Estimation of a structural model of competition in the Colombian electricity spot market

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Abstract

We formulate and estimate a structural complex-bidding auction model for the Colombian electricity market. We investigate whether the current dispatch mechanism for electricity generation in the Colombian market, a centralized unit commitment mechanism introduced by Resolution 51 (2009), led to a reduction in the aggregate cost of energy compared the a counterfactual of self unit commitment that prevailed before 2009. Our model accounts for the presence of complex bids, multi-plant firms, and the dynamic incentives of both hydro and thermal generators. Using a bootstrapping approach, we estimate the primitive parameters of the bidders' marginal cost function. These estimates are used to simulate a counterfactual experiment in which we evaluate the social gains of a centralized unit commitment dispatch by simulating the prices and outputs between August, 2011 and December, 2012, assuming that the market mechanism was the self unit commitment format without complex bids. Our findings show that, although the new dispatch mechanism is on average associated with higher bid markups, the total cost of the energy produced was substantially lower in the new mechanism. As a by product, we compare estimated marginal and opportunity costs from our model with engineering marginal costs, a standard methodology in the non-economic literature. We find that engineering costs substantially under estimate the marginal and opportunity costs of our structural model.

1 Introduction

In 1994, Colombia became the first Latin American country to design an electricity market, which followed the English model of production allocation through multi-unit auctions. Generally, electricity markets are considered complex because of the interactions that arise between the economic incentives of the agents and technical constraints of the production and distribution systems. The particular regulatory design of the Colombian electricity market makes it a case of study that should be of universal interest.

The main transactions in this market involve four types of agents. Generators produce the electricity that is sold in the wholesale electricity market of Colombia. The main generation technologies in this market are hydroelectric (hydro) and themoelectric (thermal), with more than 63% of the total installed capacity held by hydro units and 32% by thermal units. The role of retailers is just of an intermediary. They buy the electricity that is produced in the market and sell it to the final consumer. The other two agents, transmitters and distributors, are responsible for delivering the electricity from the generators to the consumers.

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The wholesale electricity market in Colombia, also known as *Mercado de Energía Mayorista* (MEM), was established in 1994 when generation and trade were deregulated. The MEM is a centralized market interconnected through the *Sistema Interconectado Nactional* (SIN), a country-wide network. It consists of two separated markets: the forward market and the spot[1] market. Most electricity is traded in the forward market through bilateral contracts between generators and retailers. However, all production decisions are centralized and defined in the spot market.

Since its establishment, two central regulatory interventions have affected the centralized planning dispatch and the rules of the Colombian spot market, which operates by receiving day-ahead bids and using those bids for dispatch decisions and calculation of the spot price. Until 2001 the spot market required generating units (plants) to self-commit generating capacity and submit hourly energy price offers along with declaration of their available capacity for each hour of the next day. Using these bids, the system operator (XM) would determine the least cost generation dispatch to satisfy demand on an hour by hour basis and determined the hourly wholesale price, as the price offered by the marginal plant (that is the highest cost plant needed to meet demand). The hourly market clearing price was used to compensate all dispatched generating units. This mechanism is similar to running an hourly uniform price auction for energy (see below for details) and subsequently handling transmission constraints through an out of market balancing mechanism. After 2001, the *Comisión de Regulación de Energía y Gas* (CREG)[2] determined that firms were allowed to bid only one price for each generation unit the own, for the entire 24-hour period.[3]

In 2009 CREG realized the possibility of productive inefficiencies of the existing market design due to the heterogeneity of generating technologies comprising hydro and thermal generating units, with very different cost structures.[4] In particular, such inefficiencies could arise from the non-convex cost structure of thermal generating units, since their start-up and shut down costs were not explicitly accounted for in the dispatch optimization algorithm. The economic and engineering literature has extensively discussed the fact that in the presence of non-convexities, self-committed uniform price auctions with energy only offer prices can lead to productive inefficiencies (Sioshansi, Oren, & O'Neill, 2008a, 2008b, 2010). From the suppliers' perspective, thermal units face an unnecessary risk when restricted to submit energy only offer prices since if a unit is dispatched, the market clearing price would need to be sufficiently high to compensate for start-up costs.

Following recommended international best practices and academic literature, the CREG undertook a redesign of the spot market and centralized energy dispatch. In broad terms the market became a pool, with multipart bids and centralized unit commitment. More precisely, generating units are now required to separate their offers into variable and quasi-fixed costs (start-up and shut down). In this way generators now submit "complex bids" consisting of three-part hourly bids for the next 24 hours: (i) bid price (the same for the next 24 hours), (ii) start-up and shut down cost (the same for a three month period) and (iii) maximum available capacity for each hour of the day. Using this information the system operator determines the least cost generation needed to satisfy demand on a daily basis, setting the market clearing price as the price offered by the marginal plant. *Ex post* the system operator determines which of the dispatched plants cannot recover their fixed costs given the energy market clearing price over the 24 hour period. Such plants are paid a "make whole payment" in addition to their energy

[1]This is rather a day-ahead market since, as we describe in following sections, the spot price is determined using bids placed un day ahead of the actual calculation. Nevertheless, as in de Castro, Oren, Riascos, and Bernal (2014), we will follow the usual practice in Colombia and refer to this market and its price as "spot market" and "spot price", respectively.

[2]Colombia's energy regulatory agency.

[3]See Resolution CREG-026 (2001).

[4]See Document CREG–011 (2009), Resolution 051 (2009) and subsequent modifications.

sales revenues, which enables them to recover their fixed costs.

While in the current centralized unit commitment the system operator can determine a more efficient dispatch, the incentives to overstate costs through the auction mechanism is still not clear as complex bids may allow for further strategic behavior. There are no theoretical studies with clear-cut results that rank the performance of one design relative the other. Therefore the question remains an empirical one. In a recent work Riascos, Bernal, de Castro, and Oren (2016) find evidence supporting the claim that the current market design resulted in a positive welfare effect at least in terms of productive efficiency. However, they also find strong evidence suggesting that the potential increase in market power allowed producers to obtain a larger share of the productive efficiency gains through higher market prices.

The goal of this study is to answer the question about whether the current dispatch mechanism for generation in the Colombian electricity market led to a reduction in the energy cost for the consumers. In particular, we propose a counterfactual experiment that allows us to compare the realized cost of the energy sold between August, 2011 and December, 2012 with the cost the system would have faced if the dispatch mechanism was a multi-unit uniform-price auction as before Resolution 051 (2009), for the same period. The results of such experiment allows us to conclude that the aggregate cost of energy sold would have been lower between August, 2011 and December, 2012 if the dispatch mechanism was a standard uniform-price auction instead of the current dispatch with complex bids.

