Electoral Systems and Economic Growth

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Abstract

Political institutions that are built to deal with conflict and arbitrate differences are likely to affect policy outcomes and economic performance. Electoral systems are part of this type of institutions. In our model, electoral systems are important for growth because, together with party systems, they shape the distribution of power in parliament, where policies that potentially affect growth are decided. Two main results emerge from our model. First, political institutions matter for economic growth. Second, the way in what they matter is not immediate. A precise ranking of these political institutions in terms of economic growth requires the knowledge of the distribution of people among different social classes in society.

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1 Introduction

In 1983, Margaret Thatcher was reelected with only 42 percent of the vote but obtained a large majority (61% of the seats) in the House of Commons under the UK majoritarian electoral system. With this majority, Thatcher was able to deepen the economic reforms that began in her first government. These reforms had important consequences on the British economy of the last two and a half decades (Card and Freeman, 2002; Blanchflower and Freeman, 1993). Given these developments, it is interesting to ask the counterfactual question: 'Would the policy and economic outcomes have been the same if the electoral system in UK were a proportional representation one?'

Electoral systems map citizens' policy preferences into public policies and public policies affect economic performance. The same preferences under different electoral systems could result in different types of parliaments and therefore, different policies and outcomes.

This paper develops an endogenous growth model where electoral systems play an important role in explaining economic outcomes. The economic model is a three-sector (-class) dynastic model with limited altruism where the engine of endogenous growth is public investment (*a la* Barro, 1990). Our political model makes the choice of the level of public investment endogenous which is something that the previous literature on endogenous growth did not.

Two types of electoral systems are allowed; a first-past-the-post majoritarian electoral (M) system and a proportional representation (PR)system. Each of these systems and their attached party systems will determine, through a pre-electoral and a parliamentary game, an equilibrium public policy. These equilibrium public policies (rules) will lead to different balanced growth equilibria.

This paper makes two main contributions to the literature. First, to our knowledge this is the first theoretical attempt to understand how electoral systems affect economic growth. This paper establishes a link between the literature on the consequences of alternative electoral systems on public policy (e.g. Persson, Roland and Tabellini, 2007 and Persson and Tabellini, 2004) and the literature on public policy and growth (e.g. Barro, 1990, and Barro and Sala-i-Martin, 1992).

Secondly, it contributes to the debate on the consequences of democratic vs. non-democratic political institutions on growth. Our model suggests that it is necessary to know the type of democratic or non-democratic regime to be able to make accurate predictions about growth. As Acemoglu (2008a, p.996) rightly foresees, "to understand how different political institutions affect economic decisions and economic growth we will need to go beyond the distinction between democracy and nondemocracy."

There are two main conclusions from our work. First, *per se* the PR and M systems do not necessary imply different economic growth. Secondly, it is not true that democracy will always deliver faster growth than dictatorships. The first result could explain why some recent works fail in finding differences in growth performance across electoral systems (e.g. Persson, 2005). The second result can explain the mixed evidence regarding the hypothesis that democracies deliver higher growth than dictatorships (Persson and Tabellini, 2008, and Przeworski, 2004).

Our model predicts the following ranking in terms of economic growth (from higher to lower): i.) PR in a society with relative majority of the rich class and rich class dictatorships; ii.) Middle class dictatorships, M systems and PR in a society with relative majority of the middle class; and finally iii.) Poor class dictatorships and PR in a society with relative majority of the poor class.

In what follows, section 2 briefly reviews the related literature. Section 3 develops the model. Section 4 provides the main results. Section 5 derives the optimal investment policy and growth under the rule of an utilitarian social planner. Section 6 develops some extensions. Section 7 presents comparative statics and dynamics. Section 8 presents some anecdoctical evidence that illustrates the potential of the theory. Finally, in section 9 the conclusions are presented.

2 Review of the literature

To our knowledge, there is no single paper comparing the growth consequences of alternative electoral systems.

Marsiliani and Renström (2007) is the only paper that is relatively close to our aims. In this paper the authors try to analyze the consequences on growth of two types of parliamentary democracy under a proportional representation electoral system.

But of course, the literature on the political economy of growth is extensive. Summaries of the first wave of this literature can be found in Aghion and Howitt (1998, ch.9), Drazen (2000, ch.11) and Persson and Tabellini

(2000, ch.14). Daron Acemoglu (2008a) devotes the last 2 chapters of his economic growth book to the discussion of the more recent political economy of growth literature.

Most of the early literature revolves around the effects of income inequality on growth via redistribution. Works along this line include, among others, Perotti (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), Glomm and Ravikumar (1992). Surveys are presented in Benabou (1996), Perotti (1996) and Aghion, Caroli and García-Peñalosa (1999). The first of these papers also presents unifying models for this literature. However, it also includes models of political instability (Devereux and Wen, 1998), special interest and rents (Tornell and Velasco, 1992, Tornell, 1997, Krusell and Ríos-Rull, 1996)).

Any conflict between individuals or classes in this literature is resolved without the mediation of any political system. In most of the papers the assumption of direct democracy, together with a majoritarian electoral rule and some version of the median voter theorem are used to find the political equilibrium (e.g. Alesina and Rodrik, 1994, Glomm and Ravikumar, 1992, Benabou, 1996, Bertola, 1993). In others the "political" equilibrium is basically the solution of a game between two or more groups of people (e.g. Banabou, 1996, Benhabib and Rustichini, 1996). Again, this is done without the mediation of any explicit political institution.

The most recent literature has focused on the role of institutions in economic development and growth. Acemoglu et al. (2005) presents a review of this literature. Most of this literature is empirical or it is not formalized in mathematical models. Some exceptions are the models of Acemoglu and Robinson (2000) and Llavador and Oxobi (2005) on enfranchisement; Persson and Tabellini (2007) and Acemoglu (2008b) on the debate democracy vs. non-democracy and economic performance, and Malley, Philippopoulos and Woitek (2007) and Economides, Philippopoulos and Price (2003) on elections, fiscal policy and growth.

There is also related literature on the consequences of political institutions on economic policy, (especially on fiscal policy), that has some level of development (Persson, 2004, Milesi-Feretti, Perotti and Rostagno, 2002, Persson, Roland, and Tabellini, 2000, 2007). Only very recently this literature has focused its attention on the study of the consequences of electoral rules on economic policy (Ticchi and Vindigni, 2005, Iversen and Soskice, 2006, Persson and Tabellini, 2006a).

3 Theory

The basic assumptions of our model are as follows.

Society. We assume a society is populated by a continuum of dynasties (of mass one). Each dynasty consists of just one individual at a time. Each individual lives for two periods. At the end of their life, an offspring (with the same preferences and technology) takes the place of the parent in the dynasty. Therefore, the dynasty is infinitely lived. These dynasties can be grouped according to their initial level of wealth into three different social classes (poor, middle and rich classes). There is no population growth or overlapping between generations. In this society, each individual is simultaneously a consumer and a producer.

Preferences. Individuals care about their consumption of private goods and the bequest (initial wealth) of their children (bequest-as-a-consumption or bequest-as-a-joy-of-giving approach).[1] For instance, this kind of approach has been used by Acemoglu (2008a) and Benabou (1996).

Technology. As in Barro (1990), our constant returns to scale production function incorporates two factors of production: private and public capital. The total factor productivity is different across social classes. There is a direct and positive correspondence between initial wealth of the dynasty and total factor productivity.

Credit and labor market. There is no credit market or possibility of transference of money across classes. The only resource available for consumption, investment and tax payments at the beginning of life is the inherited wealth. There is no labor market.

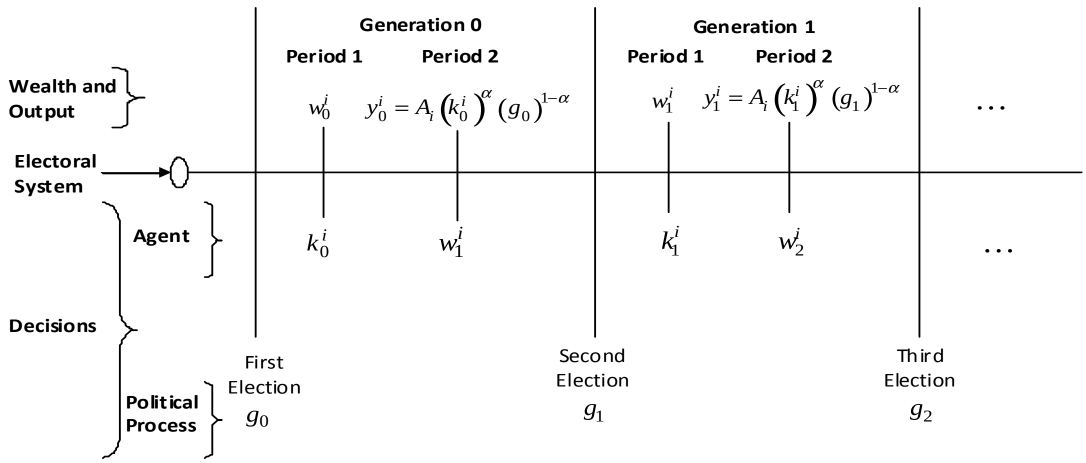
Public policy. The government only provides public capital (infrastructure) financing it with a uniform lump-sum tax.

Electoral systems. There are two possible electoral systems: a majoritarian electoral system and a proportional representation electoral system. Electoral systems are exogenous and are given at the beginning of history. As we will see later on, each electoral system is associated with a specific type of party system.

The timing of the events in our model is as follows. First, at the beginning of history (generation 0), the electoral system, the initial level of wealth and total factor productivity of each dynasty are given. Second, each individual

[1] Note that this is not the same as to care for the offspring utility (i.e. altruist motive). See Abel and Warshawsky (1988) for a discussion of the links between joy of giving motive and altruism.

Figure 1: Timing. Public policy and private decisions



votes once at the beginning of their life. The votes and the electoral system will determine a particular configuration of power in parliament. Third, public policy is the result of a bargaining process among different groups in parliament. Fourth, once public policy is implemented, each citizen decides in the first period of their life how much to invest and in the second period how much they will bequeath (or what it is the same, the initial level of wealth of the next generation of the dynasty). Production takes place in the second period of life. For generation t (for $t > 0$) the timing of events is exactly as before, the only difference is that now the initial wealth is inherited from the previous generation.

3.1 The economic model

3.1.1 Assumptions

The society is formed by 3 different classes: p, m and r (poor, middle, and rich class) with size s^p, s^m, s^r respectively, (for simplicity assume $\sum_{j=p,m,r} s^j = 1$). We also assume that $\max\{s^p, s^m, s^r\} < \frac{1}{2}$. With this assumption we avoid having the "uninteresting" case of a "natural majority" in the society. Note that the previous assumption implies that $s^i + s^j > \frac{1}{2}$ $\forall i \neq j$ $(i, j = p, m, r)$, (i.e. the number of people in any two classes is more than half of the total population).

The initial level of wealth only differs across social classes. By definition (of social classes) we have that $w_0^p < w_0^m < w_0^r$, where w_0^i is the initial wealth of dynasties belonging to class i ($i = p, m, r$).

In our model, individuals are simultaneously consumers and producers.

The utility function of an individual belonging to class i and generation t is

$$u_t^i = \log(c_{1,t}^i) + \rho\gamma \log(c_{2,t}^i) + \rho(1-\gamma)\log(w_{t+1}^i), \tag{1}$$

where $c_{j,t}^i$ is the private consumption of an individual belonging to class i and generation t in period $j = 1, 2$, w_{t+1}^i is the bequest that generation t gives to generation $t+1$ (or the initial level of wealth of generation $t+1$), ρ ($\rho < 1$) is a discount factor and γ ($0 < \gamma < 1$) is a parameter that measures the relative importance of consumption and bequest in the utility function.

The production function is class specific and is given by the following Cobb-Douglas technology

$$y_t^i = A_i (k_t^i)^{\alpha} (g_t)^{1-\alpha}, \tag{2}$$

where k_t^i is the private capital[2] of an individual belonging to class i and generation t, g_t is the public capital (e.g. infrastructure) in per capita terms[3], A_i is a class specific total factor productivity parameter, α (with $0 < \alpha < 1$) is a parameter. In our model capital and investment are synonyms, since we are assuming complete depreciation in one generation. Production takes place at the beginning of period 2.

We are assuming that the total factor productivity A depends positively on the initial level of wealth of dynasties, i.e. $A_p < A_m < A_r$, and are inherited by the following generations. The first assumption can be interpreted as ability differentials across dynasties. There is some evidence that supports the assumption of different total productivity across social classes. For example Duflo (2006) and Banerjee and Duflo (2007a,b) show that poor families

[2]Probably the best way of interpret it is in terms of a composite index of physical and human capital.

[3]Implicitly we are assuming some kind of congestion. As Barro and Sala-i-Martin (1992) points out most of public goods and services suffer from some kind of congestion, and this is typically the case with roads and education. However, note that because there is no population growth in our model, this is not an important assumption.

have significantly lower productivity than other families in some poor countries. Many reasons can explain this fact, credit constraints, inexistence of insurance markets, land tenancy arrangements, small scales of production, not enough intake of calories, etc. With respect to the second assumption, Black *et al.* (2005) presents evidence that suggests that ability can be inherited.

In the first period of their life, after w_t^i is inherited, consumer-producers decide how much to invest, k_t^i, and they pay lump sum taxes T_t ($T_t \geq 0$). In period 2, after production is realized, they decide the level of bequest for the next generation, w_{t+1}^i. Under these assumptions, the budget constraints of an individual belonging to class i and generation t are

$$c_{1,t}^i = w_t^i - T_t - k_t^i, \tag{3}$$

$$c_{2,t}^i = y_t^i - w_{t+1}^i. \tag{4}$$

We are assuming an every period balanced public budget, then

$$g_t = T_t. \tag{5}$$

There is no credit or labor market in this economy. Therefore, bequests are indispensable for the propagation of the dynasty.

