the vote-buying competition is a zero-sum game, which implies that the marginal benefit of changing v_i^1 must be equal for A and B. That plus the assumed symmetry in the marginal cost of changing opinions and the scale-free property of the contest success function imply that in equilibrium both persuaders spend the same percentage of resources on each voter ($a_i = b_i$), although the percentage might be different across voters.

If B has less resources than A, she will not be able to prevent A from increasing her probability of winning. This does not guarantee that A will win, since votes are stochastic, so B can still win with positive probability. Even at a disadvantage, persuader B does benefits from spending her resources, because it reduces the probability A wins. If B did not spend any resources A would win with probability one. Furthermore, scaling-up the amount spent benefits the side with less resources. For example, if $R_A > R_B$ then $\pi^*(R_A, R_B) < \pi^*(\lambda R_A, \lambda R_B)$ for any $\lambda > 1$. This happens because the marginal impact of a dollar spent decreases as resources are scaled up.

Proposition 1 gives a necessary but not sufficient condition to find a pure strategy nash equilibrium. This is complemented by Proposition 2 which uses a concavity condition (γ has to be small enough) to show that the stated strategies are indeed an equilibrium. Furthermore, under this condition I can show the equilibrium is unique.[17]

Proposition 2 (Existence and uniqueness)**.** *There exists $\bar{\gamma} > 0$ such that for all $\gamma < \bar{\gamma}$, the strategies stated in Proposition 1 are the unique equilibrium of the game.*

Proof. See Appendix B. □

[17]For a larger γ the stated strategies might still be an equilibrium, but there might be other equilibria as well. From Proposition 1 we know these would necessarily be in mixed strategies. Since this is a zero-sum game, from the minimax theorem we know that all equilibria would be payoff equivalent. (See Mas-Colell, Whinston and Green (1995).) As $\gamma \to \infty$, the stated strategies cannot continue be an equilibrium, because the game approaches a standard Colonel Blotto game which has no pure strategy equilibria.

3.1 The equilibrium without network updating.

Without the network, persuaders target pivotal voters. A voter is *pivotal* for the election if, conditional on the realized votes of the others, changing his vote changes the outcome. Because the votes are uncertain, persuaders target the voters with the highest probability of being pivotal. Let q_i represent the *probability voter i is pivotal* under $\mathbf{p^T}$. This is given by

$$q_i = \sum_{\substack{S \subset N \setminus \{i\} \\ |S| = \bar{N}-1}} \prod_{j \in S} p_j^T \prod_{\substack{j' \notin S \\ j' \neq i}} \left(1 - p_{j'}^T\right)$$

Pivotal voters are important because persuaders only care about winning, but not by how many votes. This implies influencing pivotal voters has the highest expected marginal benefit. *Ex-post*, spending money to change a vote that is not pivotal is a waste of resources. *Ex-ante*, persuaders target voters by their probability of being pivotal. Applying Proposition 1, in equilibrium we have

$$a_i^* = b_i^* = \frac{q_i v_i^1 (1 - v_i^1)}{\sum q_j v_j^1 (1 - v_j^1)}$$

Where q_i and v_i^1 are calculated as if the persuaders spend (R_A, R_B) on each voter.

3.2 The equilibrium with weak network effects.

With one round of network updating persuaders target voters according to a network multiplier that averages the pivotality of a voter with the pivotality of his neighbors. Since the network changes the voting probabilities, it also changes the pivotality of each voter. Measuring how network influence changes pivotality is analytically difficult because pivot probabilities are complicated objects. Conceptually, though, it is straightforward. Let q_i denote the probability voter i is pivotal under $\mathbf{p^2}(R_A, R_B)$. That is, we calculate v_i^1 as if persuaders spend (R_A, R_B) on each voter, then do one round of network updating and calculate the pivot probabilities. Since the persuaders know the network, they have enough information to do this calculation. From here we get

$$\frac{\partial \pi}{\partial v_i^1} = \sum_k \frac{\partial \pi}{\partial p_k^2} \cdot \frac{\partial p_k^2}{\partial v_k^2} \cdot \frac{\partial v_k^2}{\partial v_i^1} = \frac{1}{2}\Big(M_{ii}q_i + \sum_{k \neq i} M_{ki}q_k\Big)$$

Remember that M_{ki} is how much voter k listens to voter i. In equilibrium persuaders A and B spend according to

$$a_i^* = b_i^* = \frac{(M_{ii}q_i + \sum_{k\neq i} M_{ki}q_k)v_i^1(1-v_i^1)}{\sum_j (M_{jj}q_j + \sum_{k\neq j} M_{kj}q_k)v_j^1(1-v_j^1)}$$

So spending on a voter is proportional to a weighted average of his pivotality and the pivotality of his neighbors.

3.3 The equilibrium with strong network effects.

We could repeat the calculation in the previous subsection for any finite T. Each time we would adjust the calculation of q_i. The cumbersome part would be calculating all the direct and indirect influences after $T-1$ rounds of updating. Instead, in this section I study the limit as $T \to \infty$ to understand what happens when the network effects are very strong. In the limit I get a surprising result: the pivotality of a voter does not matter at all, only his network influence matters. Furthermore, network influence only depends on structure of the network, not on the initial opinions.

The following result by DeGroot (1974) is necessary to solve for equilibria: under mild conditions on the network, in the long run all opinions converge to a consensus that is a weighted average of the initial opinion of every voter. The influence of a voter's initial opinion on the final consensus is given by the DeGroot weight of the voter, as defined below.

Definition 3 (**The DeGroot Weights**). *Let M be a directed weighted network which is row-stochastic. Suppose the network is path-connected and aperiodic. Define the* ***DeGroot weights of network influence****, or simply the* ***DeGroot weights****, as the unique left eigenvector of matrix M that corresponds to the eigenvalue 1 and whose entries have been normalized to sum to one. I denote it by $\mathbf{w}$. In mathematical terms, $\mathbf{w}$ is the unique vector such that*

$$\mathbf{w}M = \mathbf{w} \textit{ with } \sum w_i = 1$$

Theorem 4 (DeGroot 1974). *Let M be a path-connected, aperiodic network which is row-stochastic. For any initial vector of opinions $\mathbf{v^1} \in \mathbb{R}^N$ we have*

$$\lim_{t\to\infty} M^t \mathbf{v} = v^* \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \textit{ with } v^* = \sum w_i v_i^1$$