We propose a structural model of bidding behavior that accounts for the presence of complex bids and the dynamic incentives of both hydro and thermal generators. Our model is based on the works by Reguant (2014) and Balat, Carranza, and Martin (2015). The model allows us to estimate marginal production costs for thermal units using the first order conditions implied by firm profit-maximizing behavior. In a first stage, we use simple bids to estimate marginal costs, using the identification strategies suggested by Wolak (2007). For marginal costs, the identification strategy relies on the fact that variations in the observed contract sales position affect firms' markups but not their costs. To approximate the firms' expectations about the market outcomes we rely on the bootstrapping method suggested by Hortacsu and McAdams (2010). To generate the bootstrapped moments we use the computational model introduced by Camelo, de Castro, Papavasiliou, Riascos, and Oren (2016) which can is able to approximate the complexity of the Colombian electricity market algorithm for the ideal dispatch.

Recent studies that estimate marginal costs or markups include those by Hortacsu and Puller (2008), Wolak (2000, 2003), Gans and Wolak (2008), Ciarreta and Espinosa (2010). However, these works focus only on the static version of the competition model, ignoring the dynamic incentives that arise due to the presence of non-convexities in the costs of thermal units or to the ability of storing water for large hydro units. Most of the works that completely characterize the dynamic incentives of hydro generators are basically theoretical and restrict the empirical analysis to test the predictions of the underlying models using observed bidding data (Garcia, Campos, & Reitzes, 2005; Garcia, Reitzes, & Stacchetti, 2001; Stacchetti, 1999; Vegard Hansen, 2009). Riascos et al. (2016) and Camelo et al. (2016) are successful in analyzing the dynamic incentives of the thermal units in a setting with complementary bidding mechanisms. However they restrict their analysis to reduced-form estimates of the costs and bidding functions to address their empirical questions. To our knowledge only the work by Reguant (2014) estimates the costs of thermal units controlling for the dynamic incentives that arise due to the presence of start-up costs. Conversely, Balat et al. (2015) and Martin (2015) are the only studies to fully characterize the dynamic incentives of hydro generators in a setting of multi-unit auctions and provide an empirical strategy to identify the implied opportunity cost of the water based on observed bidding data. Nevertheless, these papers do not jointly account for the interactions between the dynamic incentives associated with both technologies.

Given the parameter estimates that we obtain for the marginal costs function of each generating unit, we first compare our estimated series of the average marginal costs with those engeneering series of margninal costs computed by de Castro et al. (2014). In most cases the engineering approximation underestimates the firm's economic valuation for each KWh to be sold since the engineering measure does not account for the incentives associated with each unit's technological restrictions, as well as unobserved opportunity costs associated with fuel prices and exchange rates. Then, we compare the estimated average marginal costs with the observed bids, by fuel type. The results suggest that the highest markups are charged in average by fuel oil units, while coal units charge the lowest markups.

The main purpose of estimating the structural parameters of the model is to perform a counterfactual experiment, which is intended to evaluate the current auction design by comparing the aggregate costs of energy produced in the observed equilibrium with those simulated in a market structure with the previous auction setting that was used in the Colombian electricity market between 2001 and 2009, which resembled a uniform-price auction. However, the computation of the counterfactual equilibrium is usually complicated. The main complications arise due to the existence of multiple equilibria and on the extrapolation of the distribution of firms' expectations over an unboserved combination of market structure and state space. To our knowledge, there are no empirical studies that are able overcome these features in order to perform counterfactual experiments in which firms play a game other than a Vickrey-equivalent mechanism.

In this study, we contribute to the empirical literature on energy markets by overcoming these difficulties by making two major assumptions. Specifically, we assume that firms always play the same symmetric equilibrium for the respective game that we observe in the data for each of the two dispatch mechanisms (i.e. the previous auction format and the current dispatch with complex bids). In other words, we assume that, outside of the estimation sample, the conditional distribution of expectations can be extrapolated as a game-specific function of the observed state variables.

Our findings show that the current dispatch mechanism is associated with higher spot prices in average. That is, before taking into account start-up costs, the aggregate cost of energy sold is systematically lower under the current dispatch mechanism. However, we also find that the aggregate firms' revenue of the energy sold is substantially higher under the new mechanism. Overall, the results of such experiment allows us to conclude that the current dispatch mechanism used in the Colombian electricity market between August, 2011 and December, 2012, worked in benefiting the generating firms over consumers.

The objectives of this document are to introduce the structural model for bidding behavior, to propose an estimation methodology based on the available data of the Colombian electricity market and to answer the question about the cost-efficiency of the current dispatch mechanism through a counterfactual experiment. The structure of the document is as follows. In the next section we present a description of the structural characteristics, auction rules and the database of the Colombian electricity market used in this study. Section 3 presents a the auction model of bidding behavior from which we characterize the firms' profit-maximizing optimality conditions. Section 4 presents the estimation methodology. The estimation results are presented in Section 5. Then, Section 6 describes the simulation procedure and results of our counterfactual experiment. Finally, section 7 concludes with some final remarks.

2 The Colombian Electricity Market

In this section we present a brief description of the electricity market in Colombia and the auction data used for our empirical analysis. We focus on the wholesale market, called the *Mercado de Energía Mayorista* (MEM), where the price and quantity of produced electricity are defined (see Carranza, Riascos, Morán, and Bermeo (in press) for a detailed description).

The wholesale electricity market in Colombia was established in 1994 when generation and trade were deregulated. The MEM is a centralized market interconnected through the *Sistema Interconectado Nactional* (SIN), a country-wide network. The main transactions in this market involve four types of agents. Generators and retailers are the only active agents of the MEM. Generators produce the electricity that is sold in the MEM. Retailers buy that electricity to sell it to the final consumer. The other two agents, transmitters and distributors, are completely owned by the State. Competition in transmission and distribution activities is possible only in projects for the expansion of the network.

Trade and operation in MEM are coordinated by the *Centro Nacional de Despacho* (CND), the market operator. The CND is responsible for the planning, supervision and control of the integrated operation of generation resources and the transmission connectivity of the SIN. A subsidary of the CND, the *Administrador del Sistema de Intercambios Comerciales* (ASIC), administrates all monetary transactions made by the active agents of the MEM. Since 2005, both ASIC and CND are administrated by XM, a subsidiary of *Interconexión Eléctrica S.A.* (ISA). Finally, all transactions are monitored by the *Comisión de Regulación de Energía y Gas* (CREG), the regulatory agency.