3.1.2 Policy

Each individual can directly choose the level of investment and bequest and indirectly (voting) the fiscal policy. Let us first find the optimal investment and bequest functions for a generic individual.

Taking into account (1)-(5) the problem to be solved for an individual belonging to class i and generation t is

$$\max_{k_t^i, w_{t+1}^i} u_t^i = \log(w_t^i - g_t - k_t^i) + \rho\gamma \log\left[A_i (k_t^i)^\alpha (g_t)^{1-\alpha} - w_{t+1}^i\right] + \rho(1-\gamma)\log(w_{t+1}^i), \tag{6}$$

From the first order conditions we have that the optimal investment and bequest functions are

$$k_t^i = \frac{\rho\alpha}{1+\rho\alpha}(w_t^i - g_t) \text{ for } g_t \leq w_t^i, \text{ 0 otherwise}, \tag{7}$$

$$w_{t+1}^i = (1-\gamma) y_t^i. \tag{8}$$

It can be verified that the second order condition for a maximum is in place for this problem.

As a consequence of the assumption of non-existence of credit markets, the level of wealth of individual i is relevant to determine the optimal level of capital and bequest.[4]

From (7), (8) and (2) we have

$$y_t^i = A_i \left(\frac{\rho\alpha}{1+\rho\alpha} \right)^{\alpha} (w_t^i - g_t)^{\alpha} (g_t)^{1-\alpha}, \tag{9}$$

$$w_{t+1}^i = (1-\gamma) A_i \left(\frac{\rho\alpha}{1+\rho\alpha} \right)^{\alpha} (w_t^i - g_t)^{\alpha} (g_t)^{1-\alpha}. \tag{10}$$

With these results the indirect utility function, v_t^i, of an agent belonging to class i and generation t, for $w_t^i > g_t > 0$, can be written as

$$v_t^i(w_t^i, g_t) = D_i + (1+\rho\alpha)\log(w_t^i - g_t) + \rho(1-\alpha)\log(g_t), \tag{11}$$

where $D_i \equiv \log\left\{ \left[A_i \left(\frac{\rho\alpha}{1+\rho\alpha} \right)^{\alpha} (1-\gamma)^{(1-\gamma)} \gamma^{\gamma} \right]^{\rho} \left(\frac{1}{1+\rho\alpha} \right) \right\}$. At $g_t = 0$, $v_t^i(w_t^i, g_t = 0) = \log(w_t^i)$ $(= u_t^i(w_t^i, g_t = 0))$.

If we maximize (11) with respect to g_t we obtain

$$g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i, \tag{12}$$

the preferred public policy of a citizen belonging to class i and generation t.[5] This implies $g_t^p < g_t^m < g_t^r$ as long as $w_t^p < w_t^m < w_t^r$.

Note that the level of wealth is important to determine the desired level of public capital. This implies that different social classes could eventually

[4] Note that in some sense there is a contradiction between the assumption that it is not possible to transfer savings from period 1 to 2, but it is possible to transfer wealth from generation t to generation $t+1$. However, this can be easily avoided (without any changes on the results) assuming that in the second period the decision is how much to spend in education and not how much wealth to leave to the offspring. With this assumption and the additional assumptions that the production function of "education" is linear in wealth, and that the level of education of the offspring enters into the utility function instead of their initial wealth, we will arrive to identical results. For the sake of simplicity, we will keep this "asymmetric assumption" about the possibility of transference of resources within and between generations.

[5] Again, it can be verified that the second order condition for a maximum is in place.

vote for different public policies (or parties). Richer classes will prefer higher public investment.

The intuition behind this result is the following: Richer classes want to transfer more resources to period 2 of their life, but for this they need to produce more output and because private and public capital are complementary, they want a higher level of both. This implies in particular that they prefer a higher level of public investment.[6]

3.2 The politico-institutional model

In our model there are only two possible electoral systems: the majoritarian system and the proportional representation system. The most accepted way of characterizing these systems is via the electoral formula associated to them, or in other words, how votes are counted to distribute seats.[7] The M system is characterized by the first-past-the-post principle, the winner takes all the seats of the relevant electoral district. Under a PR system seats are distributed according to the proportion of votes obtained by each candidate/party in the relevant electoral district.

Norris (2004), in chapter 2 of her book Electoral Engineering: Voting Rules and Political Behavior, summarizes the main characteristics of these two electoral systems.

"The aim of majoritarian electoral systems is to create a 'natural' or a 'manufactured' majority, that is, to produce an effective one-party government with a working parliamentary majority while simultaneously penalizing minor parties, especially those with spatially dispersed support. In 'winner take all' elections, the leading party boosts its legislative base, while the trailing parties get meager rewards."

"[P]roportional representation electoral systems focus on the inclusion of all voices, emphasizing the need for and bargaining and compromise within parliament, government, and the policymaking process. The basic principle of proportional representation (PR) is that parliamentary seats are allocated

[6]Remember that production is the only way of transferring resources from one period to another.

[7]Of course we can classify electoral systems according to a vector of characteristics; but as Norris (1997, p.299) points out even when we can include for this classification "district magnitude, ballot structures, effective thresholds, malapportionment, assembly size, and open/closed lists, ... the most important variations concern electoral formulas".

according to the proportion of votes cast for each party."[8]

Then, majoritarian electoral systems tend to generate an overrepresentation in parliament of the party with most votes in the election, (more seats in parliament than votes in the election), while PR systems generate a distribution of seats in parliament that it is closer to the proportion of votes obtained by each party in the election.

Electoral systems also have important implications in terms of party systems' structure. Majoritarian electoral systems tend to be associated with two-party political systems, while PR electoral systems to multi-party systems. This is the so called 'Duverger's law' (Lijphart, 1994).

In order to capture in the simplest possible way these two characteristics of "real world" political systems, we will make the following assumptions.

M system: i. single nation-wide electoral district, ii. existence of two office-motivated parties (D and R), iii. sincere voting.[9]

PR system: i. single nation-wide electoral district, ii. existence of three policy-motivated parties (or class-specific parties, call them p, m and r), they have preferences similar to those of the classes that they represent, except that they enjoy an extra constant utility if they are in power, iii. sincere voting, iv. party types are observed.

The assumption of a single electoral district implies that only one party will be represented in parliament under a M electoral system (an extreme type of overrepresentation). In other words, the party with the most votes in this single district takes all the seats in parliament.

Under a PR system, as we will see in the next sections, we will have perfect proportional representation in parliament.

[8] Colomer (2004, p.10) also describes the characteristics of these systems along similar lines:

> "...electoral systems based on the majority principle, ... tend to produce a single, absolute winner and subsequent absolute losers, ... proportional representation, [is] a principle forged to create multiple partial winners and much fewer losers than majority rule."

[9] This assumption implies that each citizen votes for the policy (or candidate) that brings him/her the maximum utility, ignoring the possible effects that his/her decision and those of others could have on the election outcome. We can justify the sincere voting assumption by noticing that we have a continuum of (or infinite) agents in our model and therefore, the probability of an agent being pivotal tends to zero. Then voters vote for their first best option without any strategic consideration.

3.2.1 Political competition under M system

The sequence of events for any generation t is the following: 1.) Parties announce a political platform, g_t^j; 2.) Voting takes place; and 3.) The winner implements the promised platform (the political platform is binding).

Voters vote sincerely. Therefore voter i vote for party D (R) if $v_t^i(w_t^i, g_t^D) > v_t^i(w_t^i, g_t^R)$ ($v_t^i(w_t^i, g_t^R) > v_t^i(w_t^i, g_t^D)$), where g_t^D (g_t^R) is the political platform of party D (R). Otherwise she is indifferent between the 2 parties, and we will assume in this case that there is 0.5 of probability of voting for D.

Now note that if $g_t^p < g_t^m < g_t^r$ (and we will prove later that this is always the case) and because the indirect utility function is concave and $\max\{s^p, s^m, s^r\} < \frac{1}{2}$ and $s^i + s^m > \frac{1}{2}$ ($i = p, r$), then the median voter belongs to the middle class. In other words, class p (r) prefers g_t^m to g_t^r (g_t^p), and taking into account the distribution of people among classes, the middle class is pivotal here.

Both parties, D and R, want to maximize their probability of being elected, q^i ($i = D, R$), and of course this probability will depend on (g_t^D, g_t^R), (i.e. the platforms announced by both parties). Parties behave strategically.

At this point, it is easy to see that we are in a context where the median voter theorem applies.

3.2.2 Political competition under PR system

The sequence of events for any generation t is as follows: 1.) Parties announce a political platform, g_t^j; 2.) Voting takes place; and 3.) Public policy, g_t, is decided in a process of bargaining between parties that have parliamentary representation.

Under PR, we have parties that are policy-motivated and have similar preferences to those of the classes that they represent. In particular we assume that the indirect utility function of party i is $v_t^i(w_t^i, g_t) + Z$ ($i = p, m, r$), with $Z > 0$ if the party has parliamentary representation and $Z = 0$ otherwise (Z can be interpreted as ego rents). We will assume that Z is big enough as to imply that all parties will participate in the election.

The policy formation in our model is consequence of a process of bargaining between political parties in parliament. We will assume that to pass legislation it is necessary the achieve majority of votes in parliament. The protocol of bargaining is very simple. The representative (randomly appointed) of the party having relative majority in parliament, say party i, put forward a policy proposal, g_t^{ij}, to the head of one other party, say party j. If the proposal is accepted, a coalition is formed and the agreed proposal is implemented. If the proposal is rejected a default policy is implemented: $g_t = 0$.

4 Politico-economic equilibrium

A politico-economic equilibrium simultaneously involves two types of equilibria; a political equilibrium and an economic equilibrium. These equilibria are defined as follows.

Definition 1 (*Political equilibrium or equilibrium public policy*) *An equilibrium public policy is a policy that is the equilibrium result of the bargaining game in parliament.*

Definition 2 (*Economic equilibrium or balanced growth equilibrium*) *A balanced growth equilibrium is characterized by a pair of constant relative levels of wealth, $\left[\left(\frac{w^p}{w^m}\right)^*, \left(\frac{w^r}{w^m}\right)^*\right]$, such that $\frac{w_{t+1}^i}{w_t^i} = \frac{y_{t+1}^i}{y_t^i} = \frac{k_{t+1}^i}{k_t^i} = \frac{g_{t+1}}{g_t} = \mu \ \forall i$ $(i = p, m, r)$, (i.e. such that all the variables in the economy are growing at the same constant rate, $\mu - 1$).*

4.1 Majoritarian electoral system

What follows is just an application of the median voter theorem.

In section 2.2 we already found the optimal investment and bequest functions conditional on g_t, and also the preferred policy for each class.

Note that the policy space in our model is unidimensional (g_t) and preferences are single piked in this policy space, then the median voter theorem applies to this problem. This implies that the policy preferred by the median voter will defeat any other alternative in a pairwise vote.

Subsequently, both parties will choose the median voter preferred policy as their political platform. Note that the winner will have majority (all the seats) in parliament, and the (equilibrium) implemented fiscal policy at time t will be that preferred by the median voter no matter which party wins the election (electoral platforms are binding).

Note that we have assumed that the electoral platform is binding but we could assume alternatively that in the first stage of the game parties not only choose a platform but also candidates. Therefore, even if the platform is not binding, choosing candidates that want to implement it is a way of solving the commitment problem. In other words, parties can choose middle class candidates in order to reassure the median voter that its preferred policy will be implemented once the party is in power.

We know for $t = 0$ that $w_0^p < w_0^m < w_0^r$, and then given (12), we have $g_0^p < g_0^m < g_0^r$ and because the sum of the members of any two classes is greater than 1/2, we know that the median voter belongs, in this case, to the middle class. The problem is that for generations $t > 0$, w_t^i is endogenous, so *a priori* we do not know if the so called middle class at time 0 will continue to be the middle class in the future and then we cannot be sure that the median voter belongs to this class for $t > 0$.

It can be proved that there is no social mobility in our model, (i.e. $w_t^p < w_t^m < w_t^r \; \forall t$; see appendix 1). In which case, the median voter is always an individual belonging to class m.

The intuition behind the no social mobility result is simple. Richer classes are more productive and subsequently they always produce more output for the same level of g_t, but because all classes have the same utility function, they will leave the same proportion of their output for the next generation (see (8)). Therefore, richer classes will leave more bequest in absolute terms. However, this bequest is nothing else than the next period wealth. So, the relative position of each class in the society is preserved over time. In other words, there is no social mobility in our model.

With these results we can state our first proposition.

Proposition 1 *Under a majoritarian electoral system the model has a unique political equilibrium. Both parties announce and the winner implements a platform that coincides with the middle class preferred policy:* $g_t = g_t^m = \frac{\rho(1-\alpha)}{1+\rho} w_t^m \; \forall t$.

Now, let us proceed with finding the balanced growth equilibria of our model. We will study the economic equilibria of the model under two conditions.

Condition 1 $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho} \left(\frac{\rho\alpha}{1+\rho\alpha} \right)^{\alpha}$.

This condition will guarantee the existence of an equilibrium.

Condition 2 $x_1 + d \leq \frac{w_0^p}{w_0^m}$.

x_1 and x_2, with $x_1 \leq x_2$, are the solutions to the equation $x = -d + b_{pm}(x)^{\alpha}$, where $d \equiv \frac{\rho(1-\alpha)}{1+\rho}$ and $b_{pm} \equiv \frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}$.

Condition 2 ensures that the economy will converge to an equilibrium. The condition implies a restriction on the initial distribution of wealth: w_0^p must be relatively close to w_0^m (for more on this condition see appendix B).

First, note that the median voter belongs to the middle class. Then, from (10) and the equilibrium fiscal policy (Proposition 1) we have that

$$\frac{w_{t+1}^m}{w_t^m} = A_m(1-\gamma)\frac{\rho}{1+\rho}\alpha^{\alpha}(1-\alpha)^{1-\alpha}. \tag{13}$$

The wealth of the middle class is growing at a constant rate. Therefore, if an equilibrium exist it will necessarily imply $\mu = \mu_m \equiv A_m(1-\gamma)\frac{\rho}{1+\rho}\alpha^{\alpha}(1-\alpha)^{1-\alpha}$. We will assume that the parameters are such that $\mu_m > 1$ (positive growth rate).

