Cracks in the Glass Ceiling and Gender Equality: Do Exports Shatter the Glass Ceiling? *

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Abstract

Do female managers improve workplace outcomes for female employees? The evidence is mixed. We use Brazilian administrative employer-employee matched data over the large and unanticipated 1999 Brazilian *real* exchange rate devaluation and find that the share of female employees grows in female-led firms, but the male-female wage gap grows in both female- and male-led firms. This behavior is inconsistent with both theoretical models of taste-based employer discrimination and third-degree oligopsonistic labor markets by gender. Outcomes only differ between female- and male-led firms regarding managerial and supervisory employees, where female-led firms hire more female managerial and supervisory employees, yet the gender wage gap widens relative to male-led firms. We conclude that exports further crack the glass ceiling in female-led firms, though at the cost of a more expansive gender wage gap.

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1 Introduction

What are the implications of the growing female presence in corporate boards of directors and the corporate C-suite[1] for female employees? Many observers suggest that these cracks in the glass ceiling will be a fundamental step towards gender equality, as a larger female presence in management will cause women to act as "agents of change" and foster a gender equity supporting environment to redress past inequities in female representation and the gender wage gap (Cohen and Huffman, 2007). The empirical evidence of whether female managers act as "agents of change" regarding workforce balance and the gender wage gap is mixed (Flabbi, Macis, Moro and Schivardi, 2019; Bertrand, Black, Jensen and Lleras-Muney, 2019; Kunze and Miller, 2017).[2]

Estimating the causal effects of female managers as a visible signal of a gender equity-supporting environment on workplace outcomes is complicated by endogeneity and the possibility that outside factors, such as climate or a desire for positive publicity, may simultaneously cause both the decision to hire female managers and more equitable treatment of female employees. Using female leadership as a proxy for a gender equity supportive environment,[3] we believe this study is the first to exploit an exogenous export shock to obtain causal effects of the differential responses of female- versus male-led firms on gender related labor outcomes.

Brazil in the 1990s and early 2000s has many advantages as a venue for our study. First, the Brazilian economy experienced a large and unexpected exchange rate devaluation in January of 1999, which induced an export surge of manufactured goods and an expansion of employment in manufacturing (Araújo and Paz, 2014). Second, as Latin America's largest economy and most populous country, Brazil has a diversified manufacturing sector with significant gender inequality in labor market outcomes. Using data from the Brazilian 2000 census, Paz and Ssozi (2021) find a female employment share of 30 percent and a 40 percent male-female wage gap in manufacturing. However, these figures vary widely depending on the manufacturing industry. Third, government policies to promote gender equality were practically non-existent in this period. Finally, and of great importance, Brazil has very rich administrative employer-employee matched data from annual forms mandated by the Brazilian government. This data includes worker demographics, the industry of affiliation,

[1] Corporate leadership positions containing the word "chief," such as Chief Executive Officer, Chief Operating Officer, Chief Financial Officer, etc.

[2] See also Cardoso and Winter-Ebmer (2010), Hensvik (2014), Hirsch (2013), Pitts, Orozco-Aleman and Rezek (2014), and Stojmenovska (2019).

[3] Boards of directors in Brazil appeared more likely to choose a CEO who matched their gender profile (Madalozzo, 2011).

occupation, and wages, which enable us to estimate the dynamics of various aspects of gender inequality that occurred in response to the unanticipated devaluation of the *real*. We examine the proportion of female employees and the wage gap at the firm-level and for five occupational groups: 1. Unskilled White Collar, 2. Professional or Managerial, 3. Skilled Blue-Collar, 4. Technical or Supervisory (Skilled White Collar), and 5. Unskilled Blue Collar.

In the data of Brazilian manufacturing firms, we uncover several stylized facts relating to gender-based labor market outcomes. At first glance, larger firms have a higher proportion of female employees than smaller firms, but disaggregating by the gender of firm leadership reveals an interesting heterogeneity. The proportion of female employees increases in firm size for female-led firms but decreases for male-led firms.. The male-female wage gap is positively correlated with firm size for both genders of firm leadership; curiously, the male-female wage gap is greater in female-led firms than in male-led firms of a similar size. However, this behavior is consistent with a theoretical model of firms with monopsonistic power. We then use this model to predict the effects of an exchange rate shock-induced increase in the unit price of the output good. More specifically, we extend the Card, Cardoso, Heining and Kline (2018) model to employees by gender. In our model, third-degree oligopsonistic firms exploit the heterogeneous valuation of non-pecuniary amenities by gender to create a gender wage gap even when female and male labor are perfect substitutes.[4] Relative to male workers, female workers place a higher value on non-wage amenities such as workplace flexibility (Goldin and Katz, 2010; Vick, 2017; Corradini et al., 2025; Sharma, 2023) or a gender-equity supportive environment. Hence, we assume that only female workers consider female leadership a non-wage amenity. High amenity firms, proxied by those with female leadership, can both hire a larger proportion of female employees and have a wider wage gap than low amenity male-led firms. Despite these differences, our model predicts that additional hiring in response to an exogenous export shock will not change the gender wage gap or the proportion of female workers in female versus male-led firms.

Our empirical work builds upon Verhoogen (2008), who used the 1994 devaluation of the Mexican *peso* to estimate causal relationships between firm exports and wages, taking into account heterogeneity by firm size. Verhoogen's (2008) estimates are causal because manufacturing firms must quickly hire additional labor to meet the growing demand for their products in export markets caused by the exogenous and unanticipated devaluation. We

[4] Corradini, Lagos and Sharma (2025) provide direct evidence that female workers value non-pecuniary amenities more than male workers in Brazil. This gender-based heterogeneous valuation of amenities implies a lower female wage elasticity of labor supply relative to male workers, as found in the U.S. (Ransom and Oaxaca, 2010; Webber, 2015) and Brazil (Vick, 2017).

apply Verhoogen's (2008) methods to estimate changes in the proportion of female employees and the male-female wage gap around the 1999 devaluation of the Brazilian *real* separately for firms with male and female leadership. We extend this methodology to a second dimension of firm heterogeneity — the gender of firm leadership gender — and estimate triple differences to infer the differential behavior of female-led relative to male-led firms of similar size. Female-led firms significantly increase the proportion of female employees, but male-led firms do not. The male-female wage gap further widens in all firms regardless of the gender of firm leadership. Relative to male-led firms, additional increases in the wage gap by female-led firms are positive but imprecisely estimated. Disaggregated by occupational category, we find that the change in the percentage of females and the wage gap among Professional and Managerial, Skilled Blue-Collar, and Technical or Supervisory employees further increases in female-led firms. Relative to male-led firms, the increase is only significant for Professional or Managerial employees, which suggests that the 1999 devaluation further cracked the glass ceiling only in female-led firms in the manufacturing sector in Brazil. More precisely, females in leadership positions did help only other females in professional and managerial positions to go through the cracks, but did not necessarily improve the gender wage gap. This implies a constructive role for additional policies to address the gender gap.

In the remainder of our paper, we review related literature, describe the Brazilian data, and develop a series of stylized facts to characterize the relationship between gender-based outcomes and firm size. We develop a theoretical model and an empirical strategy to estimate testable predictions from theory and stylized facts, discuss our empirical results, and conclude.

2 Literature Review

Goldin and Katz (2010) examine the evolution of workplace flexibility, gender workforce balance, and the gender wage gap in medicine, veterinary medicine, business, and pharmacy. As changes in technology and firm ownership (corporate vs sole proprietorship) have reduced the cost of workplace flexibility, workforce gender balance and the gender wage gap have improved. From this perspective, it is of great importance to consider the role of organizational climate on both the gender of firm leadership and gender-based labor market outcomes. A "female-friendly" climate may cause a firm to crack its glass ceiling, have female leadership, offer more equal wages, and employ a greater share of female employees. Beyond issues of causality, it is challenging to construct objective measurements of firm climate and infeasible to completely dissociate it from the gender of firm leadership. Hence, we follow the extant literature and consider female leadership and a female-friendly organizational climate to be equivalent. Bøler, Javorcik and Ulltveit-Moe (2018) find that Norwegian exporting firms exhibit a differential gender wage gap, which seems to be related to a lack of workplace schedule flexibility of female workers with a college degree at childbearing age.

Opportunities are rare to directly measure firm-level responses to exogenous changes in the gender of firm leadership.[5] Scholars have used government mandates that increase gender equity in corporate leadership to estimate the effects of female leadership on corporate outcomes, but have found little impact on gender equity below the executive level (Bertrand et al., 2019; Matsa and Miller, 2013).[6]

In 2003, Norwegian legislation mandated that the proportion of female members on corporate boards of directors remain between 40 and 60 percentage points. Several studies use this legislation to estimate changes in gender-based outcomes as firms were required to increase the proportion of female directors sharply. Bertrand et al. (2019) document that the gender gap in earnings among board members declined dramatically. Nonetheless, these benefits did not extend to other female employees of firms affected by the legislation. In contrast, Kunze and Miller (2017) find that increased female corporate leadership benefits women lower in the corporate hierarchy. Finally, Matsa and Miller (2013) observe that Norwegian firms with a mandated increase in female directors implement fewer workforce reductions, which increases relative labor costs and employment levels and reduces short-term profits relative to firms unaffected by the legislation. Matsa and Miller (2013) attribute these outcomes to hiring like-minded executives. Similarly, Maida and Weber (2019) study the 2011 Italian law mandating step-wise increases in female membership of corporate boards and find only moderate and imprecisely estimated spillover effects.

Important insights can be gleaned from examining the employment and compensation decisions of firms in response to exogenous economic shocks as opposed to direct government mandates. Black and Strahan (2001) examine changes in rent-sharing with female versus male employees as U.S. states legalized interstate branch banking in the late 1970s and 80s. After deregulation, they find a much larger reduction in male employee salaries than in female salaries. Black and Brainerd (2004) find that globalization caused the salaries of lower skilled workers in manufacturing industries to decline but also appeared to benefit

[5] See Baron, Ganglmair, Persico, Simcoe and Tarantino (2024) for a natural experiment that exploits random variation in the gender composition to the committee that nominates members to the Internet Engineering Task Force.

[6] An additional study of the effects of exogenous policy changes on firm outcomes is Huber, Lindenthal and Waldinger (2021), who estimates reductions in the stock price, dividends, and return on assets in firms that were forced to dismiss Jewish managers in Nazi Germany. Yet, there is no gender dimension to this study.

women by increasing their share in employment. Most importantly, none of these studies have examined the differential response of female-led and male-led firms.

3 Data and Stylized Facts

The data used in this study comprises two firm-level confidential administrative databases covering 1995-2004, which were merged by firm tax identification codes. The first dataset contains records of firm-level export operations provided by the *Secretaria de Comércio Exterior* (Secretary of Foreign Trade) of the *Ministério do Desenvolvimento, Indústria e Comércio Exterior* (Ministry of Development, Industry and Foreign Trade). It contains information about each firm-level export transaction, including the product code, value, and destination country of each exported good. The second dataset is the RAIS (*Relação Anual de Informações Sociais*) from the *Ministério do Trabalho e Emprego* (Ministry of Labor and Employment). The RAIS form contains each firm's tax identification code and each employee's identification number, gender, age, level of educational attainment, and occupation. Every firm is required by law to annually fill out and file this RAIS form disclosing the labor contracts for all of its employees, even if the duration of employment were as brief as a single day. A firm with no employees must still file a RAIS form stating it has no employees. These characteristics make the RAIS dataset effectively cover all formal employment except self-employed and informal workers.[7]

We consider a worker employed at a specific firm during a given year if she (or he) worked for at least one day in December of that year.[8] We retain only firms with at least 20 employees. Additionally, we restrict the scope of our data to manufacturing firms (Brazilian Industry Classification CNAE version 1.0 codes ranging from 15 to 37) as they produce tradable goods that can be exported (Black and Brainerd, 2004; Tsou and Yang, 2019). These firms are more likely to be directly affected by a large currency devaluation, the quasi-exogenous source of variation exploited by our empirical strategy.

At the employee level, we dropped from the sample workers who were younger than 15 years of age or older than 65 years of age. Worker records with missing observable characteristics and wages are also excluded. The characteristic with the highest proportion of missing values is occupation, with approximately 1.5% of the employee records missing. These missing observations exhibit no discernible pattern in industry, year, worker characteristics, or

[7]The use of informal workers in exporting firms is highly unlikely, as most informal jobs are created by firms typically employing less than 30 employees. For a detailed study of informal workers in Brazil, see Paz (2014).

[8]For further detail on data sources and the construction of our data set, see Appendix A.1.

firm size. Our balanced panel contains 60,095 firms that employed, on average, three million workers annually.

Within firms, we group employees into broad categories based on skill level. We define a skilled worker as an employee who has graduated from high school. We further disaggregate employees by firm into the following classification of occupations: Professional and Managerial, Skilled White-Collar, Unskilled White-Collar, Skilled Blue-Collar, and Unskilled Blue-Collar (Helpman, Itskhoki, Muendler and Redding, 2017). Even though these categories are broad, this classification is not significantly affected by misclassification problems present in highly disaggregated occupational data (Kambourov and Manovskii, 2009).

To classify the gender of firm leadership, we define a firm as having male leadership if, between 1995 and 2004, females were never employed as either managers or directors, equivalent to C-suite positions in the USA. [9] A firm is designated as having female leadership if it employs females as either managers or directors in at least one of the years between 1995 and 2004. By this definition, a firm is considered female-led for the duration of our study if it employs just one female in a leadership position for a single year. This is because hiring a female leader indicates an environment more amenable to female workers, which likely existed before hiring the female leader. Most importantly, it is unobservable whether or not this firm had previously hired a female leader due to search and matching frictions.[10]

Table 1 presents the firm-level descriptive statistics for all firms and the subsamples of female-led and male-led firms for 1995 and 2004. Using the criterion above, the sample has 49,803 female-led firms that, as a group, employed an average of 1,225,842 workers per year and 10,292 male-led firms that, as a group, employed an average of 1,813,944 workers per year. Columns 4 and 8 report a difference of means t-test between female- and male-led firms. Even though female-led firms are more numerous in our sample, male-led firms have dramatically more employees and are more export-intensive, both of which are conducive to paying higher wages (Verhoogen, 2008; Frías, Kaplan and Verhoogen, 2012; Araújo and Paz, 2014). Female-led firms have a much smaller proportion of employees with college degrees and a smaller proportion of skilled workers.[11] The remainder of Table 1 reports the proportion of female employees by selected worker attributes and occupations. Male-led firms employ a larger share of female workers in four occupational categories: Unskilled White-Collar (1995 only), Professional or Managerial, Skilled Blue-Collar, and Unskilled

[9] The occupation codes for these positions are listed in Appendix Table A.1

[10] We conduct robustness checks of our results using more stringent definitions of firm-level leadership gender.

[11] In a cross-country study, Ashraf, Delfino, Glaeser and Solmone (2025) finds that female-led firms are also typically smaller and less human-capital intensive than male-led firms.

Blue-Collar. Next, we describe how we construct the other outcome of interest: the gender-based wage gap.

3.1 Male-Female Wage Gap

We follow Berik, Rodgers and Zveglich (2004) to construct the male-female wage gap. We first estimate a Mincer-type wage equation using data from male employees separately for each year of the data sample.

$$w_{ijt} = \theta_0 + \mathbf{x'_{it}}\gamma_\mathbf{t} + \epsilon_{ijt} \tag{1}$$

where w_{ijt} is the logarithm of the hourly wage received by worker i employed by firm j in year t, θ_0 is a constant, $\mathbf{x_{it}}$ is a vector of the worker's observable characteristics (age, age squared, years of education, college degree indicator, tenure at firm j, and state of residence fixed effects), and a stochastic disturbance term, ϵ_{ijt}.[12] The estimated coefficients of regressions based on Equation (1) are reported in Appendix Table A.2. We find evidence that the hourly wage is increasing in the worker's age at a decreasing rate and increasing in years of tenure. Each year of additional schooling adds approximately seven percent to the hourly wage. Workers with a college degree earn 70% higher hourly wages. These estimates are comparable to those from extant literature.

Using the estimated coefficients from Equation (1), we calculate the firm- and occupation by firm-level average male-female wage gap. For both male and female workers, we compute predictions of the worker's return to her observable characteristics component, $\widehat{s}_{it} \equiv \mathbf{x'_{it}}\widehat{\gamma}_\mathbf{t}$. Equation (2) specifies the firm-level male-female wage gap as the difference between the average residual wages for male and female employees by firm and by year. $N^{♂}_{jt}$ is the number of male employees of firm j in year t. This difference in residuals captures the potential effect of discrimination on the returns of both observable and unobservable worker characteristics:

$$(\widehat{w^{♂} - w^{♀}})_{jt} \equiv \frac{1}{N^{♂}_{jt}} \sum_{k=1}^{N^{♂}_{jt}} (w_{ijt} - \widehat{s}_{kjt}) - \frac{1}{N^{♀}_{jt}} \sum_{k=1}^{N^{♀}_{jt}} (w_{ijt} - \widehat{s}_{kjt}) \tag{2}$$

In addition to the average male-female wage gap, we compute the occupation-firm wage gap as the difference between the average residuals of male and female workers for each occupation-firm unit. We also calculate the firm-level wage gap for workers with and without

[12] We do not include any controls for the worker's occupation or firm characteristics because the job held and its characteristics may be themselves the result of gender discrimination.

college degrees and for skilled and unskilled workers. These measurements of the wage gap are only defined for firms with male and female employees in the category, which results in a smaller number of wage gap estimates for some categories.

Table 2 reports summary statistics for the male-female wage gap by occupation, skill, and education categories. Relative to male-led firms, the wage gap is larger in female-led firms for Unskilled White-Collar employees and Professional or Managerial employees. It is also smaller in female-led firms for Skilled Blue-Collar employees and Technical or Supervisory employees. When dividing employees by education level, we find that employees with college degrees experience a higher wage gap in female-led firms. Employees without college degrees exhibited a lower wage gap in female-led firms than in male-led firms in 1995 but a higher gap in 2004. Unskilled employees experience a smaller wage gap in female-led firms, while the wage gap for skilled workers in male-led firms was larger in 1995 and smaller in 2004 than that of female-led firms. The final row of Table 2 shows that male workers earned, on average, approximately 25 percent more than female workers. Female-led firms exhibited a narrower wage gap than male-led firms in 1995, which was reversed in 2004.

3.2 Stylized Facts by Firm-Size and Gender Outcomes

The empirical literature includes studies in which male-led and female-led firms differ in gender-based outcomes, such as the share of female employees and the gender wage gap. Interestingly, no clear pattern emerges – female-led firms exhibit a wider wage gap in some studies and a narrower gap in others. While these studies differ in methodology and sample characteristics, none account for firm heterogeneity, especially in firm size as measured by the number of employees. Evidence shows that firm size is related to firm-level employment composition outcomes, such as skills, in both Mexico (Verhoogen, 2008) and Brazil (Araújo and Paz, 2014). The relationship between firm size and gender-related labor outcomes has yet to be explored. Heterogeneity in the employment level of female-led and male-led firms (Table 1, Row 1) could contribute to these divergent results. Suppose that the workforce of female-led firms contains a higher proportion of female workers than male-led firms of the same size, or a higher female share. Nevertheless, if male-led firms are generally larger than female-led firms, then it is possible in aggregate that they appear to employ a greater share of female employees. With this in mind, we examine firm size and gender outcomes using a cross-sectional linear regression of gender labor outcomes on firm size with industry and state fixed effects. Table 3 reports the coefficients of these regressions using data for 1995 and 2000. The first row of Table 3 shows a positive and statistically significant estimated coefficient of the firm size for the share of female workers in the all-firm sample, similar to results found in U.S. data (Carrington and Troske, 1998).[13] After splitting our sample by the gender of firm leadership, female-led firms exhibit a positive correlation between firm size and female share. In contrast, male-led firms display a negative and significant correlation. We state these as the following three stylized facts.

Stylized Fact 1. *The proportion of female workers increases in overall firm employment for female-led firms.*

Stylized Fact 2. *The proportion of female employees decreases in overall firm employment for male-led firms.*

Stylized Fact 3. *The proportion of female employees is larger in female-led firms than in male-led firms of similar size.*

We conduct similar estimates with data disaggregated by educational and skill categories. Employment in the firms we study is predominantly non-college degree workers and unskilled workers. The share of female workers in these categories is increasing in firm size for female-led firms and decreasing for male-led firms. The share of female skilled workers increases with firm size in female-led firms. In contrast, the relationship in male-led firms is positive and much smaller for 1995 and negative for 2000.[14] The female share among skilled and college degree holders is also increasing in firm size. The relationship is larger for female workers with college degrees in male-led firms than in female-led firms.[15]

The remaining rows of Table 3 report cross-sectional regressions of several male-female wage gap measurements on firm size. In the sixth row of Table 3, the estimated coefficient for firm size is positive and statistically significant for the average wage gap. This motivates the following stylized fact.

Stylized Fact 4. *The male-female wage gap is positively related to overall firm employment.*

The estimated coefficients of firm size on the wage gap are positive and significant for each category of education and skill level for firms in Rows 7 through 10. Next, we contrast the magnitudes of the estimated effect of firm size on the wage gap by the gender of firm leadership, in Columns 2, 3, 5, and 6. All estimated coefficients are positive and highly significant for both male- and female-led firms in the pre- and post-devaluation periods. In

[13] The estimated coefficients for the regressions using data for the remaining years have the same sign and statistical significance, albeit there is some variation in magnitudes.

[14] This is the only outcome in Table 3 that exhibits alternating signs by year.

