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# Contracts, Firm Dynamics and Aggregate Productivity

Bernabe Lopez-Martin<sup>†</sup>      David Perez-Reyna<sup>‡</sup>

## Abstract

We construct a framework of firm dynamics to evaluate the impact of the enforcement of contracts between final goods producers and their intermediate goods suppliers on firm life-cycle growth, technology accumulation and aggregate productivity. We build upon the tractable contracts model of Acemoglu et al. (2007), where the final goods firm chooses technology in contractible activities conducted by suppliers of intermediate inputs. Suppliers select investments in noncontractible activities, anticipating the payoffs of a bargaining game with the producer of the final goods. We show that contractual incompleteness implies a *wedge* on profits for producers of final goods, potentially dependent on the level of technology of the firm, which disincentives technology accumulation at the firm level in our dynamic model. We evaluate this mechanism in general equilibrium to analyze its quantitative implications. Our model accounts for differences in output per worker of up to 33 percent across economies with complete and incomplete contracts. The impact on firm life-cycle growth, the age and size distribution of firms is quantitatively significant.

**JEL Classification:** D86, E23, O11, O40.

**Keywords:** size dependent-distortions, contracts, aggregate productivity, firm dynamics.

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# Contratos, dinámica de empresas y productividad agregada

Bernabe Lopez-Martin      David Perez-Reyna

## Resumen

En este artículo proponemos un macro teórico de dinámica de empresas para evaluar el impacto del cumplimiento de contratos entre productores del bien de consumo y sus proveedores de bienes intermedios sobre el crecimiento de empresas, la acumulación de tecnología y la productividad agregada. Nos basamos en el modelo de Acemoglu et al. (2007), donde la empresa que produce el bien de consumo escoge tecnología en actividades para las cuales puede firmar un contrato y que son ofrecidas por proveedores de insumos intermedios. Los proveedores escogen cuánto invertir en actividades para las que no se puede firmar un contrato, anticipando los pagos en una negociación con el productor del bien de consumo. Demostramos que la incompletitud contractual implica una cuña en los beneficios de los productores del bien de consumo, potencialmente dependiente del nivel de tecnología de la empresa, que desincentiva la acumulación de tecnología al nivel de firmas en nuestro modelo dinámico. Analizamos este mecanismo en equilibrio general para analizar las implicaciones cuantitativas y encontramos que nuestro modelo puede explicar diferencias en producto per cápita de hasta 33% al comprar economías con y sin contratos completos. El impacto en el crecimiento de las empresas a lo largo de su ciclo de vida es cuantitativamente significativo.

**JEL Classification:**D86, E23, O11, O40.

**Palabras clave:** distorsiones que dependen del tamaño, contratos, productividad agregada, dinámica de empresas.

# 1 Introduction

Idiosyncratic distortions not only affect the allocation of inputs of production across firms but also the incentives to invest in technology and productivity within the firm. Both channels potentially have, at least in theory, a significant impact on aggregate productivity. Identifying the sources of these distortions is of paramount importance to assist the design of economic policies aiming at promoting economic development. In turn, the development of quantitative frameworks provides useful insights that contribute to understand the mechanisms and the potential impact of different distortions faced by firms on aggregate outcomes.<sup>1</sup>

We construct a dynamic framework of heterogeneous firms to evaluate the impact of contract enforcement on firm life-cycle growth and aggregate productivity. We build upon the model of Acemoglu et al. (2007), who provide a tractable structure where firms that produce final goods need to procure intermediate goods from suppliers. Producers of final goods (henceforth, firms) can choose the investment levels in contractible activities by the suppliers of intermediate goods (henceforth, suppliers). However, suppliers choose investment in noncontractible activities, a decision that anticipates the results of a bargaining game. This results in an allocation of resources that is not efficient. In a static setup, Acemoglu et al. (2007) show that contractual incompleteness has a negative impact on technology adoption. Furthermore, they show in a number of applications that this mechanism can generate sizable productivity differences across countries.

We deepen the analysis of this friction by analyzing its impact in a framework of firm dynamics (Hopenhayn, 1992; Hopenhayn and Rogerson, 1993). This approach allows us to make a contribution that we outline in the following manner. First, we show that the friction under study implies a *wedge* (or distortion equivalent to a tax) on profits, that is dependent on the technology level of the firm. Second, we describe how this wedge affects not only the size of the firm but also the dynamic incentives to invest in technology and productivity within the firm, which will determine the life-cycle growth profile of firms and aggregate productivity. Additionally, we demonstrate its impact on the age and size distribution of firms. Third, our analysis allows us to connect our quantitative results with the literature that studies alternative frictions in similar theoretical frameworks. For example, an extensive literature has studied the role of financial frictions, by examining alternative specifications, calibrations and margins through which they affect aggregate productivity.<sup>2</sup> A similar comparison can be made with the literature that studies firm entry costs. Based on our quantitative results, which we summarize next, we make the point that frictions that distort the ability of firms to make contracts with suppliers deserve similar attention. To the best of our knowledge, we are the first to explore the role of firm-supplier contract enforcement in a quantitative framework of firm dynamics.

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<sup>1</sup>We provide a brief overview of the literature below.

<sup>2</sup>This literature is extensive, some examples are: Amaral and Quintin (2010), Buera et al. (2011), D'Erasmus and Moscoso Boedo (2012), Midrigan and Xu (2014), Moll (2014), Lopez-Martin (2016), Hill and Perez-Reyna (2017).

For our quantitative analysis we consider the US economy as a benchmark and calibrate our model under the assumption that there is contract completeness. Some of the parameters are standard and obtained from the literature of firm dynamics, while others are calibrated to replicate key statistics of the US economy, such as firm exit rates, firm life-cycle growth, and the distribution of employment by age of the firm. We then document how the economy performs, in general equilibrium, as the range of contractible activities is reduced. This affects the investment in technology at the firm level, the age and size distribution of firms and aggregate productivity. Our model explains up to a 33% difference in output per worker across economies, which is comparable to losses generated by financial frictions in similar quantitative models. Furthermore we attain considerable differences in firm growth when comparing economies with and without contract incompleteness: average firm size for 26 to 30 year old firms is 2.6 times that of young firms when contracts are complete (this figure is replicated by calibration in the baseline reference), while firm growth is negligible when contracts are incomplete. Finally, the role of key parameters of the model is assessed.

## 2 Relation to the Literature

Our work is related to different strands of the literature on firm dynamics, misallocation and aggregate productivity. It is connected to the literature that evaluates the effects of idiosyncratic distortions, in models where productivity is endogenous (see Bhattacharya et al., 2013; Gabler and Poschke, 2013; Hsieh and Klenow, 2014; Ranasinghe, 2014; Álvarez Parra and Toledo, 2015; Buera and Fattal-Jaef, 2016; Bento and Restuccia, 2017; Da-Rocha et al., 2017). The analysis of these models has shown that assuming an exogenous distribution of firm productivity can lead to the underestimation of the consequences of distortions that affect the allocation of resources across production units. Distortions can affect incentives to improve productivity, which adds to the effect on the allocation of resources across firms, thus generating an amplification mechanism. This effect can be particularly detrimental when distortions are more severe for the most productive firms, often termed *correlated distortions*, as in Bento and Restuccia (2017).<sup>3</sup>

Related to the previous line of research, we contribute to the strand of the literature that aims to identify the sources of size dependent distortions and distortions faced by firms in general. For example, D’Erasmus and Moscoso Boedo (2012), Ulyssea (2018), López (2017), and Lopez-Martin (2016), among others, analyze tax evasion or the informal sector.<sup>4</sup> Mukoyama and Osotimehin (2017) evaluate the effects of worker firing taxes on overall innovation and productivity growth. Cole et al. (2016) develop a dynamic costly state verification model of venture capital. This financial friction affects the incentives to invest in different technologies that

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<sup>3</sup>Hopenhayn (2014) provides theoretical foundations for understanding the quantitative relevance of the correlation between distortions and productivity in a setup with an exogenous productivity distribution.

