Banking Competition and Stability: The Role of Leverage*

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Abstract

This paper reexamines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting the different types of risk and the role of leverage. By means of a simple model we explore whether competition increases portfolio risk, insolvency risk, liquidity risk and systemic risk. We show that the effect of competition on financial stability depends crucially on banks' liability structure, on whether banks are financed by insured retail deposits or by uninsured wholesale investors, whether indebtness is exogenous or endogenous and on the degree of competition in the banking industry. In particular we show that, while in a classical originate-to-hold banking industry competition might increase financial stability, the opposite might be true for an originate-to-distribute banking structure characterized by a larger fraction of market short term funding. This leads us to revisit the existing empirical literature using a more precise classification of risk. Our theoretical model, thus clarifies a number of apparently contradictory empirical results and proposes new ways to analyze the impact of banking competition on financial stability.

Keywords: Banking Competition, Financial Stability, Leverage

JEL Classification: G21, G28

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1 Introduction

This paper reexamines the classical issue of the possible trade-offs between banking competition and financial stability by highlighting the different types of risk and the role of leverage. By means of a simple model we show how competition affects portfolio risk, insolvency risk, liquidity risk and systemic risk in different ways. The relationships depend crucially on banks' liability structure, on whether banks are financed by insured retail deposits or by uninsured wholesale investors, whether this is exogenous or endogenous and on the degree of competition in the banking industry. In particular, we show that, while in a classical deposit insurance funded banking economy competition might increase financial stability, the opposite might hold true for an originate-to-distribute banking structure with a larger fraction of market short term funding.

Understanding the link between bank competition and financial stability is essential to the design of an efficient banking industry and its appropriate regulation. Because of the relevance of this topic, a large body of literature devoted to the issue has developed, with important contributions from both theoretical and empirical perspectives. Yet, in spite of the critical importance of the subject and notwithstanding today's improved understanding of its complexity, there is no clear-cut consensus on the impact of competition on banks' risk taking and on the resulting overall financial stability. This has been the major motivation for this paper.

On this issue, two main theoretical modeling approaches contend: the charter value view and the risk shifting view. The charter value theory, first put forward by Keeley (1990), assumes that banks choose their level of risk and argues that less competition makes banks more cautious in their investment decisions, as in case of bankruptcy they will lose the present value of the future rents generated by their market power. Instead, proponents of the risk shifting hypothesis, which originated with Boyd and De Nicolo (2005), postulate that risks result from the borrowing firms' decisions and point out that higher interest rates will lead firms to take more risk and therefore will increase the riskiness of the banks' portfolio of loans.

The theoretical debate on the impact of banking competition cannot be simply solved by resorting to the empirical evidence, which, as mentioned, is often ambiguous and contradictory.[1] Part of the ambiguity stems from the difficulty in the choice of measurements for "financial stability". Indeed, a bank's risk has multiple dimensions.

[1] See Beck, Jonghe, and Schepens (2011), who show that the relationship between competition and financial stability is ambiguous and displays considerable cross-country variation.

The empirical literature reveals a great diversity in the concepts of "competition" and "financial stability" , with analysis differing in both the measures of competition and risk.[2] This is why a first requirement for the analysis of the link between banking competition and financial stability is the building of a model encompassing the different types of banking risks: portfolio risk, banks' insolvency risk, illiquidity risk and systemic risk. A second requirement is to consider the endogeneity of leverage, a point the literature has largely ignored.[3] Because leverage affects banks' solvency, liquidity and financial contagion, it constitutes a central hub that connects all types of banking risk and plays a key role in the analysis of the impact of competition on financial stability. Thus, for instance, safer portfolios can lead banks to take on more debt, and, consequently, the insolvency risk of banks is not necessarily reduced, while their funding liquidity risk and systemic risk is increased.

Our approach builds on a large body of literature on banking competition that starts with the seminal paper of Keeley (1990). As mentioned before, Boyd and De Nicolo (2005) rightly point out that the intrinsic countervailing force of firms' risk-shifting can make the relationship between competition and financial stability ambiguous. Martinez-Miera and Repullo (2010) further refine Boyd and De Nicolo's argument by showing that the low profit resulting from competition reduces banks'buffer against loan losses and can, therefore, jeopardize financial stability. Wagner (2009) considers both banks' and entrepreneurs' incentives to take risk on the portfolio side: once entrepreneurs and banks move sequentially, the overall effect coincides with the charter value hypothesis. The fact that all these contributions focus solely on insolvency risk and take the simplifying assumption of exogenous leverage has been one of the main motivations for our paper.

The study of banks' leverage in a competitive environment is addressed in Allen, Carletti, and Marquez (2009). The authors show that as competition decreases charter values, banks' incentives to monitor borrowers are reduced. To provide banks with the proper incentives to monitor, one way is to require them to hold more capital.

Because our objective is to explore the impact of competition on the different types of risk, our starting point has to be the microfoundations of borrowing firms' risk taking. Following Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010), we assume that firms' investment decisions are subject to moral hazard, such that a higher interest rate leads them to

[2]Table 3 offers a synthetic survey of the different choices in the measures of competition and risk in the empirical contributions to the analysis of the competition-financial stability link.

[3]For example, Boyd and De Nicolo (2005) considers banks solely financed by debt. Martinez-Miera and Repullo (2010) assume the cost of equity to be independent of banks' risk.

take riskier investment projects. Consequently, greater banking competition decreases portfolio risk but reduces the banks' buffer provided by market power. Liquidity risk is then introduced through a global games approach and contagion risk through the lower market price of banks' assets. Using this framework, we study the impact of banks' competition on financial stability, both when bank leverage is exogenous and when it is endogenous.

Our main result is that the relationship between insolvency risk and banking competition crucially depends on the degree of banks' market power and their liability structure. The use of a specific model allows us to solve the model analytically and show that the risk shifting hypothesis is satisfied for a low level of insured deposits and high levels of market power while the charter value is correct in the opposite case. So the impact of banking competition on financial stability could be the opposite in a classical originate-to-hold banking industry and in an originate-to-distribute one. This result is helpful in understanding the apparent contradictions in the empirical results; it is also relevant to test the impact of banking competition as it predicts that the results should vary depending on the characteristics of the banking industry.

In addition, our analysis of the different types of risks allow us to establish the impact of increased banking competition on each type of risk. First, when leverage is exogenous, which can be interpreted as capital ratios being binding, competition will always increase liquidity risk. If instead, leverage is endogenous, it presents a countervailing force to portfolio risk, moving always in the opposite direction, as when the portfolio of assets becomes safer the bank will increase its leverage and its solvency and liquidity risks and conversely[4]. Nevertheless, the total credit risk of a bank, defined as the sum of solvency risk plus funding liquidity risk is dominated by the impact of competition on insolvency risk.

We extend the model to incorporate systemic risk and find the results to be robust: although the overall risk is higher, the impact of competition on financial stability still depends upon de degree of banks' market power and their liability structure.

The paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the benchmark case, exploring how various risks are affected by banking competition under the assumption of exogenous leverage. In section 4, we determine endogenous bank leverage and analyze its impact on banks' insolvency, illiquidity and systemic risk. The results contrast with those under exogenous leverage. We devote section 5 to the empirical literature, reinter-

[4]This is clearly illustrated in the extreme case where a bank's strategy is to maintain a given insolvency risk, which is in line with the idea of "economic capital" . In this case, any changes in portfolio risk are exactly offset by the bank's leverage adjustment.

preting the empirical findings with the refined definition of "financial stability" and forming new testable hypotheses. Relevant policy implications are discussed in section 6. Section 7 concludes.

2 Model Setup

2.1 Portfolio risk and competition

We consider a one-good three-date ($t = 0, 1, 2,$) economy where all agents are assumed to be risk neutral. There are three types of active agents: entrepreneurs, banks and banks' wholesale financiers, and one type of purely passive agents: retail depositors. There is a continuum of entrepreneurs in the economy. They are penniless but have access to long-term risky projects. A project requires one unit of investment. It yields a gross return of $x > 1$ if succeeds, and 0 if fails. The projects are subject to moral hazard: Each entrepreneur chooses the probability of success $P \in (0, 1]$ in order to maximize his expected utility,

$$E(U) = P(x - r) - \frac{P^2}{2b}. \tag{1}$$

Here $r \in (1, x)$ is the gross loan rate charged by banks. $b \in (0, B]$ represents an entrepreneur's type, with a higher b implying a lower marginal cost of efforts. Entrepreneur types are private information, and in particular, unknown to banks, which hold prior beliefs that b is uniformly distributed in the interval $[0, B]$. Entrepreneurs' reservation utility is normalized to zero.

Because idiosyncratic risk diminishes in a bank's well diversified portfolio of loans, we dispense with modeling this type of risk and focus, instead, on bank level risk that affects the whole portfolio in the following way: Whether a project succeeds or not is jointly affected by entrepreneurs' choice P and a bank-level risk factor z. The risk is assumed to be identical for all loans in a bank's portfolio, but can change across banks. It is assumed that z follows a standard normal distribution. Following Vasicek (2002) and Martinez-Miera and Repullo (2010), we assume the failure of a project is represented by a latent random variable y. When $y < 0$, the project fails.[5] The variable y takes the following form

$$y = -\Phi^{-1}(1 - P) + z, \tag{2}$$

where Φ denotes the c.d.f. of standard normal distribution. A project defaults either because of the entrepreneur's moral hazard (a low P) or an unfortunate risk realization that affects the bank's whole portfolio (a low z). For the sake of consistency, note that the probability of success P is given by:

$$Prob(y \geq 0) = 1 - Prob(y < 0) = 1 - Prob(z < \Phi^{-1}(1 - P)) = 1 - \Phi(\Phi^{-1}(1 - P)) = P.$$

Banks are assumed to invest in a continuum of projects. We further assume the loan market is fully covered and all types of entrepreneurs are financed. The loan portfolio generates a random cash flow that we denote by θ.