For the other classes we have that, again using (10) and the equilibrium fiscal policy,

$$\frac{w_{t+1}^i}{w_t^i} = cA_i\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w_t^m}{w_t^i}\right)^{\alpha}\left(\frac{w_t^m}{w_t^i}\right)^{1-\alpha}, \tag{14}$$

where $c \equiv (1-\gamma)\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}\left(\frac{\rho(1-\alpha)}{1+\rho}\right)^{1-\alpha}$.

Imposing the balanced growth equilibrium condition $\frac{w_{t+1}^i}{w_t^i} = \mu_m$ to (14), and solving for $\frac{w_t^m}{w_t^i}$, we can find the equilibrium relative (to middle class) level of wealth for an individual of class i. The existence of this relative wealth is enough to ensure that the growth rate of wealth and of other variables is the same across social classes, i.e. that an equilibrium exist. As it is shown in Appendix B, Condition 1 will guarantee the existence of such an equilibrium. In appendix B we also prove that under Condition 2 there is convergence to equilibrium.

Finally, given that $\frac{w_{t+1}^i}{w_t^i} = \mu_m$ $\forall i$, it is easy to show that all the other variables of the economy will be growing at the same rate (see appendix B).

Proposition 2 *Under a majoritarian electoral system and Condition 1, a.) exist two balanced growth equilibria*[10]*, in both equilibria all the variables of*

[10]These 2 equilibria are defined by 2 different $\frac{w_t^p}{w_t^m}$ ratios.The equilibrium defined by the smallest ratio is unstable, while the other is stable.

the economy are growing at rate $(\mu_m - 1)$*. b.) If Condition 2 is also in place the economy converge to an equilibrium.*

Proof. Appendix B. ■

4.2 PR electoral system

Let us first analyze the political equilibrium of the model. Note first that parties have no incentive to present a platform different from their preferred one because their type is observed and there is no commitment technology (in the parliamentary game they will always try to maximize their utility, which is identical to that of the class that they represent, except for a given ego rent term). Given that parties type are observed and the assumption of sincere voting, each class will vote for the party that has its same preferences. Proposition 3 states the political equilibrium under PR.

Proposition 3 *Under a PR electoral system the model has a political equilibrium where: 1.) each party announces a platform that it is according to its preferences, (i.e. party* p*,* m *and* r *announce* g_t^p*,* g_t^m*, and* g_t^r *respectively); and 2. each class vote for the party that has its same preferences, (i.e. class* p*,* m *and* r*, vote for party* p*,* m *and* r *respectively).*

Corollary 1 *Party (Class)* i *will have a proportion* s^i *of seats in parliament.*

Corollary 1 is a direct consequence of proposition 3 and the single electoral district assumption.

Let us now analyze the parliamentary game and the economic equilibria.

First note that from Corollary 1 we have that the distribution of seats in parliament maps perfectly the distribution of people among classes in society: s^p, s^m, s^r.

Because *a priori* we do not want to impose further restrictions on the distribution of people among social classes, we will find the equilibrium under three different alternatives (we are ruling out the possibility of equal-size groups): i. $\max\{s^p, s^r\} < s^m$ ii. $\max\{s^m, s^r\} < s^p$ iii. $\max\{s^m, s^p\} < s^r$.

Before proceeding to analyze each of these cases, let us discuss first the conditions under which a proposal is accepted in the parliamentary game.

Note that when class (party) i receives the offer of forming a coalition with class (party) j, it will accept it as long as the proposed policy g_t^{ij} gives it an utility greater than the default policy $g_t = 0$. The region of acceptance is defined by $g_t \in \left[\underline{g}_t^i, \bar{g}_t^i\right]$, where $\underline{g}_t^i$, $\bar{g}_t^i$, with $\underline{g}_t^i < \bar{g}_t^i$, are such that the default utility (i.e. $u_t^i(w_t^i, g_t = 0)$) is equal to the indirect utility evaluated at these points, i.e. $\underline{g}_t^i$, $\bar{g}_t^i$ are defined by $\log(w_t^i) = v_t^i(w_t^i, g_t)$ (note that $u_t^i(w_t^i, g_t = 0) = \log(w_t^i)$). For g_t in the interval $\left[\underline{g}_t^i, \bar{g}_t^i\right]$ the utility is equal or greater than the default utility (because the indirect utility is concave).

Now note that $u_t^i(w_t^i, g_t = 0) = v_t^i(w_t^i, g_t = 0)$, then $\underline{g}_t^i = 0\ \forall i$. Even though it is not possible to find analytically $\bar{g}_t^i$, we can say more about it. Note that by concavity of the indirect utility function $0 = \underline{g}_t^i < g_t^i < \bar{g}_t^i$, where $g_t^i = \arg\max_{g_t} v_t^i(w_t^i, g_t)$. Additionally, $\bar{g}_t^p < \bar{g}_t^m < \bar{g}_t^r$ (see appendix D).

4.2.1 The middle class has relative majority ($\max\{s^p, s^r\} < s^m$).

Note that because $\bar{g}_t^m < \bar{g}_t^r$ and $0 < g_t^m < \bar{g}_t^m$, then $0 < g_t^m < \bar{g}_t^r$ and therefore $g_t^m \in \left[\underline{g}_t^r = 0, \bar{g}_t^r\right]$. Given the last result, class m will be able to choose its first best policy, g_t^m, since this policy is better for r than the default policy. So, there is always at least one party (party r) that will accept g_t^m.

Proposition 4 *Under a PR electoral system where* $\max\{s^p, s^r\} < s^m$ *the political equilibrium implies:* $g_t = g_t^m = \frac{\rho(1-\alpha)}{1+\rho} w_t^m\ \forall t$.

The proof of the following proposition is analogous to that of proposition 2 (appendix B).

Proposition 5 *Under a PR electoral system where* $\max\{s^p, s^r\} < s^m$ *and Condition 1 a. two balanced growth equilibria exist*[11]*, in both equilibria all the variables of the economy are growing at rate* $(\mu_m - 1)$*. b. If additionally Condition 2 is verified the economy will converge to an equilibrium.*

[11] These 2 equilibria are defined by 2 different $\frac{w_t^p}{w_t^m}$ ratios.The equilibrium defined by the smallest ratio is unstable, while the other is stable.

In terms of the evolution of the economy, we are exactly in the same case as under a M system. Then, if the middle class has a relative majority, PR and M are equivalent in terms of economic outcomes. In other words, when the middle class has a relative majority in the society, the electoral system is irrelevant for growth.

4.2.2 The poor class has relative majority ($\max\{s^m, s^r\} < s^p$).

Following the same reasoning that in previous case, p will be able to choose its first best policy g_t^p. For both, m and r, g_t^p is better than the default policy.

Proposition 6 *Under a PR electoral system where* $\max\{s^m, s^r\} < s^p$ *the political equilibrium implies:* $g_t = g_t^p = \frac{\rho(1-\alpha)}{1+\rho} w_t^p \ \forall t$.

Note that from (10) and (12) for p we have

$$\frac{w_{t+1}^p}{w_t^p} = A_p(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}, \tag{15}$$

and then if an equilibrium exist (and it can be proved that it always exist, see appendix A.5.1) it will imply necessary that the economy will be growing at rate $(\mu_p - 1)$, where $\mu_p \equiv A_p(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$.

For the other classes we have that

$$\frac{w_{t+1}^i}{w_t^i} = cA_i\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w_t^p}{w_t^i}\right)^\alpha \left(\frac{w_t^p}{w_t^i}\right)^{1-\alpha}. \tag{16}$$

Under Condition 3 can be shown that there exists $\frac{w_t^p}{w_t^i}$ (for $i =, m, r$) such that $\frac{w_{t+1}^i}{w_t^i} = \mu_p \ \forall i$, i.e. an equilibrium exist (see appendix E). Again, given this result, it can be shown that all the other variables in the economy are also growing at rate $(\mu_p - 1)$. Additionally, it also can be shown that there is always convergence to the equilibrium.

The following proposition summarizes these results.

Proposition 7 *Under a PR electoral system where p has relative majority, a.) exists a unique balanced growth equilibrium where all variables in the economy are growing at rate* $(\mu_p - 1)$*; and b.) there is always convergence to this equilibrium.*

Note that because $A_m > A_p$ then $\mu_m > \mu_p$. Therefore, the economic growth under a PR electoral system where $\max\{s^m, s^r\} < s^p$ will be lower than that under a M system.

4.2.3 The rich class has relative majority ($\max\{s^m, s^p\} < s^r$).

Suppose the rich class has a relative majority, so they can make a proposal. Without having prior knowledge of the parameters and initial distribution of wealth of the model, we cannot know if they will be able to find a partner for their first best policy.

However, if the initial distribution of wealth is such that $g_0^r < \bar{g}_0^m$, r will be able to find a coalition partner for its first best policy. In this case we know that at least for the middle class g_0^r is better than the default policy.

Proposition 8 *Under a PR electoral system where* $\max\{s^m, s^p\} < s^r$ *and* $g_0^r < \bar{g}_0^m$ *the political equilibrium implies:* $g_t = g_t^r = \frac{\rho(1-\alpha)}{1+\rho} w_t^r \ \forall t$.

Now, let us proceed to find the balanced growth equilibrium of our model under the following two conditions.

Condition 3 $\frac{A_r}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$.

This condition will guarantee the existence of an equilibrium.

Condition 4 $x_1^{'} + d \leq \frac{w_0^p}{w_0^r}$ *and* $x_1^{''} + d \leq \frac{w_0^m}{w_0^r}$.[12]

$x_1^{'}$ and $x_2^{'}$, with $x_1^{'} \leq x_2^{'}$, are now the solutions to the equation $x^{'} = -d + b_{pr}\left(x^{'}\right)^{\alpha}$, where $b_{pr} \equiv \frac{A_p}{A_r}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}$; and $x_1^{''}$ and $x_2^{''}$, with $x_1^{''} \leq x_2^{''}$, are the solutions to the equation $x^{''} = -d + b_{mr}\left(x^{''}\right)^{\alpha}$, where $b_{mr} \equiv \frac{A_m}{A_r}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}$.

Condition 4 ensures that the economy will converge to an equilibrium. Condition 4 is sufficient for the existence of an equilibrium where the economy is growing at rate $(1-\mu_r)$, with $\mu_r \equiv A_r(1-\gamma)\frac{\rho}{1+\rho}\alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

Proposition 9 *Under a PR electoral system where* r *has relative majority,* $g_0^r < \bar{g}_0^m$*, and Condition 3 is in place, a.) up to 4 balanced growth equilibria exist*[13] *in all of them all the variables of the economy are growing at rate* $(\mu_r - 1)$*. b.) If Condition 4 is also in place the economy will converge to the stable equilibrium.*

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[12] Conditions 2 and 5 are enough to avoid the disappearance of a class (i.e. the case where $T_t \geq w_t^i$ for some i).

[13] If condition 3 is verified with strict inequality there will be 4 equilibria. These 4 equilibria are defined by the 4 possible combinations of 2 equilibrium values of $\frac{w_t^p}{w_t^r}$ and 2 equilibrium values of $\frac{w_t^m}{w_t^r}$. Only the equilibrium defined by the maximum value of both ratios is stable.

Then, in this case the economy will grow faster than in the preceding cases.

If $g_0^r > \bar{g_0^m}$, then we have that the restrictions are active (i.e. $\bar{g_0^p}, \bar{g_0^m} < g_0^r$). In this case, because $\bar{g_0^p} < \bar{g_0^m} < g_0^r$ and v^r is concave (i.e. because both $\bar{g_0^p}, \bar{g_0^m}$ are at the left of the maximum, and $v^r(\bar{g_0^p}) < v^r(\bar{g_0^m})$), r will choose m as coalition partner (the cheapest class to buy as coalition partner) and will offer them $g_t^{rm} = \bar{g_t^m} > g_t^m$, the minimum utility for participation, and the offer will be accepted. Unfortunately, in this case it is not possible to find a closed form solution.

5 Social planner

It could be interesting to compare the previous equilibrium policies and economic performance with those of an utilitarian social planner.

The social planner solve the following problem, where i now denote a particular social class,

$$\begin{aligned} \max W_t &= \sum_i s^i u_t^i, \qquad (17) \\ & s.t. \end{aligned}$$

$$\begin{aligned} \sum_i s^i(c_{1,t}^i + T_t + k_t^i - w_t^i) &= 0, \\ \sum_i s^i(c_{2,t}^i + w_{t+1}^i - y_t^i) &= 0, \\ \sum_i s^i \left[y_t^i - A_i(k_t^i)^\alpha (g_t)^{1-\alpha}\right] &= 0, \\ g_t - T_t &= 0. \end{aligned}$$

The social planner can separate production from consumption at individual level and can then potentially increase the global productivity in the production of goods in the economy.

The Lagrangian of this problem and the first order conditions (FOC) are presented in appendix G.

The first FOC (35) implies that

$$\frac{y_t^j}{k_t^j} = \frac{y_t^i}{k_t^i}, \forall i, j, \tag{18}$$

i.e. the marginal product of capital must be equal across social classes. Because the productivities of different classes are different, the amount of capital that is allocated to each class vary. However, they will generally be allocated in such a way so as to equalize the marginal productivity across classes.

After applying simple algebra, using the Cobb-Douglas production function, we find out that the capital is distributed in such a way that

$$k_t^m = \left(\frac{A_m}{A_r}\right)^{1/(1-\alpha)} k_t^r, \tag{19}$$

$$k_t^p = \left(\frac{A_p}{A_r}\right)^{1/(1-\alpha)} k_t^r. \tag{20}$$

Consequently, more capital will be allocated to those classes with higher productivity.