Because of the Degroot consensus, in the limit as T tends to infinity, π becomes a monotone transformation of $\tilde{\pi} = \sum w_i v_i^1$. Therefore we can restate the persuaders' problem as maximizing or minimizing $\tilde{\pi}$.[18] From here I get that in equilibrium:

$$a_i^* = b_i^* = \frac{w_i v_i^1 (1 - v_i^1)}{\sum_j w_j v_j^1 (1 - v_j^1)}$$

The long-run consensus of opinions is not an artifact of the myopic updating. Rational agents who share information must also converge to a common posterior in a finite number of steps. (They cannot "agree to disagree".) DeMarzo, Vayanos and Zwiebels show that rational individuals on a network sharing posteriors that are derived from a normal prior and normal signals converge to the optimal bayesian consensus belief in at most N^2 steps. Furthermore, the work by Acemoglu, Ozdaglar, and Parandeh-Gheibi show that even if

[18] This argument is only true for pure strategy equilibria.

network influence is random, in the sense that it depends on the probability each pair of individuals meet and on the probability they persuade each other, the long-run opinions converge almost surely to a consensus, although the value of the consensus depends on the realized pattern of meetings. In their model, the expected value of the consensus corresponds to the DeGroot consensus when the network is deterministic.[19]

Even though voting probabilities reach a consensus, not all voters are equally likely to be pivotal because of the ideological dimension. When voters have the same opinion but different ideologies, they have different voting probabilities and, hence, different pivot probabilities. In this context, it is interesting to analyze what would happen if the timing of the game was flipped so that voters first exchanged opinions through the network and persuaders spent to influence opinions just before the final vote. This would be equivalent to the model with $T = 1$ where the initial opinions just happened to be the same. In equilibrium persuaders would target pivotal voters without considering their position in the network, which emphasizes that pivotal voters always have the highest marginal benefit for the persuaders.

Why, then, does the network shift spending from pivotal to influential voters? The shift happens because the network prevents resources from being targeted. As $T \to \infty$, persuaders cannot change the opinion of voters independently, because their opinions are strongly influenced by the opinion of their neighbors. Persuaders must then spend to change the DeGroot consensus. The most efficient way of doing this is to target the influential voters.

Is there a systematic relationship between pivotal and influential voters? Theoretically no, these two concepts are orthogonal. One can always construct a network where pivotal voters are the same as influential voters and one can construct a network where influential and pivotal voters are completely different. This is a consequence of linear updating. Under linear updating, influence is independent of opinions, but the probability of being pivotal

[19] See DeMarzo, Vayanos and Zwiebel (2003); Acemoglu, Ozdaglar and Parandeh-Gheibi (2010).

crucially depends on them. This is shown in the following example.

3.4 A parent and child example.

A parent and a child have to decide between two almost identical products: A and B. The only difference between them is that A is sponsored by a popular cartoon character. Initially, the child is very much convinced that A is better than B: $v^0_{child} \approx 1$. The parent is of the opposite state of mind. For symmetry, assume $v^0_{parent} = 1 - v$, $\theta_{child} = v$ and $\theta_{parent} = 1 - v$.

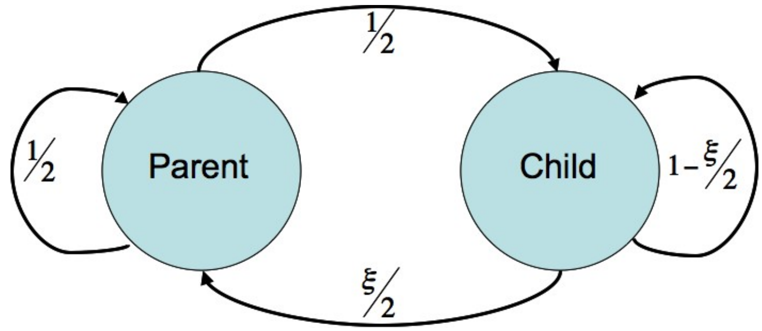
To decide which product they want, the parent and the child are going to take a vote. Product B is the status quo object. Product A is only chosen if both the parent and the child vote for it.

Suppose the persuaders, firms A and B, have the same amount of resources to spend on advertising. Because of the unanimity rule, a voter is pivotal only if the other voter choses A. Therefore, without network influence, the parent is pivotal with probability v and the child with probability $1 - v$. Since it's much more likely that the parent's vote will be decisive for the election, firms react rationally by heavily targeting the parent. In equilibrium, each firms spends a fraction v of it's budget on persuading the parent and a fraction $1 - v$ on persuading the child.

Suppose instead that before taking the decision the parent and the child deliberate. The parent feels it's important to give an equal weight to his child's opinion. The child, being a childish, pays very little attention to the parent. She places $\xi/2 \approx 0$ weight on the parent's opinion and $1 - \xi/2$ on her own.

The matrix representation of the network is

$$M = \begin{pmatrix} M_{parent,parent} & M_{parent,child} \\ M_{child,parent} & M_{child,child} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ \xi/2 & 1 - \xi/2 \end{pmatrix}$$



The corresponding DeGroot weights are

$$\mathbf{w} = \begin{pmatrix} w_{parent} \\ w_{child} \end{pmatrix} = \begin{pmatrix} \frac{\xi}{1+\xi} \\ \frac{1}{1+\xi} \end{pmatrix}$$

Given this, if the parent and the child talk for long enough, the opinion of the child will almost completely prevail. Knowing this, the firms would spend a large fraction of their resources on the child: $^1/_{(1+\xi)} \approx 1$.

Which is the right model? Different products might have different levels of communication. The parent might not be willing to debate with the child what is the right type clothes for playing in the snow. On the other hand, the car drive from San Francisco to LA gives the child ample time to convince his parent they should go to Disneyland instead of the LA Museum of Contemporary Art.

4 Testing the model against PAC contributions in the US. House of Representatives.

To test the model empirically I use data on campaign contributions by business and labor Political Action Committees (PACs) in the U.S. House of Representatives. I wish to test two hypothesis:

Hypothesis 1. *Network influence has no effect on how legislators' vote and therefore has no predictive power over contributions.*

Hypothesis 2. *Network influence completely determines how legislators' vote, therefore pivotality has no predictive power over contributions.*

To test the two hypothesis I regress campaign contributions on network influence and on pivotality to see which is a significant predictor. Finding a significant effect for network influence would reject Hypothesis 1, while finding a significant effect on pivotality would reject Hypothesis 2.