In order to focus on bank leverage and risks, we dispense with the specific modeling of loan market competition and consider the loan rate r as a sufficient statistic for the degree of competition. With lower r associated greater competition, our setup captures the driving force for risk reduction in Boyd and De Nicolo (2005) and is consistent mainstream competition models that predict competition leading to lower spreads.[6] The assumption also fits the empirical results, as low interest margins are found to be associated with less concentrated markets, Degryse, Kim, and Ongena (2009).

2.2 Funding and liquidity risk

Each bank holds a unit portfolio of loans, and finances it with debt and equity. At $t = 0$, a bank finances its total investment of size 1 by raising F from insured retail deposits, V_D from short-term wholesale creditors, and the rest from equity holders. Because retail depositors are insensitive to the banks' risks and play a purely passive role, we assume that their supply is

[5] This additional complexity is necessary to analyze the role of banks' leverage. Indeed, under the two classical assumptions of either zero or perfect correlation, a bank's capital level does not affect its insolvency risk. In the first case, law of large numbers leads to zero portfolio risk for a well diversified portfolio. In the second case, capital buffer is insufficient to cope with loan default shocks.

[6] The opposite relationship may be obtained in models based on Broecker (1990) where an increase in the number of banks raises the probability for a bad borrower to get funded in equilibrium which implies an increase in the equilibrium interest rate.

fixed and inelastically set equal to F, and that safety net of deposit insurance is offered to banks at no cost.[7]

Debt issued to wholesale financiers promises a face value D at $t = 2$, is demandable, uninsured, and risky. It is raised in a competitive market where investors are risk neutral and require the market interest rate that is here normalized to zero. Each bank's debt is jointly financed by a continuum of creditors. The short-term nature of the debt allows creditors to withdraw at $t = 1$, before banks' risky investment matures. In that case an creditor receives qD, where $1 - q \in (0, 1)$ represents an early withdrawal penalty. Alternatively, the same debt contract can simply be viewed as promising an interest rate qD/V_D at time $t = 1$ and D/V_D at time $t = 2$.

The bank's risky loan portfolio takes two periods to mature. When the bank faces early withdrawals, it has to sell part (or all) of its portfolio in a secondary market at a fire-sale discount:[8] for one unit asset with cash flow θ, the bank obtains only

$$\frac{\theta}{1+\lambda}.$$

Here $\lambda > 0$ reflects the illiquid nature of banks' long-term assets that can be attributed to moral hazard, e.g., banks' inalienable human capital in monitoring entrepreneurs, or adverse selection, e.g., buyers are concerned with banks selling their 'lemon' loans. The maturity mismatch and fire-sale discount together expose banks to the risk of bank runs. We focus on the natural case where runs make it more difficult for a bank to repay its debt, which occurs when the discount on the value of assets is greater than that on liabilities.

$$\frac{1}{1+\lambda} < q \tag{3}$$

If condition (3) is not satisfied, the scenario will be paradoxical in the sense that a bank can more easily meet its debt obligations in a fire sale, and an insolvent bank with $\theta < D + F$ will be saved by a run, provided $(1 + \lambda)qD + F < \theta$.[9]

Once a bank's cash flow is insufficient to repay its debt, it declares bankruptcy and incurs a bankruptcy cost. For simplicity, we assume the bankruptcy cost is sufficiently high such

[7] Assuming a flat deposit insurance premium that is based on the expected equilibrium debt ratio will not change our results.

[8] The alternative assumption of banks using collateralized borrowing generates similar results. See Morris and Shin (2009).

[9] Note also that condition (3) is always true as q approaches 1.

that once bankruptcy happens, the wholesale creditors get zero payoffs and a senior deposit insurance company that represents retail depositors gets the residual cash flow.

At $t = 1$ each wholesale creditor privately observes a noisy signal $s_i = \theta + \epsilon_i$, where ϵ_i is pure noise that follows a zero-mean continuous distribution on a small interval. Based on the signal, the wholesale creditors play a bank-run game. Each player has two actions: to wait until maturity or to withdraw early. For a creditor who chooses to withdraw early, she receives nothing if the bank fails at $t = 1$, and qD if the bank does not fail on the intermediate date. If the bank is only able to pay early withdrawals at $t = 1$ but goes bankrupt at $t = 2$, creditors who wait receive nothing. If the bank does not fail at $t = 2$, depositors who wait receive the promised repayment D.

2.3 Leverage

Banks will choose their leverage so as to maximize the equity value of banks' existing shareholders.[10] In particular, each bank chooses to issue an amount of debt that promises a repayment qD at $t = 1$ and D at $t = 2$, with $q < 1$. We assume that capital is costly due to market frictions or tax shield. Because of the existence of bankruptcy costs, the optimal leverage ratio will trade off between the cost of equity and the expected cost of bankruptcy. The existence of a liquidity risk makes the choice of leverage more complex as banks take into account both insolvency risk and illiquidity risk.

2.4 Time line

The timing of the model is summarized in the figure below.

t = 0	t = 1	t = 2
1. Banks choose capital structure (D). 2. Entrepreneurs choose P for a given r.	1. Upon signals, wholesale financiers decide whether to run or not. 2. Banks facing a run sell assets at discount. 3. Banks fail or survive.	1. Returns realize for surviving banks. 2. Wholesale financiers who have not run on surviving banks get paid.

[10] It can be shown that this assumption in turn implies the maximization of leveraged firm values.

3 Banking risks with exogenous leverage

In this section, we analyze various bank risks for a fixed level of leverage. We move upward the spectrum of types of risk: from bottom (individual loan default risk) to the top (systemic risk).

3.1 Loan default and risk shifting

In the spirit of Boyd and De Nicolo (2005), we first show that bank competition reduces loan default risk by curbing entrepreneurs' moral hazard. Note that entrepreneurs' utility maximization yields the following probability of success.[11]

$$P_b^*(r) = \begin{cases} 1 & \text{if } b \in [1/(x-r), B] \\ b(x-r) & \text{if } b \in (0, 1/(x-r)) \end{cases}$$

While an entrepreneur of type $b \geq 1/(x-r)$ will not default for any finite realization of z, loans to entrepreneurs of lower types can default. This makes a natural partition between risk-free and risky loans. Given the uniform distribution of b, it implies that a fraction α of loans

$$\alpha \equiv 1 - \frac{1}{B(x-r)} \tag{4}$$

are risk free, and the complementary fraction $1-\alpha$ of loans are risky and have positive probabilities of default. The riskiness of a bank's loan portfolio is fully captured by α.

As in Boyd and De Nicolo (2005), the risk of the portfolio decreases with bank competition. When banks charge lower loan rates under fierce competition, entrepreneurs have more 'skin in the game' and therefore take less risk, so that banks' pool of safe loans grows.

$$\partial\alpha/\partial r = \frac{-1}{B(x-r)^2} < 0 \tag{5}$$

3.2 Portfolio risk: loan loss and cash flow

In order to characterize a bank's portfolio risk we now derive the distribution of loan losses and cash flows. A bank's portfolio risk is driven both by entrepreneurs' risk-taking and the

[11] Note $U_E(P_b^*) \geq 0$ so that entrepreneurs' participation constraints are always satisfied for P^*.

bank-level risk factor z. Denote the fraction of non-performing loans *in the risky pool* by γ. We show that γ follows a uniform distribution on $[0, 1]$.

Lemma 1. *The loan loss γ, defined as the fraction of defaults in the risky pool, follows a uniform distribution on* $[0, 1]$.

Proof. See Appendix A.1. $\square$

Lemma 1 suggests that the expected loan loss in the risky pool is always $1/2$. The riskiness of loan portfolio only depends on the size of its risky pool. As it shrinks with a lower loan rates r, the bank's portfolio risk drops.

A loan portfolio generates the following cash flow θ whose stochastics is driven by the random loan loss γ.

$$\theta \equiv \alpha r + (1-\alpha)[0 \cdot \gamma + r \cdot (1-\gamma)] = r\left[1 - \frac{\gamma}{B(x-r)}\right]$$

With γ entering the expression in a linear way, the cash flow θ follows a uniform distribution on $[\alpha r, r]$. Figure 1 depicts two distribution functions of cash flows, associated with different levels of competition (r increases to r'). When competition intensifies, the distribution function becomes steeper, implying a smaller volatility. Analytically, the variance of θ decreases with a lower r.

Lemma 2. *A bank loan portfolio generates a random cash flow $\theta \sim U(\alpha r, r)$. When banking competition reduces the loan rate r, the volatility of the cash flow decreases.*

3.3 Insolvency risk

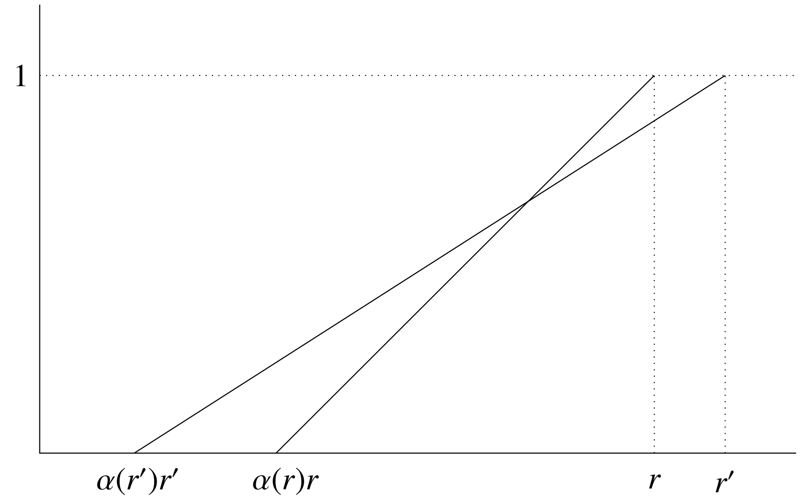
In this subsection, we define a bank's insolvency risk for a given level of debt. A bank is solvent if its cash flow meets its liability,

$$\theta = r - (1-\alpha)r\gamma \geq F + D.$$

The inequality gives a critical level of loan loss $\hat{\gamma}$.

