Resource Curse or Blessing? Sovereign Risk in Resource-Rich Emerging Economies*

Franz Hamann†
Enrique G. Mendoza‡
Paulina Restrepo-Echavarria§

July 10, 2018

Abstract

In this paper we document the stylized facts about the relationship between international oil price swings, sovereign risk and macroeconomic performance of oil-exporting economies. We show that even though being a bigger oil producer decreases sovereign risk–because it increases a country's ability to repay–having more oil reserves increases sovereign risk by making autarky more attractive. We develop a small open economy model of sovereign risk with incomplete international financial markets, in which optimal oil extraction and sovereign default interact. We use the model to understand the mechanisms behind the empirical facts, and show that it supports them.

*The views expressed in this document are those of the authors and not those of the Banco de la República and/or the Federal Reserve Bank of St Louis or The Federal Reserve System. We would like to thank Maria Arias, Sergio Arango, Leonardo Barreto and Brian Reinbold for their excellent research assistance. And give special thanks to Juan Camilo Mendez for his research assistance which goes beyond the line of duty.

†Banco de La Republica
‡University of Pennsylvania, and NBER
§Federal Reserve Bank of St. Louis

1 Introduction

International oil price volatility has affected the macroeconomic performance of oil exporting countries through changes in incentives. More specifically, changes in oil prices affect a country's desire to exploit their natural resource–a real asset–affecting reserves and extraction of oil. Changes in oil prices also affect a country's desire to consume and borrow or lend in financial assets, and finally they also affect incentives to repay or default, which at the same time affect sovereign risk and asset prices.

In this paper we document the stylized facts about the relationship between international oil price swings, sovereign risk and macroeconomic performance of oil-exporting economies. We show that even though being a bigger oil producer decreases sovereign risk–because it increases a country's ability to repay–having more oil reserves increases sovereign risk by making autarky more attractive. As we document in the next section using a dynamic fixed effects regression, the short-run elasticity of country risk with respect to changes in oil production is 0.04% and the long-run elasticity with respect to oil reserves is -0.14%. That is, when oil production increases by 1% country risk decreases by 0.04%, and when oil reserves increase by 1%, country risk increases by 0.14%.

We then develop a small open economy model of sovereign risk with incomplete international financial markets, in which optimal oil extraction and sovereign default interact. We use the model to understand the mechanisms behind the empirical facts.

Examining data for the 30 largest emerging market oil exporters over the period 1979-2016, we found that these countries hold an average external public debt[1] to GDP ratio of around 30% and sixteen countries in the sample have experienced between one and five default episodes. We highlight three features of the relationship between country risk and the size of the oil sector: First, as is natural to expect, a given oil exporting country is perceived by investors as less risky, the higher their oil production and the higher oil prices, allowing its public sector to support higher levels of public debt. Second, and perhaps less natural to expect, in the long run, country risk perception increases the higher the level of oil reserves of the country. This may reflect the fact that having a large stock of oil increases a country's outside option (the value of autarky), making default more appealing. Third, the data also shows that during default episodes, the median oil exporting country increases net oil exports. This evidence suggests that a country in default and excluded from international financial markets, increases its oil exports to withstand the consequences of financial autarky.

When we explore the relationship between oil price changes and macro performance, we find that increases in oil prices are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price decreases are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk perception and a higher number of default events.

We build a small open economy model with two types of goods: a tradable and non-storable

[1] We use external public debt data from the World Bank where public and publicly guaranteed debt comprises long-term external obligations of public debtors, including the national government, political subdivisions (or an agency of either), and autonomous public bodies, and external obligations of private debtors that are guaranteed for repayment by a public entity. Data are in current U.S. dollars.

consumption good and oil. The sovereign government owns and delegates oil extraction to an oil producing company—which transfers its profits back at the end of the period—and can trade non-state contingent bonds with risk neutral competitive foreign lenders in international financial markets but cannot commit to repaying its debt. The relative price of oil and the consumption good are exogenously given.

We find that theory predicts that long-run reserves of the resource have two opposing effects in determining the long-term sovereign risk premium. Higher stock of reserves allow the country to have a higher extraction rate to support debt repayments, lowering default risk. However, they also allow the country to use the resource during default times, making the value of default more attractive. The tension between these two forces determines the long run default risk premium.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the calibration, solution method, and quantitative results and Section 5 concludes.

2 Empirical Evidence

2.1 Data

We have collected data for oil GDP, non-oil GDP, oil reserves, oil consumption, oil net exports, total public debt, total external public debt, net foreign assets, default episodes and country risk, for Saudi Arabia, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam. The common feature among these thirty countries is that they are the group of emerging market countries with the largest oil exports.

As an indicator of country risk we use the Institutional Investor Index (III from now on). The III country credit rating, is a measure of sovereign debt risk that is published biannually in the March and September issues of the Institutional Investor magazine. It is also commonly known as the Country Credit Survey. More specifically, the III is an indicator used to identify and measure country risk, where country risk refers to a collection of risks related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. We have biannual data for the 1979-2010 period. The index is based on information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents have graded each country on a scale of zero to 100, with 100 representing the least likelihood of default. Respondents responses are weighted according to their institutions' global exposure.

In this literature country risk is usually measured using country spreads, and what is most commonly used is the Emerging Markets Bond Index (EMBI). The problem with using the EMBI is that it is only available since 1994 and only for a certain number of countries. As a result it doesn't provide us with enough observations to carry out the empirical analysis that we desire. For this reason we use the III and in the EMBI for those years in which both are available. Also, in Appendix A we show

how the III relates to other different measures of country risk.

We use data on oil reserves, oil production, oil consumption and oil net exports (in thousands of barrels per day) for 219 countries, as well as Brent spot prices (in USD per barrel), from 1980 to 2013 reported by the US Energy Information Administration (EIA).

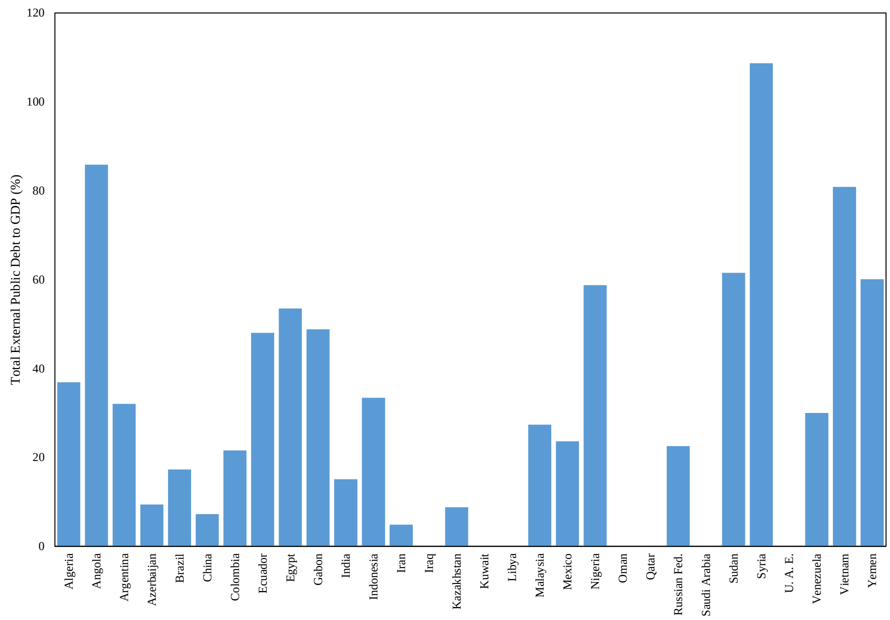
Total public debt data comes from the World Development Indicators tables (WDI) and the World Economic Outlook database (WEO). We have information, covering 1979-2010 period, for direct government fixed-term contractual obligations to others outstanding on a particular date. It includes domestic and foreign liabilities such as currency and money deposits, securities other than shares, and loans. In this case, information is not reported for Iraq and Libya. Total public external debt data is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Canada, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, and Oman. In the case of Russia, we use data reported by the Ministry of Finance.[2]

We use the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane and Milesi-Ferreti (2007) to obtain information on net foreign asset positions. It contains data for the 1970-2011 period and for 188 countries (including those in our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i) the accumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between accumulated flows of "debt assets", and the stock of debt measured by the World Bank (or the OECD).

Default data is from Borensztein and Panizza (2006), for the 1980-2010 period. We include sovereign defaults on foreign currency bond debt and foreign currency bank debt. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary.

[2]Russian total public external debt data is available for a monthly frequency. Since GDP information is reported only at the end of each year, we use debt information in December, in order to construct public external debt-to-GDP ratios. Russian data is only available since 1990.

Figure 1: Average External Public Debt of Net Oil Exporters (1979-2010)



2.2 Stylized Facts

We start this section by illustrating that countries who are net oil exporters, do hold external public debt and default on it. Figure 1 shows the average external public debt as a percentage of GDP for twenty three countries in our sample. Those countries that show up as having no external public debt is because there is no data available. This subset of countries holds a minimum average external public debt to GDP ratio of around 5% (Iran) and a maximum of around 110% (Syria). On average, these countries have a mean external public debt to GDP ratio of roughly 25% which is not negligible.

Figure 2 shows the number of default episodes for the same set of countries. In this case a value of zero means that it was a country that did not default on its debt during this period. We have this data for all of our sample. The number of default episodes ranges between zero and 5. The fact that these countries do hold external public debt and default on it shows that it is relevant to better understand the connection between oil prices, macroeconomic performance and sovereign default.

We now turn our attention to the relationship between sovereign risk—measured by the III—and different variables. Specifically, Table 1 shows the unconditional correlation between the III and oil reserves, oil prices, external public debt and total public debt for all the countries in our sample. We can see that as expected, both external and total public debt are negatively correlated with sovereign risk and this relationship is statistically significant in nearly all of the cases. On the other hand, oil prices are positively correlated with the III, meaning that higher oil prices are associated with lower sovereign risk. This is natural, as higher oil prices imply a greater ability to repay. Finally, we can see that when we don't condition on anything, the sign of the correlation between oil reserves and sovereign risk is mixed. For some countries it is positive and for some it is negative.

Figure 2: Net Oil Exporters Number of Default Episodes (1979-2010)

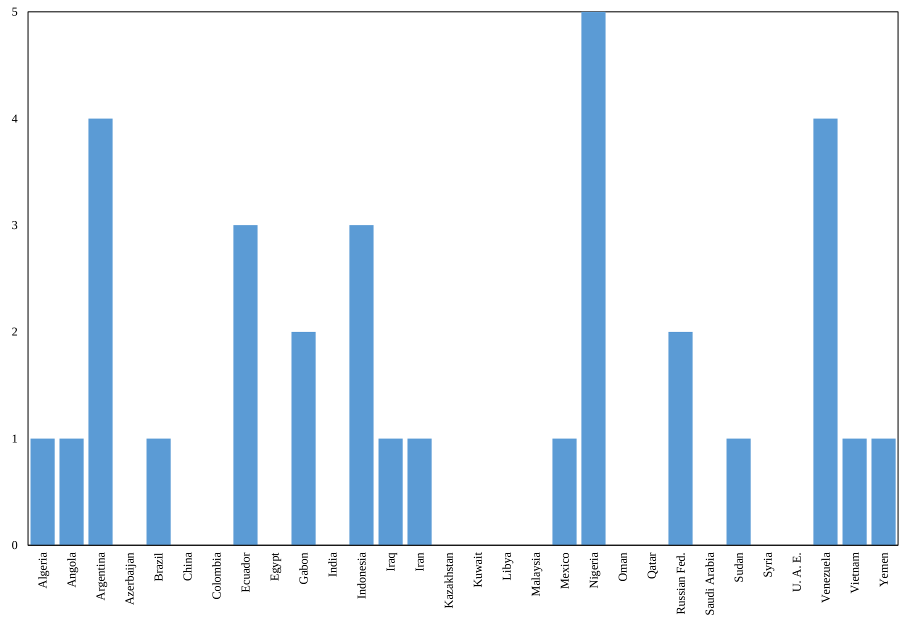


Table 1 contains contemporaneous correlations reflecting short-term relationships, now, we take a look at long-run unconditional correlations. To do so, we run a between effects regression of the III on oil production and on oil reserves to production.[3] The between effects regression is just an OLS regression on the cross-sectional equation over the average value of each variable for a given country. Figure 3 shows the in between effects regression results for the III on oil production. Again, each point in the plot represents the average values over time for a given country. The results show that in the long-run countries that have maintained high oil production have been characterized by a higher average credit rating: the correlation is 0.28. As mentioned earlier, this is expected as higher oil production implies a greater ability to repay debt.

On the other hand Figure 4 shows the in between effects regression for the III on oil reserves to production. In this case the long-run unconditional correlation is -0.02. This result points to the fact that having oil might have two opposing effects on country risk. High oil production increases a country's ability to repay and lowers country risk, but possessing a large stock of oil increases the value of autarky triggering a "Limited Commitment" mechanism: a country can be excluded from financial markets but it can still sell their real asset—oil in this case—in international markets, making autarky more attractive.

[3]Note that reserves to production (oil extraction) represents the number of years that it would take a country to deplete its reserves assuming that there are no new discoveries.

Table 1: Unconditional Correlations

Country	Oil Reserves	Real Oil Prices	Total External Public Debt to GDP	Total Public Debt to GDP
Algeria	0.4098**	0.7744***	-0.7498***	-0.7413***
	(0.1588)	(0.0767)	(0.1135)	(0.1225)
Angola	0.8044***	0.8597***	-0.7438***	-0.7002***
	(0.1034)	(0.0619)	(0.1364)	(0.1908)
Argentina	-0.0059	0.3587***	-0.6159***	-0.6353***
	(0.1741)	(0.1132)	(0.1351)	(0.1434)
Azerbaijan	0.5799**	0.9004***	-0.8409***	-0.8994***
	(0.2456)	(0.0907)	(0.1632)	(0.1652)
Brazil	0.8174***	0.8856***	-0.7383***	-0.2157
	(0.1003)	(0.0563)	(0.1157)	(0.1783)
China	-0.5770***	0.8812***	-0.8191***	0.5051***
	(0.1422)	(0.0573)	(0.1014)	(0.1726)
Colombia	-0.1126	0.8565***	-0.8025***	-0.5751***
	(0.173)	(0.0626)	(0.1023)	(0.1519)
Ecuador	0.0083	0.5082***	-0.5903***	-0.6466***
	(0.1741)	(0.1044)	(0.1384)	(0.1393)
Egypt	-0.4703***	0.3351***	-0.6809***	-0.5912***
	(0.1536)	(0.1143)	(0.1256)	(0.1498)
Gabon	-0.3720**	0.6194***	-0.7295***	-0.6806***
	(0.1616)	(0.0952)	(0.1173)	(0.1338)
India	0.0708	0.8168***	-0.8436***	0.1667
	(0.1736)	(0.07)	(0.0921)	(0.1934)
Indonesia	0.2101	0.4709***	-0.5926***	-0.9226***
	(0.1702)	(0.107)	(0.1381)	(0.0704)
Iraq	-0.5215***	0.5946***		
	(0.1508)	(0.0975)		
Iran	0.7418***	0.0991	0.1229	-0.5640***
	(0.1186)	(0.1207)	(0.1782)	(0.1533)
Kazakhstan	0.8081***	0.8809***	-0.5395***	-0.7951***
	(0.1473)	(0.073)	(0.1837)	(0.1516)
Kuwait	0.0469	0.8257***		-0.9062***
	(0.1766)	(0.0684)		(0.113)
Libya	0.4660***	0.7069***		
	(0.154)	(0.0858)		
Malaysia	0.2187	0.7522***	-0.4409***	-0.3123*
	(0.1699)	(0.0799)	(0.1539)	(0.1764)
Mexico	-0.8381***	0.7638***	-0.7459***	-0.7085***
	(0.095)	(0.0783)	(0.1142)	(0.1311)
Nigeria	0.4223**	0.8194***	-0.7410***	-0.6734***
	(0.1578)	(0.0695)	(0.1152)	(0.135)
Oman	0.7066***	0.6547***		-0.4931***
	(0.1232)	(0.0917)		(0.1588)
Qatar	0.9091***	0.8442***		-0.4797**
	(0.0725)	(0.065)		(0.2013)
Russian Federation	0.7410***	0.6909***	-0.7539***	-0.8704***
	(0.1679)	(0.0877)	(0.1434)	(0.1194)
Saudi Arabia	-0.3566**	0.8461***		-0.9083***
	(0.1652)	(0.0646)		(0.1118)
Syria	0.4064**	0.3173***	0.8676**	-0.7369***
	(0.1591)	(0.115)	(0.2224)	(0.1234)
Sudan	0.5003***	0.6235***	-0.4986***	-0.8144***
	(0.1555)	(0.0948)	(0.1487)	(0.1407)
United Arab Emirates	0.3769**	0.7213***		0.3344*
	(0.1637)	(0.084)		(0.1721)
Venezuela	-0.2995*	0.3915***	-0.4065**	-0.7707***
	(0.1687)	(0.1116)	(0.159)	(0.1301)
Vietnam	0.3896*	0.8493***	-0.8134***	-0.7283***
	(0.201)	(0.0805)	(0.1269)	(0.1713)
Yemen	0.4929	-0.4010*	0.4628	-0.5306
	(0.2751)	(0.2048)	(0.2803)	(0.346)

Standard errors in parenthesis
*** p<0.01, ** p<0.05, * p<0.1

Figure 3: In between effects regression of the Institutional Investor Index (X-Axis) on average oil production (Y-Axis): 1979-2010.

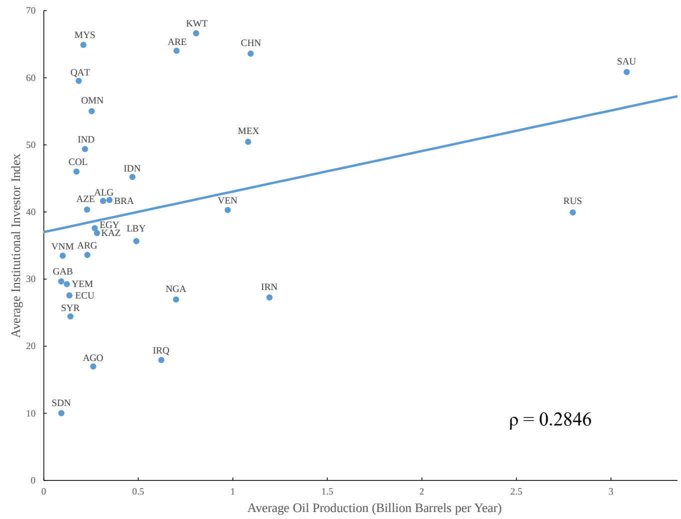
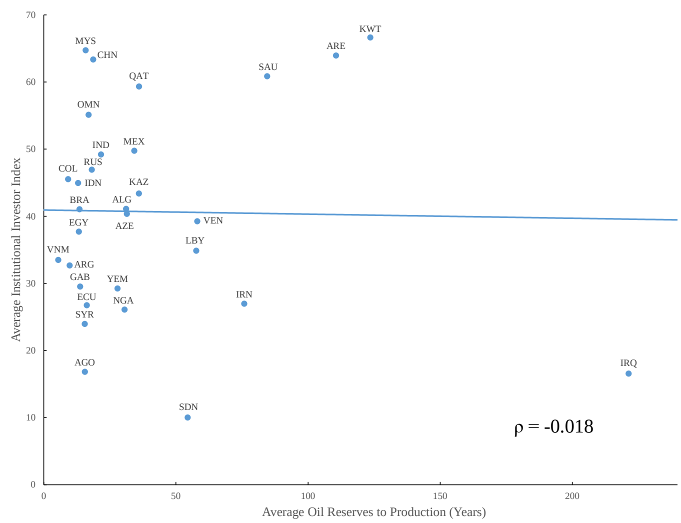


Figure 4: In between effects regression of the Institutional Investor Index (X-Axis) on average oil production (Y-Axis): 1979-2010.



Notice that our analysis has two dimensions. We want to study the short versus long-run effects of both production and reserves. To summarize, so far, these unconditional correlations point towards two mechanisms. First, extracting more oil (production) increases a country's ability to repay its debt, decreasing country risk. Second, owning a larger stock of oil (reserves) seems to be positively correlated with country risk, and this goes in line with the idea that if a country has a larger stock of a real asset, then autarky becomes a more attractive option.

In order to study more formally the presence of these two mechanisms in the data, and establish conditional correlations we run a dynamic fixed effects estimation[4] of long-run, short-run and convergence coefficients. These allows us to put all the previous results together and be able to establish statistical significance of the relevant variables and timing. The results are shown in Table 2.[5]

Note that in each model the convergence coefficient has the expected sign and is statistically significant at the 1% level. Since the estimated coefficients take a value between -0.2 and -0.3, convergence in the III runs at an annual rate of about 0.25%, which means that each year the III covers about 0.25% of its distance to the "steady state". It should also be noted that convergence is slightly slower in Model (2), where the net foreign assets-to-GDP ratio is included and default is excluded.

