The endogenous dynamics of crime structure: Heracles' lessons on how to fight the hydra

David Bardey, Daniel Mejia and Andres Zambrano

Universidad de los Andes

May 2, 2013

1 Introduction

One of the main problems that goverment authorities have to confront everyday is that ofchoosing how to allocate scarce enforcement resources to reduce violence and illegal activities. In such a process, there is an underlying strategic interaction with criminals, who react to government choices by changing their operations to reduce the chances of being caught or their illegal businesses from being disrupted. In this paper we study this problem by proposing a dynamic model where a governemnt has to decide the allocation of enforcement resources across heterogeneos drug traffickers (DTs). We assume that DTs become more efficient in running their business, the more they have produced in the past. In other words, we assume a learning-by-doing process whereby DTs become more efficient the more drugs they have smuggled in the past.

The contribution of this paper is twofold. First, on positive grounds, our paper provides a rationale as for why short-sighted governments usually put much more effort in catching big fish (*e.g.*, DT who run a larger business and are thus more experienced), despite the fact that they know that they will be replaced by lower ranked members in their organization. In other words, if the structure of the illegal drug industry is pyramidal, our model explains why, when the government is short-sighted or myopic, it focuses on a so-called behading strategy, whereby it puts most enforcement resources in catching the player in the top of the pyramid.

We claim that this has been the norm in drug producing and transit countries such as Colombia or Mexico, where goverments have repetitively over time focused on arresting the most experienced and visible drug traffickers: the top of the pyramid. This is not to say that this policy pursue is wrong itself but, rather, to point out that this might not be the optimal policy if the aim is to disrupt the drug trafficking business. The second contribution of this paper refers precisely to this normative question: Where should a non-myopic goverment put its enforcement resources if the aim is to disrupt this illegal bussines?

2 Set up

Consider a mass 1 of population of drug trafficking organizations (DTOs) and a government authority whose main aim is to disrupt illegal drug trafficking by capturing drug traffickers. A drug trafficker (DT hereafter) maximizes the discounted sum of profits over time by choosing the amount of drugs to be smuggled and sold every period. The more drugs he has smuggled over time, the more efficient he becomes at producing them. In other words, we assume that there is an underlying learning-by-doing process through which DTs become more efficient as they accumulate more experience over time. On the other hand, a government that lives forever aims to minimize the amount of DTs by spending limited resources each period on catching them and, by doing this, disrupting their business. We assume that the govornment faces more social pressure to catch the more experienced DTs, that is those who have smuggled more drugs over time. We also assume that the probability of catching a DT increases with the amount of drugs he smuggles (*e.g.*, the probability of detecting and arresting a DT is increasing in the size of his operations).

2.1 Drug traffickers

The instantaneous profit of a DT is given by $\pi(q, Q, \varepsilon)$, where $q \in [0, \bar{q}]$ is a control variable of the DT. It can denote the quantity of drugs smuggled but also the amount of violence he uses to secure his operations. For the sake of simplicity, we shall restrict our attention to the quantity interpretation. The variable Q represents the total quantity of drugs that the DT has smgugled since he has been in the industry, and we will refer to a DT with a higher Q as a more experienced one. The parameter ε represents an idiosyncratic productivity shock that is i.i.d. and drawn from an absolute continuous distribution $f(\cdot)$ with support in $\mathbb{R}^{++}$.

The profit π is a differentiable concave function of q; the concavity captures a non-increasing marginal benefit from the DTs point of view. This is usually the case when DTs face an inelastic downward sloping demand function. We also assume for simplicity that each DT has its own demand and we abstract from strategic interactions among DTs. At a given quantity, a more experienced DT obtains higher profits, *e.g.* the profit function is increasing in Q, although such marginal return is also decreasing, $\partial^2\pi/\partial Q^2 < 0$. *Ceteris paribus*, a DT benefits from a greater experience to manage a drug network distribution, although such benefit decreases as he gets more expericed in running its business.