The MEM consists of two separated markets: the forward market and the spot market.[5] Most electricity is traded in the forward market through bilateral contracts between generators and retailers. However, the role of the forward market is merely a financial one. All production decisions are centralized by the CND and cleared in the spot market.

Procurement in the spot market settle using a mechanism similar to a multi-unit uniform-price auction where generators submit supply schedules to satisfy load demand in an hourly-period basis. The bidding structure and the definition of the market price (spot price) differ across three different periods since 1995. For our empirical analysis we focus on the 2010–2015 period when the auction design incorporates complex bids.

2.1 Productive structure

The Colombian electricity generation technology is primarily hydroelectric (hydro) and themo-electric (thermal). During the sample period, the dominant production technology was hydro with more than 63% of the total installed capacity of the SIN (see Figure 2). More than 95% of hydro capacity was operated by plants that use dams, while the reminder 5% belonged to run-of-river plants. Thermal plants accounted for 32% of the total installed capacity, most of which are fueled by natural gas.[6] The rest of the capacity of the SIN belonged to producers using eolic technology (0.14%) and *cogeneration* (0.18%), a technology that produces electricity from the thermal energy generated from other productive activities.[7]

In terms of aggregate production, the share of hydro generation is even higher. Between 2000 and 2013, the yearly generation was between 41,278 and 62,197 GWh, with an average growth rate close to 4% (see Figure 1a). Under normal hydrological conditions hydro plants can reach up to 91% of this generation. This productive structure, however, makes the Colombian electricity industry very vulnerable to water scarcity periods, as pointed out by Stacchetti (1999). We illustrate this in Figure 2a. In periods of droughts as those caused by *El Niño* in 1992-1993 and 2009-2010, hydro generation share was close to 51% and 46%, respectively. Consequently, the spot price can also be severely affected by these extreme weather conditions. Figure 2b shows

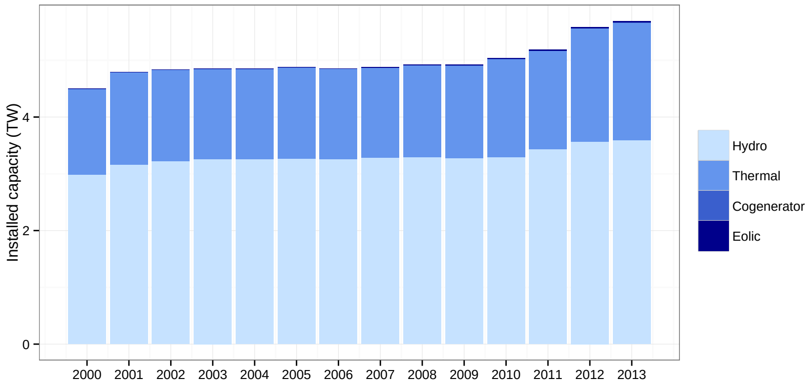
[5]As mentioned before, this is rather a day-ahead market but we will follow the usual practice in Colombia and refer to this market and its price as "spot market" and "spot price", respectively.

[6]We include combined cycle gas turbine power plants in the set of thermal technology.

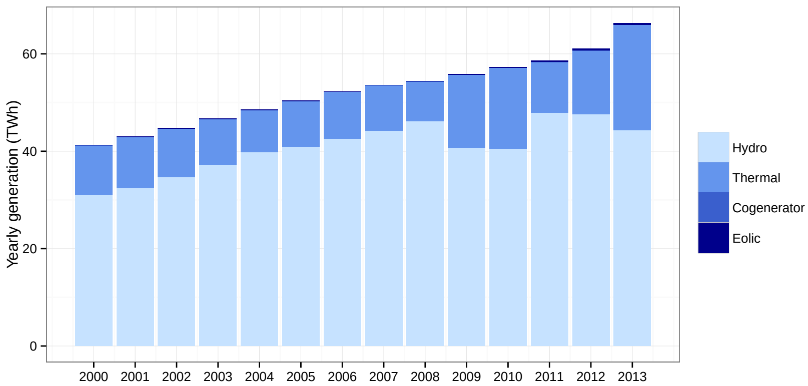
[7]In Colombia the main source of cogeneration is the sugar industry.

Figure 1: Evolution of the Productive Structure

(a) Installed Capacity



(b) Aggregate Generation



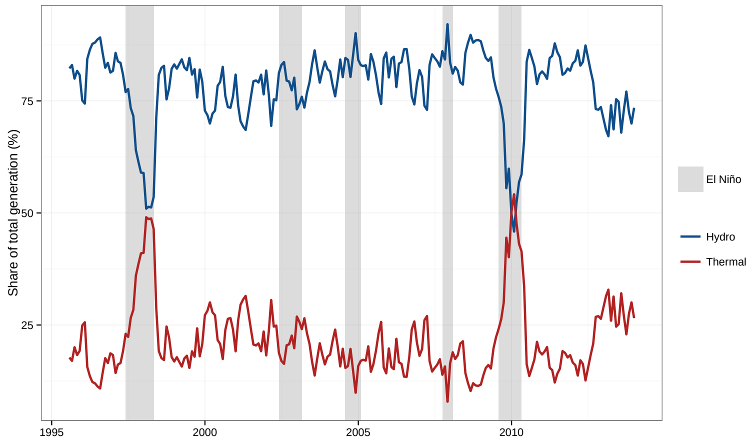
the evolution of the monthly average spot price. During the most severe events of *El Niño* in Colombia, the monthly average spot price increased 3.5 times from June, 1997 to February, 1998 and 1.2 times from April, 2009 to April, 2010.

Producers in the MEM are registered as *generators*. A generator definition depends on whether it uses hydro or thermal technology. In general, a generating firm may own more than one plant. Within a given plant there may be more than one generation unit. Several hydro plants operating with the same dam or river form a *hydro chain*. Thus a hydro generator is defined as a plant or hydro chain (if that is the case) while a thermal generator is a generation unit of a thermal plant.

Generators in the MEM are also classified by size. This classification determines whether a generator is subject to central dispatch, that is, if the generator must participate in the electricity auction. Large generation units with a net effective capacity (NEC) above 20 MW are classified as *major* generators. Major generators are always centrally dispatched. Generators with a NEC below 20 MW are called *minor*. Generally, minor generators are not subject to central dispatch; however, when having a NEC between 10 an 20 MW, a minor generator may decide whether to be centrally dispatched or not. During our period of study most generators in the SIN were minor, accounting for 61% of the all generators and 4% of the installed capacity of the SIN, while major generators accounted for about 34% of all the generators and almost 96% of total capacity (see Figure 3). The third group consists of all generators that use cogeneration and those, not connected to the SIN, that produce electricity for self-consumption called *autogenerators*. Neither autogenerators nor cogenerators are centrally dispatched.