[15] This is consistent with the cross-country study of Ashraf et al. (2025).

almost all cases, the estimated coefficients for female-led firms are greater than those for male-led firms. This leads to the following stylized fact.

Stylized Fact 5. *The male-female wage gap is wider in female-led firms than in male-led firms of similar size.*

Table 4 presents cross-sectional estimates for the five broad occupational groups. The female share is increasing in the firm size for each occupational group. It is greater in female-led firms as in Stylized Fact 3, except for Unskilled Blue-Collar employees, where the coefficients for female- and male-led firms do not significantly differ. Turning to the male-female wage gap by occupational group, the estimated coefficients of firm size are positive, indicating a wider wage gap in larger firms, as in Stylized Fact 4. Interestingly, the estimated coefficients for female-led firms exceed those of male-led firms, consistent with Stylized Fact 5. The coefficients in Table 4 indicate that the previous Stylized Facts cannot be attributed to differences in occupational composition between male- and female-led firms.

For most outcomes, the estimated coefficient using 1995 data (pre-devaluation) differs significantly from estimates using 2000 data (post-devaluation) in Tables 3 and 5. While this hints at the possible effects of the 1999 exchange rate devaluation, we stress that these estimates should only be interpreted as simple correlations. Section 5 introduces an empirical strategy to estimate the causal effects of an unanticipated export shock on the gender-based labor outcomes of male- and female-led firms based on the theoretical model developed in the following section.

4 Oligopsony Framework

Evidence exists that labor markets are less than perfectly competitive, such as firm-level wage premiums in Brazil (Krishna, Poole and Senses, 2012; Araújo and Paz, 2014), Mexico (Frías et al., 2012), and in Italy (Macis and Schivardi, 2016). These deviations from a competitive market wage are sometimes attributed to unionization, worker accumulation of firm-specific human capital, worker heterogeneity, and monopsony power.

Under the assumption that both search costs for workers and recruitment costs for firms exist in the presence of heterogeneity, firms can exploit idiosyncratic tastes for non-wage amenities such as commuting time and work schedule flexibility to accommodate family needs to exercise monopsony power, or pay employees less than their marginal revenue product.[16]

[16] See Azar and Marinescu (2024), Manning (2011), Manning (2021), and Ashenfelter, Card, Farber and Ransom (2022) for a review of monopsony power in labor markets.

Recent studies find evidence of monopsony power in Brazilian labor markets (Vick, 2017; Corradini et al., 2025; Sharma, 2023).[17] To frame our study of export-oriented Brazilian manufacturing firms, we look to a basic model of third-degree oligopsony, where firms in an industry exploit heterogeneous preferences of non-wage amenities to offer female and male wages below the marginal revenue product of labor. Relative to male workers, female workers place a higher value on non-wage amenities such as workplace flexibility (Goldin and Katz, 2010; Vick, 2017; Corradini et al., 2025; Sharma, 2023) or a gender-equity supportive environment. In light of this, we view female firm leadership as an indicator of a gender-equity supportive environment valued by female but not male workers.[18] Firms with monopsonistic power can exploit heterogeneous values placed on non-wage amenities by female and male employees to offer different wages by gender, even in a setting where the marginal revenue product of female and male labor is equal.

We begin with Figure 1, representing wage and employment outcomes for firms with monopsony power. Panel (a) depicts a firm with low non-wage amenities. Female labor supply is less wage elastic than male labor supply.[19] This is reflected by a steeper labor supply and marginal cost of labor (MCL) curve for female vs male labor and a firm-level MCL, which is the horizontal summation of female and male MCL curves for each wage level. Employment by gender is determined by equating MCL to the marginal revenue product of labor (MRPL). The oligopsonistic firm pays the minimum necessary wage to female and male employees to secure the desired levels of employment. Panel (b) represents an identical firm to Panel (a) except for a higher level of non-wage amenities. This is represented by a downward shift in female labor supply and female MCL with appropriate adjustment to firm MCL. Male labor supply and MCL are assumed to be unaffected by the increased non-wage amenities. The downward shift in MCL causes the firm to have higher total employment than in Panel (a). Because of the lower reservation wages of female employees due to higher amenities, the firm in Panel (b) employs more women than in Panel (a). The lower MCL

[17] See Berger, Herkenhoff and Mongey (2022) for evidence of monopsonistic labor markets in the U.S. and Hirsch, Jahn and Schnabel (2018) for Germany.

[18] One possible justification could be an absence of taste-based discrimination against female workers by female managers. Another possibility is that female leaders are seen as having a better assessment of the productivity of female workers (Flabbi et al., 2019). Alternative explanations for the preference of female workers for female leadership include the idea expressed in the Sociology literature of female leaders as "agents of change" who promote gender-equalizing hiring practices and pay grades.

[19] Recent empirical estimates of the firm-level wage elasticity of labor supply for male and female workers find lower elasticities for female than male workers using Norwegian data (Barth and Dale-Olsen, 2009), Australian data (Booth and Katic, 2011), U.S. data (Ransom and Oaxaca, 2010; Webber, 2015), German data (Hirsch, Schank and Schnabel, 2010), and in a meta-study (Sokolova and Sorensen, 2021) Of particular interest to our study, Vick (2017) uses matched employer-employee data from Brazil and finds male elasticities from 1.638 to 2.175 and female elasticities from 1.22 to 1.502.

implies that the firm in Panel (b) will employ fewer men and pay a lower male wage than in Panel (a). A high amenity (female-led) firm will exhibit both a wider wage gap and a more gender-balanced workforce than a low amenity (male-led) firm, as in Stylized Facts 3 and 5.

Theorem 1. *High amenity firms are characterized by a wider gender wage gap relative to low amenity firms.*

Proof: See Appendix $A.3$

Theorem 2. *High amenity firms are characterized by a higher proportion of female employees than low amenity firms.*

Proof: See Appendix $A.3$

An outward shift in the marginal revenue product of labor represents the effects of an increase in productivity, holding amenities constant. We illustrate this in Figure 1 by contrasting Panel (c) to Panel (a) for a low amenity firm, and Panel (d) to Panel (b) for a high amenity firm. As a result of higher MRPL, all wages and employment levels increase. Intuitively, the increase in output price causes profit-maximizing producers to expand their output level. This necessitates an expansion in employment (size). If amenities are fixed in the short run, employment can only be increased by raising wages. Given the greater wage elasticity of labor supply for male workers, the marginal expense of additional male workers is less than that of additional female workers. Because of this, the proportion of female employees will decline. As applied to higher productivity firms, this result is consistent with Stylized Fact 3 for low-amenity (male-led) firms as shown in Columns (3) and (6) of Table 3.[20] An alternative explanation from the oligopsony framework is in the relationship between firm-level elasticity of labor supply, ϵ_{jk}, and the industry-level elasticity, ϵ_k,

$$\epsilon_{jk} = \frac{\epsilon_k}{s_j} \tag{3}$$

where s_j is firm j's share of employment. (Azar and Marinescu, 2024) An implication of this is that larger firms have greater monopsony power over their employees than smaller firms. This result can be stated in theorem form.

Theorem 3. *The workforce will be less gender-balanced in larger firms.*

[20]Nonetheless, we find a positive coefficient for female-led firms instead in columns (2) and (5). Note that these cross-section estimates for female-led firms capture both the effects of Theorems 2 and 3, as the conflicting implications of these theorems may lead to these positive estimated coefficients.

Proof: See Appendix A.4

Although an increase in MRPL causes both female and male wages to increase, the effect on the gender wage gap is more difficult to assess graphically or with intuition. The oligopsony model predicts a widening gender gap, consistent with Stylized Fact 4.

Theorem 4. *The gender wage gap will be wider in larger firms.*

Proof: See Appendix A.4.

To analyze the effects on firm-level gender labor outcomes of an export-inducing shock, we derive four additional theorems. This shock is modeled as a price increase, which causes a shift in the MRPL curve to the right-hand side. As a result, firms adjust their labor demand accordingly, as shown in the following two theorems.

Theorem 5. *An exogenous output price increase will reduce the firm-level share of female employees.*

Proof: See Appendix A.5.

Theorem 6. *An exogenous output price increase will cause the gender wage gap to widen further.*

Proof: See Appendix A.5.

Note that for a linear MRPL curve, as would be the case for perfectly competitive output markets, the effects of a positive price shock on labor market outcomes are comparable to those of an exogenous increase in productivity. The MRPL curve will no longer be linear if the output market is monopolistically competitive (Jha and Rodriguez-Lopez, 2021). In a monopolistic competitive market, firm-level increasing returns to scale will cause more productive (larger) firms to further reduce the share of female employees and increase their wage gap in response to an export shock.

Finally, we contrast the behavior of female- and male-led (high vs low amenity) firms in response to an exogenous output price shock. In a perfectly competitive market structure, differences between the behavior of female- and male-led firms should not exist. (Becker, 1957) However, in an oligopsonistic framework, differences between the responses of high- and low-amenity firms to an exogenous output price shock must be carefully derived. Despite systematic differences between high and low-amenity firms in the gender wage gap and share of female employees, our mathematical model in Appendix A.5 does not predict any additional response in female versus male-led export-oriented firms to an exogenous output price increase. Both results are in stark contrast with the predictions of an employer taste-based model of discrimination (Becker, 1957), which predicts a reduction in gender balance and widening of the wage gap only for male-led firms.

Theorem 7. *High and low-amenity firms of similar size have equivalent changes in the gender wage gap in response to an exogenous output price shock.*

Proof: See Appendix A.5.

Theorem 8. *High and low-amenity firms of similar size have equivalent changes in the percentage of female employees in response to an exogenous output price shock.*

Proof: See Appendix A.5.

The absence of distinct behavior between high and low amenity firms in the last two theorems rests upon our assumption of an exogenous level of firm amenities.[21] Suppose firms are legally required to increase amenity levels, including workplace flexibility, as the number of employees grows. Since the mandated increase applies equally to female- and male-led firms, the increase would not necessarily cause differences in gender labor outcomes between male- and female-led firms. However, if female-led firms were to increase amenities beyond the mandated amounts (and those of male-led firms), female-led firms would experience both a larger gender wage gap and a larger proportion of female employees relative to male-led firms.

5 Methodology

To estimate the additional effect of a gender equity supportive environment, proxied by female leadership, on the gender wage gap and workforce balance, we make use of the macroeconomic shock caused by the large and unexpected 1999 Brazilian *real* exchange rate devaluation. This shock increased the local currency price received by Brazilian firms for foreign sales, leading to greater output levels. We begin this section with an overview of the 1999 devaluation of the *real*, followed by the econometric methodology used in this study.

[21]To carefully derive expected differences in the behavior of female- and male-led firms if the amenity level were endogenous requires the assumption of a specific mechanism for amenity provision. As our data does not include amenity-related variables other than the gender of leadership, it is beyond the scope of our current paper to investigate amenity provision mechanisms, which is left for future research.

5.1 Quasi-Natural Experiment

The fundamental barrier in identifying the effect of exports on employment and wages is the simultaneity of firm choices regarding export status, wage policies, and workforce composition. Idiosyncratic productivity shocks could generate a spurious correlation between export status and wages if positive productivity shocks induce firms to expand the quantity of exports, necessitating higher wages and additional hiring (Schank, Schnabel and Wagner, 2007). Exogenous variation in exports is needed to identify the effect of increased exports on the outcomes of interest.

Export surges have been preceded by large real exchange rate devaluations in several developing countries, including Brazil (Freund and Pierola, 2012). The use of a large real exchange rate devaluation as an export shock has gained considerable attention in the literature and has been used to investigate other international trade-related issues in Mexico (Verhoogen, 2008), Argentina (Brambilla, Lederman and Porto, 2012), and Brazil (Araújo and Paz, 2014; Almeida and Poole, 2017).

The Brazilian government implemented a new macroeconomic stabilization plan called the *Plano Real* in July 1994. This plan successfully reduced the annual inflation rate from 2,500 percent in 1993 to a maximum of 12 percent in 2002. One of its prominent features was establishing a crawling-peg exchange rate system to restore public confidence in the domestic currency. After implementing the plan, the Brazilian government was unable to control budget deficits. This slowly eroded the government's ability to sustain the crawling-peg system. In January 1999, a large, unexpected speculative attack on the domestic currency left the Brazilian government with no choice but to abandon the crawling peg and adopt a free-floating exchange rate system. Between the 13^{th} and the 29^{th} of January, the *real* depreciated by approximately 50 percent. This large real exchange rate devaluation was not reversed in the following years, as shown in Figure 2.[22]

Exports increased after the devaluation by more than 10 percent in 2000 relative to 1998 and by another 17 percent by 2002, as seen in Figure 3. This strong export growth continued in subsequent years and surpassed the seven percent per year threshold used to characterize export surges. (Freund and Pierola, 2012) To ensure that exports are causing the labor force reallocation, the exchange rate shock must also increase the share of firm output that is exported. In the case of the Brazilian *real* devaluation, the export share of production increased more than 30 percent between 1998 and 2004. Moreover, this increase in share was

[22]This sudden exchange rate regime change intended to curb the depletion of Brazilian foreign currency reserves. It was complemented by an IMF loan package with many conditionality clauses. None of these clauses addressed the reduction in gender disparities in labor markets.

not simply a substitution from domestic to foreign demand. While manufacturing output remained roughly constant between 1995 and 1999, it increased by more than 20 percent between 1999 and 2004.[23] Thus, firms experienced an export-induced increase in demand and presumably had to adjust their employment level accordingly.[24]

For a devaluation to be effective in changing exporting firm behavior, it must be both unanticipated and perceived as permanent. Black markets for foreign currencies are a ubiquitous feature of developing economies. As the black market is free of any government controls, black market rates reflect agents' expectations. The absence of any substantial deviation between black-market exchange rates and the official exchange rate before the devaluation, as seen in Figure 4, is further evidence that the devaluation was not widely anticipated and that the exchange rate shock should be considered exogenous. Furthermore, we observe two factors in support of the Brazilian devaluation being recognized as permanent by exporting firms. The crawling peg exchange rate system was replaced with a floating exchange rate. And the crawling-peg exchange rate misalignment was widely perceived as being caused by economic imbalances that were unaddressed by the Brazilian government.

5.2 Empirical Methodology

We utilize the quasi-natural experiment created by the 1999 exchange rate shock to compare differences in firm behavior before and after the shock with the theoretical predictions developed in our oligopsonistic framework. Even though firms are exposed to the same export shock, their responses to the export shock may differ by firm size or the gender of firm leadership. We develop empirical specifications to assess the magnitude and significance of each.

The first empirical specification is designed to evaluate Theorems 5 and 6. We employ the same methodology that Verhoogen (2008) developed to assess how an export shock affected Mexican firms differentially according to their productivity. Since productivity is not observable in our data, we use the firm-level size measured as $\ln(\text{employment})$ as a proxy for productivity due to the strong positive correlation between productivity and employment in Brazilian firms (De Negri, 2005).[25] This econometric specification comprises a system of two

[23] The manufacturing output and export share figures are available upon request.

[24] Another potential concern is the existence of a credit crunch in the aftermath of a major devaluation. From Figure A.1 in the Online Appendix, there was no contraction in the amount of outstanding loans in the period of interest and no break in the trend of credit expansion.

[25] Unfortunately, the available data contain no other variable that could be used as a proxy for productivity. Nonetheless, Verhoogen (2008) and Frías et al. (2012) uses different proxies for Mexico (employment, sales, and sales per worker) and find comparable results.

equations, as depicted below. In the first equation, the change in the outcome of interest between 1995 and 1998 (pre-devaluation period) is regressed on the initial period (1995) firm-level size. In the second equation, the change between 2004 and 2000 (post-devaluation period) in the outcome of interest is regressed on the post-devaluation initial period (2000) firm-level size,

$$\Delta y_{98-95,j} = \mu_{pre} + \varphi_{1995,j}\eta_{pre} + X_j\beta_{pre} + \epsilon_{pre,j} \tag{4}$$

$$\Delta y_{04-00,j} = \mu_{post} + \varphi_{2000,j}\eta_{post} + X_j\beta_{post} + \epsilon_{post,j} \tag{5}$$

where $\Delta y_{98-95,j}$ is the change in firm j's outcome of interest before the devaluation in 1999 and $\Delta y_{04-00,j}$ after the devaluation. μ is the intercept, $\varphi_{0,j}$ is the initial period firm-level size, X_j is a matrix of additional regressors, including industry and state fixed effects, and ϵ_j is the stochastic disturbance term.

The identifying assumption of this specification is that before the shock (pre-devaluation period), the relationship between the outcome variable and firm size is stable, or $\widehat{\eta_{pre}} = 0$. Nonetheless, in the post-devaluation period, the response to the export shock is proportional (or inversely proportional) to firm size. Under this specification, the effect of the export shock on firm-level outcomes is computed as the difference between the post- and the pre-shock estimated coefficient of the initial period firm size, $\Delta\eta = \eta_{post} - \eta_{pre}$.[26] Theorem 5 predicts a negative $\Delta\eta$ for the female share of employment. Theorem 6 implies a positive $\Delta\eta$ for the male-female wage gap outcome.

Our identification strategy also assumes that the effect of unanticipated real exchange rate devaluation on exports is uncorrelated with other shocks that could differentially affect firms according to their size (Verhoogen, 2008). Under these conditions, while differential trends between small and large firms may exist, they are eliminated through differencing, leaving consistent estimates of the effect of the exchange rate shock.[27] Another concern is that firm productivity may be subject to time-varying shocks (Verhoogen, 2008). Although all firms receive such shocks, larger (or more productive) firms may benefit more (Acemoglu, Aghion and Zilibotti, 2006). Because of this, the productivity innovation will be correlated with the

[26] This specification can also be interpreted as small firms being untreated by the devaluation of the *real* and $\Delta\eta \approx 0$, while large firms receive treatment in the form of expanded exports and potentially experience $\Delta\eta \neq 0$ (Verhoogen, 2008). All firms face the same shock, but their responses are heterogeneous, as the export (price) shock affects firms in proportion to their productivity. (Melitz, 2003) Thus, this approach implements a difference-in-differences methodology to estimate an Average Treatment Effect.

[27] One germane example of a differenced-out trend would be larger firms implementing policies to address gender disparities.

initial firm size and $\widehat{\eta}$ will not be consistently estimated by ordinary least squares (OLS). However, if both the distribution of the productivity shocks and their effects on outcomes are time-invariant, then the bias in $\widehat{\eta}$ is removed when differencing between the pre- and post-devaluation periods. Under this assumption of time-invariance, the OLS estimates of Equations (4) and (5) are consistent (Verhoogen, 2008).

Following Verhoogen (2008), we estimate Equations (4) and (5) as a Seemingly Unrelated Regression (SUR) and cluster standard errors at the industry level to account for industry-level heteroskedasticity and shocks. (Bertrand, Duflo and Mullainathan, 2004) Our specifications use long differences (1998–1995 and 2004–2000) in contrast to differences in adjacent years because firms may respond slowly to an unanticipated export shock. The percentage of female employees and wage gap outcome variables are themselves estimates. Because of this, we further correct for heteroskedasticity by weighting each firm-level observation by the square root of firm size as measured by the number of employees.

Recent developments in the difference-in-differences methodology literature have called into question the statistical reliability of conventional panel fixed effects methods.[28] The average treatment effect, computed as a weighted average of the estimated individual firm or unit treatment effects, can contain negative weights. The likelihood of negative weights increases with differing lengths of treatment. When combined with heterogeneous treatment effects, the possibility exists for estimated average treatment effects of misleading magnitude or even mathematical sign. In our specific case, all firms are subject to the same length of treatment, decreasing the likelihood of negative weights when estimating an average treatment effect. In contrast to a conventional difference-in-differences design in which there are separate treatment and control groups, all firms receive the treatment of the devaluation. However, smaller non-export-oriented firms would not benefit from treatment.[29]

Our second empirical specification is designed to directly estimate the effect of the gender of firm leadership on the gender wage gap and workforce balance. The evaluation of Theorems 7 and 8 requires contrasting the responses of female and male-led firms of similar size, allowing firm responses to be heterogeneous by both the gender of firm leadership and productivity. To do so, we estimate our previous difference-in-difference specifications separately for female and male-led firms to account for the firm size heterogeneity, and estimate an additional difference across leadership gender, $\widehat{\Delta^2\gamma} = \widehat{\Delta\eta^{♀}} - \widehat{\Delta\eta^{♂}}$. We implement this as a four-equation seemingly-unrelated regression (SUR). Coefficients estimated by this triple

[28]See De Chaisemartin and d'Haultfoeuille (2020), Callaway and Sant'Anna (2021), and Goodman-Bacon (2021).

[29]Verhoogen (2008) explains that if there were two distinct groups of manufacturing firms with respect to productivity instead of a continuum, his empirical methods could be characterized as a triple-difference.

difference estimator are robust with respect to several possible deficiencies, including the endogeneity of female leadership and heterogeneous time trends by firm leadership gender. Theorem 7 predicts no difference in the change to the male-female wage gap by the gender of firm leadership, or $\widehat{\Delta^2 \gamma} = 0$. Likewise, Theorem 8 predicts no difference in the change to the share of female employees by the gender of firm leadership, or $\widehat{\Delta^2 \gamma} = 0$. In the next section, we discuss the estimates obtained using this methodology with Brazilian data and present several robustness checks of our main results.