Moreover, we assume that the more drugs a DT has produced, the lower is the marginal cost of drugs distribution, *e.g.* $\partial\pi/\partial q$ is increasing in Q. Again, this assumption can be interpreted as a learning-by-doing effect: A DT that has operated for a longer period of time is more efficient in the distribution when the amount of drugs he smuggles increases. Roughly speaking, it is less costly to increase the size of a network of drugs' distribution for a more experimented DTs. The productivity shock ε increases profits and the marginal benefit of producing higher quantities.

A DT maximizes his discounted inter-temporal expected profit by choosing the quantity q after observing the productivity shock that occurs each period. We assume that the DT dies exogenously with a probability δ or he can be captured by the government with probability p, which will be endogenously determined as described later on. In both cases we assume that the DT gets a normalized utility of 0. With the complementary probability, $1-p-\delta$, the DT follows up in the drug business and accumulates more expertise $Q+q$.

We assume that the probability of being captured in a given period, $p \equiv p\left(q,e\right)_{q\in[0,\bar{q}],e\in[0,\bar{e}]}$, depends on the quantity of drugs smuggled in that period (q) and on the amount of effort e undertaken by the government. Both, q and e, depend on Q; therefore, Q could be interpreted as the type of a DT. We assume that $p(.)$ is an increasing function in both arguments, concave and convex in e and q respectively. Moreover, the cross derivative p_{eq} is assumed to be positive. In words, the marginal effect of government's effort on the probability of capturing a DT increases with the quantities smuggled by the DT. Time is discrete and the individual value function can be written recursively as:

$$v(Q,\varepsilon) = \max_{q} \pi(q,Q,\varepsilon) + (1-p(e,q)-\delta)\, E_{\varepsilon'}\left[v\left(q+Q,\varepsilon'\right)\right]. \tag{1}$$

Assuming free entry in the drugs business, let us define v^e as the expected value of a potential DT entrant. The potential entrant decides between staying outside of the drug business and earning an outside utility $v_o > 0$, or entering in the business (with no experience and a realization of the shock) and obtaining a utility $v(0, \varepsilon)$. Formally,

$$v^e = \max\{v(0, \varepsilon), v_o\}. \tag{2}$$

2.2 The government authority

Let us now turn to the deterrence program solved by the government. We assume that the government lives forever and discounts the future at a rate β. Each period the government gets social pressure $z(Q)$ to catch a DT with expertise Q. The government's problem is to decide the amount of effort, $e(Q)$, to be spent to increase the probability of catching a DT with experience (type) Q.[1] Let $\mu(Q)$ be the amount of DTs with expertise Q; therefore μ represents the density of DTs in the society in that period, the aggregate state of this economy. Such distribution is endogenously determined by the optimal policies of both the DTs and the government. We assume that the government's instantaneous loss function is given by the amount of DTs weighted by the preasure it faces and the density:

$$L(q, e, \mu) = \int_{\varepsilon} \int_{Q} z(Q) \left(1 - p\left(q, e\left(Q\right)\right)\right) \mu(Q) f(\varepsilon)\, dQ d\varepsilon, \tag{3}$$

where q summarizes the quantities chosen for each possible realization of ε and each possible expertise Q. Similarly, e denotes the vector of efforts for each possible Q.

We assume that pressure $z(Q)$ is increasing in Q, meaning that society puts more preassure on the governemnt to capture more experienced DTs. We also assume that the government has a fixed enforcement budget, M, to capture DTs, and this budget should be distributed each period across different DTs with the aim of minimizing the loss function. Thus, the government objective function can be expressed recursively as:

[1] It is assumed that productivity shocks are not observed by the government. Or, alternatively, that they are drawn at the same time government chooses effort. Otherwise, effort could also be a function of the shock.