Figure 2: Evolution of Hydro and Thermal Generation Shares and The Spot Price

(a) Generation



(b) Spot Price

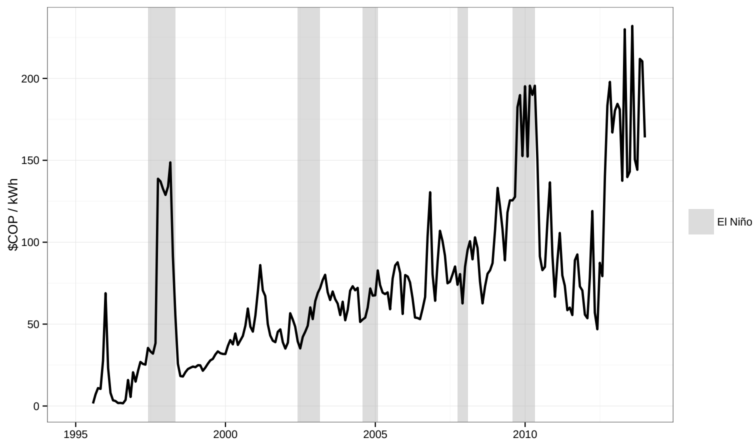


Table 1 presents the distribution of plants and installed capacity across the different types of generation technologies at the end of 2015. The data shows that the majority of production capacity is owned by less than 20% of the firms. Three large companies: *Emgesa*, *Empresas Públicas de Medellín* (EPM) and *Isagen* dominate the productive structure. These firms owned more than 56% of the SIN's installed net capacity and almost 70% of the total water storage capacity. The rest of production capacity was operated by 3 medium-size and 32 small firms. This structure has not changed much since then.

Figure 3: Distribution of Installed Capacity

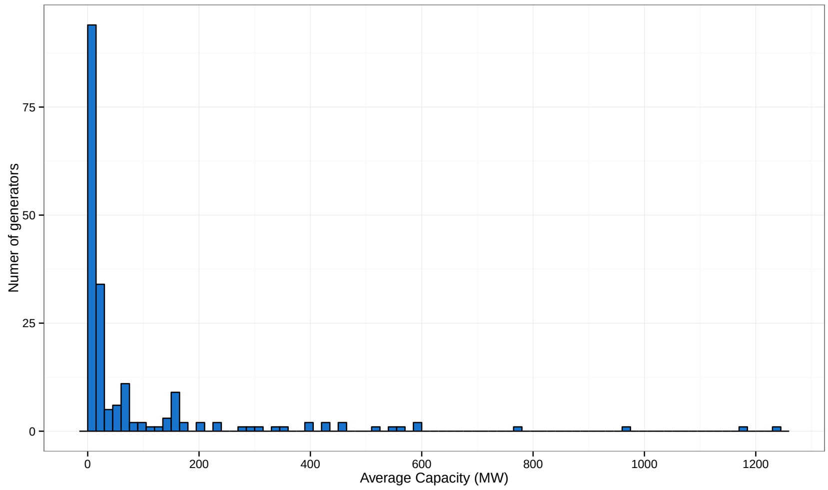


Table 1: Distribution of the Installed Capacity, December 2015

Type of generation		Number Firms[a]	Number Generators	Installed Capacity (MW)	Share (%)
Hydro	Dam	7	18	8,495	63.03
	Run-of-river	22	84	502	3.72
Thermo	Gas	14	17	3,551	26.34
	Coal	3	3	700	5.19
	Fuel[b]	1	1	187	1.39
Eolic		1	1	18	0.14
Cogeneration		5	7	25	0.18
Total		37	131	13,478	100

Source: Authors' calculations based on data from XM. [a] The number of firms is defined as the number of agents that operate the plants in each category (row) as registered in the MEM.
[b] Includes plants that use diesel, fuel-oil or a mix of gas and fuel.

2.2 The Rules of the Spot Market

From 2009 to the present, procurement in the spot market has been made through a daily optimization process that resembles a uniform-price multi-unit auction. All centrally dispatched generators (units) are required to participate by submitting day-ahead bids consisting of a unique price and an estimate of the maximum available capacity they expect to have for each hour of the next day. Additionally, at the beginning of every quarter of the year, firms are also required to submit a monetary start-up cost bid for each thermal unit the operate.[8] The auction is conducted by the CND who defines a daily generation schedule that satisfies demand at minimum generation costs.

The process is described as follows. Every day before 8:00 AM, firms submit a day-ahead bid

[8]Noncentrally dispatched generators, on the order hand, are not supposed to participate in the auction. Instead, they are asked to submit an hourly power schedule they are willing to sell as price takers.

schedule for each unit they owned.[9] Using these bids, the CND calculates a generation schedule that ensured energy supply at minimum production costs. This schedule, called *economic dispatch*, consists of the amount of electricity every generator is required to produce in order to satisfy the expected demand for each hour of the next day. The purpose of the economic dispatch is to define a day-ahead operation plan that aims to satisfy the forecasted demand based on the expected production availability and network performance. This dispatch does not involve the definition of any monetary variable.

During the operation day, the CND is responsible for adjusting the economic dispatch for available capacity changes, network restrictions and deviations of real demand from the forecast. The schedule that accounted for these adjustments is called *real dispatch*. The objective of the real dispatch is to coordinate supply and demand according to the technical constraints that have to be met in a real time basis. The main difference in respect to the economic dispatch is the realization of demand and unexpected technical flaws of generating units and network congestion.

The day after, the CND computes the hourly spot price by solving the dispatch optimization problem taking into account the realized demand and actual supply but assuming ideal network conditions. The resulting schedule is called *ideal dispatch*. The last generation unit dispatched is called the marginal generator and is only dispatched for the residual demand not covered by the other dispatched units. The objective of the ideal dispatch is to define the monetary variables of the market. In particular, the hourly spot price is set equal to the highest price among the flexible dispatched units. Finally, similar to a uniform-price auction, all ideally dispatched units are paid with the spot price for every kWh produced in the respective hour.

2.3 Description of the Database

For our empirical analysis, we use information of centrally dispatched generators on bid prices, aggregate demand, available capacity, water storage and inflow levels, among other market variables from 2010 to 2016. The data is provided by XM and is of public domain. The data also includes information on fossil fuel prices from the *Unidad de Planeación Minero Energética* (UPME).[10] We observe 1826 days, 27 firms and 63 generation units (20 hydro and 43 thermal). The final database is an unbalanced panel of 146,542 observations.