$$\hat{\gamma} \equiv \frac{r - (F + D)}{(1-\alpha)r}$$

Figure 1: Cash flow distribution under two different levels of competition



A bank with realized loan losses greater than $\hat{\gamma}$ will be insolvent. For $\gamma \sim U(0,1)$, this implies that the solvency probability is equal to $\hat{\gamma}$. The bank's pure insolvency risk, i.e., the risk of failure in the absence of bank run, ρ_{SR}, takes the following form.

$$\rho_{SR} \equiv 1 - \hat{\gamma} = \frac{(F+D) - \alpha r}{w} \tag{6}$$

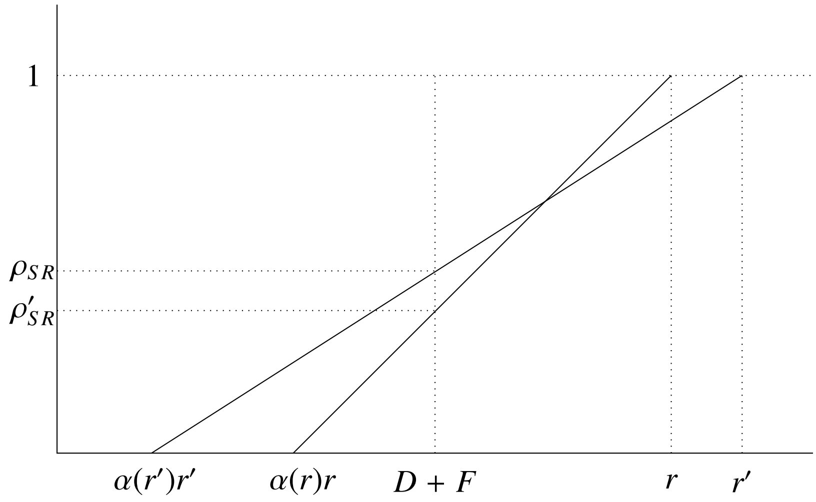
Note that insolvency risk is not monotonic in r. The reason is the same as in Martinez-Miera and Repullo (2010). Banking competition has two countervailing effects on insolvency risk: On the one hand, lower loan rates reduce entrepreneurs risk-taking so that the portfolio losses decrease (risk-shifting reduction). On the other hand, competition also makes the interest margin thinner and banks less profitable, reducing the buffer available to absorb loan losses (buffer reduction). The overall effect is characterized by the following proposition.

Proposition 1. *For a given capital structure, a bank's insolvency risk is reduced by competition if and only if* $r^2 > x(F+D)$.

Proof. See Appendix A.2. □

Graphically, $r^2 > x(F+D)$ is equivalent to two conditions: (1) $\partial(\alpha r)/\partial r > 0$ so that the distribution function satisfies a single crossing condition, and (2) the face value of debt should lie to the left of the crossing point. Figure 2 illustrates such a scenario: As banking competition weakens and the loan rate rises from r to r', the solvency probability drops from ρ_{SR} to ρ'_{SR}.

Figure 2: Change of insolvency risk under exogenous leverage



3.4 Funding liquidity risk and bank run

In this section we use the global games approach of Carlsson and Van Damme (1993) to examine banks' funding liquidity risk. We derive a critical level of cash flow below which a bank becomes solvent but illiquid: being able to repay its $t = 2$ liability in full if no one runs on it at $t = 1$, but going to default if sufficient many wholesale creditors withdraw early.

A bank can fail either at $t = 1$ or $t = 2$. In the former case, the liquidation value of all assets is insufficient to repay early withdrawals. In the latter case, while partial liquidation generates sufficient cash to pay early withdrawals, the residual portfolio is insufficient to pay creditors who wait until $t = 2$. Denote by L the fraction of wholesale financiers who run on the bank. A bank survives $t = 1$ withdraws but fails at $t = 2$ if the fraction of early withdrawals exceeds the following threshold.

$$L > \frac{\theta - F - D}{[(1+\lambda)q - 1]D} \tag{7}$$

For simplicity, we focus on the second case where bank runs occur because of agents anticipating a lack of funds at $t = 2$. In other words, we assume that banks can cover their liquidity needs at $t = 1$ even when all wholesale creditors run on the bank, $\theta/(1+\lambda) > qD$; runs are triggered only because the cost in terms of the depletion of bank asset is high enough. And the inequality should be satisfied for the lowest possible θ. That is, $\alpha r > (1+\lambda)qD$.[12]

A creditor's decision to run or not depends on both her signal $s_i = \theta + \epsilon_i$ and her belief concerning other agents' actions. Creditors play a switching strategy: They run on the bank if

[12] One can verify that this condition is satisfied at the optimal debt level in section 4.1.

the observed signal is smaller than a critical level. The equilibrium is characterized by a unique critical level s^*.

The lack of common knowledge leads to the so-called Laplacian property of global games: No matter what signal a player i observes, he has no information on the rank of his signal as compared to the signals observed by the other players. Denote by M the fraction of players that player i believes to observe a higher signal than his. The Laplacian property implies $M \sim [0, 1]$.[13] As the Laplacian property holds for all players and in particular for the creditor who observes the critical signal s^*, the creditor will hold a belief that $M \sim U(0, 1)$ fraction of players will not run on the bank and the rest $(1 - M)$ will. Combined with condition (7), he anticipates the bank to survive with a probability $(\theta - F - D)/[(1 + \lambda)q - 1]D$.

The critical signal s^* is characterized by the indifference between running on the bank or not.

$$Prob(t = 1 \text{ survival}|s = s^*) \cdot qD = Prob(t = 2 \text{ survival}|s = s^*)D$$

Given $Prob(t = 1 \text{ survival}|s = s^*) = 1$ and $Prob(t = 2 \text{ survival}|s = s^*) = (\theta - F - D)/[(1 + \lambda)q - 1]D$, the indifference condition can be rewritten as the following.

$$q = \frac{\theta - F - D}{[(1 + \lambda)q - 1]D}$$

This implies the following critical cash flow θ^*.

$$\theta^* = F + D + q[(1 + \lambda)q - 1]D$$

A run successfully happens when a bank's cash flow θ falls below the critical level θ^*. Define $\mu \equiv 1 + q[(1 + \lambda)q - 1]$. A bank is solvent but illiquid if

$$F + D < \theta \leq F + \mu D,$$

where $\mu > 1$ because $(1 + \lambda)q > 1$. In order to survive potential bank runs, a bank has to make more profit than what is required to be barely solvent. Note that the critical cash flow increases in λ and D: Greater fire-sale losses and more exposure to unstable short-term funding lead to a higher chance of illiquidity.

[13] More detailed discussion of the property can be found in Morris and Shin (2001) and we reproduce the proof in Appendix B.

Proposition 2. *There exists a critical level* $\theta^* = F + \mu D$*,* $\mu = 1 - q[1 - (1 + \lambda_1)q] > 1$*, such that a bank that has cash flow* $\theta \in [F + D, \theta^*]$ *is solvent but illiquid.*

Proposition 2 states that banks with $\theta \in [F + D, F + \mu D]$ face pure liquidity risk: These banks are solvent in the absence of bank runs but insolvent if a run occurs. Because a bank has cash flow $\theta \sim U[\alpha r, r]$, the pure liquidity risk, defined as the probability of a bank being solvent but illiquid, is as follows.[14]

$$\rho_{IL} \equiv \frac{(\mu - 1)D}{(1 - \alpha)r} \tag{8}$$

When a bank's debt obligation is exogenous, its funding liquidity risk increases with competition. The result follows directly from the first order derivative,

$$\frac{\partial \rho_{IL}}{\partial r} = (\mu - 1)\frac{-D}{(1 - \alpha)^2}\frac{\partial(1 - \alpha)}{\partial r} < 0.$$

Competition contributes to illiquidity by reducing the expected cash flows. Intuitively, for a given level of fire-sale losses (λ) and a given level of leverage ($D + F$), the lower cash flow due to intensified competition provides a thinner buffer against fire-sale losses. Creditors who withdraw early cause a greater loss to those who wait. As the negative externalities aggravate, the coordination failure intensifies, and bank runs are more likely.

In practice, it is hard to distinguish bank failures due to insolvency and those due to illiquidity. The observational equivalence makes it sensible to examine a bank's total credit risk, the summation of pure insolvency and illiquidity risk that measures a bank's probability of going bankrupt for either solvency or liquidity reasons. Denoting the total credit risk by $\rho_{TCR} = \rho_{SR} + \rho_{IL}$, we have

$$\rho_{TCR} = \Pr(\theta < \theta^*) = \frac{(F + \mu D) - \alpha r}{(1 - \alpha)r}. \tag{9}$$

Banking competition reduces total credit risk ($\partial \rho_{TCR}/\partial r > 0$) if and only if

$$r^2 > x(F + \mu D).$$

[14] Contrary to the pure insolvency risk, the amount of stable funds provided by insured deposits, F, is absent from the above measure of risk, as retail depositors do not have any incentive to run on the bank. The same would hold true for long-term debts, as by definition their contract make it impossible for the claim holders to run the bank.

Note that this condition is more stringent than the condition in Proposition 1. Thus for a parameter constellation satisfying $x(F + \mu D) > r^2 > x(F + D)$ banking competition would decrease pure insolvency risk but increase total credit risk, once liquidity risk is taken into account. In other words, when illiquidity risk is considered, the set of parameters where the result of Boyd and De Nicolo (2005) applies will shrink. The following proposition summarizes our result.