$$T(\mu, q) = \max_e \{-L(q, e, \mu) + \beta T(\mu', q')\} \quad (4)$$

$$\text{s.t.} \quad \int_Q e(Q)\,\mu(Q)\,dQ = M. \quad (5)$$

2.3 Distribution of DTs

The law of motion for the density of the DTs is given by

$$\mu_{t+1}(Q) = \int_0^Q \left(1 - p\left(e\left(\hat{Q}\right), Q - \hat{Q}\right) - \delta\right) h\left(Q|\hat{Q}\right) \mu_t\left(\hat{Q}\right) d\hat{Q}$$

where $h\left(Q|\hat{Q}\right)$ is the conditional distribution induced by the shock distribution, formally $h\left(Q|\hat{Q}\right) = f(\varepsilon)$ when $q\left(\hat{Q}, \varepsilon\right) + \hat{Q} = Q$. In words, the amount of DTs with experience Q in the next period is given by the surviving DTs with previous experience $\hat{Q}$, and that next period will have experience Q given their optimally chosen quantities after observing the shock.

We assume that such conditional distribution satisfies de likelihood ratio order with respect to $\hat{Q}$. This condition means that it is more likely to observe a higher Q when the previous $\hat{Q}$ was higher. Since the probability of surviving is decreasing in Q, then this assumption implies that the density distribution $\mu(Q)$ is decreasing in Q. That is, the crime structure is pyramidal where there are fewer DTs with more experience.

A stationary distribution is such that $\mu_{t+1} = \mu_t = \mu$. The existence and uniqueness of the distribution is obtained using Hopenhayn and Prescott (1992). This is true given the monotonicity of the distribution.

2.4 Equilibrium

We will focus on the Stationary Markov Perfect Equilibrium (SMPE) as the solution to this dynamic game given the recursive nature of the DT's and governemnts problem. A SMPE is a vector $(q^*(Q, \epsilon), e^*, \mu^*)$ such that:

1. The policy function $q^*(Q, \varepsilon)$ solves the DTs problem (1)

2. The policy function e^* solves the governments problem (4)

3. A stationary distribution $\mu(\cdot)$ induced by the latter policy functions

3 Myopic Equilibrium

We will first solve the equilibrium when the governemnt is completely myopic (e.g., when $\beta = 0$) and each period the government minimizes the static loss function. Given the simultaneity of the DTs and the government's choices within each period, a Nash equilibrium is defined in each period. In the next subsection, we solve for the DT's best response and then solve for the governments' best response. In each case, we compute the relevant comparative static exercices. Finally, we characterize the myopic equilibrium. The predictions will show that this type of equilibrium is similar to the one we observe in practice.

3.1 Best response for a DT

Let us first solve the inter-temporal problem for the DTs. Assuming $p\left(q,e\right)$ is a differentiable function, we know there exists a solution to this problem using Weierstrass theorem. If it is an interior solution, the first order condition yields:

$$\pi_q(q^*, Q, \varepsilon) + (1 - p(e,q) - \delta)\, E_\varepsilon \left[\frac{\partial v\left(Q+q,\varepsilon\right)}{\partial q}\right] - p_q(q^*, e) v\left(Q+q\right) = 0, \quad (6)$$

where,by the envelope condition, $\frac{\partial v(Q+q,\varepsilon)}{\partial q} = \pi_Q(q^{'}, Q+q, \varepsilon)$. In words, the amount of drugs smuggled each period is obtained by balancing the marginal instantaneous benefit plus the discounted future benefit of increasing expertise with the marginal opportunity cost of being arrested and forgone the future value of surviving in the drug trafficking business. However, it is worth noticing that the solution may also be a corner one, *i.e.* the DT does not produce or produces $\bar{q}$.