To measure network influence I construct a measure of the bilateral influence between each pair of legislators (the weights of the network) using data on cosponsorship patterns in the House. Using these bilateral weights I calculate the DeGroot weight of each legislator. To measure pivotality I use the voting probabilities of the dw-nominate model to simulate the vote on each bill thousands of times and measure how frequently each legislator is pivotal for the vote. This gives an estimate of the ex-ante likelihood each legislator would have been pivotal for each bill. In the following sections I describe the data and how I estimate network influence and pivotality before proceeding to the results of the estimation.

4.1 The contributions data.

As a proxy for the spending in the model, I use the campaign contributions by PACs to Representatives in the U.S. House. The data comes from the Federal Elections Committee (FEC) and covers the period from 1990 to 2006.[20] I only use contributions by business and labor PACs. The unit of observation is the contributions of a given PAC to a given legislator during an electoral cycle. Of the 919 Representatives in my sample, 912 received contributions from PACs in every electoral cycle.

Many PACs donate to a relatively small number of legislators. This is problematic for the regression because I do not get enough variation in the dependent variable. To solve

[20] I downloaded the data from the Center for Responsive Politics. http://www.opensecrets.org.

this, in my sample I only keep PACS that contributed to at least 8 legislators in at least one electoral year. Although these correspond to roughly 50% of the PACs in my sample, they are responsible for 95 percent of the contributions in the data, so their behavior is representative of the vast majority of contributions in the House. There were initially 7,171 PACs in my data, at the end I was left with 2,966.

Even after restricting the PACs, corner solutions remain a first order concern to describe the data. The average PAC in my sample only gave to 8 percent of the legislators in a chamber.[21] In the model, interest groups spend on influential legislators because they equate the marginal benefit and the marginal cost across legislators. This optimality condition need not hold for corner solutions. Suppose that legislator i has a larger influence than legislator j relative to the marginal costs of changing their opinion. PACs would benefit by reassigning resources in the margin, decreasing the contributions of legislator j to increase the contributions of legislator i. But, if PACs are spending zero on j, there is nothing to reassign and the difference in influence does not translate into a reallocation of contributions.[22] Therefore the theory predicts a very different relationship between network influence and campaign contributions depending on whether interest groups donate a positive amount or not. Running a linear specification would not capture this. Instead, I run a censored regression.[23]

PAC contributions are an imperfect measure of the expenditures in the model because they are not the only way interest groups spend resources on legislators: they can also hire full time lobbyists. By law, interest groups are required to disclose how much they spend on full-time lobbyists, but they are not required to disclose on what bills or over which

[21]The FEC data only show reports for positive contributions, so to account for corner solutions I had to "fill in" the data by adding a zero if a PAC did not contribute to a legislator in a year where the PAC was actively donating to other members of the House. To get a panel without gaps, I also added zeros in years where a PAC did not contribute to any legislator but had contributed in both a previous year and a later year in the sample.

[22]The data also have a second source of censoring because by law PACs cannot give more than 10,000 dollars to a legislator during a given electoral year. Although I also incorporate the bound in my regression, this second source of censoring is much less important, since it only affects 0.3 percent of the contributions in my sample.

[23]The model ruled out corner solutions by assuming that the marginal benefit when contributions are small is infinitely large. This is just a tool to get closed form solutions.

legislators the lobbyists focus their efforts, so I cannot use this data to measure targeting. Even though the predictions of the model refer to the total resources interest groups spend on a legislator, PAC contributions to a legislator can serve as a proxy as long as they are positively correlated with the resources (time, effort, etc.) lobbyists spend on the legislator.

In each electoral year, many bills are presented and many different interest groups compete over separate issues. I interpret each bill as an independent realization of my model, with an interest group on each side of the issue that spends according to either network influence or pivotality. Even if there are many interest groups spending to influence a single bill, once the content of the bill is fixed there are only two sides to the issue, so multiple interest groups spending in a coordinated matter should spend as in my model.

4.2 Measuring network influence.

To build the network I use data on the cosponsorship of bills in the U.S. Congress.[24] Every time a bill is presented in Congress it must be sponsored by a legislator, while other legislators can sign up as cosponsors. This data is very convenient because it has a direction of influence. Every time legislator j cosponsors one of legislator i's bills, I interpret that i has some influence over j. Because legislators cosponsor together many times within and across legislatures, I can build a weight for each link.

To construct the network for the electoral year t, I take each pair of legislators i, j who served in year t and measure the weight j puts on i's opinion by counting the number of times j cosponsored a bill sponsored by i in any congress where they both served together up to year t. After counting all the cosponsorships, I divide the cosponsorships from j to i by the total number of times j cosponsored with anybody else. This makes the matrix row-stochastic. The influence of i over j is then just the frequency with which j cosponsors i's bills relative to how often j cosponsors with anybody else. Legislators become more influential as they get more cosponsors on their bills except if those cosponsors frequently

[24]The data was collected and made available by Fowler (2006a,b). The data has all the bills, resolutions and amendments between 1972 and 2006, from the 93rd to the 109th Congress.

cosponsor with other legislators. Links in the network accumulate over time for legislators that remain in Congress. Since this strongly biases the DeGroot weights in favor of more senior legislators, I control for seniority in the regression.

A problem with the data is that some bills are cosponsored by almost everybody in the chamber. Bills with a number of cosponsors larger than half the chamber probably involve signaling by the majority party instead of influence by the sponsoring legislator. To deal with this I do two things: First I drop the bills that were cosponsored by more than half the House. This deletes about 10% of the bills. Next, I weigh down the links between cosponsors and sponsors by the number of cosponsors on each bill. If j cosponsored i's bill along with 9 other legislators, I assign a weight of 1/10 from j to i.[25]

Since I do not observe self links in the data, I need to make some assumption on the weight each legislator puts on his previous opinion to identify the DeGroot weights. I assume that all legislators put the same weight on themselves. As long as this weight is positive, it doesn't matter which value I choose, the DeGroot weights will be the same.[26]

4.3 Measuring pivotality.

To estimate pivot probabilities I simulate the vote on each bill using the dw-nominate model.[27] The model fits the following stochastic utility model to the data:

$$p_{ij} = \Phi\left[\beta\left\{e^{\left(-\frac{1}{2}\sum_{s=1}^{2} w_s d_{ijs1}^2\right)} - e^{\left(-\frac{1}{2}\sum_{s=1}^{2} w_s d_{ijs0}^2\right)}\right\}\right]$$

Where p_{ij} is the probability legislator i will vote in favor of bill j and Φ is the c.d.f of a standard Normal. The first exponential term in the left hand side is the utility legislator i gets from voting for bill j; the second is the utility he gets from voting against it; and the d^2

[25]Running the regression without these adjustments yields similar coefficients to those reported below, but they are no longer statistically significant.