Now, from the first and second FOC (37-38) we have that

$$\alpha \frac{y_t^j}{k_t^j} = (1-\alpha)\frac{y_t}{g_t}, \forall j, \tag{21}$$

where $y_t \equiv \sum_{i=p,m,r} s^i y_t^i$. Equation (21) tells us that in equilibrium the marginal productivity of private capital for each producer must be equal to the global marginal productivity of the public capital. The social planner takes into account the global effect (for every individual) of an increase in the public investment and not only the individual (or class) effect as in some forms of democracy (for example in the M system case above, only the effect on class m is taken into account).

Now FOCs (39)-(40) imply $c_{1,t}^i = c_{1,t}^j$, $c_{2,t}^i = c_{2,t}^j$, $w_{t+1}^i = w_{t+1}^j$, $c_{2,t}^i = \frac{\gamma}{(1-\gamma)} w_{t+1}^i \ \forall i, j$. In words, because all citizens have the same utility function and the utility function is strictly concave, maximization of social utility implies that everybody must be consuming the same quantity of goods and leaving the same bequests for next generation. Distributional decisions are completely independent of production decisions.

Proposition 10 *Under a social planner the optimal public investment is* $g_t = \frac{\rho(1-\alpha)}{1+\rho} w_t$*, and the growth rate of the economy is* $(\mu_s - 1)$*, where*

$$\mu_s = A_r B^{1-\alpha} (1-\gamma) \frac{\rho}{1+\rho} \alpha^{\alpha} (1-\alpha)^{1-\alpha}, \tag{22}$$

$B \equiv \left[s^p \left(\frac{A_p}{A_r} \right)^{1/(1-\alpha)} + s^m \left(\frac{A_m}{A_r} \right)^{1/(1-\alpha)} + s^r \right]$*, and* $w_t \equiv \sum_{i=p,m,r} s^i w_t^i$.

Proof. See appendix G.4. ■

Note that this implies that the growth rate under a social planner is smaller than that is the result of a PR system where the rich class has relative majority (assuming $g_0^r < \bar{g_0^m}$), since $B^{1-\alpha} < 1$, and greater than that is the result of a PR system where the poor has relative majority $\left(A_r B^{1-\alpha} = A_p \left[s^p + s^m \left(\frac{A_m}{A_p} \right)^{1/(1-\alpha)} + s^r \left(\frac{Ar}{Ap} \right)^{1/(1-\alpha)} \right]^{1-\alpha} > A_p \right)$. However, it is not possible to establish if the growth rate of the economy under a social planner is greater or smaller than that corresponding to a M system or a PR system where the middle class has relative majority without knowing the exact distribution of people among classes (the s^i) and the value of the parameter α.

Finally, note that the optimal level of investment under a social planner, $g_t = \frac{\rho(1-\alpha)}{1+\rho} w_t$, is a weighted average of the optimal investment for each class.

6 Extensions

6.1 Dictatorships

What would be the growth rate of the economy under a dictatorship? If we assume, oversimplifying, that the basic characteristic of a dictatorship is that the dictator can choose freely (without any political constraint) the public policy, the answer will depend on the type of dictatorship that we have at hand.

Assuming that dictators that belong to class i have the same utility function that the others members of this class and that they can decide the economic policy without any restriction, we can state the following proposition.

Proposition 11 *A poor (middle) [rich] class dictator will choose g_t^p (g_t^m) [g_t^r] and the economy will grow at rate $\mu_p - 1$, ($\mu_m - 1$) [$\mu_r - 1$] repectively.*

Proof. First, note that the dictators are unconstrained to choose their first best policy. So a poor (middle) [rich] class dictator will choose g_t^p (g_t^m) [g_t^r] respectively (see (12)). Second, if conditions 1-4 are verified, we know that there will be convergence to a balanced growth equilibrium where the economy will be growing at rate $\mu_p - 1$, ($\mu_m - 1$) [$\mu_r - 1$] if the dictator belongs to the poor (middle) [rich] class respectively. ■

It is interesting to note, that these different performances according to the type of the dictator, could explain why the evidence on the economic performance of democratic vs. non-democratic regimes is inconclusive about which is the best regime in terms of economic growth (see Przeworski, 2004).[14] Moreover, proposition 11 can also help to explain why some dictatorships were or are successful in terms of establishing high rates of economic growth (e.g. China, South Korea), while others can only deliver a very poor performance (e.g. many African dictatorships).

6.2 An alternative default policy

Let's assume now a more realistic default policy. We will now assume that if an agreement is not reached in parliament the public investment is maintained at the level of the previous period, i.e. the default policy is $g_t = g_{t-1}$, and for period 0 the default policy is $g_0 = 0$.

Under the majoritarian electoral system, the middle class can freely choose its best policy. Subsequently, results under this electoral system are the same as before.

The results under a PR system are also exactly the same as before when the middle class or the poor class have relative majority (see appendix H). The intuition is the following. Class r always prefers a public investment that it is bigger than that which is optimal for classes p and m. For period 1 the default policy is 0, therefore r will accept the offer of class i in period 1. Now, in period two, (because we are assuming that the parameters of the model are such that there is economic growth), the optimal public investment for class i is bigger than its preferred (the equilibrium policy) in period 1. Therefore, we will have the following situation in period 2: $g_1{=}g_1^i < g_2^i < g_2^r$.

[14] As we will discuss in ch. 4, methodological problems can be also behind these results.

Then for class r, is better to accept g_2^i than to have the default policy $g_1 = g_1^i$. By complete induction it can be proved that this is also valid for any $t > 2$.

Therefore, the optimal policy of class i (for $i = p, m$) is an equilibrium public policy, and from here the same results of section 4.2 follow.

The case of PR with r having relative majority is more complex now. For the first period the public policy (or r offer) will be such that makes m indifferent between accepting or rejecting it. This could imply $g_0 = g_0^r$ or $g_0 = \bar{g}_0^m$ depending on the parameters' values, as in section 4.2. The difference now is that even if $g_0 = g_0^r$, there is no guarantee that in the next periods we will continue to have $g_t = g_t^r$. The only thing that can be proved is that $g_t^m \leq g_t \leq g_t^r$. Note that $g_t < g_t^m$ cannot be an equilibrium because both are better off increasing g_t at least up to g_t^m, and $g_t > g_t^r$ cannot be an equilibrium because again both will be better off reducing g_t to the level g_t^r. With the functional forms that we have, it is not possible to find an analytical solution, but how close will be g_t of g_t^m will depend on how close is g_{t-1} of g_t^m. In other words, as closer the default policy gets to the optimal policy of m, more will need r to offer m to achieve a coalition partner.

6.3 Endogenous party system (or citizen-candidates)

In section 3.2, we assumed that the party system was exogenous and it was simply attached in an *ad-hoc* manner to the electoral system. In this section we will make it endogenous (as in Ticchi and Vindigni, 2005). As we will see the results in terms of economic policy under alternative electoral systems will be unchanged. Therefore, the results in terms of economic growth will be the same as before.

The difference now is that under a majoritarian electoral system we will only have policy-oriented candidates that belong to the middle class (the number of candidates remains undetermined). This is a result, not an assumption. Additionally, candidates from the three classes will be participating in the electoral process under a PR system, (but again how many of them will remain undetermined). Finally, under a majoritarian electoral system only the middle class has seats in parliament. Meanwhile, the proportion of seats in parliament of class i is s^i (for $i = p, m, r$) under a PR system (same as before).

Let us now assume that the number of candidates is endogenous (in a similar approach to that of Osborne and Slivinsky, 1996 and Besley and Coate, 1997). Each voter (consumer-producer) can choose to become a candidate at the election. By participating as a candidate, he incurs in a utility loss of C (leisure loss) and if he wins the election, he gets an extra utility of Z (ego-rents), where $Z > C$. The number of candidates is endogenous and a citizen runs for office if and only if the expected return of doing so is greater than the associated costs.

The game has 3 stages: 1.) entry of candidates stage: each citizen decides whether or not to become a candidate (knowing s^i $\forall i$); 2.) election stage: the members of the parliament are elected in a single nation-wide electoral district where every citizen has the right to vote; 3.) parliamentary stage: at least one half of the parliament must approve the policy to be implemented.

6.3.1 Majoritarian electoral system

We will use backward induction to find the political equilibrium.

Parliamentary stage. Note that because there is only one electoral district and the winner takes all the seats, the government is formed only with one class, say class i, and the policy to be implemented is its preferred one, i.e. $g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i$.

Election stage. Note that because we assumed sincere voting, the voter j will vote for a candidate $f \in \Omega$ (the set of candidates) if f is such that $v_t^j(g_t^f, w_t^j) > v_t^j(g_t^i, w_t^j)$ $\forall i \in \Omega$, where g_t^f represents the optimal policy for candidate f and g_t^i for candidate i. Of course this condition implies that if a candidate of the same class of voter j is available, then the vote of j goes to this candidate. Otherwise, the vote goes to the candidate that maximizes j's utility, given that a candidate of his class is not available. Note that voter j could be indifferent between two candidates if their optimal policy is the same, i.e. if they are from the same class. If this is the case, we assume that every candidate of the same class has the same probability of receiving the vote.

Entry of candidates stage. First, note that in the election, potentially, can be candidates of only one class, two classes or even three different classes. What we will prove now is that the model has a unique equilibrium where only candidates belonging to the middle class (those who prefer the median policy) will participate in the election. First, note that because in our model the median voter theorem applies, in any pairwise vote the median policy (and the m class candidate) will win. Then candidates belonging to the other classes (p, r) will not participate if a candidate of the m class is participating, and in this way they will avoid incurring in a net cost of C. Now we will prove that having an election with candidates belonging to the three classes is not an equilibrium. Note that if candidates of all three classes participate, because one of the classes has relative majority, this class will win the election with certainty, then the candidates belonging to the other classes do better falling out of the race and avoiding to pay C. Then the only possible equilibrium is the one with only candidates of the m class.

Second, we need to prove that the set of candidates is not empty. Denote as q^m the probability of victory for a particular middle class candidate (in a symmetric equilibrium will be the same across candidates of this class). A middle class candidate will be running for office if his/her expected gain exceed its cost. Then, to prove that the set of candidates is not empty, it is enough to prove that this net gain is positive when there is only one candidate. If there is only one candidate $q = 1$ and his/her participation constraint can be written as $[v_t^m(w_t^m, g_t^m) - v_t^m(w_t^m, g_t^i)] + Z - C \geq 0$ (the term in square brackets represents the gain of implementing the preferred policy). Because g_t^m maximizes v_t^m (by definition) the expression in square brackets is always non-negative and because $Z > C$, then $[v_t^m(w_t^m, g_t^m) - v_t^m(w_t^m, g_t^i)] + Z - C > 0$. Of course it could exist more than one middle class candidate, and because there is free entry of candidates, in general there will be as many candidates as needed to make the expected net gain of participating in the election equal to zero.

After this discussion we are able to state the following proposition.

Proposition 12 *Under a majoritarian electoral system the model has a unique political equilibrium with the following features. Only middle class candidates run for office and the equilibrium policy is:* $g_t^m = \frac{\rho(1-\alpha)}{1+\rho} w_t^m$.

Therefore, this case is equivalent in terms of equilibrium policy to the case of two exogenous vote-maximizing parties competing in a single-district majoritarian electoral system (proposition 1).

6.3.2 Proportional representation electoral system

The assumption of sincere voting implies that if a candidate of our class is available we will vote for her/him. This assumption together with the assumption of a PR system implies that there are opportunities for existence of candidates of the three different social classes (they all have now a positive probability of being elected). [15]

Additionally, if we assume that the parliament is large enough as to imply that a single additional seat for any of the parties does not affect the policy outcome, then the only variables that matter at the time of deciding participation are Z, C and the (endogenous) probability of being elected, q^i. A candidate will run for office if $q^i Z > C$. In equilibrium (if there is perfect competence) we will have enough candidates of each class as to make $q^i = C/Z$, and each class will win exactly s^i of the seats (more formally we are assuming that the parliament is composed by a continuum of legislators of mass δ, $0 < \delta < 1$).[16]

Note that at the parliamentary stage we will have exactly the same cases and results described in section 4.2.

6.4 Income, consumption and inheritance tax

Given our particular time separable log utility function, if we alternatively impose a proportional income, consumption or inheritance tax to finance public capital we arrive at the result that there is no disagreement across social classes about the optimal tax rate. This is consequence of the multiplicative characteristic of the tax rates and the log utility function. These results will not hold with more complex utility functions.

Therefore, under these alternative tax systems and the log utility function assumed here the electoral system does not matter for economic growth (see appendix I).

7 Comparative statics and dynamics

7.1 Comparative statics

For our purposes the most important comparative statics are those that come from variations in the electoral system and the distribution of the population

[15] Note that each class under PR can can win up to s^i of the seats of the parliament.

[16] Note that the number of candidates of class i will be greater than $s^i\delta$. $s^i\delta$ candidates will be elected with probability $q^i = 1$, and this will imply that each candidate will get an expected gain of $Z - C > 0$. However, if there is free entry and perfect competence, new candidates will arrive until $q^i Z - C = 0$ and then the number of candidates must be greater than $s^i\delta$. This implies that there are enough candidates as to elect $s^i\delta$.

across classes. In table 1 the results are summarized.

It is important to note that *per se* the PR and the M systems do not necessarily imply different economic growth. For example, if the middle class has a relative majority in the society both systems will generate a growth rate of $A_m E$. In order to produce a precise ranking, we need to know the distribution of people among classes. This could be one of reasons why recent empirical works (e.g. Persson, 2005) fail to find a clear link between electoral systems and growth.[17] In other words, we need to take into account the distribution of people among classes before making any prediction of the impact of the electoral system on growth.

