Public Development Banks and Credit Market Imperfections[*]

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Abstract

This paper is devoted to understanding the role of public development banks in alleviating financial market imperfections. We explore two issues: 1) which types of firms should be optimally targeted by public financial support; and 2) what type of mechanism should be implemented in order to efficiently support the targeted firms' access to credit. We model firms that face moral hazard and banks that have a costly screening technology, which results in a limited access to credit for some firms. We show that a public development bank may alleviate the inefficiencies by lending to commercial banks at subsidized rates, targeting the firms that generate high added value. This may be implemented through subsidized ear-marked lending to the banks or through credit guarantees which we show to be equivalent in "normal times". Still, when banks are facing a liquidity shortage, lending is preferred, while when banks are undercapitalized, a credit guarantees program is best suited. This will imply that 1) there is no "one size fits all" intervention program and 2) that any intervention program should be fine-tuned to accommodate the characteristics of competition, collateral, liquidity and banks capitalization of each industry.

Keywords: Public development banks; governmental loans and guarantees; costly screening; credit rationing

JEL codes: H81, G20, G21, G23

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Bancos Públicos de Desarrollo e Imperfecciones en el Mercado Crediticio

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Resumen

Este artículo estudia el papel que los bancos públicos de desarrollo juegan para aliviar restricciones en el acceso a crédito de las empresas. Exploramos dos dimensiones primordiales: 1) hacia cuáles empresas debería enfocarse el apoyo financiero público; y 2) qué tipo de mecanismo es más eficiente para apoyar el acceso a crédito de estas empresas. Modelamos firmas que enfrentan riesgo moral y bancos que tienen acceso a una tecnología costosa de evaluación de sus clientes potenciales, lo que resulta en menor acceso a crédito del que sería óptimo para algunas empresas. Mostramos que un banco público de desarrollo puede aliviar esta ineficiencia prestando a bancos comerciales a tasas subsidiadas para líneas focalizadas hacia empresas y proyectos que generan alto valor agregado. El subsidio se puede implementar a través de créditos a los bancos o de programas de garantías públicas de crédito. Mostramos que ambos mecanismos son equivalentes en tiempos normales. La política de prestar a los bancos comerciales es superior cuando estos bancos enfrentan restricciones de liquidez, mientras que los programas de garantías públicas son óptimos so los bancos tienen dificultades para responder con requerimientos legales de capital. El análisis implica que cualquier intervención debe ser ajustada al grado de competencia entre bancos, la disponibilidad de colateral, las condiciones de liquidez del sistema bancario, y el grado de capitalización de los bancos.

Palabras clave: Bancos públicos de Desarrollo; préstamos y garantías del gobierno; racionamiento de crédito; evaluación costosa de clientes.
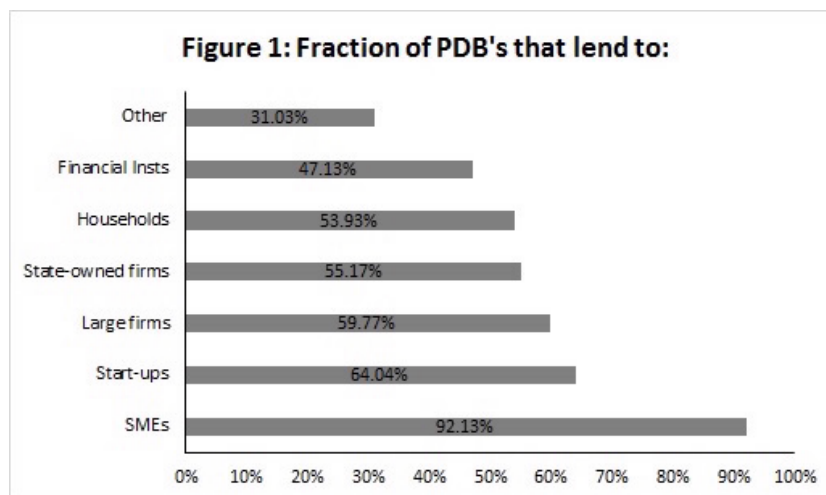
JEL codes: H81, G20, G21, G23

Figure 1: Fraction of PDB's that lend to:

Other	31.03%
Financial Insts	47.13%
Households	53.93%
State-owned firms	55.17%
Large firms	59.77%
Start-ups	64.04%
SMEs	92.13%

Figure 1:

1 Introduction

The financing of firms by specialized public institutions is a pervasive feature of financial markets, whether in less developed economies, emerging or developed ones. Regional and global associations of development banks have over 280 members around the world, some of them large players in the credit markets of their respective countries.[1] The activities of these institutions are varied both in scope and focus. As illustrated in Figure 1, some of them offer financing to a broad base of clients, while many others target particular types of firms, such as Small and Medium Enterprises (SMEs), startups, nascent or weak sectors. They also differ in the way they intervene: while some lend directly to businesses, others offer loans that are intermediated by private financial institutions (Figure 2). Many–73%, according to the Global Survey of Development Banks–offer public guarantees instead of, or in addition to, providing credit.

It is natural to think of the development of such institutions as a response to a financial market imperfection that generates credit rationing. Still, it is not clear which financial frictions the provision of support by Public Development

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[1]Respondents of the World Bank´s Global Survey of Development Banks report participations in assets of between 9% and 19% in the respective market (Luna-Martínez and Vicente, 2012). Lazzarini et al (2014) report that the Brazilian Public Development Bank, BNDES, represents over 20% of loans in the Brazilian credit market, and amount to almost 10% of GDP.

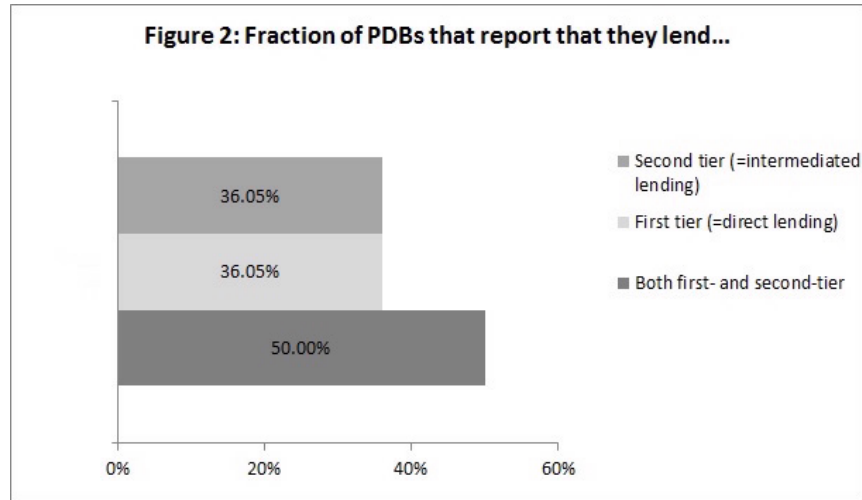**Figure 2: Fraction of PDBs that report that they lend...**

Figure 2:

Banks (PDB) is intended to remedy and which instrument, among those used by these institutions, is best suited for dealing with those frictions. In brief, the public funding of specific enterprises is a practice in search of a theory.

This paper provides a theoretical framework to answer the following questions: 1) should the public finance of firms take the form of direct lending, risk sharing, or direct subsidies to either firms or banks? 2) what types of firms, if any, should be the target of a particular public support programs?; 3) to what extent public development banks should lend to banks at subsidized rates, and under what conditions a program of public guarantees (i.e. loss sharing) might be preferred.

We consider an asymmetric information framework leading to financial markets imperfections that prevent some firms from accessing the credit market. In particular, we elaborate on standard moral hazard and costly information extraction, two major building blocks that have been developed to justify the existence of banks and their role in financial markets. Their application to heterogeneous firms will allow us to study how public support to credit rationed firms can improve efficiency.

The model will consider firms that require funding in order to implement their projects. Firms can be good or bad, and only good firms have positive net present value projects. They belong to "industries", which are characterized by a risk profile, so that "industries" may correspond to sectors or types of firms (young, SMEs,...). Certain "industries" may be characterized by higher risk, less transparency, or higher moral hazard incentives.

The type of a firm is not directly observable to either banks or the govern-

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ment. Still, commercial banks have access to a costly screening technology that yields a signal that may or may not be informative (Ruckes 2004). Certain positive net present value, but high risk, firms are credit rationed because the cost of perfect screening is exceedingly high. In addition to the assumption of costly screening we consider firms' moral hazard, as in Holmstrom and Tirole's (1997) classical paper. Moral hazard implies further rationing of firms with insufficient pledgeable cash flows. There is a tension between the two types of financial market imperfections we introduce. For any given firm, the bank and the firm will share the project net present value. A low return for the bank will imply low incentives to screen the project. Reciprocally, a low return for the firm will imply the firm has a higher incentive to implement the high private benefits and negative net present value project.

These core inefficiencies (namely imperfect screening and moral hazard) imply in equilibrium some good firms will be credit rationed, so that there is room for public intervention. A key result in our analysis is that, in this framework, the equilibrium level of banks' screening activity is inefficiently low. The reason for the underprovision of screening is that banks do not take into account the externality they create when facilitating firms' access to credit and the rents they generate. Because of this, credit is underprovided, and the underprovision is more severe for types of firms where the rents the bank cannot appropriate are larger. The focus of our research is in evaluating the potential effectiveness of different instruments to deal with the credit market failures previously mentioned and to identify the types of firms that should be targeted. These are characterized by sufficiently high expected firms' profits, where banks' profits are low which reduce banks' incentives to screen and with low per granted loan screening costs.

We evaluate welfare, measured by net expected output, under alternative mechanism of public financial support, considering the effects of each mechanism on firms and banks' behavior (i.e. moral hazard and screening, respectively), as well as the implied costs of subsidies. We derive the optimal conditions for subsidies to firms and banks, as well as for direct government lending, and compare the relative merits of the different arrangements above. Whether one or the other institution is optimal depends, in turn, on the returns and risk profiles of the projects that can benefit from the public policy. This has implications for the optimal targeting of government programs, including whether SME's or other usual suspects are the best possible target.

We extend our basic framework so as to analyze credit rationing that is driven by liquidity and/or solvency restrictions for banks, and show that such restriction have completely different policy implications, a relevant issue for less developed financial markets we also address.

The empirical literature has shown that financing constraints affect more starkly particular types of firms. For instance, SMEs report higher financing obstacles than large firms, and the effect of these financing constraints is stronger for them compared to more established firms (See Beck et al. (2008), Beck et al. (2005); Beck et al.,(2006) and Beck and Demirguc-Kunt, 2006 for an overview). Nevertheless, there is also heated debate about whether the high

growth potential of SME's and the more intense obstacles to growth they seem to face are indeed a feature that derives from firms being small or rather from other characteristics of these firms that are correlated with size. Haltiwanger et al. (2013), for instance, show that the high growth believed to characterize small firms no longer correlates with firm size once firms' age is controlled for. These findings have been used to argue that targeting government support to young rather than small firms may be a better policy strategy. Our framework will contribute to this discussion, and related ones, by identifying features of firms that make them a better target of policies aimed at alleviating credit rationing. As a concrete example, we investigate the optimal targeting of subsidies and government loans across industries characteristics.

The theoretical literature on banking provides a number of models where firms with a limited credit history (Diamond, 1991), lack of collateral (Holmstrom and Tirole, 1997, Ruckes, 2004) or, simply, risky (Bolton and Freixas, 2000) will not have access to funding in spite of the fact that the project they want to finance has a positive net present value. It also provides models in which credit rationing may arise as a consequence of the need to screen and/or moral hazard (Holmstrom and Tirole, 1997), or because of liquidity constraints in the financial market (Armendáriz, 1999). Our results will contribute to understand the optimal policy responses to these different financial market imperfections and the type of firms that should be targeted.