<sup>4</sup>In some of these studies, the informal sector refers to the *extensive margin*, while the intensive margin refers to firms that are registered but do not fully comply with regulation and tax obligations.

determine the life cycle growth of firms, the age and size distribution of firms, and aggregate productivity: the pattern of technology adoption is a function of monitoring efficiency and the extent of the cash-flow control problem. In line with this area of research, we analyze a particular source of distortions, potentially correlated with firm productivity or technology, which generates disincentives for investment in innovation and firm growth.

Mukoyama and Popov (2015) is perhaps most closely related to our work. They embed the contract incompleteness setup of Acemoglu et al. (2007) in a dynamic general equilibrium model with evolving institutions during the process of industrialization. They show that incompleteness of contracts leads to two types of misallocation that generate production inefficiency: unbalanced use of inputs and unbalanced production of different goods. In their model, the government is allowed to invest in enforcement institutions to improve the contractual environment, which allows them to analyze how different types of governments choose different patterns of institutional investment over time. Boehm and Oberfield (2018) use microdata on Indian manufacturing firms to show that production and sourcing decisions appear systematically distorted in states with weaker enforcement. We find these works, as well as their forceful motivation of the study of contract enforcement, complementary to ours.

### 3 Quantitative Framework

We consider an economy where a continuum of firms produce an homogeneous final good. We will refer to these production units as *firms*, as opposed to intermediate good suppliers. These firms purchase intermediate goods from suppliers, while suppliers need to invest in a range of activities to deliver the intermediate goods. Firms invest each period to improve their technology level, this level of technology refers to the measure of intermediate goods (a higher level of technology implies a larger range of intermediate goods). We first describe the static problem and the contracting problem faced by firms following Acemoglu et al. (2007), in our version of the model the technology level is given in any period. Then we describe the dynamic problem of firms, that decide how much to invest in improving their technology level for the next period. We assume that there is a representative household endowed with a unit of time that is inelastically supplied to firms as labor.

#### 3.1 Technology and Payoffs

Denote the technology level of a firm by  $n \in \mathbb{R}_+$ , which represents the range of intermediate goods the firm can use in production. In this sense, a higher  $n$  represents a more complex final good. For each  $j \in [0, n]$ ,  $\bar{x}(j)$  is the quantity of intermediate input  $j$ . The output function derived from the production technology follows Acemoglu et al. (2007), to which we add a term with decreasing returns to

scale in labor:<sup>5</sup>

$$y = z^{1-\beta} n^{\beta(\kappa+1-1/\alpha)} \left[ \int_0^n \bar{x}(j)^\alpha dj \right]^{\beta/\alpha} \cdot l^\nu \quad (1)$$

with  $\kappa > 0$  and  $0 < \alpha < 1$ . Parameter  $\alpha$  determines the degree of complementarity between inputs, so that the elasticity of substitution is  $1/(1 - \alpha)$ . Parameter  $\kappa$  controls the elasticity of output with respect to the level of the technology, while  $\nu$  governs the decreasing marginal productivity of labor.

There is a large number of profit-maximizing suppliers that produce the intermediate goods, who have an outside option  $\omega$ . The supplier of an intermediate input makes a relationship-specific investment, with constant marginal cost  $c_x$  for each activity necessary for production, which we consider to be in terms of the cost of labor.<sup>6</sup> The production function of intermediate inputs is Cobb-Douglas and symmetric in the activities is given by:

$$\bar{x}(j) = \exp \left[ \int_0^1 \ln x(i, j) di \right], \quad (2)$$

where  $x(i, j)$  is the level of investment in activity  $i$  performed by the supplier of input  $j$ . Payment to supplier  $j$  consists of two parts: an ex ante payment  $\tau(j) \in \mathbb{R}$  before the investments  $x(i, j)$  take place and payment  $s(j)$  after these investments are completed. The payoff to supplier  $j$ , taking into account her outside option:

$$\pi_x(j) = \max \left\{ \tau(j) + s(j) - \int_0^1 c_x x(i, j) di, \omega \right\}.$$

The profits of the firm are:

$$\pi = y - \int_0^n [\tau(j) + s(j)] dj - w l,$$

where  $w$  is the wage rate.

### 3.2 Equilibrium under Complete Contracts

We first consider a benchmark economy where contracts are complete (i.e. the first best). With complete contracts a firm pays each supplier the outside option: it makes a contract offer  $[\{x(i, j)\}_{i \in [0,1]}, \{s(j), \tau(j)\}]$  for every input  $j \in [0, n]$ .

We consider a subgame perfect equilibrium, that can be represented as a solu-

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<sup>5</sup>We will later consider a version of the model with physical capital.

<sup>6</sup>In general equilibrium the wage level will go down as contract institutions worsen, reducing the marginal cost of the activities of suppliers and, to some extent, moderating the negative effects of more adverse conditions (in this sense, the results are conservative).



tion to the following problem:

$$\max_{\{\{x(i,j)\}_{i,j},\{s(j),\tau(j)\}_{j,l}\}} y - \int_0^n [\tau(j) + s(j)] dj - w l$$

subject to (1), (2) and the participation constraint of suppliers:

$$s(j) + \tau(j) - c_x \int_0^1 x(i,j) di \geq \omega \quad \forall j \in [0, n].$$

This last condition is satisfied with equality in equilibrium, so there are no rents for suppliers. Since all activities are symmetric, the firm chooses the same investment level  $x$  for all activities in all intermediate inputs. With this condition the problem becomes:

$$\pi(z, n) \equiv \max_{\{x,l\}} z^{1-\beta} n^{\beta(\kappa+1)} x^\beta l^\nu - n(x c_x + \omega) - w l. \quad (3)$$

Notice that (3) is strictly concave in  $x$  and  $l$  as long as  $1 - \beta - \nu > 0$ .

Lemma 1 in Appendix A shows that the values for activities and labor under complete contracts are given by:

$$x = \frac{1}{n} \left[ \left( \frac{\nu}{w} \right)^\nu \left( \frac{\beta}{c_x} \right)^{1-\nu} z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}} \quad \text{and} \quad l = \left[ \left( \frac{\nu}{w} \right)^{1-\beta} \left( \frac{\beta}{c_x} \right)^\beta z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}} \quad (4)$$

and production is:

$$y = \left[ \left( \frac{\nu}{w} \right)^\nu \left( \frac{\beta}{c_x} \right)^\beta z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}}. \quad (5)$$

### 3.3 Equilibrium under Incomplete Contracts

We now consider an economy with incomplete contracts. Contract incompleteness is modeled as the fraction of activities that are not contractible. That is, for every intermediate input, we define  $\mu \in [0, 1]$  such that investments in activities  $0 \leq i \leq \mu$  are observable and contractible, while  $\mu < i \leq 1$  are not contractible. The contract stipulates investments for the contractible activities but not for the  $1 - \mu$  noncontractible activities: suppliers will decide investment in  $1 - \mu$  in anticipation of the ex-post distribution of revenue.

The timing is as follows:

- $z$  and  $n$  are fixed at the beginning of the period.
- The firm hires labor  $l$ , offers contract  $[\{x_c(i,j)\}_{i=0}^\mu, \tau(j)]$  for every intermediate input  $j \in [0, n]$ , where  $x_c(i, j)$  is investment level in a contractible activity,  $\tau(j)$  is an upfront payment to supplier  $j$  (can be positive or negative).