Proposition 3. *For a given level of debt obligation, the probability for a bank to be solvent but illiquid monotonically increases with competition. The total credit risk, the risk of bank failures due to either insolvency or illiquidity, decreases with competition if and only if $r^2 > x(F + \mu D)$.*

3.5 Contagion and systemic risk

The next step is to explore what happens once we add the risk of contagion. This is illustrated in a two-bank setup: We make a stylized assumption that when both banks need to sell, the fire-sale discount increases from λ to λ'. The assumption captures the observation that the secondary market price tends to fall further when more banks fail and sell, due to either cash-in-the-market pricing or informational contagion. In the former case, market prices are driven down by the limited supply of cash. In the latter, a high number of bank failures leads investors to update their expectations for banks' common risk exposures and lower their willingness to pay for the assets. The exposure to the same asset price provides a channel of financial contagion: When the first bank goes under and sells, the asset price is driven down. This magnifies coordination failure among debt holders of the other banks', leading to further bank runs.[15]

Following the same procedure of the last section, one can derive a critical cash flow level

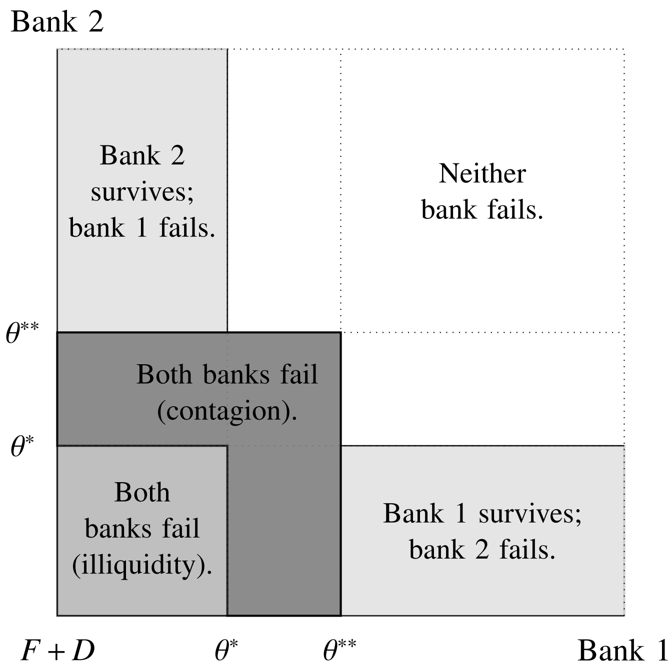
$$\theta^{**} = F + \mu' D > \theta^*,$$

where $\mu' = 1 - q[1 - (1 + \lambda')q] > \mu$. A bank whose cash flow falls between $[\theta^*, \theta^{**}]$ will be solvent and liquid if the other bank does not face a run, but will become illiquid if runs happen to the other bank. In particular, a bank whose cash flow falls between $[F + \mu D, F + \mu' D]$ is exposed to contagion and will fail, if the other bank's cash flow falls below $F + \mu' D$. We therefore define the exposure to contagion

$$\rho_{CTG} \equiv Prob(\theta^* < \theta < \theta^{**}) = \frac{(\mu' - \mu)D}{(1 - \alpha)r}. \tag{10}$$

[15]For a full-fledged model that shows asset prices drop with bank runs, see Li and Ma (2012).

Figure 3: Illustration of Various Bank Risk



In the two-bank setup, the systemic risk, is captured by the probability that both banks fail at the same time.[16]

$$\rho_{SYS} = Prob(\theta < \theta^{**})^2 = \left(\frac{\theta^{**} - \alpha r}{(1-\alpha)r}\right)^2 \tag{11}$$

Note that since competition reduces banks' buffer against fire-sale losses, $\partial(1-\alpha)/\partial r > 0$, the exposure to contagion increases with competition. On the other hand, competition reduces systemic risk if and only if $r^2 > x(F + \mu' D)$, which forms a counterpart to condition (3.4).

$$\frac{\partial \rho_{SYS}}{\partial r} = 2Prob(\theta < \theta^{**})B\Big[-(F + \mu' D)\frac{x}{r^2} + 1\Big] > 0 \quad \text{iff} \quad r^2 > x(F + \mu' D)$$

Proposition 4. *For a given level of debt obligation, banks' exposure to contagion always increases with competition. The risk of a systemic crisis decreases with competition if and only if* $r^2 > x(F + \mu' D)$.

Proof. See Appendix A.3. □

The liquidity risk, exposure to contagion, and systemic risk are illustrated in Figure 3. As competitive environment changes, the critical cash flow level θ^* and θ^{**} shift, leading to the corresponding changes in bank risk.

[16]The extension to n banks is straightforward.

Therefore, even if banks do not adjust their leverage to the changing competitive environment, we show banking competition can affect different types of risk differently. In particular, for $x(F+D) < r^2$, competition always reduces pure insolvency risk; but increases pure liquidity risk if $x(F+D) < r^2 < x(F+\mu D)$; and increases systemic risk if $x(F+D) < r^2 < x(F+\mu' D)$. So the general implication of the analysis is that focusing on solely one dimension of risk can lead to biased judgment for the overall effects.

4 Endogenous leverage and its impacts

Although banks' leverage decision are restricted by solvency regulation, banks still have the ability to choose the buffer they hold above and beyond the regulatory capital requirements as well as the structure of maturities of their debt. It is important to notice that portfolio risk is not identical to the default risk of a bank; leverage plays a crucial role: a low risk portfolio financed with high leverage may end up generating a high insolvency risk. Consequently, endogenous leverage may play a countervailing role to any reduction in the portfolio risk due to competition. It is therefore crucial to study whether our results are robust to the introduction of endogenous leverage.

4.1 Endogenous leverage

We assume that the cost of capital is larger than the cost of debt and denote by k the equity premium, so that the expected return on capital is $1+k$. The classical justification of the risk premium would be the tax benefits of debt; alternatives are the dilution costs à la Myers and Majluf (1984) and the renegotiation costs à la Diamond and Rajan (2000).[17]

The optimal level of debt is determined by its marginal cost being equal to its marginal benefit: on the one hand, a higher debt level entails a greater chance of bankruptcy; on the other hand, a higher debt level saves on costly capital. Banks rationally set their leverage by taking into account the probability of bankruptcy, caused either by insolvency or by illiquidity.[18]

[17] When a corporate tax is levied at a constant rate τ and debt repayments are exempted, k reflects the cost of losing tax shields. With $1+k = 1/(1-\tau)$ or $\tau = k/(1+k)$, the model will provide the familiar expression that firms trade off between tax shields and bankruptcy costs.

[18] To simplify the analysis, we assume that regulators bail out banks in a systemic crisis: when both banks fail, they both will be bailed out. So banks are assumed not to take into account the systemic risk of contagion in their leverage choice. Relaxing this assumption would imply negative externalities of leverage: a bank that is leveraged and fails is contagious to other banks. Because such cost is not taken into account in private decisions, banks are induced to lever more.

4.1.1 The general case

Banks choose their capital structure to maximize the leveraged firm value to existing shareholders. If ω is the fraction of the bank that is sold to outside shareholders, the old shareholders obtain

$$(1-\omega)\int_{F+\mu D}^{\overline{\theta}} \Big[\theta - F - D\Big] h(\theta, r) d\theta,$$

where bank cash flow θ follows a density function $h(\theta, r)$ and has a support on $[\underline{\theta}, \overline{\theta}]$. The bank will raise V_E from new shareholders, V_D from wholesale short term creditors, and F from insured depositors. And the three sources of funding should provide the required amount of investment. That is, $V_E + V_D + V_F = 1$. Consequently the optimal capital structure is the solution to

$$\begin{aligned}
&\max_{\omega, D} \left\{(1-\omega)\int_{F+\mu D}^{\overline{\theta}} \Big[\theta - F - D\Big] h(\theta, r) d\theta\right\} \\
s.t. \qquad & V_E = \frac{\omega}{1+k}\int_{F+\mu D}^{\overline{\theta}} \Big[\theta - D - F\Big] h(\theta, r) d\theta \\
& V_D = \int_{F+\mu D}^{\overline{\theta}} D h(\theta, r) d\theta \\
& V_F = F \\
& V_E + V_D + V_F = 1
\end{aligned}$$

Adding the three constraints to the objective function we obtain the unconstrained optimization, with $\int_{\underline{\theta}}^{F+\mu D} F h(\theta, r) d\theta$ reflecting the subsidy of deposit insurance.

$$\max_{D}\left\{\int_{F+\mu D}^{\overline{\theta}} [\theta + k(D+F)] h(\theta, r) d\theta + (1+k)\int_{\underline{\theta}}^{F+\mu D} F h(\theta, r) d\theta - (1+k)\right\} \tag{12}$$

The optimization program yields the following first order condition.

$$-\mu\Big[(F+\mu D) + k(F+\mu D)\Big] h(F+\mu D, r) + \int_{F+\mu D}^{\overline{\theta}} k h(\theta, r) d\theta + (1+k)\mu F h(F+\mu D, r) = 0$$

This can be written compactly as

$$-\mu(\mu+k) D h(F+\mu D, r) + \int_{F+\mu D}^{\overline{\theta}} k h(\theta, r) d\theta = 0,$$

or, with H denoting the c.d.f. of θ,

$$D^* = \frac{k[1 - H(F + \mu D^*, r)]}{\mu(\mu + k)h(F + \mu D^*, r)}.$$

4.1.2 Application to our setup

The uniform distributions in the current paper simplify the model. It is especially convenient to work with the stochastic loan losses $\gamma \sim U[0, 1]$. To facilitate exposition, we denote

$$\hat{\gamma}_\mu \equiv \frac{r - (F + \mu D)}{(1 - \alpha)r}.$$

$\hat{\gamma}_\mu$ is a counterpart of $\hat{\gamma}$: It denotes the critical loan loss the bank will survive once liquidity risk is taken into account. The optimization program transforms into the following form.

$$\begin{aligned}
&\max_{\omega,D} \left\{(1-\omega)\int_0^{\hat{\gamma}_\mu}\Big[\theta - F - D\Big]d\gamma\right\} \\
s.t. \quad & V_E = \frac{\omega}{1+k}\int_0^{\hat{\gamma}_\mu}\Big[\theta - D - F\Big]d\gamma \\
& V_D = \int_0^{\hat{\gamma}_\mu} D d\gamma \\
& V_F = F \\
& V_E + V_D + V_F = 1
\end{aligned}$$

After substituting the constraints, the program simplifies to:

$$\max_D \left\{ \int_0^{\hat{\gamma}_\mu}\Big[\theta + k(D + F)\Big]d\gamma + (1+k)\int_{\hat{\gamma}_\mu}^1 F d\gamma - (1+k)\right\} \tag{13}$$

Recall that $\theta = r - (1-\alpha)r\gamma$. The maximization program has the following first order condition[19]

$$\left[r\frac{\partial\hat{\gamma}_\mu}{\partial D} - \frac{(1-\alpha)r}{2}2\hat{\gamma}_\mu\frac{\partial\hat{\gamma}_\mu}{\partial D}\right] + k\hat{\gamma}_\mu + k(F + D)\frac{\partial\hat{\gamma}_\mu}{\partial D} - (1+k)F\frac{\partial\hat{\gamma}_\mu}{\partial D} = 0$$

which yields the optimal level of risky debt

$$D^* = \frac{r - F}{\mu^2/k + 2\mu}. \tag{14}$$

The result is summarized in the following theorem.