6 Results

In Table 5, we report estimates of Equations (4) and (5) and a difference-in-differences using data from all firms. Column 1 estimates the effects of the exchange rate shock on firm-level exports using $\ln(1+\text{exports}_{jt})$ as the dependent variable. The estimated coefficient for firm size is positive and statistically significant in both the pre- and post-devaluation periods, as found in Mexico (Verhoogen, 2008) and Brazil (Araújo and Paz, 2014). The coefficient for the post-devaluation period is substantially larger, as seen by the large and highly significant difference-in-differences estimated coefficient in the third row. Consistent with our identifying assumption, these estimates indicate that in response to the devaluation of the *real*, firm-level exports increase, and more so in larger firms.

The remaining columns of Table 5 report difference-in-difference estimates for the percentage of female employees for all classifications of employees at all firms. For all workers, as reported in Column 2, the change in the percentage of female employees in all firms was not significantly affected by the positive output price shock caused by the devaluation of the *real*. This result remains when disaggregating by whether an employee has earned a college degree. When disaggregating by whether a position is skilled, the percentage of female employees significantly increases in response to the output price shock for both skilled and unskilled employees, indicative of export effects among employees with high school degrees or less education only. Each of these results conflicts with Theorem 5, which predicted a decline in the percentage of female employees. However, our empirical findings are consistent with the mixed results of the extant literature.

Table 6 reports estimated changes in the male-female wage gap. The overall wage gap in Column (1) further widened due to the devaluation, consistent with Theorem 6. Further disaggregation shows the impact on workers without college degrees and unskilled workers. In contrast, the wage gap for college-educated workers and skilled workers did not significantly change.

Table 7 presents estimates for employees by the five broad occupational categories. The third row contains changes in firm behavior for the percentage of female employees in odd columns, which are positive and highly significant for Professional or Managerial employees and Technical or Supervisory employees, marginally significant for Unskilled White Collar employees, and otherwise do not differ from zero. As with the more aggregated estimates, these conflict with Theorem 5. In contrast, the estimated effects on the male-female wage gap in even columns do not significantly differ from zero for all occupational categories with one exception. The wage gap increased significantly for Skilled Blue Collar employees as predicted by Theorem 6.

We repeat the previous difference-in-differences analyses for the subsample of female-led and male-led firms and estimate a triple difference, which is the additional response of female-led firms to the exogenous output price shock relative to male-led firms. We report estimated coefficients in Tables 8 through 10. We first examine the change in firm exports, reported in Table 8, Column (1). In Panel A, exports increase in female-led firms, though imprecisely estimated. In contrast, the exports of male-led firms significantly increase in response to the devaluation (Panel B). The difference in the reaction of female-led and male-led firms is negative and highly significant, as seen in Panel C. Male-led firms are more responsive than female-led firms to the export-induced demand shock.[30]

The remainder of Table 8 presents estimates of changes in the percentage of female employees. Panel A shows that the export shock induced female-led firms to increase their share of female workers across all categories except for workers with college degrees. In contrast, there was no significant change in the percentage of female employees in male-led firms (Panel B). Results in both female- and male-led firms conflict with Theorem 5, which predicted a decrease in the percentage of female workers. In Panel C, we find no significant difference in the responses of female- and male-led firms in aggregate, for workers without college degrees, for skilled and unskilled workers, consistent with Theorem 8. The percentage of female workers increases more in female-led than in male-led firms for workers with college degrees and significantly so, in conflict with Theorem 8.

Table 9 reports differential changes by the gender of firm leadership for the male-female wage gap outcomes. From Panel A, the export shock induced female-led firms to widen the average wage gap and the wage gap of the other subcategories, consistent with Theorem 5. However, the change was insignificant for skilled workers. The only case of a narrowing wage

[30] This suggests that male-led firms may be different from female-led firms in other dimensions, such as access to capital. Unfortunately, our dataset does not have information to further explore this. Yet, as long as these potential factors are time invariant, they will be differenced out in our estimated model.

gap took place among college workers. Interestingly, a similar pattern emerges for male-led firms, though the magnitude of the coefficients is smaller in almost all cases. As a result, all the triple difference estimated coefficients in Panel C are positive but do not differ from zero, consistent with Theorem 7.

Table 10 reports estimates disaggregated by occupational category. In Panel A, the change in the percentage of female employees in female-led firms increased for employees in the Professional and Managerial, Skilled Blue Collar, and Technical or Supervisory categories, all in conflict with Theorem 5. Yet in these same categories, the male-female wage gap significantly widened, consistent with Theorem 6. The absence of a similar pattern in Panel B is a fascinating contrast between female- and male-led firms. By comparison, the only significant results for male-led firms are a widening wage gap for Skilled Blue Collar and an increase in the female share for Technical and Supervisory employees. In Panel C, female and male-led firms do not differ in their response to the exogenous price shock for four of five categories, consistent with Theorems 7 and 8. The notable exception, at a 1% level of significance, is Professional and Managerial employees, where female-led firms have both a larger increase in the percentage of female employees and the male-female wage gap.

6.1 Discussion

The academic study of discrimination in labor markets gained prominence with Becker (1957). Within the Becker framework, firm owners who engage in taste-based discrimination face higher labor costs than those who do not. The ability of a firm to discriminate is tempered within competitive markets and can only persist in less competitive markets. Thus, firms with positive economic profits (perhaps due to market power) have more ability to discriminate than less profitable firms. If male-led firms were engaging in taste-based discrimination against female employees, a positive export shock would reduce the female share of employment and widen the gender wage gap. We find some evidence of a growing wage gap in male-led firms in Panel B of Table 9, but no evidence of a declining female share of employment in Table 8. If, instead, female-led firms were engaging in taste-based discrimination that favors female employees, a positive export shock would cause the opposite outcomes: improved gender balance and a narrowing wage gap. We find strong evidence of an increasing female share of employment in female-led firms in Panel A of Table 8, but evidence of a growing overall wage gap in Table 9. Whether male-led firms discriminate against female employees or female-led firms discriminate in favor of female employees, we should expect the differential response of female versus male-led firms regarding the percentage of female employees to be positive and the gender wage gap to be negative. We find that female-led firms do increase the share of college-educated female employees and female professional or managerial employees more than male-led firms, but not in aggregate. We find that relative changes in the gender wage gap differ from the predictions of a taste-based discrimination model: the wage gap further widens in female-led firms relative to male-led firms, with no changes for other categories or in aggregate. We conclude that the signs and significance levels of our estimated coefficients mostly conflict with a model of taste-based employer discrimination.[31]

While Becker's analysis of discrimination is constructed within an idea of costly discrimination that fades away with increased competition, we model the behavior of individual firms within an oligopsonistic market in which discrimination is profit-maximizing due to the different values male and female workers place upon non-pecuniary amenities. From Equation 3, firms have monopsony power proportional to market share, or equivalently, size. As a result, we predict greater discrimination in larger firms than in smaller firms (Theorems 3 and 4), in contrast to Becker (1957). Our results in Tables 5 and 6 are evidence of this behavior. Moreover, we find empirical evidence broadly consistent with the theorems we develop regarding the male-female wage gap. Our findings in Tables 6 and 9, Panels A and B of a positive demand shock causing additional wage discrimination — predicted in Theorem 6 — are consistent with the reduced wage discrimination found by Black and Strahan (2001) in response to a negative demand shock to locally owned independent banks.[32] However, there is less continuity between our theoretical predictions and empirical results regarding the female share of the firm workforce. Theorem 5 predicts that a positive output price shock will lead to a less gender-balanced firm workforce. This is consistent with Black and Brainerd (2004), who find that the negative shock from globalization led to a greater female share in manufacturing employment in the United States. In contrast, we find that a positive shock increased the share of female employees.

We return to whether a larger female presence in management will cause women to act as "agents of change" and foster a gender equity supporting environment to redress past inequities in female representation and the gender wage gap (Cohen and Huffman, 2007). Despite the predictions of our theoretical model of differences in the wage gap and workforce balance between female- and male-led firms, and further deterioration in each as a result

[31] Both Paz and Ssozi (2021) using Brazilian industry-level household data and the 2000s China shock were inconsistent with Becker's model.

[32] While this result conflicts with Flabbi, Piras and Abrahams (2017), who find a smaller gender wage gap in female-run Latin American and Caribbean manufacturing firms (Flabbi et al., 2017), though their study does consider the effect of firm size on the gender wage gap.

of a positive export shock, our model does not predict that female and male-led firms will behave any differently in response to an export shock. Our estimates indicate that the responses of female-led firms generally do not significantly differ from those of male-led firms, as predicted by our model, but with several notable exceptions. The percentage of female workers with college degrees increases more rapidly at female-led than male-led firms. Disaggregating total employment by occupational category, both the percentage of female professional and managerial employees and the male-female wage gap for professional and managerial employees grow more rapidly in female-led firms than in male-led firms. We note that professional and managerial employees are college-educated.

We conclude that the evidence on whether female managers act as agents of change is mixed. Female-led firms do increase gender workforce balance, but only for college-educated females and for professional and managerial occupations, at the cost of a more expansive gender wage gap. Within the context of firms with monopsony power, additional policies to address the gender gap may be constructive.

6.2 Robustness Checks

We use two approaches to examine the robustness of our findings that female and male-led firms differ in their responses to an exogenous demand shock. First, we implement an alternate statistical inference methodology where the gender of firm leadership is counterfactually assigned by random, or randomization-based inference. In the second robustness check, we implement an alternate definition of a female-led firm and re-estimate the main specifications of our model.

Athey and Imbens (2017) argue that randomization-based inference methods should be used in laboratory and natural economic experiments. The data sets in these experimental designs contain the universe of all available observations and not a random sample from a larger population. It follows that stochastic variation within the universe of all possible observations is caused by the assignment of treatment status and not sampling. To implement randomization inference methods within our study, we re-estimate the triple-difference specifications 5,000 times with random reassignment of the treatment status, whether an individual firm is female-led. In the bottom row of Tables 8, 9, and 10, we report each estimated coefficient's inverse empirical cumulative distribution in square brackets with statistical significance indicated by the number of $^+$ signs. In Table 8, Column (1), the inverse empirical cumulative distribution of the quad difference estimated parameter -0.110 is indicated in square brackets as 0.003 or $[F^{-1}(-0.110) = 0.003]$, meaning that 0.3% of the

5,000 estimated coefficients using randomly assigned counterfactual treatment status were less than -0.110, the coefficient found using actual treatment status. This result is significant at a 1% level, as indicated by $^{+++}$.[33]

Using randomization-based inference methods, we find significance levels broadly consistent with those found using clustered standard errors with two noteworthy exceptions. In Table 10 Column 4, the change in the male-female wage gap for Professional and Managerial employees is significant at the 1% level using clustered standard errors but insignificant using randomization methods. In Columns 8 and 9, the coefficients are significant at the 10% level using randomization but insignificant using clustered methods.In summary, we find broad agreement in the significance levels of our results, whether using conventional clustered standard errors or randomization inference methods. We find the two points of wider disagreement between the two inference methodologies interesting, but a careful examination of the difference is beyond the scope of this paper.

In the next robustness check, we define female-led firms as those that exhibited at least one female in a leadership position for at least one year between 1995 and 1998, while male-led firms are defined as before. As a result, firms with females in leadership positions only after 1998 are excluded from the sample, and the number of female-led firms declines to 10,993. We then re-estimate the triple-difference econometric models reported in Tables 8 through 10. These new estimates are in the Online Appendix in Tables A.3 through A.5, respectively. Only the estimates in Panels A and C of these new tables will differ from those using the original female-led firm definition. The results in the third row of Panel A in Table A.3 are comparable to those in Table 8, except that the coefficients for the female share of college and skilled workers do not significantly differ from zero. Except for the female share of skilled workers, the other triple-difference coefficients in Panel C are positive, although imprecisely estimated. This is not unexpected, given the complexity of our models and the drastic reduction in sample size.

For the wage gap outcomes, Table A.4 reports in the third row of Panel A estimates with comparable signs to Table 9. Nevertheless, for the unskilled worker wage gap, the estimated coefficient is now negative and of a much smaller magnitude. Only the estimated effect on the gender wage gap for employees with a college education is statistically significant. In Panel C, most coefficients are near zero and lack statistical significance. Finally, at the occupational level, the estimates displayed in the third row of Panel A in Table A.5 present a few sign reversals relative to those in Table 10. In Panel C, the results indicate a narrowing wage gap

[33]We implement two-tailed tests of significance. The result is significant at the 1% level if $F^{-1}(-0.110) < 0.005$ or $F^{-1}(-0.110) > 0.995$.

for Managerial and Professional, and Unskilled Blue Collar employees, and an increase in the female share of Unskilled Blue Collar employees. These occupational-level findings further indicate that, although firms with sustained female leadership did not significantly increase the proportion of female employees – except within the category of Unskilled Blue Collar workers – they nonetheless contributed to the advancement of gender equity by reducing the gender wage gap across numerous occupational groups. Taken together, these results imply that females in leadership positions may have a role in promoting gender equity, but improvements do not seem to be either immediate or applied to both employment and compensation.

The estimated coefficients of this robustness exercise indicate that removing the firms that only hired female managers after the devaluation did not substantially change the estimated response to the exchange rate devaluation of the remaining female-led firms. However, the coefficients had a smaller magnitude and were imprecisely estimated, possibly caused by the large reduction in sample size. Our results are robust to the definition of a female-led firm.

7 Conclusion

We use Brazilian administrative employer-employee matched data to address whether female managers improve workplace outcomes for female employees. Our data includes the census of formal employees in manufacturing industries. From this data, larger firms appear to have a higher proportion of female employees than smaller firms. Yet, we observe that firms with female leadership do appear to hire a higher proportion of female employees than a male-led firm of similar size. We find that the proportion of female employees is negatively associated with firm size for male-led firms, but this association is positive for female-led firms. Curiously, the male-female wage gap appears to be increasing in firm size, and greater in female-led firms than in male-led firms.

To reconcile these seemingly contradictory stylized facts, we introduce a model of employees who value both monetary compensation and non-wage amenities, including workplace flexibility and the gender of firm leadership. Firms use heterogeneous valuation of non-wage amenities to offer different wages to female and male employees. These third-degree oligopsonistic firms offer employee wages that exceed those available in the perfectly competitive sector, yet less than their marginal revenue product of labor. Equilibrium results from this model are broadly consistent with the stylized facts we observe in the data.

Since the equitable treatment of female employees and the gender of firm leadership may be endogenous, we observe how firms respond to the plausibly exogenous devaluation of the Brazilian real in 1999, and compute the additional responses of female-led versus male-led firms in the gender wage gap and proportion of female employees. Our triple-difference model corrects for various endogeneity and selection issues by comparing changes in the gender wage gap and percentage of female firm employees before versus after the large unanticipated devaluation of the Brazilian *real* in manufacturing firms with female versus male leadership. We find empirical estimates broadly consistent with the theoretical predictions of our oligopsonistic model concerning the gender wage gap. As a result of the export shock, the gender wage gap further widened in both female- and male-led firms, but the response did not substantially differ by the gender of firm leadership. We find that the percentage of female employees increased in female-led firms but not in male-led firms – both in conflict with our model, which predicts a declining share of female employees. The responses of female- and male-led firms did not significantly differ, which is consistent with the predictions of our model.

We find significant differences between the behavior of female- and male-led firms for professional or managerial employees. Female-led firms do increase the percentage of female employees more than male-led firms, yet the gender wage gap widens more in female-led firms. Our empirical findings conflict with the predictions of a model of employer taste-based discrimination (Becker, 1957), yet are broadly consistent with the behavior of third-degree oligopsonistic firms that discriminate because it is profit maximizing. We conclude that exports do crack the glass ceiling for professional and managerial employees in female-led firms. However, the gender wage gap does not appear to be mitigated by exports or female leadership and may require policy interventions.

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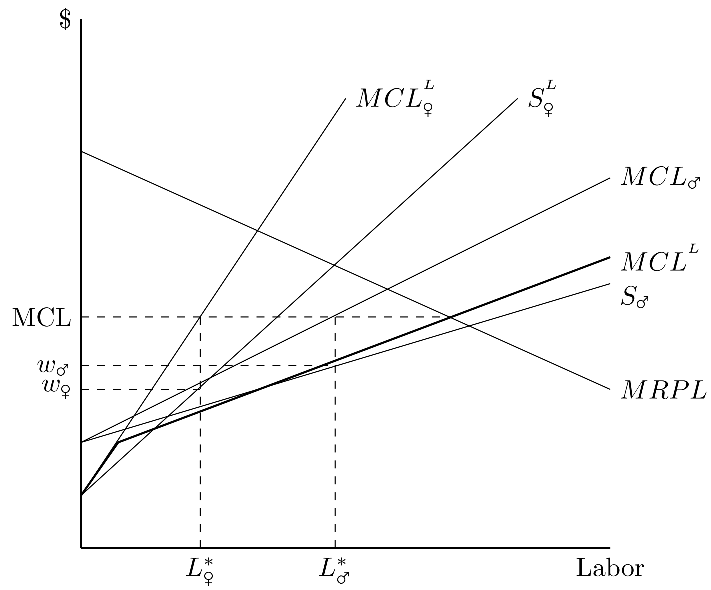
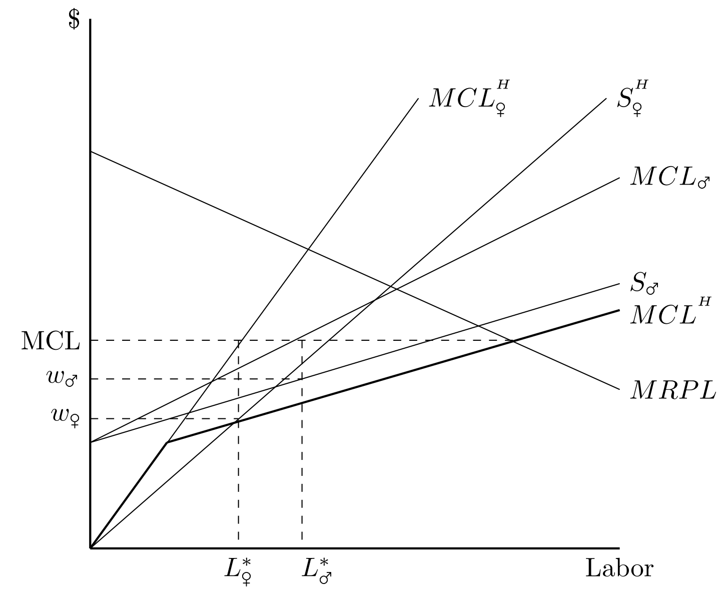
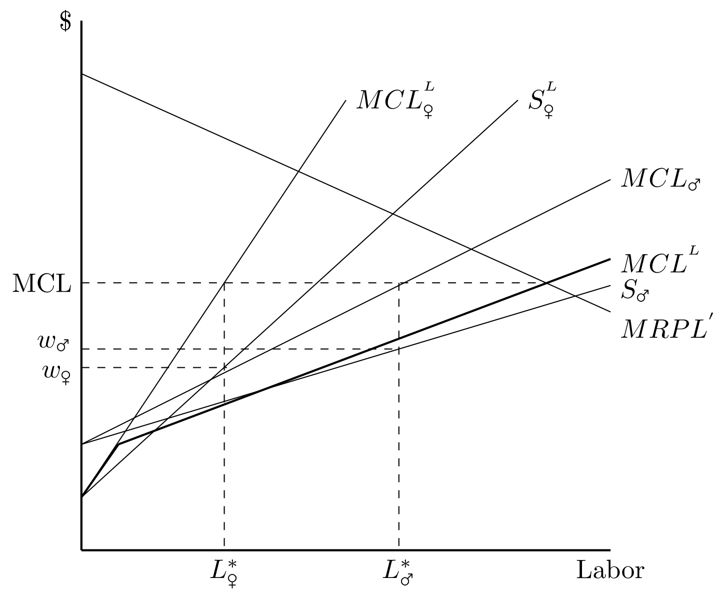
Figure 1: Oligopsonistic Firms – Exogenous Output Price Increase by Amenity Level



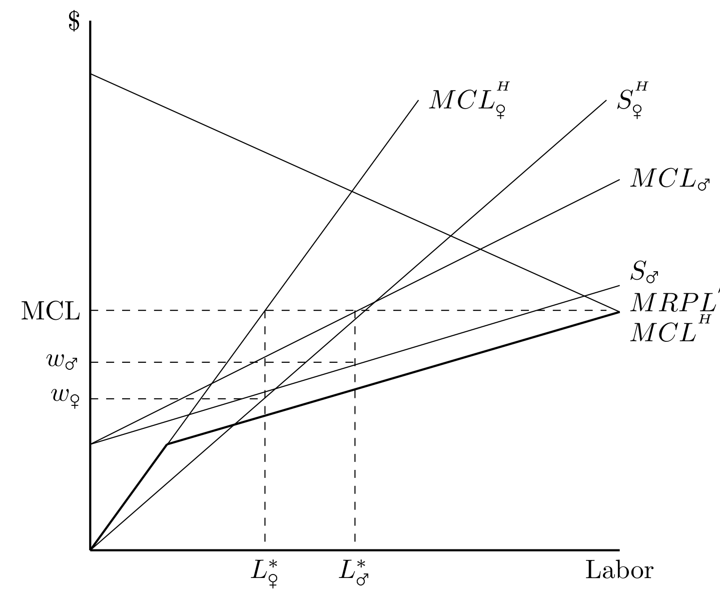
(a) Baseline - Low Amenity



(b) Baseline - High Amenity



(c) Output Price Shock - Low



(d) Output Price Shock - High

Notes: Superscripted "L" and "H" represent low and high-amenity firms. A higher marginal revenue product of labor, represented as $MRPL'$, can represent the effects of an exogenous increase in technology or an exogenous output price increase.