Direct lending by a Public Development Bank (PDB), public guarantees or subsidies to lending have been previously studied from a theoretical perspective within different setups. Hainz and Hakenes (2012) investigate the relative merits of public guarantees vs. direct lending, in a context where the market failure to be addressed is the underprovision of funding to projects with positive externalities but with negative net present value. By contrast, our approach disregards positive externalities and focus on interventions addressed to solving credit rationing that arises from asymmetric information both on the part of banks and firms. This allows us to address the most outstanding issues relating imperfections in the provision of credit to businesses, and the natural intervention of the government in the market of loans to firms. Arping et al (2010) do consider credit rationing due to entrepreneurs facing moral hazard incentives but, because they assume perfect information on firms' types, screening and the provision of credit by banks cannot be addressed in their setting.

In the next section we will describe our model and the financial market imperfection it implies, then turn to direct lending as a benchmark case. Section 3 will be devoted to a second best policy of subsidization to firms and banks. Section 4 will consider the impact of banks competition and its implications. Section 5 extends the analysis to explore the role of collateral, liquidity shortages and banks' capital shortages. Section 6 considers business cycles and Section 7 is devoted to the robustness of the qualitative results our framework delivers. Section 8 concludes.

2 The model

Consider an economy where all agents are risk neutral. Interest rates are normalized to zero. Different industries are characterized by risk parameters p, where p captures the potential probability of success of projects in the industry. Within industries, there are two types of firms, good and bad, in proportions μ and $1 - \mu$. Good firms are at the industry's potential, facing probability of success p, while bad firms have a lower probability of success p_-. If successful, a project undertaken by a good firm yields an outcome of y per unit of investment, with constant returns to scale up to its full size I, so that a successful project of size I yields yI, while a null return is obtained if the project is unsuccessful. Moreover, firms are able to engage in moral hazard by choosing a project that yields private benefits B at the expense of a lower probability of success, $p - \Delta p$ or $p_- - \Delta p$. Projects by good firms that do not engage in moral hazard behavior yield a positive expected return ($py > 1$), while projects with a probability p_- or $p - \Delta p$ (and a fortiori $p_- - \Delta p$) yield a negative net present value ($yp_- < 1$ and $(p - \Delta p)y < 1$).

Neither the type (good or bad) of a firm nor its choice to engage in moral hazard behavior are observable to the bank or the government. The value of p, by contrast, is observed by the bank and by the public entity. The bank's role in the economy is to screen firms and, thus to weed out bad firms. We assume the banks are perfectly solvent, so that the behavior of PDB during a crisis is disregarded. Solvency problems and the countercyclical role of PDBs are addressed in extensions to the model.

2.1 Inefficiency in the market for credit

In order to fund their projects, firms approach banks that have a screening technology. For every industry/risk p, by paying a sunk cost $C(q)$, banks obtain a perfect signal on the firm's type good or bad (p or p_-) with probability q while, with probability $1 - q$, they obtain no signal. We assume $C(q)$ satisfies $C'(q) > 0, C''(q) > 0, C(0) = 0$ and $C'(0) = 0$. If the bank receives a signal it will lend to good firms and deny credit to bad ones. If the bank does not receive a signal, we will assume it does not grant a loan, which occurs when μ is low (namely when $[\mu p + (1 - \mu)p_-]\, y < 1$). We assume that screening costs are independent of the firm's project size. Alternative assumptions can be formulated, so that the results we obtain regarding size are to be considered in connection with this critical assumption. We justify our assumption because we associate an increase in size to higher complexity in the structure (balance sheet, multiple business lines,...), but this is compensated by a higher transparency. Because we assume the marginal screening cost is here relevant, to interpret our framework as our focusing on relationship lending, as we think lending based on credit scoring techniques is better characterized by a zero marginal screening cost.

The loan repayment per unit is $R(p) \le y$, and depends upon the structure of competition in the credit market. We take $R(p)$ as given for the time being;

a later section analyzes the setting of $R(p)$ and its implications for our central problem. We assume that all banks share the same technology, so that they obtain exactly the same signal or absence of signal. At this stage, we also assume that there is no collateral, an extension we consider later on.

Banks maximize their profits, choosing a level of screening for every type of risk (industry) p :

$$\max_{q(p)} \mu q(p)(pR(p) - 1)I - C(q(p))$$

This is a concave maximization problem with the following first order condition[2]

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\begin{aligned}
\mu(pR(p) - 1)I &= C'(q(p)) \text{ for an interior solution} & (1) \\
\mu(pR(p) - 1)I &> C'(1) \text{ for corner solution } q = 1 \\
\mu(pR(p) - 1)I &< C'(0) \text{ for corner solution } q = 0
\end{aligned}
$$

The screening level is thus increasing in the banks' return $pR(p)$, so that banks with higher market power will tend to finance more firms, even if they are riskier. This fact is in line with our model's results on bank competition, as a higher market power will imply a higher $pR(p)$ and this, in turn, will lead to a higher $q(p)$. Notice that the assumptions $C'(0) = 0$ and $pR(p) > 1$ allow us to disregard the corner solution at $q = 0$.

For a given repayment $R(p)$, the firm will choose the high probability of success project, rather than enjoying the private benefits if and only if:

$$p(y - R(p))I \geq (p - \Delta p)(y - R(p))I + B \qquad (2)$$

that is,

$$R(p) \leq y - \frac{B}{I\Delta p} \qquad (3)$$

In other words, for firms to avoid engaging in moral hazard behavior, banks must leave a sufficient rent to the firm. The maximum repayment must be in line with the pledgeable income $y' \equiv y - \frac{B}{I\Delta p}$. We will assume condition (3) is satisfied for any p larger than some floor level $\underline{p}$. In other words, we assume that $(y - R(p))$ increases with p.

In order to identify the financial market imperfections, it is useful to compare the market and the efficient allocation of credit. In doing so, we show that, in equilibrium, banks underprovide screening, with a consequent underprovision of credit in comparison with the efficient allocation level.

The efficient solution, in the perfect information case, results from the maximization of the aggregate output net of the production cost, where the central planner aggregates over industries (risk profiles):

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[2]The convexity of $C(q)$ jointly with $C(0) = 0$ allow us to dispense with the banks participation constraint, $\mu q(pR(p) - 1) \geq C(q)$. At the optimal point this constraint will always be satisfied.

$$\max_{q(p)} \int_0^1 \left[\mu q(p)(py-1)I - C(q(p)) \right] dF(p)$$

The solution to this problem shows that the efficient level of screening is obtained for

$$\mu (py-1) I = C'(q(p)) \text{ for an interior solution} \qquad (4)$$
$$\mu (py-1) I > C'(1) \text{ for corner solution } q = 1$$

We can now state our first result:

Proposition 1 *For levels of μ, such that $(\mu p + (1-\mu)p_-)y < 1$, if $\mu (pR(p) - 1) I < C'(1)$ market equilibrium leads to underprovision of screening by banks. The size of the inefficiency grows with $p(y\text{-}R(p))$, the rent that the bank cannot appropriate.*
Proof. *Convexity of $C()$ and the feasibility condition $R(p) \leq y$ yield the result, by direct comparison of (1) and (4), except in the case where $\mu (pR(p) - 1) I > C'(1)$, i.e. the marginal cost of screening is so low that the market already provides full screening.* ∎

The intuition behind Proposition 1 is simply that screening generates an externality: the good firm that is screened and obtains funding creates an additional output $y - R(p)$ with probability p, an expected profit the bank ignores[3]. So, the discrepancy between the market and the efficient provision of screening is precisely given by $\mu p(y - R(p))$, the benefits that the bank does not fully internalize. Moral hazard exacerbates the inefficiency by requiring an even larger $(y - R(p))$ gap. Still, whether this inefficiency can be partially dealt with, and how, depends upon the instruments available to the government.

We now study some such instruments. As a benchmark, we first examine the problem of a PDB that directly lends to firms. Later, we solve the second best problem where the government, because of asymmetric information, moral hazard or imperfect corporate governance, cannot efficiently lend directly to firms, but is able to act as a principal and design mechanisms to support access to credit by subsidizing banks and firms activities.

2.2 The Direct Lending benchmark

The most straightforward way to channel credit to those firms that are credit rationed is to structure the PDB as a financial institution, with access to funding and equipped with a screening technology. This means the PDB directly lends to firms. We study in this section a PDB that has access to the same screening

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[3]Notice this is not due to the use of debt as the banks' financial instrument. Any other type of contract would generate the same effect, as the bank screening incentives would come from the from the fraction of the firm net expected profit it appropriates.

technology that other banks have. Still, we argue that the PDB may depart from maximizing the sum of total net output minus the cost of intervention.

The simplest way to model the cost of intervention is to introduce a fixed distortion due to taxation. This is to be interpreted as the marginal cost of raising taxes when the tax scheme is optimal. Alternatively, the same parameter may reflect the shadow cost of the PDB budgetary restriction . Denote by λ the distortion associated to the raising of taxes. Because the public bank obtains revenues $pR(p) - 1$ on each dollar lent, the cost of screening is an adjusted $[C(q(p)) - \lambda \mu q I(pR(p) - 1)]$. One benefit of direct lending by the PDB, compared to the market solution, is that the public institution internalizes the screening externality. Moreover, the public institution chooses its rates, so it can provide direct subsidies to firms, of an amount $S_F(p)$ per dollar lent. In our context, subsidies to firms are welfare enhancing if they generate incentives for firms to abstain from engaging in moral hazard behavior

Departure of a PDB from maximizing net output minus the cost of subsidies is to be considered because of its potential lack of independence from politicians, because of imperfect corporate governance, limits to the remuneration policy and other characteristics of many public banks that may lead to a higher screening cost. Critics of public development banks (PDBs) worry that lending by these institutions may end up being inefficiently allocated due to political or institutional constraints. An abundant body of empirical evidence points at cases where this allocation seems to follow political considerations rather than seeking to maximize efficiency. Direct lending by PDBs has been found to increase in election years, and to be targeted to politically valuable costumers or regions, especially in election years (Carvalho, 2014; Cole, 2009; Dinc, 2005; Khwaje and Mian, 2005; Lazzarini et al, 2014; Sapienza, 2004).

We take into account the possibility that the PDB's agenda departs from strict welfare maximization by assuming that, instead of maximizing the net surplus $(py - 1)$, it maximizes a biased objective function $(p(y + \chi(p)) - 1)$. The generality of this formulation has the benefit of being open to a number of interpretations. Indeed, $\chi(p)$ (that we assume could be also be negative) may be interpreted as capturing measurement errors, institutional weakness, corruption or opportunistic behavior by politicians seeking election.

In our current formulation, the political rents $\chi(p)$ are lost if the project is not successful. Notice, nevertheless that, if we redefine $\chi'(p)$, as $\chi'(p) = p \chi(p)$, or $\chi'(p) = \mu p \chi(p)$ the alternative interpretation, of political rents unrelated to the success of the project, is obtained (Still, the extreme case of subsidies without screening in exchange for potential or actual campaign support is not covered). In what follows, we refer to the $\chi(p)$ bias as the "political economy drift", with the acknowledgement that alternative interpretations might fit better some environments than others.[4]

Assume, first, the political rents are such that $(\mu p + (1 - \mu)p_-)(y + \chi(p))$

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[4]Notice, nevertheless that this formulation do not cover cases of bribery, where by providing a subsidy to a firm the politician obtains a kickback, which would imply $\chi(p) = G(S_F(p))$, where G is an increasing function. Instead, the λ cost of any subsidy is accounted for, independently of the political drift $\chi(p)$.

< 1, so that without screening there would be no credit.