Additionally, note that in our model it is not true that democracy will deliver always faster growth than dictatorships. In fact, the raking in terms of economic growth and public investment (from higher to lower) is:

1. PR in societies with relative majority of the rich class and rich class dictatorships, $A_r E$,

2. middle class dictatorships, M systems and PR in societies with relative majority of the middle class, $A_m E$, and finally

3. poor class dictatorships and PR in societies with relative majority of the poor class, $A_p E$.

It is important to note that in our model higher growth does not imply necessarily higher welfare, so our theory cannot be interpreted as an argument in favor of a pro-rich class government. Distributional considerations are important in our criterion of welfare. Moreover, even when we only consider growth rates, the social optimum implies a growth rate equal to $B^{1-\alpha} A_r E$, which is smaller than the growth rate of an economy under a dictatorship of the rich class, $A_r E$. Therefore, a dictatorship of the rich class cannot be social optimal.

7.2 Comparative dynamics

In this subsection we will calibrate and simulate our model. We calibrate the model with parameters estimated in Malley et al. (2007) and other works for the U.S. economy.

[17] There are also methodological problems in some of these works. We will discuss this on chapter 4.

Table 1: Political institutions and economic growth

Political Institution	Growth $(\mu)^{(1)}$	Public investment $(g_t)^{(2)}$
Democracy		
Majoritarian	$A_m E$	$F w_t^m$
Proportional Representation		
i. $\max\{s^p, s^r\} < s^m$	$A_m E$	$F w_t^m$
ii. $\max\{s^m, s^r\} < s^p$	$A_p E$	$F w_t^p$
iii.a $\max\{s^m, s^p\} < s^r$ and $g_0^r < \bar{g}_0^m$	$A_r E$	$F w_t^r$
Dictatorship		
Poor class dictator	$A_p E$	$F w_t^p$
Middle class dictator	$A_m E$	$F w_t^m$
Rich class dictator	$A_r E$	$F w_t^r$
Social Planner	$B^{1-\alpha} A_r E$	$F w_t$

Note: ${}^{(1)}$ $E \equiv (1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}$; $A_p < B^{(1-\alpha)}A_r < A_r$, ${}^{(2)}$ $F \equiv \frac{\rho(1-\alpha)}{1+\rho}$.

Malley *et al.* (2007) estimates a $(1-\alpha) \simeq 0.3$, Romp (2007, ch.5) summarizes many other studies that generally find values below 0.4. Again, Malley *et al.* (2007) presents an estimate of $\rho \simeq 0.96$. However, note that in our case c_1 and c_2 are a kind of total consumption of periods 1 and 2 of the life of an individual thus, each of these periods can be of 30 or 40 years. Therefore, a more appropriate discount factor for our model would be $\rho \simeq (0.96)^{35}$.

From our model we know that the optimality conditions imply $\frac{w_{t+1}^i}{c_{2,t}^i} = \frac{1-\gamma}{\gamma}$. Most of the studies have found that inherited wealth is in the range of 15% to 40% of total bequeathable wealth (see Hurd and Smith, 2002, Kopczuk and Lupton, 2007, and references therein). This bequeathable wealth is estimated at $191,000 by Hurd and Smith (2002), while Kopczuk and Lupton (2007) estimate the wealth of the elderly at $170,731, both for the year 1998. Consequently, the bequest as a proportion of the yearly consumption can be anywhere between 1.35 and 4 (considering the consumption per capita of U.S. in 1998). Taking into account that $c_{2,t}^i$ is the consumption in the second period of life (say in 35 years), a round value of $(1-\gamma) = 0.1$ seems to be in the admissible range and good as any other in the range.

Pressman (2007), defining the middle class as those households with income between 0.75 and 1.25 of the median household income and found that for the U.S. in the year 1999 $s^p = 0.35$, $s^m = 0.29$, and $s^r = 0.36$. The share of household incomes by quintiles for this year in U.S. was 3.6%, 8.9%, 14.9%, 23.2% and 49.4% (US Census Bureau, 2000). Thus, the top 40% had 72.6% of the income, the bottom 40%, 12.5% and the middle 20%, 14.9%. Therefore, on average each household percentile in the top 40%, middle 20% and bottom 40% had 1.815%, 0.745% and 0.3125% of the total income, respectively. Thus, based on this distribution we will calibrate the model with $w_0^p = 42$, $w_0^m = 100$, $w_0^r = 244$.

Unfortunately we don't have estimations of total factor productivity differentials across social classes. We will calibrate our model assuming a relatively small productivity differential of approximately 10% from one class to another (the absolute values were chosen as to generate a long-run growth of approximately 3% per year under a M system; note that 3% is the average growth rate of the U.S. economy in the period 1978-2007).

In sum, to analyze the short run dynamics of the model and the convergence to equilibrium we will take the following parameterization of the model: $\rho = 0.24$, $\gamma = 0.9$, $\alpha = 0.7$, $A_p = 682$, $A_m = 750$, $A_r = 825$, $w_0^p = 42$, $w_0^m = 100$, $w_0^r = 244$, and $s^p = 0.35$, $s^m = 0.29$, $s^r = 0.36$.[18]

The dynamics shown in the following graphs come from iterating in the following equations (equations (10) and (12)),

$$
\begin{aligned}
g_t^j &= \frac{\rho}{1+\rho}(1-\alpha)w_t^j, \text{where } j \text{ is the class that "chooses" the policy,} \\
w_{t+1}^i &= (1-\gamma)A_i\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}(w_t^i - g_t^j)^{\alpha}(g_t^j)^{1-\alpha}, \text{ for } i,j = p,m,r,
\end{aligned}
$$

and computing $w_t = \sum_i s^i w_t^i$.

Under the proportional representation system, the rich class chooses the fiscal policy while under the majoritarian electoral system the middle class chooses the policy. The different economic performance is completely due to the difference in total factor productivity between the middle class and rich class. This implies that under a proportional representation system, the economy will grow a 10% more in a generation than under a majoritarian electoral system (or what it is the same an additional 0.14 percentage points per year; lifetime=70 years).

[18]Using this parameter configuration, it can be verified that the utility of the middle class is greater under the best policy for the rich class that under the default policy. At t=0, the default utility is 2 while the utility under g_o^r is 2.8. Then we are in case iii.a., where the rich class can implement its optimal policy.

The graphs show the yearly growth rate of wealth (=GDP) for generations 1 to 20. Even thought the impact of the different electoral systems in the long run growth is relatively small, (since we are assuming a modest productivity differential), they imply a completely different growth path of the wealth of the different classes during the first 20 generations. This evidence points to an important fact. When analyzing particular country cases, it is sometimes more relevant to understand the transitional dynamics of the system (that can last 20 generations, as in our model!) than comparing two equilibrium situations.

Interestingly, under both systems the inequality is reduced if we compare the equilibrium situation with the initial point. The relative wealth go from $\frac{w_0^p}{w_0^m} = 0.42$, $\frac{w_0^r}{w_0^m} = 2.44$ to $\frac{w_t^p}{w_t^m} = 0.644$, $\frac{w_t^r}{w_t^m} = 1.472$ under a PR electoral system and to $\frac{w_t^p}{w_t^m} = 0.680$, $\frac{w_t^r}{w_t^m} = 1.435$ under a majoritarian electoral system (as $t \to \infty$). Note that the majoritarian electoral system implies a convergence to a more equal society with the current configuration of parameters.

Figure 2: Majoritarian electoral system

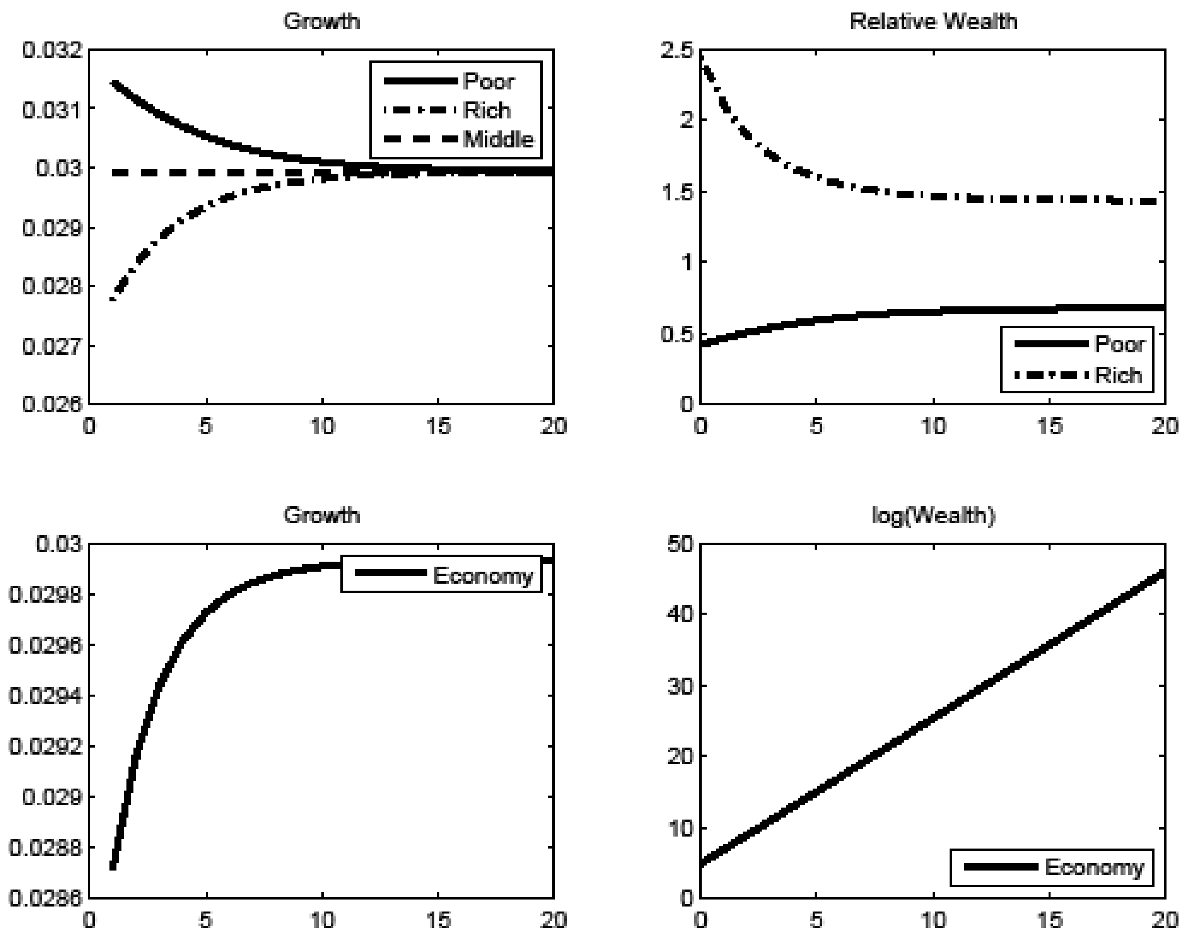
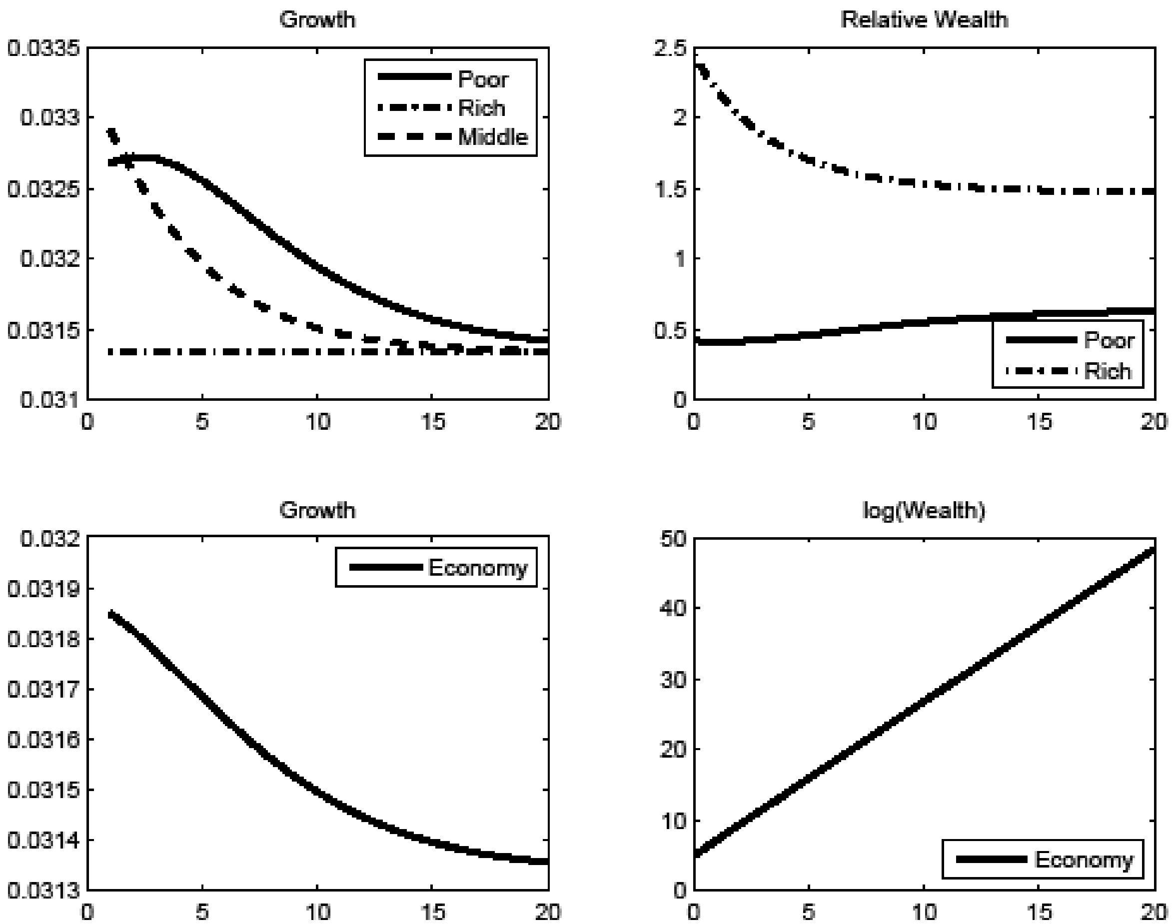


Figure 3: Proportional representation electoral system



8 Some illustrations

8.1 China

China perfectly illustrates that growth is not *per se* dependent upon whether the government is a dictatorship or a democracy. We can use communist China as an example of a dictatorship that delivers high economic growth if we look at the recent history of the country (1978-), or an example of exactly the opposite if we focus the attention on the period 1949-1977.