3 A multi-unit auction model with complex bids

Consider a wholesale energy market. There are $i = \{1, \ldots, N\}$ firms who operate $j = \{1, \ldots, J_i\}$ generation units, and compete in a daily multi-unit auction for the right to produce electricity. There are two main generation technologies, hydro and thermal. For every firm i, each thermal unit j's hourly production is represented by $q_{ijh} \in [\underline{q}_{ij}, \overline{q}_{ij}]$.

3.1 The auction rules

The auction design follows the rules of the Colombian electricity spot market. Here and after we refer to simple bids as the collection of price and quantity bids and complex bids as the quasi-fixed cost component, that is, the start-up cost bid.

[9] Units that did not submit their bids before 8:00 AM entered in the auction with the bid schedules they submitted in the last auction.

[10] UPME is a special administrative unit attached to *Ministerio de Minas y Energía* (the Ministry of Mines and Energy) responsible for planning energy mining development. See more at: http://www1.upme.gov.co

Every three months τ firms submit a set of complex bids consisting of a monetary start-up and shut-down costs for each thermal unit the own. The complex bid for unit j owned by firm i is denoted by A_{ij} and the collection of all complex bids is represented by $\mathbf{c}$. Then, every day within a given quarter, firms submit for each unit a set of simple bids which consist of a unique daily price and 24 available (declared) capacity values, one for each hour of the day. The simple bid submitted by firm i for unit j is represented as $\{b_{ij}, g_{ijh}\}$ and the collection of all simple bids is denoted by the array $\mathbf{b}$.

The market clearing algorithm searches all technically feasible combinations of units and production schedules that satisfy demand at the minimum cost. Both simple and complex bids are used by the market operator to define the daily dispatch (also known as *ideal dispatch*), which is defined as the combination of generating units and production schedules that minimize the daily cost of energy supply. Below we formally define this algorithm. To ease notation we temporarily omit the day and quarter subscripts.

Define demand for electricity D_h at hour h, as the sum of a deterministic price-inelastic component, D_h, and a stochastic component, ε_h, that is $\tilde{D}_h \equiv D_h + \varepsilon_h$. While D_h is known by all agents, firms are *ex ante* uncertain about the realization of ε_h. However, the process that generates ε_h is common knowledge and is represented by $F_\varepsilon(\varepsilon_h)$.

Let $\mathbf{k}^1_{ijh}$ and $\mathbf{k}^2_{ijh}$ be a set of multiple nonlinear vector functions specific to firm i's unit j at hour h, and let $\mathbf{r}_{ij}$ be the collection of all technical parameters for unit j. The equilibrium dispatch is defined as the feasible combination of generating units s and daily production schedule $\{q_{ijh}\}$ that solves the following optimization problem

$$\min_{\{q_{ijh}\}} \quad \sum_{h=0}^{23}\sum_{i=1}^{N}\sum_{j=1}^{J_i} b_{ij}q_{ijh} + A_{ij}\mathbf{1}^{\text{start}}_{ijh} \tag{1a}$$

subject to

$$\left\{\sum_{i=1}^{N}\sum_{j=1}^{J_i} q_{ijh} - \tilde{D}_h\right\} \geq \mathbf{0} \tag{1b}$$

$$\left\{\mathbf{k}^1_{ijh}(q_{ijh}, s_{ijh}, \mathbf{r}_{ijh})\right\} = \mathbf{0} \tag{1c}$$

$$\left\{\mathbf{k}^2_{ijh}(q_{ijh}, s_{ijh}, \mathbf{r}_{ijh})\right\} \geq \mathbf{0} \tag{1d}$$

Equation (1b) represents the market clearing condition, which is a usual restriction in most energy auctions. On the other hand, equations (1c) and (1d) are the most particular characteristic of represent the set of technical restrictions that need to be satisfied in order for the dispatch to be technically feasible.

Given the equilibrium dispatch the market clearing price p_h is computed as the lowest price among all flexible dispatched units.

3.2 Profits of the firm

All dispatched units are paid p_h for each kWh produced at the respective hour of the day.

Additionally, given the production schedule and submitted bids, the market operator defines for each unit a daily minimum revenue requirement characterized by the submitted bid price and start-up cost. According to the rules of the Colombian electricity market, every thermal unit whose gross revenue, characterized by the hourly market clearing price, is below its minimum revenue requirement is also paid with an uplift for each kWh produced during the day. Formally, let

$$R_{ij} = \sum_{h=1}^{23} p_h q_{ijh} \quad \text{and} \quad \underline{R}_{ij} = \sum_{h=1}^{23} b_{ij} q_{ijh} + A_{ij} \mathbf{1}_{ijh}^{\{\text{start}\}}, \tag{2}$$

be the daily gross and minimum revenue of unit j, respectively. Then every thermal unit j for which $R_{ij} < \underline{R}_{ij}$, is also paid with an uplift to the hourly price, denoted by ΔI, which depends on the market outcomes and is defined as follows:

$$\Delta I(\mathbf{b}, \mathbf{c}) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{J_i} \Big(\max\{0, \underline{R}_{ij} - R_{ij}\} + \sum_{h=0}^{23} d_{ijh} q_{ijh} [\max\{p_h, RP_{ij}\} - p_h] \Big)}{\sum_{h=0}^{23} \tilde{D}_h} \tag{3}$$

where RP_{ij} is the positive reconciliation price.[11]

At the time of bidding firms are still uncertain about other firms' strategies as well as the realization of ε_h. Therefore, firm i will choose a bidding strategy in order to maximize its expected profits, conditional on a given distribution of other firms' bids as well as on a set of common public information and independent private shocks.

We assume that the set of public information common to all firms includes demand forecasts, dams' water storage levels and inflows, fossil fuel prices as well as the technical parameters of all generating units. We denote the set of public information known to all firms at the time of bidding by ω. On the other hand, a given firm's private values may consist on information such as maintenance strategy or unit unavailabilities and bilateral contracts. Given the available information set, firm i's expectations about the market outcomes of are taken over a taken over its own beliefs about other firms' strategies.