The PDB will then maximize:

$$\max_{S_F(p),q(p),p^*} \int_{p^*}^1 \begin{Bmatrix} \mu q(p)\left[p(y+\chi(p))-1\right]I - \left[C(q(p))+\lambda\mu q(p)I(pR(p)-1)\right] \\ +\lambda\mu q(p)C(q(p)-\lambda\mu q(p)pIS_F(p) \end{Bmatrix} f(p)dp$$

$$\left[y + S_F(p) - R(p)\right]I\Delta p \geq B \qquad\qquad (5)$$

$$S_F(p) \;\;\geq\;\; 0; \qquad 1 \geq q(p);$$

Denote by $\gamma(p)$ the Lagrangian multiplier associated to constraint (5) and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$

The first order conditions with respect to $S_F(p), q(p)$ and p^* are, respectively:

$$-\lambda\mu q f(p) + \gamma(p)\Delta p \;\;\leq\;\; 0 \qquad (6)$$

$$\mu I\left\{(p(y+\chi(p))-1)I - \lambda\left[pS_F(p)+(1-pR(p)]\right\} - C'(q(p)) - \frac{\delta(p)}{f(p)} \;\;=\;\; 0 \qquad (7)$$

$$\left[\mu q(p^*)(p^*(y+\chi(p^*))-1)-C(q(p^*))-\lambda(pS_F(p^*))\right] \;\;=\;\; 0$$

Abstracting first from political rents, it is clear that, unless the distortions from raising taxes to fund firms are excessive, direct lending by the PDB increases screening, and subsequently lending, bringing them closer to the first best solution. In particular, with direct PDB lending, and focusing for simplicity in the case with $S_F = 0$ and $q < 1$, condition (7) implies that the equilibrium screening level will be characterized by:

$$\mu I(py-1) = C'(q(p)) + \mu I\lambda(1-pR(p)$$

The left hand side of this equation highlights the fact that the PDB fully internalizes the benefits of funding the positive value projects, while the last term of the right hand side captures the cost of the intervention compared to the first best. This equilibrium implies higher q than the market solution, but, because λ is strictly positive, the first best cannot be reached. If, in turn, the cost of intervening is high enough that q under intervention is lower than in the market solution, the PDB should abstain from intervening.

Even if λ is low, however, PDB intervention can do more harm than good in the presence of what we have called political rents. It is clear that these rents will lead to two biases with respect to the optimal policy in the $\chi(p) = 0$ benchmark case. Regarding the level of screening, expression (7) states that a positive $\chi(p)$ will lead to an excess of screening $q(p,\chi(p))$ while a negative value for $\chi(p)$ will lead to an underprovision of screening.

An extreme case arises if $\left(\mu p + (1-\mu)p_-\right)(y+\chi(p)) > 1$, then, because the firm will always end up being financed, the PDB will choose not to screen and all "bad" projects, in proportion $1 - \mu$ will be financed. That is, even lending to bad firms may yield political benefits that make it attractive to the PDB, leading to inefficient lending. Interestingly, whether this holds or not depends

on the industry's p. Certain industries or types of firms may yield particularly high political rents to the politician. The implication is an additional source for inefficiency: the credit allocation is distorted towards these politically attractive groups of firms.

Notice the question of whether PDB lending is a substitute or a complement of commercial banks' activity only makes sense when referring to direct lending, as credit guarantees or intermediated lending will only complement banking activity. In our framework, the answer to this question is straightforward: when it comes to direct lending, the activity of the PDB is a substitute and directly competes with commercial bank lending in so far as it lends to firms that generate a sufficiently high pledgeable cash flow; it is a complement only for firms that receive a subsidized loan because of the moral hazard it faces, in which case they would never have been financed by banks. A different issue is whether the implementation of subsidized lines of credit to banks by the PDB will substitute market funding. As we will see below this will indeed be the case, although it will not be in competition with market funding and it will increase banks' profit, so that claims of unfair competition are not supported by this analysis.

3   Second best

An alternative to direct lending by the PDB is public lending intermediated by a private financial institution. The benefit of direct lending is that it limits the political drift that may be inherent to direct lending. There are several reasons why this is so:

- Lending occurs only if the banks deem it profitable

- Firms are selected by banks, not by the PDB

- The lending or credit guarantees programs do not target specific firms but specific characteristics

Intermediated lending may be subsidized, or not, depending on the conditions banks and firms face. We will now assume the government is able to subsidize banks and firms and determine to what extent and under which conditions it is optimal to set positive subsidies[5]. We will denote by $S_B(p)$ the per dollar loan subsidy to the bank and by $S_F(p)$ the per dollar of loan subsidy to the firm that is successful, so that the total cost of the subsidies to industry p will be $\lambda\mu qI(S_B(p) + pS_F(p))$.

We assume the industry characteristics, p, and subsequently y and $R(p)$ are observable. It is thus possible to implement a policy of subsidies to banks and firms that are industry (or risk) dependent. As it is obvious, unconditional subsidies will not affect the agents behavior and, consequently, we directly consider subsidies that are related to the granting of a loan, in the case of SB, and to

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[5]Of course, subsidizing banks may imply that the tax structure should be rearranged. If so banks may receive a subsidy on their lending activity while taxed on their profits.

the success of the project, in the case of SF. The assumption that p is equally observable to both banks and the government is a useful starting point, but we later discuss the implications of relaxing it to address questions of central interest, such as how does optimal public intervention change when the bank has better information than the government about its client firms.

Consider the problem with λ, as before, the distortion associated to the raising of taxes. This approach may overestimate the cost of the subsidies as the profit the bank obtains from the subsidy will, presumably, be subject to taxation.

$$\max_{S_B(p), S_F(p), q(p), p^*} \int_{p^*}^1 \left[\mu q(p)(py-1)I - C(q(p)) - \lambda \mu q(p) I (S_B(p) + p S_F(p)) \right] f(p) dp$$

$$\mu(pR(p) + S_B(p) - 1)I - C'(q(p)) = 0 \tag{8}$$

$$[y + S_F(p) - R(p)] I \geq \frac{B}{\Delta p} \tag{9}$$

$$S_B(p) \geq 0; \qquad S_F(p) \geq 0; \qquad 1 \geq q(p);$$

Where constraint (8) holds with equality because the government should subsidize only up to the point where the bank is just induced to provide the second best's q. The positivity constraint for p^* plays no role, as constraint (8) implies $p^* > 0$. We also abstract from the possible case $q = 0$ which we deem irrelevant for a problem in which the government should intervene only if this intervention generates some additional financing of firms.

Denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraints (8) and (9), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_B(p), S_F(p), q(p)$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0 \tag{10}$$

$$-\lambda \mu p q(p) f(p) + \gamma(p) \leq 0 \tag{11}$$

$$\mu I \left[py - 1 - \lambda \left(S_B(p) + p S_F(p) \right) \right] - C'(q(p)) - \tag{12}$$

$$-\frac{\nu(p) C''(q(p)) + \delta(p)}{f(p)} = 0 \tag{13}$$

$$[\mu q(p^*) I \left[p^* y(p^*) - 1 - \lambda (S_B(p^*) + p S_F(p^*)) \right] - C(q(p^*))] = 0$$

We now examine the different possible combinations of $S_F(p)$ and $S_B(p)$ for a sector characterized by p. We structure the discussion to address first the case in which p is such that the government only subsidizes banks ($S_F(p) = 0$) and later address the complementary case $S_F(p) > 0$. Setting $S_F(p) = 0$ is optimal if $p > \underline{p}$, where $\underline{p}$ is the unique solution to $[y - R(p)] I(p) \Delta p = B$. The government optimally sets $S_F(p) = 0$ when $p > \underline{p}$, because in this region constraint (3) holds with strict inequality for any level of S_F. That is, firms in sectors characterized by $p > \underline{p}$ abstain from engaging in moral hazard behavior even without government intervention, so there is no reason for the government to subsidize them.

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3.1 $p \geq \underline{p}$ case:

We begin with the case of industries for which $S_F(p) = 0$.

Focusing first on the interior solution for S_B, such that $S_B > 0$ and (10) holds with equality, we can replace (10) and $S_F = 0$ into (12) to obtain:

$$\mu I \left[py - 1 - \lambda S_B(p) \right] - C'(q(p))$$
$$-\lambda q C''(q(p)) - \frac{\delta(p)}{f(p)} = 0$$

Substracting constraint (8) we obtain:

$$\mu(p(y - R(p)) - S_B(p)) - \mu \lambda S_B(p)) - \lambda q \frac{C''(q(p))}{I} - \frac{\delta(p)}{If(p)} = 0 \qquad (14)$$

We thus have

$$S_B(p) = \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p)) \right] \frac{1}{1 + \lambda} \text{ if } q(p) < 1 \qquad (15)$$

$$S_B(p) < \left[p(y - R(p)) - \frac{\lambda}{\mu I} C''(1) \right] \frac{1}{1 + \lambda} \text{ if } q(p) = 1 \qquad (16)$$

where $S_B(p)$ and $q(p)$ satisfy (8).

The interior solution (15) subsidy $S_B(p)$ compensates for the upside the bank ignores when it takes its screening decision, so it depends upon $y - R(p)$. Derivation of expression (8) with respect to q yields the marginal cost of driving q up via subsidizing the bank, given by $\frac{\partial S_B}{\partial q} = \frac{C''}{\mu I}$. The level of the subsidy is greater the lower is this marginal cost (see (15)). The larger the distortionary cost of taxation λ, the lower the optimal subsidy . Notice, in fact, that the second best $q(p)$ will always be lower than the first best because $\lambda > 0$. If there was no distortion associated to the use of fiscal revenue, $\lambda = 0$, then the first best would, obviously, be obtained.[6]

Proposition 2 *The second best efficient solution requires to set a subsidy to bank lending that is increasing in the externality associated with banks screening; decreasing in $C''(q(p))$ and decreasing in the distortions associated with using fiscal resources, λ; and increasing in μ and p.*

To understand the implications of this result in terms of the industries that should be targeted, notice that condition $S_B(p) > 0$, together with (15) and (16), imply

$$p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p)) \qquad (17)$$

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[6] The distortion λ plays a key role later in our discussion of the merits of direct vs. intermediated public funding.

That is, subsidies are granted to banks on loans directed to p profiles for which the externality is stronger and the cost of subsidizing is smaller, where the latter happens for larger projects and when the second best q is sufficiently high. It is also the case that subsidizing banks is optimal when the probability of finding a good project is sufficiently high and the cost of taxation sufficiently low.

Proposition 3 *In the second best solution the bank will be subsidized for loans to firms that satisfy $p(y - R(p)) \geq \frac{\lambda q}{\mu I} C''(q(p))$, where $q(p) = Min\{1, q^*\}$ and q^* is the solution[7] to: $\mu I \left\{ pR(p) + \left[p(y - R(p)) - \frac{\lambda q}{\mu I} C''(q(p)) \right] \frac{1}{1+\lambda} - 1 \right\}) = C'(q(p)).$*

Notice that intervention may optimally bring banks to fully screen in cases where the market solution implies $q < 1$. To fulfill constraint (8) the government need only provide S_B up to the point that satisfies:

$$\mu I(pR(p) + S_B(p) - 1) = C'(1) \tag{18}$$

implying $\mu(pR(p) - 1)I - C'(1) < 0$. That is, the banks would not have reached the $q(p) = 1$ level without the subsidy.

Notice also that, in the market solution, the bank's choice of $q(p) = 1$ implies

$$\mu I(pR(p) - 1) - C'(1) > 0$$

If this condition is satisfied, then any $S_B > 0$ would violate constraint (8). In other words, if banks fully screen in the market solution then optimal policy would imply no subsidization of banks ($S_B = 0$) in this case.