What seems clear about the Chinese economic revolution is that behind the proximate factors that account for growth (capital, labor, land) lay important institutional transformations.

China's economic reform was led since 1978 by a more "right-wing" (and probably more pro-rich class) government in comparison with its predecessors. It is also interesting to note that the distribution of people across classes seemed to change accordingly. In 1970, the distribution of people according to classes (p, m and r) were 0.28, 0.38 and 0.34 respectively but in 1978 this distribution changed to 0.26, 0.32 and 0.42 which represents a change from a relative majority of the middle class to a majority of the rich class. This could potentially explain the change in growth rates according to our theory.[19]

If we have to trust the predictions of our model and assuming that we have indeed a more pro-rich government since 1978, we should look for a more pro-middle or pro-rich class policy after the seventies. This should also imply higher public investment as proportion of GDP and faster economic growth.

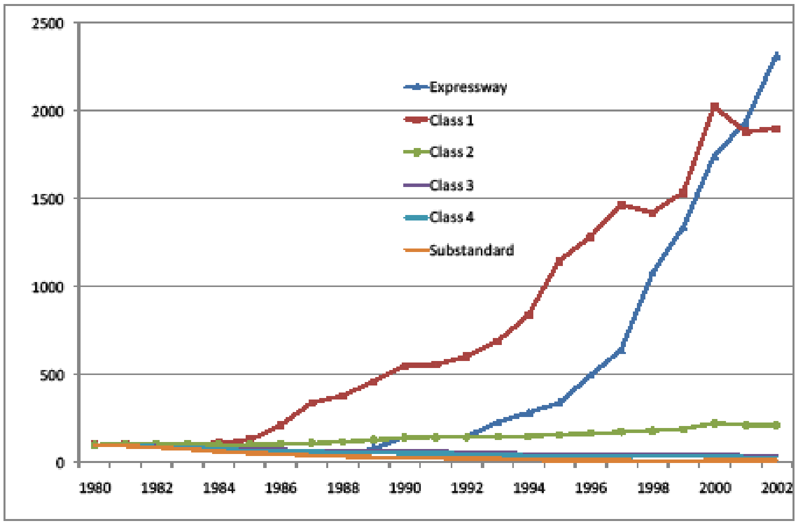
Chow (2007, ch.5) estimates that the reforms introduced after 1978 account for 2.8 annual percentage points of increase in the growth rate of the Chinese economy. This evidence supports the prediction of our theory.

With respect to the public investment, China had undergone a vertiginous process of infrastructure construction since mid-80s. Figures 4 and 5 illustrate the evolution in the length and change in length of roads by class

[19] Own calculations based on data from Sala-i-Martin (2006). Middle class was defined as the population with incomes between 0.75 and 1.25 of the median income; poor class as the population with less than 0.75 of the median income and rich class as the population with income greater than 1.25 of the median income.

Just as an additional piece of information, the distribution was: 0.34, 0.21 and 0.46 in the year 2000.

Figure 4: China: Length of roads relative to GDP (1980=100) (Fan *et al.*, 2005)

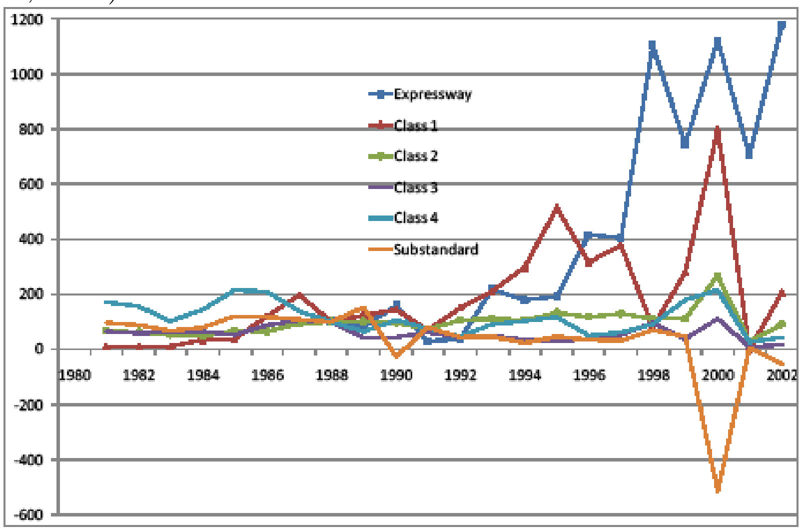


relative to the GDP.[20] These variables are proxies to the importance of public capital and investment as a proportion of GDP in China. As can be seen in both graphs, high standard roads (i.e. expressway and class 1 and 2) have had and important increase relative to GDP since mid-80. These kinds of roads were non-existent prior to the 1980s. Again, this is consistent with the prediction of our model.

There is also more anecdotal evidence of the very high level of public investment in pos-reform China—namely, China's mega projects. In 1994 China began the construction of the Three Gorges Dam on the Yangtze River. This dam is the world's largest hydro-electric power station with an estimated investment of 30 billion USD. In 2005 the Shanghai's Deep-water Port began operations and will be fully operative in 2020. This is one of the world's largest port projects that has ever been undertaken. In April 2008, China opened the world's longest cross-sea bridge, the Hangzhou Bay Bridge, measuring 36km and linking the cities of Shanghai and Ningbo. The estimated investment was 1.8 billion USD.

[20] The source of these data is Fan and Chan-Kang (2005).

Figure 5: China: Change in lengths of roads relative to GDP (1988=100) (Fan *et al.*, 2005)



8.2 New Zealand

New Zealand was one of the purest examples of a majoritarian electoral system during for most of the 20th century. In 1993 New Zealand held a nationwide referendum to choose one of two alternative electoral systems—a M system or a PR system. The PR was chosen with 54% of the votes (Vowles, 2005, Nagel, 2004).

Should we expect any change in the growth rate of the economy based on our theory? To answer this question we have to first know the distribution of people among classes in New Zealand. Table 2 presents this information.

As can be seen in table 2, we are in a case where $\max\{s^m, s^p\} < s^r$, i.e. the rich class has relative majority in society. According to our model, we should expect a middle class policy under the M system and rich class policy under a PR system. Therefore we should observe a higher economic growth rate in New Zealand after the 1993 reform.

Figure 6 shows the growth rate of New Zealand's GDP. As can be seen in the graph, the year 1993 seems to be an inflection point. In the period 1993-2004 the economy grew at 2.4% per year, while the annual growth rate for the previous eleven years (1982-1993) was 0.8%. If we take a longer perspective, say the period 1950-1993, we find out that the average annual growth rate

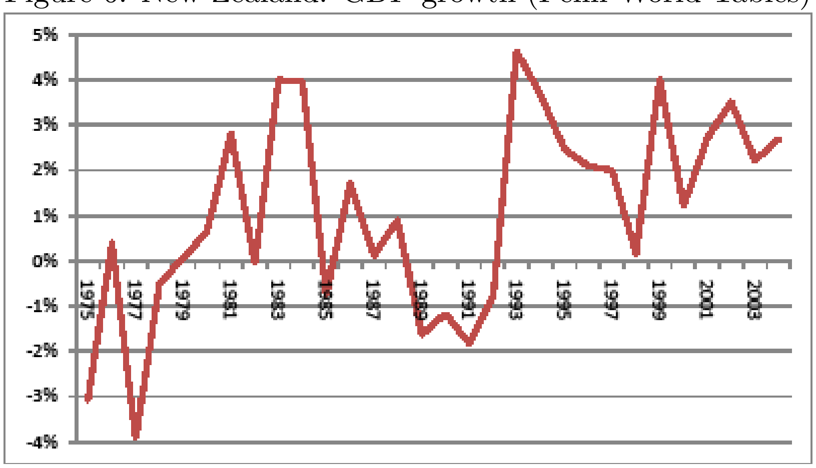
Table 2: Social classes in New Zealand

Social Classes (% population)	1970	1992	2000
Poor	28.9	30.5	33.6
Middle	27.8	25.8	24.0
Rich	43.3	43.6	42.4

Source: own calculations using Sala-i-Martin's (2006) data.

was 1.3%. Therefore the average growth rate in the period 1993-2004 was more than 1 percentage point higher than the historical growth rate under the M system.[21]

Figure 6: New Zealand: GDP growth (Penn World Tables)



Of course, this is anecdotal evidence at best but it suggests how our theory could be used to understand structural changes.

9 Conclusions

Is democracy better than dictatorship in terms of economic growth? Do different types of democracies deliver different economic growth?

[21] Of course we must be careful in attributing this additional growth only to the electoral system change since other factors are not fixed.

The distribution of people among classes seems to be the key factor to be taken into account before answering those questions.

This is not surprising. After all, political institutions are means of "summarizing" preferences and as such, they could be biased towards different classes and therefore deliver different policies and economic outcomes under different social structures.

Our model generates the following ranking in terms of economic growth: 1.) PR in societies with a relative majority of the rich class and rich class dictatorships; 2.) PR in societies with a relative majority of the middle class, M systems and middle class dictatorships; and 3.) PR in societies with a relative majority of the poor class and poor class dictatorships.

Our model represents only the first step towards comprehending how democratic and non-democratic political institutions affect economic growth. As such, there are many possible ways forward.

For example, it would be interesting to include social mobility in the model. This could advance our knowledge of how social dynamics interacting with institutions affect economic growth.

Secondly, we expect more extreme results in a dictatorship of the poor or rich class than under a PR system where the poor or the rich class has relative majority in society since the last type of institution has at least some checks and balances in place to avoid the "exploitation of one class by another." A more complex process of bargaining in parliament seems to be a possible way forward to obtain more intermediate results under a PR system.

Finally, the form of government (parliamentary vs presidential) is one of the most important democratic political institutions and as such, we expect it to influence economic policy and growth. Therefore, the analysis of the links between the form of government and economic growth is a very relevant line of research.

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A No-social-mobility property

We have to prove that $w_t^p < w_t^m < w_t^r$ $\forall t$. This is true for period 0 by assumption. By complete induction we will prove that this is also true for any $t > 0$.

First, let us prove that if $w_0^p < w_0^m < w_0^r$ then $w_1^p < w_1^m < w_1^r$. From (10) we have that $\frac{w_{t+1}^i}{w_{t+1}^m} = \frac{A_i}{A_m}\left(\frac{w_t^i - g_t}{w_t^m - g_t}\right)^\alpha$, then $\frac{w_1^i}{w_1^m} = \frac{A_i}{A_m}\left(\frac{w_0^i - g_0}{w_0^m - g_0}\right)^\alpha$. Now because $A_p < A_m < A_r$ and $w_0^p < w_0^m < w_0^r$, then $\frac{w_1^r}{w_1^m} = \frac{A_r}{A_m}\left(\frac{w_0^r - g_0}{w_0^m - g_0}\right)^\alpha > 1$ and $\frac{w_1^p}{w_1^m} = \frac{A_p}{A_m}\left(\frac{w_0^p - g_0}{w_0^m - g_0}\right)^\alpha < 1$. Therefore $w_1^p < w_1^m < w_1^r$.

Second, following the same steps as before it can be proved that if $w_{t-1}^p < w_{t-1}^m < w_{t-1}^r$, given that $A_p < A_m < A_r$, then $w_t^p < w_t^m < w_t^r$.

Then by complete induction, $w_t^p < w_t^m < w_t^r$ $\forall t$.

B Propositions 2 and 5

B.1 Existence of a steady-growth equilibrium and convergence

First note that because (from (13)) w_t^m is always growing at rate $\mu_m - 1$, if the ratio $\frac{w_t^m}{w_t^i}$ is constant then w_t^i must also be growing at rate $\mu_m - 1$. Using this argument we will prove that under some conditions an equilibrium exist.

From (13) and (14) we have that

$$\frac{\frac{w_{t+1}^i}{w_t^i}}{\frac{w_{t+1}^m}{w_t^m}} = \frac{cA_i}{\mu_m}\left(1 - \frac{\rho(1-\alpha)}{1+\rho}\frac{w_t^m}{w_t^i}\right)^\alpha \left(\frac{w_t^m}{w_t^i}\right)^{1-\alpha}, \tag{23}$$

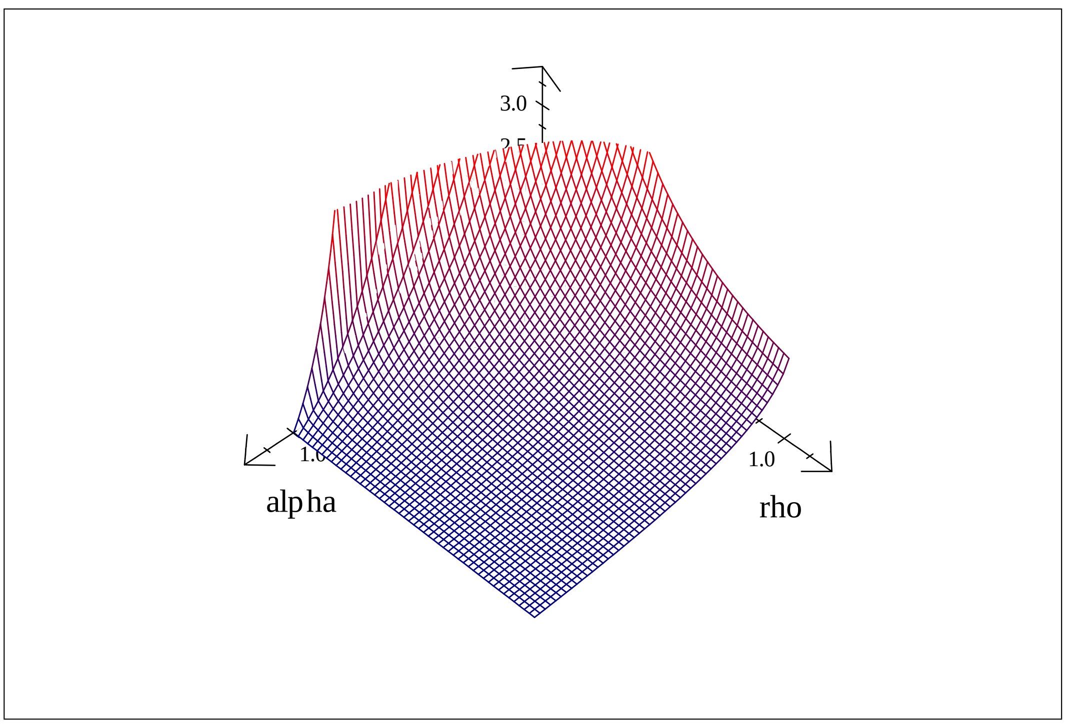
and this expression can be rewritten as

$$x_{t+1}^i = -d + b_{im}\left(x_t^i\right)^\alpha, \tag{24}$$

where $d \equiv \frac{\rho(1-\alpha)}{1+\rho}$, $b_{im} \equiv \frac{cA_i}{\mu_m} = \frac{A_i}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^\alpha$ and $x_t^i \equiv \frac{w_t^i}{w_t^m} - d$. Note that $0 < d < 1/2$, and $b > 0$.