[19] It is straightforward to check that the second order condition is satisfied, $-(1-\alpha)r\left(\frac{\partial\hat{\gamma}_\mu}{\partial D}\right)^2 + 2k\frac{\partial\hat{\gamma}_\mu}{\partial D} < 0$.

Proposition 5. *A bank that maximizes its value by trading off the benefits of debts versus bankruptcy cost sets its debt* $D^* = (r - F)/[\mu^2/k + 2\mu]$.

The risky debt that a bank issues is proportional to its *maximum* residual cash flow after paying insured deposits F. In particular, the coefficient c increases in the cost of capital, $\partial c/\partial k > 0$, and decreases in the liquidity risk, $\partial c/\partial \mu < 0$, and so does the bank's optimal debt level. Note also that

$$\lim_{\mu \searrow 1} c = \frac{1}{1/k + 2} < 1.$$

For c monotonically decreases in μ_1, it holds that $c < 1/(1/k+2) < 1$: a bank cannot issue more risky debt claims than its maximum cash flow after paying the risk-free F and is unwilling to issue risky debt more than c fraction of $(r - F)$.

4.2 Risk under endogenous leverage

With exogenous leverage D replaced by endogenous D^*, the different bank risks under endogenous leverage are defined analogously as in equations (6) and (8) - (11). Accordingly we denote the risks by a superscript star. Substituting in D^*, one can write insolvency risk, liquidity risk, and total credit risk as follows.

$$\rho^*_{SR} \equiv 1 - \frac{r - F - D^*}{(1-\alpha)r} = 1 - \left[1 - \frac{1}{\mu^2/k + 2\mu}\right]\frac{r - F}{(1-\alpha)r} \tag{15}$$

$$\rho^*_{IL} \equiv (\mu - 1)\frac{D^*}{(1-\alpha)r} = \frac{\mu - 1}{\mu^2/k + 2\mu}\frac{r - F}{(1-\alpha)r} \tag{16}$$

$$\rho^*_{TCR} \equiv 1 - \frac{r - F - \mu D^*}{(1-\alpha)r} = 1 - \frac{\mu + k}{\mu + 2k}\frac{r - F}{(1-\alpha)r} \tag{17}$$

The endogenous leverage has a crucial impact on the various risks already identified. In some instances, it reverses the results obtained under exogenous leverage.

Proposition 6. *Under endogenous leverage, pure insolvency and liquidity risk always move in the opposite direction, with the latter dominant in determining total credit risk. In particular, for* $r^2 > xF$*, pure insolvency and total credit risk decreases as competition intensifies, whereas funding liquidity risk increases. Otherwise, the result reverses.*

Proof. See Appendix A.4. □

The impact of competition on banks' exposure to contagion (ρ^*_{CTG}) and the risk of a systemic crisis (ρ^*_{SYS}) can also be analyzed following the the definitions.

$$\rho^*_{CTG} \equiv \frac{(\mu' - \mu)D^*}{(1-\alpha)r}$$
$$\rho^*_{SYS} \equiv Prob(\theta < \theta^{**})^2 = \left(1 - \frac{r - F - \mu' D^*}{(1-\alpha)r}\right)^2$$

Proposition 7. *For $r^2 > xF$, while banks' exposure to financial contagion increases with competition, the risk of a systemic crisis decreases, provided $\mu' c < 1$.*

Proof. See Appendix A.5. □

Overall, our results state that the impact of competition on financial stability critically depend on the type of banking industry that is considered. Two possible cases emerge. The case $r^2 > xF$, corresponds to less productive firms facing high borrowing costs, while banks obtain high margins and raise funding in the market (low level of insured deposits). When this is the case, total credit risk is reduced with competition. As a limit case, $F = 0$ can be interpreted as investment banking. More competition means safer investment banking. Alternatively, the case $r^2 < xF$, corresponds to highly productive firms facing low borrowing costs, with banks mainly financed through deposits. In such environment, the opposite result holds: banking competition reduces financial stability. This correspond to classical retail banking with low margents and prudent funding through insured deposits.

4.3 Interpretation

Although our model does not pretend to provide robust results that hold true in every environment, it is worth noticing the key ingredients that determine here the impact of bank competition on the different types of financial stability. As shown by inequality (A.21), banks' liability structure, and in particular the amount of short term wholesale funding, is central to the relationship between competition and bank risk. Our model's conclusions provide a much richer view of the link between banking competition and financial risk than is usually considered.

1. To begin with, notice that the result depends upon the borrowing firms' project returns x. For a given level of deposits and banks' market power, the effect of banking competition on financial stability depends upon how productive the firms are. In highly productive economies,

bank competition constitutes a threat to financial stability. The impact of moral hazard is reduced, and the key determinant of the link between bank competition and financial stability is the role of the buffer generated by banks' market power. Comparing Proposition 1 and Proposition 6 we observe that the threshold for x that inverts the relationship from banking competition to financial stability is reached much earlier if we take into account the endogeneity of banks' leverage. This is the case because banks will be more conservative in their choice of leverage, so that the strength of the Boyd and De Nicolo's argument is weakened and the charter value dominates. The argument can be reinterpreted considering the business cycle. In a boom, banking competition jeopardizes financial stability, while, in a bust it reduces banks' risks.

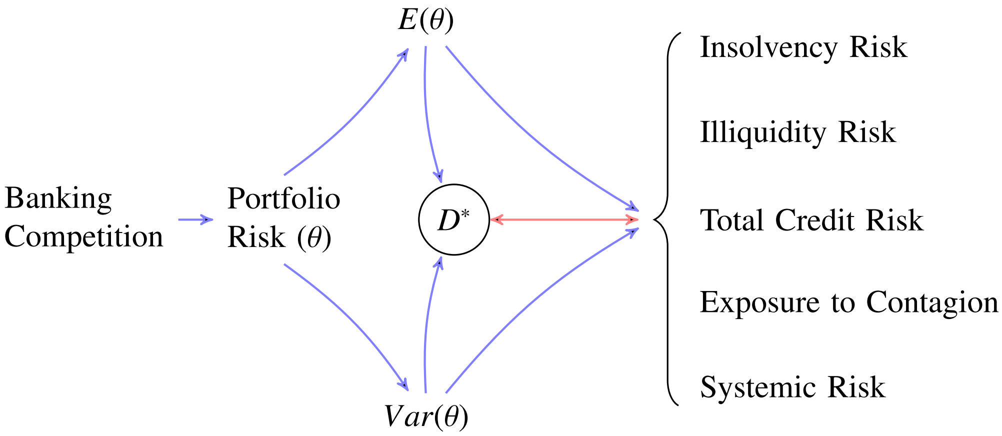
2. The level of market power is also essential in our framework. For high market power competition reduces bank fragility, nevertheless a threshold may exist (provided that $xF > 1$) beyond which the result is reserved. This is interesting from a policy perspective as it provides a more nuanced prescription than the usual one: in order to sustain financial stability, it might be interesting to promote competition up to a certain threshold, but beyond that point, competition will lead to higher banking risk.

3. The role of stable funds is critical for our result. In a traditional banking industry funded through deposits and long term bonds (equivalent in our context to insured deposits) where $xF > r^2$, competition will be detrimental to financial stability. Instead in a banking industry where wholesale short term (possibly interbank) funding is prevalent, the Boyd De Nicolo argument will prevail.

4. More generally, two types of banks, corresponding to the two possible signs of $xF - r^2$, may coexist and will react in a different way to an increase in competition. For banks that rely less on stable funding $xF < r^2$, in particular for investment banks, an increase in competition will increase financial stability. Instead, for banks with high levels of deposits and lower market power, for which the inequality $xF > r^2$ is fulfilled, the opposite occurs and banking competition's main effect is to reduce the banks buffer and to encourage higher leverage.

Figure 3 sets out the channels through which banking competition affects risk, either directly through cash flow riskiness, or indirectly through the changing leverage. It emphasizes how banking competition determines both cash flow characteristics and bank leverage and how risks are jointly determined by the optimization behavior of banks. It should also be acknowledged that despite our efforts to build a comprehensive model, the current paper still understates

Figure 4: Linkages between Banking Competition, Leverage and Various Risk



the complexity of the issue, because competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, cash hoarding, and so on, issues which are absent from the current setup.

4.4 Comparison to the exogenous leverage case

To emphasize the crucial impact of endogenous leverage, we summarize and tabulate in Table 1 and 2 the results under exogenous and endogenous bank leverage for a side-by-side comparison. The '**+**' sign denotes that competition increases the bank risk considered; and the '**-**' sign denotes that competition reduces that type of risk.