In the previous analysis, the optimal choice of S_B has only been implicitly solved for, as condition (15) defines S_B in terms of endogenous variable $q(p)$. As an example with a closed form solution, suppose that $C(q) = \alpha q$. The market allocation would be:

$$q = 1 \text{ for } pR(p) - 1 > \frac{\alpha}{\mu I} \tag{19}$$

$$q = 0 \text{ for } pR(p) - 1 < \frac{\alpha}{\mu I} \tag{20}$$

If (19) holds, then the government would abstain from subsidizing the bank ($S_B = 0$). But if instead we have a corner market solution without screening (20), the government might offer a subsidy just enough to bring the bank to fully screen:

$$S_B(p) = \frac{\alpha}{\mu I} - pR(p) + 1$$

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[7] We are assuming a solution exists, which is generally the case. There are exceptions, one of which is simply the linear cost function where $q(p)$ will neither appear in the left hand side nor in the right hand side.

For this subsidy to be optimal, however, condition (16) should hold, so that the social cost of the subsidy has to be lower or equal to the social benefit, $p(y(p) - R(p))$

$$S_B(p)(1 + \lambda) \leq p(y(p) - R(p))$$

Replacing $S_B(p)$ we obtain the locus of industries for which it is efficient to subsidize:

$$p(y - R(p)) \geq \left\{ \frac{\alpha}{\mu I} - (pR(p) - 1) \right\}(1 + \lambda)$$

These are industries for which the value of the externality being addressed $p(y - R(p))$ exceeds the (net) cost of screening, adjusted by the distortionary cost of taxing. Suppose for instance that banks obtain a constant markup over each loan (so R decreases with p to guarantee that $(pR(p) - 1)$ is constant). This last expression makes it clear that, conditional on $pR(p) - 1 < \frac{\alpha}{\mu I}$ so market screening is not perfect, only high upside value industries (high p, high y) are subsidized, because it is for them that the externality to be addressed is largest, with a constant subsidy $S_B = \frac{\alpha}{\mu I} - markup$. High banking competition reduces the markup for banks, increasing the size of optimal subsidies to incentivize screening. This discussion, as noted, holds if $pR(p) - 1 < \frac{\alpha}{\mu I}$, so that decreased risk (increased p) makes subsidies more likely only for cases in which risk and screening costs are not so low that the market already provides perfect screening.

The quadratic cost function case provides another useful illustration. Assuming $C(q) = \frac{1}{2}\beta q^2$, it is also easy to solve for $q(p)$ and $S_B(p)$:

$$\begin{aligned} S_B(p) &= \frac{p(y - R(p)) - \lambda(pR(p) - 1)}{1 + 2\lambda} \qquad (21) \\ q(p) &= \frac{\mu I}{\beta}\left[\frac{py - 1 + \lambda(pR(p) - 1)}{1 + 2\lambda}\right] \end{aligned}$$

The parameter constellations for which $S_B(p) > 0$ are illustrated in Figure 3. Consistent with our discussion of the linear case, the optimal subsidy is positive if p is large enough, and the size of the subsidy also increases with p. The range of p's satisfying this condition expands if y is higher (light grey line rather than black line). In turn, higher distortionary costs of taxation, lambda, reduce the range of subsidized p's and the size of the subsidy (dashed line).[8]

p<p̲ case:
If $p < \underline{p}$ a positive subsidy to the firm $(S_F(p) > 0)$ opens up as a possibility. $S_F(\underline{p}) = 0$ would still be optimal, however, in the specific case where $q(\underline{p}) = 0$. In this case, the sector $(\underline{p})$ that just fulfills the no moral hazard condition would

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[8]To generate this figure, we choose parameter values that satisfy the modelling assumption that $p\mu y > 1$. In particular, we assume $\mu = 0.7$; $y = 3$ in the baseline and $y = 4.5$ in the high y case; and $p \geq 0.48$.
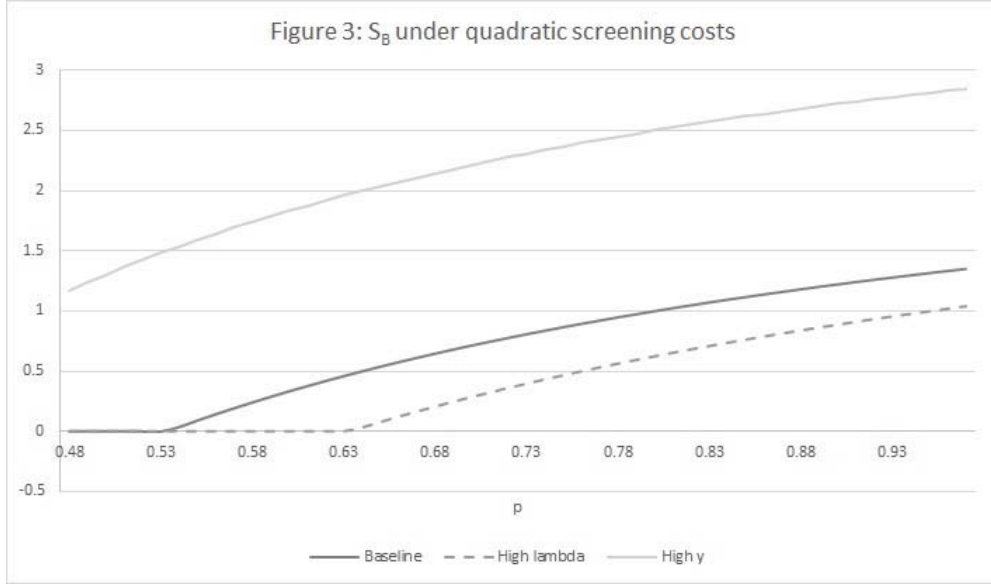
Figure 3: S_B under quadratic screening costs

Figure 3:

not be granted credit. Subsidizing sectors with p even marginally below $\underline{p}$ so that they abstain from moral hazard behavior is worthless, as moral hazard is not the reason why they are not granted credit. If the opposite holds, that is if $q(\underline{p}) > 0$, then it is optimal to the government to subsidize firms so that they behave, which in turns makes them credit worthy. This is stated in the following Proposition.

Proposition 4 *If $C(q)$ is strictly concave and banks make positive profits in industry $\underline{p}$ ($\underline{p}R(\underline{p}) + S_B(\underline{p}) > 1$), then $p^* < \underline{p}$*

Proof. $S_F\left(\underline{p}\right) = 0$, as otherwise it would imply a strict inequality $[y + S_F(p) - R(p)]\,I(p)\Delta p > B$. Plugging $S_F\left(\underline{p}\right) = 0$ into the first order condition (13) implies:

$$\mu I\left[\underline{p}y(\underline{p}) - 1 - \lambda S_B\left(\underline{p}\right)\right] = C'(q\left(\underline{p}\right)) + \lambda q C''(q(\underline{p}))$$

On the other hand, maximization with respect to p^* implies:

$$\mu I(p^*y(p^*) - 1 - \lambda(S_B(p^*))) = \frac{C(q^*(p^*))}{q^*(p^*)}$$

Because $pR(p) + S_B(p) > 1$ we have $q\left(\underline{p}\right) > 0$ and the strict concavity of $C(q)$ implies $C'(q\left(\underline{p}\right)) + \lambda q C''(q(\underline{p})) > \frac{C(q^*(p^*))}{q^*(p^*)}$, we have $\underline{p} \neq p^*$, implying $p^* < \underline{p}$. ∎

The above proposition states that, as long as $q(\underline{p}) > 0$, there is always a fringe of firms that it is worth subsidizing (a range of $p < \underline{p}$ such that $S_F > 0$). The

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intuition is simply that a very small subsidy will allow the firm to be financed (provided, of course $q(p) > 0$, for which a sufficient condition is the existence of positive profits for the bank) and this will bring an increase in both banks and firms' profits.

If the subsidy S_F is positive, then $\gamma(p) = \lambda\mu pq(p)If(p)$ and this implies the associated constraint is binding, so that

$$S_F(p) = \frac{B}{\Delta p} - y + R(p)$$

The positivity of $S_F(p)$ implies that subsidies go only to firms that would not have been financed otherwise, as their profit $y - R(p)$ would be lower than $\frac{B}{\Delta p}$. That is, $S_F(p) > 0$ only for $p\epsilon(p^*, \underline{p})$

Now, using (14), we derive the value for $S_B(p)$

$$\mu(p(y - R(p)) - S_B(p)) - \mu\lambda(S_B(p) + pS_F(p)) - \lambda q\frac{C''(q(p))}{I} - \frac{\delta(p)}{f(p)} = 0$$

implying, for the interior solution:

$$S_B(p) = \left[p(y - R(p)) - \lambda pS_F(p) - \frac{\lambda q}{\mu I}C''(q(p))\right]\frac{1}{1 + \lambda}$$

Consequently, there is some trade-off between the two subsidies, as each dollar of additional subsidy to the firm leads to a decrease of $\frac{\mu\lambda}{1+\lambda}$ in the bank's subsidy. The intuition is obvious: because the a subsidy to a firm creates a distortion $\mu\lambda pS_F(p)$, this comes as a reduction in the benefits $p(y - R(p))$ from a banking subsidy.

3.2   Economic Interpretation

Our setup highlights the central role of the externality that leads to screening underprovision: financiers do not fully internalize the benefits of lending because they cannot appropriate them (i.e. $y - R(p) > 0$). By pinpointing this specific market failure, the analysis makes clear that a subsidy to banks, conditional on their granting a loan, is a natural intervention. Direct subsidies to ex post successful firms may also be optimal, but only for relatively high risk firms ($p < \underline{p}$), as a way to reduce the moral hazard incentives that prevent these firms from accessing credit. The analysis allows us to clarify which types of firms/loans should be targeted.

In particular, condition (17) implies that the subsidies S_B should target industries characterized by:

$p(y - R(p)) \geq \frac{\lambda q}{\mu I}C''(q(p))$ (condition 17), but only as long as $\mu I(pR(p) - 1) - C'(1) < 0$,(eq. 18) so that $q(p) < 1$ in absence of the subsidy. This implies the targeted firms are characterized by:

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1. Sufficiently high expected firms' profits (i.e. high $\mu p\left(y - R\left(p\right)\right)$) as this reflects the inefficiency of credit rationing the subsidies intend to remedy, proportional to the benefits not internalized by the bank. In turn, high expected profits are not only related to high upside value y, but also generally to low risk (high p) unless that risk is so low that the market would provide perfect screening in absence of subsidies. Moreover, since y reflects market conditions, and a highly competitive industry will deliver low y levels, subsidies may be optimally granted to industries were competition is still limited and potential rents high, so that there is room to increase its screening.

2. Industries/types of firms for which the markup that banks obtain on loans are low (low $pR(p) - 1$), as these are the industries where the market q may be below 1, and therefore screening is underprovided. Fierce bank competition may, therefore, justify bank subsidies S_B.

3. Though we have so far assumed that screening costs are not industry-specific, if these costs were to vary across industries our analysis would suggest that subsidies should be directed only to industries/clients for which the marginal screening cost is sufficiently high for there to be screening underprovision.

4. Projects with sufficiently large financing needs I and with a high proportion of good firms μ .

Some of these implications challenge the conventional wisdom about valid targets for the public financing of enterprises. Credit for firms/projects with high expected returns is frequently deemed unworthy of subsidizing, under the expectation that they will be particularly well served by the market. Our results, however, point out that these projects may be, in fact, more acutely underserved, because the externality that reduces screening is particularly stark for these projects. Low risk (high p) and high y industries are, in consequence, plausible targets of S_B, except in the extreme case where their risk is sufficiently low that the market would grant $q = 1$ without subsidies.