[26]To see this let $\alpha > 0$ be the weight each legislator puts on himself and let M be the network matrix whose main diagonal is zero and whose rows sum to one. The true network would be $\alpha I + (1-\alpha)M$, but the largest left eigenvector of $\alpha I + (1-\alpha)M$ is also the largest left eigenvector of M.

[27]See Poole and Rosenthal (2007), Poole (2005) and www.voteview.com.

parameters are the ideological distance between the legislator's ideal point and the location of the competing policies in a two-dimensional policy space. The parameters β, w and d^2 parameters are all estimated from row call data.[28]

Using these voting probabilities I simulate a vote on each bill thousands of times using independent realizations of the vote for each legislator with the given voting probability.[29] This gives me an estimated pivot probability for each legislator on each bill. Since I cannot match PAC contributions to specific bills, I use the average pivot probability of each legislator across all bills in a given electoral year as my measure of his pivotality. This averaging might generate an attenuation bias in the pivotality coefficient, but there is no clear solution to the problem.

To test if weak network effects are present, in the regression I also include the weighted average of the pivot probability for the network neighbors for each legislator.

As seen in the model section, there is no theoretical reason to expect a relationship between pivot probabilities and DeGroot weights. It turns out that the empirical relationship is also very weak. The correlation between pivotality and DeGroot weights is 0.04. The correlation between the pivotality of a legislator and the pivotality of his neighbors is also not very strong: 0.1.[30]

4.4 The specification.

To account for corner solutions in the regression I run a tobit specification with Chamberlain-style random effects. The specification is as follows. For a PAC k, a legislator i and an

[28]For the US. House the estimated value for β is 7.366, the parameter w_1 is normalized to 1 and the parameter w_2 is 0.4063. The voting probabilities can be directly downloaded at www.voteview.com/dwnomin_choices.htm.

[29]I first ran a simulation of 5,000 votes and deleted 87 percent of the bills that did not show a significant probability of having a pivotal vote. I then ran 100,000 simulations on the remaining bills.

[30]The relationship between pivotality and ideological extremism (measured by the absolute distance of the dw-nominate scores) is also very weak. When I regress pivotality over the absolute value of the dw-nominate scores for each legislator I get an R-squared of 0.06.

electoral year t, there is a latent variable for contributions denoted by $y^*_{k,i,t}$ as follows

$$y^*_{k,i,t} = \alpha_i + \beta_1 DeGroot_{i,t} + \beta_2 Pivotality_{i,t} + \beta_3 NeighborPivot_{i,t} + \beta_{\mathbf{X}} \mathbf{X}_{i,t} + e_{k,i,t}. \quad (1)$$

The mean-zero errors $e_{k,i,t}$ are assumed to be i.i.d Normal across legislators, but I allow correlation within each legislator by using clustered standard errors. The matrix $X_{i,t}$ is a group of controls that includes the following variables:

1. **Seniority and seniority squared.** Measured from the first time a legislator entered the House. It's particularly important to control for seniority because the measure of network influence accumulates over time, so network influence is strongly correlated with this variable.

2. **Leadership dummies.** I include dummies for the House Speaker, the Majority and the Minority leaders and whips.

3. **Committee dummies.** I include dummies for members of influential committees: Appropriations; Ways and Means; Energy and Commerce; and Banking and Finance.

4. **Majority dummy.** Previous work by Cox and Magar had found that being in the majority party is a significant predictor of campaign contributions.[31] It is also significantly correlated with pivotality.

5. **Party dummies.**

6. **Ideological distance to the center.** Measured by the absolute value of the first dimension dw-nominate score. I include it to guard against the possibility that pivotality or network influence proxy for ideological moderation, but if legislators with more extreme ideologies are harder to persuade, this measure might be a proxy for heterogeneity in the marginal cost of persuading a legislator.

[31] See Cox and Magar (1999).

7. **Electoral year dummies.** My model does not give a prediction about the total amount PACs would spend, it only predicts the relative amount spent on each legislator. In the data I observe a lot of year-to-year variation in contributions. (The average year-to-year standard deviation per legislator is 35 per cent of the average contributions.) Adding these dummies helps reduce the noise from these variations.

As an outside researcher I only observe the censored contributions, $y_{k,i,t}$, which are equal to:

$$y_{k,i,t} = \max\{0, \min\{y^*_{k,i,t}, \bar{y}_t\}\}$$

Where $\bar{y}_t$ is the contribution limit for the electoral year t expressed in real dollars.

A concern with the specification is that unobserved legislator characteristics might bias my estimates of network influence and pivotality. For example, if the legislator's ability to raise campaign resources is correlated with his ability to get cosponsors, the DeGroot weights might be working as a proxy for the fund-raising abilities of the legislator instead of measuring network influence. To control for this I include the legislator specific random effects as follows.

Let α_i be an intercept for each legislator. To be able to remain within the tobit framework, I must assume the intercepts are normally distributed, but I allow them to be correlated with the time-average of the explanatory variables.

$$\begin{aligned}\alpha_i = &\gamma_1 \frac{1}{T_i} \sum_t DeGroot_{i,t} + \gamma_2 \frac{1}{T_i} \sum_t Pivotality_{i,t} \\ &+ \gamma_3 \frac{1}{T_i} \sum_t NeighborPivot_{i,t} + \gamma_{\mathbf{X}} \frac{1}{T_i} \sum_t \mathbf{X}_{i,t} + \nu_i\end{aligned}$$

Where T_i is the number of legislatures that legislator i served in the sample and ν_i is i.i.d. $Normal(0, \sigma_\alpha)$. This specification exploits the multiple observations I have for each legislator over different electoral years. As long as the legislator's intrinsic fund-raising ability, and any other unobserved characteristics, are constant, I can remove the bias in the network and pivotality coefficients by including the random effects. Identification of the network effect comes from comparing the changes in network influence across electoral years with the changes in campaign contributions.[32]

The DeGroot weights of a legislator change over time for two reasons: legislators sponsor and cosponsor new bills, and the cosponsors of a legislator might leave congress. Since legislators get a higher DeGroot weight if their cosponsors have a high DeGroot weight, the influence of a legislator significantly decreases when an influential cosponsor leaves. For example, during the 1994 Republican take-over of Congress, the Democrat Richard Gephardt lost 28% of his DeGroot influence, falling 16 places in the ranking of legislators. Pivot probabilities change from year to year because the composition of the chamber changes and because different bills are included in the agenda.