- Potential suppliers decide whether to apply for the contracts.
- Suppliers  $j \in [0, n]$  choose investment levels  $x(i, j)$  for all  $i \in [0, 1]$ , in contractible activities  $i \in [0, \mu]$ , investment is  $x(i, j) = x_c(i, j)$ .
- Suppliers and firm bargain over the division of revenue (suppliers can withhold their services in noncontractible activities).
- Output  $y$  is produced and distributed.

Following to Acemoglu et al. (2007), we consider a symmetric subgame perfect equilibrium (SSPE) and we denote hired labor, investment in contractible activities, investment in noncontractible activities, and upfront payment to suppliers by  $\{\hat{l}, \hat{x}_c, \hat{x}_n, \hat{\tau}\}$ . A SSPE is solved by backward induction, at the penultimate stage of the game given  $l$  and  $x_c$ .

We are interested in constructing a symmetric equilibrium, suppose  $x_n(-j)$  is investment in noncontractible activities for all suppliers other than  $j$ , while investment by supplier  $j$  is  $x_n(j)$ . Denote the Shapley value of supplier  $j$  by  $\bar{s}_x[l, x_c, x_n(-j), x_n(j)]$ , for which an explicit expression is derived below. In equilibrium, symmetry is satisfied  $x_n(j) = x_n(-j)$ , so  $x_n$  is a fixed point given by:

$$x_n = \arg \max_{x_n(j)} \bar{s}_x(l, x_c, x_n, x_n(j)) - (1 - \mu) c_x x_n(j). \quad (6)$$

Let  $s_x(l, x_c, x_n) \equiv \bar{s}_x(l, x_c, x_n, x_n)$ . In a symmetric equilibrium output of the firm is given by  $y = z^{1-\beta} (n^{\kappa+1} x_c^\mu x_n^{1-\mu})^\beta l^\nu$ . The Shapley value of the firm is obtained as a residual:

$$s(l, x_c, x_n) = z^{1-\beta} (n^{\kappa+1} x_c^\mu x_n^{1-\mu})^\beta l^\nu - n s_x(l, x_c, x_n)$$

The contract offered by the final-good firm has to satisfy the participation constraint for suppliers:

$$\bar{s}_x(l, x_c, x_n, x_n) + \tau \geq \mu c_x x_c + (1 - \mu) c_x x_n + \omega \quad (7)$$

The maximization problem of the (final good) firm is:

$$\max_{\{l, x_c, x_n, \tau\}} s(l, x_c, x_n) - n\tau - wl \quad \text{s.t. (6) and (7).}$$

We can obtain  $\tau$  from the participation constraint that will be satisfied with equality in equilibrium, then:

$$\max_{\{l, x_c, x_n\}} s(\cdot) + n[\bar{s}_x(\cdot) - \mu c_x x_c - (1 - \mu) c_x x_n] - \omega n - wl$$

s.t. condition (6), and the upfront payment needs to satisfy:

$$\hat{\tau} = \mu c_x \hat{x}_c + (1 - \mu) c_x \hat{x} + \omega - \bar{s}_x(\hat{l}, \hat{x}_c, \hat{x}_n, \hat{x}_n)$$

Acemoglu et al. (2007) show that  $s_x(l, x_c, x_n) = \frac{\beta}{\alpha+\beta} y/n$  and  $s(l, x_c, x_n) = \frac{\alpha}{\alpha+\beta} y$ .  $\frac{\alpha}{\alpha+\beta}$  is interpreted as the bargaining power of the firm, increasing in  $\alpha$  and

decreasing in  $\beta$ . The role of these parameters is discussed with more detail below.

### 3.3.1 Characterization of Equilibrium

Using the incentive compatibility constraint, the problem of the supplier is given by

$$x_n = \arg \max_{\{x_n(j)\}} \frac{\beta}{\alpha + \beta} z^{1-\beta} \left[ \frac{x_n(j)}{x_n} \right]^{(1-\mu)\alpha} x_c^{\beta\mu} x_n^{\beta(1-\mu)} n^{\beta(\kappa+1)-1} l^\nu - (1-\mu) c_x x_n(j).$$

In this problem there are two differences with respect to the first best. First, the supplier receives a fraction  $\frac{\beta}{\alpha+\beta}$ , so the supplier is not a full residual claimant of the return to its investment in noncontractible activities and thus underinvests relative to the optimal level. Second, multilateral bargaining distorts the concavity of the private return. The solution is obtained from the first-order condition of the problem and solving for the fixed point  $x_n(j) = x_n$ , this results in a unique  $x_n$ :

$$x_n = \bar{x}_n(x_c, l) = \left[ \frac{\alpha\beta}{\alpha + \beta} \times (c_x)^{-1} x_c^{\beta\mu} z^{1-\beta} n^{\beta(\kappa+1)-1} l^\nu \right]^{1/[1-\beta(1-\mu)]}. \quad (8)$$

Taking this as given the problem of the firm is:

$$\begin{aligned} \pi_i(z, n; \mu) \equiv \max_{\{x_c, l\}} & z^{1-\beta} [x_c^\mu \bar{x}_n(x_c, l)^{1-\mu}]^\beta n^{\beta(\kappa+1)} l^\nu \\ & - c_x n \mu x_c - c_x n (1-\mu) \bar{x}_n(l, x_c) - \omega n - w l \end{aligned} \quad (9)$$

In Appendix A we prove that

$$l_i = h_1(\mu) \cdot l, \quad x_c = h_1(\mu) \cdot x, \quad x_n = h_2(\mu) \cdot x_c$$

and

$$y_i \equiv z^{1-\beta} n^{\beta(1+\kappa)} x_c^{\beta\mu} x_n^{\beta(1-\mu)} l_i^\nu = h_3(\mu) y, \quad (10)$$

where

$$\begin{aligned} h_1(\mu) &\equiv \left[ \frac{1}{\alpha + \beta} \left( \frac{\alpha + \beta - \alpha\beta(1-\mu)}{1 - \beta(1-\mu)} \right)^{1-\beta(1-\mu)} \alpha^{\beta(1-\mu)} \right]^{\frac{1}{1-\nu-\beta}} \\ h_2(\mu) &\equiv \alpha \frac{1 - \beta(1-\mu)}{\alpha + \beta - \alpha\beta(1-\mu)} \quad h_3(\mu) \equiv h_1(\mu)^{\beta+\nu(1-\mu)} \cdot h_2(\mu)^{(1-\mu)\beta}. \end{aligned}$$

Notice that  $h_1(1) = 1$  and  $h_1'(\mu) > 0$ , so  $x_c \leq x$  and  $h_2(1) = \frac{\alpha}{\alpha+\beta}$  and  $h_2'(\mu) > 0$ , so  $x_n < x_c$ . This implies,  $y_i < y$ . Furthermore, since  $n c_x x = \beta y$  and  $w l = \nu y$ , we can express profits under complete and incomplete contracts as, respectively:

$$\pi = (1 - \beta - \nu) y - \omega n \quad \text{and} \quad \pi_i = (1 - \beta - \nu) h_1 y - \omega n. \quad (11)$$

We discuss below how incomplete contracts generate a distortion that depends

on the technology level of the firm, in the spirit of Bento and Restuccia (2017).

### 3.4 Dynamic Problem of the Firm

We now describe the dynamic problem of firms. Technology  $n$ , a state variable, is accumulated over time with investment in a stochastic innovation technology. The dynamic problem of the firm can be written in recursive form as follows:

$$v(z, n) = \max_{\{e\}} \pi(z, n) - e - c_f \tag{12}$$

$$+ \gamma(1 - \phi) \sum_{\{n', z'\}} \Lambda(z' | z) \cdot P(n' | n, e) \cdot \max\{v(z', n'), \underline{v}\}$$

where  $\pi(z, n)$  is the level of profits, whether with complete or incomplete markets, that depends on the level of technology  $n$  and the stochastic productivity shock  $z$ ,  $e$  are expenditures in the innovation technology,  $\gamma$  is the discount parameter and  $\phi$  is an exogenous exit shock. The per-period fixed cost of production  $c_f$  generates exit of firms while the exit value when a firm decides to close down is  $\underline{v}$ . Firm productivity evolves according to a discrete Markov process  $\Lambda(z' | z)$ .