Figure 2: Monthly real exchange rate index

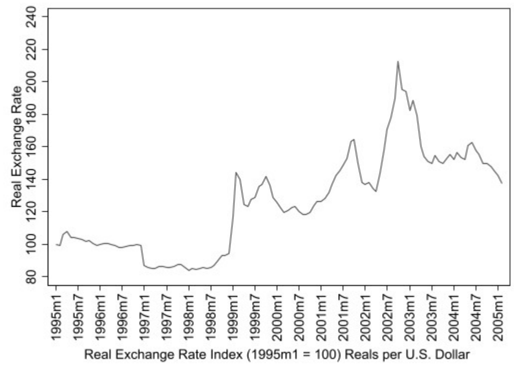


Figure 3: Nominal export volume of manufacturing in billions of USD and Export share of domestic manufacturing output for Brazil.

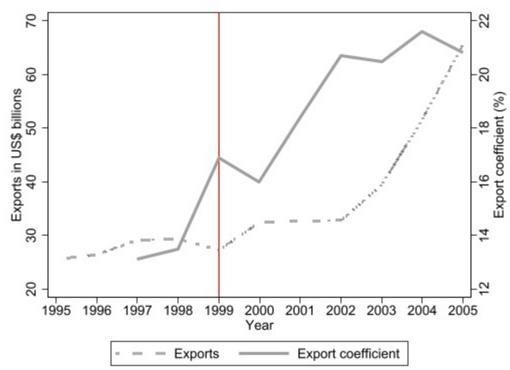


Figure 4: Nominal official and nominal black market exchange rate.

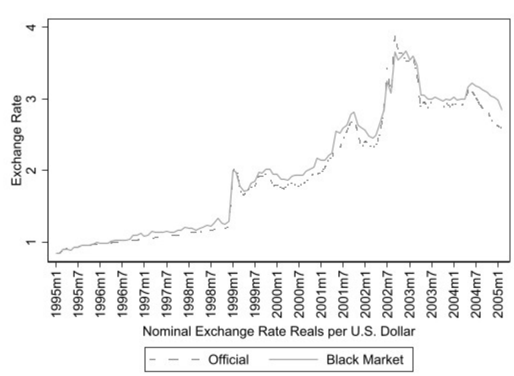


Table 1: Summary Statistics: Balanced Panel

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	1995 Sample				2004 Sample			
VARIABLES	Sample	Female	Male	t	Full	Female	Male	t
Employment	51.791	25.741	177.850	-19.000	53.461	24.459	193.802	-21.750
	(356.068)	(115.228)	(810.509)	***	(346.408)	(106.996)	(788.389)	***
	[60,095]	[49,803]	[10,292]		[60,095]	[49,803]	[10,292]	
ln(1+Exports)	0.985	0.605	2.822	-40.576	1.280	0.776	3.724	-48.374
	(3.386)	(2.635)	(5.412)	***	(3.852)	(2.983)	(6.032)	***
	[60,095]	[49,803]	[10,292]		[60,095]	[49,803]	[10,292]	
College Degree	0.015	0.011	0.030	-25.288	0.025	0.019	0.056	-35.086
	(0.063)	(0.061)	(0.071)	***	(0.078)	(0.071)	(0.102)	***
	[60,095]	[49,803]	[10,292]		[60,095]	[49,803]	[10,292]	
Skilled Employee	0.111	0.101	0.161	-28.590	0.262	0.240	0.367	-45.831
	(0.191)	(0.188)	(0.195)	***	(0.268)	(0.265)	(0.254)	***
	[60,095]	[49,803]	[10,292]		[60,095]	[49,803]	[10,292]	
Proportion of Female Employees								
Unskilled White Collar	0.358	0.352	0.369	-3.264	0.377	0.375	0.381	-1.183
	(0.373)	(0.391)	(0.329)	***	(0.388)	(0.406)	(0.345)	
	[21,012]	[14,475]	[6,537]		[24,253]	[16,763]	[7,490]	
Professional or Managerial	0.378	0.369	0.400	-6.077	0.359	0.292	0.507	-50.469
	(0.383)	(0.401)	(0.337)	***	(0.378)	(0.379)	(0.330)	***
	[23,322]	[16,293]	[7,029]		[30,811]	[21,248]	[9,563]	
Skilled Blue Collar	0.211	0.196	0.281	-22.647	0.195	0.183	0.250	-18.299
	(0.331)	(0.328)	(0.340)	***	(0.323)	(0.320)	(0.327)	***
	[56,322]	[46,503]	[9,819]		[54,576]	[44,968]	[9,608]	
Technical or Supervisory	0.513	0.516	0.506	2.140	0.516	0.519	0.508	2.794
	(0.390)	(0.407)	(0.338)	**	(0.377)	(0.393)	(0.314)	***
	[30,549]	[22,683]	[7,866]		[40,997]	[31,719]	[9,278]	
Unskilled Blue Collar	0.181	0.166	0.228	-9.413	0.191	0.175	0.235	-11.300
	(0.315)	(0.309)	(0.328)	***	(0.320)	(0.314)	(0.329)	***
	[12,911]	[9,704]	[3,207]		[19,133]	[14,018]	[5,115]	
College Degree	0.345	0.334	0.362	-4.171	0.416	0.380	0.471	-15.210
	(0.364)	(0.387)	(0.327)	***	(0.386)	(0.412)	(0.336)	***
	[11,484]	[6,913]	[4,571]		[15,860]	[9,673]	[6,187]	
No College Degree	0.264	0.245	0.355	-33.043	0.281	0.262	0.372	-34.735
	(0.318)	(0.318)	(0.305)	***	(0.311)	(0.312)	(0.288)	***
	[59,978]	[49,696]	[10,282]		[59,985]	[49,708]	[10,277]	
Skilled	0.412	0.403	0.440	-8.501	0.403	0.385	0.467	-22.892
	(0.366)	(0.384)	(0.305)	***	(0.354)	(0.367)	(0.293)	***
	[29,793]	[22,238]	[7,555]		[44,131]	[34,516]	[9,615]	
Unskilled	0.250	0.233	0.336	-29.592	0.253	0.236	0.334	-28.439
	(0.324)	(0.322)	(0.319)	***	(0.323)	(0.323)	(0.313)	***
	[59,094]	[48,913]	[10,181]		[58,023]	[47,921]	[10,102]	
All Employees	0.265	0.246	0.357	-33.653	0.285	0.265	0.382	-37.627
	(0.317)	(0.317)	(0.302)	***	(0.309)	(0.311)	(0.282)	***
	[60,095]	[49,803]	[10,292]		[60,095]	[49,803]	[10,292]	

Notes: Standard deviations appear below each mean in parentheses. All statistics are annual data reported at the firm level. The first four rows contain data that is not gender specific. The following rows report the percentage of female employees by occupation, skill, and educational level. Columns 4 and 8 contain an equality of female and male sample means t test with unequal variances. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 2: Summary Statistics: Male-Female Wage Gap

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		1995 Sample				2004 Sample		
VARIABLES	Sample	Female	Male	t	Full	Female	Male	t
Unskilled White Collar	0.228	0.239	0.214	2.934	0.188	0.195	0.178	2.184
	(0.427)	(0.462)	(0.376)	***	(0.403)	(0.428)	(0.370)	**
	[9,676]	[5,420]	[4,256]		[10,520]	[5,825]	[4,695]	
Professional or Managerial	0.400	0.401	0.399	0.187	0.341	0.387	0.294	9.965
	(0.556)	(0.601)	(0.493)		(0.544)	(0.563)	(0.520)	***
	[10,446]	[5,860]	[4,586]		[13,488]	[6,747]	[6,741]	
Skilled Blue Collar	0.216	0.204	0.249	-8.813	0.226	0.215	0.254	-8.019
	(0.349)	(0.361)	(0.316)	***	(0.316)	(0.324)	(0.295)	***
	[20,492]	[14,687]	[5,805]		[18,843]	[13,328]	[5,515]	
Technical or Supervisory	0.223	0.208	0.248	-4.899	0.228	0.219	0.245	-4.286
	(0.494)	(0.525)	(0.436)	***	(0.451)	(0.480)	(0.382)	***
	[14,271]	[8,994]	[5,277]		[21,493]	[14,522]	[6,971]	
Unskilled Blue Collar	0.192	0.193	0.190	0.253	0.186	0.180	0.195	-2.017
	(0.365)	(0.374)	(0.347)		(0.283)	(0.292)	(0.268)	**
	[3,936]	[2,527]	[1,409]		[5,878]	[3,610]	[2,268]	
College Degree	0.440	0.464	0.419	2.849	0.359	0.394	0.332	4.707
	(0.567)	(0.625)	(0.510)	***	(0.537)	(0.584)	(0.497)	***
	[5,308]	[2,477]	[2,831]		[7,050]	[3,055]	[3,995]	
No College Degree	0.234	0.232	0.243	-2.907	0.208	0.217	0.180	11.132
	(0.348)	(0.366)	(0.286)	***	(0.312)	(0.324)	(0.271)	***
	[35,769]	[27,031]	[8,738]		[40,456]	[30,849]	[9,607]	
Skilled	0.257	0.246	0.277	-4.093	0.202	0.204	0.198	1.155
	(0.481)	(0.515)	(0.414)	***	(0.421)	(0.447)	(0.357)	
	[15,501]	[9,892]	[5,609]		[25,992]	[17,962]	[8,030]	
Unskilled	0.220	0.212	0.242	-7.468	0.226	0.224	0.232	-1.996
	(0.345)	(0.360)	(0.296)	***	(0.329)	(0.336)	(0.309)	**
	[32,033]	[23,845]	[8,188]		[32,184]	[23,737]	[8,447]	
All Employees	0.251	0.247	0.263	-4.073	0.238	0.243	0.222	5.871
	(0.365)	(0.384)	(0.297)	***	(0.344)	(0.358)	(0.291)	***
	[36,100]	[27,309]	[8,791]		[41,089]	[31,364]	[9,725]	

Notes: Standard deviations appear below each mean in parentheses. The number of observations appears below each standard deviation in square brackets. All statistics are annual data reported at the firm level. Missing observations result from firms without female employees of the indicated category or occupation. Columns 4 and 8 contain an equality of female and male sample means t test with unequal variances. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 3: Cross-Sectional Estimates

	Firm Size$_{1995}$			Firm Size$_{2000}$		
	All	Female Leadership		All	Female Leadership	
Outcome / Regressor	Firms	Y	N	Firms	Y	N
	(1)	(2)	(3)	(4)	(5)	(6)
% Female Employees	0.010***	0.013***	-0.011***	0.007***	0.010***	-0.024***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
% Female Employees — College Degree	0.045***	0.038***	0.049***	0.056***	0.046***	0.052***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
% Female Employees — no College Degree	0.010***	0.013***	-0.012***	0.007***	0.010***	-0.024***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
% Female Employee — Skilled	0.063***	0.069***	0.034***	0.055***	0.066***	-0.006***
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
% Female Employee — Unskilled	0.011***	0.014***	-0.008***	0.010***	0.014***	-0.018***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Male-Female Wage Gap	0.022***	0.026***	0.018***	0.013***	0.014***	0.017***
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
Male-Female Wage Gap — College Degree	0.032***	0.054***	0.025***	0.043***	0.054***	0.051***
	(0.006)	(0.010)	(0.007)	(0.006)	(0.010)	(0.007)
Male-Female Wage Gap — No College Degree	0.019***	0.023***	0.015***	0.009***	0.012***	0.013***
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
Male-Female Wage Gap — Skilled	0.043***	0.050***	0.037***	0.027***	0.025***	0.039***
	(0.003)	(0.004)	(0.004)	(0.002)	(0.004)	(0.003)
Male-Female Wage Gap — Unskilled	0.026***	0.031***	0.019***	0.019***	0.022***	0.018***
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
ln(1+Exports)	1.100***	0.792***	1.684***	1.272***	0.890***	2.020***
	(0.008)	(0.008)	(0.024)	(0.009)	(0.010)	(0.029)
Observations	60,095	49,803	10,292	60,095	49,803	10,292

Notes: Two-digit industry and state fixed effects are included in each specification. Columns (1) through (3) specifications are estimated with data for 1995, whereas Columns (4) through (6) are estimated with data for 2000. Columns (1) and (4) report estimated coefficients using data from all firms. Subsamples in Columns (2) and (5) contain firms in which at least one female was present in the C-Suite in any year between 1995 and 2004. Columns (3) and (6) contain firms without females in the C-Suite in any year between 1995 and 2004. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 4: Cross-Sectional Estimates by Occupational Group

	Firm Size$_{1995}$			Firm Size$_{2000}$		
	All	Female Leadership		All	Female Leadership	
Outcome / Regressor	Firms	Y	N	Firms	Y	N
	(1)	(2)	(3)	(4)	(5)	(6)
Occupation 1: Unskilled White Collar						
% Female Employees	0.055***	0.056***	0.039***	0.060***	0.063***	0.029***
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
Male-female wage gap	0.015***	0.022***	0.017***	0.013***	0.017***	0.015***
	(0.003)	(0.005)	(0.004)	(0.003)	(0.006)	(0.005)
Occupation 2: Professional or Managerial						
% Female Employees	0.057***	0.058***	0.034***	0.060***	0.064***	0.000
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
Male-female wage gap	0.041***	0.061***	0.027***	0.042***	0.055***	0.043***
	(0.004)	(0.006)	(0.005)	(0.004)	(0.006)	(0.005)
Occupation 3: Skilled Blue Collar						
% Female Employees	0.017***	0.018***	0.009***	0.017***	0.019***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Male-female wage gap	0.028***	0.030***	0.021***	0.019***	0.021***	0.013***
	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)
Occupation 4: Technical or Supervisory						
% Female Employees	0.068***	0.081***	0.024***	0.064***	0.079***	-0.003
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
Male-female wage gap	0.044***	0.051***	0.037***	0.038***	0.040***	0.040***
	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)
Occupation 5: Unskilled Blue Collar						
% Female Employees	0.017***	0.015***	0.018***	0.017***	0.015***	0.018***
	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Male-female wage gap	0.013***	0.018***	0.016***	0.003	0.012**	-0.004
	(0.004)	(0.005)	(0.006)	(0.004)	(0.006)	(0.006)
Observations	60,095	49,803	10,292	60,095	49,803	10,292

Notes: Two-digit industry and state fixed effects are included in each specification. Columns (1) through (3) specifications are estimated with data for 1995, whereas Columns (4) through (6) are estimated with data for 2000. Columns (1) and (4) report estimated coefficients using data from all firms. Subsamples in Columns (2) and (5) contain firms in which at least one female was present in the C-Suite in any year between 1995 and 2004. Columns (3) and (6) contain firms without females in the C-Suite in any year between 1995 and 2004. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 5: Difference-in-Difference Estimates: Pooled Sample by Percentage of Female Employees

		Δ% Female Employees				
			College Degree		Skilled	
	Δ ln(1+Exports)	All	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)	(6)
	coeff	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)	(se)
Firm Size (pre)	0.026***	-0.002***	0.000	-0.002***	0.001	-0.002***
	(0.006)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Firm Size (post)	0.076***	-0.001***	-0.000	-0.001***	0.004***	-0.001***
	(0.007)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Δ Firm Size	0.050***	0.001	-0.000	0.001	0.003***	0.002**
	(0.014)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
Observations	120,190	120,190	20,144	119,904	60,457	116,175

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 6: Difference-in-Difference Estimates: Pooled Sample by Wage Gap

	Δ Male-Female Wage Gap				
		College Degree		Skilled	
	Mean	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)
	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)
Firm Size (pre)	-0.004***	0.002	-0.004***	-0.005***	-0.003***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)
Firm Size (post)	0.001**	-0.004	0.001*	-0.002*	0.008***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)
Δ Firm Size	0.006***	-0.006	0.005**	0.003	0.011***
	(0.002)	(0.005)	(0.002)	(0.003)	(0.004)
Observations	69,136	8,885	67,990	30,227	55,053

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 7: Difference-in-Difference Estimates: Pooled Sample by Occupational Group

	Unskilled White Collar		Professional or Managerial		Skilled Blue Collar		Technical or Supervisory		Unskilled Blue Collar	
	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)
Firm Size (pre)	-0.002***	-0.003	0.001	0.000	-0.001***	-0.007***	0.001	-0.007***	-0.004***	0.002
	(0.001)	(0.002)	(0.001)	(0.002)	(0.000)	(0.001)	(0.000)	(0.002)	(0.001)	(0.003)
Firm Size (post)	0.002**	0.001	0.009***	0.005**	-0.000	0.009***	0.007***	-0.006***	-0.002	0.008*
	(0.001)	(0.003)	(0.001)	(0.003)	(0.000)	(0.001)	(0.001)	(0.002)	(0.001)	(0.004)
Δ Firm Size	0.004*	0.004	0.008***	0.005	0.001	0.016***	0.007***	0.002	0.002	0.006
	(0.002)	(0.006)	(0.002)	(0.005)	(0.001)	(0.004)	(0.001)	(0.004)	(0.004)	(0.009)
Observations	34,732	14,163	41,348	16,265	108,246	31,017	58,016	23,608	17,403	4,257

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table 8: Triple-Difference Estimates: Percentage of Female Workers

		Δ% Female Employees				
			College Degree		Skilled	
	Δ ln(1+Exports)	All	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)	(6)
	coeff	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)	(se)
	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$
Panel A: Female Leadership						
♀ Firm Size (pre)	0.010	-0.005***	-0.003	-0.005***	-0.003***	-0.005***
	(0.007)	(0.000)	(0.002)	(0.000)	(0.001)	(0.000)
♀ Firm Size (post)	0.025***	-0.002***	0.003	-0.002***	0.003***	-0.002***
	(0.008)	(0.000)	(0.002)	(0.000)	(0.001)	(0.000)
Δ♀ Firm Size	0.015	0.002***	0.006	0.002***	0.006***	0.003***
	(0.016)	(0.001)	(0.004)	(0.001)	(0.002)	(0.001)
Observations	99,606	99,606	11,149	99,351	44,580	95,998
Panel B: Male Leadership						
♂ Firm Size (pre)	0.001	-0.001**	-0.000	-0.001***	0.002**	-0.001***
	(0.019)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)
♂ Firm Size (post)	0.127***	-0.000	-0.002**	-0.001	0.004***	0.000
	(0.024)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)
Δ♂ Firm Size	0.126***	0.000	-0.002	0.001	0.002	0.001
	(0.030)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Observations	20,584	20,584	8,995	20,553	15,877	20,177
Panel C: F - M Leadership						
Δ♀ Firm Size	-0.110***	0.002	0.008**	0.002	0.003	0.001
-Δ♂ Firm Size	(0.034)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)
	$[0.003]^{+++}$	[0.851]	$[0.973]^{+}$	[0.818]	[0.902]	[0.807]

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$. The inverse empirical cumulative distribution function relative to 1,000 randomized draws of treatment status is given in square brackets. $^{+++}$ significant at a 1% level, $^{++}$ significant at a 5% level, $^{+}$ significant at a 10% level. See Section 6.2 for further detail.

Table 9: Triple-Difference Estimates: Male-Female Wage Gap

	Δ Male-Female Wage Gap				
		College Degree		Skilled	
	Mean	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)
	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)
	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$
Panel A: Female Leadership					
♀ Firm Size (pre)	-0.002	-0.006	-0.002*	-0.006**	-0.002
	(0.001)	(0.008)	(0.001)	(0.003)	(0.001)
♀ Firm Size (post)	0.008***	-0.009	0.006***	-0.001	0.008***
	(0.001)	(0.008)	(0.001)	(0.002)	(0.001)
Δ♀ Firm Size	0.010**	-0.003***	0.008**	0.005	0.010**
	(0.004)	(0.000)	(0.004)	(0.005)	(0.005)
Observations	51,328	3,465	50,401	18,529	39,578
Panel B: Male Leadership					
♂ Firm Size (pre)	-0.005***	0.004	-0.005***	-0.005**	-0.003**
	(0.001)	(0.004)	(0.001)	(0.002)	(0.001)
♂ Firm Size (post)	0.000	-0.003	0.002*	-0.001	0.008***
	(0.001)	(0.004)	(0.001)	(0.002)	(0.002)
Δ♂ Firm Size	0.005*	-0.008	0.007**	0.004	0.012*
	(0.003)	(0.006)	(0.003)	(0.004)	(0.006)
Observations	17,808	5,420	17,589	11,698	15,475
Panel C: F - M Leadership					
Δ♀ Firm Size	0.005	0.005	0.001	0.002	-0.002
-Δ♂ Firm Size	(0.008)	(0.010)	(0.008)	(0.007)	(0.013)
	[0.934]	[0.521]	[0.737]	[0.683]	[0.301]

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$. The inverse empirical cumulative distribution function relative to 1,000 randomized draws of treatment status is given in square brackets. $^{+++}$ significant at a 1% level, $^{++}$ significant at a 5% level, $^{+}$ significant at a 10% level. See Section 6.2 for further detail.