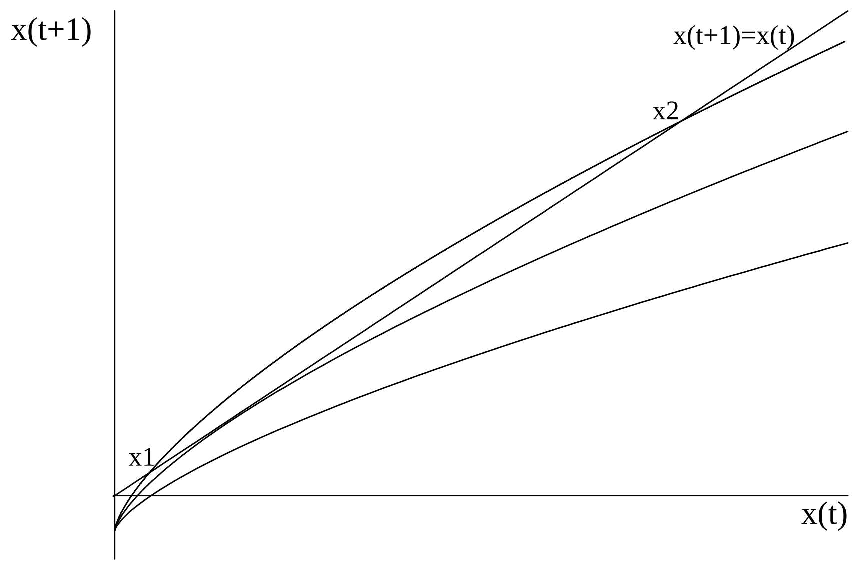
Let us analyze this non-linear first order difference equation. First note that because $0 < \alpha < 1$, this function is concave, and so we can have from 0 to

2 equilibria (see plot with the 3 different cases).[22] Call $f(x_t^i) = -d + b_{im}\left(x_t^i\right)^{\alpha}$. The bifurcation point (where we pass from 0 to 2 equilibria), is defined by $f'(x_t^i) = 1$, and takes the coordinates: $(x_t^i, x_{t+1}^i) = \left((\alpha b_{im})^{\frac{1}{1-\alpha}}, (\alpha b_{im})^{\frac{1}{1-\alpha}}\right)$. Therefore, the condition for existence of an equilibrium is $f\left((\alpha b_{im})^{\frac{1}{1-\alpha}}\right) \geq (\alpha b_{im})^{\frac{1}{1-\alpha}}$, or $-d + b_{im}(\alpha b_{im})^{\frac{\alpha}{1-\alpha}} \geq (\alpha b_{im})^{\frac{1}{1-\alpha}}$. In terms of parameters this condition is equivalent to: $\frac{A_m}{A_i} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. Note that because $A_r > A_p$, it is enough to impose the condition $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$ to guarantee the existence of an equilibrium. Note that for $\rho, \alpha \in (0,1)$, $1 < \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$, (see graph A.1, the z axis begins at z=1) therefore for any value of ρ, α it will exist A_m and A_p such that $A_m > A_p$ and $\frac{A_m}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. Thus, for A_m sufficiently close to A_p we know that an equilibrium exists.



[22] The two equilibria case uses the following parameterization: $\rho = 0.24$, $\gamma = 0.9$, $\alpha = 0.7$, $A_p = 682$, $A_m = 750$ (same as in simulations of chapter 2). The case of a unique equilibrium uses $A_p = 555$, and finally the case of no equilibrium $A_p = 400$.

Figure 7: Equilibria



Assuming that this condition is in place with strict inequality, we will have two balanced growth equilibria, x_1^i and x_2^i, both defined by the solution to the equation $x^i = -d + b_{im}(x^i)^\alpha$. Because the function $f(\cdot)$ is concave, the first one, x_1, is unstable and second one, x_2, is stable (note that at x_1, $f'(x_t^i) > 1$ while at x_2, $f'(x_t^i) < 1$).

Now, it is easy to verify that if x_0^i is at the right of x_1, the economy will converge to x_2. If the initial distribution of wealth implies a x_0^i that is at the left of x_1, then there is no convergence.

However, note that $x_0^r \equiv \frac{w_0^r}{w_0^m} - d$ is always at the right of x_1. To see this note first that $\frac{w_0^r}{w_0^m} > 1$, then $x_0^r > 1 - d$. Now, if we can prove that $f(1-d) > 1-d$, then we will know that $1-d$ is at the right of x_1 (more precisely $x_1 < 1 - d < x_2$) and because $x_0^r > 1 - d$ then x_0^r will be as well at the right of x_1. It can be verified that $f(1-d) > 1-d$ if and only if $\frac{A_r}{A_m} > 1$[23], and the later condition is true by definition. Thus, $x_0^r > x_1$ and the relative wealth of the rich class will always converge to the stable equilibrium x_2.

[23] $f(1-d) > 1-d \iff -d + b_{rm}(1-d)^\alpha > 1-d \iff b_{rm}(1-d)^\alpha > 1$ $\iff \frac{A_r}{A_m}(\frac{1+\rho}{1+\rho\alpha})^\alpha(\frac{1+\rho\alpha}{1+\rho})^\alpha > 1 \iff \frac{A_r}{A_m} > 1$ QED.

Unfortunately the same cannot be said about x_0^p. Note that $0 < \frac{w_0^p}{w_0^m} < 1$, therefore x_0^p is restricted by definition to the interval $(-d, 1-d)$, i.e. $-d < x_0^p \equiv \frac{w_0^p}{w_0^m} - d < 1 - d$. To ensure convergence it is necessary to impose additionally the condition $x_1 < x_0^p$. The question is, does such x_0^p exist? The answer is yes it does. We must prove that exist x_0^p such that $x_1 \leq x_0^p < 1-d$. In other words we must prove that $x_1 < 1-d$ or that the interval $(x_1, 1-d)$ is not empty.

First, it can be verified that $f(1-d) < 1-d$ if $\frac{A_p}{A_m} < 1$ (and this is the case here), then $1-d < x_1$ or $1-d > x_2$. Second, the following inequality $(\alpha b_{pm})^{\frac{1}{1-\alpha}} < 1-d$ is also always verified for $\frac{A_p}{A_m} < 1$[24], and $x_1 < (\alpha b_{pm})^{\frac{1}{1-\alpha}}$ (since we are assuming $f\left((\alpha b_{pm})^{\frac{1}{1-\alpha}}\right) > (\alpha b_{im})^{\frac{1}{1-\alpha}}$ and therefore $x_1 < (\alpha b_{im})^{\frac{1}{1-\alpha}} < x_2$), then $x_1 < 1-d$ (in fact $x_1 < x_2 < 1-d$). Therefore, the interval $(x_1, 1-d)$ is not empty QED.

B.2 Growth rates of other variables

All the variables are linked to wealth. Let us prove that if the wealth is growing at rate $\mu_m - 1$, then all the other variables of the economy will be growing at the same rate.

1. Growth rate of g_t. Note that

$$g_t = g_t^m = \frac{\rho}{1+\rho}(1-\alpha)w_t^m. \tag{25}$$

Dividing this expression by the equation lagged one period, and using $\frac{w_t^m}{w_{t-1}^m} = \mu_m$, we have that

$$\frac{g_t}{g_{t-1}} = \frac{w_t^m}{w_{t-1}^m} = \mu_m. \tag{26}$$

[24] $\frac{A_p}{A_m} < \frac{1/\alpha+\rho}{1+\rho} \Leftrightarrow \frac{A_p}{A_m} < \frac{1}{\alpha}\frac{1+\rho\alpha}{1+\rho}$
$\Leftrightarrow \frac{A_p}{A_m} < \frac{1}{\alpha}\left(\frac{1+\rho\alpha}{1+\rho}\right)^{1-\alpha}\left(\frac{1+\rho\alpha}{1+\rho}\right)^{\alpha} \Leftrightarrow \alpha\frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha} < \left(\frac{1+\rho\alpha}{1+\rho}\right)^{1-\alpha}$
$\Leftrightarrow (\alpha b_{pm})^{\frac{1}{1-\alpha}} = \left[\alpha\frac{A_p}{A_m}\left(\frac{1+\rho}{1+\rho\alpha}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}} < \left(\frac{1+\rho\alpha}{1+\rho}\right) = 1-d$. Note that the first inequality is verified for $\frac{A_p}{A_m} < 1$.

2.Growth rate of k_t^i for $i = p, m, r$. From (7) we have that

$$k_t^i = \frac{\rho\alpha}{1+\rho\alpha}(w_t^i - g_t). \tag{27}$$

Now, using the fact that $g_t = \mu_m g_{t-1}$ (31) and $w_t^i = \mu_m w_{t-1}^i$, the (1+) growth rate of the investment is

$$\frac{k_t^i}{k_{t-1}^i} = \frac{(w_t^i - g_t)}{(w_{t-1}^i - g_{t-1})} = \mu_m \frac{(w_{t-1}^i - g_{t-1})}{(w_{t-1}^i - g_{t-1})} = \mu_m. \tag{28}$$

3.Growth rate of y_t^i for $i = p, m, r$. Because the production functions have constant returns to scale with respect to k_t^i and g_t and both variables are growing at the same rate $\mu_m - 1$, y_t^i will grow at this rate.

C Properties of the indirect utility function

The indirect utility function for an individual of class i for $w_t^i > g_t$ is

$$v_t^i(w_t^i, g_t) = D_i + (1+\rho\alpha)\log(w_t^i - g_t) + \rho(1-\alpha)\log(g_t). \tag{29}$$

Because $D_p < D_m < D_r$ (since $A_p < A_m < A_r$) and $w_t^p < w_t^m < w_t^r$ we will have that $v_t^p(w_t^p, g_t) < v_t^m(w_t^m, g_t) < v_t^r(w_t^r, g_t)$, i.e. for a given level of g_t the utility is higher as higher is the wealth.

We already know that the indirect utility functions are concave and have a maximum at $g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i$, then $g_t^p < g_t^m < g_t^r$. Additionally, $\lim_{g_t \longrightarrow w_t^i} v_t^i(w_t^i, g_t) = -\infty$, and when $g_t = 0$, $u_t^i = \log(w_t^i)$ (since $k_t^i = y_t^i = 0$).

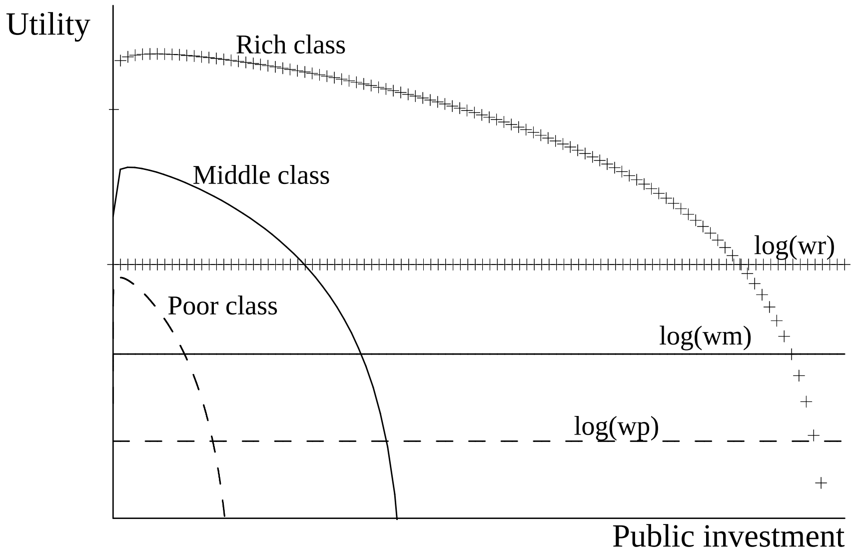
The following figure shows how the graph of the indirect utility function looks like for three individuals of three different social classes. We also included in the plot the "default" utility $u_t^i = \log(w_t^i)$ for $i = p, m, r$.[25]

D Proof that $\bar{g}_t^p < \bar{g}_t^m < \bar{g}_t^r$

We will prove that $\bar{g}_t^p < \bar{g}_t^m < \bar{g}_t^r$.

[25] The plot uses the same parameter's and initial variable's values assumed for the simulations of section 7.2.