If we focus on the short-run coefficients, we observe that an increase in oil GDP decreases country risk, and this result is statistically significant at a 5% level. The same for an increase in non-oil GDP, but this result is significant to a 1% level. In the short run, a positive change in oil reserves (which can happen if extraction is lower than discoveries of oil in a given period), decreases country risk. In the first model this result is significant to a 10% level and in the second and third models to a 5%. As is expected, increases in external public debt increase country risk, and this result is statistically significant to a 1% level in the second model and to a 5% in the first and third model. Finally, a positive *change* in net-foreign assets increases country risk.

When looking at the long-run coefficients, as shown in Pesaran et al. (1999), the usual interpretation—when series are in logs—is that of an elasticity. Then, the long-run oil GDP elasticity is 0.09 in the first model, 0.12 in the second, and 0.09 in the third, which means that when oil GDP increases by 1%, the III is between 0.09% and 0.12% higher in the long-run. With respect to non-oil GDP, long-run elasticities are positive, and country risk rating increases around 0.12% because of a 1% increment in non-oil GDP.

Moreover, a significant negative relationship between oil reserves and the III was found. A rise in oil reserves worsens our measure of country risk in the long term. Thus, an oil exporting economy is perceived as more risky in the future when it boosts its reserves today. This elasticity is statistically different from zero at a 5% level for the second model, at a 1% level for the first model and at a 1% level for the third model where we control for net-foreign assets, default, and oil discoveries.

As expected, in the long-run, external public debt still has a negative effect on country risk and

[4] See Appendix C to see the different methods we used for the estimation and the Hausman test that determined that dynamic fixed effects was the dominant approach.

[5] Due to data limitations, in Model (1) Yemen was excluded from the sample. In Model (2) Iraq, Libya, Yemen and Azerbaijan were excluded, and in Model (3) Iraq, Russian Federation, Yemen and Azerbaijan. Consequently, the estimation is performed taking into account 887, 785 and 828 observations, respectively.

Table 2: Dynamic Fixed Effects Regression Results for Institutional Investor Index

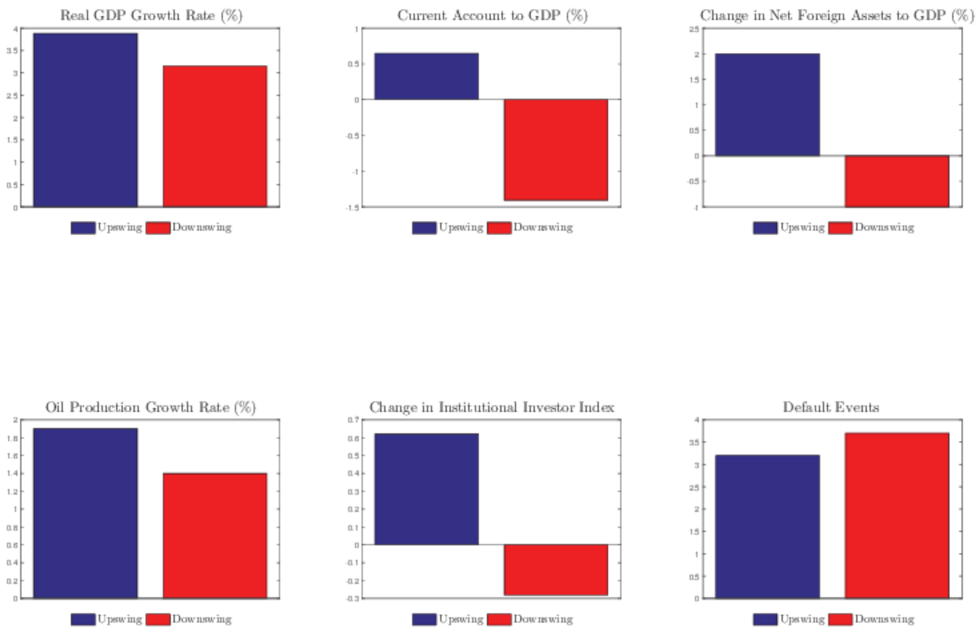
	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence coefficient			
Inst. Investor Index (-1)	-0.259***	-0.230***	-0.262***
	(0.023)	(0.023)	(0.023)
Short-run coefficients			
Δ Oil GDP	0.035**	0.042**	0.039**
	(0.016)	(0.017)	(0.016)
Δ Non-Oil GDP	0.386***	0.411***	0.376***
	(0.074)	(0.076)	(0.074)
Δ Oil Reserves	0.048*	0.053**	0.055**
	(0.026)	(0.027)	(0.026)
Δ Ext. pub. debt to GDP	-0.103**	-0.177***	-0.157**
	(0.043)	(0.066)	(0.064)
Δ Oil Discoveries	-0.018	-0.013	-0.018
	(0.019)	(0.020)	(0.019)
Δ NFA		-0.083**	-0.077*
		(0.042)	(0.041)
Long-run coefficients			
Oil GDP	0.085**	0.115***	0.087**
	(0.037)	(0.042)	(0.036)
Non-oil GDP	0.117**	0.163***	0.119**
	(0.052)	(0.061)	(0.051)
Oil Reserves	-0.130***	-0.110**	-0.144***
	(0.044)	(0.051)	(0.044)
Ext. pub. debt to GDP	-0.596***	-0.615***	-0.527***
	(0.093)	(0.172)	(0.148)
Default	-0.277***		-0.278***
	(0.051)		(0.050)
Oil Discoveries	0.172*	0.184	0.177*
	(0.099)	(0.115)	(0.098)
NFA		0.165	0.110
		(0.120)	(0.101)
Constant	0.463***	0.200	0.468***
	(0.180)	(0.177)	(0.179)
Observations	512	509	509

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

is again statistically significant to a 1% level for the three models. Different from the short-run, in the long run the *level* of net-foreign assets does matter for country risk. Finally, as expected, being

Figure 5: Oil Price Swings and Macro Performance



in default increases country risk. When the probability of default increases by 1%, the III drops by 0.28%. This last result is statistically significant to a 1% level.

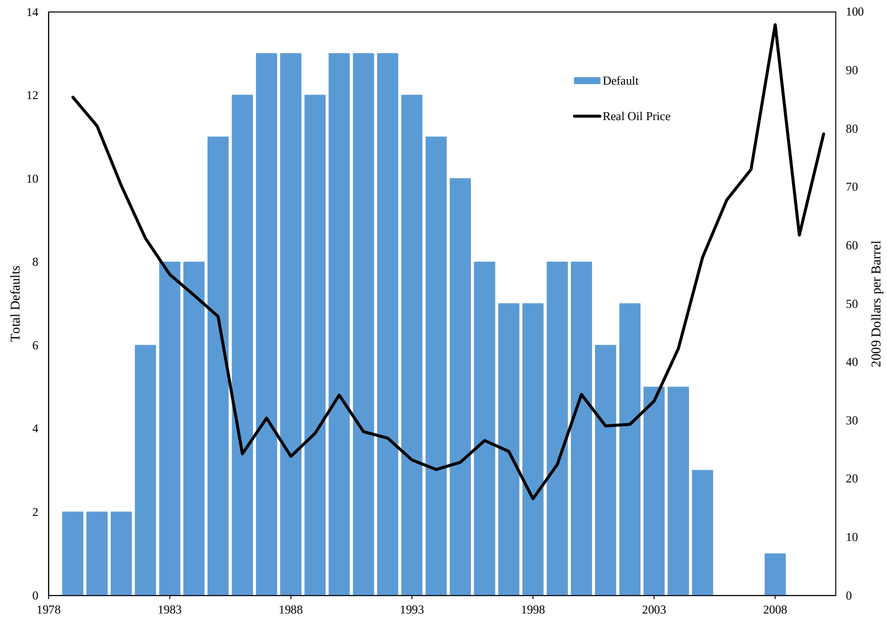
The effect of oil discoveries is only significant to a 10% in the first and third model, where an increase of 1% in oil discoveries, decreases country risk by about 0.17%.

These results support the two mechanisms that we believe are behind the unconditional correlations presented above. Oil production decreases country risk by increasing a country's ability to repay, but greater oil reserves increase country risk by making autarky more attractive.

We now look at the relationship between oil price swings and macro performance. To construct Figure 5, we classified all the years in our sample as corresponding to an upswing (a positive change in prices) or downswing (a negative change in prices), and then constructed interval periods for the two. Table 16 in Appendix D shows how the intervals were constructed. We then averaged the different macroeconomic variables over the upswings and downswings and Figure 5 shows the results.

We can see that oil price upswings are associated with higher oil extraction and higher GDP growth rates, trade balance and current account improvement, lower sovereign risk perception and lower number of default events. Likewise, oil price downswings are associated with lower oil extraction and lower GDP growth rates, trade balance and current account deterioration, higher sovereign risk

Figure 6: Oil Prices and Default Episodes



perception and higher number of default events.

To expand on the relationship between the price of oil and default episodes, in Figure 6 we plot the total number of default episodes observed in the data by year (left axis) and the evolution of the real oil price over the same years (right axis). We can see that as the price of oil drops the number of default episodes increases. It is particularly interesting to see how between 2005 and 2007 there are no defaults—the price is high—and as soon as the price drops in 2008 a default is triggered. This supports the fact that for these countries, oil increases their ability to repay, and hence lower oil prices are related to more default episodes.

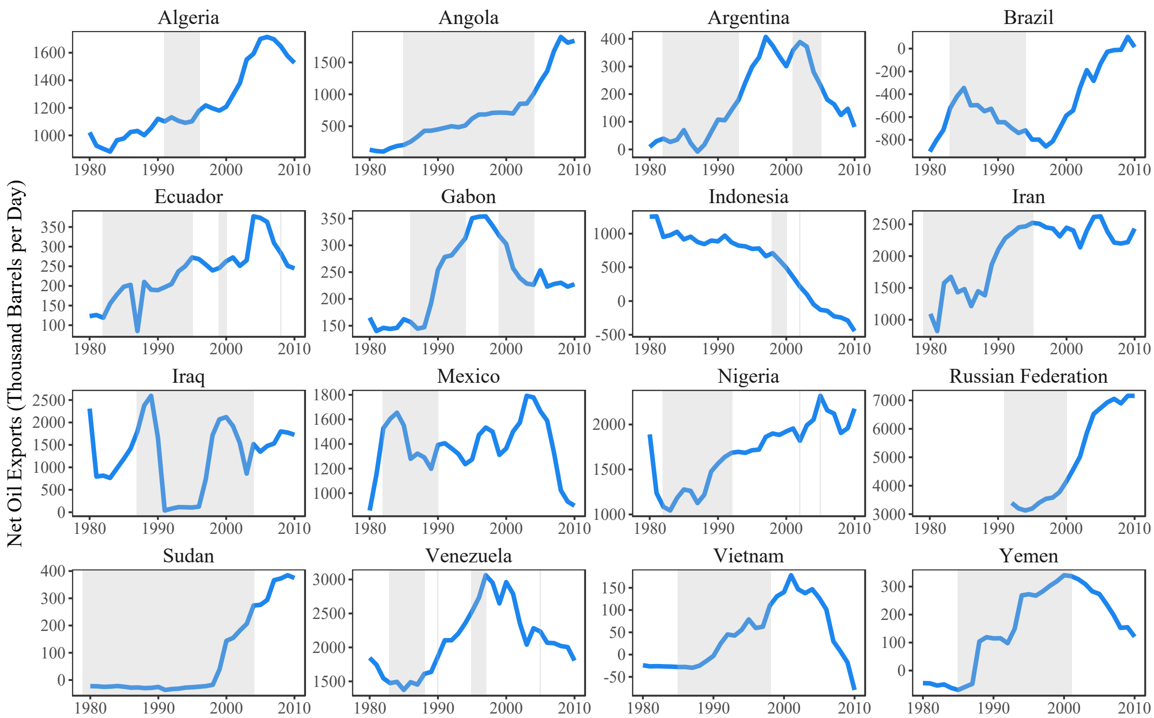
Just as lower oil prices are related to more default episodes, we are also interested in seeing what happens with oil exports during default episodes. Figure 7 shows this for those countries in our sample that have defaulted. The grey shaded areas represent the default episodes. As we can see in general during default episodes, countries increase the amount of oil exported, in order to compensate for the lower oil price. Summarizing, lower prices are related to more default episodes, and during those default episodes usually countries increase oil exports to compensate for the lower price.

3 A Model of Sovereign Default and Oil Extraction

We present an off-the-shelf general equilibrium small open economy model with sovereign default and modify it so that the sovereign receives an important part of its income from the oil industry, and the oil industry's extraction decisions are affected by country risk.

There are two agents in the model and two types of goods: a company that produces oil and

Figure 7: Oil Exports During Default Episodes



Note: Shaded area means the country is in default

the sovereign government who receives a tradable non-storable consumption good. We assume that the price of oil, p, is determined by international markets and is taken as given by the oil company, and the tradable non-storable consumption good received by the government, y, is also exogenously determined. At the beginning of the period both $p$and $y$are realized, and initial oil reserves, s, and sovereign debt standing, b, are given. At that point the sovereign government decides whether to default on its debt or repay. If the sovereign decides to default, there is no bond market that period and the oil company decides how much oil to extract, x, and sells it in the international oil market facing a trade penalty because of the default of the sovereign. Then consumption is determined.

If the sovereign government decides to repay, the bond market opens, the oil company takes as given the price of bonds, q, and decides how much oil to extract and oil reserves for tomorrow. Simultaneously the government chooses its debt level for tomorrow and consumption is determined.

In the next subsections we formalize this problem.

The problem of the oil producing company

Given today's realization of the price of oil, p, the oil company decides how much oil to extract optimally, x, from the available stock of proven oil reserves, s, by maximizing the present discounted value of their payoffs:

$$w(s,p)=\max_{\{x,s'\}}\left\{f(x,s,p)+q\left(B',S,y,p\right)E\left[W\left(s',p'\right)\right]\right\}$$

subject to a transition function for oil reserves and oil prices respectively,

$$s'=g(x,s,p),\ p'=z_p(p), \tag{1}$$

and a feasibility constraint,

$$0\leq x\leq s+d. \tag{2}$$

The payoff function is given by $f(x,s,p)=px-e\left(x,s\right)$, where px are the revenues generated by selling the oil at the international price and $e\left(x,s\right)$ are the extraction costs that the firm faces for extracting x units of oil. The transition function $g(x,s,p)=s-x+d$, states that tomorrow's oil reserves are equal to today's stock, minus what is extracted plus d, which stands for oil discoveries, and are assumed to be constant. Finally, the feasibility constraint (2) guarantees that extraction is less than today's reserves plus the discoveries, and that once oil has been extracted from the ground it can't be returned.

The extraction cost function $(e\left(x,s\right))$ is expressed in units of the consumption good and has the following properties: $e_s<0$, $e_x>0$ and $e_s\left(0,s\right)=0$. Total extraction costs fall with oil reserves but increase with the extraction rate. Also, the marginal cost of an additional unit of reserves, conditional on not extracting oil is zero. We assume that $e(x,s)$ is given by:

$$e(x,s)=\psi\left(\frac{x}{s}\right)^{\gamma}x. \tag{3}$$

The oil company discounts the future at a rate $q\left(B',S,y,p\right)$, where B' is tomorrow's optimal debt level chosen by the sovereign, and is taken as given by the firm. As will be clear from the next Subsection, the optimal level of debt chosen by the government will depend on the optimal extraction policy of the firm, but the firm doesn't endogenize that effect, as such, we denote it by a capital letter in the problem of the firm. Likewise, note that $q$is a function of (capital) S , signaling that the firm does not endogenize the effect that their choice of reserves has on their discount factor.

The discount factor, $q\left(B',S,y,p\right)$, corresponds to the equilibrium price of the risky bond issued by the government, and as such depends on the probability of default of the sovereign such that extraction decisions are affected by the spread of the country. This assumption implies, that if the oil company were to acquire debt from foreign lenders they would face a risk premia like the one the sovereign government faces. In Appendix D, we show that this is observed in the data. Credit ratings of oil companies are highly correlated with the credit ratings of its country of origin.

The optimality conditions of the firm imply the following Euler equation

$$q\left(B',S,y,p\right)E\left[p'-e_x\left(x',s'\right)-e_s\left(x',s'\right)\right]=p-e_x\left(x,s\right). \tag{4}$$

Note that Equation 4 states that optimal extraction, $x^*(p, s; B')$, is such that the marginal benefit of extracting a unit of oil today (right hand side of the equation) has to be equal to the discounted expected benefit of extracting a unit of oil tomorrow (left hand side of the equation). Whenever q is different from zero, the firm's decision is inter-temporal, and the tradeoff between extracting today or tomorrow is affected by the discount factor q, which is determined in equilibrium by the probability of default of the sovereign government. More on this to follow.

Finally, at the end of each period the oil company transfers its profits $\pi^*(s, p; B')$ to the sovereign government.

The problem of the sovereign

The sovereign is a benevolent government who maximizes the utility of private agents. He receives a tradable non-storable consumption good, y, and the profits generated by the oil company, $\pi^*(s, p; b')$. He issues all possible debt abroad by issuing one-period, non-state contingent discount bonds, b', to be repaid tomorrow but he cannot commit to repaying its debt. The set of bond face values is $B = [b_{min}, b_{max}] \subset R$ where $b_{min} \leq b_{max} = 0$. As in the Eaton-Gersovitz model, when the country defaults it does not repay in the current period and the punishment is exclusion from world credit markets in the same period. Next period the country can re-enter world financial markets with a probability λ. Consequently, the planner's payoff is given by:

$$V(b, S, y, p) = \max\left\{v^{nd}(b, S, y, p), v^d(S, y, p)\right\},$$

where $v^{nd}(b, S, y, p)$ is the value of no-default and $v^d(S, y, p)$ is the value of default.

The value of no-default solves the following constrained maximization problem:

$$v^{nd}(b, S, y, p) = \max_{\{c, b'\}} \left\{u(c) + \beta E\left[V\left(b', S'^*(s, p; b'), y', p'\right)\right]\right\} \tag{5}$$

subject to the budget constraint

$$c - b + A = y + px^*(s, p; b') - q(b', S, y, p)\, b', \tag{6}$$

where $x^*(s, p; b')$ and $S'^*(s, p; b')$ correspond to the optimal oil extraction and tomorrow's optimal stock of oil reserves chosen by the oil company.

$q(b', S, y, p)$ is the pricing function for the risky bond. Recall that in the previous subsection, the discount factor of the firm was denoted by $q(B', S, y, p)$ to emphasize that the oil company doesn't internalize the fact that tomorrow's optimal debt level—which determines the price of the risky bond—will be affected by their optimal extraction choice, but make sure to note that $q(B', S, y, p) = q(b', S, y, p)$. Also note that the stock of oil inside the pricing function is a capital S, meaning that the sovereign takes that level of reserves as given, however in equilibrium $S = s$.

We assume that the oil company pays the extraction cost, $e(x^*(s, p; b'), s)$, to the government so it doesn't show up in the budget constraint. Finally, A, adjusts absorption given that the model doesn't have investment or government spending.

If the government chooses to default, then he solves the following constrained optimization problem:

$$v^d(S,y,p) = \max_{\{c\}} \left\{u(c) + \beta (1-\lambda) Ev^d \left(S'^d, y', p'\right) + \beta\lambda EV \left(0, S'(p,s;0), y', p'\right)\right\} \tag{7}$$

subject to the budget constraint

$$c + A = y + \theta p x^d (s,p). \tag{8}$$

We assume that if the sovereign defaults on its debt today, they will be seized a fraction $(1-\theta)$ of the oil revenues.[6] Furthermore, just as in the case of no default they receive $px^*(s,p;b')$ from the oil company, we denote by $px^d(s,p)$ what they receive in the case of default, $x^d(s,p) = x^*(s,p;0)$. This comes from the fact that whenever the sovereign defaults, there is no bond market and $q(b',S,y,p) = 0$. In this case, the oil company's optimality condition (Equation 4) reduces to $p - e_x(x,s) = 0$, the problem of the oil company becomes intra-temporal, and the optimal extraction policy is given by

$$x^d(s,p) = \left[\frac{p}{\varphi(1+\gamma)}\right]^{\frac{1}{\gamma}} s. \tag{9}$$

In other words, the extraction policy under default is constant for all pairs, $\{s,p\}$. Intuitively what is happening is that when there is a default the price of debt goes to zero and its yield goes to infinity. Then arbitrage requires that the return on oil production goes to infinity as well, and this can be accomplished by setting current extraction so that the marginal extraction cost equals the current price (see Equation 4).

Equation 7 states that the value of default is equal to todays consumption utility plus the weighted discounted expected continuation values. The weight on the continuation values is given by λ, the probability with which the sovereign can re-enter financial markets. If they do re-enter international financial markets, they do so with a debt $b' = 0$. On the other hand, with probability $(1-\lambda)$ the sovereign will remain in the default state.