4.5 The results.

Table 1 presents the results of the regression. After controlling for potential confounds, I find that both the DeGroot weights and the pivot probabilities are statistically significant predictors of campaign contributions. (Respectively $p = 0.03$ and $p = 0.00$.) Therefore I can reject that network influence has no role in determining PAC contributions (Hypothesis 1), but I can also reject that pivotality has no effect over PAC contributions (Hypothesis 2).[33] This is not a contradiction because different bills can have different amounts of consultation among legislators. When legislators influence each other a lot, network influence should matter more. When legislators vote independently, pivotal legislators should be more important.

To get a sense of which of the two variables explains more variation in the data I estimate the marginal effect of increasing each by one standard deviation. (See Table 2). My specification predicts that an increase in the DeGroot weight of a legislator is associated with an

[32]The regression also includes seniority and seniority squared which implicitly allow for quadratic time trends.

[33]The pivotality of neighbors is not statistically significant and has the wrong sign, although the magnitude is not substantively significant.

increase of about 26,000 dollars, or 6% of the average campaign contributions in my sample. Pivotality has a larger effect per unit of standard deviation (9% vs. 6%) but the difference is not statistically significant.

The effect of DeGroot weights on contributions is about half the effect of becoming the House Speaker (11%). The direct effect of becoming the Speaker is huge (921,000 dollars!) but it happens to very few legislators, so it doesn't explain as much variation in the data. Increasing the DeGroot weights has a larger effect per unit of standard deviation than becoming the Majority Leader (3%) or joining the powerful Ways and Means Committee (3%).

As can be seen by comparing column (2) and column (4) in Table 1, the most important channel by which the DeGroot weights influence campaign contributions is by increasing the number of PACs that contribute, rather than increasing the contributions from PACs that are already donating. Since $E(y|\mathbf{X}) = Prob(y > 0|\mathbf{X}) * E(y|y > 0, \mathbf{X})$, the percentage increase in campaign contributions equals the percentage increase of the campaign contributions of those PACs that are already donating plus the percentage increase in the number of PACs that donate. Increasing the DeGroot weights of the average legislator by one unit (when the total influence in the chamber has been normalized to 100) increases his campaign contributions by 27% but only increases the contributions from PACs that were already donating by 4.3%. Accounting for the corner solutions in the data is crucial to distinguish between these effects.

5 Conclusion

I proposed a model of strategic persuasion over social networks. This is one of the first models to address the role of network influence in electoral competitions. The model is tractable and allows me to solve for the equilibrium spending on each voter.

In equilibrium, the expenditure on a voter is proportional to his network influence. This contrasts with previous models of strategic spending for voting competitions, which predict that equilibrium spending targets voters who are more likely to be pivotal for the outcome of an election.

Network influence displaces pivotality because network influence hampers targeting. When opinions are frequently updated through a social network, it's impossible to change the opinion of a voter without also changing the opinion of his neighbors. Parties react by spending on voters with influential positions on the network, even if they do not directly care about their vote.

The model also predicts that the relevant measure of network influence is based on eigenvector centrality measure: the DeGroot weights. Measures of network influence based on eigenvectors are self-referential: individuals are influential if influential individuals listen to them. As such, this measure highlights the quality rather than the quantity of connections.

For political campaigns, the model proposes a parsimonious way to process the highly detailed information that is being generated by social-networking sites. Instead of spending resources on traditional local leaders, the model suggests that political campaigns should look at the structure of social relationships to identify who holds influential positions, as measured by the DeGroot weights.

To test my model I combined data on campaign contributions by Political Action Committees with data on cosponsorship networks in the US. Congress for the electoral cycles from 1990 to 2006. After controlling for several confounding variables, I find that network influence is a significant predictor of campaign contributions for the House of Representatives.

According to my estimations, an increase in network influence of a legislator by one standard deviation is associated an increase of 26,000 dollars ($p = 0.03$) in the campaign contributions of a Representative. This amount corresponds to 6% of the average campaign contributions. An increase in the pivotality of a legislator is associated with a larger increase of 39,000 dollars (9% of the average, $p = 0.00$), but the difference is not statistically significant.

The results are evidence of a relationship that had not been documented before between the networks of relationships in Congress and campaign contributions. Unfortunately, even though my model predicts that network influence causes larger campaign contributions, the reverse causality cannot be ruled out, since legislators who are better fund-raisers might also be able to leverage their connections with PACs to develop relationships with other legislators. More work is needed to disentangle these effects.

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Appendix A Proof for Proposition 1.

Proof. This proof is an adaptation of the proof by Shubik and Weber.

I first prove that a pure strategy equilibrium must be in the interior. I do this by the contrapositive. Suppose that $b_i = 0$. Then A can spend an arbitrarily small quantity on i to obtain $v_i^1 = 1$. Since $v_j^1(\cdot, b_j)$ is continuous for $a_j > 0$ and A must be spending somewhere, A can decrease v_j^1 for some j by an arbitrarily small amount to increase π by a discrete amount. Since this is true for an arbitrarily small change in a_j, persuader A has no best-response and the strategies cannot constitute an equilibrium.

Next I show that persuaders spend the same percentage on each voter. From the First Order Conditions (FOCs) for A, I obtain

$$\frac{\partial v_i^1 / \partial a_i}{\partial v_j^1 / \partial a_j} = \frac{\partial \pi / \partial v_j^1}{\partial \pi / \partial v_i^1}$$

From the FOCs for B we get

$$\frac{\partial v_i^1 / \partial b_i}{\partial v_j^1 / \partial b_j} = \frac{\partial \pi / \partial v_j^1}{\partial \pi / \partial v_i^1}$$

$$\Rightarrow \frac{\partial v_i^1 / \partial a_i}{\partial v_i^1 / \partial b_i} = \frac{\partial v_j^1 / \partial a_j}{\partial v_j^1 / \partial b_j}; \forall i, j$$

On the other hand, by the homogeneity of v^1, I can apply Euler's law to get

$$\begin{aligned} a_i \frac{\partial v_i^1}{\partial a_i} + b_i \frac{\partial v_i^1}{\partial b_i} &= 0 \\ -\frac{\partial v_i^1 / \partial a_i}{\partial v_i^1 / \partial b_i} &= \frac{b_i}{a_i} \end{aligned}$$