In every period firms can invest in the innovation good  $e$  to increase the stock of technology.<sup>7</sup> Three outcomes are possible every period, depending on the amount of investment in the innovation good in the previous period: technology may increase by a proportion  $\psi$ , it may remain constant, or decrease by  $\psi$ .

Technology is defined on the grid  $\{\underline{n}, \underline{n}(1 + \psi), \underline{n}(1 + \psi)^2, \dots, \bar{n}\}$ , where  $\underline{n}$  and  $\bar{n}$  are the lowest and highest possible levels of technology, respectively. The probability of a successful outcome is given by:

$$P(n' = n(1 + \psi) | n, e) = \frac{(1 - \xi) \cdot (e/n)}{1 + (e/n)}.$$

There are diminishing returns to innovation investment  $e$ . Fixing a probability of success in innovation,  $P(n(1 + \psi) | n, e)$ , the necessary investment in innovation goods  $e$  to increase the productivity of the firm by a fixed percentage is proportional to technology  $n$ . Parameter  $\xi$  determines the expected return to investment in innovation. The probability of a negative outcome is given by:

$$P(n' = n/(1 + \psi) | n, e) = \frac{\xi}{1 + (e/n)}.$$

The level of technology level  $n$  summarizes the history of investment and success in innovations and governs the size of the firm (Klette and Kortum 2004). Furthermore, it is lost when the firm closes, regardless of whether exit is due to an exogenous exit shock or it is optimal to close the firm. Finally, technology is assumed to be firm-specific and there is no market for its trade.

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<sup>7</sup>The stochastic innovation process builds on Pakes and McGuire (1994) and Farias et al. (2012). For related stochastic specifications see Klette and Kortum (2004) and Atkeson and Burstein (2010).

### 3.5 Entry of New Firms

A new firm enters with an initial level of technology  $\underline{n}$ . The value of a potential entering firm, net of the entry cost, is given by:

$$v_e = \int v(z, \underline{n}) dF(z) - c_e$$

where  $F(z)$  is the unconditional distribution of idiosyncratic firm productivity  $z$ . In equilibrium a break-even condition needs to be satisfied  $v_e = 0$ .

### 3.6 Representative Household

We close the model by assuming that there is an infinitely lived representative household with preferences over consumption sequences given by:

$$\sum_{t=0}^{\infty} \gamma^t u(c_t)$$

with  $c_t$  denoting consumption in period  $t$ ,  $\gamma \in (0, 1)$  is the discount factor,  $u(c)$  is assumed to satisfy standard conditions.

The household has a unit endowment of labor that is inelastically supplied in the market. Resources for the household are  $c = d + w - \overline{en} + \overline{ex}$ , where  $\overline{en}$  denotes aggregate creations costs,  $\overline{ex}$  is the aggregate exit value of firms,  $d$  denotes aggregate dividends from the firms and suppliers. We focus on the stationary equilibrium of this economy, where prices and aggregate variables are constant.

## 4 Parameters and Calibration

We start our analysis with the baseline model. As is standard in the literature, we set parameter values that jointly contribute to replicate key statistics of the U.S. economy. The critical institutional parameter  $\mu$  represents the share of activities, of each intermediate input, for which investment is observable and contractible. For the undistorted economy we assume  $\mu$  is equal to 1, which implies perfect markets. In the quantitative financial development literature, for example, assuming perfect markets is standard for the U.S.

### 4.1 Predetermined Parameters

We first enumerate the set of predetermined parameters in Table 1, assigning standard values in the literature. In the model, the length of a time period represents one year. The discount factor  $\gamma$  of 0.99, jointly with an exogenous death rate of firms of 0.04 (which is a calibrated parameter discussed below), determine an

*effective discount value* of 0.95 for the firms, which is within the range of commonly used values.

description/role of parameter	symbol	value
discount factor	$\gamma \cdot (1 - \phi)$	0.95
exponent on technology and intermediate inputs	$\beta$	0.45
prod. function exponent on labor	$\nu$	0.40
elasticity of substitution intermediate inputs	$\alpha$	0.50
elasticity of output w.r.t. technology	$\kappa$	0.30
exog. productivity process: autocorrelation	$\rho$	0.60
exog. productivity process: volatility	$\sigma_\varepsilon$	0.25

The returns to scale in the production function are jointly determined by  $\nu$  and  $\beta$ . In Acemoglu et al. (2007), the authors consider a monopolistic competition framework, where  $\beta$  determines the elasticity of demand. Their benchmark value for this parameter is 0.75, in a model without labor or physical capital. This number is consistent with the generally accepted range of the elasticity of substitution between final-good varieties.

In our setup, we need to take into account several issues. First, the returns to scale are determined by  $\nu$  and  $\beta$ , so that their sum should be in line with *span-of-control* values in the literature or its equivalent curvature in monopolistic competition models (e.g. Restuccia and Rogerson, 2008). Second, the weight given to intermediate inputs is larger than the weight on labor and capital (e.g., Gopinath and Neiman, 2014). Third, as we will show below, we require  $\nu + \beta(\kappa + 1) < 1$  in order to have a wedge that is increasing in the level of technology  $n$ , which is the relevant case. With these considerations in mind, we set  $\nu$  and  $\beta$  equal to 0.40 and 0.45, respectively. Nevertheless, we discuss below how our main results change with different values.

The value of  $\alpha$  determines the degree of complementarity between intermediate inputs. This parameter is not relevant for calibration, since it does not enter the problem of the firms under complete contracts. However, it does affect the impact of worse judicial institutions given its role in the bargaining process: as  $\alpha$  increases, intermediate inputs become more substitutable, and the magnitude of the effects diminishes. Given that there is no obvious way of interpreting this parameter from the data, we follow Acemoglu et al. (2007) in fixing its central value at 0.50, and provide a discussion of how quantitative results change within a range of values. Parameter  $\kappa$  controls the elasticity of output with respect to the level of the technology. We set a baseline value of 0.30, in the range considered by Acemoglu et al. (2007), and below discuss how it affects the main quantitative results.

The exogenous productivity component of the production function  $z$  follows an AR(1) process, with an autocorrelation parameter of 0.60 and a volatility parameter of 0.25, which are in middle of the ranges in the literature, respectively, for their values (for a discussion see Lopez-Martin, 2016). These parameters are not quantitatively relevant for our quantitative results since we look at the size and productivity-growth of firms in the long run. In our setup they will, jointly with other parameters, contribute to determine moments such as exit rates by age and size, and the size and age-distribution of firms.<sup>8</sup>

## 4.2 Calibration

We now turn to our calibration approach, which we show in Table 2. The per-period fixed cost of production  $c_f$ , jointly with the exogenous probability of firm exit, denoted by  $\phi$ , determine firm-exit rates in our model. In a stationary equilibrium, total exit and entry rates of firms are equal, we target a level of 0.10, consistent with the literature (e.g., Gabler and Poschke, 2013). Large and productive firms are less likely to exit endogenously in this type of models, and thus their exit rates are mainly generated by exogenous shocks. The range for this moment is approximately 0.04-0.05 (Hsieh and Klenow, 2014; D’Erasmus and Moscoso Boedo, 2012 and Ranasinghe, 2014); our value of 0.04 is at the lower bound of this range, in line with D’Erasmus and Moscoso Boedo (2012).

description/role of parameter	symbol	value
per-period fixed cost of production	$c_f$	3.761
exogenous firm death rate	$\phi$	0.040
innovation technology: size innovation steps	$\psi$	0.500
innovation technology: <i>success</i> probability	$\xi$	0.673
value of suppliers	$\omega$	0.020

The three remaining calibrated parameters mainly govern the growth dynamics of firms. The proportional size of each technology step is given by  $\psi$ , while the probability of an increase in technology, for a given level of investment, is determined by  $\xi$ . We target the growth pattern of firms using the data from Hsieh and Klenow (2014) for the U.S., at two points of their life-cycle: the size of survivors of age 6-10 relative to age 1-5, and the size of survivors of age 31-35 relative to age 1-5 (see Table 3). Firms grow faster when they are young, which requires a larger  $\psi$ ; their growth moderates afterwards.<sup>9</sup> Parameter  $\omega$ , which represents the outside option

<sup>8</sup>These parameters are relevant in the literature of financial constraints, since they govern the dispersion of the marginal products of capital. In our model there is no dispersion in the marginal product of labor across firms (or capital, in the alternative version of the model).