The definition of the default set and the probability of default are as in the Eaton-Gersovitz model. For a debt position $b < 0$, default is optimal for the set of realizations of $\{y,p\}$ for which $v^d(S,y,p)$ is at least as high as $v^{nd}(b,S,y,p)$:

$$D(b,S) = \left\{\{y,p\} : v^{nd}(b,S,y,p) \le v^d(S,y,p)\right\}. \tag{10}$$

The probability of default tomorrow perceived as of today, $d(b', S'^*(b';p,s), y, p)$, can be calculated from the default set and the transition probability functions of the endowment $z_y(y' \mid y)$ and oil prices $z_p(p' \mid p)$ as follows:

$$d\left(b', S'^*(b';p,s), y, p\right) = \int\int_{D(b',S)} dz_y(y' \mid y)\, dz_p(p' \mid p). \tag{11}$$

[6] This is essentially a trade penalty as observed in the data and will be discussed further in the calibration section.

Hence, the risky bond price is determined by the following non-arbitrage condition

$$q\left(b';S,y,p\right)=\bar{q}\left(1-d\left(b';S,y,p\right)\right),$$

where $\bar{q}$ is the risk free rate.

It is important to note that the sovereign knows how the optimal policies of the oil company respond to changes in debt through its effect on q, and he takes into account these effects when choosing tomorrow's debt. This can be seen through Equations 5 and 6, where the sovereign evaluates how q affects x^* and s'^* and takes it into account when choosing b'. In this sense the setup is like a Stackelberg leader where the government is the leader and chooses optimal plans internalizing the optimal plans of the oil company.

3.1 Two period example

In this Subsection we study a two period version of the model described above to illustrate the implications of the model.

We assume that there is a fixed amount of oil reserves, s_0, at the beginning of the first period and there are no discoveries. The price in the first period is known and the only uncertainty comes from the price in the second period.

Today the sovereign chooses a level of debt for the next period, and in the second period after observing the oil company's extraction decision for that period—and hence the oil price—it decides whether to default or not.

The equilibrium bond price depends on the probability of default of the sovereign, which at the same time depends on the realization of the oil price in the second period. This is because in the first period, the optimal extraction of the oil company depends on the price today relative to the expectation on tomorrow's price. If the price in the first period is high, the oil company will prefer to extract more today, as expectations on tomorrow's price being lower than today's are high. As such, today's optimal extraction policy is increasing in p_1 and decreasing in p_2. Likewise, extraction in the second period is increasing in p_2 and decreasing in p_1. Figure 8 illustrates the optimal extraction policies for a given q.

This means that for a given level of debt, there is a $\tilde{p_2}$ threshold, under which the sovereign will default for sure because it implies low oil revenue in the second period. This threshold depends on today's price relative to the expectation on tomorrow's price. The lower the price today, the more likely it is that tomorrow's price will be higher that today, so oil extraction is differed to tomorrow and the probability of default goes down.

Also, today's extraction, x^*, is decreasing in the sovereign bond price (see Equation 18 in Appendix G). This is intuitive because a higher q implies that the yield of the oil company drops and the opportunity cost of extracting oil falls, giving the oil producer the incentive to adjust extraction plans to reduce the return on "investing" in oil production (i.e. investing in oil reserves) to match the lower return on bonds. This implies extracting less today, because extracting less today reduces the marginal cost of extraction (for a given p), and hence increases the net marginal revenue on oil extraction today (that is $p - e_x(x,s)$ rises), reducing the rate of return on oil reserves. This is all working through Equation 4.

Figure 8: Optimal extraction policies

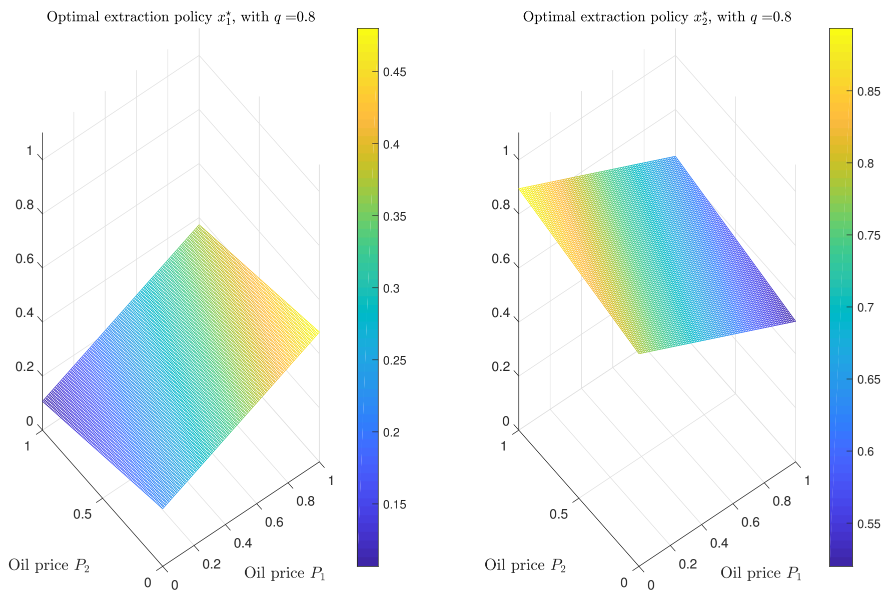
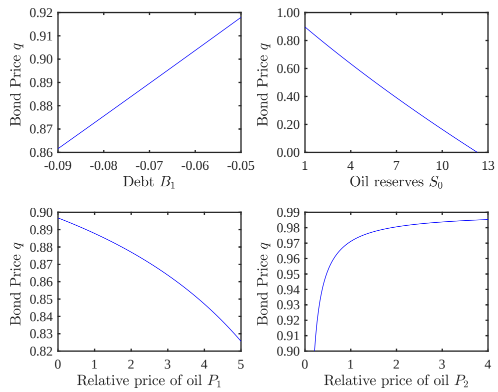


Figure 9 shows the comparative statics for debt, reserves, and oil prices. As is expected, higher levels of debt imply lower $q's$, or higher risk premia (first subplot in Figure 9).

As was previously explained, a higher price today would imply that the threshold price tomorrow that is needed to repay is higher, so a higher price today implies a lower q(left bottom subplot in Figure 9), and a higher price tomorrow implies a higher ability to repay and hence lower risk premia, higher q(right subplot in the bottom of Figure 9).

But more surprising is the relationship between oil reserves and sovereign risk. As can be seen from the right upper subplot in Figure 9, the more oil reserves, the higher sovereign risk. So much so, that there is a level of oil reserves such that at that point the sovereign will default for sure. This is pretty intuitive, and is in line with what we found in the empirical section, if the sovereign has enough oil then its not necessary to resort to debt to smooth out consumption, as such higher oil reserves increase the value of autarky and sovereign risk goes up.

Figure 9: Comparative Statics



4 Quantitative Analysis

In this Section we study the model's quantitative predictions. We start by calibrating the model to Russian data. Then we solve the model and use the results to perform three tasks: First, we compare the statistical moments that the model produces with their data counterparts; second, we study the dynamics of the model around default events; and third, we explore the quantitative features of key elements of the model's solution (decision rules, default sets, default costs, etc) to shed light on the mechanisms driving the results.

4.1 Calibration and Solution Method

We calibrate the model to Russian data at an annual frequency. On the sovereign's side, the model has six parameters: the sovereign's discount factor, β, the coefficient of relative risk aversion, μ,[7] the price of the risk-free bond, $\bar{q}$, the trade penalty incurred when in default, θ, the probability of reentering financial markets when in the default state, λ, and a constant amount of autonomous spending, A, that accounts for investment and government spending in the resource constraint, which are not included in the model but are needed in order for the model's national accounts to be consistent with the data.

The values of μ and $\bar{q}$ are set to standard values from the literature. Accordingly, we set $\mu = 2$ and $\bar{r} = 0.00775$. The latter corresponds to the average ex-post, US-CPI deflated yield on a 3-month U.S. Treasury bill for the 1955-2014 period (see Bianchi, Liu and Mendoza (2016)).

We set the probability of reentry at $\lambda = 0.332$ for a yearly frequency, according to Dias and Richmond (2007) who using a sample of 128 episodes of sovereign default on foreign currency bank debt and foreign currency bonds during the period 1980-2005, find that the median period of exclusion for partial financial market of a country is three years after default.

We set the fraction of oil exports that can be exported while the economy is in default to $\theta = 0.2$,

[7]The utility function is assumed to be $u(c) = \frac{c^{1-\mu}}{1-\mu}$

which implies a trade penalty of 8% of the steady state GDP,[8] matching the average trade penalty estimate in terms of GDP found by Rose (2005). We set the value of β using the simulated method of moments, as we discuss later in this Section.

The autonomous spending parameter, A, was set to $A = 1 + rb - c$, where b and c are the mean external debt to GDP ratio and the mean private consumption share in GDP respectively, for the period 1989-2016. The value of r is set to $\bar{r} + spread$, where *spread* corresponds to the average value of JP Morgan's EMBI+GSS spread for the period 1979-2016 (777 bp). Since the JP Morgan data starts in 1998 but we have the Institutional Investor Index for a longer sample, we estimate a linear regression between the Institutional Investor Index and the EMBI for the period 1998-2016, and then use the regression and the observed values of the Institutional Investor Index to estimate the Russian EMBI spread for the period 1979-1997.

The oil company's optimization problem requires three parameters: two parameters associated to the extraction costs, φ and γ, and the oil discoveries, d. From the dynamics of oil reserves, we know that in the deterministic steady state $x = d$ and by the normalization of of oil prices $p = 1$. Also since we normalize total GDP to one, $y + px = 1$, the share of oil production in GDP is $\frac{px}{y+px} = \frac{1d}{1} = d$, so $d = 0.1$ to match the share of oil production in Russia's GDP which is 10%.

In the simulated method of moments we estimate β and γ to match as closely as possible two moments in the data: a default rate of 3% and a 20% ratio of external debt stocks (public and publicly guaranteed) to GDP for the period 1992-2015. We find $\beta = 0.78$ and $\gamma = 2.25$.

The remaining parameter, φ, is set such that in the deterministic steady-state the extraction cost per unit is 32%. Table 3 summarizes all the parameter values mentioned above.[9]

Table 3: Summary of Parameters

Parameter	Description	Value
β	discount factor	0.78
μ	risk aversion	2
$\bar{q}$	risk-free debt price	0.99
θ	trade penalty	0.2
d	discovery rate	0.10
λ	redemption probability	0.33
γ	curvature extraction cost	2.25
φ	scale extraction cost	224
A	rescaling parameter	0.331

Recall that there are also two exogenous processes. One for the non-oil output (the non-storable consumption good received by the sovereign) and the price of oil. We estimate a VAR for these two variables using annual data for the real Brent price of crude oil (average of the daily price) and the real GDP (excluding oil rents) as a proxy of our non-oil endowment for the period 1979-2016. The

[8] Oil GDP represents 10% of total GDP for Russia, so by imposing a $\theta = 0.2$, it means that the trade penalty costs the countrty 80% of its oil output, and given that oil output is 10% of the total, it represents a lose of 8% of total output.

[9] In the deterministic steady-state $\varphi = n^{\gamma}/(1 + \gamma - (1/(1 + r))\gamma/n)$, where n is the ratio of Russian oil reserves to production (in years) for the period 1997-2016. In accordance with the US Energy Information Administration (EIA) we set $n = 18.5$ years.

estimated VAR(1) model for p_t and y_t, imposing the restriction that Russian non-oil GDP does not impact international oil prices, is:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} \rho_p & 0 \\ \rho_{py} & \rho_y \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_p e_{p,t} \\ \sigma_y e_{y,t} \end{bmatrix}$$

where $e_{p,t}$ and $e_{y,t}$ are mean-zero, i.i.d. random variables. The diagonal of the estimated covariance matrix of the innovations is a matrix with variances σ_p^2 and σ_y^2. Table 4 shows the results of the estimation.

Table 4: VAR Process for Non-Oil Output and Oil Prices

Parameter	Description	Value
ρ_p	oil price auto-correlation	0.89
ρ_y	non-oil output auto-correlation	0.67
ρ_{py}	oil price non-oil output correlation	0.15
σ_p^2	variance oil price	0.062
σ_y^2	variance non-oil output	0.002

We use the following algorithm to solve the model.

1. Assume an initial value for the price of the bond $q_0(B', S, y, p)$ as well as initial values for the functions v_0^d and V_0.

2. Use q_0 and solve equation (4) to find the optimal extraction policy, $x_0^*(p, s; B')$. We use a time-iteration algorithm to find the function x_0^*.

3. Use q_0 and x_0^* to solve the sovereign problem, that is solve (5) subject to (6) and (7) subject to (8), and update the value functions v_1^d and V_1.

4. Find the default set using equation (10) and compute the probability of default using (11).

5. Update the price of the bond using the following equation:

$$q_1 = \bar{q}\left(1 - d\left(b'; S, y, p\right)\right).$$

6. Use the updated price of the bond q_1 and the updated value functions v_1^d and V_1 to repeat steps 1-5 until $\|q_1 - q_o\| < \epsilon$, $\left\|v_1^d - v_o^d\right\| < \epsilon$ and $\|V_1 - V_o\| < \epsilon$, where ϵ is a small number.

4.2 Comparison of statistical moments

Table 5 compares model versus data moments for the standard moments examined in the default literature. As explained in the previous Section, the mean debt ratio and the default frequency were used as calibration targets using the simulated method of moments, so the model comes very close to matching the data moments (a mean debt ratio of about 20% of GDP and a default frequency of 3%).

The rest of the model moments shown in the Table are model predictions that can be contrasted with the data moments to gauge the model's ability to match the data.

The standard deviation of oil GDP in the data is 40% and the one generated by the model is 56%.[10] The model generates average reserves in years of 21 while in the data its 18.5. This number represents how many more years the country has left extracting at their historical rate.

Finally, we calculate the model spread as $\frac{q}{\bar{q}}$ and compare it to the volatility of the EMBI spread in the data. The model underestimates both the mean and volatility of the EMBI, the mean is 3.5% in the model versus 8% in the data and the standard deviation is 8% in the data versus 3% in the model. Even though we have used the Institutional Investor Index as our measure of country risk, it is not a price but an index, so we considered appropriate to compare our results against an actual spread such as the EMBI.

Table 5: Data vs Model Moments

Description	Data	Model
Average External Debt to GDP	0.20	0.21
Standard Deviation of Oil GDP	0.41	0.56
Average Reserves (in years)	18.5	21
Default Rate	0.03	0.03
Average Spread	0.08	0.035
Standard Deviation of Spread	0.08	0.03

Estimates of the proven reserves for Russia correspond to those of the US Energy Information Administration.

Table 6 shows the comparison between other volatilities and correlations. The correlation between oil GDP and total GDP is 55% in the data and 54% in the model. The volatility of the overall GDP is 7% in the data and 9% in the model.

Total consumption (including government) has a correlation with total GDP of 71% in the data and in the model it is 87%, while its volatility in the data is 5% but 15% in the model. As pointed out by Chatterjee and Eyigungor (2012) one-period sovereign debt default models exhibit a counterfactual consumption volatility. Debt refinancing of all outstanding debt during a given period implies that changes in bond prices q tend to imply large changes in consumption, especially if b is a relatively high share of output. In our benchmark calibration debt as a share of GDP is relatively large (20% of GDP) compared to other models of short-term sovereign debt (usually at most 10% of GDP). Introducing long-term debt smooths consumption because it reduces refinancing needs, for a given volatility of bond prices. We do not introduce long-term debt as it is beyond the scope of this paper.

[10] Oil GDP in the data is defined as the share of oil rents times real GDP, and in the model it is given by px. Both observed and model generated data are filtered using the Hodrick and Prescott filter with a 6.25 lambda.

Table 6: Summary of Comovements - Russia vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.		Correlation vs GDP	
	Data	Model	Data	Model
Oil GDP	0.41	0.56	0.55	0.54
Total GDP	0.07	0.09	1.00	1.00
Total Consumption	0.05	0.15	0.71	0.87
Trade Balance	0.05	0.06	-0.48	-0.43
Spread	0.08	0.035	-0.58	-0.16

*Except TB which is relative to GDP and EMBI which is in levels (basis points).

The model does very well in terms of the cyclicality and volatility of the trade balance. It has a standard deviation of 5% in the data compared to 6% in the model, and in the data the correlation of the trade balance to GDP is -0.48 versus -0.43 in the model. Finally, even though the model misses in terms of the volatility of the spread it still captures the counter cyclicality.

4.3 Dynamic of Default Episodes

This model is different from the usual sovereign default model in the literature. We know that in order to induce default in equilibrium, existing models introduce an adhoc default cost on output. This cost ranges from just loosing a percentage of output, to more complicated linear or non-linear loss functions. In the more involved cases, this cost is zero for output values below a certain threshold and is increasing with output above that threshold as to induce default in bad times.

Recall that in our model total output is $y^t = y + px$, and we don't impose any default cost on y, we only introduce a trade penalty, θ, on oil such that in a state of default the oil revenue becomes θpx. If we want to define the default cost, for a given $y$and p, we need only look at x. Let us denote by x^f the optimal extraction policy when there is no default risk, and by x^d the extraction corresponding to the default state—which is constant for a given price and level of reserves and equal to the expression in Equation 9.

Figure 10 depicts the default cost in terms of extraction $-\left(\frac{\theta x^d}{x^f} - 1\right)$. In this model we know that $x^d > x^f$, and so when $\theta = 1$, there is no trade penalty, there is a default benefit that goes from 60% for the lowest price, to close to zero with the highest price (note that a negative value in the plot represents a gain). With a $\theta = 0.2$—the benchmark value—the curve moves into the positive quadrant and becomes flatter, generating costs between 70% and 80%. The default cost is concave because as the price increases, so does extraction but there is an upper bound because the oil company can't extract more than the available reserves. Above a certain price, x^d and x^f are equal because all available reserves are being extracted. From the plot we can see that this occurs at $p > 1.5$—a cost of 80% corresponds to $-(\theta - 1)$. These are prices that are more than two standard deviations above the mean.

Figure 10: Default Cost in terms of Extraction

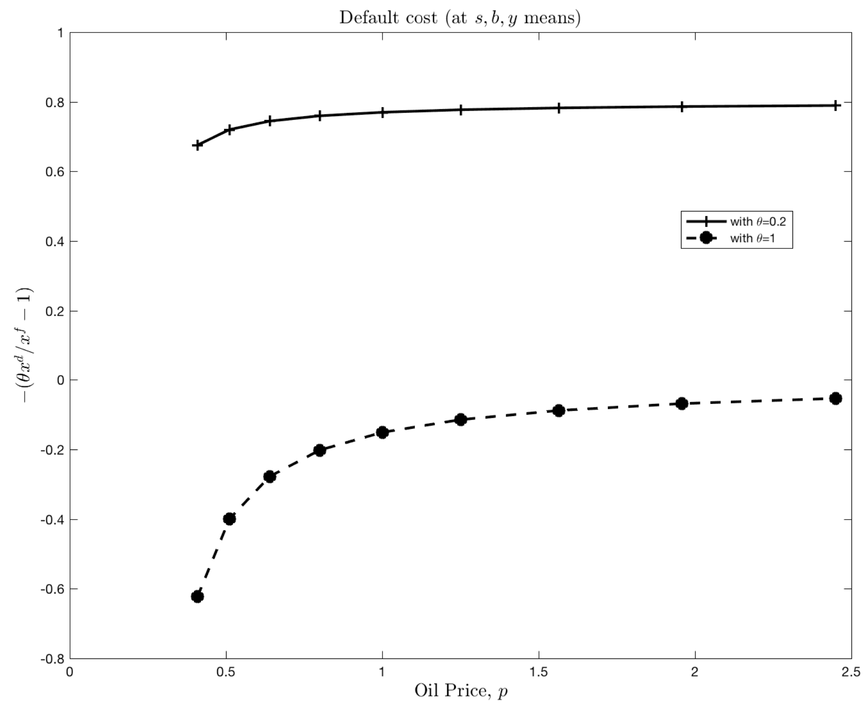


Figure 11 depicts the default costs in terms of total GDP $\left(-\left(\frac{y+\theta p x^d}{y+p x^f} - 1\right)\right)$, it incorporates the effect of prices and non-oil output. The case for $\theta = 1$ takes us back to the original default model where there is basically a default cost that is proportional and constant, with the difference that here it is not a default cost, but a default benefit of around 2%. The fact that it barely changes with $p$means that it doesn't make default incentives stronger at higher prices.

With $\theta = 0.2$, there are costs of default that range between 1% and 22%, and the cost is increasing in p, and slightly convex. This is happening because of the non-oil output term that is additive. While x^d and x^f converge as $p$rises (i.e. they both go to $x = s$ as prices increase), the value of the non-oil GDP in units of oil (i.e. $\frac{y}{p}$) keeps falling until it reaches zero in the limit as $p \to \infty$.