Table 1: Banking competition and risk under exogenous leverage

	$r^2 < x(F+D)$	$r^2 \in [x(F+D), x\theta^*)$	$r^2 \in [x\theta^*, x\theta^{**})$	$r^2 > \theta^{**}$
Pure insolvency risk	+	-	-	-
Pure liquidity risk	+	+	+	+
Total credit risk	+	+	-	-
Exposure to contagion	+	+	+	+
Systemic risk	+	+	+	-

Table 2: Banking competition and risk under endogenous leverage

	$r^2 < xF$	$r^2 > xF$
Pure insolvency risk	+	-
Pure liquidity risk	-	+
Total credit risk	+	-
Exposure to contagion	-	+
Systemic risk	+	-

To a large extent, the model presented is a special case where all important comparative statics depend on the sign of $r^2 - xF$. Yet the result conveys the key messages of the paper: (1) banking competition affects different types of risk differently; and (2) endogenous leverage is a central hub that both reflects changes in the cash flow riskiness and affects all different aspects of banking risk.

5 Reinterpreting the empirical literature

The difficulties in analyzing the link between competition and financial stability exponentially increase when we turn to the empirical studies. The empirical analysis has led to a multiplicity of results that are sometimes difficult to reconcile and susceptible to alternative interpretations. As we have emphasized in this paper there are multiple measures of financial stability just as there are multiple measures of competition, ranging from franchise or charter value (Tobin's Q), to market structure (e.g., HHI, C-n), to structural measurement (i.e., P-R H-stat., Lerner's index, Boone's indicator), and to institutions (contestability of the market, e.g., activity and entry restrictions).[20] The industrial organization literature does not provide an unambiguous answer to which measurement reflect competition best. In the context of banking, some empirical studies, e.g., Claessens and Laeven (2004) and Schaeck, Cihák, and Wolfe (2009), show that concentration measures are poor proxies for bank competition, but this still leaves a wide range of possible measures.

The empirical implication of our model is that banks' risks should be measured at four fundamentally different levels: first, at the level of banks' assets; second, at the level of banks' solvency; third, at the level of banks' liquidity risk and fourth at the level of the overall systemic risk and contagion. Competition directly affects the riskiness of banks' cash flows; but because banks react to these changes by altering their leverage, all other types of risks are also affected.

[20]See Degryse, Kim, and Ongena (2009) for a comprehensive review on measuring banking competition.

Indeed, banks' endogenous leverage constitutes a central hub that connects these three types of risk. The implication is that depending on the magnitude of the direct and indirect forces, a diverse range of predictions can rise.

As a consequence, our reading of the results of the empirical literature introduces drastic differences depending on whether the evidence concerns the riskiness of banks' assets, or the riskiness of banks themselves, either their solvency or their liquidity, or systemic risk.

Our theoretical framework suggests a progressive approach to the understanding of the impact of competition on banks' risk-taking by refining the questions that are asked as successive layers.

1. Does competition increase the safety of a banks portfolio of assets? In other words is Boyd & De Nicolo's basic result true?

   Next, once we take into account the optimal reallocation of assets and the optimal choice of leverage by banks, the following issues are to be addressed:

2. Does competition increase the risk of bank insolvency?
3. Does competition increase the liquidity risk of banks?
4. Does competition increase banks' systemic risk?

Revisiting the empirical literature through this filter leads us to regroup the empirical results in a more complete and orderly way, refusing to find the different measures either equivalent or complementary in the assessment of the impact of competition on financial stability, without taking into account the changes in leverage it produces. In the end of the section, we summarize in table 2 the empirical literature by highlighting the different key contributions, the measures of risk and competition they utilize and the results regarding the impact of competition on financial instability they obtain.

5.1 Portfolio risk: non-performing loans

The basic postulate of the Boyd and De Nicolo (2005) approach is that competition will reduce the riskiness of banks' portfolio, an issue independent of the banks' leverage reaction. The alternative hypothesis put forward by the proponents of the charter value approach is that the banks' overall investment strategy will be more risky as the opportunity cost of bankruptcy is lower. So knowing whether Boyd & De Nicolo's basic assumption is in line with empirical evidence is a crucial step forward. In order to measure the riskiness of assets, measures like stock volatility in Demsetz, Saidenberg, and Strahan (1996); Brewer and Saidenberg (1996) are contaminated by leverage. The non-performing loans (NPLs) ratio is the variable that reflects most accurately the riskiness of banks' assets, and a bulk of literature appears to support this view, as it analyzes non-performing loans as one of the key variables in the analysis of the competition-financial stability link.

Restricting the measurement of risk to NPLs implies focusing on a very specific dimension of the broad link between competition and financial stability where we might hope for some consensus on the empirical results. Unfortunately even with this drastic reduction the evidence is mixed. So, in spite of the fact that the charter value and risk shifting theories have completely opposite predictions regarding the impact of banks' competition on non-performing loans, empirical studies give no definitive answer on which one should be the predominant view.

The initial paper on the charter value Keeley (1990) did not consider NPLs measures but rather estimates of overall bank risk of failure. The prediction on NPLs is backed by more recent works, such as the analysis of Salas and Saurina (2003) and Yeyati and Micco (2007). The authors found an increase in non-performing loans as bank competition increased in Spain and in eight Latin American countries respectively. Support for the risk shifting hypothesis comes from Boyd et al. (2006) and is corroborated by Berger et al.(2008) who find an interesting set of results based upon both loan risk and overall bank risk. Using cross-sectional data on 29 developed countries for the years 1999 through 2005, they find that banks with a higher degree of market power exhibit significantly more loan portfolio risk.

The impact of the US introduction of Nationwide banking also leads to contradictory results: while Jayaratne and Strahan (1998) report that "Loan losses decrease by about 29 basis points in the short run and about 48 basis points in the longer run after statewide branching is permitted", Dick (2006) finds out that "charged-off losses over loans (...) appears to increase by 0.4 percentage point following deregulation".[21]

Some caveats are also in order regarding the accuracy of this measurement. First, banks can manipulate NPL by rolling over bad loans. Second, a risky loan granted today will only default

[21] It has been argued that institutional changes can be a more robust measurement than market structure, for its exogeneity facilitates statistical analysis to establish causal relationships. The instrumental approach bears its value for banking deregulation is usually associated with a removal of barriers to entry that will increase competition. Yet, it is not only associated to the removal of barriers to entry, as it might also affect the range of financial products banks are allowed to invest in and the structure of financial institutions. As pointed out by Cubillas and Gonzalez (), bank liberalization has not only an effect on banks' competition, but also an indirect effect on banks' strategies other channels. This implies that the "banking deregulation" measure of market power explores the effect of a package of measures related to market power on financial stability, but market power is only an undistinguishable part of it.

in the future (e.g. after a two-year lag if we follow Salas and Saurina (2003) and the rate of default will depend upon the business cycle (Shaffer 98). Although the latter might be corrected by the introduction of macroeconomic risk controls, such as the GDP growth rate, the time lag may be more difficult to correct because of the persistence of the non-performing loans ratios. Third, the riskiness of assets could also be altered by changing the portfolio allocation among the different classes of risks. A bank with higher market power may be willing to take more risks on its assets that will result in higher NPLs in order to obtain a higher expected return while its market power on, say, deposits provides a natural buffer that prevents its financial distress.

5.2 Individual bank risk: insolvency

Because of the endogeneity of banks leverage, changes in the portfolio risk do not translate into equivalent changes on banks' default risk: a poorly capitalized bank can have a high probability of failure, even if its portfolio risk is low. Such divergence between the impact of competition on the riskiness of banks' assets (as measured by NPL) and its overall risk is perfectly illustrated in Berger, Klapper, and Turk-Ariss (2009): in spite of finding confirmation of the risk shifting hypothesis in the NPL their analysis shows that banks with a higher degree of market power have lower overall risk exposure mainly due to their higher equity capital levels.

Since Keeley (1990) the literature has been focusing on the risk of individual bank failure. In his classic paper, Kelley considers the market-value capital-to-asset ratio and the interest cost on large, uninsured CD's. Following his approach, Demsetz, Saidenberg, and Strahan (1996) use seven different measures of BHCs' risks and in each of them franchise value is statistically significant providing support to the charter value theory.[22] Brewer and Saidenberg (1996) found also corroborating evidence that the standard deviation of stock returns volatility was negatively related to S&L franchise values as measured by the market-to-book asset ratio. Also confirming the charter value perspective, Salas and Saurina (2003) show that capital ratio increases with Tobin's Q, thus providing some evidence on the possible behavior of the (endogenous) leverage ratio.

[22]The risk measurements include annualized standard deviation of weekly stock returns, systematic risk, firm-specific risk, capital-to-assets Ratio, loans-to-assets ratio, commercial and industrial loans-to-assets ratio and loan portfolio concentration.

In our judgement, pure insolvency risk can be best measured by Z-scores,[23] and many empirical works take that as the main risk measurement. Still, there are important nuances in these results. Beck, Jonghe, and Schepens (2011) show on average, a positive relationship between banks' market power, as measured by the Lerner index, and banks' stability, as proxied by the Z-score. Nevertheless, they find large cross-country variation in this relationship. Jiménez, Lopez, and Salas (2010) report empirical evidence that supports the franchise value paradigm but only if market power is measured by Lerner indexes based on bank-specific interest rates and bank risk.

Opposing this view, Boyd and Jalal (2009) provided cross-country empirical evidence supporting the risk-shifting model using several proxies to measure bank risk, including using the Z-score.[24] Using a US sample and a cross-country one they consistently find that banks' probability of failure is negatively and significantly related to measures of competition. Confirming this view, De Nicolo and Ariss (2010) analyze the impact of large deposit and loan rents and show that they predict higher probabilities of bank failures and lower bank capitalization.