Our results also make it clear that subsidizing loans for large projects/firms may in fact be optimal given the large expected benefits of these loans. Loans to sectors facing particularly dynamic demand growth, or those to firms with risky but high upside value projects, are plausible targets of this policy.

It is also clear from these results that external positive effects on other firms (other than the one receiving the loan), often deemed as the justification behind the government financing of enterprises, are not a necessary condition for subsidies to be optimal. Even in their absence, the fact that the financier cannot fully internalize the benefits of lending leads to loan underprovision. Of course, when externalities over third firms are in fact present they represent an additional reason for an intervention that subsidizes loans.

The relationship between optimal subsidies to firms and risk profiles is also interesting. While–with the noted exception of industries where the market

already provides perfect screening–low risk (high p) firms are the optimal target of subsidies to banks, subsidies to firms should be directed to groups of firms exhibiting relatively high risk. This is the case under the plausible assumption that high risk increases the temptation to engage in moral hazard, by increasing $R(p)$. There are also circumstances (types of projects, for instance) where higher risk is correlated with higher upside value, y. Under these circumstances, it may be optimal to grant subsidies to banks for loans to particularly risky industries, if the upside value is high enough.

3.3   Implementation: Subsidized Lending vs. Credit Guarantees Programs

To begin with, notice that a pure subsidy to a firm, unconditional on the success of the project would have no effect on the moral hazard constraint, as it would be added both to the left and right hand side of condition (2). The conditional subsidy $S_F(p)$ can be implemented, first through a reduction of rates, so that the firm net repayment, if successful, is $R(p) - S_F(p)$. This could be a reimbursement to the firm or to the bank which in the latter case is conditional on the bank offering the rate $R(p) - S_F(p)$.

Regarding the bank subsidy, its simplest interpretation is as a direct subsidy $S_B(p)$, per dollar of loan, which makes it conditional on the loan being granted. This is not the most usual practice. Still, the subsidy could be reached alternatively by funding the bank in conditions that entail an implicit subsidy. As what is relevant here is to reach a level of $pR(p) + S_B(p) - 1$ that would lead banks to increase their level of screening to the second best level, a policy of subsidized funding to banks at below the market rate, $1 - \delta$, will lead to the same result provided the terms of the loan, $R(p)$, are agreed beforehand. Otherwise cheaper funding might simply be a source of rents for the bank. By setting $\delta = S_B(p)$, the second best allocation will be reached.

Combining the previous two subsidies, a possible way to implement them is to consider funding for the bank with a discount $\delta = S_B(p) + pS_F(p)$ under the condition that the bank lends at the rate $R(p) - S_F(p)$. Notice that, because $R(p) - S_F(p)$ is strictly positive only for firms in the range $(p^*, \underline{p})$ the policy may imply two different bank funding schemes, one for firms that receive the subsidy and another for firms that are not subsidized.

Alternatively a policy of credit guarantees will also allow to reach the second best allocation. A credit guarantees policy will be defined by a payment to the bank in case the firm defaults. So, under this scheme, the bank receives $G(p)$ for every firm that defaults. We are assuming here that $G(p)$ is not too large, as otherwise the bank will prefer to lend to bad firms and therefore not to screen. In terms of the bank incentives, credit guarantees means that the bank return on a loan will be $pR(p) + (1-p)G(p) - 1$. Consequently, the subsidy to the bank can be implemented in this way, by setting $G(p)$ so that $(1-p)G(p) = S_B(p)$, or $G(p) = \frac{S_B(p)}{1-p}$. For $S_F(p) > 0$, the implementation implies that the bank commits to making a loan at the rate $R(p) - S_F(p)$, and it is compensated

through a credit guarantees scheme, by setting simply $G(p) = \frac{S_B(p)+pS_F(p)}{1-p}$. Notice that a credit guarantees contract with the firm (rather than with the bank) would have a negative impact on the moral hazard constraint. Indeed, it would increase the attractiveness of the private benefits and low probability of success project, because if the project fails, the firm will still obtain a positive profit.

Notice that in our framework there is no difference between a credit guarantees program and selling a credit default swap at the adequate price.

To summarize, a credit support policy program should

1. Develop the information available to the PDB, so that it has the best possible information on industries characteristics and screening costs, while banks have an efficient screening procedure (a credit registry greatly reduces screening costs).

2. The PDB should identify the level of credit rationing in each industry, which measures q, and disregard industries with no credit rationing ($q = 1$).

3. The PDB should identify the industries with the higher upside potential $p(y - R(p)$ that are facing credit rationing.

4. The PDB should determine the marginal impact of a subsidy on the banks screening level (that depends upon competition and the shape of the screening cost function $C''(q)$.

3.4 Comparing direct government lending vs. second best

In order to compare direct government lending and the second best allocation obtained through subsidies, it is useful to consider, first, the hypothetical case of an unbiased public bank, that is with $\chi = 0$. In this case, it is easy to see that direct lending is superior to the second best allocation.

Remark 5 *When direct lending is unbiased ($\chi = 0$), it is superior to the second best allocation. This is the case because the value of the objective function is higher, as the cost of intervention is reduced because $pR(p) - 1 - C(q(p)) > 0 > -\mu q S_B$ while the direct lending maximization problem feasibility set is larger as the bank incentive constraint is dropped from the program.*

By continuity, the previous remark implies that for small levels of political drift χ, direct lending is preferred, while for larger levels the indirect intervention through the subsidization of banks and firms will be preferred.

Nevertheless, the empirical evidence already mentioned seems to suggest that, at least for some PDBs, the bias χ is quite significant. One possible reason is that the PDB does not have access to the same information about risk profiles p than private banks. Another is institutional weakness leading to the direct lending process not being autonomous with respect to the government and

the objectives and constraints of its leaders. Others are corruption and more stringent legal constraints that bind public agencies compared to private institutions. In contexts where any of these reasons weigh sufficiently, the distortion that χ brings to PDB lending outweights its benefits and the implementation of a direct lending program by the PDB will be inefficient.

4   Competition and Credit Market Equilibrium

So far, we have simplified the analysis by assuming an exogenous loan rate, $R(p)$, but, of course the market equilibrium may imply that this rate is itself affected by subsidies to lending, and it may be the case that part or all of the subsidy is passed down to firms. To deal with these concerns, we now study the market equilibrium and its implications for optimal subsidies to loans.

Modeling credit market competition in an imperfect screening framework requires obtaining the optimal interest rate and screening level setting strategies. The difficulty is, that , as in Broecker(1990) and Ruckes(2004), and contrary to Freixas et al.(2007), there is no pure strategy equilibria in a Nash equilibrium for the loan repayments.   (An equilibrium in pure strategies cannot obtain because in this set up the screening signal is perfect and a profit making pure strategy repayment would always be slightly undercut, as undercutting it allows for "cream skimming").

Assume N banks are active in the market. The probability of a bank j, $j \neq i$, not granting a loan to a good firm will be the probability of either getting a good signal but too high a repayment or getting no signal, $1 - q$. Restricting the analysis to the symmetric equilibrium case, for bank i to be able to grant a loan, it has to be the case that the $N-1$ other banks j are either quoting larger repayments or obtained no signal. Thus, the probability of granting a loan at rate R_i is $[q(1 - F(R_i)) + 1 - q]^{N-1}$.

Consequently, when quoting R_i, a bank i confronted with $N - 1$ competing banks will have an expected revenue[9] equal to:

$$\Pi(R_i) = \mu q I(p R_i - 1) \left[q(1 - F(R_i)) + 1 - q \right]^{N-1} \qquad (22)$$

Because in a mixed strategy equilibrium all strategies yield the same expected profit, the equality $\Pi(R_i) = K$ allow us to obtain the common cumulative probability distribution $F(R)$, that satisfies $K = \mu q I(p R_i - 1) \left[q(1 - F(R)) + 1 - q \right]^{N-1}$

The repayment R_i is bounded below by the zero profit lower bound, $R_i \geq \frac{1}{p}$ and above by the pledgeable cash flow $y - \frac{B}{I \Delta p}$ that we denote by y'. Because this upper limit is a possible strategy that satisfies $1 = F(y')$ we have:

$$K = \mu q I(p y' - 1) \left[1 - q \right]^{N-1}$$

so that, using the assumption of a symmetric equilibrium, condition (22) can be rewritten as $(p y' - 1) \left[1 - q \right]^{N-1} = (p R - 1) \left[q(1 - F(R)) + 1 - q \right]^{N-1}$

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[9] Because the banks signals are perfect, they are perfectly correlated, and once a firm is known to be good, it is known to be good for all banks receiving the signal.

From which $F(R)$ is obtained

$$F(R) = \frac{1}{q} \left\{ 1 - (1-q) \left[\frac{py' - 1}{pR - 1} \right]^{\frac{1}{N-1}} \right\}$$

Denote by $\underline{R}$ the lower bound for R_i, which is the solution to $F(R) = 0$. Thus, $\underline{R}$ satisfies $1 = (1-q) \left[\frac{py'-1}{p\underline{R}-1} \right]^{\frac{1}{N-1}}$ so that

$$p\underline{R} = 1 + (1-q)^{N-1} (py' - 1)$$

Remark 6 *The solution therefore leads to positive profits* $\mu q I (1-q)^{N-1}(py' - 1)$ *even for the lowest bound* $\underline{R}$, *provided* $q < 1$.

Remark 7 *Notice that banks' per dollar profits are larger than their average costs because of the convexity of* $C(q)$, *so that banks participation constraint is always satisfied.*

Remark 8 *Banks will quote repayments* R *in the range* $(\underline{R}, y')$, *and good firms will choose the best offer, provided they have at least one offer, which occurs with probability* $1 - (1-q)^N$. *The spread of prices depends upon the difference* $y' - \underline{R}$, *which, itself depends upon* p. *Firms with* $y' < \underline{R}$ *will receive no offer as they would have no incentives to choose the right project. Replacing* $\underline{R}$ *by its value, we observe* $y' < \frac{1+(1-q)^{N-1}(py'-1)}{p}$ *is equivalent to* $y' < \frac{1}{p}$.[10] *Not surprisingly, it is risky firms that will be rationed because of moral hazard.*

4.1 Equilibrium Screening Level

Given this equilibrium pricing strategy, it is easy to obtain the optimal level of screening in the absence of a subsidy. The bank maximizes

$$\max_{\widehat{q}} \int_{\underline{R}}^{y'} \Pi(R_i) dF(R_i) - C(q)$$
$$\widehat{q} \leq 1$$

But, because $\Pi(R_i) = K = \mu q I (py' - 1) [1 - q]^{N-1}$ (with y' equal to the firm's pledgeable income) the problem is simplified and only an interior solution exists, that satisfies

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[10] If $py' < 1 + (1-q)^{N-1}(py' - 1)$, then

$$(1-q)^{N-1}(py' - 1) > py' - 1$$

But this implies

$$\left[(1-q)^{N-1} - 1 \right] (py' - 1) > 0$$

Because $\left[(1-q)^{N-1} - 1 \right] < 0$, the condition is equivalent to $py' - 1 < 0$

$$\mu I(py' - 1)(1 - q)^{N-1} = C'(\widehat{q}) \tag{23}$$

where $\widehat{q}$ is the bank's optimal screening level given other banks' screening q. Notice that $q = 1$ will never hold in a symmetric equilibrium. Consider, as an example, the case of linear screening costs, $C(q) = \alpha q$. While in absence of competition screening in this case, if any, is $q = 1$, the market solution with competition implies $(1 - q)^{N-1} = \frac{\alpha}{\mu I(py'-1)}$, so that $q < 1$. The equilibrium level of screening decreases with N. Despite this fact, more firms may end up screened because, for any given q,, the likelihood of being served by at least one bank grows with the number of banks.