In any case, let $q^*\left(Q\right)$ denote the optimal quantity chosen by a DT with expertise Q, $\pi^*\left(Q\right)$ the associated optimal profits and $p^*\left(Q\right)$ the equilibrium probability of being captured. Since there is no interaction among DTs, these optimized variables must be equal among DTs with same expertise. That is, for each Q, optimal drug smuggling should be the same. From the implicit function theorem, we have:

$$\frac{\partial q}{\partial e} = -\frac{-p_e(e,q)\frac{\partial v(Q+q)}{\partial q} - p_{qe}(q^*, e) v\left(Q+q\right)}{\pi_{qq}(q^*, Q) + (1 - p(e,q) - \delta)\frac{\partial^2 v(Q+q)}{\partial q^2} - 2p_q(e,q)\frac{\partial v(Q+q)}{\partial q} - p_{qq}(q^*, e) v\left(Q+q\right)} < 0.$$

In words, the best response function of a DT with expertise Q implies that he will reduce the amount of drugs distributed when the government's effort in catching him increases. The value function can be expressed as:

$$v(Q)=\pi^{*}(Q)+\left(1-p\left(q^{*},e\right)-\delta\right)v\left(Q+q\right). \tag{7}$$

3.2 The best response for a myopic government

A myopic government minimizes the (static) loss function subject to its budget constraint. The first order condition for $e\left(Q\right)$ yields

$$z(Q)p_{e}(.)-\lambda=0, \qquad \forall Q,$$

where λ denotes the Lagrange multiplier. This last condition implies:

$$p_{e}\left(q\left(Q\right),e\left(Q\right)\right)=\frac{z(Q')}{z(Q)}p_{e}\left(q\left(Q'\right),e\left(Q'\right)\right) \qquad \forall\left(Q,Q'\right)$$

Therefore we can express each $e\left(Q'\right)$ as a function of some $e\left(Q\right)$, its relative weight $z\left(Q\right)/z\left(Q'\right)$, quantities $q\left(Q\right)$ and $q\left(Q'\right)$. Using the resource constraint, the best response for $e\left(Q\right)$ is obtained implicitly from:

$$\int_{Q'}e\left(Q'\right)\left(e\left(Q\right),\frac{z\left(Q\right)}{z\left(Q'\right)},q\left(Q\right),q\left(Q'\right)\right)\mu\left(Q\right)=M.$$

From these equations we can compute comparative statics of the best response such as:

$$\begin{aligned}\frac{\partial e\left(Q\right)}{\partial q\left(Q\right)} &= -\frac{p_{eq}\left(Q\right)}{p_{ee}\left(Q\right)}>0,\\ \frac{\partial e\left(Q\right)}{\partial q\left(Q'\right)} &= -\frac{\int_{Q'}-\frac{p_{eq}\left(Q'\right)}{p_{ee}\left(Q'\right)}dQ'}{\int_{Q'}\frac{z(Q)p_{ee}(Q)}{z(Q')p_{ee}(Q')}dQ'}<0.\end{aligned}$$

In words, the government wants to increase the effort on catching DTs with more experience, and it wants to decrease such effort the more drugs the other DTs produce. The results are intuitive since catching a DT is easier the more drugs he produces, so that the government can exert effort more efficiently to catch him. The second result is also expected, since resources are limited.

3.3 Equilibrium Characterization

Given the previous best responses we can obtain the existence and uniqueness of the Nash equilibrium within each period. Moreover, we can provide some comparative statics that are of special interest in order to compare the predictions of the model to what we observe in reality.

Lemma 1 *Within each period, there exists a unique Nash equilibrium characterized by:*

$$\begin{aligned}\pi_q(q^*, Q) + (1 - p(e(Q), q) - \delta)\frac{\partial v(Q+q)}{\partial q} - p_q(q^*, e(Q))v(Q+q) &= 0, \\ \int_{Q'} e(Q')\left(e(Q), \frac{z(Q)}{z(Q')}, q(Q), q(Q')\right)\mu(Q)\, dQ' &= M.\end{aligned}$$

Proof. See Appendix. ■

Lemma 1 determines the conditions that must hold at the Nash equilibrium within each period. The following proposition provides comparative static exercises with respect to Q.