The third line in the plot corresponds to the case where oil production is fixed at a constant level. It shows that having endogenous oil production decreases the cost of default below a certain price, and it increases it above a certain price relative to an endowment economy with the same trade penalty, θ. This shows that having endogenous extraction strengthens the mechanism that triggers defaults in periods with low prices.

Figure 12 depicts the same default cost $\left(-\left(\frac{y+\theta p x^d}{y+p x^f} - 1\right)\right)$ as Figure 11, but now in the x-axis we have reserves changing and each line represents the default cost for the lowest price, the mean price and the highest price. Note that default costs are lower at the lowest price and higher at the highest

Figure 11: Default Cost in terms of GDP

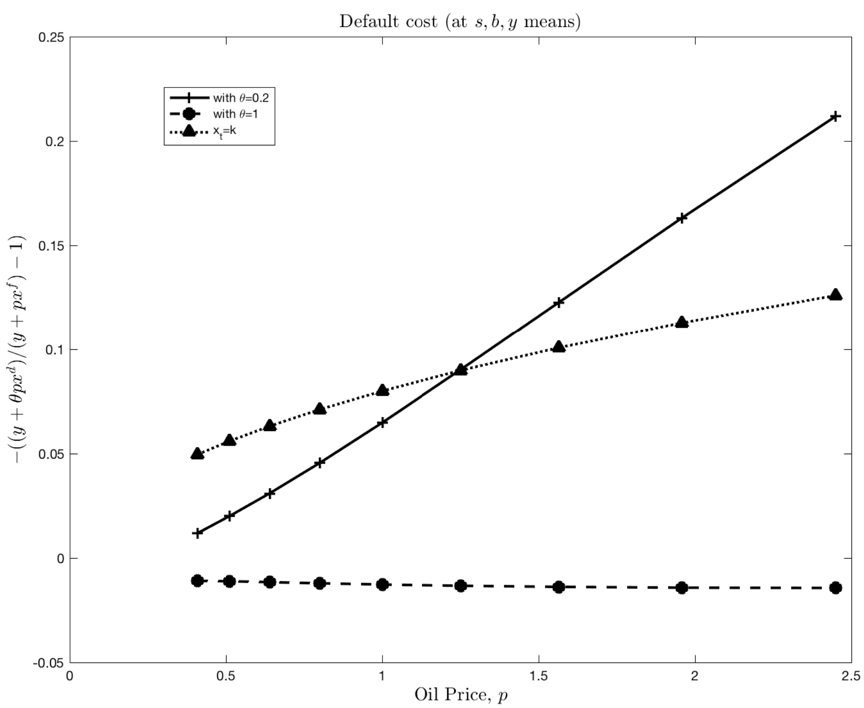
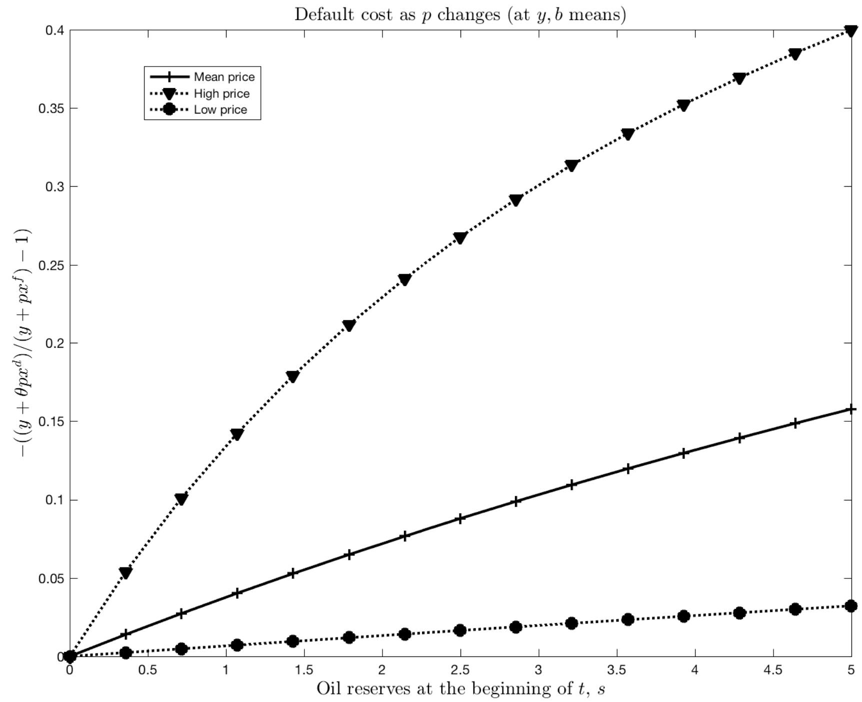


Figure 12: Default Cost in terms of GDP as Reserves Change for different Price Levels



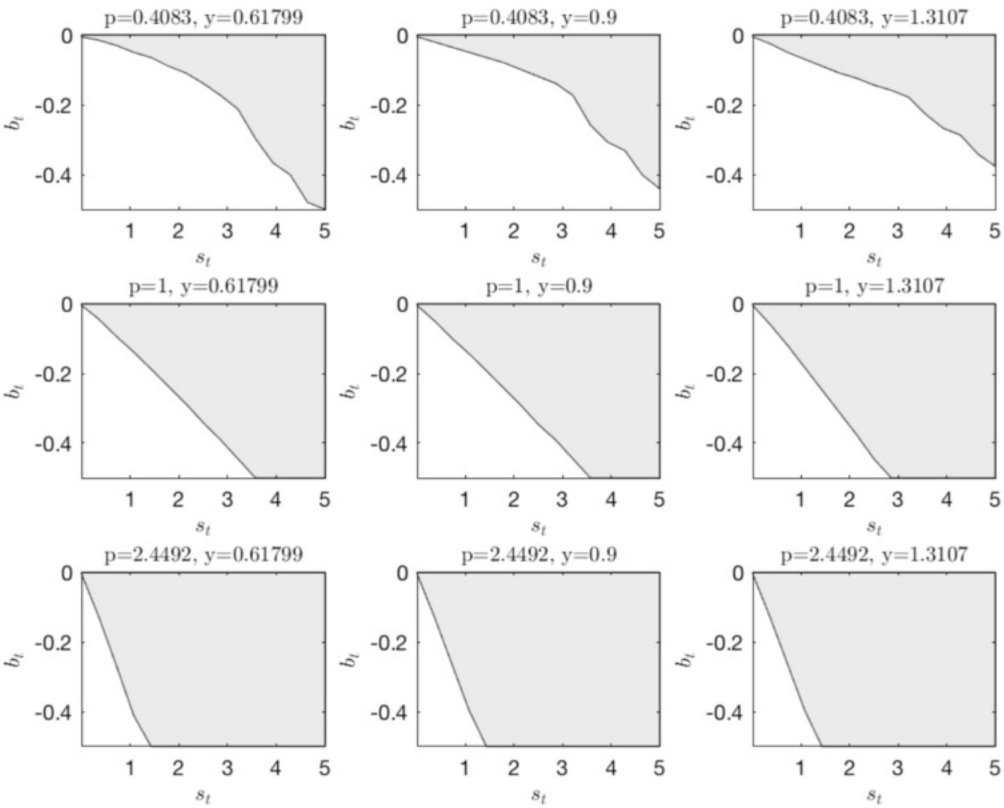
price. This plot shows that at a higher level of reserves, a drop in the price from high to the mean or the mean to low, represents a larger drop in the default cost. This exemplifies the fact that a drop in prices is more likely to trigger a default when reserves are high.

Figure 13 shows the resulting default sets associated with high, medium and low oil prices as well as high, medium and low non-oil output. The white areas represent the default set and the grey area is the area of repayment. The default set is increasing in debt and oil reserves for all states. It is also true that the effect of a higher non-oil output is not independent of oil prices. Notice that for the low price (first row in the plot) a higher non-oil output implies a larger default set (larger white area). This is the case because low prices decrease the countries ability to repay but the non-oil output increases the value of autarky, generating a larger default set.

For medium prices (the middle row in the plot), the default set is decreasing in the non-oil output reflecting the fact that it increases the ability to repay. For high prices (the bottom row) the default set is pretty much constant in non-oil output.

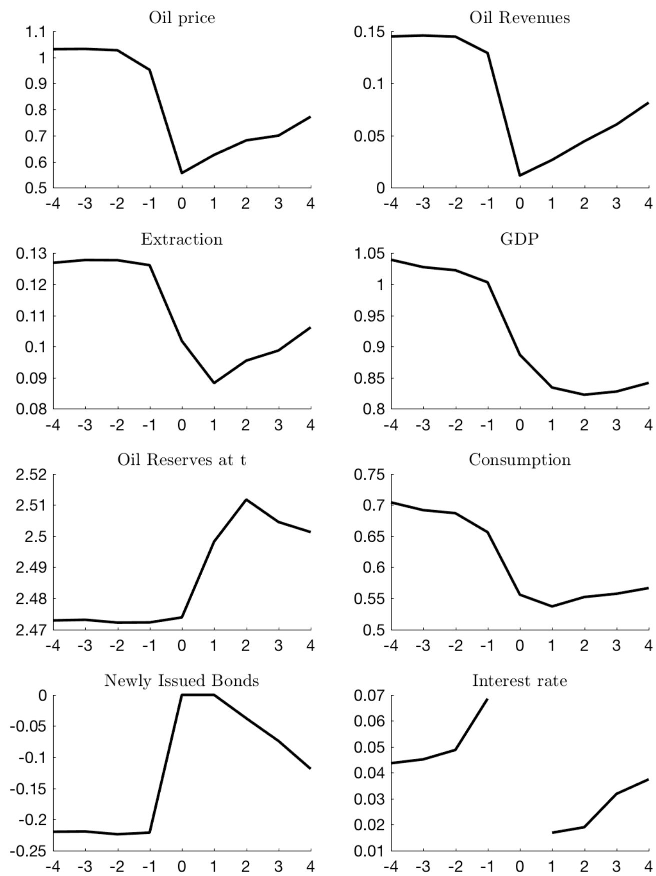
This non-monotonicity in the size of the default sets comes from the non-linearity of the "default cost" or benefit in our case shown in Figure 10.

Figure 13: Default Sets



We now use the model to study the dynamics around default episodes. Using simulated data, we average out over the default episodes and look at the path of the different variables four periods (years) before and four periods after. Figure 14 shows the results. Not surprisingly the main driver of a default in the model is the price of oil and interestingly, the dynamics of the model resemble the typical boom-boost pattern of a financial crises in an emerging economy: the price of a commodity is booming—oil in this case—, oil extraction is high and reserves are falling, consumption grows faster that GDP implying trade and current account deficits. When oil prices collapse, oil extraction falls and there is a sharp adjustment in both consumption and GDP. The sovereign defaults, bonds go to zero and debt prices collapse.

Figure 14: Model Dynamics Four Years Before and After Default Episodes



In this model for a given price of oil, x^d (extraction under default) is higher than that under repayment (x^*) which in turn is weakly higher than that under the risk free rate (x^f): $x^d(p) > x^*(p) \geq x^{rf}(p)$. So in this model—with $\theta = 1$— as was shown earlier, there is no default cost, in fact there is an incentive to default because oil revenues for a given state p are higher under default. However there is a difference between oil output, x, and oil revenues, px. Suppose that as is shown in Figure 14 we come from a long sequence of drawing the mean oil price $\bar{p}$ and low default risk so that oil output x^* is not very different from x^f. Hence at $t-1$ we have $\bar{p}$, and $x^* \approx x^f$ which implies $\bar{p}x^f$. Then a negative shock of around two standard deviations hits p, taking it to p_l, and triggers a default. Now at t we have p_l, $x^d(p_l)$, and $p_l x^d(p_l)$. So what can we say about the values of t relative to $t-1$?

We know for sure that $p_l < \bar{p}$, that is p at t is lower than at $t-1$, but it is less obvious what happens to the output and oil revenues, because that hinges on whether $x^d(p_l)$ is higher or lower than $x^f(\bar{p})$.

We only know that $x^f(p_l)$ is lower than both $x^f(\bar{p})$ and $x^d(p_l)$, but this leaves open the possibility that $x^d(p_l)$ could be higher than $x^f(\bar{p})$. In other words, while it can be that oil output falls at t relative to $t-1$—as is the case in Figure 14—it can also be higher.

To make this point, note that in equilibrium extraction can be written as $x(p, q(b', y, s, p)) : p$ affects directly through the marginal benefit of extracting an extra unit as well as indirectly via the effect of p on country risk. As such, we can write the derivative $\frac{dx}{dp}$ in the following way:

$$\frac{dx}{dp} = \frac{\Delta x}{\Delta p} + \frac{\Delta x}{\Delta q}\frac{\Delta q}{\Delta p}.$$

In this expression we know that $\frac{dx}{dp} > 0$, $\frac{dx}{dq} < 0$, and $\frac{dq}{dp} \geq 0$. This last one is greater or equal to zero because in the interval where the probability of default is zero, $q = \frac{1}{1+\bar{r}}$—where $\bar{r}$ is the risk free rate—and q is independent of p. This implies that the sign of $\frac{dx}{dp}$ is ambiguous. When the probability of default is zero or close to zero, $\frac{dq}{dp}$ is zero or very small, and $\frac{dx}{dp} > 0$. But as q goes down— and given that q has a steep slope—$\frac{dq}{dp}$ can be high which implies that $\frac{dx}{dp} < 0$ when country risk is high. This means that in the equilibrium dynamics, extraction can increase or decrease in response to a lower price, and it is more likely that it increases when the price drop occurs in a state where country risk is higher.

On the side of the oil revenue, something similar happens:

$$\frac{d(px)}{dp} = x + p\left[\frac{\Delta x}{\Delta p} + \frac{\Delta x}{\Delta q}\frac{\Delta q}{\Delta p}\right],$$

the difference is now that it is harder for $\frac{d(px)}{dp}$ to be negative when $\frac{\Delta q}{\Delta p}$ is large, because x is adding up to the expression. In other words, in terms of oil revenues one would expect them to go down with the oil price, even though extraction were to go up.

4.4 Model and Data

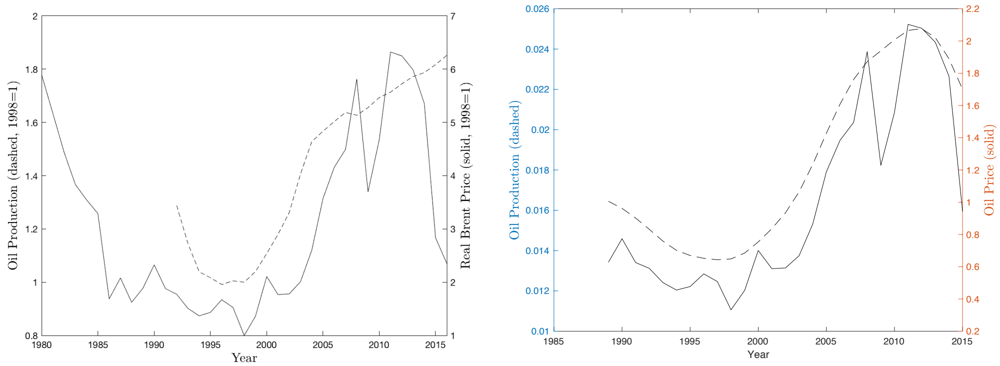
In this subsection we go further in comparing the model generated data with the observed data. To do so, we start by feeding into the model the observed sequence of prices in the data to see what is the resulting extraction in the model.

Figure 15 shows in the left panel oil production versus oil prices in the data and the panel in the right shows the same but for the model. We can see that in both cases extraction has a trend that is very similar to that of prices, and the model replicates closely what we observe in the data.

To tie down our theoretical results with the empirical ones, we take model generated data for 10,000 periods, and bootstrap thirty year windows to run the same regression shown in Table 2. The results are shown in Table 7. As the measure of country risk we use the price of the risky bond (as opposed to the III in the empirical counterpart), we can't control for default episodes because of multicollinearity (i.e. the price of the risky bond is zero when there is a default episode), and we can't control for discoveries because in the model they are constant. Everything else is the same as in Section 2.

The most remarkable thing is that all coefficients have the same sign, with only one exception,

Figure 15: Oil Production and Prices: Model versus Data



Oil GDP. We can see that in the short run, the change in oil GDP, non-oil GDP, and reserves are all positive meaning that increases in these variables reduces country risk, and as expected the change in external public debt to GDP has a negative effect.

In the long run, non-oil GDP has a positive coefficient and oil GDP, oil reserves and external public debt have a negative coefficient. This result shows, that in the model it is also true that although higher oil production decreases sovereign risk, having a larger stock of oil increases it.

5 Counterfactuals

In this section we run several counterfactuals to check the robustness of the results and have a better understanding of the mechanisms behind the model. Tables 8 to 10 show the results.

The column labeled "profits" stands for the case where the extraction costs of the oil company are not paid to the sovereign, and so in the sovereign's budget constraint we observe the oil company's profits instead of its revenues. In this case the model sustains less external debt (because the sovereign has a lower ability to repay), and as such the default probability goes down and so does the average spread and its standard deviation. Apart from these there are no mayor differences.

The column labeled SDF, stands for the case where the discount factor of the oil company is the marginal rate of substitution instead of the price of the risky bond. In this case although the model sustains 4% more debt, the default rate goes down to 1% and the spread becomes slightly pro-cyclical.

Table 8: Target and model moments

Description	Data	Benchmark	Profits	SDF	$x = k$	$q = \bar{q}$
Average external Debt to GDP	0.20	0.21	0.14	0.25	0.23	0.20
Std dev of oil GDP	0.41	0.56	0.54	0.44	0.36	0.50
Average reserves (in years)	18.5	21	21	19	25	23
Default rate	0.03	0.03	0.02	0.01	0.01	0.02
Average spread	0.08	0.035	0.025	0.01	0.01	0.02
Std dev of spread	0.08	0.03	0.025	0.01	0.01	0.02

Estimates of the proven reserves for Russia correspond to those of the US Energy Information Administration.

Table 7: Model Short versus Long-run Coefficients

	Δ Price Risky Bond
Convergence coefficient	
Price Risky Bond (-1)	-0.727***
	(0.016)
Short-run coefficients	
Δ Oil GDP	0.043***
	(0.007)
Δ Non-Oil GDP	0.009
	(0.006)
Δ Oil Reserves	0.005
	(0.084)
Δ Ext. pub. debt to GDP	-0.027**
	(0.011)
Long-run coefficients	
Oil GDP	-0.042***
	(0.010)
Non-oil GDP	0.080***
	(0.007)
Oil Reserves	-2.343***
	(0.555)
Ext. pub. debt to GDP	-0.161***
	(0.013)
Constant	1.478***
	(0.341)

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 9: Summary of Comovements - Russia vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.						Correlation vs GDP					
	Data	Benchmark	Profits	SDF	$x_t = k$	$q = q_f$	Data	Benchmark	Profits	SDF	$x = k$	$q = \bar{q}$
Oil GDP	0.41	0.56	0.54	0.44	0.36	0.50	0.55	0.54	0.64	0.59	0.36	0.56
GDP	0.07	0.09	0.09	0.09	0.07	0.09	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.05	0.15	0.13	0.14	0.11	0.14	0.71	0.87	0.93	0.91	0.90	0.90
Trade Balance	0.05	0.06	0.06	0.09	0.08	0.09	-0.48	-0.43	-0.35	-0.44	-0.37	-0.43
Spread	0.08	0.035	0.025	0.01	0.01	0.02	-0.58	-0.16	0	0.09	0.31	0.07

*Except TB which is relative to GDP and EMBI which is in levels (basis points).

The column labeled $x = k$, stands for the case when oil production is exogenous and constant. In this case we can see that the default rate goes down to 1% and the spread becomes pro-cyclical. The pro-cyclicality comes from the higher (lower) cost when p decreases (increases), because this changes the incentives to default in bad versus good times, meaning that the risk premia will go up in good times with a higher probability.

Table 10: Summary of Comovements in no-default periods - Russia vs. Model

1979-2016. HP filter Log-Data*

	Std. Dev.						Correlation vs GDP					
	Data	Benchmark	Profits	SDF	$x = k$	$q = q_f$	Data	Benchmark	Profits	SDF	$x = k$	$q = \bar{q}$
Oil GDP	0.41	0.37	0.37	0.22	0.40	0.37	0.55	0.68	0.75	0.67	0.49	0.65
GDP	0.07	0.09	0.09	0.07	0.06	0.09	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.05	0.15	0.13	0.11	0.05	0.14	0.71	0.87	0.95	0.91	0.90	0.90
Trade Balance	0.05	0.09	0.08	0.07	0.05	0.09	−0.48	−0.39	−0.35	−0.42	−0.36	−0.42
Spread	0.08	0.03	0.02	0.001	0.01	0.02	−0.58	−0.18	0	0.08	0.3	0.08

*Except TB which is relative to GDP and EMBI which is in levels (basis points).