Remark 9 *It is interesting to observe the connection between firms' moral hazard and screening, because at the limit point $y' = \frac{1}{p}$ equation (23) implies the screening level is zero, so that banks will not lend anyway. The linear screening cost example makes this connection clear: the equilibrium screening level is directly related to the pledgeable income.*

Remark 10 *Because $(1 - q)^{N-1}$ is decreasing in N, the impact of increased competition, due to a larger number of banks N on the symmetric equilibrium $(\widehat{q} = q(N))$ is to decrease $q(N)$. Still, since the measure of firms that are financed is $(1 - (1 - q)^N)$, the overall effect of competition is to improve the efficiency of credit allocation.*

Remark 11 *From equation (23) it is easy to derive the impact of changes in the other banks' screening level q on the bank optimal level $\widehat{q}$ and show that banks' screening strategies are strategic substitutes.*

4.2   Optimal Subsidy Policy

In order to determine how competition changes the optimal subsidies, recall, first, that the probability of a good firm not being granted credit is $(1 - q)^N$, so that the probability of a firm being financed in equilibrium is $\mu(1 - (1 - q)^N)$. Second, only firms such that $y' < \frac{1}{p}$ are susceptible of receiving a subsidy S_F. In addition, it is clearly inefficient to leave a rent to the firm above $\frac{B}{\Delta p}$. As a consequence, if a firm receives a subsidy S_F, the subsidy will satisfy $y' + S_F(p) = \underline{R}$. That is, of the support of interest rates that constitute the banks' mixed strategy competition equilibrium, $\underline{R}$ (but not higher interest rates) will be made feasible by the subsidy to firms.[11] This implies the mixed strategy distribution becomes a pure strategy.

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[11] Recall that, for industries with p such that $y' < \frac{1}{p}$, no equilibrium with screening will be feasible in absence of S_F.

The PDB problem may now be written:

$$\max_{S_B(p),S_F(p),q(p),p^*} \int_{p^*}^1 \left\{ \mu I \left[1 - (1-q(p))^N \right] [py - 1 - \lambda(S_B(p) + pS_F(p))] - NC(q(p)) \right\} f(p)dp$$

$$\mu I(1-q)^{N-1}(py' + S_B - 1) - C'(q(p)) = 0 \tag{24}$$

$$p(y' + S_F(p)) \geq 1$$

$$S_B(p) \geq 0; \qquad S_F(p) \geq 0; \qquad 1 \geq q(p);$$

As before, denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraint (24) and to the moral hazard constraint, and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$. The Lagrangian conditions become

$$-\lambda(1 - (1-q(p))^N)f(p) + (1-q)^{N-1}\nu(p) \leq 0$$

$$-\lambda\mu I(1 - (1-q(p))^N)f(p) + \gamma(p) \leq 0$$

$$\mu IN(1-q(p))^{N-1}(py - 1 - \lambda(S_B(p) + pS_F(p))) - NC'(q(p)) -$$

$$-\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0$$

$$\mu I(1 - (1-q(p^*))^N)(p^*y - 1 - \lambda(S_B(p^*) + pS_F(p^*))) - NC(q(p^*)) \quad = \quad 0$$

For firms with $p > \frac{1}{y'}$ it is optimal to set $S_F = 0$, and because $q < 1$, we have, for $S_B > 0$ and $\nu(p) = \frac{\lambda(1-(1-q(p))^N)f(p)}{(1-q)^{N-1}}$:

$$\mu I \left[(1-q(p))^{N-1}(py - 1) - \lambda S_B(p) \right] - C'(q(p)) - \lambda \frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}} C''(q(p)) = 0$$

Substracting (24) leads to

$$\mu I \left[(1-q(p))^{N-1} (p(y - y') - (1+\lambda)S_B) \right] - \lambda \frac{(1 - (1-q(p))^N)}{N(1-q)^{N-1}} C''(q(p)) = 0$$

So that the optimal subsidy satisfies

$$S_B = \frac{1}{1+\lambda} \left\{ p(y - y') - \frac{\lambda \frac{(1-(1-q(p))^N)}{N(1-q)^{N-1}} C''(q(p))}{\mu I(1-q(p))^{N-1}} \right\} \tag{25}$$

For firms with $p < \frac{1}{y'}$ a subsidy $S_F > 0$ satisfying $p(y' + S_F) = 1$ is enough for the firm to choose the good project. Substituting into the first order conditions, we obtain:

$$S_B = \frac{1}{1+\lambda} \left\{ p(y - y') - \lambda(1 - py') - \frac{\lambda \frac{(1-(1-q(p))^N)}{N(1-q(p))^{N-1}} C''(q(p))}{\mu I(1-q(p))^{N-1}} \right\}$$

24

Firms will receive subsidies, if any, when they satisfy both $py' < 1$ and $p\epsilon(p^*, \frac{1}{y'})$, but at an interest rate that implies a zero profit for the bank and $q(p) = 0$, so that the subsidy to the firm is ineffective if not accompanied by a subsidy to the bank.

Consider again the example of linear screening costs, $C(q) = \alpha q$, and assume $p > \underline{p}$, so that $S_F = 0$. Optimal policy implies $S_B = \frac{p(y-y')}{1+\lambda}$ and an increased level of screening with respect to the market solution, given by $(1-q)^{N-1} = \frac{\alpha}{\mu I \left(py' + \frac{p(y-y')}{1+\lambda} - 1 \right)}$. The link that competition introduces between screening and moral hazard–highlighted in remark 9 is evident again in the fact that, even with $p > \underline{p}$, the optimal subsidy depends on pledgeable income y', since the support of equilibrium interest rates depends on y'. The convex costs case, $C = \frac{\beta C^2}{2}$, meanwhile, yields $S_B = \frac{\lambda\beta}{\mu I (1+\lambda)} \left(\frac{1-(1-q)^N}{N^2} - \frac{N^2(1-q)^{N-1}}{N^2} \right)$. The number of banks, N, has an ambiguous effect on S_B: while greater competition reduces the effectiveness of the subsidy to increase banks incentives to screen–because part of the subsidy is passed on to firms via reduced prices–, competition also increases the need for the subsidy.

5 Extensions: Collateral, liquidity and capital shortages.

So far, we have considered subsidies to banks and firms in a market where firms cannot pledge collateral and banks are able to issue any type of liability and face no constraint, either on their liquidity or on their solvency. When this is not the case, the analysis of optimal subsidy policy changes. To simplify the analysis, we assume away the mixed strategies characteristic of competition, and assume each bank faces a market repayment.

5.1 Collateral

So far, we have assumed that a bank receiving no signal on a firm will not finance its project. Nevertheless, this need not be the case if the firm is to post collateral[12]. In this case, however, it is possible that the amount of the loan the firm obtains is constrained by the availability of collateral and the firm's project has to be downsized. We extend now our analysis to the case where agents are endowed with some exogenously given amount of collateral, which we denote V[13].

As it is standard, we will assume collateral is costly, as the V value of the asset to the bank is lower than its value to the firm, $(1+\delta)V$, where $\delta > 0$. In

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[12]If property rights do not provide legal certainty to pledging and repossession, however, collateral based credit may be quite limited.

[13]This amount will depend, among other factors, upon the legal and institutional features of the economy.

the present setup, collateral will play two related roles: as a signalling device and in mitigating credit risk.

Signalling allows good firms to separate from bad firms, if the latter are not willing to post collateral. Let $R_V(p, V)$ be the per dollar repayment on a loan I collateralized with an asset valued V to the bank. Because firms know their types[14], when the value of collateral V is larger than some threshold, only good firms will be ready to pledge their collateral. Define ν_B as the collateral per dollar of loan that leaves the bad firms indifferent between a partially collateralized loan and abstaining from demanding a loan. That is, v_B satisfies the following condition:

$$p\_(y - R_V(p, V)) - (1 - p\_)(1 + \delta)\nu_B = 0$$

Then, any loan contract with a collateral to loan ratio $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$ will deter bad firms from applying for a loan. Because downsizing has an opportunity cost for the firms, efficient contracts will be characterized by the maximum loan per unit of collateral, that is the minimum $\frac{V}{I}$ that satisfies $\nu_B \leq \frac{V}{I}$. This implies the good firm individual rationality constraint is trivially satisfied, for any contract characterized by a collateral to loan ratio v_B. This ratio, jointly with V will determine the maximum size I at which the firm will be able to develop its project.

Notice that whenever the above inequality is satisfied it is unnecessary for banks to screen firms for collateralized lending. The use of collateralized loans implies that all good firms have their projects funded so that there is no credit rationing due to banks' insufficient screening.

Still, depending on the availability of collateral V and on the cost $(1 - p)\delta$ of pledging it, the firm may prefer to be screened by the bank. This will be the case if the firm's profits are higher with an uncollateralized loan, that is:

$$p(y - R(p))I^* > p(y - R_V(p))\frac{V}{\nu_B} - (1 - p)(1 + \delta)V$$

where I^* is the size of the loan required to finance the project without downsizing. The condition is obviously met when collateral is scarce. Still, even if collateral is plentiful, if its cost δ is sufficiently high in comparison to the cost of screening, the condition is also fulfilled[15]. In the following we will assume the condition is so that both firms and banks prefer to screen, so that banks'

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[14] If firms do not know their type, under our assumption of an expected negative present value for the average firm, $(\mu p + (1 - \mu)p_-)y < 1$, if banks break even, firms will make losses and, therefore will abstain from asking for a collateralized loan.

[15] Because in equilibrium per dollar expected profits should be equal across banks, we have $pR(p)) - \frac{C(q(p))}{\mu I} = pR_V(p) + (1 - p)\nu_B$

A sufficient condition for the above inequality to be satisfied is:

$$\frac{C(q(p))}{\mu I} \leq (1 - p)(1 + \delta)\nu_B$$

because, in this case, the firm prefers an uncollateralized loan, even in the absence of any downsizing, simply because the expected cost to the firm of losing its collateral is higher than the screening cost to the bank.

screening and public support to firms are still an issue. Notice, though, that when this condition is not satisfied, and the firm prefers to borrow collateralized because it has sufficient collateral, the policy implication is clear: the PDB should abstain from any intervention.

The introduction of collateral changes the objective function of the bank. If the bank obtains a non-informative signal, which occurs with probability $(1-q)$, it will still be able to grant a collateralized loan. The bank profits will now become:

$\max_q \mu \left\{ q(pR(p)-1)I^* + (1-q)\left[(pR_V + (1-p)\nu_B)-1\right]\frac{V}{\nu_B} \right\} - C(q))$

The first order condition that determines the level of screening will be:

$$\mu \left\{ (pR(p)-1)I^* - \left[(pR_V + (1-p)\nu_B)-1\right]\frac{V}{\nu_B} \right\} = C'(q))$$

Consequently, the introduction of collateral decreases q through the "spare tire" effect of collateralized lending when the bank obtains no signal. Of course, this does not mean that a policy promoting the use of collateral by protecting creditors' rights to repossession should not be implemented. It simply states that it has a cost in terms of relationship banking and in the lower level of screening it generates. The result is in line with Manove et al.(2001) model of "lazy banks" and has competition policy and regulatory implications. Indeed, on the competition policy side, it implies that the lower the banks' market power in the collateralized market, $pR_V + (1-p)\nu_B - 1$, the higher the level of screening in the uncollateralized segment. On banking regulation, it implies that collateralized loans should have very low capital charge, in line with Basel II and III, and excess of caution will be costly in terms of screening incentives.

Thus, overall, the introduction of collateralized lending will, on the one hand, increase the total output but, on the other hand, diminish the bank's incentive to screen.