Table 10: Triple-Difference Estimates: Occupational Group

	Unskilled White Collar		Professional or Managerial		Skilled Blue Collar		Technical or Supervisory		Unskilled Blue Collar	
	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)	(se)
	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$
Panel A: Female Leadership										
♀ Firm Size (pre)	-0.003**	-0.001	-0.002*	-0.004	-0.003***	-0.002	-0.002**	-0.009***	-0.006***	-0.006
	(0.001)	(0.005)	(0.001)	(0.005)	(0.000)	(0.002)	(0.001)	(0.003)	(0.001)	(0.005)
♀ Firm Size (post)	-0.003	-0.003	0.017***	0.009	-0.000	0.013***	0.008***	0.007*	0.003*	0.007
	(0.002)	(0.006)	(0.001)	(0.006)	(0.000)	(0.002)	(0.001)	(0.004)	(0.002)	(0.007)
Δ♀ Firm Size	0.000	-0.003	0.019***	0.012***	0.003**	0.015**	0.010***	0.016**	0.009*	0.013
	(0.002)	(0.010)	(0.002)	(0.000)	(0.001)	(0.006)	(0.002)	(0.008)	(0.005)	(0.010)
Observations	22,367	6,902	26,936	7,576	89,110	21,327	42,225	13,791	12,400	2,461
Panel B: Male Leadership										
♂ Firm Size (pre)	-0.000	-0.007**	0.000	0.001	-0.000	-0.009***	0.002**	-0.009***	-0.003*	0.005
	(0.001)	(0.003)	(0.001)	(0.003)	(0.000)	(0.002)	(0.001)	(0.003)	(0.001)	(0.005)
♂ Firm Size (post)	0.003*	0.002	0.002**	0.009***	0.001**	0.008***	0.009***	-0.007***	-0.003	0.006
	(0.001)	(0.004)	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)	(0.003)	(0.007)
Δ♂ Firm Size	0.003	0.010	0.002	0.008	0.002	0.017**	0.007***	0.002	-0.000	0.001
	(0.003)	(0.007)	(0.002)	(0.006)	(0.002)	(0.007)	(0.002)	(0.005)	(0.006)	(0.013)
Observations	12,365	7,261	14,412	8,689	19,136	9,690	15,791	9,817	5,003	1,796
Panel C: F - M Leadership										
Δ♀ Firm Size	-0.003	-0.012	0.017***	0.004***	0.001	-0.002	0.003	0.014	0.009	0.011
-Δ♂ Firm Size	(0.006)	(0.012)	(0.005)	(0.000)	(0.006)	(0.014)	(0.005)	(0.010)	(0.013)	(0.022)
	[0.268]	[0.126]	$[0.999]^{+++}$	[0.472]	[0.682]	[0.333]	[0.782]	$[0.971]^{+}$	$[0.952]^{+}$	[0.723]

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$. The inverse empirical cumulative distribution function relative to 1,000 randomized draws of treatment status is given in square brackets. $^{+++}$ significant at a 1% level, $^{++}$ significant at a 5% level, $^{+}$ significant at a 10% level. See Section 6.2 for further detail.

A Online Appendix

A.1 Data

Our data covers the period 1995-2004 and consists of two firm-level confidential administrative databases. These datasets were cleaned and merged by the Instituto de Pesquisa Econômica Aplicada (IPEA, Institute for Applied Economics Research - Brasília). The first dataset contains firm-level administrative records of foreign trade operations, provided by the *Secretaria de Comércio Exterior* (Secretary of Foreign Trade) of the *Ministério do Desenvolvimento, Indústria e Comércio Exterior* (Ministry of Development, Industry and Foreign Trade). This dataset contains records for firm-level export transactions, which detail the product code, value, quantity, weight, and destination country. The second administrative dataset is the RAIS (*Relação Anual de Informações Sociais*) from the MTE (*Ministério do Trabalho e Emprego*) or Ministry of Labor and Employment. Every firm must fill out and file the RAIS form annually, regardless of the number of employees throughout the year. All labor contracts must be reported, even if the duration of employment were as brief as a few days. If a firm has no employees, it must file a RAIS form to inform the Brazilian government that it has no employees. Because of these requirements, the RAIS dataset effectively covers almost all formal employment, except self-employed and informal workers. The RAIS form contains the firm's tax identification code and all employees' gender, age, level of educational attainment, occupation, and tenure.

These two datasets were merged by the firms' tax identification codes by the Instituto de Pesquisa Econômica Aplicada, (IPEA, Institute for Applied Economics Research - Brasília). as described by De Negri (2005). We consider a worker employed at a certain firm in a given year if she (or he) worked for at least one day in December of that year. After merging, the data was cleaned and validated at the firm and then at the employee level. At the firm level, we retain only firms in manufacturing industries (CNAE-Brazilian Industry Classification version 1.0 codes ranging from 15 to 37) that were under the private sector law, i.e., those with *Tipo_Estb* variable equal to 1 and the *Nat_Jurid* variable different from 2151, 4073, 2178, 2135, 2160, 2992, 2046, 2100, 2097, 2089, 2127, and 2062. The effect of this is to exclude firms owned by the federal, state, or local government.[34] Finally, we also dropped firms with zero employees in December 1995, 1998, 2000, and 2004.

At the employee level, we remove workers younger than 15 years of age or older than 65 years of age from the sample. We dropped workers with missing observable characteristics

[34]These state-owned firms are excluded because their wage policies have to follow federal and state-specific laws, and hiring must take place through civil service examinations.

and wages. Approximately 1.5% of the employee records had no information about occupation. This is the variable with the largest share of missing values. Most importantly, these missing observations exhibit no discernible pattern by industry, year, worker characteristics, or firm size. Additionally, we removed from the sample temporary workers and workers who were not hired for an urban job using a labor contract under the Brazilian CLT labor law labor regulation (*Consolidaç ão das Leis do Trabalho*). If multiple jobs are recorded for a worker, we consider only the highest-paying job.

To further disaggregate employees within firms, we use Helpman et al.'s (2017) five-category classification of occupations: Professional and Managerial, Skilled White-Collar, Unskilled White-Collar, Skilled Blue-Collar, and Unskilled Blue-Collar. The occupation is reported in our data using the CBO 1994 classification for 1995-2002 and the CBO2002 classification for 2003-2004. Helpman et al.'s (2017) five categories are defined using CBO 1994 codes. We use the official correspondence table from the Ministry of Labor and Employment as a crosswalk for the CBO 2002 occupations.

A.2 Math Appendix: Oligopsonistic Firm Wage and Employment

Let firm $j \in \{1..J\}$ offer a wage of $w_{j♀}$ to female members of the labor force and $w_{j♂}$ to males. For each worker i in gender $g \in \{♀, ♂\}$, let the indirect utility of working at firm j be

$$U_{ijg} = \beta_g \ln(w_{jg} - b) + v_g(a_j) + v_{ijg} \tag{6}$$

b is the wage available to the worker in the competitive sector. a_j is the level of non-wage amenities provided by firm j, including workplace flexibility and other aspects of a gender equity supportive environment. The value placed on non-wage amenities, $v_g(a_j)$ is assumed to differ by gender (Goldin and Katz, 2010). For the sake of simplicity, we assume that amenities have a common valuation across all employees of the same gender within each firm. v_{ijg} is the idiosyncratic preferences of worker i for working at firm j, such as commuting time. In line with the findings of Vick (2017) for the Brazilian labor market, we further assume that male employees place a higher value on relative wages than do females, expressed in the notation of our model as $\beta_{♂} > \beta_{♀}$.[35] Female employees place a higher value on amenities than do male employees, $v_{♀}(a_j) > v_{♂}(a_j)$ (Corradini et al., 2025; Goldin and Katz, 2010). For ease of analysis, we normalize the values placed on amenities so that $v_{♂}(a_j) = 0$ and $v_{♀}(a_j)$ represents the additional value placed on amenities by female employees relative to

[35] For instance, Webber (2016) used U.S. LEHD data and found an elasticity of 0.94 for females and 1.09 for males.

male employees.

The probability that worker i is employed at firm j follows a multinomial logit specification (McFadden, 1973). Assuming a large number of firms J, the exponential approximation of the probability that a worker of gender g is employed at firm j is

$$p_{jg} = \gamma_g \exp(\beta_g \ln(w_{jg} - b) + v_g(a_j)) \tag{7}$$

where γ_g is a parameter shared by all firms. The approximate firm-level labor supply for workers of each gender are

$$\ln\left(S_{j♀}(w_{j♀}, a_j)\right) = \ln\left(\mathbb{S}^{♀}\gamma_{♀}\right) + \beta_{♀} \ln\left(w_{j♀} - b\right) + v_{♀}(a_j) \tag{8}$$

$$\ln\left(S_{j♂}(w_{j♂}, a_j)\right) = \ln\left(\mathbb{S}^{♂}\gamma_{♂}\right) + \beta_{♂} \ln\left(w_{j♂} - b\right) + v_{♂}(a_j) \tag{9}$$

where $\mathbb{S}^{♀}$ and $\mathbb{S}^{♂}$ are the total number of available workers by gender in the labor market and $\gamma_{♀}$ and $\gamma_{♂}$ are constants common to all firms.

Using Equations (8) and (9), the firm-level elasticity of labor supply by gender is

$$\epsilon_{jg} = \frac{\beta_g w_{jg}}{w_{jg} - b} \tag{10}$$

Following Pigou (1924), let firm j's ability to "exploit" its workers, i.e., depress wages of workers below their marginal revenue product of labor, be

$$E_{jg} = \frac{1}{\epsilon_{jg}} \tag{11}$$

A convenient measurement of the gender balance in firm j is the difference of Equations (8) and (9)

$$\ln\left(\frac{S_{j♀}(\cdot)}{S_{j♂}(\cdot)}\right) = \ln\left(\frac{\mathbb{S}^{♀}\gamma_{♀}}{\mathbb{S}^{♂}\gamma_{♂}}\right) + \beta_{♀} \ln\left(w_{j♀} - b\right) - \beta_{♂} \ln\left(w_{j♂} - b\right) + v_{♀}(a_j) \tag{12}$$

where by assumption $v_{♂}(a_j) = 0$. Each firm's gender balance is a function of the balance of the labor pool, wages offered by the firm, and the value workers place on non-wage amenities.

Let firm j have the production function

$$Y_j = f\left(S_{j♀}, S_{j♂}, T_j\right) \tag{13}$$

where T_j is a firm-specific productivity parameter and $f(\cdot)$ is a continuous production function twice differentiable in all parameters. To ensure a solution to the profit maximization problem exists, let output increase in technology at a decreasing rate with positive marginal products of labor. It is also necessary that either $\partial^2 f\left(S_{j♀}, S_{j♂}, T_j\right)/\partial S_{j♀}^2 < 0$ and $\partial^2 f\left(S_{j♀}, S_{j♂}, T_j\right)/\partial S_{j♂}^2 < 0$ or some degree of product differentiation and market power to ensure the MRPL curve is downward sloping.[36] Given technology, firms choose wages and amenity levels to maximize profit

$$\max_{w_{j♀}\ w_{j♂}} Pf\left(S_{j♀}, S_{j♂}, T_j\right) - w_{j♀} \cdot S_{j♀}(w_{j♀}, a_j) - w_{j♂} \cdot S_{j♂}(w_{j♂}, a_j) - c(a_j)$$

where $c(a_j)$ is the cost of providing non-wage amenities—such as workplace flexibility (Goldin and Katz, 2010) or female leadership (Flabbi et al., 2019)—and can be assumed exogenous with respect to wage determination. The first order condition for wages by gender can be rearranged as

$$\begin{aligned} w_{jg}\left(1 + E_{jg}\right) &= P\frac{\partial f\left(S_{j♀}, S_{j♂}, T_j\right)}{\partial S_{jg}} \\ MCL_{jg} &= MRPL_{jg}\left(S_{j♀}, S_{j♂}, T_j\right) \end{aligned} \tag{14}$$

Note that an important feature of this model is that wages are increasing in firm productivity, and labor supply is increasing in wages, by Equations (8) and (9). Together, firm-level employment (hereafter, firm size) is increasing in productivity, as in the theoretical model of Melitz (2003), and empirically in Mexico (Verhoogen, 2008) and in Brazil (De Negri, 2005). Equation (14) can be further rearranged to solve for wages by gender.

$$w_{jg} = \frac{MRPL_{jg}\left(S_{j♀}, S_{j♂}, T_j\right)}{1 + E_{jg}} \tag{15}$$

Combining this expression with Equations (10) and (11), the optimal wage for firm j is expressed as a weighted average of the reference wage from the competitive sector and the marginal revenue product of labor.

$$w_{jg} = \frac{1}{1+\beta_g} b + \frac{\beta_g}{1+\beta_g} MRPL_g\left(S_{j♀}, S_{j♂}, T_j\right) \tag{16}$$

A.3 Math Appendix: Amenities

To compare wage and employment outcomes between low and high amenity firms, we derive the equilibrium effects of an exogenous increase in amenities for a firm j by total differenti-

[36]The precise mechanism used to ensure a downward-sloping MRPL curve is inconsequential to the subsequent analysis.

ation of female and male wages, and female and male log labor supply.

$$\begin{bmatrix} \frac{dw_{j\female}}{da} \\ \frac{dw_{j\male}}{da} \\ \frac{d\ln(S_{j\female}(\cdot))}{da} \\ \frac{d\ln\left(S_{j\male}(\cdot)\right)}{da} \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{13} & A_{14} \\ 0 & 0 & A_{23} & A_{24} \\ A_{31} & 0 & 0 & 0 \\ 0 & A_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{dw_{j\female}}{da} \\ \frac{dw_{j\male}}{da} \\ \frac{d\ln(S_{j\female}(\cdot))}{da} \\ \frac{d\ln\left(S_{j\male}(\cdot)\right)}{da} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\partial\ln(S_{j\female}(\cdot))}{\partial a} \\ 0 \end{bmatrix} \tag{17}$$

The A_{ij} represent the partial derivative of variable i with respect to variable j. For example,

$$A_{13} = \frac{\partial w_{j\female}}{\partial \ln\left(S_{j\female}\right)}$$

Applying Cramer's rule, we calculate

$$\begin{bmatrix} \frac{dw_{j\female}}{da} \\ \frac{dw_{j\male}}{da} \\ \frac{d\ln(S_{j\female}(\cdot))}{da} \\ \frac{d\ln\left(S_{j\male}(\cdot)\right)}{da} \end{bmatrix} = D^{-1} \cdot \begin{bmatrix} [A_{13} - A_{42}(A_{13}A_{24} - A_{14}A_{23})] \frac{\partial\ln(S_{j\female}(\cdot))}{\partial a} \\ A_{23} \frac{\partial\ln(S_{j\female}(\cdot))}{\partial a} \\ [1 - A_{24}A_{42}] \frac{\partial\ln(S_{j\female}(\cdot))}{\partial a} \\ A_{42}A_{23} \frac{\partial\ln(S_{j\female}(\cdot))}{\partial a} \end{bmatrix} \tag{18}$$

where

$$D = 1 - A_{13}A_{31} - A_{24}A_{42} + A_{31}A_{42}(A_{13}A_{24} - A_{14}A_{23})$$

Before determining the mathematical sign of D and of each calculated effect, we make the following simplifying assumptions with respect to the A_{ij}. The partial derivatives of female and male wages with respect to both female and male labor supply can be re-expressed as

$$\begin{bmatrix} A_{13} & A_{14} \\ A_{23} & A_{24} \end{bmatrix} = PT_j \begin{bmatrix} S_{j\female} \cdot \frac{\beta_{\female}}{1+\beta_{\female}} \cdot \frac{\partial^2 f(\cdot)}{\partial S_{j\female}^2} & S_{j\male} \cdot \frac{\beta_{\female}}{1+\beta_{\female}} \cdot \frac{\partial^2 f(\cdot)}{\partial S_{j\female} S_{j\male}} \\ S_{j\female} \cdot \frac{\beta_{\male}}{1+\beta_{\male}} \cdot \frac{\partial^2 f(\cdot)}{\partial S_{j\female} S_{j\male}} & S_{j\male} \cdot \frac{\beta_{\male}}{1+\beta_{\male}} \cdot \frac{\partial^2 f(\cdot)}{\partial S_{j\male}^2} \end{bmatrix} \tag{19}$$

For the vast majority of firms in our sample, $S_{j\male} > S_{j\female}$. As supported by the extant empirical literature, we assume that the indirect utility of male workers places a higher value on the relative wage than do female workers $\beta_{\male} > \beta_{\female}$.[37] Finally, the second derivatives of the production function contained in the previous expression form the Hessian of the production

[37] See Equation (1) for further detail.

function.

$$H = \begin{bmatrix} \frac{\partial^2 f(\cdot)}{\partial S_{j\female}^2} & \frac{\partial^2 f(\cdot)}{\partial S_{j\female} S_{j\male}} \\ \frac{\partial^2 f(\cdot)}{\partial S_{j\female} S_{j\male}} & \frac{\partial^2 f(\cdot)}{\partial S_{j\male}^2} \end{bmatrix}$$

A necessary condition for a unique profit maximizing set of inputs is that H be negative definite. This, in turn, requires that

$$\frac{\partial^2 f(S_\female, S_\male, T_j)}{\partial S_\male^2} < 0 \qquad \frac{\partial^2 f(S_\female, S_\male, T_j)}{\partial S_\female^2} < 0$$

and

$$\frac{\partial^2 f(S_\female, S_\male, T_j)}{\partial S_\male^2} \cdot \frac{\partial^2 f(S_\female, S_\male, T_j)}{\partial S_\female^2} - \left[\frac{\partial^2 f(S_\female, S_\male, T_j)}{\partial S_\male \partial S_\female} \right]^2 > 0$$

If $\partial^2 f(\cdot)/\partial S_{j\female}^2$ and $\partial^2 f(\cdot)/\partial S_{j\male}^2$ are roughly similar in magnitude, then

$$\left| \frac{\partial^2 f(\cdot)}{\partial S_{j\female}^2} \right| > \left| \frac{\partial^2 f(\cdot)}{\partial S_{j\female} \partial S_{j\male}} \right| \quad \text{and} \quad \left| \frac{\partial^2 f(\cdot)}{\partial S_{j\male}^2} \right| > \left| \frac{\partial^2 f(\cdot)}{\partial S_{j\female} \partial S_{j\male}} \right|$$

The mathematical sign of both A_{14} and A_{23} depends on whether male and female labor are substitutes or complements. We assume they are perfect substitutes, resulting in a negative sign.

To aid in signing and evaluating the magnitudes of terms within Equation (18), we transform expressions of A_{ij} into elasticities.

$$\begin{aligned} A_{13}A_{31} &= \frac{\partial w_{j\female}}{\partial \ln (S_{j\female})} \cdot \frac{\partial \ln (S_{j\female}(\cdot))}{\partial w_{j\female}} \\ &= \frac{\partial w_{j\female}}{\partial S_{j\female}} S_{j\female}(\cdot) \cdot \frac{\epsilon_{j\female}}{w_{j\female}} \\ &= \underbrace{\gamma_{j\female} \epsilon_{j\female}}_{(-)} \end{aligned}$$

Let $\gamma_{j\female}$ be the elasticity of female wages with respect to female labor supply. Using Equation

(15), this elasticity can be expressed in terms of the marginal revenue product of labor.

$$\begin{aligned}\gamma_{jg} &= \frac{\partial w_{jg}}{\partial S_{jg}} \cdot \frac{S_{jg}(\cdot)}{w_{jg}} \\ &= \frac{\partial \left[MRPL_{jg}(\cdot)/(1+E_{jg})\right]}{\partial S_{jg}(\cdot)} \cdot \frac{S_{jg}(\cdot)}{MRPL_{jg}/(1+E_{jg})} \\ &= \frac{\partial MRPL_{jg}(\cdot)}{\partial S_{jg}} \cdot \frac{S_{jg}(\cdot)}{MRPL_{jg}(\cdot)}\end{aligned}$$

By the diminishing marginal product of labor, $\gamma_{jg} < 0$. We are not aware of any guidance in the literature on possible magnitudes of γ_{jg}. Under the simplifying assumption that male and female labor are perfect substitutes, the marginal product of labor and the rate at which it diminishes are equal by gender.

$$\begin{aligned}\frac{\gamma_{j♀}}{\gamma_{j♂}} &= \frac{(\partial MRPL_{j♀}(\cdot)/\partial S_{j♀}) \cdot (S_{j♀}(\cdot)/MRPL_{j♀}(\cdot))}{(\partial MRPL_{j♂}(\cdot)/\partial S_{j♂}) \cdot (S_{j♂}(\cdot)/MRPL_{j♂}(\cdot))} \\ &= \frac{S_{j♀}(\cdot)}{S_{j♂}(\cdot)} \qquad (20)\end{aligned}$$

As the observed proportion of female workers is less than 50%, we have $\gamma_{j♂} < \gamma_{j♀} < 0$.

Similarly,

$$\begin{aligned}A_{24}A_{42} &= \frac{\partial w_{j♂}}{\partial \ln(S_{j♂})} \cdot \frac{\partial \ln(S_{j♂}(\cdot))}{\partial w_{j♂}} \\ &= \frac{\partial w_{j♂}}{\partial S_{j♂}} S_{j♂}(\cdot) \cdot \frac{\epsilon_{j♂}}{w_{j♂}} \\ &= \underbrace{\gamma_{j♂}\epsilon_{j♂}}_{(-)}\end{aligned}$$

$$\begin{aligned}A_{31}A_{42}(A_{13}A_{24} - A_{14}A_{23}) &= \frac{\partial \ln(S_{j♀}(\cdot))}{\partial w_{j♀}} \cdot \frac{\partial \ln(S_{j♂}(\cdot))}{\partial w_{j♂}} \left[\frac{\partial w_{j♀}}{\partial \ln(S_{j♀})} \cdot \frac{\partial w_{j♂}}{\partial \ln(S_{j♂})} - \frac{\partial w_{j♀}}{\partial \ln(S_{j♂})} \cdot \frac{\partial w_{j♂}}{\partial \ln(S_{j♀})}\right] \\ &= \frac{\epsilon_{j♀}}{w_{j♀}} \cdot \frac{\epsilon_{j♂}}{w_{j♂}} \cdot S_{j♀}S_{j♂} \cdot \left[\frac{\partial w_{j♀}}{\partial S_{j♀}} \cdot \frac{\partial w_{j♂}}{\partial S_{j♂}} - \frac{\partial w_{j♀}}{\partial S_{j♂}} \cdot \frac{\partial w_{j♂}}{\partial S_{j♀}}\right] \\ &= \underbrace{\epsilon_{j♀}\epsilon_{j♂}}_{(+)} \underbrace{(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀})}_{(+)}\end{aligned}$$

The elasticities of labor supply with respect to wages are positive. If the Hessian is negative definite, the positive determinant ensures that the second term is also positive.