Denoting S as the set of all feasible combinations of units being dispatched,[12] the expected profits of firm i conditional on the state variables for a given day can be expressed as

$$E_{-i}[\Pi_i(\mathbf{b}, \mathbf{c}) \mid \omega] = \sum_{s \in S} \Pr(s \mid \mathbf{b}_i, \mathbf{c}_i) \, E_{-i}[\Pi_i(\mathbf{b}_s, \mathbf{c}_s) \mid \omega, s], \tag{4}$$

where $\Pr(s \mid \mathbf{b}_i, \mathbf{c}_i)$ defines the probability that a combination of units s is dispatched, conditional on firm i's own bids. Notice that, conditional on a given state $\{\omega, s\}$, the market outcomes are only determined by the set of bids that are dispatched, denoted by $\{\mathbf{b}_s, \mathbf{c}_s\}$. To simplify notation for the rest of the document we state that expectations are always taken conditional on ω. Hence, firm i's profit function at a given state s and bid strategies $\{\mathbf{b}_s, \mathbf{c}_s\}$ is given by

$$\begin{aligned} \Pi_i(\mathbf{b}_s, \mathbf{c}_s) = \Bigg[& \sum_{h=0}^{23} (Q_{ih}(\mathbf{b}_s, \mathbf{c}_s) - v_{ih}) \, p_h(\mathbf{b}_s, \mathbf{c}_s) - \Delta I(\mathbf{b}_s, \mathbf{c}_s) v_{ih} \\ & + \Delta I(\mathbf{b}_s, \mathbf{c}_s) \sum_{j=1}^{J_i} \mathcal{I}_{ij}(\mathbf{b}_s, \mathbf{c}_s) q_{ijh}(\mathbf{b}_s, \mathbf{c}_s) \Bigg] \\ & - \sum_{j=1}^{J_i} C_{ij}(\mathbf{q}_{ij}(\mathbf{b}_s, \mathbf{c}_s)), \end{aligned} \tag{5}$$

where $Q_{ih}(\cdot)$ is the total quantity produced by firm i at hour h, v_{ih} is the firm's aggregate net sales position in the market of bilateral contracts, $\mathcal{I}_{ij}$ is an indicator function defined as follows

$$\mathcal{I}_{ij} = \begin{cases} 1, & \text{if } j \text{ is thermal and } R_{ij} < \underline{R}_{ij} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

[11] For the objectives of this study, an explicit definition of this price is not relevant. See Carranza et al. (in press) for a detailed description.

[12] That is, those satisfying the market clearing conditions (1b), as well as technical restrictions (1c) and (1c).

and $C_{ij}(\cdot)$ represents the total daily costs function of unit j, which depends on the vector of hourly equilibrium unit quantities. Note that firms' dynamic incentives are summarized in their cost structures.

The dynamic problem of thermal units arises due to the existence of ramping and start-up costs. These costs represent the inflexibility of a thermal unit to rapidly change its production levels. For thermal units we use the costs specification proposed by (Reguant, 2014):

$$C_{ij}(\mathbf{q}_{ij}) = \sum_{h=0}^{23} \gamma_{ij1} q_{ijh} + \frac{\gamma_{ij1}}{2} \tilde{q}_{ijh}^{2} + \frac{\gamma_{ij3}}{4} (q_{ijh} - q_{ijh-1})^2 + \alpha_{ij} \mathbf{1}_{ijh}^{\{\text{start}\}}, \tag{7}$$

where γ_{ij1} and γ_{ij2} represent j's marginal costs of production, γ_{ij3} represents the ramping costs, $\tilde{q}_{ijh} = \max\{q_{ijh} - \underline{q}_{ij}, 0\}$ is the unit's production over its minimum level, and α_{ij} is the total cost incurred whenever j gets switched on.

On the other hand, the dynamic problem of hydro units arises because their capacity to store energy in the form of water. This implies an intertemporal opportunity cost defined as the value of future payoffs the firm gives up in order to produce energy (by releasing the water) in the current period. We also account for the dynamic problem of hydro generators, which hold most of the installed capacity in the Colombian electricity market. In order to do that, we follow the characterization proposed by Balat et al. (2015) and define the cost function for hydro units as follows

$$C_{ij}(\mathbf{q}_{ij}) = \left(\sum_{h=0}^{23} \lambda_{ij} q_{ijh} \right) + \Psi_{ij}(\mathbf{q}_{ij}, \boldsymbol{\omega}_{ij}), \tag{8}$$

where λ_{ij} is the marginal costs of production and $\Psi_{ij}(\cdot)$ represents firm i's valuation for the sum of its future expected profits associated with unit j, which depends on the production output $\mathbf{q}_{ij}$ as well as on the current state of water storage and inflows levels, $\boldsymbol{\omega}_{ij}$.

3.3 Equilibrium and optimality conditions

We characterize the equilibrium of this model as the solution to a generic two-stage game with incomplete information. In the initial stage firms make irrevocable decisions about their complex bids. Once firms have submitted their complex bid schedules, a competition stage begins. In the second stage, all firms play in a sequence of 90 repeated sub-games. Each sub-game is characterized as multi-unit auction in which profits are determined according to the rules of the Colombian Spot Market. For any given day during the competition stage, we also assume that firm i's information set also includes information about the market outcomes of the previously disputed competition sub-games.

Given the sequential nature of the game we use backward induction to characterize the optimal strategies for both simple and complex bids. That is, for each firm i we start by deriving the optimality conditions for simple bids conditional on a fixed complex bid strategy. Then, the resulting simple bid strategy is incorporated in the firm's decision problem during the first stage to compute the respective optimality conditions for complex bids.

3.3.1 Optimality conditions for simple bids

The Markovian structure of the cost functions for both hydro and thermal units allows us to solve each sub-game of the competition stage as a conditionally independent simultaneous auction. This implies that, in each day within a given quarter of the year, firms will choose simple bid strategies as to maximize their expected daily profits:[13]

$$\max_{\mathbf{b}_i} \sum_{s \in S} \Pr(s \mid \mathbf{b}, \mathbf{c})\, E_{-i}[\Pi_i((\mathbf{b}_i, \mathbf{b}_{-i}), \mathbf{c}) \mid s, \mathbf{c}_i]\,. \tag{9}$$

As usual in the literature for energy auctions we focus on the first-order conditions with respect to the price offers (Hortacsu & Puller, 2008; Kastl, 2011; Reguant, 2014; Wolak, 2003).[14] Then, the optimal strategy for simple bidding must satisfy the following first-order condition:

$$\sum_{s \in S} \Pr(s \mid \mathbf{b}, \mathbf{c}) \frac{\partial E_{-i}[\Pi(\mathbf{b}, \mathbf{c}) \mid s, \mathbf{c}_i]}{\partial b_{ij}} + \sum_{s \in S} \frac{\partial \Pr(s \mid \mathbf{b}, \mathbf{c})}{\partial b_{ij}} E_{-i}[\Pi(\mathbf{b}, \mathbf{c}) \mid s, \mathbf{c}_i] = 0. \tag{10}$$

This expression allows us to analyze separately the process that determines the combination units that are going to be dispatched form the one that defines prices and quantities.