Proposition 2 *A higher level of Q:*

i) increases the quantity q sold by DD.

ii) increases the enforcement decision e

Proof. See Appendix. ■

Proposition 2 points out that DT always increase their production over time. As long as they have accumulated more expertise, it is optimal for them to increase the quantity of drugs sold, even if this increases the probability of being catched by the goverment. The governments's enforcement also increases in the DT's experience since the drugs industry adopts a pyramidal structure. Roughly speaking, as it is observed in practice, if the mass of DTs decreases with the amount of expertise, then the government will devote more resources on arresting big players.

4 Optimal policy

5 Conclusion

6 Appendix

6.1 Proof of Lemma 1

The existence of the Nash equilibrium within each period is immediate as the State's best reply function is increasing in q, *i.e.* $\partial e\left(Q\right)/\partial q\left(Q\right) > 0$, while the DD's best reply function is decreasing, *i.e.* $\partial q/\partial e < 0$. A traditional continuity argument ensures the existence of this Nash Equilibrium. These conditions also ensure uniqueness since the best responses can only cross one time given their monotonicity.

6.2 Proof of Proposition 2

In order to provide comparative static exercises on the Nash Equilibrium in a period t, let us consider the following implicit functions built from the conditions which are satisfied at the equilibrium. We have

$$\nabla_e \equiv z(Q)p_e(.) - \lambda = 0, \qquad \forall Q,$$

while the derivative with respect to the Lagrange multiplier yields the State's budget constraint:

$$\nabla_\lambda \equiv M - \int_Q e\left(Q\right)\mu\left(Q\right)dQ.$$

From the DD's side, we have

$$\nabla_q \equiv \pi_q(q^*, Q) + \left(1 - p(e\left(Q\right), q) - \delta\right)\frac{\partial v\left(Q+q\right)}{\partial q} - p_q(q^*, e\left(Q\right))v\left(Q+q\right).$$

The total differentiation of this system gives

$$\begin{pmatrix} \nabla_{qq} & \nabla_{qe} & \nabla_{q\lambda} \\ \nabla_{eq} & \nabla_{ee} & \nabla_{e\lambda} \\ \nabla_{\lambda q} & \nabla_{\lambda e} & \nabla_{\lambda\lambda} \end{pmatrix}\begin{pmatrix} dq \\ de \\ d\lambda \end{pmatrix} = -\begin{pmatrix} \nabla_{qQ} \\ \nabla_{eQ} \\ \nabla_{\lambda Q} \end{pmatrix}dQ.$$

Using the Cramer's rule, we obtain

$$\frac{dq}{dQ} = \frac{\begin{vmatrix} -\nabla_{qQ} & \nabla_{qe} & \nabla_{q\lambda} \\ -\nabla_{eQ} & \nabla_{ee} & \nabla_{e\lambda} \\ -\nabla_{\lambda Q} & \nabla_{\lambda e} & \nabla_{\lambda\lambda} \end{vmatrix}}{\det H} = \frac{\nabla_{qQ}}{\det H} > 0$$

and

$$\frac{dq}{dQ} = \frac{\begin{vmatrix} \nabla_{qq} & -\nabla_{qQ} & \nabla_{q\lambda} \\ \nabla_{eq} & -\nabla_{eQ} & \nabla_{e\lambda} \\ \nabla_{\lambda q} & -\nabla_{\lambda Q} & \nabla_{\lambda\lambda} \end{vmatrix}}{\det H} = \frac{\nabla_{qq}\frac{\partial \mu}{\partial Q}}{\det H} > 0,$$

where

$$\det H = \begin{vmatrix} \nabla_{qq} & \nabla_{qe} & \nabla_{q\lambda} \\ \nabla_{eq} & \nabla_{ee} & \nabla_{e\lambda} \\ \nabla_{\lambda q} & \nabla_{\lambda e} & \nabla_{\lambda\lambda} \end{vmatrix}.$$