5.3 Individual bank risk: illiquidity

Funding liquidity risk has largely been overlooked by empirical studies.[25] One might argue that upon observing bank failures, it is difficult, if not impossible, to distinguish pure solvency issue from illiquidity ones, (Goodhart, 1987). However, just as insolvency risk can be measured by Z-scores, illiquidity can be measured with accounting information too. For example, in their study of bond pricing, Morris and Shin (2004) identify the extra yield due to illiquidity risk: as far as the yield reflects default probability, the liquidity risk can be reflected in bond pricing. While the relationship between funding liquidity risk and competition has not been studied, theoretical models do provide a sound guide for estimating the risk: funding liquidity risk is reduced by high returns (e.g., measured by ROA) and high asset market liquidity (e.g., the holding of reserves and cash), and aggravated by the amount of uninsured short-term funding. Morris and Shin (2009) provide a further practical guide. In the context of herding behavior Bonfim and Kim (2011) present an attempt to measure the risk by a variety of liquidity ratios.

[23]The measurement is calculated $(RoA - E/A)/\sigma(RoA)$ to capture a bank's distance from insolvency.

[24]The author also use loan losses and dummy for actual bank failures.

[25]On contrast, even though theoretical models made no prediction on how competition affects leverage, which in turn affects insolvency risk, the empirical study has taken into account insolvency risk adjusted by leverage by using measurements like z-scores.

In addition to the accounting information, funding liquidity risk can also be estimated by market data. Veronesi and Zingales (2010) suggest constructing bank run index using CDS spreads. In sum, we believe banks' liquidity risk can be measured. Yet how those liquidity measurements are related to banking competition invites much future research.

5.4 Systemic risk

The analysis of systemic risk is, obviously, even more difficult one as it often has to deal with cross-country analysis and the main driving force for changes in market power are related to banking deregulation, market entry, deposit insurance and a number of joint measures of which increased competition is only one of the consequences.[26] The precise definition of a banking crisis itself as well as its timing is subject to different interpretations. Thus, while some authors consider the intervention of exceptional measures by the Treasury, or a 10% of the banking industry being affected, others like Anginer, Demirgüç-Kunt, and Zhu (2012) and De Nicolo et al.(2004) prefer to measure the probability of systemic risk by pairwise distance to default correlation or constructing an indicator of the probability of failure for the five largest banks.

According to Beck, Demirgüç-Kunt, and Levine (2006), who analyze a sample of 69 countries over a 20 year period, more concentrated national banking systems are subject to a lower probability of systemic banking crisis. Still, they point out that concentration need not be related to market power, as already mentioned by Claessens and Laeven (2004), and that other measures of competition may lead to the opposite result. Contradicting the result of Beck, Demirgüç-Kunt, and Levine (2006), Schaek et al.(2006) show, using the Panzar and Rosse H-Statistic as a measure for competition in 38 countries during 1980-2003, that more competitive banking systems are less prone to systemic crises and that time to crisis is longer in a competitive environment even controlling concentration and the regulatory environment.

Our paper's empirical prediction states here that an increase of competition may have different effects depending upon the amount of insured retail deposits, and the profitability of projects and banks' spreads, thus suggesting new lines for future empirical research based on the differentiation of different types of banking systems. It would be interesting to pursue this research by distinguishing among different types of banks. If we interpret our model literally,

[26] It should be noted that with newly developed measurements on systemic risk such as *CoVaR* in Adrian and Brunnermeier (2010), one can also link a bank's market power to its contribution to the systemic risk, e.g., by regressing an individual bank's *CoVaR* on its Learner's index.

this would be to distinguish banks with low deposit to asset ratios from those with a high deposit to asset ratio. Still, more generally, this could be interpreted as dividing the banks according to their different access to short maturity market funds.

Table 3: Does banking *competition* lead to *instability*? Diverse risk/competition measurements and results from the empirical literature.

Paper	Risk	Competition	Results	Data Source	Comments
Keeley (1990)	Interest Cost	Tobin's q	Yes	US	also via capital
Demsetz, Saidenberg, and Strahan (1996)	Stock Volatility	Market-Book Value	Yes	US	via capital/diversification
Brewer and Saidenberg (1996)	Stock Volatility	Market-Book Value	Yes	US S&L banks	
Jayaratne and Strahan (1996, 1998)	NPLs	Deregulation	No	US	
Salas and Saurina (2003)	Loan Loss	Tobin's q	Yes	Spain	also via capital
De Nicolo and Loukoianova (2005)	Z-Score	HHI	No	Non-industrialized	interaction with ownership
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Concentration	Yes	Cross-Country	
Beck, Demirgüç-Kunt, and Levine (2006)	Crisis Dummy	Contestability	No	Cross-Country	
Dick (2006)	Loan Loss	Deregulation	Yes	US	
Yeyati and Micco (2007)	Z-Score & NPLs	P-R H-Stat.	Yes	Latin America	
Schaeck and Cihák (2010a)	Capitalization	P-R H-Stat.	No	Developed	IV
Dell'Ariccia, Igan, and Laeven (2008)	Lending standard	Number of banks	Yes	US	mortgage market
Boyd and Jalal (2009)	Loan Loss	HHI	No	US/Cross-Country	
Boyd and Jalal (2009)	Z-Score	HHI	No	US/Cross-Country	
Boyd, Nicolo, and Loukoianova (2009)	Crisis Dummy	HHI/C3	No	Cross-Country	
Berger, Klapper, and Turk-Ariss (2009)	NPLs	Lerner Index/HHI	Yes	Developed	via capital
Berger, Klapper, and Turk-Ariss (2009)	Z-Score	Lerner Index/HHI	No	Developed	via capital
Schaeck, Cihák, and Wolfe (2009)	Crisis Dummy	P-R H-Stat.	No	Cross-Country	
Schaeck, Cihák, and Wolfe (2009)	Duration until crisis	P-R H-Stat.	No	Cross-Country	
Schaeck and Cihák (2010b)	Z-Score	Boone's Indicator	No	US/EU	via efficiency
Jiménez, Lopez, and Salas (2010)	NPLs	Lerner Index	Yes	Spain	
Jiménez, Lopez, and Salas (2010)	NPLs	HHI/C5		Spain	
Beck, Jonghe, and Schepens (2011)	Z-Score	Lerner Index	Yes	Cross-Country	cross-country heterogeniety
De Nicolo and Ariss (2010)	Z-Score	Deposit market rent	Yes	Europe	
De Nicolo and Ariss (2010)	Z-Score	Loan market rent	No	Europe	
Dick and Lehnert (2010)	Personal bankruptcy	Deregulation	Yes	US	
Dick and Lehnert (2010)	NPLs/risk management	Deregulation	No	US	
Anginer, Demirgüç-Kunt, and Zhu (2012)	D-to-D Correlation	Lerner Index	No	Cross-Country	supervision/ownership

6 Discussion and policy implications

Because the aim of our paper is to clarify the multiple concepts of risk and the key role played by leverage, our model has made a number of drastic simplifying assumptions that although lead to relatively simple propositions but cannot be easily generalized. Indeed, our framework considerably understates the complexity of the issue, since competition also affects banks' portfolio choice, e.g., the correlation of their portfolios, securitization, cash hoarding, and so on, which are all abstracted from in the current setup.

Also our model's main objective is not to address the issue of the design of overall banking regulation, and consequently, from that perspective, it suffers from two serious limitations. On the one hand it does not take into account the impact of competition in increasing productivity through the Shumpeterian creative destruction process (Dick 2006) and, on the other hand, it does not consider the supply of credit, which is exogenously set as all firms are able to get financed. As a consequence, instead of the usual risk-return trade off, higher risk is associated here with a higher private and social expected cost of bankruptcy. In spite of this, it is interesting to consider the implications our results have for regulatory policies. Two main lessons can be drawn: the first one regarding the impact of competition in general, and the second one regarding capital and liquidity regulation.

The first lesson of our model regarding banking regulation is that the one-size-fits-all approach to the analysis of the link between banking competition and financial stability is insufficiently rigorous. To be more precise we conclude that the link depends among other things on the degree of market power of financial institutions. If financial institutions have a high market power, then competition reduces the level of total credit risk (that is the sum of insolvency risk and liquidity risk) in financial institutions, confirming the risk shifting hypothesis of Boyd and De Nicoló. Still, in this high market power case, we show that the impact is dampened by the increase in liquidity risk the increase in competition causes. On the other hand, once the banking industry is sufficiently competitive, the inequality is reversed and additional competition leads to financial instability, thus confirming the charter value assumption. From that perspective the policy position depends upon the fact that market power is above or below some threshold that depends upon firms' productivity as well as upon banks' liability structure.

A simple extension of our framework, consisting in distinguishing wholesale short-term market funding from long-term market funding, ($D = D_S + D_L$) where long-term market funding has a higher cost, also has implications regarding liquidity regulation. Indeed, we show that a more competitive banking industry that, in principle, should be more efficient has a higher level of liquidity risk, proportional to D_S if leverage is exogenous but not if it is endogenous. This is directly related to capital regulation, because if capital regulation is binding, leverage becomes exogenous. As a consequence, the implication is that liquidity regulation may reduce risk in competitive economies where banks have a higher cost for capital and for long term funds than for short term wholesale funds. Liquidity regulation, as suggested by the recent changes introduced in Basel III, could therefore decrease the liquidity risk that is implied by fiercer competition.

7 Concluding remarks

We develop a model to study banks' risk in competitive environments. We model explicitly the credit risk created by borrowing firms' moral hazard and examine how banks optimally adjust their leverage in the light of various risk. With the theoretical framework, we clarify the concept of financial stability: it has multiple dimensions ranging from portfolio risk to systemic risk. We show that competition can affect different types of risk differently, and the idea of an identical impact of banking competition on financial stability that would hold across types of banks and types of firms has no theoretical foundation. This can help explain the diverse findings in the empirical literature. We further suggest that banks' leverage and liability structure play a key role in determining the relationship between banking competition and financial stability. As a consequence, testing our model's prediction that the competition-financial stability link depends upon the type of bank and the state of the economy through firms self financing and productivity may lead to an important step forward in our understanding of the issue.