Because the subsidy in case of a collateralized loan is not justified, the second best problem becomes:

$$\max_{S_B(p),S_F(p),q(p),p^*} \int_{p^*}^1 \{\mu \left[q(p)(py-1)I^* + (1-q(p))\left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right)\right]$$
$$-C(q(p)) - \lambda\mu q I^*(S_B(p)+pS_F(p))\} f(p)dp$$
$$\mu \left[(pR(p)+S_B(p)-1)I^* - \left[(pR_V + (1-p)\nu_B)-1\right]\frac{V}{\nu_B}\right] - C'(q(p)) = 0$$
$$[y + S_F(p) - R(p)]\, I^*\Delta p \geq B \tag{26}$$
$$S_B(p) \geq 0; \qquad S_F(p) \geq 0; \qquad 1 \geq q(p);$$

Denote, as before, by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to the first two constraints, and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

Still, this is only an extreme sufficient condition when, in fact, downsizing has an opportunity cost that makes our hypothesis of efficient screening even more natural.

The first order conditions with respect to $S_B(p), S_F(p), q(p)$ and p^* are:

$$-\lambda q f(p) + \nu(p) \leq 0 \tag{27a}$$

$$-\lambda \mu q f(p) + \gamma(p)\Delta p \leq 0 \tag{27b}$$

$$\mu[(py-1)I^* - \left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right) - \lambda(S_B(p) + pS_F(p))]I^*$$

$$-C'(q(p)) - \frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} = 0 \tag{27c}$$

$$\mu\left[q(p^*)(p^*y(p^*) - 1)I^* + (1-q(p))\left(p(y-1)\frac{V}{\nu_B} - (1-p)\delta V\right)\right]$$

$$-C(q(p^*)) - \lambda I(S_B(p^*) + pS_F(p^*)) = 0$$

The analysis of subsidies to screened firms is the same as before.[16]

$$p(y - R_V \frac{V}{\nu_B} - (1-p)(1+\delta)V (\geq (p - \Delta p)(y - R_V \frac{V}{\nu_B} - (1 + \Delta p - p)(1+\delta)V + B$$

This implies

$$R_V \leq y + (1+\delta)\nu_B - \frac{B\nu_B}{V\Delta p} \tag{28}$$

with, as intuition suggests, a much higher pledgeable cash flow due to the value of collateral to the firm. The main impact of collateral will be on the optimal bank subsidy, as the option of collateralized lending decreases the benefits of screening.

Following the same procedure that we used to derive the optimal subsidy in the absence of collateral, we obtain, when a collateral V could be pledged with the bank:

$$S_B(p) = \{p(y - R(p)) - \left(p(y - R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*}$$

$$-\frac{\lambda q}{\mu I^*}C''(q(p))\}\frac{1}{1+\lambda} \text{ if } q(p) < 1 \tag{29}$$

$$S_B(p) \leq \{p(y - R(p)) - \left(p(y - R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*}$$

$$-\frac{\lambda}{\mu I}C''(1)\}\frac{1}{1+\lambda} \text{ if } q(p) = 1$$

This expression allows us to identify the industries that should be targeted, that is, such that $S_B(p) > 0$. These industries will be characterized by the following expression:

$$p(y - R(p)) \geq \left(p(y - R_V)\frac{1}{\nu_B} - (1-p)(1-\delta)\right)\frac{V}{I^*} + \frac{\lambda q}{\mu I}C''(q(p)) \tag{30}$$

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[16] The introduction of collateral, however, also modifies the moral hazard problem for firms receiving collateralized loans, as they will now choose the positive net present value project taking into account the possible loss of collateral:

For a given expected profit, our findings square with the argument that firms lacking the possibility of collateralizing their loans are desirable targets of public financing. In particular, low available collateral V and high minimum required collateral v_B make it more likely that the above condition is fulfilled. Small and young firms, and those in sectors holding little pledgeable assets (such as services), are likely examples of such targets.

The comparison between the level of the subsidy when there is no collateral, (15) and when there is collateral, (29) shows that the subsidy is much larger in the first case. The explanation is obviously that in the first case, the social loss of the bank not getting any signal is the loss of $p(y-1)$, while in the second case, the cost is only a lower level of funding for the firm, corresponding to $p(y-1)(I-V(1-\gamma))$, which depends upon the amount of collateral V available at the firm level. Of course, this does not mean that a policy promoting the use of collateral by protecting creditors' rights to repossession should not be implemented. It simply states that it has a cost in terms of relationship banking and in the lower level of screening it generates.

5.2 Liquidity

The decrease in the volume of credit that characterizes a crisis may result from a decrease in its demand or in its supply. In the second case, it may be due to a reduction in banks' access to funding. In our set up, the banks limited access to funds can be easily modelled through the introduction of an additional constraint limiting the bank's liquidity supply in the analysis of the second best. This, of course, disregards why and how a liquidity shortage occurs. Considering those reasons would require the modeling of the whole monetary policy framework. Implicitly, it is assumed that the supply of outside liquidity by monetary policy authorities cannot be altered, and that the PDB is not forced to support the monetary contraction policy (although a reinterpretation of λ could account for the PDB liquidity shortage) and is able to pursue its own lending policy. In such a framework the banks choice of screening will take the constraint into account, as they will now maximize

$$\max_{q(p),p^{\tilde{u}}} \int_{p^*}^{1} \left\{ \mu q(p)(pR(p) + S_B(p) - 1)I - C(q(p)) \right\} f(p)dp$$

$$\int_{p^*}^{1} \mu q(p)I f(p)dp \leq L \tag{31}$$

The solution to this problem will be, if ϕ is the Lagrangian multiplier associated to the liquidity constraint:

$$\mu(pR(p) + S_B(p) - (1 + \phi))I - C'(q(p)) = 0$$

$$\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1 + \phi))I))If(p)dp \leq L$$

$$\mu q(p^*)(p^* R(p^*) - (1 + \phi))I - C(q(p^*)) = 0$$

Not surprisingly the liquidity restriction implies a shadow cost of liquidity that can be interpreted as an interest rate.

The PDB will now solve:

$$\max_{S_B(p), S_F(p), q(p), p^*} \int_{p^*}^{1} \{\mu q(p)(py - 1)I - C(q(p)) - \lambda \mu q I(S_B(p) + pS_F(p))\} f(p)dp$$

$$\mu(pR(p) + S_B(p) - (1 + \phi))I - C'(q(p)) = 0$$

$$\int_{p^*}^{1} \mu C'^{-1}(\mu(pR(p) - (1 + \phi))I))If(p)dp \leq L$$

$$\mu q(p^*)(p^* R(p^*) + S_B(p^*) - (1 + \phi))I - C(q(p^*)) = 0$$

$$[y + S_F(p) - R(p)]I\Delta p \geq B$$

$$S_B(p) \geq 0; \qquad S_F(p) \geq 0; \qquad 1 \geq q(p);$$

Now, depending on the way the subsidies are implemented, they may imply additional liquidity. Under intermediated lending, the PDB will be able to use, in fact two instruments: $S_B(p)$ and $\Delta L(p)$, a credit line that will alleviate the liquidity constraint for loans in the industry p and the liquidity constraints.[17]. Let $\phi(p)$ be the Lagrangian multiplier associated to the liquidity constraint (31).

It is easy to prove that, if $\phi > 0$, that is, if the liquidity constraint is binding, the use of $\Delta L(p)$ will always strictly improve upon the exclusive use of subsidies implemented through instruments unrelated to liquidity. Indeed, assume the optimal structure constrained by $\Delta L(p) = 0$ is obtained. Because in the constraints of the above problem, only the expression $S_B(p) - (1 + \phi)I$ appears, it is clear that a positive $S_B(p)$ can be substituted by the equivalent decrease in $\phi(p)$ that is generated by an increase in $\Delta L(p)$. Still, while $S_B(p)$ has a λ cost, an increase in liquidity has no cost. So, even if the optimal policy may still involve a subsidy, it will be combined with a policy of intermediate lending that will alleviate the bank's liquidity constraint and thus reduce the opportunity cost of lending for some specific industries p.

5.3 Capital Shortages

The banks' lack of regulatory capital, characteristic of a credit crunch (See Bernanke and Lown, 1991) may also impose a limit to the banks' ability to lend.

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[17]The issue of firms' rationing on their long term funding, which we consider quite relevant, is beyond the scope of our analysis.

Although the equation that captures the restriction is similar to the liquidity constraint above? the effects will be quite different. Denote as β the coefficient of required capital to extend a given amount of credit, i.e. the risk weight. If all the loans to firms have the same risk weight, the constraint will be:

$$\int_{p^*}^{1} \beta\mu q(p)If(p)dp \leq E$$

If capital shortages are a key constraint, then reducing the loss given default on a loan is a feasible way to soften that constraint. Because a credit guarantees program reduces the banks' risk for the targeted loans as exposure is reduced from I to a fraction $(1-\alpha)I$, if α is the fraction of losses the PDB commits to cover. This means that the PDB credit guarantees program is the right way to intervene. Nevertheless, the impact of credit guarantees will depend upon the rating of the PDB. With an ill-rated PDB, credit guarantees by the PDB may not be credible and therefore be ineffective.

6   Business and Credit Cycles

An important issue regarding the activity of a PDB is its role in a situation where banks are constrained in their lending, and direct public financing is expected to play a particularly important countercyclical role (World Bank, 2013). Our framework provides a rationale for this expectation as recessions may be times of particularly acute liquidity and capital restrictions for the banks, specially when associated with financial crises. For this reason, funding to banks and credit guarantees will help ease liquidity and capital constraints as established in section 5, and will be particularly valuable during times of crises. However, crises may also be times when expected return from new projects is also particularly low (low py), reducing, for any given $R(p)$, the externality that implies underprovision of screening, and increasing incentives to engage in moral hazard. Our analysis thus suggests that, if the decrease in the volume of credit is demand driven, PDB interventions should be reduced. Still, if, as it seems more likely, the reduction is due to liquidity and capital constraints, the PDB will play a key countercyclical role. Lending to banks is optimal only to the extent that there are (starker) liquidity constraints associated with the crisis. If, instead, the reason for the reduction in credit is banks' capital constraints, then credit guarantees with the corresponding reduction of the banks' exposures will be the correct way to intervene. So, banks themselves will choose whether they prefer, for the same implicit level of S_B, a loan or a credit guarantee, depending on its situation. Bear in mind, however, that our static framework is not well suited to deal with dynamic costs from crisis in the presence of credit constraints. [18]

The empirical evidence shows that macroeconomic conditions have a strong impact on credit, with tightening standards associated with lower future levels

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[18] For instance, Eslava et al. (2015) estimate that there are long-lasting TFP losses from the inefficient exit of profitable but credit constrained firms.

of loans and output (Lown and Morgan, 2001 p.1581). In the context of our model, this means either that banks screen more in bad times or that, in good times, banks lend indiscriminately(Ruckes, 2004). The first is possible when, in a downturn, the supply of credit decreases more than the demand leading to higher $R(p)$, and therefore a higher screening effort q. If this is the case, the implication is that subsidies should decrease in bad times. Still, liquidity and capital shortages imply, instead that the PDB develops a more active program of intermediated lending and credit guarantees, even in the absence of a subsidy.

Consequently, the activity of a PDB may switch with the business cycle. In normal times its role will be the one described in our model, providing banks with the incentives to increase their screening and, therefore, their lending to high potential $(p(y - R(p)))$ industries. In a downturn or in a crisis, instead, the liquidity and solvency constraints may be more important, and, in this case the PDB can reduce its subsidies and concentrate on providing liquidity at the market price or providing a credit guarantees insurance at its fair value, which is equivalent to writing a credit default swaps. It is important to point, however, that our model abstracts from public lending for working capital, which may be crucial in a downturn (Eslava et al. 2015).

7   Robustness

At this stage It would be interesting to examine how robust are our qualitative results.