From the FOCs we know that the left hand side must be the constant across i. Therefore a_i / b_i must be constant for all voters. This means both A and B must be spending the same fraction of their resources on each voter: $a_i^* = b_i^*$. Now we need to find what this percentage is. Doing some manipulation on $(\partial v_i^1/\partial a_i)$ I get

$$
\begin{aligned}
\frac{\partial v^1}{\partial a_i}(a_i^* R_A, b_i^* R_B) &= \frac{\partial v_i^1}{\partial a_i}(b_i^* R_A, b_i^* R_B) && \text{(Plugging in } b^*\text{)} \\
&= \frac{1}{b_i^*}\frac{\partial v_i^1}{\partial a_i}(R_A, R_B)\Big|_{a_i=1} && \text{(By homogeneity of degree -1)} \\
&= \frac{\gamma}{b_i^*} v_i^1(R_A, R_B)\big(1 - v_i^1(R_A, R_B)\big)
\end{aligned}
$$

I now substitute this in the first order condition for A.

$$
\frac{b_j^*}{b_i^*} = \frac{({}^{\partial\pi}/_{\partial v_j^1}) v_j^1(R_A, R_B)\big(1 - v_j^1(R_A, R_B)\big)}{({}^{\partial\pi}/_{\partial v_i^1}) v_i^1(R_A, R_B)\big(1 - v_i^1(R_A, R_B)\big)}
$$

Since this is true for any two voters and $\sum a_i = \sum b_i = 1$, I conclude that

$$
a_i^* = b_i^* = \frac{({}^{\partial\pi}/_{\partial v_i^1}) v_i^1(R_A, R_B)\big(1 - v_i^1(R_A, R_B)\big)}{\sum({}^{\partial\pi}/_{\partial v_j^1}) v_j^1(R_A, R_B)\big(1 - v_j^1(R_A, R_B)\big)}
$$

□

Appendix B Proof for Proposition 2.

Proof. This proof is a strengthening of Shubik and Weber's proof, who only show that the stated strategies are a local equilibria. The proof works by showing that for a small enough γ the objective function $\pi(\cdot, \mathbf{b}^*)$ is strictly concave in the relevant parameter space. From here the FOCs characterize the unique best response to $\mathbf{b}^*$.

Let $\mathbf{a}$ be a spending strategy that potentially is a best response to $\mathbf{b}^*$. As seen in the proof for Proposition 1 we can assume $a_i > 0$ for all i. To unclutter things, in what follows v_i^1 represents $v_i^1(a_i R_A, b_i^* R_B)$. Let H denote the Hessian matrix at $\mathbf{a}$.

The entries of H are as follows:

$$\begin{aligned}
H_{ii} =& \frac{\partial}{\partial a_i}\left(\frac{\partial \pi}{\partial v_i^1}\cdot\frac{\partial v_i^1}{\partial a_i}\right)\\
=& \frac{\partial^2 \pi}{\partial^2 v_i^1}\cdot\left(\frac{\partial v_i^1}{\partial a_i}\right)^2+\frac{\partial \pi}{\partial v_i^1}\cdot\frac{\partial^2 v_i^1}{\partial^2 a_i}\\
=& \frac{\partial^2 \pi}{\partial^2 v_i^1}\cdot\left(\frac{\gamma}{a_i}\right)^2\left(v_i^1\left(1-v_i^1\right)\right)^2+\frac{\partial \pi}{\partial v_i^1}\cdot\left(\frac{\gamma}{a_i}\right)^2 v_i^1\left(1-v_i^1\right)\left(1-2v_i^1-\frac{1}{\gamma}\right)
\end{aligned}$$

$$\begin{aligned}
H_{ij} =& \frac{\partial^2 \pi}{\partial v_i^1 \partial v_j^1}\cdot\frac{\partial v_i^1}{\partial a_i}\cdot\frac{\partial v_j^1}{\partial a_j}\\
=& \frac{\partial^2 \pi}{\partial v_i^1 \partial v_j^1}\cdot\left(\frac{\gamma}{a_i}\right)v_i^1\left(1-v_i^1\right)\left(\frac{\gamma}{a_j}\right)v_j^1\left(1-v_j^1\right)
\end{aligned}$$

To verify if H is negative definite we can delete the common elements of each row and each column. The simplified entries of H become

$$\begin{aligned}
H_{ii} =& \frac{\partial^2 \pi}{\partial^2 v_i^1}+2\frac{\partial \pi}{\partial v_i^1}\cdot\frac{1-2v_i^1-\frac{1}{\gamma}}{v_i^1\left(1-v_i^1\right)}\\
H_{ij} =& \frac{\partial^2 \pi}{\partial v_i^1 \partial v_j^1}
\end{aligned}$$

By Gershgorin's theorem, the eigenvalues of H are at the union of the disks with center at H_{ii} and diameter $\sum_{j\neq i}|H_{ij}|$. The rest of the proof shows that $\partial^2\pi/\partial^2 v_i^1$ and $\partial^2\pi/\partial v_i^1 \partial v_j^1$ are finite while $\partial\pi/\partial v_i^1$ is bounded away from zero. This implies that the elements of the main diagonal of H tend to $-\infty$ as γ tends to 0. Therefore all eigenvalues of H must be negative and the matrix is negative definite. I proceed case by case:

Case I: $T = 1$. In this case $\partial\pi/\partial v_i^1$ is

$$\frac{\partial \pi}{\partial v_i^1} \propto \frac{\partial \pi}{\partial p_i^1} = Prob\Big(\sum_{k\neq i}\delta_k = \bar{N}-1\Big) \equiv q_i$$

From here, $\partial^2\pi/\partial^2 v_i^1$ is 0 and $\partial^2\pi/\partial v_i^1 \partial v_j^1$ is proportional to $Prob(\sum\delta_k = \bar{N}-2) - Prob(\sum\delta_k = \bar{N}-1)$ for $k \notin \{i,j\}$, which is bounded between -1 and 1.