Finally, the last column stands for the case where the discount factor of the oil company is the price of the risk free bond.

6 Conclusion

We have shown, that being a resource rich country—and more specifically—having oil, has two different effects on country risk. First, in the short-run it decreases country-risk because it increases a countries ability to repay and second, in the long-run, having a large stock of oil reserves increases country risk as it increases the value of autarky for the country as it helps them withstand exclusion from international financial markets.

We develop an off-the-shelf sovereign default model with oil extraction and show that the model is capable of generating the same relationships that are present in the data.

References

[1.] Barnajee, A. Panel data unit roots and cointegration: an overview. *Oxford Bulletin of Economics and Statistics*, 61: 607-629, 1999.

[2.] Bianchi, J., Liu, Chenxin and Mendoza, E. Fundamentals News, Global Liquidity and Macroprudential Policy. *Journal of International Economics,* March 2016, 99(1): S2-15.

[3.] Borensztein, E., and Panizza, U. The cost of sovereign default. *International Monetary Fund Working Paper,* 08/238, 2008.

[4.] Dias, Daniel A., and Christine Richmond, "Duration of Capital Market Exclusion: An Empirical Investigation," Working Paper, UCLA, 2007.

[5.] Eaton, J., and Gersovitz, M. Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies,* Vol. 48, No. 2 (Apr., 1981), pp. 289-309.

[6.] Im, K. S., Pesaran, M. H., and Shin, Y. Testing for unit roots in heterogeneous panels. *Journal of Econometrics*, 115: 54-74, 2003.

[7.] Lane, P. R., and Milesi-Ferretti, G. The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004. *Journal of International Economics*, 73(2): 223-250, 2007.

[8.] Mendoza, E. G., Real Business Cycles in a Small Open Economy. *American Economic Review*, 81 (September 1991): 797-818.

[9.] Pesaran, M. H., Shin, Y., and Smith, R. P. Pooled mean group estimation of dynamic heterogeneous panels. *Journal of American Statistical Association*, 94: 621-634, 1999.

[10.] Pesaran, M. H., and Smith, R. P. Estimating long-run relationships from dynamic heterogeneous panels. *Journal of Econometrics*, 68: 79-113, 1995.

Appendix A: Institutional Investor Index and Other Measures of Sovereign Risk

In this section, we show that the Institutional Investor Index (III) is a robust measure of sovereign risk by demonstrating that other measures of sovereign risk are correlated with the III.

Moody's and Fitch Credit Ratings

Credit ratings by agencies such as Moody's and Fitch are commonly used measures of sovereign risk. These agencies assign risk based on rating symbols. Tables 1 and 2 provide brief descriptions of what each symbol signifies about credit risk. Table 3 provides the date each agency first issued a credit risk rating to a given sovereign.

Table 11: Moody's Global Long-Term Rating Scale

Rating	Description
Aaa	Obligations rated Aaa are judged to be the highest quality, subject to the lowest level of credit risk.
Aa	Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.
A	Obligations rate A are judged to be upper-medium grade and are subject to low credit risk.
Baa	Obligations rated Baa are judged to be medium-grade and subject to moderate credit and as such may possess certain speculative characteristics.
Ba	Obligations rated Ba are judged to be speculative and are subject to substantial credit risk.
B	Obligations rated B are considered speculative and are subject to high credit risk.
Caa	Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk.
Ca	Obligations rated Ca are very highly speculative and are likely in, or very near, defualt with some prospect of principal and interest.
C	Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest.

Note: Moody's appends numerical modifiers 1, 2, and 3 to each generic rating classification from Aa through Caa. The modifier 1 indicates that the obligation ranks in the higher end of its generic rating category, the modifier 2 indicates a mid-range ranking, and the modifier 3 indicates a ranking in the lower end of that generic rating category.

Table 12: Fitch International Credit Rating Scale

Rating	Description
AAA	Highest credit quality. AAA ratings denote the lowest expectation of default risk. They are assigned only in cases of exceptionally strong capacity for payment of financial commitments. This capacity is highly unlikely to be adversely affected by foreseeable events.
AA	Very high credit quality. AA ratings denote expectations of very low default risk. They indicate very strong capacity for payment of financial commitments. This capacity is not significantly vulnerable to foreseeable events.
A	High credit quality. A ratings denote expectations of low default risk. The capacity for payment of financial commitments is considered strong. This capacity may, nevertheless, be more vulnerable to adverse business or economic conditions than is the case for higher ratings.
BBB	Good credit quality. BBB ratings indicate that expectations of default risk are currently low. The capacity for payment of financial commitments is considered adequate, but adverse business or economic conditions are more likely to impair this capacity.
BB	Speculative. BB ratings indicate an elevated vulnerability to default risk, particularly in the event of adverse changes in business or economic conditions over time; however, business or financial flexibility exists that supports the servicing of financial commitments.
B	Highly speculative. B ratings indicate that material default risk is present, but a limited margin of safety remains. Financial commitments are currently being met; however, capacity for continued payment is vulnerable to deterioration in the business and economic environment.
CCC	Substantial credit risk. Default is a real possibility.
CC	Very high levels of credit risk. Default of some kind appears probable.
C	Near default. A default or default-like process has begun, or the issuer is in standstill, or for a closed funding vehicle, payment capacity is irrevocably impaired.
RD	Restricted default.
D	D ratings indicate an issuer that in Fitch's opinion has entered into bankruptcy filings

Note: Within rating categories, Fitch may use modifiers. The modifiers "+" or "-" may be appended to a rating to denote relative status within major rating categories. Such suffixes are not added to AAA ratings and ratings below the CCC category.

Unlike the III that is updated each semester, credit rating changes can occur at any time for an individual sovereign. In order to merge credit ratings data with the III, we use the credit rating that has been assigned the longest to a sovereign during a particular semester and merge that rating with the respective semester III reading. Since the III is a continuous variable and credit rating are a discrete variable (i.e. factor variable over the ordinal ratings labels), we visualize their correlation with box plots.

Table 13: Credit Agency Rating's First Issued Date

Country	Moody's	Fitch
Argentina	11/18/1986	5/28/1997
Brazil	11/18/1986	12/1/1994
China	5/18/1988	12/11/1997
Colombia	8/4/1993	8/10/1994
Ecuador	7/24/1997	11/8/2002
Egypt	10/9/1996	8/19/1997
Gabon		10/29/2007
India	1/28/1988	3/8/2000
Iran		5/10/2002
Kazakstan	11/11/1996	11/5/1996
Kuwait	1/29/1996	12/20/1995
Malaysia	1/18/1986	8/13/1998
Mexico	12/18/1990	8/30/1995
Oman	1/29/1996	
Qatar	1/29/1996	
Russia	10/7/1996	10/7/1996
Saudi Arabia	1/29/1996	11/24/2004
Venezuela	12/29/1976	9/15/1997

Box plots are used to show the overall dispersion of a continuous variable over groups. In our case, the y-axis is the continuous III, and the x-axis is the agency's credit rating ranks. The credit rating ranks are ordered along the x-axis from highest to lowest credit risk (from left to right). The box plots then graphs the quartiles of III observations over each credit risk rating. The horizontal line across the middle of the box is the median. The second quartile is the region from the median line to the bottom of the box, while the third quarter is the region from the median line to the top of the box. The bottom end of the lower whisker is the smallest value excluding outliers and the top end of the upper whisker is the largest value excluding outliers. Outliers are plotted as dots above and below the whisker of the box. Outliers above the upper whisker are 1.5 times greater than the third quartile while outliers below the lower whisker are 1.5 times lower than the first quartile. Figure 1 plots the III over Moody's credit risk ratings, and figure 2 plots the III over Fitch credit risk ratings.

We can see from the distributional characteristics of III over the Moody's and Fitch credit risk ratings that each sovereign's corresponding III measure tends to increase as its credit rating improves. This indicates that the III is correlated with credit ratings.

Figure 16: Moody's Long-Term Sovereign Credit Ratings over the III

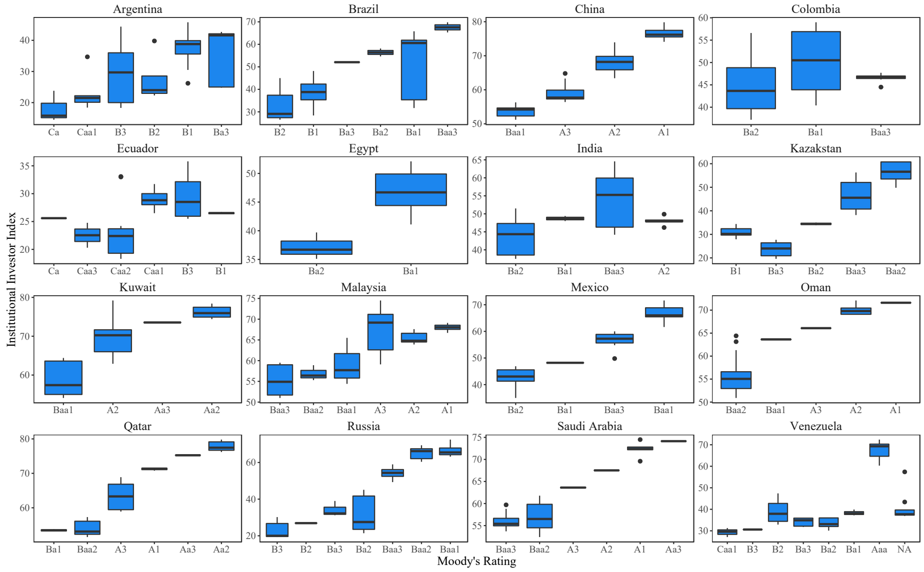
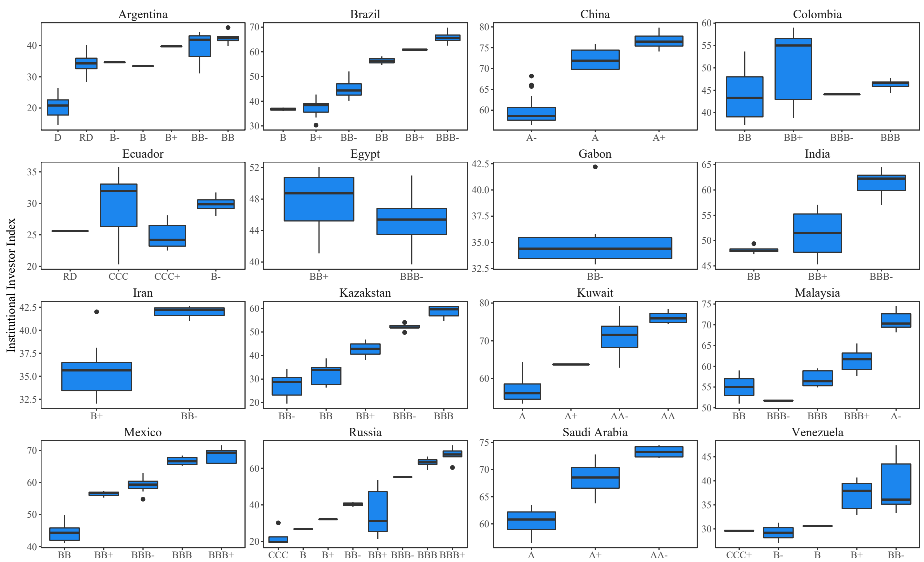


Figure 17: Fitch Long-Term Sovereign Credit Ratings over the III



Appendix B

Figure 18 plots the relationship between the III and oil production value to GDP ratio, for each country, over the period 1979-2010. One feature stands out from Figure 18: when oil production value to GDP ratio is high, the country risk index tends to improve. Note that there are countries where the correlation is not significant, such as Iran, United Kingdom, Egypt or Gabon.

Figure 18: Institutional Investor Index (X-Axis) and oil production value to GDP (%, Y-Axis).

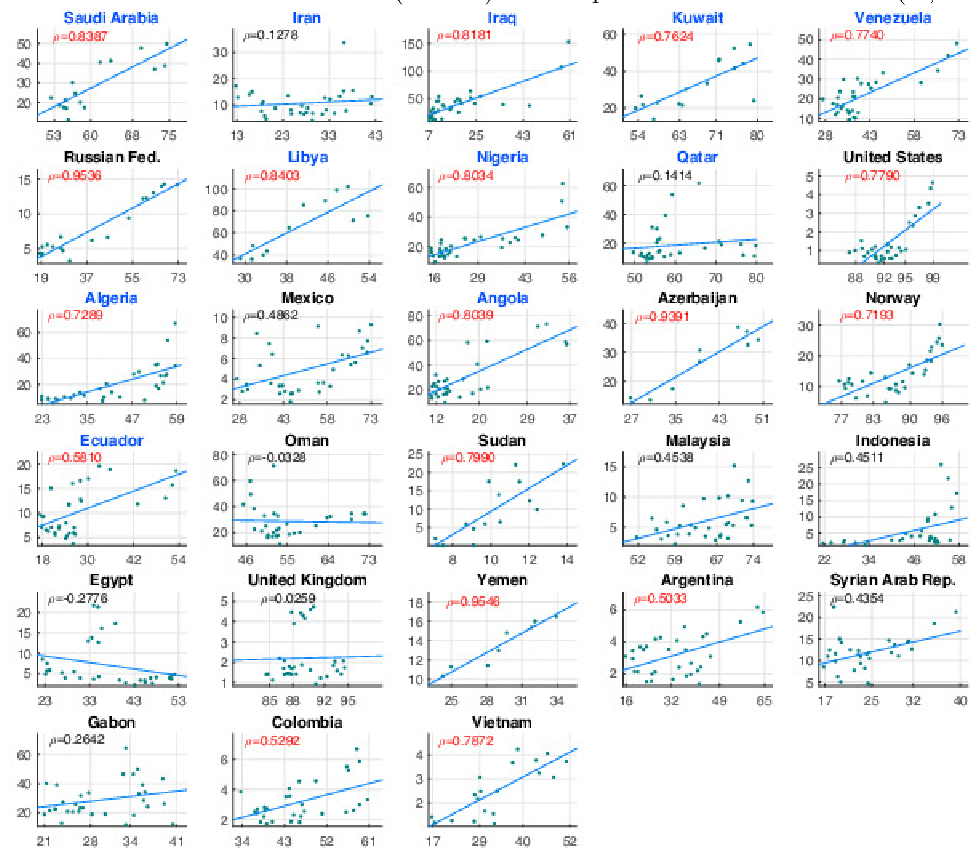
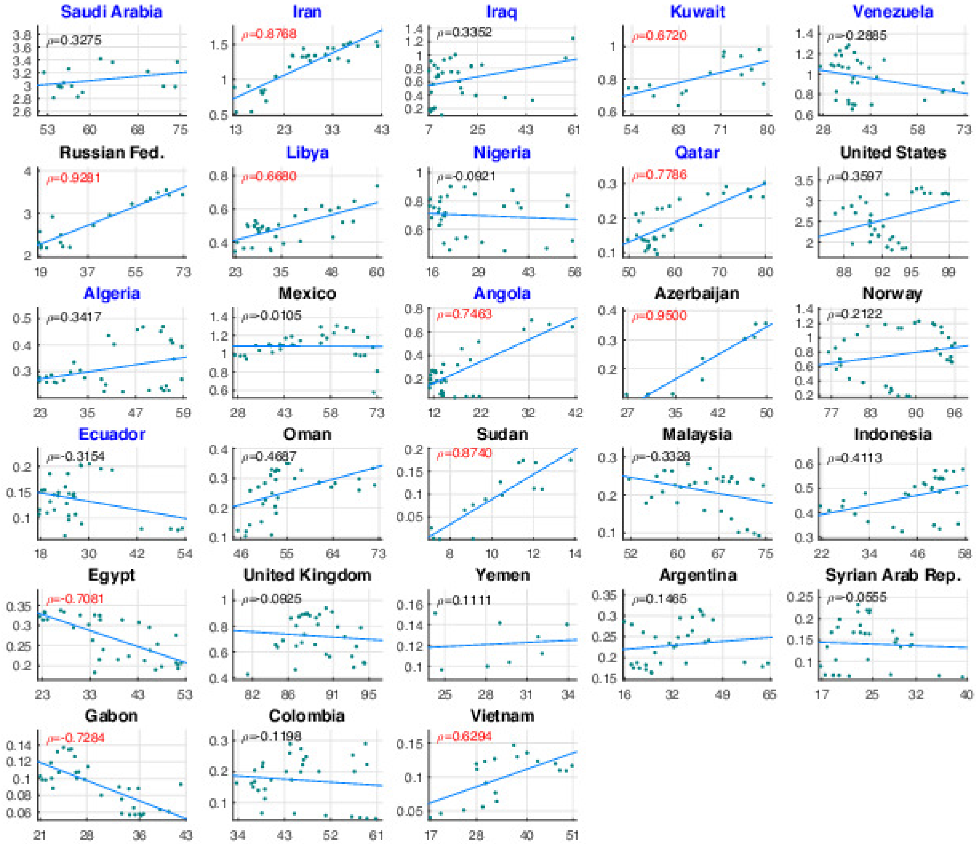


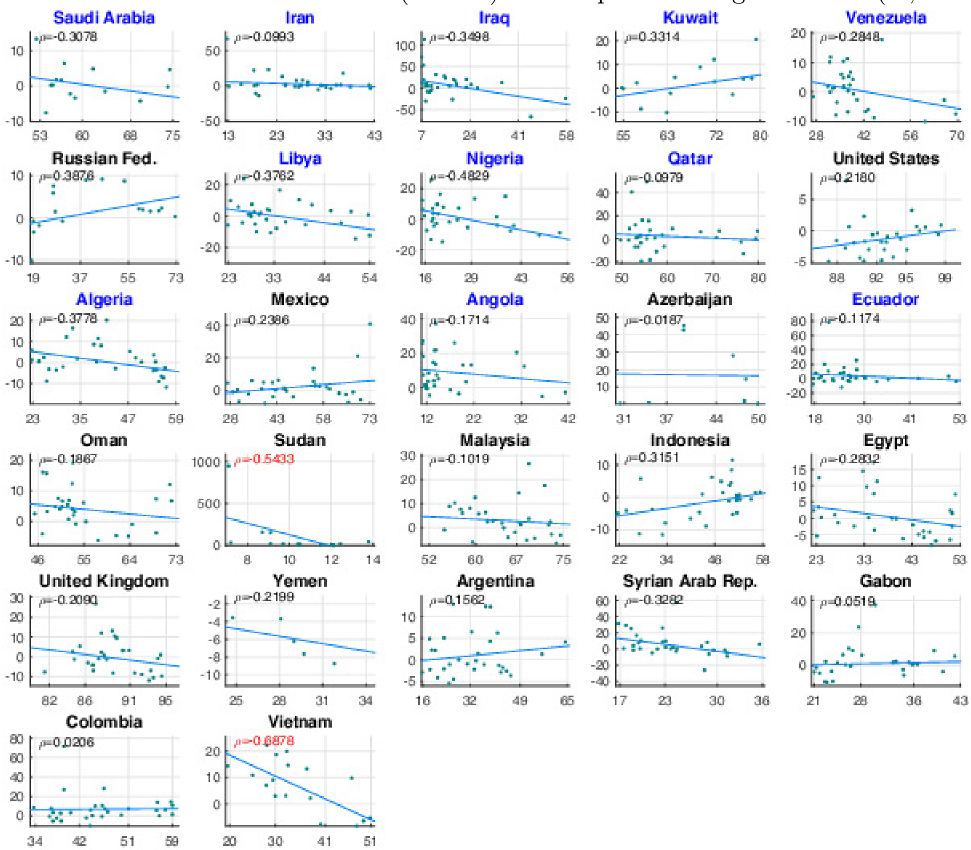
Figure 19 presents the III versus oil production (in billion barrels per year).

Figure 19: Institutional Investor Index (X-Axis) and oil production (billion barrels per year, Y-Axis).