Figure 8: Indirect utility functions



Totally differentiating $\log(w_t^i) = D_i + (1+\rho\alpha)\log(w_t^i - g_t) + \rho(1-\alpha)\log(g_t)$ we can find the effect of an increase in wealth on $\bar{g_t^i}$ (this is valid even when the increase is from w_t^p to w_t^m or to w_t^r because the indirect utility functions are identical, except for the term D_i, then we just have to take into account the additional increase due to changes in D_i when we change classes)

$$\frac{dg_t}{dw_t^i}_{g_t=\bar{g_t^i}} = \frac{\left(\frac{1+\rho\alpha}{w_t^i - \bar{g_t^i}} - \frac{1}{w_t^i}\right)}{\left(\frac{1+\rho\alpha}{w_t^i - \bar{g_t^i}} - \frac{\rho(1-\alpha)}{\bar{g_t^i}}\right)} > 0. \tag{30}$$

Note that the numerator is always positive since $1+\rho\alpha > 1$ and $w_t^i - \bar{g_t^i} < w_t^i$, and the denominator is also positive because $\bar{g_t^i} > g_t^i = \frac{\rho(1-\alpha)}{1+\rho} w_t^i$ (note that at $g_t = g_t^i$ the denominator is zero, from there up, i.e. for $\bar{g_t^i} > g_t^i$, is positive). When we go from $i = p$ to $i = m$, i.e. from w_t^p to w_t^m, we have then that $\bar{g_t^m} > \bar{g_t^p}$ (as said the increase will be bigger than (A.8) suggests because we have to add an additional positive effect due to the increase in D_i).

E Proposition 7

Similar to propositions 2 and 5.

E.0.1 $w_t^p < w_t^m < w_t^r \; \forall t.$

See no social mobility property above.

E.1 Existence

Now the condition will be $\frac{A_p}{A_m} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$. But note that this condition is always verified since $\frac{A_p}{A_m} < 1$ and $\frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha} > 1$ for $\rho, \alpha \in (0,1)$.

E.2 Convergence

Note that both $\frac{w_0^m}{w_0^p}, \frac{w_0^r}{w_0^p} > 1$. Thus, following the same lines of reasoning that in the proof of propositions 2 and 5 it can be shown that we always begin at the right of the unstable equilibrium. Therefore, there is always convergence to the stable equilibrium.

E.3 Growth rates of other variables

All variables are growing at rate $\mu_p - 1$. Proof: same as in propositions 2 and 5.

F Proposition 9

Similar to propositions 2 and 5.

This case is the most demanding in terms of the conditions that are necessary to verify for existence and convergence to equilibrium.

F.0.1 $w_t^p < w_t^m < w_t^r \; \forall t.$

See no social mobility property above.

F.1 Existence

Now the condition will be $\frac{A_r}{A_p} \leq \frac{1+\rho}{\rho}\left(\frac{\rho\alpha}{1+\rho\alpha}\right)^{\alpha}$.

F.2 Convergence

Note that both $\frac{w_0^p}{w_0^r}, \frac{w_0^m}{w_0^r} < 1$. Then for convergence we must impose the condition that both $\frac{w_0^p}{w_0^r} - d$ and $\frac{w_0^m}{w_0^r} - d$ are at the right of the unstable equilibrium.

F.3 Growth rates of other variables

All variables are growing at rate $\mu_r - 1$. Proof: same as in propositions 2 and 5.

G Social planner

The Lagrangian of this problem is

$$L = \sum_i s^i u_t^i + \lambda_1 \sum_i s^i (c_{1,t}^i + g_t + k_t^i - w_t^i) + \lambda_2 \sum_i s^i \left[c_{2,t}^i + w_{t+1}^i - A_i (k_t^i)^\alpha (g_t)^{1-\alpha} \right], \tag{31}$$

where λ_1 and λ_2 are Lagrangian multipliers.

G.1 FOC in production

The first order conditions with respect k_t^i, and g_t for $i = p, m, r$, are[26]

$$\lambda_1 = \lambda_2 \alpha \frac{y_t^i}{k_t^i}, \tag{32}$$

$$\lambda_1 = \lambda_2 \sum_i s^i (1-\alpha) \frac{y_t^i}{g_t}. \tag{33}$$

G.2 FOC in consumption and bequest

The first order condition with respect to $c_{1,t}^i, c_{2,t}^i, w_{t+1}^i$, for $i = p, m, r$, are

[26] We are assuming the existence of an interior solution.

$$c_{1,t}^i = \frac{1}{\lambda_1}, \tag{34}$$

$$c_{2,t}^i = \frac{\rho\gamma}{\lambda_2}, \tag{35}$$

$$w_{t+1}^i = \frac{\rho(1-\gamma)}{\lambda_2}. \tag{36}$$

G.3 FOC wrt λ_1 and λ_2

$$\sum_i s^i(c_{1,t}^i + g_t + k_t^i - w_t^i) = 0, \tag{37}$$

$$\sum_i s^i \left[c_{2,t}^i + w_{t+1}^i - A_i(k_t^i)^\alpha (g_t)^{1-\alpha}\right] = 0. \tag{38}$$

G.4 Economic growth under a social planner

Note that applying (19) and (20) and $c_{1,t}^i = c_{1,t}^j$, $\forall i, j$ to the first budget constraint (42) we have that

$$\sum_i s^i(c_{1,t}^i + g_t + k_t^i - w_t^i) = c_{1,t}^r + g_t + Bk_t^r - w_t = 0, \tag{39}$$

where $B \equiv s^p \left(\frac{A_p}{A_r}\right)^{1/(1-\alpha)} + s^m \left(\frac{A_m}{A_r}\right)^{1/(1-\alpha)} + s^r$ and $w_t \equiv \sum_i s^i w_t^i$.

Again using the condition $c_{1,t}^i = c_{1,t}^j$ $\forall i, j$, and $c_{2,t}^i = \frac{\gamma}{(1-\gamma)} w_{t+1}^i$, the second budget constraint (43) can be rewritten as

$$\sum_i s^i \left(c_{2,t}^i + w_{t+1}^i - y_t^i\right) = \frac{c_{2,t}^r}{\gamma} - y_t = 0, \tag{40}$$

where $y_t \equiv \sum_i s^i y_t^i$.

Now from (21) we have $g_t = \frac{(1-\alpha)}{\alpha} \frac{y_t}{y_t^r} k_t^r$, but $\frac{y_t}{y_t^r} = \frac{\sum_i s^i y_t^i}{y_t^r} = B$ (the last step follows from applying conditions (19) and (20) to y_t^i, $\forall i$), then

$$g_t = \frac{(1-\alpha)}{\alpha} Bk_t^r. \tag{41}$$

From the first, third and fourth FOCs ((37), (38) and (39)) we have $\frac{c^r_{2,t}}{c^r_{1,t}} = \rho\gamma\alpha\frac{y^r_t}{k^r_t}$, and then applying the previous result (46) to y^r_t,

$$\frac{c^r_{2,t}}{c^r_{1,t}} = C, \tag{42}$$

where $C \equiv A_r\rho\gamma\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} B^{1-\alpha}$.

Working with the previous 4 equations (44)-(47) we will be able to find the growth rate of the economy. First, using these 4 conditions we have that $\frac{c^r_{1,t}}{c^r_{2,t}} = \frac{w_t - g_t - Bk^r_t}{\gamma y_t} = \frac{1}{C}$, then $w_t - \frac{(1-\alpha)}{\alpha}Bk^r_t - Bk^r_t = \frac{\gamma}{C}y_t = \frac{\gamma}{C}A_r\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} B^{1-\alpha}Bk^r_t$ (the last result follows from $y_t \equiv \sum\limits_i s^i y^i_t$, and applying to it the production function definition and conditions (19), (20) and (46)). Rearranging terms,

$$k^r_t = \alpha\frac{1}{B}\frac{\rho}{1+\rho}w_t. \tag{43}$$

Using the previous result (48) and (46) we have that

$$g_t = (1-\alpha)\frac{\rho}{1+\rho}w_t. \tag{44}$$

Now, using the previous two results (48)-(49) in (44) we can find $c^r_{1,t}$

$$c^r_{1,t} = \frac{1}{1+\rho}w_t. \tag{45}$$

We know that $c^r_{2,t} = Cc^r_{1,t} = C\frac{1}{1+\rho}w_t$ ((47) and (50)) and also that $c^r_{2,t} = \frac{\gamma}{(1-\gamma)}w^r_{t+1}$ ((40)-(41)), and $w^i_{t+1} = w^j_{t+1}\forall i,j$ (41) (then $w_{t+1} \equiv \sum\limits_i s^i w^i_{t+1} = w^r_{t+1}$). Then $w_{t+1} = \frac{(1-\gamma)}{\gamma}\frac{1}{1+\rho}Cw_t$ and just using the definition of C and rearranging terms we have

$$w_{t+1} = A_r B^{1-\alpha}(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}w_t. \tag{46}$$

Then the (1+) growth rate of the economy under a Social Planner is

$$\mu_s \equiv A_r B^{1-\alpha}(1-\gamma)\frac{\rho}{1+\rho}\alpha^\alpha(1-\alpha)^{1-\alpha}. \tag{47}$$

H Alternative default policy

Assume that the middle class has relative majority. For period 0 this class will be able to choose $g_0 = g_0^m$, because for the rich class g_0^m is better than the default policy $g_0 = 0$. It is easy to prove this, just note that because we are in the increasing part of the utility function, any public investment g_0 strictly greater than 0 and smaller than g_0^r is better than the default public policy $g_0 = 0$. Note that $0 < g_0^m < g_0^r$ then g_0^m is preferred to 0.

Now for $t = 1$, note that because we are assuming that the parameters are such that there is economic growth, i.e. $w_1^m > w_0^m$, then $g_1^m = \frac{\rho(1-\alpha)}{1+\rho} w_1^m > g_0^m = \frac{\rho(1-\alpha)}{1+\rho} w_0^m$. Additionally, by the no-social-mobility property $w_1^r > w_1^m$ (appendix A), then $g_1^r = \frac{\rho(1-\alpha)}{1+\rho} w_1^r > g_1^m = \frac{\rho(1-\alpha)}{1+\rho} w_1^m$. Therefore $g_0^m < g_1^m < g_1^r$ and then for the same argument as before the rich class will prefer g_1^m to g_0^m.

Following the same steps as before we can prove that the rich class will prefer g_t^m to g_{t-1}^m as long as $g_{t-1}^m < g_t^m < g_t^r$ (and this is true by the no-social-mobility property).

By complete induction this will be true for any t.

Following the same lines as before, we can prove that if the poor class has relative majority, then for both m and r it is better to accept g_t^p in period t than the default policy g_{t-1}^p.

I Income, consumption and inheritance tax

I.1 Inheritance tax

Assume that the new government's budget constraint is

$$g_t = \tau_w w_t, \tag{48}$$

where τ_w is an inheritance tax.

Now the problem to be solved by agent i in period t is

$$\max_{k_t^i, w_{t+1}^i} u_t^i = \log(c_{1,t}^i) + \rho\gamma \log(c_{2,t}^i) + \rho(1-\gamma)\log(w_{t+1}^i), \tag{49}$$

s.t.

$$
\begin{aligned}
y_t^i &= A_i(k_t^i)^\alpha (g_t)^{1-\alpha}, &(50)\\
c_{1,t}^i &= (1-\tau_w)w_t^i - k_t^i, &(51)\\
c_{2,t}^i &= y_t^i - w_{t+1}^i. &(52)
\end{aligned}
$$

From the FOCs we have (we are assuming that each individual is small

in the sense that they can ignore the effect of their decision on the aggregate wealth)

$$
\begin{aligned}
k_t^i &= \frac{\rho\alpha}{1+\rho\alpha}(1-\tau_w)w_t^i, &(53)\\
w_{t+1}^i &= (1-\gamma)y_t^i. &(54)
\end{aligned}
$$

Therefore, the indirect utility function will be

$$v_t^i(w_t^i,\tau_w) = G_i + (1+\rho\alpha)\log(1-\tau_w) + \rho(1-\alpha)\log\tau_w. \qquad (55)$$

where G_i include all the terms that do not include the tax rate.

If we maximize the indirect utility function with respect to τ_w we obtain

$$\tau_w = \frac{\rho(1-\alpha)}{1+\rho}. \qquad (56)$$

Therefore, the optimal tax rate is the same for any i.

Now note that this will imply that the total public investment will be equal to

$$g_t = \frac{\rho(1-\alpha)}{1+\rho}w_t, \qquad (57)$$

and as we saw (equation (49)) this is the social optimal public policy.

I.2 Consumption tax

Assume now a consumption tax. The government budget constraint is now

$$g_t = \tau_c c_t, \qquad (58)$$

where $c_t = \sum_{i=p,m,r}\left(c_{1,t}^i + c_{2,t}^i\right)$, and τ_c is a consumption tax.

The budget constraints of dynasty i are

$$\begin{aligned}(1+\tau_c)c^i_{1,t} &= w^i_t - k^i_t, && (59)\\ (1+\tau_c)c^i_{2,t} &= y^i_t - w^i_{t+1}. && (60)\end{aligned}$$

It is easy to show that the optimal consumption tax rate is in this case

$$\tau_c = \frac{\rho(1-\alpha)}{1+\rho(\gamma+\alpha-1)}. \tag{61}$$

Again, the tax rate is independent of the level of wealth.

I.3 Income tax

Now the government's budget constraint is

$$g_t = \tau_y y_t, \tag{62}$$

where τ_y is an income tax.

The budget constraints of dynasty i and generation t are

$$\begin{aligned}c^i_{1,t} &= w^i_t - k^i_t, && (63)\\ c^i_{2,t} &= (1-\tau_y)y^i_t - w^i_{t+1}. && (64)\end{aligned}$$

In this case the optimal income tax rate is

$$\tau_y = \frac{(1-\alpha)}{1+(1-\alpha)}. \tag{65}$$

The tax rate is invariant with respect to wealth.