We can unambiguously sign the determinant as positive.

$$D = 1 - \underbrace{A_{13}A_{31}}_{(-)} - \underbrace{A_{24}A_{42}}_{(-)} + \underbrace{A_{31}A_{42}(A_{13}A_{24} - A_{14}A_{23})}_{(+)}$$

An exogenous increase in amenities unambiguously causes female wages to decline. With $D > 0$, the sign is determined by the sign of the numerator.

$$[\underbrace{A_{13}}_{(-)} - \underbrace{A_{42}(A_{13}A_{24} - A_{14}A_{23})}_{(+)}] \underbrace{\frac{\partial \ln (S_{j♀}(\cdot))}{\partial a}}_{(+)}$$

Additive terms within the brackets have been previously signed. From female labor supply, Equation (8)

$$\frac{\partial \ln (S_{j♀}(\cdot))}{\partial a} = v'_{♀}(a_j) > 0$$

An exogenous increase in amenities also causes male wages to drop, as $A_{23} < 0$. Intuitively, this happens because firms replace male workers with female workers, as we explain next. The effect of an exogenous increase in amenities is an unambiguous increase in female employment.

$$\underbrace{[1 - A_{24}A_{42}]]}_{(+)} \underbrace{\frac{\partial \ln (S_{j♀}(\cdot))}{\partial a}}_{(+)}$$

Reduced male wages and no value placed on amenities cause the male labor supply to decrease.

Summarizing,

$$\begin{bmatrix} \frac{dw_{j♀}}{da} < 0 \\ \frac{dw_{j♂}}{da} < 0 \\ \frac{d\ln(S_{j♀}(\cdot))}{da} > 0 \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{da} < 0 \end{bmatrix}$$

The effect of an exogenous increase in amenities on the gender wage gap is

$$\frac{d}{da}\left[\frac{w_{j♂}}{w_{j♀}}\right] = \frac{w_{j♀} \cdot \frac{dw_{j♂}}{da} - w_{j♂} \cdot \frac{dw_{j♀}}{da}}{w_{j♀}^2}$$

The sign of the effect on wages is determined by the sign of the numerator.

$$
\begin{aligned}
& w_{j♀} \cdot \frac{dw_{j♂}}{da} - w_{j♂} \cdot \frac{dw_{j♀}}{da} \\
& = w_{j♀} D^{-1} \left\{ A_{23} \frac{\partial \ln (S_{j♀}(\cdot))}{\partial a} \right\} - w_{j♂} D^{-1} \left\{ [A_{13} - A_{42}(A_{13}A_{24} - A_{14}A_{23})] \frac{\partial \ln (S_{j♀}(\cdot))}{\partial a} \right\}
\end{aligned}
$$

Factoring D and $\partial \ln (S_{j♀}(\cdot)) / \partial a$ from the expression above, the sign is determined by

$$
\begin{aligned}
& w_{j♀} A_{23} - w_{j♂} [A_{13} - A_{42}(A_{13}A_{24} - A_{14}A_{23})] \\
& = w_{j♀} \gamma_{j♂♀} w_{j♂} - w_{j♂} \left[\gamma_{j♀} w_{j♀} - \frac{\epsilon_{j♂}}{w_{j♂}} (\gamma_{j♀} w_{j♀} \gamma_{j♂} w_{j♂} - \gamma_{j♀♂} w_{j♂} \gamma_{j♂♀} w_{j♀}) \right] \\
& = \underbrace{w_{j♀} w_{j♂} [\gamma_{j♂♀} - \gamma_{j♀}]}_{(+)} + \underbrace{w_{j♀} \epsilon_{j♂} (\gamma_{j♀} \gamma_{j♂} - \gamma_{j♀♂} \gamma_{j♂♀})}_{(+)} \qquad (21)
\end{aligned}
$$

The sign of this expression is positive. The sign of each additive expression is determined by the necessary conditions for the Hessian to be negative definite: the determinant must be positive, and $|\gamma_{j♀}| > |\gamma_{j♂♀}|$. We conclude that an exogenous increase in amenities causes the gender wage gap to widen. Moreover, an exogenous increase in amenities will cause female employment to increase and male employment to decrease. Therefore, the gender workforce balance improves.

In summary, an exogenous increase in amenities causes both female and male wages to fall and the gender wage gap to increase. This means that increased amenities via female leadership, valued by female but not male employees, causes the gender workforce balance to improve despite a widening gender wage gap, that is, both an absolute and relative deterioration in wages. This concludes the proofs of Theorems 1 and 2.

A change in the level of amenities causes the following changes in elasticities.

$$
\begin{aligned}
\frac{\partial \gamma_{j♀}}{\partial a} &= \frac{\partial}{\partial a} \left[\frac{\partial w_{j♀}}{\partial S_{j♀}} S_{j♀} w_{j♀}^{-1} \right] \\
&= \frac{\partial w_{j♀}}{\partial S_{j♀}} \left[\frac{\partial S_{j♀}}{\partial a} \cdot w_{j♀}^{-1} + S_{j♀} \cdot \frac{\partial w_{j♀}^{-1}}{\partial a} \right] \\
&= \frac{\partial w_{j♀}}{\partial S_{j♀}} \left[v'(a) \cdot w_{j♀}^{-1} + S_{j♀} \cdot 0 \right] \\
&= v'(a) \frac{\gamma_{j♀}}{S_{j♀}} \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \gamma_{j♂}}{\partial a} &= \frac{\partial w_{j♂}}{\partial S_{j♂}} \left[\frac{\partial S_{j♂}}{\partial a} \cdot w_{j♂}^{-1} + S_{j♂} \cdot \frac{\partial w_{j♂}^{-1}}{\partial a} \right] \\
&= \frac{\partial w_{j♂}}{\partial S_{j♂}} \left[0 \cdot w_{j♂}^{-1} + S_{j♂} \cdot 0 \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \gamma_{j♂♀}}{\partial a} &= \frac{\partial w_{j♂}}{\partial S_{j♀}} \left[\frac{\partial S_{j♀}}{\partial a} \cdot w_{j♂}^{-1} + S_{j♀} \cdot \frac{\partial w_{j♂}^{-1}}{\partial a} \right] \\
&= \frac{\partial w_{j♂}}{\partial S_{j♀}} \left[v'(a) \cdot w_{j♂}^{-1} + S_{j♀} \cdot 0 \right] \\
&= v'(a) \frac{\gamma_{j♂♀}}{S_{j♀}} \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \gamma_{j♀♂}}{\partial a} &= \frac{\partial w_{j♀}}{\partial S_{j♂}} \left[\frac{\partial S_{j♂}}{\partial a} \cdot w_{j♀}^{-1} + S_{j♂} \cdot \frac{\partial w_{j♀}^{-1}}{\partial a} \right] \\
&= \frac{\partial w_{j♀}}{\partial S_{j♂}} \left[0 \cdot w_{j♂}^{-1} + S_{j♀} \cdot 0 \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \epsilon_{j♀}}{\partial a} &= \frac{\partial}{\partial a} \left[\frac{\partial S_{j♀}}{\partial w_{j♀}} w_{j♀} S_{j♀}^{-1} \right] \\
&= \frac{\partial S_{j♀}}{\partial w_{j♀}} \left[\frac{\partial w_{j♀}}{\partial a} \cdot S_{j♀}^{-1} + w_{j♀} \cdot \frac{\partial S_{j♀}^{-1}}{\partial a} \right] \\
&= \frac{\partial S_{j♀}}{\partial w_{j♀}} \left[0 \cdot S_{j♀}^{-1} - w_{j♀} S_{j♀}^{-2} \cdot v'(a) \right] \\
&= -v'(a) \frac{\epsilon_{j♀}}{S_{j♀}} \\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \epsilon_{j♂}}{\partial a} &= \frac{\partial}{\partial a}\left[\frac{\partial S_{j♂}}{\partial w_{j♂}} w_{j♂} S_{j♂}^{-1}\right] \\
&= \frac{\partial S_{j♂}}{\partial w_{j♂}}\left[\frac{\partial w_{j♂}}{\partial a}\cdot S_{j♂}^{-1} + w_{j♂}\cdot \frac{\partial S_{j♂}^{-1}}{\partial a}\right] \\
&= \frac{\partial S_{j♂}}{\partial w_{j♂}}\left[0\cdot S_{j♂}^{-1} + w_{j♂}\cdot 0\right] \\
&= 0
\end{aligned}
$$

A.4 Math Appendix: Firm size

In our theoretical model, one of the determinants of firm size measured by the firm's employment level is firm-level productivity. To derive the effects of a productivity increase in firm j's labor outcomes, we apply total differentiation to female and male wages, and female and male log labor supply.

$$
\begin{bmatrix} \frac{dw_{j♀}}{dT} \\ \frac{dw_{j♂}}{dT} \\ \frac{d\ln(S_{j♀}(\cdot))}{dT} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dT} \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{13} & A_{14} \\ 0 & 0 & A_{23} & A_{24} \\ A_{31} & 0 & 0 & 0 \\ 0 & A_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{dw_{j♀}}{dT} \\ \frac{dw_{j♂}}{dT} \\ \frac{d\ln(S_{j♀}(\cdot))}{dT} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dT} \end{bmatrix} + \begin{bmatrix} \frac{\partial w_{j♀}}{\partial T} \\ \frac{\partial w_{j♂}}{\partial T} \\ 0 \\ 0 \end{bmatrix} \tag{22}
$$

The A_{ij} represent the partial derivative of variable i with respect to variable j. For example,

$$
A_{13} = \frac{\partial w_{j♀}}{\partial \ln\left(S_{j♀}\right)}
$$

Applying Cramer's rule, we calculate

$$
\begin{bmatrix} \frac{dw_{j♀}}{dT} \\ \frac{dw_{j♂}}{dT} \\ \frac{d\ln(S_{j♀}(\cdot))}{dT} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dT} \end{bmatrix} = D^{-1}\cdot \begin{bmatrix} (1-A_{24}A_{42})\frac{\partial w_{j♀}}{\partial T} + A_{14}A_{42}\frac{\partial w_{j♂}}{\partial T} \\ A_{23}A_{31}\frac{\partial w_{j♀}}{\partial T} + (1-A_{13}A_{31})\frac{\partial w_{j♂}}{\partial T} \\ A_{31}\left[(1-A_{24}A_{42})\frac{\partial w_{j♀}}{\partial T} + A_{14}A_{42}\frac{\partial w_{j♂}}{\partial T}\right] \\ A_{42}\left[A_{23}A_{31}\frac{\partial w_{j♀}}{\partial T} + (1-A_{13}A_{31})\frac{\partial w_{j♂}}{\partial T}\right] \end{bmatrix} \tag{23}
$$

where the sign of D is positive, as shown in Appendix $A.3$.

$$D = 1 - \underbrace{A_{13}A_{31}}_{(-)} - \underbrace{A_{24}A_{42}}_{(-)} + \underbrace{A_{31}A_{42}(A_{13}A_{24} - A_{14}A_{23})}_{(+)}$$

With $D > 0$, the mathematical sign of each effect in the matrix of Equation (23) is determined by the sign of each numerator.

From Equation (16),

$$\frac{\partial w_{jg}}{\partial T} = \frac{\beta_g}{1+\beta_g}\frac{\partial MRPL_g}{\partial T} > 0 \tag{24}$$

Given $\beta_{♀} < \beta_{♂}$, $\partial w_{j♀}/\partial T < \partial w_{j♂}/\partial T$ if the effect of a change in technology has a comparable effect on the marginal revenue product of labor by gender.

The sign of $dw_{j♀}/dT$ is theoretically ambiguous.

$$\frac{dw_{j♀}}{dT} = \underbrace{(1 - A_{24}A_{42})}_{(+)}\frac{\partial w_{j♀}}{\partial T} + \underbrace{A_{14}A_{42}}_{(-)}\frac{\partial w_{j♂}}{\partial T}$$

However, if the direct effect of a change in technology is larger in magnitude than feedback effects from the opposite gender, the net effect is positive.

The sign of $dw_{j♂}/dT$ is comparable.

$$\frac{dw_{j♂}}{dT} = \underbrace{A_{23}A_{31}}_{(-)}\frac{\partial w_{j♀}}{\partial T} + \underbrace{(1 - A_{13}A_{31})}_{(+)}\frac{\partial w_{j♂}}{\partial T}$$

Although the sign is theoretically ambiguous, if direct effects dominate feedback effects via the opposite gender, the net effect is positive.

The effects on labor supply are a positive multiple of the effects on wages. Then

$$\begin{bmatrix} \frac{dw_{j♀}}{dT} > 0 \\ \frac{dw_{j♂}}{dT} > 0 \\ \frac{d\ln(S_{j♀}(\cdot))}{dT} > 0 \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dT} > 0 \end{bmatrix}$$

The effect of an exogenous increase in technology on the gender wage gap is

$$\frac{d}{dT}\left[\frac{w_{j\male}}{w_{j\female}}\right] = \frac{w_{j\female}\cdot\frac{dw_{j\male}}{dT} - w_{j\male}\cdot\frac{dw_{j\female}}{dT}}{w_{j\female}^2}$$

The sign of the effect on relative wages is determined by the sign of the numerator.

$$\begin{aligned}
& w_{j\female}\cdot\frac{dw_{j\male}}{dT} - w_{j\male}\cdot\frac{dw_{j\female}}{dT} \\
& = w_{j\female}D^{-1}\left[A_{23}A_{31}\frac{\partial w_{j\female}}{\partial T} + (1-A_{13}A_{31})\frac{\partial w_{j\male}}{\partial T}\right] \\
& - w_{j\male}D^{-1}\left[(1-A_{24}A_{42})\frac{\partial w_{j\female}}{\partial T} + A_{14}A_{42}\frac{\partial w_{j\male}}{\partial T}\right] \\
& = w_{j\female}D^{-1}\left[\gamma_{j\male\female}\epsilon_{j\female}\frac{w_{j\male}}{w_{j\female}}\frac{\beta_\female}{1+\beta_\female}\frac{\partial MRPL_{j\female}}{\partial T} + (1-\gamma_{j\female}\epsilon_{j\female})\frac{\beta_\male}{1+\beta_\male}\frac{\partial MRPL_{j\male}}{\partial T}\right] \\
& - w_{j\male}D^{-1}\left[(1-\gamma_{j\male}\epsilon_{j\male})\frac{\beta_\female}{1+\beta_\female}\frac{\partial MRPL_{j\female}}{\partial T} + \gamma_{j\female\male}\epsilon_{j\male}\frac{w_{j\female}}{w_{j\male}}\frac{\beta_\male}{1+\beta_\male}\frac{\partial MRPL_{j\male}}{\partial T}\right]
\end{aligned}$$

As both types of workers are perfect substitutes, the exogenous change in productivity affects both marginal revenue products of labor equally, $\partial MRPL_{j\female}/\partial T = \partial MRPL_{j\male}/\partial T$, this in addition to D^{-1} can be factored out of the previous expression leaving the sign unaffected.

$$\frac{w_{j\female}\beta_\male}{1+\beta_\male}(1-\gamma_{j\female}\epsilon_{j\female}-\gamma_{j\female\male}\epsilon_{j\male}) - \frac{w_{j\male}\beta_\female}{1+\beta_\female}(1-\gamma_{j\male}\epsilon_{j\male}-\gamma_{j\male\female}\epsilon_{j\female})$$

The first-order effects of this expression can be further expanded using Equation (16).

$$\begin{aligned}
\frac{w_{j\female}\beta_\male}{1+\beta_\male} - \frac{w_{j\male}\beta_\female}{1+\beta_\female} &= \left[\frac{b}{1+\beta_\female} + \frac{\beta_\female}{1+\beta_\female}MRPL_{j\female}\right]\frac{\beta_\male}{1+\beta_\male} - \left[\frac{b}{1+\beta_\male} + \frac{\beta_\male}{1+\beta_\male}MRPL_{j\male}\right]\frac{\beta_\female}{1+\beta_\female} \\
&= \underbrace{\frac{b}{(1+\beta_\female)(1+\beta_\male)}(\beta_\male-\beta_\female)}_{(+)} + \underbrace{\frac{\beta_\female\beta_\male}{(1+\beta_\female)(1+\beta_\male)}(MRPL_\female - MRPL_\male)}_{(0)}
\end{aligned} \tag{25}$$

Under the assumption that the marginal revenue products are equal by gender, given $\beta_\female < \beta_\male$, the first-order effects of an exogenous increase in technology cause the gender wage gap to widen.

The effect of an increase in technology on employee gender balance is

$$\begin{aligned}
\frac{d}{dT}\ln\left(\frac{S_{j\female}(\cdot)}{S_{j\male}(\cdot)}\right) &= \frac{d\ln\left(S_{j\female}(\cdot)\right)}{dT} - \frac{d\ln\left(S_{j\male}(\cdot)\right)}{dT} \\
&= A_{31}\cdot\frac{dw_{j\female}}{dT} - A_{42}\cdot\frac{dw_{j\male}}{dT} \\
&= \frac{\epsilon_{j\female}}{w_{j\female}}\cdot D^{-1}\left[\left(1-\gamma_{j\male}\epsilon_{j\male}\right)\frac{\beta_{\female}}{1+\beta_{\female}}\frac{\partial MRPL_{j\female}}{\partial T} + \gamma_{j\female\male}\epsilon_{j\male}\frac{w_{j\female}}{w_{j\male}}\frac{\beta_{\male}}{1+\beta_{\male}}\frac{\partial MRPL_{j\male}}{\partial T}\right] \\
&- \frac{\epsilon_{j\male}}{w_{j\male}}\cdot D^{-1}\left[\gamma_{j\male\female}\epsilon_{j\female}\frac{w_{j\male}}{w_{j\female}}\frac{\beta_{\female}}{1+\beta_{\female}}\frac{\partial MRPL_{j\female}}{\partial T} + \left(1-\gamma_{j\female}\epsilon_{j\female}\right)\frac{\beta_{\male}}{1+\beta_{\male}}\frac{\partial MRPL_{j\male}}{\partial T}\right]
\end{aligned}$$

The first-order effects are

$$D^{-1}\left[\frac{\epsilon_{j\female}}{w_{j\female}}\frac{\beta_{\female}}{1+\beta_{\female}}\frac{\partial MRPL_{j\female}}{\partial T} - \frac{\epsilon_{j\male}}{w_{j\male}}\frac{\beta_{\male}}{1+\beta_{\male}}\frac{\partial MRPL_{j\male}}{\partial T}\right]$$

If an exogenous change in technology is assumed to affect both marginal revenue products of labor equally, $\partial MRPL_{j\female}/\partial T = \partial MRPL_{j\male}/\partial T$, this in addition to D^{-1} can be factored out of the previous expression leaving the sign unaffected. Substituting in Equation (16) and simplifying, the sign of the first-order effects is equivalent to the sign of

$$\underbrace{(\beta_{\female}\epsilon_{j\female} - \beta_{\male}\epsilon_{j\male})b}_{(-)} + \underbrace{\beta_{\female}\beta_{\male}MRPL_j(\epsilon_{j\female} - \epsilon_{j\male})}_{(-)} \tag{26}$$

Given $\beta_{\female} < \beta_{\male}$ and $\epsilon_{j\female} < \epsilon_{j\male}$, the first part of the expression is unambiguously negative. Under the assumption that the marginal revenue product of labor is equal by gender, the sign of the second expression is also unambiguously negative.

In summary, if $MRPL_{\female}$ and $MRPL_{\male}$ are equal or sufficiently close which is the case under our assumptions of male and female workers being equally productive and perfect substitutes, an exogenous increase in technology (or higher productivity) will cause the gender wage gap to widen and the gender workforce balance to deteriorate. This means that more productive (larger) firms will exhibit a wider gender wage gap and a lower female share. This concludes the proofs of Theorems 4 and 3.