Proving that q_i is bounded away from zero is more challenging. In fact, it's not true over all the domain of $\mathbf{a}$. Instead, I have to restrict the domain by deleting $\mathbf{a}$'s that yield a smaller π than $\pi(\mathbf{a}^*, \mathbf{b}^*)$. To this effect define the following:

$$p_{max} = \max\left\{\frac{\theta_i + v_i^1(R_A, b_i^* R_B)}{2}\right\}_{i\in N}$$

$$p_{min} = 1 - \left(1 - \pi^*\right)^{(N-\bar{N}+1)^{-1}}$$

Where π^* is $\pi(\mathbf{a}^*, \mathbf{b}^*)$. The interpretation of p_{max} is very simple. It's the maximum probability that can be achieved by spending all of A's resources on a single voter. Since $b_i^* > 0$ we have that $p_{max} < 1$. Next I show that any strategy such that at least $\bar{N}+1$ voters have a probability smaller than p_{min} is dominated by $\mathbf{a}^*$. Therefore, without loss, we can restrict A's strategies to fulfill this condition. Take any $\mathbf{a}$ where this is true and relabel the voters such that $p_1^1 \leqslant p_2^1 \leqslant \ldots \leqslant p_N^1$. We know $p^1_{N-\bar{N}+1} < p_{min}$. From here

$$\begin{aligned}
\pi(p_1^1, \ldots, p^1_{N-\bar{N}+1}, p^1_{N-\bar{N}+2}, \ldots, p_N^1) &< \pi(p_{min}, \ldots, p_{min}, 1, \ldots, 1) \\
&= 1 - (1 - p_{min})^{N-\bar{N}+1} \\
&= \pi^*
\end{aligned}$$

Now, to show that q_i is bounded away from zero, note that

$$q_i = Prob\Big(\sum_{k\neq i} \delta_k = \bar{N} - 1\Big) \geqslant (p_{min})^{\bar{N}-1}(1 - p_{max})^{N-\bar{N}+1} > 0$$

Case II: $T = 2$. Here $\partial\pi/\partial v_i^1$ is proportional to $M_{ii}q_i + \sum_{j\neq i} M_{ji}q_j$. The proof is analogous to Case I.

Case III: $T = \infty$. Here π is a monotone transformation of $\tilde{\pi} = \sum w_i v_i^1$. The derivative of $\partial\tilde{\pi}/\partial v_i^1$ is w_i, which only depends on the network, and all the second derivatives and cross-derivatives are zero. Therefore π is strictly quasi-concave whenever $\gamma < 1$.

After solving all three cases, we conclude that a^* is the unique best-response to b^*. Mutato mutandis, b^* is the unique best-response to a^*. This shows that the strategies are indeed an equilibrium.

Uniqueness follows because equilibria for zero-sum games are interchangeable. To show this take any equilibrium of the game: (σ_a, σ_b). These are potentially mixed-strategies. It must be that (σ_a, b^*) and (a^*, σ_b) are also equilibria. Since a^* is the unique best-response to b^*, and vice-versa, we conclude that $(\sigma_a, \sigma_b) = (a^*, b^*)$. $\square$

Appendix C Competition with fundraising.

Until now I have assumed that the amount of resources was exogenous. In this section I analyze the possibility that persuaders have to raise resources at a cost. I find that in equilibrium the ratio of the resources raised by the persuaders only depends on the relative costs of raising resources. The ratio does not depend on network influence, pivotality, the specific campaign rules nor even on the distribution of initial opinions. The absolute level of resources raised does depend on these things, but in ways that are hard to characterize.

Assume each persuader j has to pay a cost $c_j \cdot (R_j)^k$ to raise resources R_j. The parameter k is greater than one and the parameter c_j is greater than zero.

In the first stage of the game, persuaders simultaneously collect resources and the amounts they rise become common knowledge. In the second stage, persuaders decide where spend it. By backward induction the spending patterns in the second stage have to be the same as in Proposition (1). Second stage pay-offs only depend on the ratio of resources collected. Let $r = {}^{R_A}/_{R_B}$ be such ratio and let $\pi(r)$ be the second stage pay-off for persuader A. To solve for the equilibrium r^*, I write persuader A's maximization problem as one that only depends on r.

$$\max_{R_A} \pi(r) - c_A R_A^k = \max_{r} \pi(r) - c_A r^k R_B^k$$

I rewrite persuader B's problem as follows.

$$\max_{R_B} \big(1 - \pi(r)\big) - c_B R_B^k = \max_{r} (1 - \pi(r)) - c_B \left(\frac{R_A}{r}\right)^k$$

The FOCs for the problem are

$$\frac{d\pi}{dr} - k c_A r^{k-1} R_B^k = 0$$

$$-\frac{d\pi}{dr} + k c_B r^{-k-1} R_A^k = 0$$

Solving this yields a solution that is independent of π.

$$r^* = \left(\frac{c_B}{c_A}\right)^{1/k}$$

If $c_A = c_B$ both persuaders raise the same amount of resources and their probability of winning does not change from that determined by the initial opinion of voters plus the network updating.

Since the marginal benefit only depends on r^* we can find the absolute level of resources by equating the marginal cost to the marginal benefit in the FOCs above. From this I can derive two easy comparative statics.

- Everything else constant, if voters are less persuadable (γ decreases) the total amount of resources raised by each persuader decreases.

- Suppose the marginal cost parameters increase proportionally. That is, (c_A, c_B) changes to $(\lambda c_A, \lambda c_B)$ with $\lambda > 1$. Then the total amount of resources raised by each persuader decreases.

The marginal benefit of resources increases with the probability the election will be decided by a pivotal vote. Persuaders spend more money on elections that are likely to be close. The network has an ambiguous effect on campaign spending because it can make the election more or less close. For example, if everybody is very likely to choose for A except for one very influential voter, the competition with the network will be more close than without it. On the contrary, one very influential voter can tilt a large number of undecided voters, making the competition less close.

Appendix D Competitions in proportional representation systems.

In this section I solve for equilibria when persuaders want to maximize the share of voters who select them. This model is especially relevant for electoral systems with proportional representation, because parties get seats in parliament in proportion to the share of votes they get in the election.

The main result is qualitatively the same as before: persuaders spend over voters in proportion to an eigenvector-based measure of network influence: Bonacich centrality, an important influence measure in the sociology literature,[34]. Pivotal voters do not matter because the persuaders do not have a threshold number of votes they wish to achieve.

To find a relationship between equilibrium spending and Bonacich centrality, assume T follows a geometric distribution. The random variable T follows a geometric distribution if the probability the game ends in $T > t$ conditional on reaching round t is a constant $\rho \in (0, 1)$ for all t.

$$Prob(T > t | T \geqslant t) = \rho; \forall t$$

Conditional on B's strategy, persuader A solves

$$\max_{(a_1,\ldots,a_N)} E_T \Big[\sum p_i^T \Big] = \max_{(a_1,\ldots,a_N)} (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \sum_i p_i^t$$

Definition 5. *Fix $\rho \in (0, 1)$. The vector $\hat{\mathbf{w}}$ of **Bonacich influence weights** for a matrix M is*

$$\hat{\mathbf{w}} = (1-\rho)(1/N, \ldots, 1/N)[I - \rho M]^{-1}$$

[34]See Ballester, Calvó-Armengol and Zenou (2006); Bramoullé, Kranton and D'Amours (2011) for the relationship between Bonacich centrality and Nash equilibria in games with linear best responses.