<sup>9</sup>The elasticity of labor with respect to the level of technology is given by  $\beta \cdot \kappa / (1 - \nu - \beta)$ . See equation (4).

for suppliers, affects the growth dynamics of larger and more productive firms as it implies a cost that is increasing in the level of technology (see equation 3). The last target we consider is the share of employment in firms of age 41 or older (Hsieh and Klenow, 2014).

<b>Table 3.</b> Baseline Model: Calibration Moments.		
target statistics	data	model
total exit (equal to entry) rate of firms	0.100	0.100
exit rate firms 500+ workers	0.042	0.040
relative size firms 6-10/1-5 years (survivors)	1.597	1.580
relative size firms 31-35/1-5 years (survivors)	2.890	2.964
share of employment at firms with 41+ years	0.280	0.304
non-target statistics	data	model
share of employment at firms with 0-10 years	0.247	0.227
share of employment at firms with 11-20 years	0.207	0.188
share of employment at firms with 21-30 years	0.146	0.148
share of employment at firms with 31-40 years	0.121	0.132
employment at firms w/500+ workers	0.496	0.467
investment in technology/final goods production	0.064	0.076

We next discuss the fit of the model along some non-target dimensions (see Table 3). Although we do not target the entire distribution of employment by age of the firm, the model replicates this properly. Furthermore, in the U.S. the upper tail of the size distribution accounts for a significant part of employment: in our model firms with more than 500 workers account for 0.467 of total employment, compared to 0.496 in the data. In the baseline calibration the ratio of investment in technology to the production of final goods is 0.076. This figure is comparable to the estimate of the ratio of investment in business intangible capital to domestic business value added of 0.064 by McGrattan and Prescott (2010) (see their Table A3).

We end this section with some clarifications related to the solution and numerical implementation of the model. The algorithm for solving this type of models consists in normalizing the wage rate, then  $c_e$  is computed as the value that, in equilibrium, satisfies the break-even condition with equality (Hopenhayn and Rogerson, 1993; D'Erasmus and Moscoso Boedo, 2012). The lower bound on the endogenous level of technology  $\underline{n}$  is normalized. The upper bound  $\bar{n}$  is set equal to a sufficiently large number so that it is not binding: we consider 35 levels of technology, while in our simulations the maximum step reached by firms is 18. The exogenous productivity component of the production function follows an AR(1) process, which is discretized following Tauchen (1986) to construct the Markov matrix  $\Lambda(z' | z)$ .



## 5 Model Mechanics

In this section we briefly discuss how contract incompleteness implies a distortion, similar to a tax or *wedge*, that affects incentives to invest in technology and, therefore, average firm productivity growth and aggregate productivity of the economy. We analyze the mechanism by distinguishing between two effects: one static and one dynamic. First, we can show that, *ceteris paribus* (and partial equilibrium), a lower  $\mu$  curtails firm size. Second, as previously mentioned, the distortion reduces incentives for the firm to invest in improving technology, this is the dynamic effect.

Notice from (10) that  $y_i$  is increasing in  $\mu$ . The result is rather straightforward if we focus on inputs of production: when  $\mu < 1$  there is a wedge,  $1 - h_3(\mu)$ , that is decreasing in  $\mu$ . A higher  $\mu$  results in more input demand, and, therefore, increased production.

Since  $n$  is a dynamic decision, to analyze the second effect we focus on the wedge on profits, which generate the incentives for the firm to invest in improving their technology level (see (12)). In our model this wedge is increasing in  $n$ . To see why, recall from (11) that  $\pi = A y - \omega n$  and  $\pi_i = h_1(\mu) A y - \omega n$ , where  $A = 1 - \beta - \nu$ ,  $h_1'(\mu) > 0$  and  $h_1(1) = 1$ . Consider

$$\frac{\pi_i}{\pi} = \frac{h_1(\mu) \cdot A \cdot y - \omega \cdot n}{A \cdot y - \omega \cdot n} = \frac{h_1(\mu) \cdot A \cdot (y/n) - \omega}{A \cdot (y/n) - \omega}.$$

As long as  $\beta(\kappa + 1) + \nu < 1$ , which is true in our benchmark parametrization, then  $g(n) \equiv (y/n)$  is strictly decreasing in  $n$ . Then

$$\frac{\partial (\pi_i/\pi)}{\partial n} = \frac{(1 - h_1(\mu)) \cdot A \cdot g'(n) \cdot \omega}{(A \cdot (y/n) - \omega)^2} < 0.$$

The wedge on profits is equal to  $1 - \frac{\pi_i}{\pi}$ , so the inequality above implies that this wedge is increasing in  $n$ . That is, bigger firms are affected more by the friction than smaller firms. As  $\mu$  increases, it is less costly to have a higher  $n$ . In our model  $\omega$ , which is the outside option available to suppliers, plays a crucial role. If  $\omega = 0$ , the wedge for firms would be equal to  $h_1(\mu)$ , which does not depend on  $n$ . We would like to stress, however, that the wedge need not be increasing in  $n$  to affect investment in technology, a constant distortion is sufficient to generate a dynamic disincentive.

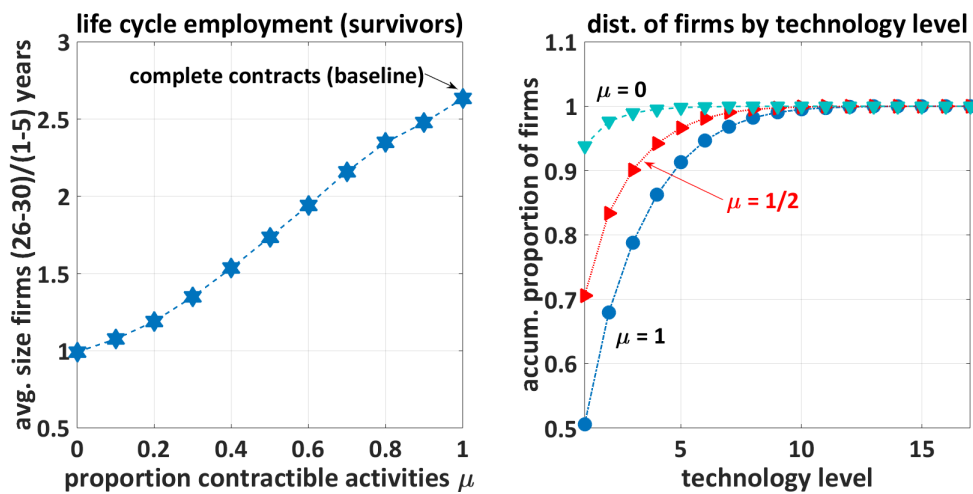
## 6 Quantitative Analysis

In this section we discuss the main quantitative results. First, we document how contract completeness affects technology accumulation and growth at the firm level, with consequences for the age and size distribution of firms in general equilibrium, as well as aggregate productivity. Second, we analyze the role of different key parameters.