The first term can be interpreted in a similar fashion as in a usual multi-unit auction setup. However, there is a important difference. In a standard uniform-price multi-unit auction, small changes b_{ij} can only affect firm i's expected profits if b_{ij} is likely to be marginal, that is, to set the market clearing price. In the Colombian auction design, even after conditioning on s, small changes b_{ij} can still affect i's profits through the uplift component ΔI, even if b_{ij} does not set the market price. For example, an optimal strategy for firm i could be to slightly decrease b_{ij} in order to increase ΔI, which will be paid to all the units that are unlikely to meet their minimum revenue requirement.

On the other hand, the second term in equation (10) arises due to the existence of complex bids and the particularities of the Colombian dispatch optimization algorithm. This term is similar to the one derived by Reguant (2014) for the Spanish market. It represents the effect of b_{ij} on i's expected profits through the probability that a particular set of units is dispatched. That is, the extent to which small changes in b_{ij} affect the probability that any unit belonging to firm i will sell a positive quantity of electricity during the day. Notice that, given a set of technical parameters, this derivative is only non-zero when b_{ij} or A_{ij} are high enough so that j is the most costly unit in s, and there is unit $l \notin s$, such that the alternative combination of units $\hat{s} = \{s_{-j}, l\}$ is technically feasible and that the cost of the resulting dispatch is sufficiently low. Since the probability that these events occur simultaneously is likely to be small, we follow Reguant (2014) by assuming that

$$\sum_{s \in S} \frac{\partial \Pr(s \mid \mathbf{b}, \mathbf{c})}{\partial b_{ij}} E_{-i}[\Pi(\mathbf{b}, \mathbf{c}) \mid s, \mathbf{c}_i] \approx 0. \tag{11}$$

This assumption allows us to express the optimality conditions for simple bids focusing only on the first term of equation (10).

Thus, based on assumption (11) and the fact that small changes in b_{ij} only affect the expected market price if b_{ij} is likely to be the marginal bid, we rearrange terms from first-order condition

[13] Notice that we also need to assume that the dynamic problem of thermal units has an horizon of one day. This is also the procedure followed by Reguant (2014).

[14] According to the Colombian regulation, firms are requested to submit an estimate of the hourly maximum available capacity, which is supposed to change only due to technical failures or maintenance. Hence, firms would not be able use it directly as a strategic variable without drawing attention form the market regulator.

(10) and express firm i's optimal simple bid for unit j, conditional on $\mathbf{c}_i$ and s as follows:

$$b_{ij} = \bar{\zeta}_{ij} - \frac{\sum_{h=0}^{23} E_{-i}\Big[Q_{ih} - (1 + \frac{\partial \Delta I}{\partial b_{ij}})v_{ih} \;\Big|\; s, p_h = b_{ij}\Big]}{\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\Big|\; s, p_h = b_{ij}\Big]} + \frac{\sum_{l=1}^{J_i}\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial \Delta I}{\partial b_{ij}} q_{ilh} + \frac{\partial q_{ilh}}{\partial b_{ij}}\Delta I \;\Big|\; s, \mathcal{I}_{il} = 1\Big]\phi_{il}}{\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\Big|\; s, p_h = b_{ij}\Big]} + \frac{\sum_{l=1}^{J_i}\sum_{h=0}^{23} E_{-i}[\Delta I \times q_{ilh} \mid s, \mathcal{I}_{il} = 1]\frac{\partial \phi_{il}}{\partial b_{ij}}}{\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\Big|\; s, p_h = b_{ij}\Big]}, \tag{12}$$

where $\bar{\zeta}_{ij}$ represents a weighted average of the daily marginal cost of unit j owned by firm i at hour h:

$$\bar{\zeta}_{ij} = \frac{\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial C_{ij}}{\partial q_{ijh}}\Big(\frac{\partial q_{ilh}}{\partial b_{ij}}\Big) \;\Big|\; s, p_h = b_{ij}\Big]}{\sum_{h=0}^{23} E_{-i}\Big[\frac{\partial Q_{ih}}{\partial b_{ij}} \;\Big|\; s, p_h = b_{ij}\Big]}, \tag{13}$$

and $\phi_{ij}(\mathbf{b}_i) \equiv \Pr(\mathcal{I}_{ij} = 1 \mid \mathbf{b}_i)$ defines the probability for unit j of being paid the extra price-uplift ΔI, conditional on the firm's simple bid strategy and the state variables of the public information set.

According to equation (12), i's optimal simple bid for unit j is equal to the average marginal cost plus a shading factor or markup. This markup consists of three separate terms. The first term is standard in static models of uniform-price auctions (Gans & Wolak, 2008; Hortacsu & Puller, 2008; Wolak, 2000, 2003). It is composed by the expected inframarginal quantity produced by the firm when the unit is accepted, divided by its effect on equilibrium quantities, which is equivalent to its effect on the residual demand. The other two terms arise due to the presence of the minimum revenue requirement and the incremental price component ΔI, which are particular to the Colombian electricity market.

3.3.2 Optimality conditions for complex bids

We now proceed to derive the optimality conditions for complex bids. In the first stage of the sequential game, firms are aware that their complex bids decisions will be committed for each of the 90 sub-games of the second stage. Firm i will choose a complex bid strategy, $\mathbf{c}_i$, so as to maximize the total sum of its expected profits during the following 90 days.

It is worth mention that, because of the imposed backward induction characterization of the equilibrium, $\mathbf{c}_i$ will affect i's profits not only directly, but through its own simple bid strategy as well. We allow firm i to account for this feature at the time of submitting its complex bids strategy. Formally, let B and C denote the space of simple and complex bids, respectively. Also, let Ω be the state space. Define firm i's optimal strategy for simple bids at auction t as the vector function $\boldsymbol{\beta} : C \times \Omega \to B$ such that, for any given $\mathbf{c}_i \in C$ and $\omega_t \in \Omega$, $\mathbf{b}_{it} = \boldsymbol{\beta}_i(\mathbf{c}_i, \omega_t)$ satisfies the conditions implied by equation (12).