Regarding the screening technology, our framework treats screening as weeding out bad firms. Would the same results hold if, instead we had a screening technology based on an imperfect signal? The answer is affirmative provided screening is costly.

- **Screening technology**

In this case, screening will provide a signal s on the firms' distribution of cash flows y, generating an ex post distribution with density function $f(y \mid s)$,which is informative about y in the sense of the Monotone Likelihood Ratio Property (MLRP), so that high signals imply a higher probability mass on the high cash flows. When this is the case, the optimal decision for the bank will be to lend whenever the signal is higher than some threshold s^*. The bank choice of screening corresponds then to the precision of the signal s, ranging from a perfect signal $y = s$ at a high cost to no precision at all (in which case $f(y \mid s) = f(y)$) at zero cost. The precision level will result from profit maximization and, again, will not take into account the benefits accruing to the firms of the choice of precision, $p(y - R(p))$.

Still, the analysis of competition will lead to different conclusions, because, signals will not be perfect any longer, so that bad firms will have a chance to be granted credit. This implies, as in Broecker(1990) that when the population of banks increase the chances of bad firms to obtain credit increases, so that for a given interest rate, the average return on a bank loan decreases.

- **Firms' Moral Hazard**

Regarding firms' moral hazard, we have assumed a unique solution $\underline{p}$ to the equation $y - R(p) = \frac{B}{\Delta p}$, but the extension to a more general case even if more cumbersome is straightforward. It will define N intervals $(p_1, p_2), ... (p_{2N-1}, p_{2N})$ such that for any $p, p\epsilon(p_{2k-1}, p_{2k}), y - R(p) < \frac{B}{\Delta p}$. Then our proof extends and it is possible to prove that it is always beneficial to subsidize firms $p_{2k-1} + \varepsilon$ and $p_{2k} - \varepsilon$ for ε sufficiently small, as a very limited subsidy allows $q(p)$ firms in this interval to be financed and generate $\mu q(p)(py - 1)$ additional output which is independent of ε. Other forms of moral hazard, as firm's effort level could be considered. In the appendix we briefly examine an alternative modeling of moral hazard, through introducing a cost of effort function at the firm level and the implications it would have, and show that, again, it would be optimal to subsidize firms with insufficient incentives to exert effort.

- **Loan Size**

We have assumed that the screening cost does not depend upon the size of the project and of the loan. This seems a reasonable yet critical assumption. Indeed, if the screening costs were to be proportional to the projects' size, it would imply that size is irrelevant in the screening decision and small firms would have the same chances of being financed as large firms.

Also, we have assumed $R(p)$ does not depend upon the size of the loan. This implies the bank choice of $q(p)$, when confronted with a repayment $R(p)$, a subsidy, $S_B(p)$, and a size I will result from the first order condition:

$\mu(pR(p) + S_B(p) - 1)I - C'(q(p)) = 0$, with the simplification that it is the marginal cost of screening per dollar of granted loan $\frac{C'(q(p))}{\mu I(p)}$ that matters. Dropping the assumption would simply imply that the bank optimal screening level will result from the total revenue $R(p, I(p))$ and total subsidy $S_B(p, I(p))$. The impact of size will then cease to be linear, but the qualitative results would remain the same.

- **Industry Specific Screening Costs**

Finally, it is often argued that screening might be more or less costly in different industries. This is the case, for instance, for SMEs. As stated by Beck et al. (2008, p.1-2)"Both high transaction costs related to relationship lending and the high risk intrinsic to SME lending explain the reluctance of financial institutions to reach out to SMEs". In addition, the scarcity of reliable data on SMEs and the possible manipulation of their financial statements make screening more costly. Finally, if the screening cost is related to the relationship lending, then a high turnover in the population of firms make the investment in the relationship less profitable. In our context, this implies considering a screening function $C(q(p), p)$, which is a straightforward extension.

8 Conclusion

The existence of specific programs of credit to firms has sometimes been justified on the basis of the positive externalities they generate or on the existence of moral hazard at the firm level. We argue that this justification of public credit support is, in fact, unrelated to the credit market and could be dealt with through a direct subsidy. Our focus is instead on the welfare costs of financial markets imperfections. We argue that, when the screening of projects is costly, banks best strategy is to limit their screening levels, which results in some good firms being credit rationing. Still, the banks profit maximizing level of screening is suboptimal because it disregards the profits the financing of a project generates at the firm level. To correct for this underinvestment in screening, a PDB can intervene in a number of ways that may vary depending on the constraints banks face. Still, the firms the PDB should target industries characterized by:

1. Some degree of credit rationing

2. A sufficiently high expected firms' profits (i.e. high $\mu p (y - R (p))$), as this reflects the benefits of screening that are not internalized by the bank.

3. Projects with sufficiently large financing needs I and with a high proportion of good firms μ .

This implies he PDB has to have access to information on the industries, so as to identify these characteristics, an information that may be facilitated by the existence of credit registries.

Regarding the way a PDB can instrument its intervention, if there is no bias in the PDB objective function, direct lending is the preferred way. Nevertheless, as the empirical evidence has shown that lack of rigorous corporate governance, government pressures and political biases makes direct lending inefficient, intermediated loans may be preferred. We show how the PDB can improve efficiency through either subsidized lending or credit guaranties as they are equivalent in good times. Still, when banks face a liquidity or capital shortage, the two programs can be adapted so as to react to the more pressing constraints.

Finally, the availability of collateral should also be taken into account. While the creation of legal certainty on collateral provides better access to the credit market, we show that it also reduces banks incentives to screen and may lead to banks reducing the size of the loans to firms, forcing them to downsize their projects.

9 References

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10   Appendix: Unobservable Firms' Efforts

An alternative common form of moral hazard is the effort model, whereby firms choose the optimal level of effort (normalized to equal the probability of success) given its quadratic cost $C(e) = \frac{e^2}{2\beta}$. Under perfect observability and contractability of effort, a firm receiving a loan would make a repayment $I(1 + \rho)$, so that the firm maximizes

$$\max_e epy - I(1 + \rho) - \frac{e^2}{2\beta}$$

and the first best effort level $e^* = \beta py$ is obtained. Under moral hazard, the chosen level of effort $\widehat{e}$, for a repayment $R(p)$ will be the solution to:

$$\max_e ep\left(y - R(p)\right) - \frac{e^2}{2\beta}$$

so that $\widehat{e} = \beta p\left(y - R(p)\right) < \beta py$, where $\widehat{e}pR(p) = I(1 + \rho)$

Consequently, a subsidy in conditional on success changes the objective function to $\max_e ep\left(y + S_F(p) - R(p)\right) - \frac{e^2}{2\beta}$ and its solution to

$$e = \beta p\left(y + S_F(p) - R(p)\right) \tag{32}$$

The equivalent second best problem will then have the added variable e and a different moral hazard constraint (32).

$$\max_{S_B(p),S_F(p),q(p),e,p^*} \int_{p^*}^1 \left[\mu q(p)(epy-1)I - C(q(p)) - \lambda\mu q(p)I(S_B(p)+epS_F(p))\right]f(p)dp$$

$$\mu(epR(p)+S_B(p)-1)I - C'(q(p)) = 0 \qquad (33)$$

$$e = \beta p\left(y + S_F(p) - R(p)\right)$$

$$S_B(p) \geq 0; \qquad S_F(p) \geq 0; \qquad 1 \geq q(p); \qquad e \leq 1$$

Denote by $\nu(p)$ and $\gamma(p)$ the Lagrangian multipliers respectively associated to constraints (33) and (32), and let $\delta(p)$ be the multiplier associated with $1 \geq q(p)$.

The first order conditions with respect to $S_B(p), S_F(p), q(p), e$ and p^* are:

$$-\lambda q f(p) + \nu(p) \qquad \leq \qquad (34)$$

$$-\lambda\mu q e I f(p) + \beta\gamma(p) \qquad \leq \qquad (35)$$

$$\mu I\left[epy - 1 - \lambda\left(S_B(p) + epS_F(p)\right)\right] - C'(q(p)) - $$
$$-\frac{\nu(p)C''(q(p)) + \delta(p)}{f(p)} \qquad = \qquad (36)$$

$$\mu q(p)I(py - \lambda p S_F(p))f(p) \qquad +\nu(p)\mu IpR(p) - \gamma(p) = (37)$$

$$[\mu q(p^*)e(p^*y(p^*)-1) - C(q(p^*)) - \lambda(S_B(p^*) + ep^*S_F(p^*))] = 0 \qquad (38)$$

Consider the case $S_B(p) > 0; S_F(p) > 0; 1 > q(p); e < 1$,

$$\nu(p) = \lambda q f(p) \text{ and} \qquad (39)$$

$$\beta p\gamma(p) = \lambda\mu q e I f(p) \qquad (40)$$

Replacing in 36) and (37) yields, respectively :

$$\mu I\left[epy - \lambda\left(S_B(p) + epS_F(p)\right)\right] - C'(q(p)) - \lambda q C''(q(p)) = 0$$

and

$$\mu q(p)I(py - \lambda p S_F(p))f(p) + \lambda q(p)f(p)\mu IpR(p) - \frac{1}{\beta}\lambda\mu q e I f(p) = 0$$

dividing by $\mu q(p)If(p)$ the expression simplifies to

$$py - \lambda p S_F(p) + \lambda pR(p) - \frac{1}{\beta}\lambda e = 0 \qquad (41)$$

$S_F(p) \geq 0$ if

$$py + \lambda pR(p) \geq \frac{1}{\beta}\lambda e \qquad (42)$$

Expression (42), can simply be interpreted as the benefits of the subsidy being larger than its costs. As the benefits of the subsidy are derived from the

incentive effect on e, we want the marginal cost of S_F, $\lambda \mu q I e p f(p)$ to be lower than the benefits it generates. Now, the for each dollar increase of S_F, the impact on e is $\frac{de}{dS_F} = \beta p$. In turn, a unit increase in e will have an direct impact on the objective function of py, and an indirect impact in the incentives for the bank to increase its screening level, because a dollar of $epR(p)$ is equivalent to a dollar increase of $S_B(p)$. So, an increase in e leads to benefits of $py + \lambda pR(p)$, wich occur with probability $\mu q(p) I f(p)$. So the net benefit condition for a subsidty is $\mu q(p) I f(p) \beta p (py + \lambda pR(p)) \geq \lambda \mu q I e p f(p)$. Simplifying we obtain expression (42).

In order to obtain a condition for the positivity of S_F without the endogenous value of e, replacing e by its value $e = \beta p (y + S_F(p) - R(p))$ in (41), we obtain:

$$y - \lambda S_F(p) + \lambda R(p) - \lambda (y + S_F(p) - R(p)) = 0 \qquad (43)$$

and

$$S_F = \frac{y(1 - \lambda)}{2\lambda} + R(p)$$

So, for $\lambda < 1$, which seems a natural assumption, all firms will be subsidized: the impact on output and the reduction in the cost of subsidizing bank loans are sufficiently strong to yield this result.

Condition $e < 1$ implies, using (32) that $1 > \beta p (y + S_F(p) - R(p))$.

If instead, e reaches the corner solution, $e = 1$, and $\beta p (y + S_F(p) - R(p)) > 1$. This implies $\gamma(p) = 0$, and consequently $S_F(p) = 0$. Consequently, the firms that will receive a subsidy will be those for which $y - R(p) < \frac{1}{\beta p}$, which is the equivalent of $y - R(p) < \frac{B}{\Delta p I}$ in our modeling approach.

Because the moral hazard problem has changed, the firms to which the subsidy will be granted has also changed. While, in the presence of the private benefits switch to private benefits the subsidy was to those firms that had insufficient rents to provide the right incentives (but were close enough), now the subsidy will go to any firm with an effort level lower than $e = 1$.