A change in the level of technology causes the following changes in elasticities.

$$\begin{aligned}
\frac{\partial \gamma_{j\female}}{\partial T} &= \frac{\partial}{\partial T}\left[\frac{\partial w_{j\female}}{\partial S_{j\female}} S_{j\female} w_{j\female}^{-1}\right] \\
&= \frac{\partial w_{j\female}}{\partial S_{j\female}}\left[\frac{\partial S_{j\female}}{\partial T}\cdot w_{j\female}^{-1} + S_{j\female}\cdot \frac{\partial w_{j\female}^{-1}}{\partial T}\right] \\
&= \frac{\partial w_{j\female}}{\partial S_{j\female}}\left[0\cdot w_{j\female}^{-1} - S_{j\female}\cdot w_{j\female}^{-2}\frac{\partial w_{j\female}}{\partial T}\right] \\
&= -\frac{\gamma_{j\female}}{w_{j\female}}\frac{\partial w_{j\female}}{\partial T} \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \gamma_{j\male}}{\partial T} &= \frac{\partial w_{j\male}}{\partial S_{j\male}}\left[\frac{\partial S_{j\male}}{\partial T}\cdot w_{j\male}^{-1} + S_{j\male}\cdot \frac{\partial w_{j\male}^{-1}}{\partial T}\right] \\
&= \frac{\partial w_{j\male}}{\partial S_{j\male}}\left[0\cdot w_{j\male}^{-1} - S_{j\male}\cdot w_{j\male}^{-2}\frac{\partial w_{j\male}}{\partial T}\right] \\
&= -\frac{\gamma_{j\male}}{w_{j\male}}\frac{\partial w_{j\male}}{\partial T} \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \gamma_{j\male\female}}{\partial T} &= \frac{\partial w_{j\male}}{\partial S_{j\female}}\left[\frac{\partial S_{j\female}}{\partial T}\cdot w_{j\male}^{-1} + S_{j\female}\cdot \frac{\partial w_{j\male}^{-1}}{\partial T}\right] \\
&= \frac{\partial w_{j\male}}{\partial S_{j\female}}\left[0\cdot w_{j\male}^{-1} - S_{j\female}\cdot w_{j\male}^{-2}\frac{\partial w_{j\male}}{\partial T}\right] \\
&= -\frac{\gamma_{j\male\female}}{w_{j\male}}\frac{\partial w_{j\male}}{\partial T} \\
&> 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \gamma_{j\female\male}}{\partial T} &= \frac{\partial w_{j\female}}{\partial S_{j\male}}\left[\frac{\partial S_{j\male}}{\partial T}\cdot w_{j\female}^{-1} + S_{j\male}\cdot\frac{\partial w_{j\female}^{-1}}{\partial T}\right] \\
&= \frac{\partial w_{j\female}}{\partial S_{j\male}}\left[0\cdot w_{j\female}^{-1} - S_{j\male}\cdot w_{j\female}^{-2}\frac{\partial w_{j\female}}{\partial T}\right] \\
&= -\frac{\gamma_{j\female\male}}{w_{j\female}}\frac{\partial w_{j\female}}{\partial T} \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \epsilon_{j\female}}{\partial T} &= \frac{\partial}{\partial T}\left[\frac{\partial S_{j\female}}{\partial w_{j\female}} w_{j\female} S_{j\female}^{-1}\right] \\
&= \frac{\partial S_{j\female}}{\partial w_{j\female}}\left[\frac{\partial w_{j\female}}{\partial T}\cdot S_{j\female}^{-1} + w_{j\female}\cdot\frac{\partial S_{j\female}^{-1}}{\partial T}\right] \\
&= \frac{\partial S_{j\female}}{\partial w_{j\female}}\left[\frac{\partial w_{j\female}}{\partial T}\cdot S_{j\female}^{-1} - w_{j\female}\cdot 0\right] \\
&= \frac{\epsilon_{j\female}}{w_{j\female}}\frac{\partial w_{j\female}}{\partial T} \\
&> 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \epsilon_{j\male}}{\partial T} &= \frac{\partial S_{j\male}}{\partial w_{j\male}}\left[\frac{\partial w_{j\male}}{\partial T}\cdot S_{j\male}^{-1} + w_{j\male}\cdot\frac{\partial S_{j\male}^{-1}}{\partial T}\right] \\
&= \frac{\partial S_{j\male}}{\partial w_{j\male}}\left[\frac{\partial w_{j\male}}{\partial T}\cdot S_{j\male}^{-1} - w_{j\male}\cdot 0\right] \\
&= \frac{\epsilon_{j\male}}{w_{j\male}}\frac{\partial w_{j\male}}{\partial T} \\
&> 0
\end{aligned}
$$

A.5 Math Appendix: Price Shock

To derive the equilibrium effects of an export shock within firm j, we apply total differentiation to female and male wages, and female and male log labor supply.

$$\begin{bmatrix} \frac{dw_{j♀}}{dP} \\ \frac{dw_{j♂}}{dP} \\ \frac{d\ln(S_{j♀}(\cdot))}{dP} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dP} \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{13} & A_{14} \\ 0 & 0 & A_{23} & A_{24} \\ A_{31} & 0 & 0 & 0 \\ 0 & A_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{dw_{j♀}}{dP} \\ \frac{dw_{j♂}}{dP} \\ \frac{d\ln(S_{j♀}(\cdot))}{dP} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dP} \end{bmatrix} + \begin{bmatrix} \frac{\partial w_{j♀}}{\partial p} \\ \frac{\partial w_{j♂}}{\partial p} \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

The A_{ij} represent the partial derivative of variable i with respect to variable j. For example,

$$A_{13} = \frac{\partial w_{j♀}}{\partial \ln\left(S_{j♀}\right)}$$

Applying Cramer's rule, we calculate

$$\begin{bmatrix} \frac{dw_{j♀}}{dP} \\ \frac{dw_{j♂}}{dP} \\ \frac{d\ln(S_{j♀}(\cdot))}{dP} \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dP} \end{bmatrix} = D^{-1} \cdot \begin{bmatrix} [1 - A_{24}A_{42}] \cdot \frac{\partial w_{j♀}}{\partial P} + A_{14}A_{42} \cdot \frac{\partial w_{j♂}}{\partial P} \\ A_{23}A_{31} \cdot \frac{\partial w_{j♀}}{\partial P} + [1 - A_{13}A_{31}] \cdot \frac{\partial w_{j♂}}{\partial P} \\ A_{31} \cdot \left\{ [1 - A_{24}A_{42}] \cdot \frac{\partial w_{j♀}}{\partial P} + A_{14}A_{42} \cdot \frac{\partial w_{j♂}}{\partial P} \right\} \\ A_{42} \cdot \left\{ A_{23}A_{31} \cdot \frac{\partial w_{j♀}}{\partial P} + [1 - A_{13}A_{31}] \cdot \frac{\partial w_{j♂}}{\partial P} \right\} \end{bmatrix} \tag{28}$$

where the sign of D is positive, as shown in Appendix $A.3$.

$$D = 1 - \underbrace{A_{13}A_{31}}_{(-)} - \underbrace{A_{24}A_{42}}_{(-)} + \underbrace{A_{31}A_{42}(A_{13}A_{24} - A_{14}A_{23})}_{(+)}$$

With $D > 0$, the mathematical sign of each effect in the system of Equation (27) is determined by the sign of each numerator. The numerator of $dw_{j♀}/dP$ expressed in terms of elasticities is

$$\begin{aligned} [1 - A_{24}A_{42}] \cdot \frac{\partial w_{j♀}}{\partial P} + A_{14}A_{42} \cdot \frac{\partial w_{j♂}}{\partial P} &= \left[1 - \frac{\partial w_{j♂}}{\partial S_{j♂}} S_{j♂} \frac{\epsilon_{j♂}}{w_{j♂}}\right] \cdot \frac{\partial w_{j♀}}{\partial P} + \frac{\partial w_{j♀}}{\partial S_{j♂}} S_{j♂} \frac{\epsilon_{j♂}}{w_{j♂}} \cdot \frac{\partial w_{j♂}}{\partial P} \\ &= [1 - \gamma_{j♂}\epsilon_{j♂}] \cdot \frac{\partial w_{j♀}}{\partial P} + \gamma_{j♀♂}\epsilon_{j♂} \cdot \frac{\partial w_{j♂}}{\partial P} \end{aligned}$$

From Equation (16),

$$\frac{\partial w_{jg}}{\partial P} = \underbrace{\frac{\beta_g}{1+\beta_g} MPL_g\left(S_{j♀}, S_{j♂}\right)}_{(+)}$$

The first-order effect of an exogenous price increase on female wages is positive. One second-order effect is positive, and the other second order effect is negative. The negative second-order effect contains the cross-elasticity of female wages with respect to male labor supply, which we speculate is small. The total effect of an exogenous price increase on $w_{j♀}$ is positive.

The mathematics for $(dw_{j♂}/dP)$ is analogous. The numerator of $(dw_{j♂}/dP)$ is

$$A_{23}A_{31} \cdot \frac{\partial w_{j♀}}{\partial P} + [1 - A_{13}A_{31}] \cdot \frac{\partial w_{j♂}}{\partial P} = \gamma_{j♂♀}\epsilon_{j♀} \cdot \frac{\partial w_{j♀}}{\partial P} + [1 - \gamma_{j♀}\epsilon_{j♀}] \cdot \frac{\partial w_{j♂}}{\partial P}$$

As for $w_{j♀}$, the first-order effect of a price increase on $w_{j♂}$ is positive. One second-order effect is reinforcing, and the counteracting second-order effect is likely small. Together, the total effect of an exogenous price increase on $w_{j♂}$ is positive.

From Equation (28)

$$\frac{d\ln\left(S_{jg}(\cdot)\right)}{dP} = \frac{\partial \ln\left(S_{jg}(\cdot)\right)}{\partial w_{jg}} \cdot \frac{dw_{jg}}{dP}$$

The effect of an exogenous price increase on the labor supply of each gender is positive.

Summarizing,

$$\begin{bmatrix} \frac{dw_{j♀}}{dP} > 0 \\ \frac{dw_{j♂}}{dP} > 0 \\ \frac{d\ln(S_{j♀}(\cdot))}{dP} > 0 \\ \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dP} > 0 \end{bmatrix}$$

The effect of a positive price shock on the gender wage gap is

$$\frac{d}{dP}\left[\frac{w_{j♂}}{w_{j♀}}\right] = \frac{w_{j♀} \cdot \frac{dw_{j♂}}{dP} - w_{j♂} \cdot \frac{dw_{j♀}}{dP}}{w_{j♀}^2}$$

The sign of the effect on wages is determined by the sign of the numerator.

$$\begin{aligned} & w_{j♀} \cdot \frac{dw_{j♂}}{dP} - w_{j♂} \cdot \frac{w_{j♀}}{dP} \\ & = w_{j♀} \cdot \left[\gamma_{j♂♀}\epsilon_{j♀} \cdot \frac{\partial w_{j♀}}{dP} + (1 - \gamma_{j♀}\epsilon_{j♀}) \cdot \frac{\partial w_{j♂}}{dP} \right] - w_{j♂} \cdot \left[(1 - \gamma_{j♂}\epsilon_{j♂}) \frac{\partial w_{j♀}}{\partial P} + \gamma_{j♀♂}\epsilon_{j♂} \frac{\partial w_{j♂}}{\partial P} \right] \end{aligned}$$

The first-order effects are offsetting and sum to zero.

$$\begin{aligned} w_{j♀} \cdot \frac{\partial w_{j♂}}{\partial P} - w_{j♂} \cdot \frac{\partial w_{j♀}}{\partial P} &= \frac{MRPL_{j♀}}{1+E_♀} \cdot \frac{\partial [MRPL_{j♂}/(1+E_♂)]}{\partial P} - \frac{MRPL_{j♂}}{1+E_♂} \cdot \frac{\partial [MRPL_{j♀}/(1+E_♀)]}{\partial P} \\ &= P \cdot \frac{MPL_{j♀}}{1+E_♀} \cdot \frac{MPL_{j♂}}{1+E_♂} - P \cdot \frac{MPL_{j♂}}{1+E_♂} \cdot \frac{MPL_{j♀}}{1+E_♀} \\ &= 0 \end{aligned}$$

We rearrange the second-order terms to facilitate comparisons.

$$[w_{j♀}\gamma_{j♂♀}\epsilon_{j♀} + w_{j♂}\gamma_{j♂}\epsilon_{j♂}] \frac{\partial w_{j♀}}{\partial P} - [w_{j♀}\gamma_{j♀}\epsilon_{j♀} + w_{j♂}\gamma_{j♀♂}\epsilon_{j♂}] \frac{\partial w_{j♂}}{\partial P}$$

If the marginal products of labor are equal by gender and given Equation (16), the second order conditions can be reexpressed as

$$MPL_j \left[w_{j♀} \left(\frac{\beta_♀}{1+\beta_♀} \cdot \gamma_{j♂♀} - \frac{\beta_♂}{1+\beta_♂} \cdot \gamma_{j♀} \right) \epsilon_{j♀} + w_{j♂} \left(\frac{\beta_♀}{1+\beta_♀} \cdot \gamma_{j♂} - \frac{\beta_♂}{1+\beta_♂} \cdot \gamma_{j♀♂} \right) \epsilon_{j♂} \right]$$

If additional female and male labor affect MRPL equally, the second-order conditions can be reexpressed as

$$MPL_j \cdot \frac{\beta_♀\beta_♂}{(1+\beta_♀)(1+\beta_♂)} \cdot \frac{\partial MRPL_j}{\partial S_j} \left(\frac{1}{w_{j♂}} - \frac{1}{w_{j♀}} \right) [w_{j♀}S_{j♀}\epsilon_{j♀} + w_{j♂}S_{j♂}\epsilon_{j♂}] \quad (29)$$

With $\partial MPL_j/\partial S_j < 0$ and $w_{j♀} < w_{j♂}$, Equation (29) is unambiguously positive. An exogenous increase in the output price causes the gender wage gap to widen. This concludes the proof of Theorem 6.

We note that the final bracketed term of Equation 29 unambiguously increases in both male and female employment, implying that the gender wage gap will further deteriorate in larger firms as a result of an exogenous increase in output price.

The effect of a demand shock on employee gender balance is

$$\begin{aligned}
\frac{d}{dP}\ln\left(\frac{S_{j♀}(\cdot)}{S_{j♂}(\cdot)}\right) &= \frac{d\ln\left(S_{j♀}(\cdot)\right)}{dP} - \frac{d\ln\left(S_{j♂}(\cdot)\right)}{dP} \\
&= D^{-1}A_{31}\cdot\left\{[1-A_{24}A_{42}]\cdot\frac{\partial w_{j♀}}{\partial P} + A_{14}A_{42}\cdot\frac{\partial w_{j♂}}{\partial P}\right\} \\
&- D^{-1}A_{42}\cdot\left\{A_{23}A_{31}\cdot\frac{\partial w_{j♀}}{\partial P} + [1-A_{13}A_{31}]\cdot\frac{\partial w_{j♂}}{\partial P}\right\} \\
&= D^{-1}\left[A_{31}(1-A_{24}A_{42}) - A_{42}A_{23}A_{31}\right]\frac{\partial w_{j♀}}{\partial P} \\
&- D^{-1}\left[A_{42}(1-A_{13}A_{31}) - A_{31}A_{14}A_{42}\right]\frac{\partial w_{j♂}}{\partial P} \\
&= (PD)^{-1}\left[\underbrace{(\epsilon_{j♀}-\epsilon_{j♂})}_{(a)} - \underbrace{\epsilon_{j♀}\epsilon_{j♂}(\gamma_{j♂}+\gamma_{j♂♀}-\gamma_{j♀}-\gamma_{j♀♂})}_{(b)}\right] \qquad (30)
\end{aligned}$$

The sign of the bracketed expression labeled (a) is negative. Substituting in the definition of the γs, (b) is equivalent to

$$\epsilon_{j♀}\epsilon_{j♂}\frac{\partial MRPL_j}{\partial S_j}(S_{j♀}+S_{j♂})\left(\frac{\beta_♂}{w_{j♂}(1+\beta_♂)} - \frac{\beta_♀}{w_{j♀}(1+\beta_♀)}\right)$$

The sign of the bracketed expression is equal to the sign of

$$\frac{w_{j♀}\beta_♂}{1+\beta_♂} - \frac{w_{j♂}\beta_♀}{1+\beta_♀}$$

Substituting in Equation (16), the expression simplifies to

$$\frac{b}{(1+\beta_♀)(1+\beta_♂)}(\beta_♂-\beta_♀)$$

which is positive. Given the diminishing marginal product of labor, Expression (b) above is negative. We conclude that the first-order effects are negative, and that the second-order effects are positive. If first-order effects dominate, then an exogenous increase in output price causes the gender workforce balance to deteriorate. We note that the magnitude of the offsetting second-order effects is increasing in firm employment, implying that large firms will see less deterioration in gender workforce balance than small firms. This concludes the proof of Theorem 5.

The difference-in-differences coefficients reported in Tables 5 through 10 represent how higher productivity firms, as proxied by firm size, respond to the exogenous increase in output price relative to lower productivity firms. The sign predicted by our theoretical model is derived by differentiating elements of Equation (28) with respect to T. As a first step, the partial of the determinant from Equation (28) with respect to technology is zero.

$$\begin{aligned}
\frac{\partial D}{\partial T} &= -\frac{\partial \gamma_{j♀}}{\partial T}\epsilon_{j♀} - \gamma_{j♀}\frac{\partial \epsilon_{j♀}}{\partial T} - \frac{\partial \gamma_{j♂}}{\partial T}\epsilon_{j♂} - \gamma_{j♂}\frac{\partial \epsilon_{j♂}}{\partial T} \\
&\quad + \frac{\partial \epsilon_{j♀}\epsilon_{j♂}}{\partial T}(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀}) + \epsilon_{j♀}\epsilon_{j♂}\frac{\partial}{\partial T}(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀}) \\
&= \underbrace{\frac{\gamma_{j♀}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}\epsilon_{j♀} - \gamma_{j♀}\frac{\epsilon_{j♀}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}}_{0} + \underbrace{\frac{\gamma_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\epsilon_{j♂} - \gamma_{j♂}\frac{\epsilon_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}}_{0} \\
&\quad + \frac{\partial \epsilon_{j♀}\epsilon_{j♂}}{\partial T}(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀}) + \epsilon_{j♀}\epsilon_{j♂}\frac{\partial}{\partial T}(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀}) \\
&= \left(\frac{\epsilon_{j♀}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}\epsilon_{j♂} + \epsilon_{j♀}\frac{\epsilon_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\right)(\gamma_{j♀}\gamma_{j♂} - \gamma_{j♀♂}\gamma_{j♂♀}) \\
&\quad + \epsilon_{j♀}\epsilon_{j♂}\left(-\frac{\gamma_{j♀}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}\gamma_{j♂} - \gamma_{j♀}\frac{\gamma_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T} + \frac{\gamma_{j♀♂}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}\gamma_{j♂♀} + \gamma_{j♀♂}\frac{\gamma_{j♂♀}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\right) \\
&= 0
\end{aligned}$$

In light of this,

$$
\begin{aligned}
\frac{\partial}{\partial T}\left[\frac{dw_{j♀}}{dP}\right] &= \frac{\partial}{\partial T}[1-\gamma_{j♂}\epsilon_{j♂}]\frac{\partial w_{j♀}}{\partial P} + [1-\gamma_{j♂}\epsilon_{j♂}]\frac{\partial}{\partial T}\frac{\partial w_{j♀}}{\partial P} \\
&\quad + \frac{\partial \gamma_{j♀♂}\epsilon_{j♂}}{\partial T}\frac{\partial w_{j♂}}{\partial P} + \gamma_{j♀♂}\epsilon_{j♂}\frac{\partial}{\partial T}\frac{\partial w_{j♂}}{\partial P} \\
&= -\left[\frac{\partial \gamma_{j♂}}{\partial T}\cdot\epsilon_{j♂} + \gamma_{j♂}\cdot\frac{\partial \epsilon_{j♂}}{\partial T}\right]\frac{\partial w_{j♀}}{\partial P} + [1-\gamma_{j♂}\epsilon_{j♂}]\frac{\beta_{♀}}{1+\beta_{♀}}\frac{\partial MPL_j}{\partial T} \\
&\quad + \left[\frac{\partial \gamma_{j♀♂}}{\partial T}\cdot\epsilon_{j♂} + \gamma_{j♀♂}\cdot\frac{\partial \epsilon_{j♂}}{\partial T}\right]\frac{\partial w_{j♂}}{\partial P} + \gamma_{j♀♂}\epsilon_{j♂}\frac{\beta_{♂}}{1+\beta_{♂}}\frac{\partial MPL_j}{\partial T} \\
&= -\left[-\frac{\gamma_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\cdot\epsilon_{j♂} + \gamma_{j♂}\cdot\frac{\epsilon_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\right]\frac{\partial w_{j♀}}{\partial P} + [1-\gamma_{j♂}\epsilon_{j♂}]\frac{\beta_{♀}}{1+\beta_{♀}}\frac{\partial MPL_j}{\partial T} \\
&\quad + \left[-\frac{\gamma_{j♀♂}}{w_{j♀}}\frac{\partial w_{j♀}}{\partial T}\cdot\epsilon_{j♂} + \gamma_{j♀♂}\cdot\frac{\epsilon_{j♂}}{w_{j♂}}\frac{\partial w_{j♂}}{\partial T}\right]\frac{\partial w_{j♂}}{\partial P} + \gamma_{j♀♂}\epsilon_{j♂}\frac{\beta_{♂}}{1+\beta_{♂}}\frac{\partial MPL_j}{\partial T} \\
&= [1-\gamma_{j♂}\epsilon_{j♂}]\frac{\beta_{♀}}{1+\beta_{♀}}\frac{\partial MPL_j}{\partial T} \\
&\quad + \left[-\frac{\gamma_{j♀♂}}{w_{j♀}}\frac{\beta_{♀}}{1+\beta_{♀}}\frac{\partial MRPL_j}{\partial T}\cdot\epsilon_{j♂} + \gamma_{j♀♂}\cdot\frac{\epsilon_{j♂}}{w_{j♂}}\frac{\beta_{♂}}{1+\beta_{♂}}\frac{\partial MRPL_j}{\partial T}\right]\frac{\partial w_{j♂}}{\partial P} \\
&\quad + \gamma_{j♀♂}\epsilon_{j♂}\frac{\beta_{♂}}{1+\beta_{♂}}\frac{\partial MPL_j}{\partial T} \\
&= \Bigg[\underbrace{\frac{\beta_{♀}}{1+\beta_{♀}}}_{(+)} + \underbrace{\frac{\beta_{♀}\beta_{♂}}{(1+\beta_{♀})(1+\beta_{♂})}\frac{\partial MRPL_j}{\partial S_j}\epsilon_{j♂}S_{j♂}\left(\frac{1}{w_{j♀}}-\frac{1}{w_{j♂}}\right)}_{(-)} \\
&\quad + \underbrace{P\gamma_{j♀♂}\epsilon_{j♂}b(\beta_{♂}-\beta_{♀})\frac{\partial w_{j♂}}{\partial P}}_{(-)}\Bigg]\frac{\partial MPL_j}{\partial T} \qquad (31)
\end{aligned}
$$

Likewise,

$$\begin{aligned}
\frac{\partial}{\partial T}\left[\frac{dw_{j\male}}{dP}\right] &= \gamma_{j\male\female}\epsilon_{j\female}\frac{\beta_\female}{1+\beta_\female}\frac{\partial MPL_j}{\partial T} + [1-\gamma_{j\female}\epsilon_{j\female}]\frac{\beta_\male}{1+\beta_\male}\frac{\partial MPL_j}{\partial T} \\
&= \Bigg[\underbrace{\frac{\beta_\male}{1+\beta_\male}}_{(+)} + \underbrace{\frac{\beta_\female\beta_\male}{(1+\beta_\female)(1+\beta_\male)}\frac{\partial MRPL_j}{\partial S}\epsilon_{j\female}S_{j\female}\left(\frac{1}{w_{j\male}}-\frac{1}{w_{j\female}}\right)}_{(+)} \\
&\quad + \underbrace{P\gamma_{j\male\female}\epsilon_{j\female}b(\beta_\female-\beta_\male)\frac{\partial w_{j\female}}{\partial P}}_{(+)}\Bigg]\frac{\partial MPL_j}{\partial T} \qquad (32) \\
&> 0
\end{aligned}$$

The sign of Equation (32) is positive, yet the sign of Equation (31) is ambiguous. Since $\beta_\male > \beta_\female$, $\frac{\partial}{\partial T}\left[\frac{dw_{j\male}}{dP}\right] > \frac{\partial}{\partial T}\left[\frac{dw_{j\female}}{dP}\right]$, or in response to an exogenous output price increase, the change in male wages will exceed the change in female wages in larger firms. Because of this, the wage gap will grow more in larger than in smaller firms.