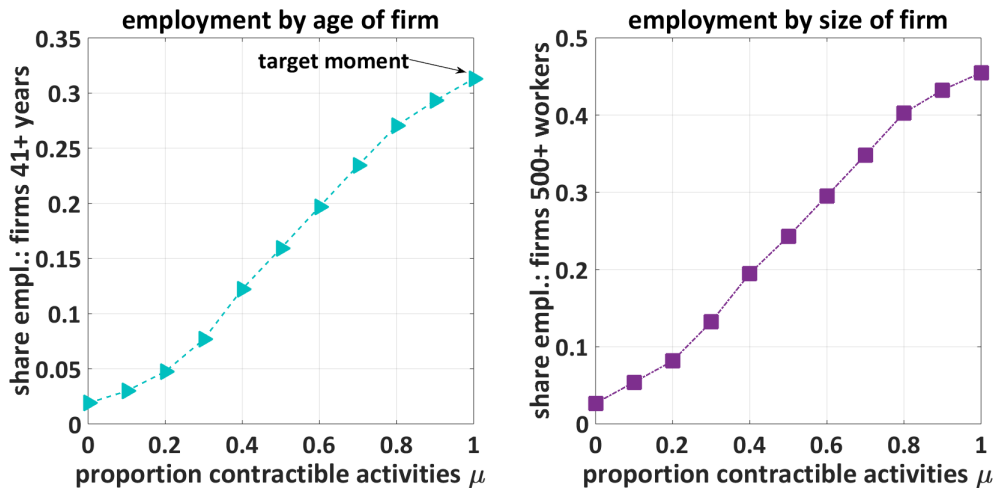
## 6.1 Baseline Quantitative Results

The main exercise consists of reducing  $\mu$ , the parameter that represents completeness of contracts, starting from the baseline calibration.<sup>10</sup> As discussed previously, as contracts are relatively *more incomplete* (i.e., we reduce  $\mu$  and compute the new equilibrium), a distortion worsens which reduces incentives to invest in technology. In the extreme case of contract incompleteness, firm growth is negligible even after 26 years (see Figure 1, left panel). This directly affects the distribution of technology in the economy (Figure 1, right panel).

**Figure 1. Impact of Contract Institutions:  
Firm Growth and Accumulation of Technology**



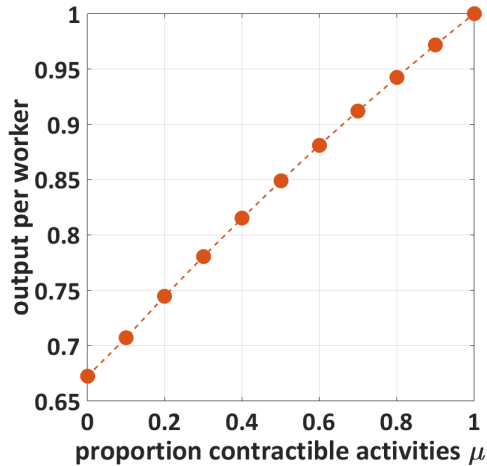
**Figure 2. Impact of Contract Institutions:  
Employment by Age and Size of Firm**



<sup>10</sup>For the quantitative analysis, and the results and graphs we report, we consider  $\mu \in (0.01, 0.99)$ .

The impact on the relevance of older and bigger firms, and the distribution of employment by age and size of firm in general, is considerable: as  $\mu$  decreases the share of employment in these firms decreases (Figure 2).

**Figure 3. Impact of Contract Institutions:  
Output per Worker**



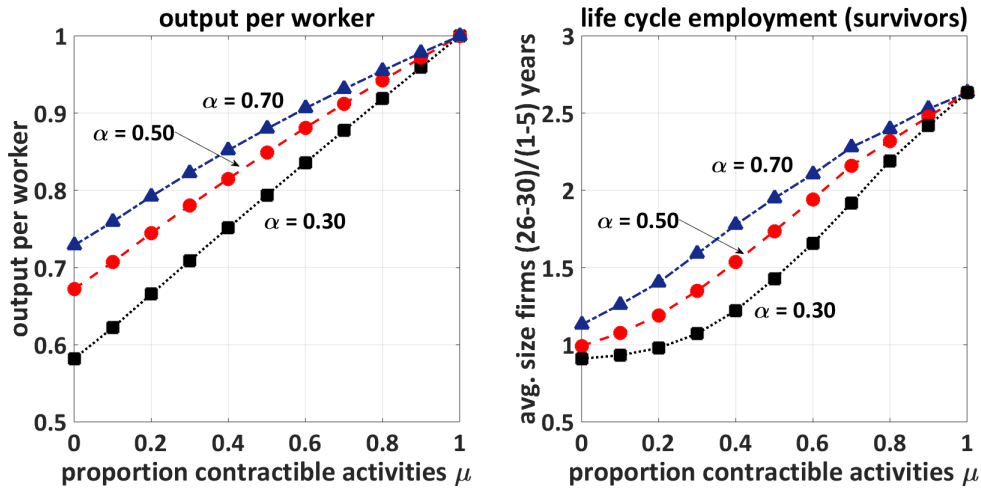
The economic consequences of contract incompleteness are economically significant: in the extreme case of contract incompleteness output per worker falls by more than 30 percent relative to the baseline scenario (Figure 3). These losses are comparable to those found in the literature of financial frictions.

## 6.2 Sensitivity Analysis

Next, we discuss the role of different key parameters for our quantitative results. Parameter  $\alpha$  determines the degree of complementarity between inputs. As discussed by Acemoglu et al. (2007), a higher  $\alpha$  implies a higher elasticity of substitution between different intermediate inputs, thus every individual supplier becomes less essential in production, increasing the bargaining power of the firm producing a final good. As discussed in a previous version, the distortion faced by the firm is decreasing in  $\alpha$ . In our model, this effect influences the incentives to invest in more advanced technologies and therefore life-cycle growth of firms (Figure 4).

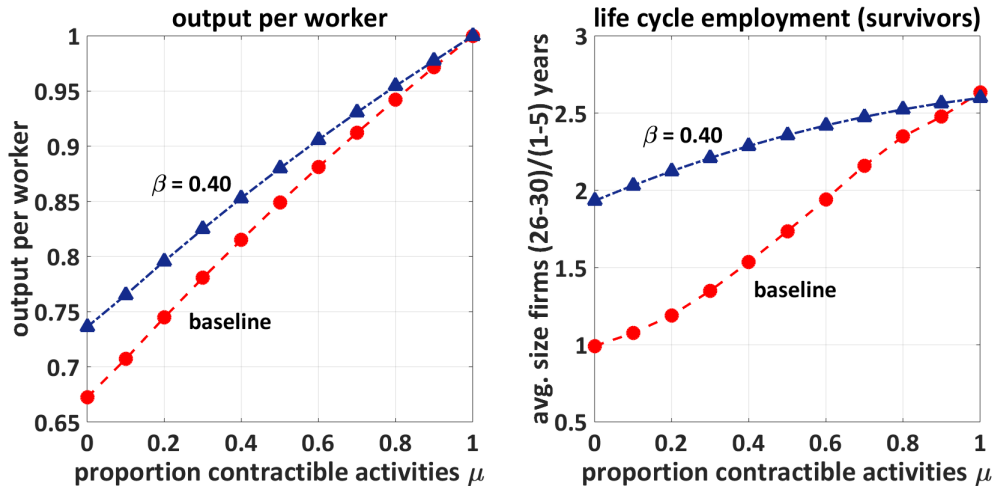
Parameter  $\alpha$  does not affect the allocation of resources with complete contracts, thus we do not have to recalibrate other parameters to analyze its role. This is not the case for parameter  $\beta$ . This will make the comparison of the different parameterizations less transparent as we need to modify other parameters to replicate

**Figure 4. Impact of Contract Institutions:  
The Role of Parameter  $\alpha$**



the target moments discussed for the calibration. We keep the number of modified parameters to a minimum as described next.