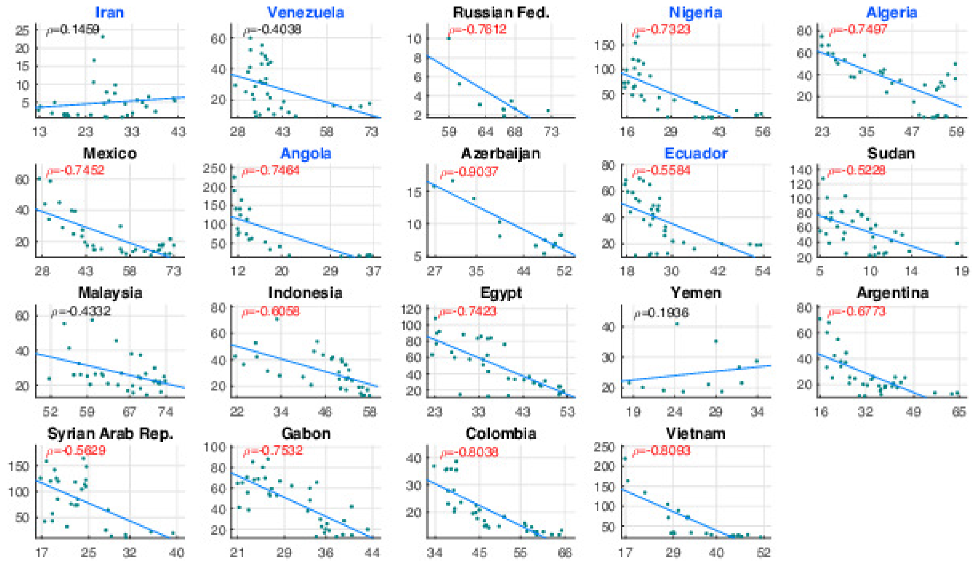
In this figure, absolute value of correlation coefficients greater than 0.5 are displayed in red. As we can see, there is not a clear pattern, since there are some countries for which the relationship is clearly positive, while for others it is negative or zero. This suggests that oil price is the "main driving force" behind changes in the country risk index (and not oil production). In Figure 20 we document the association between III and the oil production growth rate.

Figure 20: Institutional Investor Index (X-Axis) and oil production growth rate (%, Y-Axis).



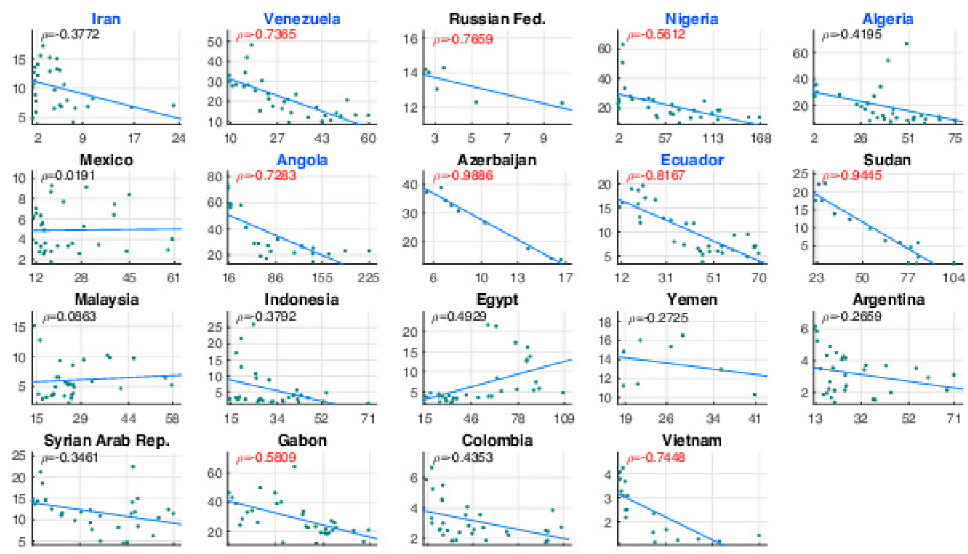
In this case, correlation coefficients lower than -0.5 are displayed in red. The results point in the direction that there is not any association between these two variables, although a negative relationship is observed for Sudan and Vietnam. Additionally, Figure 21 shows the relationship between the III and total public external debt to GDP ratio.

Figure 21: Institutional Investor Index (X-Axis) and total external public debt to GDP (%, Y-Axis).



Note that for most countries, correlation coefficients are displayed in red, which means that these are lower than -0.5. As we can see, III goes down when total public external debt increases. Additionally, Figure 22 shows the association between total external public debt to GDP ratio and oil production value to GDP ratio.

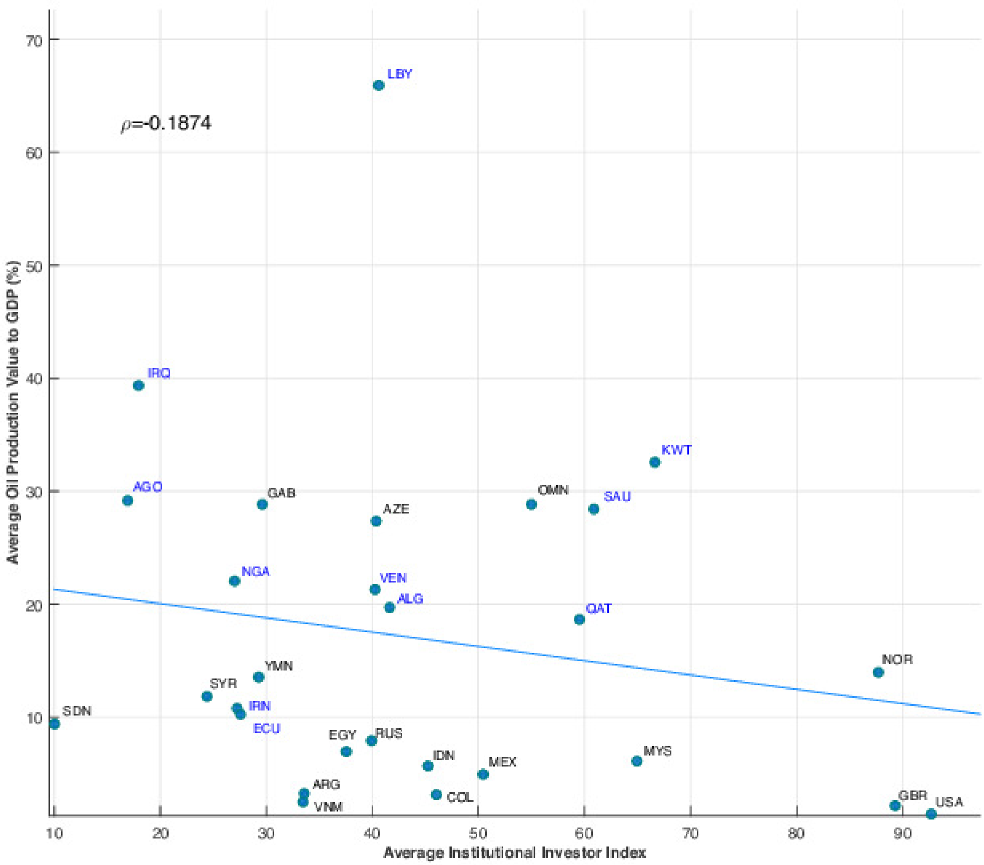
Figure 22: Total external public debt to GDP (%, X-Axis) and oil production value to GDP (%, Y-Axis).



As we can see, for 9 countries there is a negative correlation, which implies than when oil production value to GDP is high, total public external debt tends to be low. Nevertheless, such a contention is not reinforced by the rest of countries in the sample, since no significance is observed. Moreover, in the case of Egypt, the estimated coefficient shows almost a positive and statistically strong effect.

Moreover, Figure 23 plots the average III against average oil production value to GDP: In this case, we compute a low correlation coefficient (-0.187). The negative trend indicates that countries with high oil production value to GDP over time show a high country risk (or a low average III). It is important to mention that average oil production value to GDP may be low because historical GDP is very high when compared with the historical oil production value, such as in USA or Norway. Furthermore, this negative relationship may also be driven by exceptional cases such as Libya or Iraq, which have average oil production value to GDP of about 67 and 39 percent, and average III of about 41 and 17, respectively.

Figure 23: Average Institutional Investor Index (X-Axis) and average oil production value to GDP (Y-Axis): 1979-2010.



Appendix C: Estimation Approach

Before proceeding to dynamic panel data models, we need to verify that all variables are integrated of the same order. In doing so, we have used the test of the panel unit root of Im. *et al.* (2003, IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF henceforth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregressive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{12}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where N refers to the number of countries in the panel and T refers to the number of observations over time. In this case, y_i stands for each variable under consideration in our model (for example, III, oil GDP or non-oil GDP), α_i is the individual fixed effect and p is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all i, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where N_1 denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{13}$$

where t_i^{ADF} is the ADF t-statistic for country i, taking into account the country specific ADF regression, given by (12). The $\bar{t}$ statistic has been shown to be normally distributed under H_0. Table 1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both T and N are large. At one extreme, the usual practice is either to estimate N separate regressions and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran and Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error variances are constrained to be the same. In this case, the model controls for all time-invariant differences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant

Table 14: Im *et al.* (2003) panel unit root test outcome: 1979-2010

	Levels		Logs	
	t-statistic	P-value	t-statistic	P-value
Inst. Inv.	-1.396	0.635	-1.356	0.741
Δ Inst. Inv.	-3.600	0.000	-3.613	0.000
Oil GDP	-0.603	1.000	-1.208	0.949
Δ Oil GDP	-3.486	0.000	-3.745	0.000
Non-oil GDP	0.630	1.000	-1.150	0.981
Δ Non-oil GDP	-2.839	0.000	-3.249	0.000
Oil reserves	-0.974	0.999	-1.234	0.940
Δ Oil reserves	-3.812	0.000	-3.871	0.000
Ext. pub. debt to GDP	-1.397	0.572	-1.277	0.855
Δ Ext. pub. debt to GDP	-2.926	0.000	-2.940	0.000
NFA	-1.227	0.943	-	-
Δ NFA	-3.044	0.000	-	-

Note: When computing NFA outcome, we excluded Iraq because of data limitations.

characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran *et al.* (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i} + \theta_{1i}OilGDP_{it} + \theta_{2i}NonOilGDP_{it} + \theta_{3i}OilR_{it} + \theta_{4i}X_{it} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad (14)$$

Again, each observation is subscripted for the country i and the year t. In this case, $X \in \{ExtPubD, OilDisc, NFA\}$. The variable III is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $ExtPubD$ is the external public debt to GDP ratio, $OilDisc$ is the log of oil discoveries, NFA corresponds to net foreign assets to GDP ratio, and $Default$ is a dummy variable that the country is in default. Additionally, μ_i is a set of country fixed effects (such as geographical or institutional factors) and ϵ_{it} is the idiosyncratic error term.

Now, with a maximum lag of one for all variables except $Default$, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1,1,0) dynamic panel specification of (14):

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i}OilGDP_{it} + \delta_{11i}OilGDP_{i,t-1} + \delta_{20i}NonOilGDP_{it} + \\ \delta_{21i}NonOilGDP_{i,t-1} + \delta_{30i}OilR_{it} + \delta_{31i}OilR_{i,t-1} + \delta_{40i}X_{it} + \delta_{41i}X_{i,t-1} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad (15)$$

Then, the error correction equation of (15) is:

$$\Delta III_{it} = \phi_i \Big(III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} - \hat{\theta}_{5i} Default_{it} \Big) - \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it} \quad (16)$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i} + \delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i} + \delta_{21i}}{1-\lambda_i}$$

$$\hat{\theta_{3i}} = \frac{\delta_{30i} + \delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i} + \delta_{41i}}{1-\lambda_i}; \hat{\theta_{5i}} = \frac{\theta_{5i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, ϕ_i is the error correction speed of adjustment parameter, and we would expect ϕ_i to be negative if the variables exhibit a return to long-run equilibrium[11].

Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (16). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for China, India, and Brazil since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The

[11] Replacing $\hat{\theta}_i$-parameters and ϕ_i in equation (14) we get:

$$\Delta III_{it} = -(1-\lambda_i)\Big(III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i} OilR_{it} - \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i} X_{it} - \frac{\theta_{5i}}{1-\lambda_i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Removing similar terms, the above expression is as follows:

$$\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} + (\delta_{40i}+\delta_{41i}) X_{it} + \theta_{5i} Default_{it} - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Rewriting:

$$\begin{aligned} III_{it} - III_{i,t-1} = & -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} \\ & + (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}(OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i}(NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ & - \delta_{31i}(OilR_{it} - OilR_{i,t-1}) - \delta_{41i}(X_{it} - X_{i,t-1}) + \theta_{5i} Default_{it} + \epsilon_{it} \end{aligned}$$

Again, simplifying this equality we obtain:

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \theta_{5i} Default_{it} + \mu_i + \epsilon_{it}$$

Note that this expression is equivalent to (15). For a long-run relationship to exist, we require that $\phi \neq 0$.

Table 15: Hausman test outcome: 1979-2010

	Model (1)		Model (2)		Model (3)	
	χ^2-stat	P-value	χ^2-stat	P-value	χ^2-stat	P-value
MG vs. DFE	0.02	1.000	0.01	1.000	0.06	1.000
PMG vs. DFE	0.03	1.000	0.03	1.000	0.03	1.000
MG vs. PMG	4.42	0.491	5.05	0.537	8.99	0.174

null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = (\beta_{DFE} - \beta_{PMG})^{'} \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} (\beta_{DFE} - \beta_{PMG}) \sim \chi^2$$

where β_j is the vector of coefficients and $\text{var}(\beta_j)$ is the covariance matrix of β_j, estimated using the $j-$technique, for j =DFE, PMG. Under the null hypothesis, H has asymptotically the χ^2 distribution. Table 2 reports the results of Hausman test, in which Model (1) corresponds to equation (16), excluding NFA from X_i, while Model (2) excludes $Default$. Model (3) includes all variables in X_i into the regressors.

Under the current specification, the hypothesis that the country risk equation (equation (16)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in table 15 suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 1, which indicates that DFE is more efficient estimator than MG and PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are 1. Because of this, we choose to employ the DFE estimator.

Appendix D: Price Upswings and Downswings

Table 16: Oil Price Upswings and Downswings

Downswings		Upswings	
Period	Number of Months	Period	Number of Months
NOV 75 - OCT 78	36	NOV 78 - JAN 81	27
FEB 81 - JUL 86	66	AUG 86 - JUL 87	12
AUG 87 - NOV 88	16	DEC 88 - OCT 90	23
NOV 90 - DEC 93	38	JAN 94 - OCT 96	34
NOV 96 - DEC 98	26	JAN 99 - SEP 00	21
OCT 00 - DEC 01	15	JAN 02 - JUL 08	79
AUG 08 - MAY 10	22		
TOTAL	219	TOTAL	196

Appendix E: Are all Oil Exporting Countries Price Takers?

In this appendix we check whether the countries in our sample are price takers. This exercise was carried out by Norberto Rodriguez-Niño from the research department at the Banco de la República de Colombia.

We examine causality between a country's extraction and oil prices using two strategies, both in a bivariate context. First, we test on the levels, using a modified version of the Granger causality test proposed by Toda and Yamamoto (1995). Second, we test causality using the Granger test on the first differences of both series.

For the causality test a modified Wald test (MWALD) is used as proposed by Toda and Yamamoto (1995) that avoids the problems associated with the ordinary Granger causality test by ignoring any possible non-stationary or cointegration between series when testing for causality.[12]The Toda and Yamamoto (1995) approach fits a standard vector autoregressive model in the levels of the variables (rather than the first differences, as the case with Granger causality tests) thereby minimizing the risks associated with the possibility of wrongly identifying the order of integration of the series.

The basic idea of this approach is to artificially augment our bivariate VAR or order k, by the maximal order of integration, one in this case. Once this is done, a (k+1)-th order of VAR is estimated and the coefficients of the last one lagged vector is ignored. The application of the Toda and Yamamoto (1995) procedure ensures that the usual Wald test statistic for Granger causality has the standard asymptotic distribution hence valid inference can be done.

Lag length for VAR are chosen based on information criteria (Akaike, Schwarz and Hanna-Quinn), when there is not agreement between those indicators, pormanteau (bivariate Lung-Box statistic) test is used to decide. This statistics joint with its P-values are contained and third and four columns of tables 2 and 4.

Data

We used monthly data of crude oil for the 20 major exporting countries; the sample period cover from January 2002 to November 2016. The data source is Joint Oil Data Initiative (JODI) Database (available at http://www.jodidb.org/TableViewer/tableView.aspx). For Colombia, the figures have source Banco de la República and are based on DIAN-DANE. Units are thousand barrels per period. Exports the top 20 countries accounted for approximately 96% of reported crude oil exports at the JODI base in 2015.

[12]As quoted from Wolde (2005) "... given that unit root and cointegration tests have low power against the alternative, these tests can be misplaced and can suffer from pre-testing bias (see Pesaran et al., 2001; Toda & Yamamoto, 1995). Moreover, as demonstrated by Toda and Yamamoto (1995), the conventional F-statistic used to test for Granger causality may not be valid as the test does not have a standard distribution when the time series data are integrated or cointegrated."

Results

Unit root test results (not presented here but available up to request) show that all the variables are integrated of order one.

Table 1 shows the results for the TY test. It is worth to remain that the null hypothesis in this as next table is that of non-causality. Table 2 presents results for Granger causality test, for the series in differences. Results in both tables coincide signaling oil exports from United Arab Emirates, Oman, Brazil and Azerbaijan causing (in Granger sense) oil prices. TY shows that exports from Canada also G-cause prices, and model in differences indicated that Kuwait G-cause oil prices.

Table 17: Taro-Yamamoto test results for series in levels

	2[2]*Lag	Lung-Box		Jarque-Bera		Taro-Yamamoto		
Country		Q-Stat	P-Value	Stat	P-Value	Statistic	P-Value	Decision
Saudi Arabia	2	26.75	0.32	164.12	0.00	1.47	0.48	
Russia	2	30.14	0.18	75.23	0.00	1.90	0.39	
Iraq	2	28.48	0.24	50.22	0.00	1.28	0.53	
U. Arab Emir.	2	29.43	0.20	31.25	0.00	17.32	0.00	Cause
Canada	2	26.31	0.34	70.50	0.00	7.30	0.03	Cause
Nigeria	2	17.33	0.83	13.42	0.01	0.99	0.61	
Kuwait	2	21.64	0.60	23.36	0.00	1.20	0.55	
Angola	4	23.88	0.09	17.30	0.00	7.86	0.10	
Venezuela	2	23.61	0.48	46.25	0.00	5.17	0.08	
Iran	2	27.83	0.27	66.94	0.00	5.00	0.08	
Mexico	2	21.95	0.58	14.50	0.01	4.19	0.12	
Norway	3	18.43	0.56	6.47	0.17	3.45	0.33	
Oman	2	18.92	0.76	4320.42	0.00	9.10	0.01	Cause
Brasil	7	3.17	0.53	24.80	0.00	16.69	0.02	Cause
Azerbaijan	2	20.98	0.64	1171.78	0.00	13.11	0.00	Cause
Uni. Kingdom	2	28.88	0.22	22.93	0.00	0.10	0.95	
Algeria	2	29.15	0.21	12.62	0.01	4.89	0.09	
Qatar	2	20.04	0.69	127.50	0.00	1.84	0.40	
USA	3	21.69	0.36	332.23	0.00	1.19	0.76	
Colombia	3	25.90	0.17	13.44	0.01	0.81	0.85	

Table 18: Granger tets results for series in diferences

	2[2]*Lag	Lung-Box		Jarque-Bera		Taro-Yamamoto		
Country		Q-Stat	P-Value	Stat	P-Value	Statistic	P-Value	Decision
Saudi Arabia	7	6.69	0.15	49.06	0.00	10.77	0.15	
Russia	6	14.45	0.07	132.17	0.00	7.56	0.27	
Iraq	2	34.37	0.08	6.76	0.15	3.41	0.18	
U. Arab Emir	6	9.38	0.31	71.06	0.00	18.78	0.00	Cause
Canada	6	12.82	0.12	5.86	0.21	7.58	0.27	
Nigeria	1	37.67	0.10	25.24	0.00	0.33	0.57	
Kuwait	6	6.57	0.58	14.00	0.01	13.63	0.03	Cause
Angola	6	8.01	0.43	342.62	0.00	10.84	0.09	
Venezuela	1	26.57	0.54	16.27	0.00	2.14	0.14	
Iran	2	34.39	0.08	95.29	0.00	2.96	0.23	
Mexico	2	28.31	0.25	32.99	0.00	2.65	0.27	
Norway	2	32.64	0.11	20.19	0.00	3.26	0.20	
Oman	6	10.13	0.26	13053.21	0.00	26.42	0.00	Cause
Brazil	7	8.94	0.06	265.77	0.00	18.39	0.01	Cause
Azerbaijan	2	32.15	0.12	1029.34	0.00	12.68	0.00	Cause
Uni. Kingdom	6	14.49	0.07	27.07	0.00	5.27	0.51	
Algeria	2	33.76	0.09	7.44	0.11	3.82	0.15	
Qatar	6	7.20	0.51	87.55	0.00	12.24	0.06	
USA	3	35.85	0.02	33.07	0.00	2.56	0.46	
Colombia	2	29.64	0.20	18.90	0.00	1.43	0.49	

References

- Pesaran, M. H., Shin, Y., & Smith, R. (2001). Bounds testing approach to the analysis of level relationships. Journal of Applied Econometrics, 16, 289–326.
- Toda, H. Y., & Yamamoto, T. (1995). Statistical inference in vector auto-regressions with possibly integrated process. Journal of Econometrics, 66, 225–250.
- Wolde-Rufael, Y. (2005) “Energy demand and economic growth African experience”, Journal of Policy Modeling 27 (2005) 891–903.