To compute the response of labor supply, note from Equation 28 that

$$\begin{bmatrix} \frac{d\ln(S_{j\female}(\cdot))}{dP} \\ \frac{d\ln\left(S_{j\male}(\cdot)\right)}{dP} \end{bmatrix} = D^{-1}\cdot\begin{bmatrix} \frac{\epsilon_{j\female}}{w_{j\female}}\frac{dw_{j\female}}{dP} \\ \frac{\epsilon_{j\male}}{w_{j\male}}\frac{dw_{j\male}}{dP} \end{bmatrix}$$

Then, given the signs and magnitudes of Equations 31 and 32, it is likely that the proportion of female employees will decrease more in larger firms.

The triple-difference coefficients in Panel C of Tables 8 through 10 represent the additional response of high-amenity (or female-led) firms over low-amenity (male-led) firms. The marginal effects predicted by our theoretical model are derived by differentiating Equations

(31) and (32) with respect to amenities.

$$\begin{aligned}
\frac{\partial^2}{\partial a \partial T}\left[\frac{dw_{j♀}}{dP}\right] &= \left[\frac{\beta_♀}{1+\beta_♀}\frac{\partial}{\partial a}(1-\gamma_{j♂}\epsilon_{j♂}) + \frac{\beta_♂}{1+\beta_♂}\frac{\partial \gamma_{j♀♂}\epsilon_{j♂}}{\partial a}\right]\frac{\partial MRPL_j}{\partial T} \\
&= \left[\frac{\beta_♀}{1+\beta_♀}\left(-\frac{\partial \gamma_{j♂}}{\partial a}\epsilon_{j♂} - \gamma_{j♂}\frac{\partial \epsilon_{j♂}}{\partial a}\right) + \frac{\beta_♂}{1+\beta_♂}\left(\frac{\partial \gamma_{j♀♂}}{\partial a}\epsilon_{j♂} + \gamma_{j♀♂}\frac{\partial \epsilon_{j♂}}{\partial a}\right)\right]\frac{\partial MRPL_j}{\partial T} \\
&= \left[\frac{\beta_♀}{1+\beta_♀}\left(\underbrace{-0\cdot\epsilon_{j♂} - \gamma_{j♂}\cdot 0}_{0}\right) + \frac{\beta_♂}{1+\beta_♂}\left(\underbrace{0\cdot\epsilon_{j♂} + \gamma_{j♀♂}\cdot 0}_{0}\right)\right]\frac{\partial MRPL_j}{\partial T} \qquad (33) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2}{\partial a \partial T}\left[\frac{dw_{j♂}}{dP}\right] &= \left[\frac{\beta_♀}{1+\beta_♀}\frac{\partial \gamma_{j♂♀}\epsilon_{j♀}}{\partial a} + \frac{\beta_♂}{1+\beta_♂}\frac{\partial}{\partial a}(1-\gamma_{j♀}\epsilon j♀)\right]\frac{\partial MRPL_j}{\partial T} \\
&= \left[\frac{\beta_♀}{1+\beta_♀}\left(\frac{\partial \gamma_{j♂♀}}{\partial a}\epsilon_{j♀} + \gamma_{j♂♀}\frac{\partial \epsilon_{j♀}}{\partial a}\right) - \frac{\beta_♂}{1+\beta_♂}\left(\frac{\partial \gamma_{j♀}}{\partial a}\epsilon_{j♀} + \gamma_{j♀}\frac{\partial \epsilon_{j♀}}{\partial a}\right)\right]\frac{\partial MRPL_j}{\partial T} \\
&= \left[\frac{\beta_♀}{1+\beta_♀}\left(\underbrace{\frac{\gamma_{j♂♀}}{S_f}v'(a)\cdot\epsilon_{j♀} - \gamma_{j♂♀}\cdot\frac{\epsilon_{j♀}}{S_f}v'(a)}_{0}\right)\right. \\
&\qquad \left. - \frac{\beta_♂}{1+\beta_♂}\left(\underbrace{\frac{\gamma_{j♀}}{S_f}v'(a)\cdot\epsilon_{j♀} - \gamma_{j♀}\cdot\frac{\epsilon_{j♀}}{S_f}v'(a)}_{0}\right)\right]\frac{\partial MRPL_j}{\partial T} \qquad (34) \\
&= 0
\end{aligned}$$

Then,

$$
\begin{aligned}
\frac{\partial^2}{\partial a \partial T}\frac{d\ln S_{j\female}}{dP} &= \frac{\partial^2}{\partial a \partial T}\left[\epsilon_{j\female} w_{j\female}^{-1}\frac{dw_{j\female}}{dP}\right] \\
&= \frac{\partial^2}{\partial a \partial T}\left[\epsilon_{j\female} w_{j\female}^{-1}\right]\frac{dw_{j\female}}{dP} + \epsilon_{j\female} w_{j\female}^{-1}\frac{\partial^2}{\partial a \partial T}\left[\frac{dw_{j\female}}{dP}\right] \\
&= \frac{w_{j\female}}{dP}\left[\frac{\partial^2 \epsilon_{j\female}}{\partial a \partial T} w_{j\female}^{-1} - \epsilon_{j\female} w_{j\female}^{-1}\frac{\partial^2 w_{j\female}}{\partial a \partial T}\right] \\
&= w_{j\female}^{-1}\frac{dw_{j\female}}{dP}\frac{\partial}{\partial T}\frac{\partial}{\partial a}\left[\frac{\beta_{\female} w_{j\female}}{w_{j\female} - b}\right] \\
&= \frac{\beta_{\female}}{w_{j\female}}\frac{\partial w_{j\female}}{\partial P}\frac{\partial}{\partial T}\left[\frac{\partial w_{j\female}}{\partial a}\cdot (w_{j\female} - b)^{-1} - w_{j\female}(w_{j\female})^{-2}\cdot \frac{\partial w_{j\female}}{\partial a}\right] \\
&= \frac{\beta_{\female}}{w_{j\female}}\frac{\partial w_{j\female}}{\partial P}\frac{\partial}{\partial T}\left[0\cdot (w_{j\female} - b)^{-1} - w_{j\female}(w_{j\female})^{-2}\cdot 0\right] \\
&= 0 \qquad (35)
\end{aligned}
$$

$$(36)$$

By similar calculations, $\frac{\partial^2}{\partial a \partial T}\frac{d\ln S_{j\male}}{dP} = 0$

Using these results, we linearize to solve for the equilibrium effect.

$$
\begin{bmatrix}
\frac{d^3 w_{j\female}}{dadTdP} \\
\frac{d^3 w_{j\male}}{dadTdP} \\
\frac{d^3 \ln(S_{j\female}(\cdot))}{dadTdP} \\
\frac{d^3 \ln\left(S_{j\male}(\cdot)\right)}{dadTdP}
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & A_{13} & A_{14} \\
0 & 0 & A_{23} & A_{24} \\
A_{31} & 0 & 0 & 0 \\
0 & A_{42} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{d^3 w_{j\female}}{dadTdP} \\
\frac{d^3 w_{j\male}}{dadTdP} \\
\frac{d^3 \ln(S_{j\female}(\cdot))}{dadTdP} \\
\frac{d^3 \ln\left(S_{j\male}(\cdot)\right)}{dadTdP}
\end{bmatrix}
+
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0
\end{bmatrix}
\qquad (37)
$$

The A_{ij} represent the partial derivative of variable i with respect to variable j. For example,

$$A_{13} = \frac{\partial w_{j\female}}{\partial \ln\left(S_{j\female}\right)}$$

Applying Cramer's rule, we calculate

$$\begin{bmatrix} \frac{d^3 w_{j\female}}{dadTdP} \\ \frac{d^3 w_{j\male}}{dadTdP} \\ \frac{d^3 \ln(S_{j\female}(\cdot))}{dadTdP} \\ \frac{d^3 \ln\left(S_{j\male}(\cdot)\right)}{dadTdP} \end{bmatrix} = D^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

The model predicts no difference in the gender wage gap or workforce balance between female- and male-led firms as they respond to an exogenous export shock, correcting for firm size. This concludes the proof of Theorems 7 and 8

Figure A.1: Outstanding nominal credit volume of the banking sector in Brazil

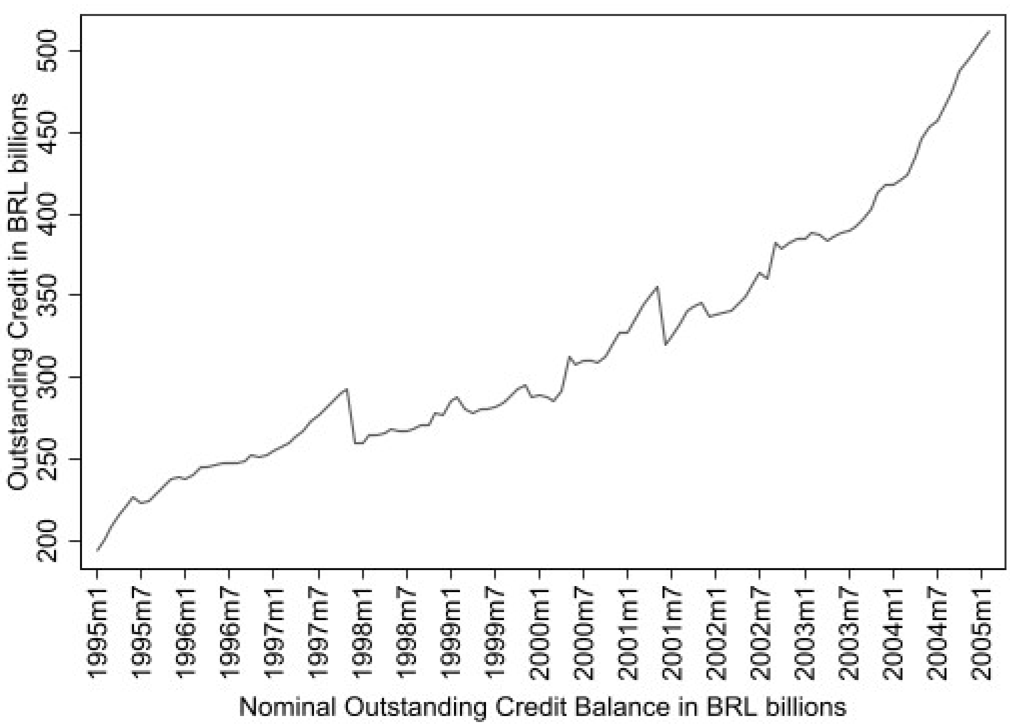


Table A.1: List of occupation codes for managerial and director (C-suite) positions.

	Classification		
Level	CBO 1994	CBO 1994	CBO 2002
Main subgroup	23		12
Occupations	2491	2422	1411
	4102	2424	1412
	4103	2432	1413
	4215	2415	1414
	5002	2413	1415
	5003	2433	1416
	5002	2414	1417
	5003	2436	1418
	2492	2437	1421
	2493	2435	1422
	3522	2434	1423
	3523	8410	1424
	2435	8340	1425
	2494	3444	1426
	2432	2423	1427
	2434	1912	
	2433	2429	
	2436	2439	
	2412	1912	
	2412	8929	

Table A.2: Cross-Sectional Estimates of Equation (1)

	(1)	(2)	(3)	(4)
Year	1995	1998	2000	2004
Age	0.098***	0.093***	0.087***	0.071***
	(0.000)	(0.000)	(0.000)	(0.000)
Age squared	-0.116***	-0.106***	-0.098***	-0.077***
	(0.000)	(0.000)	(0.000)	(0.000)
Tenure	0.005***	0.004***	0.004***	0.004***
	(0.000)	(0.000)	(0.000)	(0.000)
Years of schooling	0.079***	0.077***	0.076***	0.068***
	(0.000)	(0.000)	(0.000)	(0.000)
College	0.552***	0.597***	0.694***	0.809***
	(0.002)	(0.002)	(0.002)	(0.001)
Constant	-1.085***	-0.964***	-0.845***	-0.534***
	(0.008)	(0.007)	(0.007)	(0.005)
Observations	4,902,590	4,326,263	4,381,068	5,293,120
R^2	0.522	0.517	0.516	0.522

Notes: State fixed effects are included in each specification. The sample used contains only male workers. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table A.3: Triple-Difference Estimates: Percentage of Female Workers under More Restrictive Definition of Female-led Firms

		Δ% Female Employees				
			College Degree		Skilled	
	Δ ln(1+Exports)	All	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)	(6)
	coeff	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)	(se)
	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$	$[F^{-1}(\cdot)]$
Panel A: Female Leadership						
♀ Firm Size (pre)	-0.074	-0.001	0.001	-0.001	0.003	-0.001
	(0.117)	(0.002)	(0.004)	(0.002)	(0.002)	(0.002)
♀ Firm Size (post)	0.176	0.004**	0.000	0.004**	0.002	0.007***
	(0.131)	(0.002)	(0.004)	(0.002)	(0.003)	(0.002)
Δ♀ Firm Size	0.250	0.005*	-0.000	0.005*	-0.001	0.008***
	(0.158)	(0.003)	(0.005)	(0.003)	(0.004)	(0.003)
Observations	1,302	1,302	1,064	1,301	1,251	1,286
Panel B: Male Leadership						
♂ Firm Size (pre)	0.001	-0.001**	-0.000	-0.001***	0.002**	-0.001***
	(0.019)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)
♂ Firm Size (post)	0.127***	-0.000	-0.002**	-0.001	0.004***	0.000
	(0.024)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)
Δ♂ Firm Size	0.126***	0.000	-0.002	0.001	0.002	0.001
	(0.030)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Observations	20,584	20,584	8,995	20,553	15,877	20,177
Panel C: F - M Leadership						
Δ♀ Firm Size	0.124	0.004	0.002	0.004	-0.004	0.006
-Δ♂ Firm Size	(0.150)	(0.006)	(0.008)	(0.006)	(0.008)	(0.006)

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \le p < 0.05$, * $0.05 \le p < 0.1$.

Table A.4: Triple-Difference Estimates: Male-Female Wage Gap under More Restrictive Definition of Female-led Firms

	Δ Male-Female Wage Gap				
		College Degree		Skilled	
	Mean	Y	N	Y	N
	(1)	(2)	(3)	(4)	(5)
	coeff	coeff	coeff	coeff	coeff
	(se)	(se)	(se)	(se)	(se)
Panel A: Female Leadership					
♀ Firm Size (pre)	-0.013**	0.020*	-0.012**	-0.004	-0.010
	(0.005)	(0.012)	(0.005)	(0.007)	(0.007)
♀ Firm Size (post)	-0.008	-0.018	-0.005	0.000	-0.015*
	(0.006)	(0.011)	(0.006)	(0.006)	(0.009)
Δ♀ Firm Size	0.004	-0.037**	0.006	0.004	-0.005
	(0.010)	(0.016)	(0.011)	(0.010)	(0.018)
Observations	1,249	815	1,228	1,136	1,112
Panel B: Male Leadership					
♂ Firm Size (pre)	-0.003***	0.001	-0.003***	-0.005***	-0.001*
	(0.001)	(0.004)	(0.001)	(0.001)	(0.001)
♂ Firm Size (post)	0.002***	-0.002	0.001*	-0.001	0.009***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)
Δ♂ Firm Size	0.005**	-0.003	0.004*	0.003	0.011***
	(0.002)	(0.006)	(0.002)	(0.003)	(0.004)
Observations	67,887	8,070	66,762	29,091	53,941
Panel C: F - M Leadership					
Δ♀ Firm Size	-0.000	-0.034	0.002	0.001	-0.016
-Δ♂ Firm Size	(0.051)	(0.030)	(0.053)	(0.033)	(0.072)

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \leq p < 0.05$, * $0.05 \leq p < 0.1$.

Table A.5: Triple-Difference Estimates: Occupational Group under More Restrictive Definition of Female-led Firms

	Unskilled White Collar		Professional or Managerial		Skilled Blue Collar		Technical or Supervisory		Unskilled Blue Collar	
	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap	Δ% Female Employees	Δ M-F Wage Gap
	(1) coeff (se) $[F^{-1}(\cdot)]$	(2) coeff (se) $[F^{-1}(\cdot)]$	(3) coeff (se) $[F^{-1}(\cdot)]$	(4) coeff (se) $[F^{-1}(\cdot)]$	(5) coeff (se) $[F^{-1}(\cdot)]$	(6) coeff (se) $[F^{-1}(\cdot)]$	(7) coeff (se) $[F^{-1}(\cdot)]$	(8) coeff (se) $[F^{-1}(\cdot)]$	(9) coeff (se) $[F^{-1}(\cdot)]$	(10) coeff (se) $[F^{-1}(\cdot)]$
Panel A: Female Leadership										
♀ Firm Size (pre)	0.003	-0.012	0.001	0.019*	-0.002	-0.017*	0.007*	-0.016*	-0.007	-0.002
	(0.004)	(0.012)	(0.003)	(0.011)	(0.002)	(0.009)	(0.003)	(0.009)	(0.005)	(0.019)
♀ Firm Size (post)	-0.004	0.039***	-0.001	-0.031**	0.003	-0.022*	0.011***	-0.012	0.009	-0.032
	(0.005)	(0.014)	(0.004)	(0.013)	(0.003)	(0.013)	(0.004)	(0.009)	(0.012)	(0.029)
Δ♀ Firm Size	-0.007	0.051**	-0.002	-0.050***	0.005	-0.005	0.004	0.004	0.016***	-0.030
	(0.007)	(0.025)	(0.005)	(0.019)	(0.004)	(0.025)	(0.007)	(0.016)	(0.000)	(0.036)
Observations	1,106	849	1,202	1,017	1,239	759	1,191	1,003	517	188
Panel B: Male Leadership										
♂ Firm Size (pre)	-0.000	-0.007**	0.000	0.001	-0.000	-0.009***	0.002**	-0.009***	-0.003*	0.005
	(0.001)	(0.003)	(0.001)	(0.003)	(0.000)	(0.002)	(0.001)	(0.003)	(0.001)	(0.005)
♂ Firm Size (post)	0.003*	0.002	0.002**	0.009***	0.001**	0.008***	0.009***	-0.007***	-0.003	0.006
	(0.001)	(0.004)	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.003)	(0.003)	(0.007)
Δ♂ Firm Size	0.003	0.010	0.002	0.008	0.002	0.017**	0.007***	0.002	-0.000	0.001
	(0.003)	(0.007)	(0.002)	(0.006)	(0.002)	(0.007)	(0.002)	(0.005)	(0.006)	(0.013)
Observations	12,365	7,261	14,412	8,689	19,136	9,690	15,791	9,817	5,003	1,796
Panel C: F - M Leadership										
Δ♀ Firm Size -Δ♂ Firm Size	-0.010	0.041	-0.004	-0.059**	0.003	-0.022	-0.003	0.002	0.016***	-0.031***
	(0.014)	(0.036)	(0.011)	(0.029)	(0.008)	(0.047)	(0.014)	(0.023)	(0.000)	(0.000)

Notes: Observations are weighted by the square root number of firm employees. Two-digit industry and state fixed effects are included in each specification. Standard errors are clustered at the industry level and are reported in parentheses. *** $p < 0.01$, ** $0.01 \le p < 0.05$, * $0.05 \le p < 0.1$.