Proposition 6. *Suppose each persuader wants to maximize the percentage of voters that selects him. Then the unique pure strategy nash equilibrium, if it exists, is*

$$a_i^* = b_i^* = \frac{\hat{w}_i v_i^1(R_A, R_B)\big(1 - v_i^1(R_A, R_B)\big)}{\sum \hat{w}_j v_j^1(R_A, R_B)(1 - v_j^1(R_A, R_B))}$$

If $\gamma < 1$, this is the unique equilibrium of the game.

Proof. Take $(\mathbf{a}, \mathbf{b}) \in (0,1)^n$. I simply show that the objective function of each persuader is monotone transformation of $\sum \hat{w}_i v_i^1$. The rest of the proof follows the logic in the proof for Proposition 1 and 2. Setting up persuader A's maximization problem we have

$$\begin{aligned}
\max_{a_1,\ldots,a_N} (1-\rho)\sum_{t=1}^{\infty} \rho^{t-1} \sum_i p_i^t &= \max_{a_1,\ldots,a_N} \frac{1}{2}\sum_i \theta_i + \frac{1}{2}(1-\rho)\sum_{t=1}^{\infty} \rho^{t-1}(1,\ldots,1)M^{t-1}\mathbf{v}^1 \\
&\sim \max_{a_1,\ldots,a_N} (1-\rho)(1/N,\ldots,1/N)[I-\rho M]^{-1}\mathbf{v}^1 \\
&= \max_{a_1,\ldots,a_N} \hat{\mathbf{w}} \cdot \mathbf{p}^1
\end{aligned}$$

□

To conclude, let me point out a well known result of markov chains. In the limit as $T \to \infty$, Bonacich weights converge to DeGroot weights. So in the limit as $\rho \to 1$, strategic spending in proportional representation systems converges to the equilibrium for majoritarian systems.

Summary Statistics for the House of Representatives				
Variable	Mean	Std. Dev.	Min	Max
Campaign contributions*	427	352	-6	4,480
DeGroot Weights (sum=100)	0.23	0.24	0.0	1.8
Pivot Probability (times 100)	0.28	0.55	0.2	3.8
Seniority	10.8	8	2	52
Bills Sponsored	17.6	14	0	158
Number of Cosponsors	205	103	0	429
PACs per Legislator	166	93	0	677
Legislators per PAC	37	49	1	402
*In thousands of 2006 dollars				

Random-effects tobit estimation on PAC contributions.

Marginal effects at the mean.

VARIABLES	(1) MFX on $E(y)$	(2) As % of $E(y)$	(3) MFX on $E(y\|y>0)$	(4) As % of $E(y\|y>0)$
Pivot probability	71 000***	17%	135 000***	2.7%
(Times 100)	(19 000)	(4.6%)	(35 000)	(0.7%)
Pivotality of neighbors	−1 500	−0.4%	−3 000	−0.07%
(Times 100)	(1 600)	(0.4%)	(3 000)	(0.07%)
DeGroot weights	112 000**	27%	211 000**	4.3%
(Times 100)	(52 000)	(12.7%)	(98 000)	(2.0%)
DW-Nominate1	−474 000***	−115%	−899 000***	−18.3%
(Abs value)	(139 000)	(33.8%)	(262 000)	(5.3%)
Majority dummy	38 000***	9%	71 000***	1.4%
	(11 000)	(2.7%)	(22 000)	(0.4%)
Party dummy	−26 000	−6%	−49 000	−1.0%
(Republican=1)	(75 000)	(18.3%)	(142 000)	(2.9%)
Seniority	−7 000*	−2%	−13 000*	−0.3%
(Including seniority squared)	(4 000)	(1%)	(7 000)	(0.1%)
House Speaker	921 000***	224%	1 210 000***	24.6%
	(139 000)	(33.8%)	(142 000)	(2.9%)
Majority Leader	291 000***	70.1%	468 000***	9.5%
	(113 000)	(27.5%)	(159 000)	(3.2%)
Minority Leader	384 000***	93.5%	595 000***	12.1%
	(67 000)	(16.3%)	(88 000)	(1.8%)
Majority Whip	280 000***	68.2%	453 000***	9.2%
	(68 000)	(16.6%)	(96 000)	(2.0%)
Minority Whip	275 000***	70%	446 000***	9.1%
	(55 000)	(13.4%)	(78 000)	(1.6%)
Appropriations	30 000	7%	56 000	1.1%
	(21 000)	(5.1%)	(38 000)	(0.8%)
Ways and Means	183 000***	45%	315 000***	6.4%
	(36 000)	(8.8%)	(56 000)	(1.1%)
Energy and Commerce	71 000***	17%	129 000***	2.6%
	(24 000)	(5.8%)	(42 000)	(0.9%)
Banking	47 000**	11%	87 000**	1.8%
	(23 000)	(5.6%)	(41 000)	(0.8%)
Observations	7,166,690	7,166,690	7,166,690	7,166,690

Clustered standard errors in parentheses. *** p<0.01 ** p<0.05 * p<0.1

Table 1: Campaign contributions in thousands of 2006 dollars.

Marginal effects per unit of standard deviation.

VARIABLES	(1) MFX per std. dev. on $E(y)$	(2) As % of $E(y)$	(3) MFX per std. dev. on $E(y\|y>0)$	(4) As % of $E(y\|y>0)$
DeGroot weights	26 000	6%	50 000	1.0%
(Times 100)	(12 000)	(2.9%)	(23 000)	(0.5%)
Pivot probability	39 000	9%	74 000	1.5%
(Times 100)	(10 000)	(2.4%)	(19 000)	(0.4%)
DW-Nominate1	−81 000	−20%	−153 000	−3.1%
(Abs value)	(24 000)	(5.8%)	(45 000)	(0.9%)
Majority dummy	19 000	5%	35 000	0.7%
	(5 000)	(1.2%)	(11 000)	(0.2%)
House Speaker	45 000	11%	59 000	1.2%
	(7 000)	(1.7%)	(7 000)	(0.1%)
Majority Leader	14 000	3%	23 000	0.5%
	(6 000)	(1.5%)	(8 000)	(0.2%)
Ways and Means	13 000	3%	23 000	0.5%
	(2 000)	(0.6%)	(4 000)	(0.1%)
Observations	7,166,690	7,166,690	7,166,690	7,166,690

Clustered standard errors in parentheses.

Table 2: Marginal effect at the mean multiplied by the standard deviation of each variable. Campaign contributions in thousands of 2006 dollars.