Appendix F: Correlation Between Sovereign and Oil Firm Credit Ratings

In this section, we show that there is evidence that oil firms' credit ratings are related to their respective headquartered sovereign's credit risk ratings. We demonstrate this using Credit Default Swap (CDS) spreads. The CDS is a type of financial instrument in which the seller agrees to compensate the buyer in the event that the sovereign defaults on its debt during the given term. The spread is a series of payments the buyer makes to the seller to insure their investment. Therefore a higher CDS spread reflects greater risk.

In figures 3 through 11, we look at corporate CDS spreads of oil firms (x-axis) over their respective headquartered sovereign CDS spreads (y-axis) for various CDS terms. Each panel corresponds to the oil firms headquartered in any given country. Each column corresponds to a different CDS term (1-year, 5-year and 10-year) while each row is the specified oil firm. We see that under various CDS terms, oil firm credit risk and sovereign credit risk are correlated.

In figures 12 through 26, we do a similar exercise using data from Moody's. Moody's calculates a measure called a 5 Year Fair Value CDS (FVS-CDS) spread. These are modeled CDS spreads that can be directly compared with observed 5-year CDS spreads. This measure allows us to expand coverage of the total number of oil firms and the time frame. We look at oil firm corporate 5-year FVS-CDS spreads (x-axis) over their respective headquartered sovereign 5-year CDS spreads (y-axis). We also look at oil firm corporate 5-year CDS spreads over their respective headquartered sovereign 5-year CDS spreads for comparison. Each panel corresponds to one country and the oil firms headquartered there. Each column corresponds to the specified oil firm. The top row is oil firm's 5-year CDS spread, and the bottom row is Moody's calculated 5-year FVS-CDS for oil firms. We observe again that oil firm credit risk and sovereign credit risk are correlated under both spreads.