Appendix A Proof of propositions

Appendix A.1 Proof of lemma 1

To derive the uniform distribution of loan loss γ, take a risky type $\tilde{b} < 1/(x-r)$; and define the fraction of entrepreneurs below $\tilde{b}$ in the risky pool by $\tilde{\gamma}$. We have

$$\tilde{\gamma} = \frac{\tilde{b} - 0}{1/(x-r) - 0} = \tilde{b}(x-r). \tag{A.18}$$

Consider the critical realization $\tilde{z} = \Phi^{-1}(1 - P^*_{\tilde{b}})$ such that an entrepreneur of $\tilde{b}$ does not default but all types $b < \tilde{b}$ do. So for $z = \hat{z}$, one will have $\gamma = \tilde{\gamma}$. To derive the distribution of γ, notice that

$$\begin{aligned} F(\tilde{\gamma}) &\equiv Prob(\gamma < \tilde{\gamma}) = Prob(z > \tilde{z}) = 1 - Prob(z < \tilde{z}) \\ &= 1 - \Phi(\Phi^{-1}(1 - P^*_{\tilde{b}})) = P^*_{\tilde{b}} \\ &= \tilde{b}(x - r). \end{aligned}$$

By equation (A.18), we have $\tilde{b} = \tilde{\gamma}/(x - r)$. Substitution yields

$$F(\tilde{\gamma}) = \tilde{\gamma},$$

implying $\gamma \sim U(0, 1)$.

Appendix A.2 Proof of proposition 1

On the comparative statics of insolvency risk, computation is simplified if we consider its complementary probability, $1-\rho_{SR} = [r-(F+D)]/(1-\alpha)r$. Examining its first order derivative with respect to r, we obtain:

$$\begin{aligned} \frac{\partial(1-\rho_{SR})}{\partial r} &= \frac{1}{(1-\alpha)^2 r^2}\left[(1-\alpha)r - \frac{\partial[(1-\alpha)r]}{\partial r}\Big[r-(F+D)\Big]\right] \\ &= \frac{1}{(1-\alpha)^2 r^2}\left[(1-\alpha)r - \Big[(1-\alpha) - \frac{\partial\alpha}{\partial r}r\Big]\Big[r-(F+D)\Big]\right] \\ &= \frac{1}{(1-\alpha)^2 r^2}\left[\frac{\partial\alpha}{\partial r}r^2 + \Big[(1-\alpha) - \frac{\partial\alpha}{\partial r}r\Big](F+D)\right]. \end{aligned}$$

Recall that $\partial\alpha/\partial r = -1/B(x-r)^2$ and $(1-\alpha) = 1/B(x-r)$. Taking out the common factor, we will have

$$\begin{aligned} \frac{\partial(1-\rho_{SR})}{\partial r} &= \frac{1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}\left[r^2 + \Big[-(x-r) - r\Big](F+D)\right] \\ &= \frac{1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}\Big[r^2 - x(F+D)\Big] \end{aligned}$$

Therefore,

$$\frac{\partial\rho_{SR}}{\partial r} = \frac{-1}{(1-\alpha)^2 r^2}\frac{\partial\alpha}{\partial r}\Big[r^2 - x(F+D)\Big].$$

Pure insolvency risk is reduced by competition if and only if

$$r^2 > x(F + D). \tag{A.19}$$

Appendix A.3 Proof of proposition 4

A systemic crisis takes place if both banks' cash flow fall below θ^{**}, i.e., $\rho_{SYS} = Prob(\theta < \theta^{**})^2$. This allows us to obtain:

$$\begin{aligned}
\frac{\partial \rho_{SYS}}{\partial r} &= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}Prob(\theta < \theta^{**}) \\
&= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}\left(1 - \frac{r - \theta^{**}}{(1-\alpha)r}\right) \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\left[(1-\alpha)r - [(1-\alpha) - \frac{\partial \alpha}{\partial r}r](r - \theta^{**})\right] \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\frac{\partial \alpha}{\partial r}[r^2 - x\theta^{**}] \\
&= 2Prob(\theta < \theta^{**})\frac{-1}{(1-\alpha)^2 r^2}\frac{\partial \alpha}{\partial r}\left[r^2 - x(F + \mu' D)\right].
\end{aligned}$$

As $\partial\alpha/\partial r < 0$, the sign of the comparative statics is determined by $r^2 - x(F + \mu' D)$: the risk of a systemic crisis decreases with competition if and only if $r^2 > x(F + \mu' D)$.

Appendix A.4 Proof of proposition 6

Denote $c \equiv 1/(\mu^2/k + 2\mu)$. The pure insolvency risk, illiquidity risk, and total credit risk can be written as the following, with again total credit risk being the summation of pure insolvency risk and illiquidity risk. The comparative statics with respect to r follow from the definitions.

$$\begin{aligned}
\rho^*_{SR} &\equiv 1 - \frac{r - F - D^*}{(1-\alpha)r} = 1 - \frac{(1-c)(r-F)}{(1-\alpha)r} \\
\rho^*_{IL} &\equiv (\mu - 1)\frac{D^*}{(1-\alpha)r} = \frac{c(\mu-1)(r-F)}{(1-\alpha)r} \\
\rho^*_{TCR} &\equiv 1 - \frac{r - F - \mu D^*}{(1-\alpha)r} = 1 - \frac{(1-\mu c)(r-F)}{(1-\alpha)r}
\end{aligned}$$

(1) Comparative statics: Insolvency risk

$$\frac{\partial \rho^*_{SR}}{\partial r} = -(1-c)\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right)$$

We have shown $c < 1$. So the expression shares the same sign as

$$\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right). \tag{A.20}$$

(2) Comparative statics: Liquidity risk

$$\frac{\partial \rho^*_{IL}}{\partial r} = (\mu - 1)c\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right),$$

With $\mu > 1$, the sign will be opposite to that of expression (A.20).

(3) Comparative statics: Total credit risk

$$\frac{\partial \rho^*_{TCR}}{\partial r} = -(1-\mu c)\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right)$$

Note that $\mu c = 1/(\mu/k + 2) < 1$. The comparative statics of total credit risk is again determined by the sign of expression (A.20).

Therefore when competitive environment changes, pure insolvency risk moves in the opposite direction as pure liquidity risk. With the latter dominating, total credit risk changes in the same direction as that of pure insolvency. Now we characterize the condition that

$$\begin{aligned}
\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right) &= \frac{1}{(1-\alpha)^2r^2}\left[(1-\alpha)r - \left[-\frac{\partial\alpha}{\partial r}r + (1-\alpha)\right](r-F)\right] \\
&= \frac{1}{(1-\alpha)^2r^2}\left[\frac{\partial\alpha}{\partial r}r^2 + \left[-\frac{\partial\alpha}{\partial r}r + (1-\alpha)\right]F\right] \\
&= \frac{1}{(1-\alpha)^2r^2}\frac{\partial\alpha}{\partial r}[r^2 - rF - (x-r)F] \\
&= \frac{1}{(1-\alpha)^2r^2}\frac{\partial\alpha}{\partial r}(r^2 - xF).
\end{aligned}$$

With $\partial\alpha/\partial r < 0$, competition increases insolvency risk, decreases liquidity risk, and increases total credit risk if and only if

$$r^2 > xF. \tag{A.21}$$

Appendix A.5 Proof of proposition 7

The proof resembles that of proposition 6. Comparative statics again hinge on the sign of

$$\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right).$$

Note that for the exposure to contagion

$$\frac{\partial \rho^*_{CTG}}{\partial r} = (\mu' - \mu)\frac{\partial}{\partial r}\left(\frac{D^*}{(1-\alpha)r}\right) = (\mu' - \mu)c\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right),$$

and for the risk of a systemic crisis

$$\begin{aligned}
\frac{\partial \rho^*_{SYS}}{\partial r} &= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}Prob(\theta < \theta^{**}) \\
&= 2Prob(\theta < \theta^{**})\frac{\partial}{\partial r}\big(1 - \frac{r - F - \mu' D^*}{(1-\alpha)r}\big) \\
&= -2Prob(\theta < \theta^{**})(1 - \mu' c)\frac{\partial}{\partial r}\left(\frac{r-F}{(1-\alpha)r}\right).
\end{aligned}$$

Therefore, when $r^2 > xF$, loan competition leads to greater exposure to contagion yet a smaller chance of systemic crisis.

Appendix B The Laplacian property

In the model, the noisy signal received by representative creditor i has a structure

$$s_i = \theta + \epsilon_i.$$

We assume ϵ_i follows a continuous distribution with c.d.f. G.

Denote the critical signal for creditor i to switch from "wait" to "run" by s^*. And upon observing s^*, the creditor i believes a M fraction of creditors observing signals higher than hers. We prove $M \sim U(0,1)$.

Proof. For the continuous distribution G, the fraction of creditors who observes signal higher than s^* equals the probability that a creditor j's signal $s_j > s^*$. Then, we have

$$\begin{aligned}
M &= Prob(s_j > s^* | s_i = s^*) = Prob(\theta + \epsilon_j > s^* | s_i = s^*) \\
&= Prob(\epsilon_j > s^* - \theta | s_i = s^*) \\
&= 1 - G(s^* - \theta)
\end{aligned}$$

The randomness of M is rooted in the fact that by observing $s_i = s^*$, creditor i is uncertain about the realization of θ. As the *perceived* value of θ is random, so is the *perceived* M. Now we derive the distribution function of M. For $\hat{M} \in [0,1]$, we have

$$\begin{aligned} Prob(M < \hat{M}|s_i = s^*) &= Prob(1 - G(s^* - \theta) < \hat{M}|s_i = s^*) \\ &= Prob(\theta < s^* - G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= Prob(s^* - \epsilon_j < s^* - G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= Prob(\epsilon_j > G^{-1}(1 - \hat{M})|s_i = s^*) \\ &= 1 - G(G^{-1}(1 - \hat{M})) \\ &= \hat{M} \end{aligned}$$

Note that $M = 1 - G(s^* - \theta) \in [0,1]$. Therefore for $\hat{M} < 0$, $Prob(M < \hat{M}) = 0$; and for $\hat{M} > 1$, $Prob(M < \hat{M}) = 1$. We prove M follows a uniform distribution on $[0,1]$. □

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