**Figure 5. Impact of Contract Institutions:  
The Role of Parameter  $\beta$**



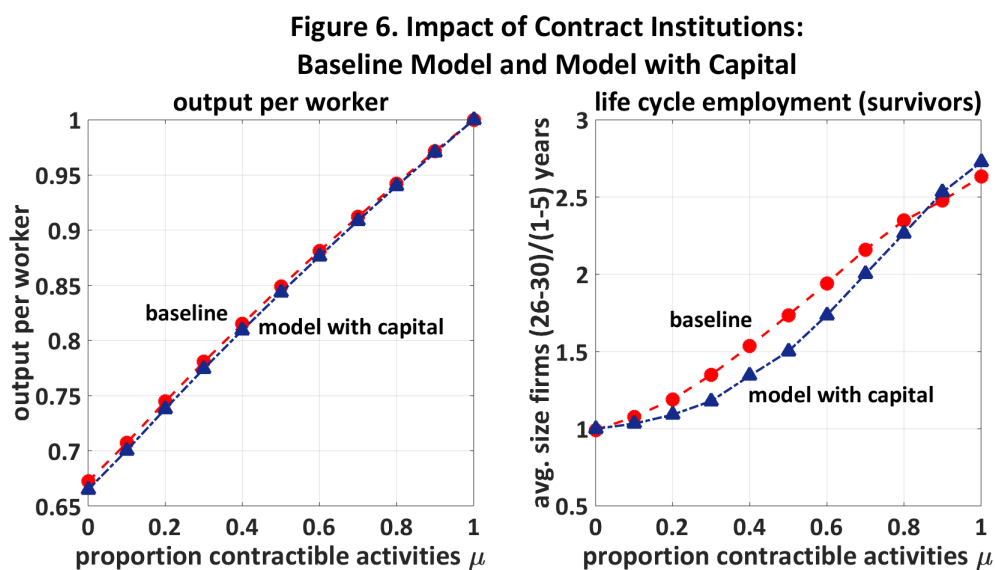
In our model, parameter  $\beta$  determines the weight of the production function given to technology and intermediate inputs.<sup>11</sup> Relative to the baseline calibration we reduce  $\beta$  to 0.40, and decrease the per-period fixed cost of production  $c_f$  and innovation parameter  $\xi$ , to keep exit rates and firm life-cycle growth on target. In

<sup>11</sup>Acemoglu et al. (2007) consider a monopolistic competition model where  $\beta$  governs the elasticity of demand. A span-of-control model (our approach) is isomorphic to the monopolistic competition model.

particular, note that a significant reduction in  $\xi$  is required, to 0.12 from the baseline value of 0.673. With a lower  $\beta$ , less weight is given to technology  $n$  and intermediate inputs, thus the negative effect of contract incompleteness is reduced relative to the baseline calibration (Figure 5). It has also been shown that the bargaining power of the firm is decreasing in  $\beta$ .

### 6.3 Model with Production Capital

We modify the model by introducing capital in the production function, considering  $f(k,l)^\nu$  with  $f(k,l) = k^\theta l^{1-\theta}$ , using a standard parameter of  $\theta$  equal to  $1/3$ .<sup>12</sup> The quantitative results are largely unchanged (Figure 6).



## 7 Final Comments

We have constructed a dynamic framework of heterogeneous firms to evaluate the impact of the enforcement of contracts between final goods producers and suppliers on technology accumulation, firm life-cycle growth and aggregate productivity. We have shown this friction implies a *wedge* on profits that is dependent on the technology level of the firm, and that this wedge affects not only the size of the firm but also the dynamic incentives to invest in technology and productivity within the firm. This determines the life-cycle growth profile of firms and aggregate productivity, as well as the age and size distribution of firms. Exploiting a framework similar to those used in the literature that studies financial frictions and firm entry costs, among alternative frictions, we find an economically significant impact of contract

<sup>12</sup>The main parameters are unchanged.

enforcement.

In principle, firms could potentially mitigate the distortion caused by contractual incompleteness through vertical integration (for applications in trade see Antràs and Chor, 2013; Antràs and Helpman, 2006). This possibility, however, forces the firm to confront a myriad of other problems, particularly in developing economies, that will limit its growth. First, contractual imperfections and monitoring technologies are important in explaining the lack of managerial delegation in developing economies (Laeven and Woodruff, 2007; Caselli and Gennaioli, 2013; Cole et al., 2016; Akcigit et al., 2016). Second, vertical integration is costly and inefficient (Boehm and Oberfield, 2018), and implies the firm is forced to invest to develop a product for which it has not accumulated know-how and human capital. Additionally, this production may be at a suboptimal scale if the production of the intermediate good is only for its own use. Third, as already discussed above, financial frictions will restrict firm growth and additional size-dependent distortions will become more severe as the firm becomes larger.

To the best of our knowledge, we are the first to explore the role of firm-supplier contract enforcement in a quantitative framework of firm dynamics, and we believe there is ample room for further research. For example, different multilateral repeated bargaining procedures could be considered.<sup>13</sup> Based on our quantitative results, we have made the point that frictions that distort the ability of firms to make contracts with suppliers deserve more attention.

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<sup>13</sup>Note that repeated bargaining does not eliminate inefficiencies. We will not attempt to present a representative set of references on multilateral bargaining. To provide just one example, Cai (2003) studies a complete-information alternating-offer bargaining game where some of the Markov Perfect Equilibria exhibit wasteful delays. Furthermore, the maximum number of delay periods that can be supported in this type of equilibria increases in the order of the square of the number of players. Cai (2003) provides additional references and a brief enumeration of potential different sources of inefficiencies in these models. Wolinsky (2000) analyzes a model of contracting and recontracting between a firm and its workers, where the unique stationary equilibrium is inefficient. In many multilateral bargaining setups the share of the surplus obtained by the player making a proposal is decreasing in the number of players (Serrano, 2005), this can generate a *wedge* with similar disincentives to improve technology at the firm level (in our model technology is associated with the number of suppliers, or intermediate inputs of production).

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## Appendix A Mathematical Appendix

Lemma 1 derives the benchmark values for activities, labor and production.

**Lemma 1.** *The equilibrium values for activities, labor and production are given by*

$$x = \frac{1}{n} \left[ \left( \frac{\nu}{w} \right)^\nu \left( \frac{\beta}{c_x} \right)^{1-\nu} z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}},$$

$$l = \left[ \left( \frac{\nu}{w} \right)^{1-\beta} \left( \frac{\beta}{c_x} \right)^\beta z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}} \quad \text{and} \quad y = \left[ \left( \frac{\nu}{w} \right)^\nu \left( \frac{\beta}{c_x} \right)^\beta z^{1-\beta} n^{\beta\kappa} \right]^{\frac{1}{1-\nu-\beta}}.$$

*Proof.* The first order condition of (3) with respect to  $x$  is:

$$\beta z^{1-\beta} n^{\beta(\kappa+1)-1} x^{\beta-1} l^\nu = c_x \quad (\text{A1})$$

while the the first order condition with respect to  $l$  is:

$$\nu z^{1-\beta} n^{\beta(\kappa+1)} x^\beta l^{\nu-1} = w \quad (\text{A2})$$

Take the ratio of (A1) and (A2):

$$l = \frac{c_x \nu}{w \beta} n x; \quad (\text{A3})$$

replace in (A2):

$$\nu z^{1-\beta} n^{\beta(\kappa+1)} x^\beta \left[ \frac{c_x \nu}{w \beta} n x \right]^{\nu-1} = w$$

then:

$$x^{1-\nu-\beta} = \frac{\nu z^{1-\beta} n^{\beta(\kappa+1)}}{w} \left[ \frac{c_x \nu}{w \beta} n \right]^{\nu-1}. \quad (\text{A4})$$

(A3) and (A4) yield the result.  $\square$

Lemma 2 shows sufficient conditions to guarantee that the objective function in (9) is strictly concave.