Figure 24: Brazil

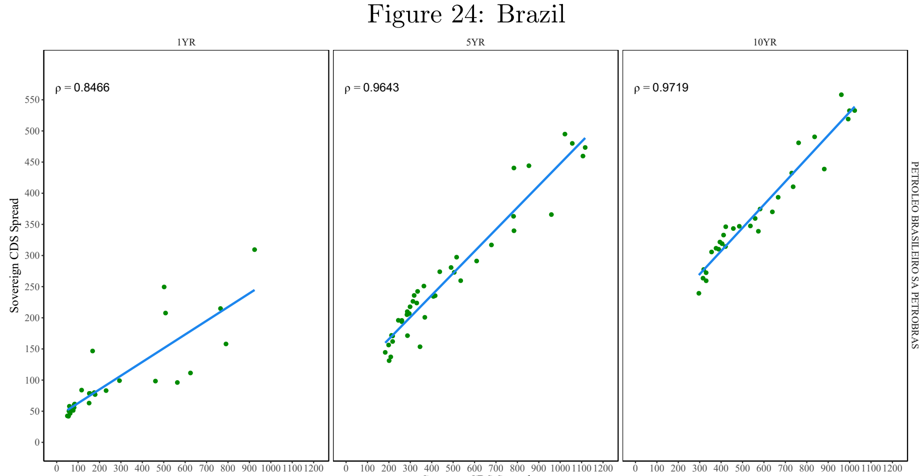


Figure 25: France

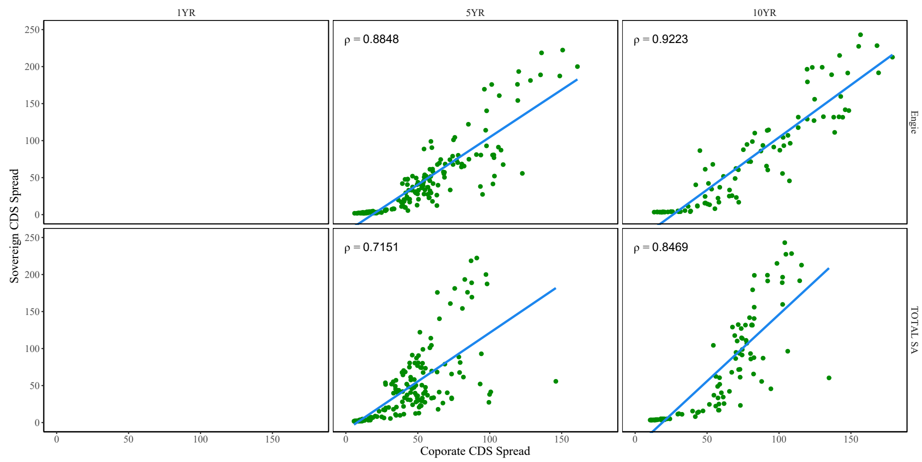


Figure 26: Italy

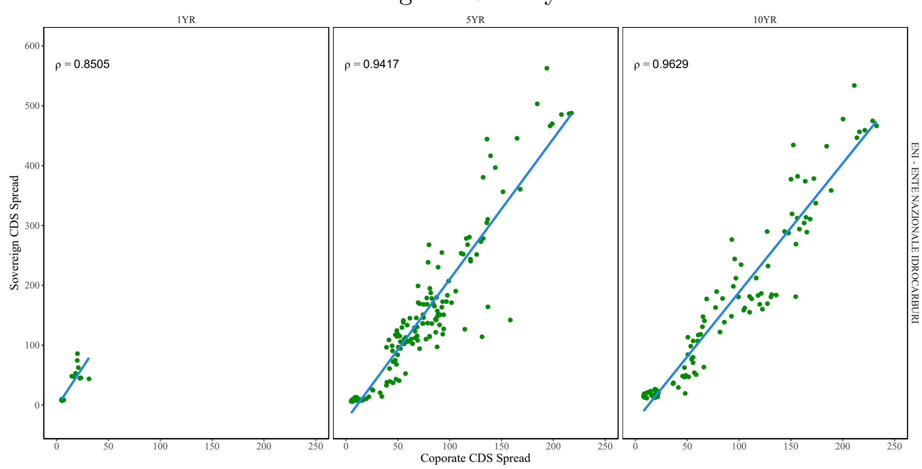


Figure 27: Mexico

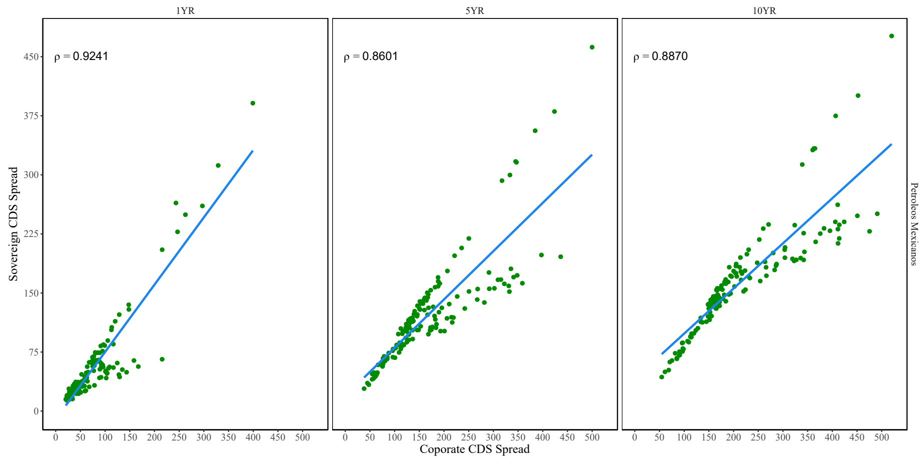


Figure 28: Netherlands

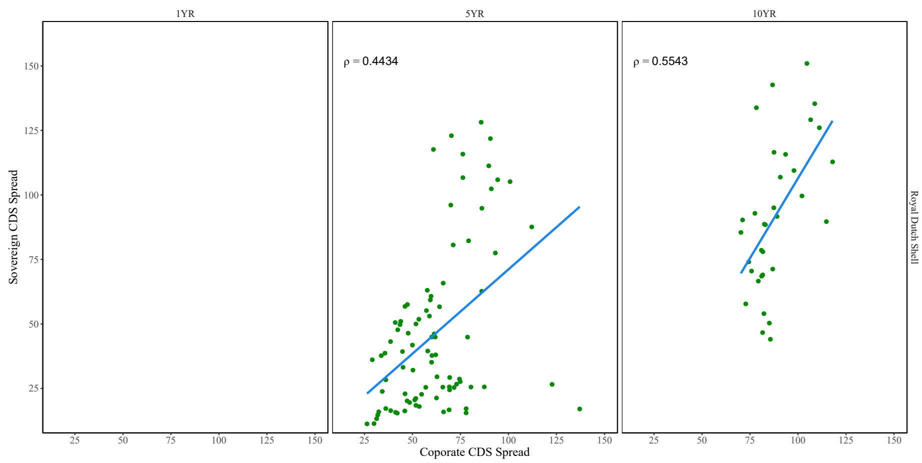


Figure 29: Norway

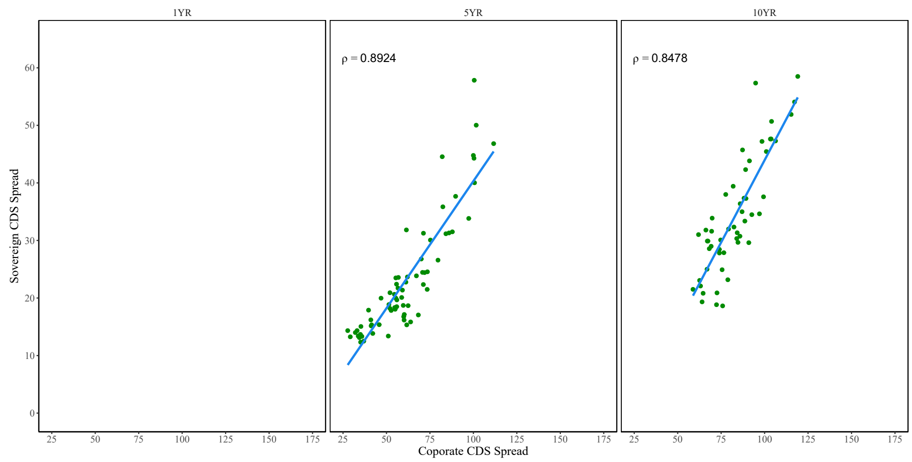


Figure 30: Spain

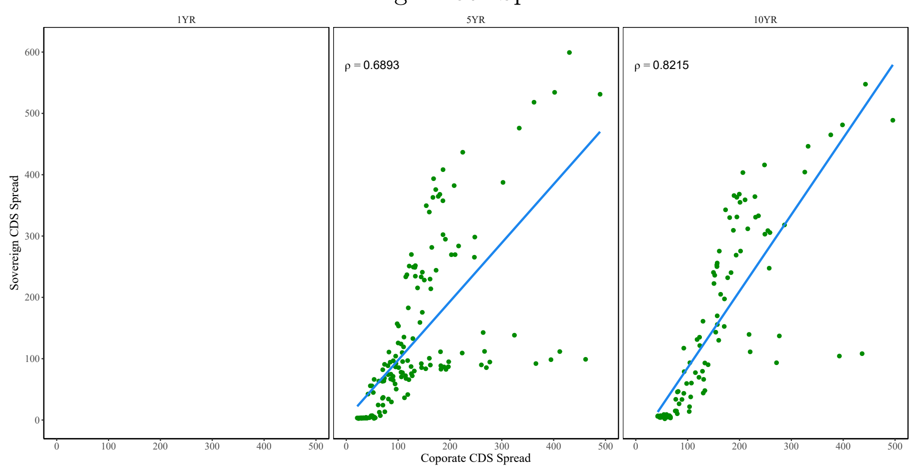


Figure 31: United Kingdom

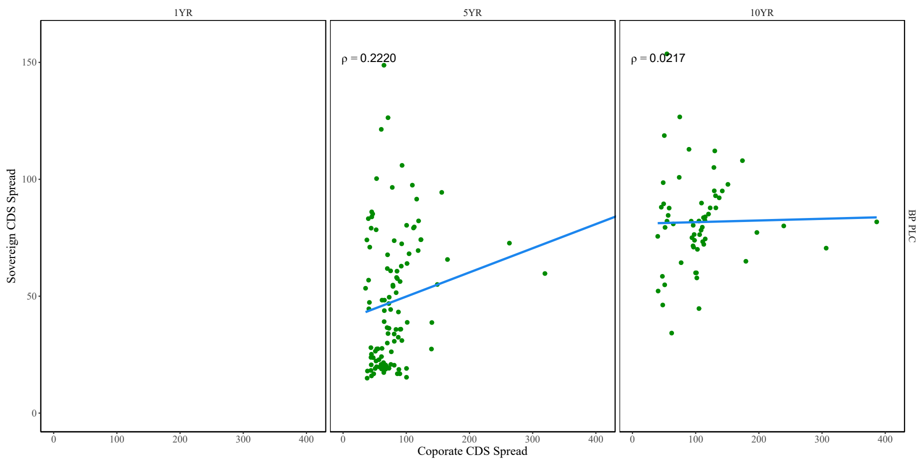


Figure 32: United States

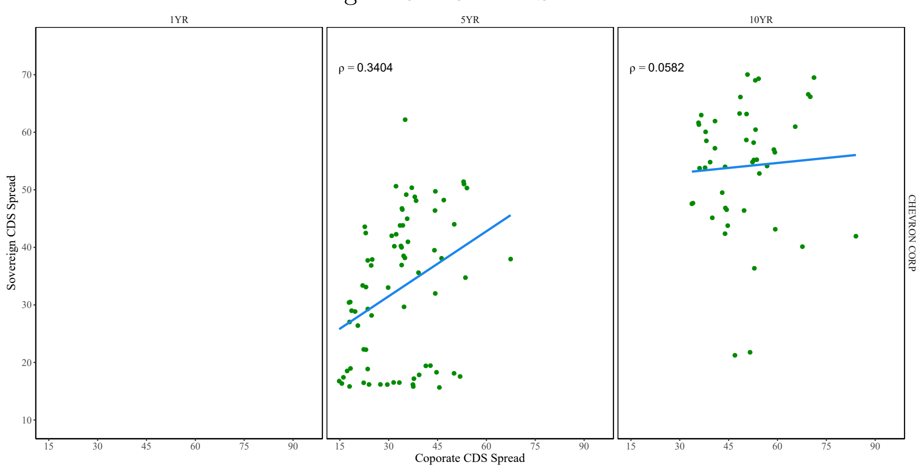


Figure 33: Argentina

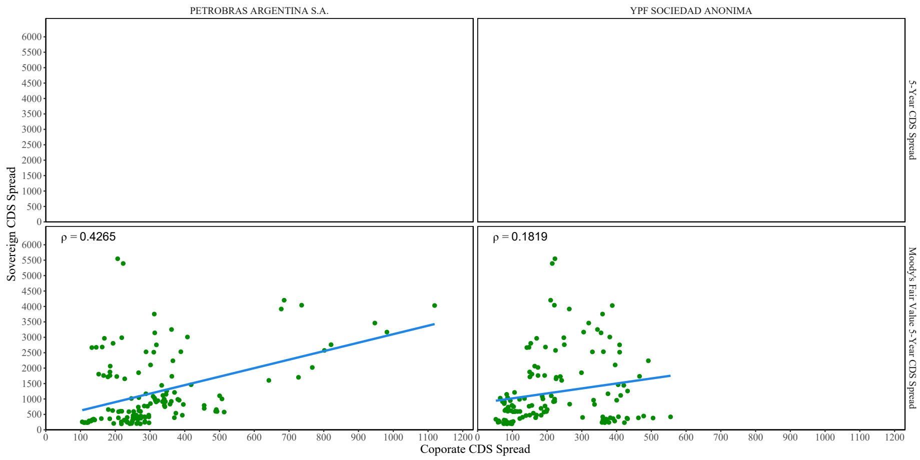


Figure 34: Brazil

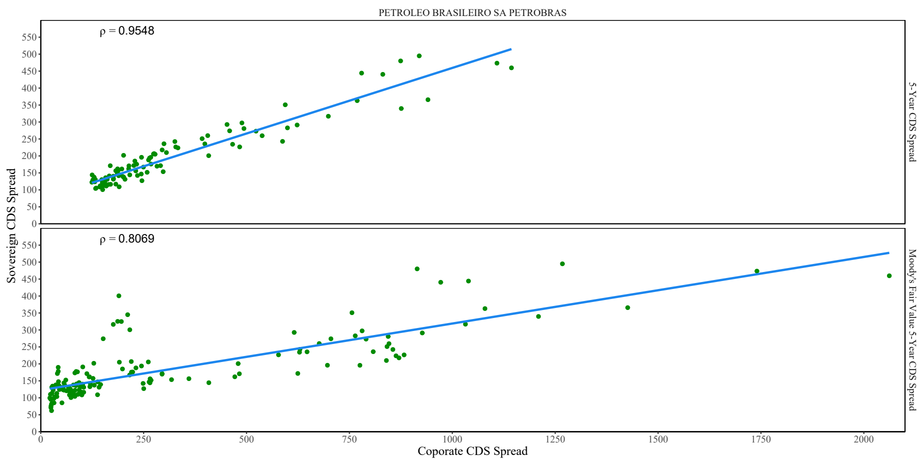


Figure 35: China

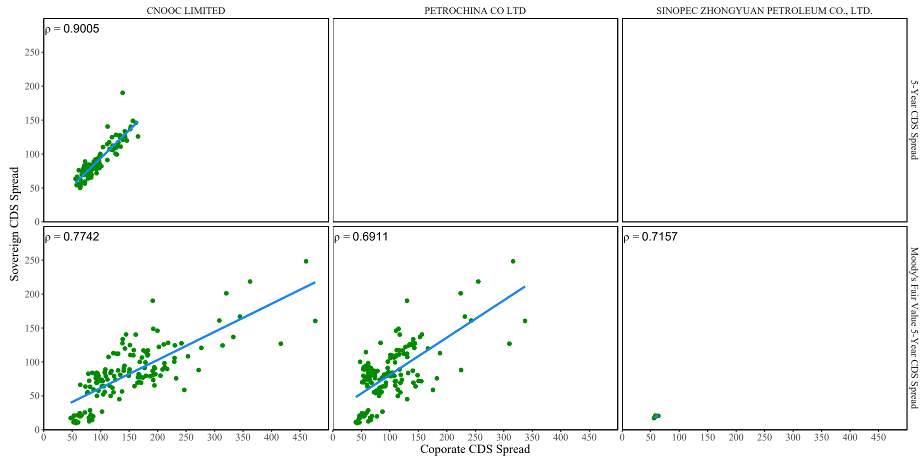


Figure 36: Colombia

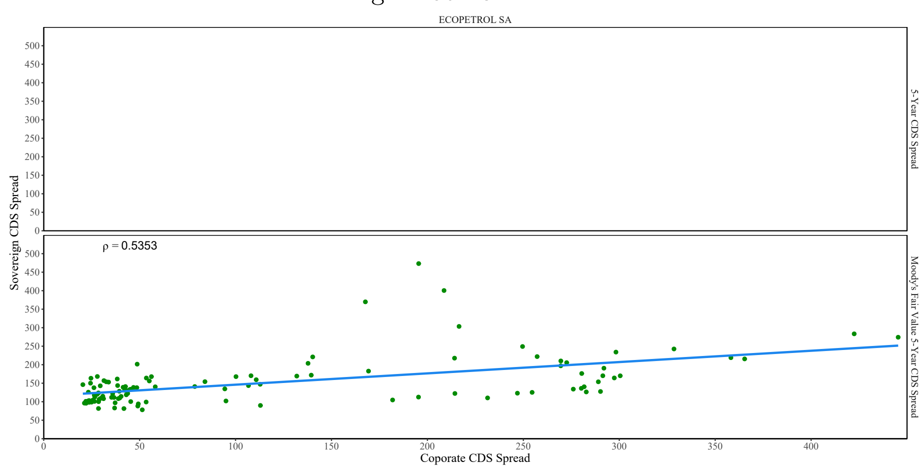


Figure 37: France

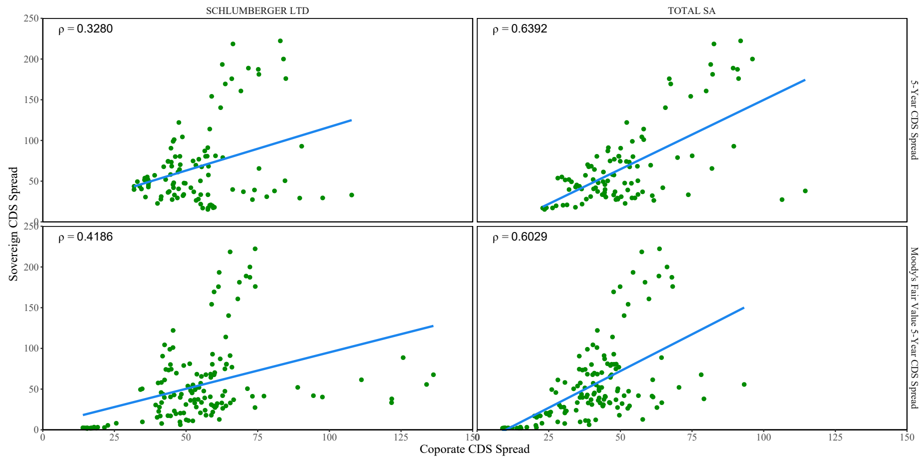


Figure 38: Greece

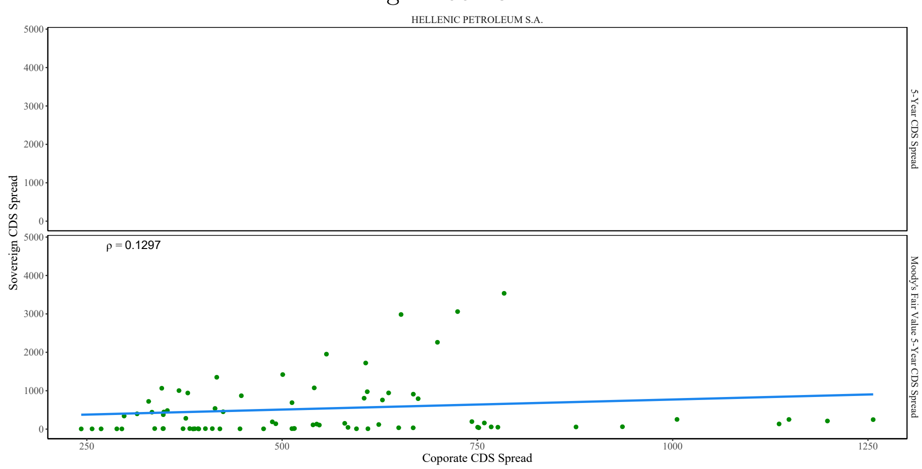


Figure 39: Italy

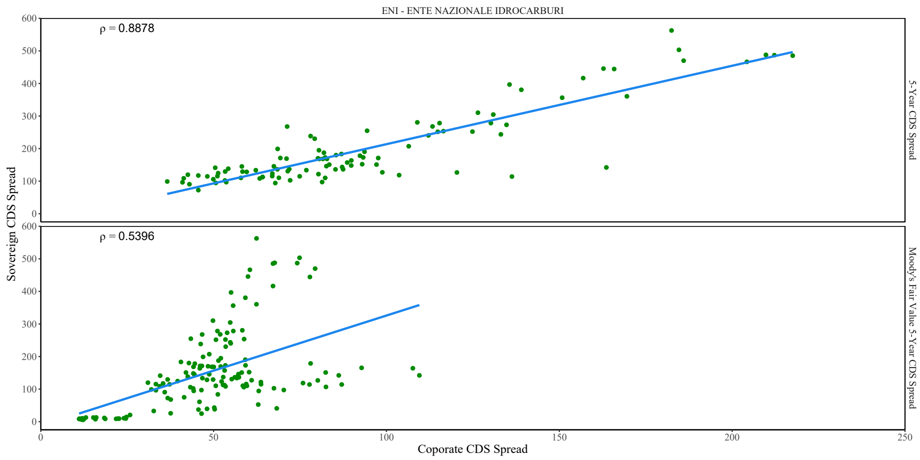


Figure 40: Malaysia

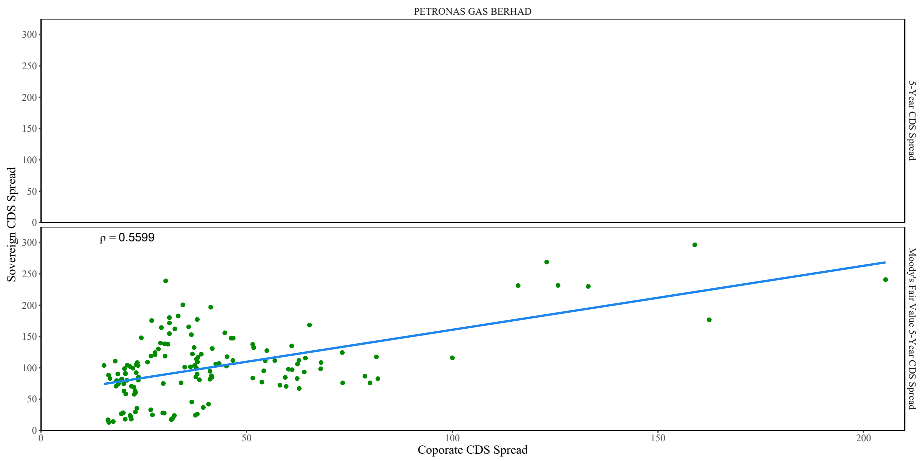


Figure 41: Norway

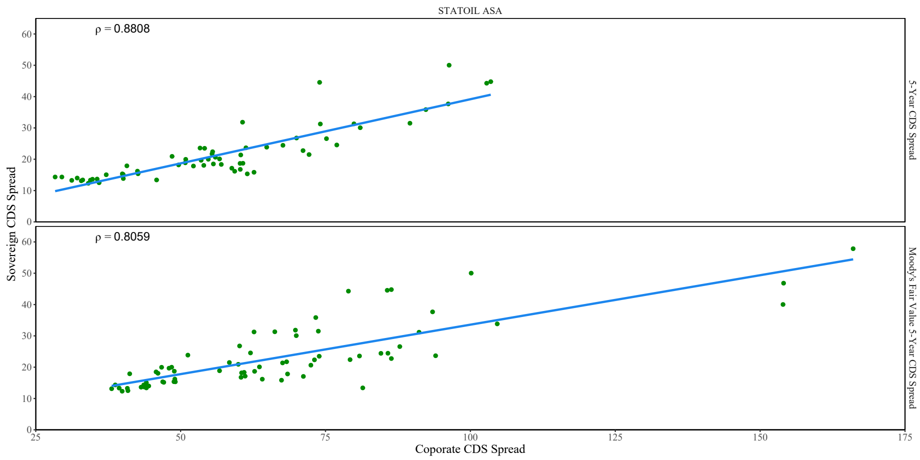


Figure 42: Poland

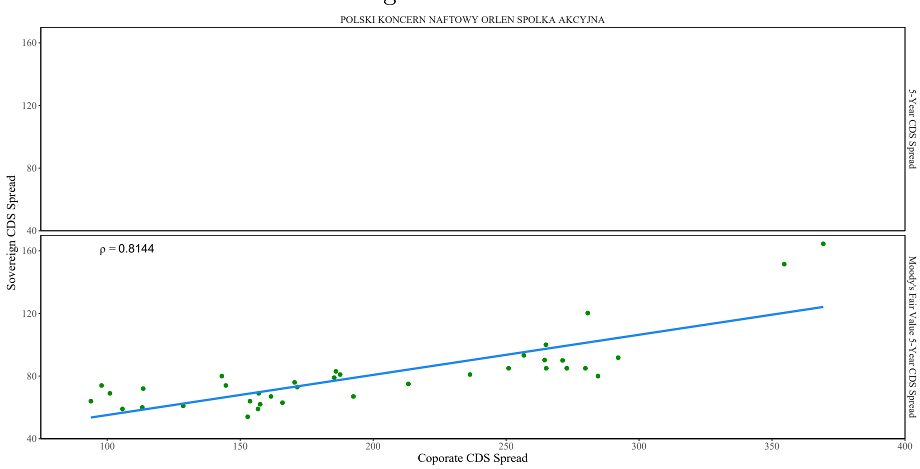


Figure 43: Russia

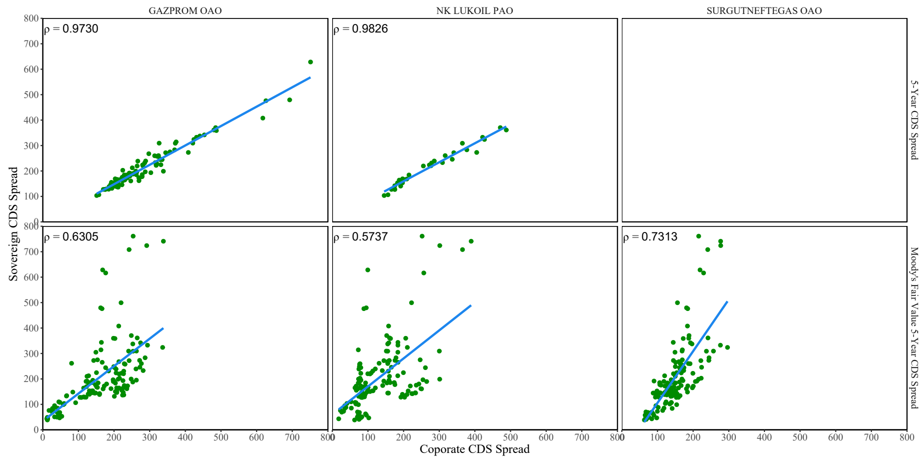


Figure 44: Spain

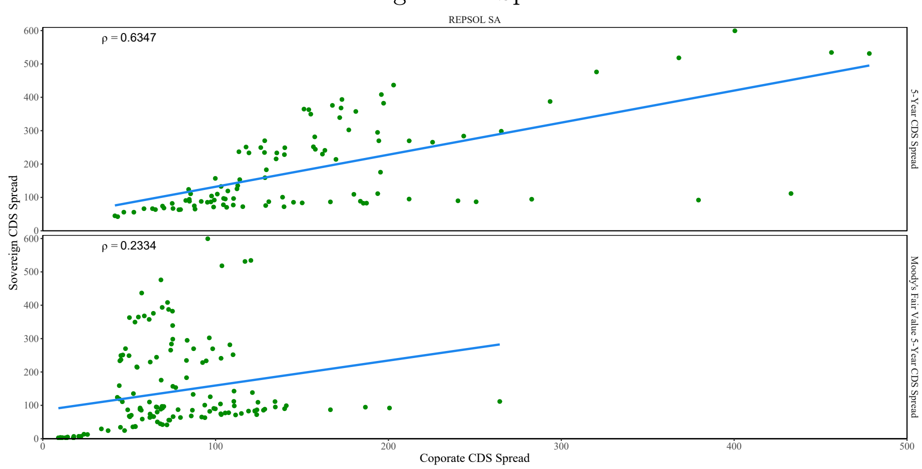


Figure 45: Thailand

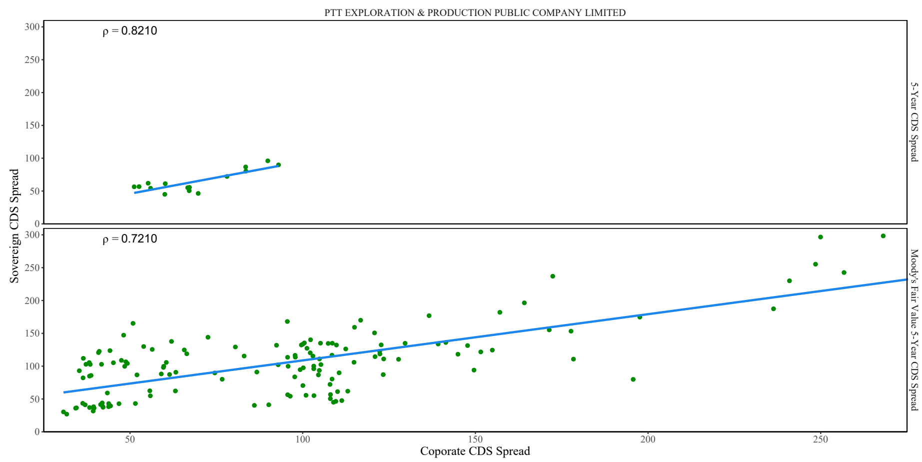


Figure 46: United Kingdom

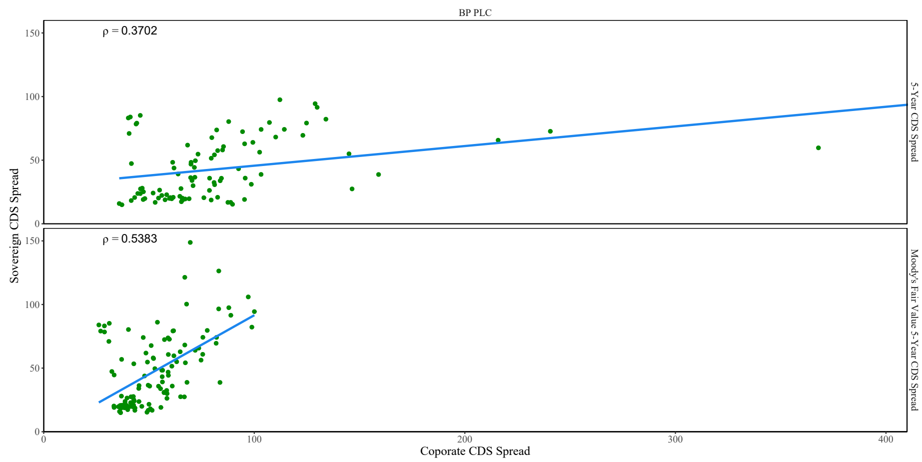
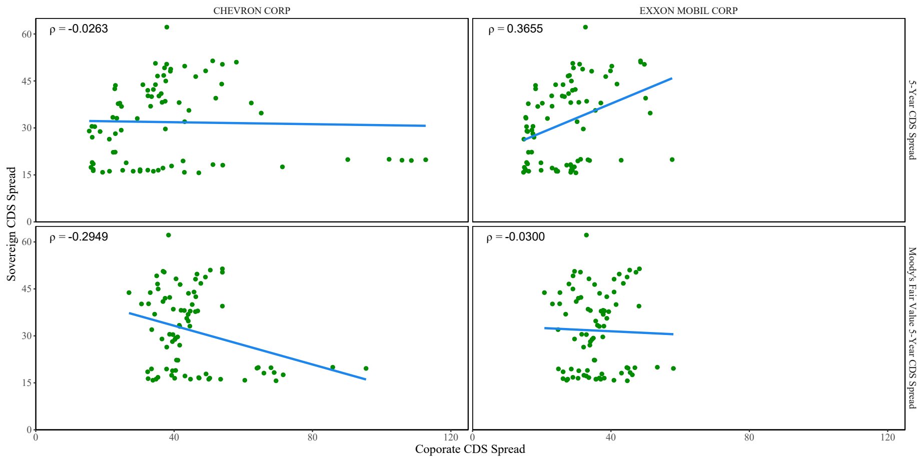


Figure 47: United States



Appendix G: The Two Period Model

We assume that reserves are fixed at $\bar{s}$, there are no discoveries and the endowment of the non-tradable good that the sovereign receives is parametric and equal in both periods. In the first period there is no uncertainty—p is known—and the oil company chooses extraction today, x, and extraction tomorrow, x', given the expected price for tomorrow

$$\max_{x\geq 0, x'\geq 0} px - e\left(x,\bar{s}\right) + q\left(b'\right) E_{p'}\left[p'x' - e\left(x'\right)\right] \tag{17}$$

subject to the resource constraint

$$x + x' = \bar{s}.$$

Recall that oil extraction costs are given by Equation 3 and in the two period case, $e\left(x,\bar{s}\right) = \frac{\varphi}{2}\left(\frac{x}{\bar{s}}\right)^{\gamma} x$ and $e\left(x'\right) = \frac{\varphi}{2}x'$ (because $x' = \bar{s} - x$). Furthermore, we assume that p' is a random variable distributed uniformly over the interval $[0,\bar{p}']$, such that its density function is

$$h\left(p'\right) = \begin{cases} \frac{1}{\bar{p}'} & \text{if } 0 < p' \leq \bar{p}' \\ 0 & \text{otherwise} \end{cases},$$

and its expected value is $E\left(p'\right) = \frac{\bar{p}'}{2}$. Under this assumption, the solution to the problem in Equation 17 implies that the optimal extraction policies (assuming an interior solution) are given by

$$x^{*} = \max\left\{\min\left[\left[\frac{2p - q\left(b'\right)\left(\bar{p}' - \varphi\right)}{\left(1+\gamma\right)\varphi}\right]^{\frac{1}{\gamma}}\bar{s}, \bar{s}\right], 0\right\}, \tag{18}$$

and extraction in the second period is given by

$$x'^{*} = \max\left\{\min\left[\left(1 - \left[\frac{2p - q\left(b'\right)\left(\bar{p}' - \varphi\right)}{\left(1+\gamma\right)\varphi}\right]^{\frac{1}{\gamma}}\right)\bar{s}, \bar{s}\right], 0\right\}. \tag{19}$$

At the beginning of the second period p' is realized and x'^{*} is observed by the sovereign who decides whether to repay or default based on the relative utilities

$$d = \begin{cases} 1 & \text{if } u\left(y' + \theta p' x'^{*}\right) \geq u\left(y' + p'x'^{*} + b'\right) \\ 0 & \text{otherwise} \end{cases}$$

This implies that for a given level of b', the realization of the price tomorrow, p', has to be above a threshold $\tilde{p'}$ for the sovereign to repay. This threshold satisfies the condition

$$\tilde{p'} x'^{*} = \frac{b'}{\left(\theta - 1\right)}. \tag{20}$$

Because the default set is defined by

$$\mathcal{D}\left(b', p, \bar{s}\right) = \left\{p' \in \mathcal{P} : d\left(b'; p, p', \bar{s}\right) = 1\right\},$$

then the probability of default is determined by the price realizations between zero and $\tilde{p'}$

$$d\left(b', p, \bar{s}\right) = \int_0^{\tilde{p'}} d\left(b'; p, p', \bar{s}\right) h\left(p'\right) dp'. \tag{21}$$

This allows us to write the problem of the sovereign who chooses debt to solve the following problem

$$\begin{aligned} \max_{b'} \quad & u\left(y + px^* - q\left(b', p, \bar{s}\right) b'\right) \\ & +\beta \left\{ d\left(b', p, \bar{s}\right) \int_0^{\tilde{p'}\left(b', p, \bar{s}\right)} u\left(y' + \theta p' x'^*\right) h(p') dp' \right. \\ & \left. + \left[1 - d\left(b', p, \bar{s}\right)\right] \int_{\tilde{p'}\left(b', p, \bar{s}\right)}^{\infty} u\left(y' + p' x'^* + b'\right) h(p') dp' \right\} \quad . \end{aligned} \tag{22}$$

Note that for $x \in (0, \bar{s})$, today's optimal extraction, x^*, is increasing in today's price, p, and the stock of available oil reserves, $\bar{s}$, but decreasing in future prices, p'. Also, today's extraction, x^*, is decreasing in the sovereign bond price. This is intuitive because a higher q implies that their yield drops and the opportunity cost of extracting oil falls, giving the oil producer the incentive to adjust extraction plans so as to reduce the return on "investing" in oil production (i.e. investing in oil reserves) to match the lower return on bonds. This implies extracting less today, because extracting less today reduces the marginal cost of extraction (for a given p), and hence increases the net marginal revenue on oil extraction today (that is $p - e_x(x, s)$ rises), hence reducing the rate of return on oil reserves. This is all working through Equation 4.

As mentioned earlier, the planner is aware of the relationship mentioned above, so in choosing debt issuance he takes into account how an increase in b' will reduce q resulting in more extraction today. Finally, also notice, that there is a level of today's price $\hat{p}$ above which it is optimal to extract all the available oil reserves today

$$\hat{p} = q\left(\frac{\bar{p}' - \varphi}{2}\right) + \frac{\varphi\left(\gamma + 1\right)}{2}.$$

Likewise, for $x \in (0, \bar{s})$, tomorrow's optimal extraction, x'^*, is increasing in future prices, p', and $\bar{s}$, but decreasing in p. Similarly, there is a level of the future expected price $E\hat{p'}$ such that at prices higher than this level it is optimal to extract all the available oil reserves in the second period

$$E\hat{p'} = 2\frac{p}{q} + \varphi.$$

Substituting for the optimal extraction policy in the second period in Equation 20, and letting H

be the cdf of p' we can compute the probability of default $H\left(\tilde{p'}\left(b', \bar{s}, p\right)\right)$

$$\delta = \tilde{H} = \frac{\tilde{p'}}{\bar{p}'} = \frac{\phi}{\left(1 - \left[\frac{2p - q(b')(\bar{p}' - \varphi)}{(1+\gamma)\varphi}\right]^{\frac{1}{\gamma}}\right)\bar{s}\bar{p}'},$$

where $\phi = \frac{b'}{(\theta - 1)}$.

Since in equilibrium the price of sovereign bonds must satisfy the arbitrage equation, $q = \bar{q}\left(1 - \tilde{H}\right)$ then,

$$q = \bar{q}\left(1 - \frac{\phi}{\left(1 - \left[\frac{2p - q(b')(\bar{p}' - \varphi)}{(1+\gamma)\varphi}\right]^{\frac{1}{\gamma}}\right)\bar{s}\bar{p}'}\right).$$

Figure 48 shows the solution to q:

Figure 48: Bond price implied by the no-arbitrage condition