**Lemma 2.**  *$1 > \beta + \nu$  is a sufficient condition for the objective function in (9) to be strictly concave.*

*Proof.* If we plug in (8) into (9), we can write the objective function as

$$B x_c^{\frac{\beta\mu}{1-\beta(1-\mu)}} l^{\frac{\nu}{1-\beta(1-\mu)}} - c_x n \mu x_c - \omega n - w l, \quad (\text{A5})$$

where

$$B \equiv \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{1 - \beta(1 - \mu)} \left[ \frac{1}{\alpha + \beta} \left( \frac{\alpha\beta}{c_x} \right)^{\beta(1 - \mu)} z^{1 - \beta} n^{\beta(\kappa + \mu)} \right]^{\frac{1}{1 - \beta(1 - \mu)}}.$$

(A5) is a Cobb-Douglas function having  $x_c$  and  $l$  as inputs, so it is strictly concave in  $x_c$  and  $l$  as long as

$$\frac{\beta\mu}{1 - \beta(1 - \mu)} + \frac{\nu}{1 - \beta(1 - \mu)} < 1,$$

which is equivalent to  $1 > \beta + \nu$ .  $\square$

Proposition 1 shows that  $\mu$  governs the wedge between input demand, labor and profits under complete contracts and under incomplete contracts. This wedge is decreasing in  $\mu$  and disappears when  $\mu = 1$ . One consequence of this proposition is that input demand, labor and profits are increasing in  $\mu$ .

**Proposition 1.** *Let*

$$h_1(\mu) \equiv \left[ \frac{1}{\alpha + \beta} \left( \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{1 - \beta(1 - \mu)} \right)^{1 - \beta(1 - \mu)} \alpha^{\beta(1 - \mu)} \right]^{\frac{1}{1 - \nu - \beta}}$$

$$h_2(\mu) \equiv \alpha \frac{1 - \beta(1 - \mu)}{\alpha + \beta - \alpha\beta(1 - \mu)}$$

and denote by  $x_c(n, z; \mu)$  the demand for contractible inputs,  $x_n(n, z; \mu)$  the demand for noncontractible inputs,  $l_i(n, z; \mu)$  the demand for labor and  $y_i(n, z; \mu)$  production under incomplete contracts. Similarly, let  $x(n, z)$  be the demand for inputs,  $l_c(n, z)$  the demand for labor and  $y(n, z)$  production under complete contracts. Then

$$x_c(z, n, k; \mu) = h_1(\mu) x(z, n, k) \quad x_n(z, n, k; \mu) = h_2(\mu) x_c(z, n, k; \mu)$$

$$l_i(z, n, k; \mu) = h_1(\mu) l(z, n, k) \quad \text{and} \quad y_i(n, z; \mu) = h_1(\mu)^{\beta + \nu} h_2(\mu)^{(1 - \mu)\beta} y(n, z).$$

Furthermore,  $h_1'(\mu) > 0$ ,  $h_1(1) = 1$  and  $h_2'(\mu) > 0$ ,  $h_2(1) = \frac{\alpha}{\alpha + \beta}$ .

*Proof.* First we will prove the properties of  $h_i(\mu)$ . Noting that  $h_1(1) = 1$  and  $h_2(1) = \frac{\alpha}{\alpha + \beta}$  is straightforward. To prove that  $h_1'(\mu) > 0$  consider

$$f_1(\mu) \equiv (1 - \beta(1 - \mu)) [\ln(\alpha + \beta - \alpha\beta(1 - \mu)) - \ln(1 - \beta(1 - \mu))] + \beta(1 - \mu) \ln \alpha.$$

$f_1'(\mu) > 0$  is equivalent to proving that  $h_1'(\mu) > 0$ . Notice that

$$f_1'(\mu) = \beta \left[ \ln \left( \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{\alpha(1 - \beta(1 - \mu))} \right) - \frac{\beta}{\alpha + \beta - \alpha\beta(1 - \mu)} \right].$$

Let

$$a \equiv \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{\alpha(1 - \beta(1 - \mu))} = 1 + \frac{\beta}{\alpha(1 - \beta(1 - \mu))}.$$

Since  $\beta \in (0, 1)$ , then  $a > 1$ . Additionally,

$$\frac{\beta}{\alpha + \beta - \alpha\beta(1 - \mu)} = 1 - \frac{1}{a},$$

so proving that  $f'_1(\mu) > 0$  is equivalent to proving that  $g(a) = \ln a - 1 + \frac{1}{a} > 0$  for  $a > 1$ . Notice that  $g(1) = 0$  and  $g'(a) = (a - 1)/a^2 > 0$  for  $a > 1$ .

To prove that  $h'_2(\mu) > 0$  consider  $f_2(\mu) \equiv \ln(1 - \beta(1 - \mu)) - \ln(\alpha + \beta - \alpha\beta(1 - \mu))$ .  $f'_2(\mu) > 0$  is equivalent to proving that  $h'_2(\mu) > 0$ . Notice that

$$f'_2(\mu) = \frac{\beta}{1 - \beta(1 - \mu)} - \frac{\alpha\beta}{\alpha + \beta - \alpha\beta(1 - \mu)}.$$

$f'_2(\mu) > 0$  if and only if  $\beta > 0$ , which holds by assumption.

To complete the proof we plug in (8) into (9).<sup>14</sup> Taking first order conditions with respect to  $x_c$  and  $l$  yields:

$$\beta\mu \frac{\Psi}{x_c} = c_x n \mu \tag{A6}$$

$$\nu \frac{\Psi}{l} = w, \tag{A7}$$

where

$$\Psi \equiv \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{1 - \beta(1 - \mu)} \left[ \frac{1}{\alpha + \beta} \left( \frac{\alpha\beta}{c_x} \right)^{\beta(1 - \mu)} z^{1 - \beta} x_c^{\beta\mu} n^{\beta(\kappa + \mu)} l^\nu \right]^{\frac{1}{1 - \beta(1 - \mu)}}.$$

If we divide (A6) over (A7) we get:

$$l = \frac{c_x \nu}{w \beta} n x_c. \tag{A8}$$

Plugging (A8) into (A6) and solving for  $x_c$  yields:

$$x_c^{1 - \nu - \beta} = \frac{1}{\alpha + \beta} \left[ \frac{\alpha + \beta - \alpha\beta(1 - \mu)}{1 - \beta(1 - \mu)} \right]^{1 - \beta(1 - \mu)} \alpha^{\beta(1 - \mu)} \tag{A9}$$

$$\times \frac{\nu z^{1 - \beta} n^{\beta(\kappa + 1)}}{w} \left[ \frac{c_x \nu}{w \beta} n \right]^{\nu - 1}$$

We can then use (A8) and (A9) to get an expression for  $l$ .

The result follows from plugging (A3) and (A4) into (A8) and (A9), and then plugging into (8). □

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<sup>14</sup> $1 > \beta + \nu$  is a sufficient condition to guarantee that the objective function in (9) is strictly concave. This result is stated as a lemma and proven in Lemma 2.