Then, adding the day subscript, we can write firm i's optimization problem at the first stage of the game as follows:

$$\max_{\mathbf{c}_i} \; E_{-i}\left[\sum_{t=1}^{90} \Pi_i(\mathbf{b}_t, \mathbf{c})\right], \quad \text{s.t. } \mathbf{b}_{it} = \boldsymbol{\beta}_{it}(\mathbf{c}_i) \equiv \boldsymbol{\beta}_i(\mathbf{c}_i, \omega_t). \tag{14}$$

Consequently, the first-order necessary conditions for this optimization problem are given by,

$$\sum_{t=1}^{90}\sum_{s\in S}\Pr\left(s \mid \boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}\right)\frac{\partial E_{-i}[\Pi_i(\boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}) \mid s]}{\partial A_{ij}}+ \sum_{t=1}^{90}\sum_{s\in S}\frac{\partial \Pr\left(s \mid \boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}\right)}{\partial A_{ij}}E_{-i}[\Pi_i(\boldsymbol{\beta}_{it}(\mathbf{c}_i),\mathbf{c}) \mid s]=0. \tag{15}$$

Notice that, complex bids affect firm i's daily profits through both probability of having any of its unit dispatched and through the definition of prices and quantities.

The derivative of the first left-hand term of equation (15) incorporates the effect of small changes in A_{ij} on firm i's daily profits. As we mentioned earlier, this effect consists of a direct effect, which is associated with the determination of quantities and the uplift ΔI, as well as an indirect effect, which is driven by the definition of the simple bid optimal strategies as functions of complex bids. To better define these two effects, we expand the first derivative term of equation (15) by applying the chain rule as follows:

$$\begin{aligned}
\frac{\partial E_{-i}[\Pi_i(\mathbf{b}_t,\mathbf{c}) \mid s]}{\partial A_{ij}} =& \sum_{h=0}^{23} E_{-i}\Bigg[\left(\frac{\partial p_{ht}(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial p_{ht}(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)(Q_{iht}(\mathbf{c}_i)-v_{iht})\\
&+\left(\frac{\partial Q_{iht}(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial Q_{iht}(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)p_{ht}(\mathbf{c}_i)\\
&-\left(\frac{\partial \Delta I_t(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial \Delta I_t(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)v_{iht}\,\Bigg|\, s\Bigg]\\
&+\sum_{l=1}^{J_i}\Bigg(E_{-i}\Bigg[\left(\frac{\partial \Delta I_t(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial \Delta I_t(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)q_{ilht}(\mathbf{c}_i)\\
&+\left(\frac{\partial q_{ilht}(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial q_{ilht}(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)\Delta I_t(\mathbf{c}_i)\,\Bigg|\, s,\mathcal{I}_{ijt}=1\Bigg]\phi_{ijt}(\mathbf{c}_i)\\
&+\left(\frac{\partial \phi_{ijt}(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial \phi_{ijt}(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)E_{-i}\Bigg[\Delta I_t(\mathbf{c}_i)q_{ilht}(\mathbf{c}_i)\,\Bigg|\, s,\mathcal{I}_{ijt}=1\Bigg]\Bigg)\\
&-E_{-i}\Bigg[\bar{\zeta}_{ilht}(q_{ilht}(\mathbf{c}_i))\left(\frac{\partial q_{ilht}(\mathbf{c}_i)}{\partial A_{ij}}+\left[\frac{\partial q_{ilht}(\mathbf{c}_i)}{\partial \mathbf{b}_{it}}\right]'\boldsymbol{\beta}'_j(\mathbf{c}_i)\right)\,\Bigg|\, s\Bigg],
\end{aligned} \tag{16}$$

where $\boldsymbol{\beta}'_j(\cdot)$ represents the vector of derivatives of $\boldsymbol{\beta}(\cdot)$ with respect to A_{ij}.

4 Estimation

In this section we propose an estimation methodology to recover the structural parameters of the costs functions for each centrally dispatched generator based on the observed bidding data of the Colombian electricity market. In particular, we estimate the structural parameters of the cost function defined in equation (7) for every firm i:

$$\boldsymbol{\theta}_i = \{\alpha_i, \gamma_i, \lambda_i\}\,. \tag{17}$$

As is usual in the empirical literature for energy auctions, we use the empirical moments implied by the optimality conditions of the bidding game defined by equations (10) and (15). Below we discuss the methodology employed for the construction of these empirical moments as well as the intuition behind identification.

4.1 Estimation of firms' expectations

To estimate the unit-specific costs parameters, we use the generalized method of moments procedure based on the first order conditions implied by equation (10). Specifically, we adapt the procedure used in previous studies in the multi-unit auction literature to the particular auction design of the Colombian electricity spot market.

To construct the empirical analog of first order conditions for simple and complex bids, we first need to estimate each firm's expectations terms. We follow the bootstrapping procedure used by Hortacsu and McAdams (2010) and Kastl (2011), which consists of simulating firm i's beliefs about other firms' strategies based on the available data. For a particular day, firm i's strategies are held fixed. For other firms different form i, strategies are randomly drawn form the sample data, approximating the uncertainty that the firm faces at the time of bidding. For a particular draw, and given the dispatch algorithm defined in (1a), we compute simulated equilibrium outcomes which determine the firm i's profits. Repeating this procedure for a sufficiently large number of times, it is possible to obtain consistent estimates of the firm's expected profits.

The consistency of the estimators for firm's beliefs with this procedure highly depends how the sampling step is done. For example, when firm i's beliefs are not generated by the same distribution between auctions, estimators of the expectation terms might be biased if the sampling of other firms' strategies does not condition on similar auctions. Therefore, we control for all variables included in the information set available for the firm at the time of bidding that are observed in the data. In particular, we condition the sampling set of similar days on water inflows, fuel prices, demand forecasts, average prices of bilateral contract sales, as well as on the day of the week.[15]

Another feature of the data that can bias the estimators is the existence of affiliated private values. Given the time series nature of the data, auctions are likely to be serially correlated. Conditioning on observed state variables of the information set is also useful when dealing with a potential bias driven by this feature. In fact, we assume that after controlling for the observed variables that are potentially serially correlated, the distribution of firm i's beliefs at the time of bidding is stochastically independent between auctions.

The bootstrapping algorithm we employ for this study can be summarized as follows:

1. Fix bidder i's strategies in auction t
2. Randomly draw strategies of other firms $k \neq i$ from a sample of N similar days, conditioning on a set of observed state variables
3. Compute the market equilibrium using the computational algorithm proposed by Camelo et al. (2016)
4. Repeat steps 2-3 M times to obtain a distribution of market outcomes

There are two aspects that make our bootstrap simulation procedure similar to the one used by Reguant (2014) and different from other applications. First, the market clearing is defined as the solution to a complex optimization problem and cannot be necessarily replicated through a standard uniform-price multi-unit auction. Second, as in the model introduced by Reguant (2014), firms also face uncertainty over their own equilibrium supply curve as the set of units that will be dispatched is also random due to the presence of complex bids.

However, our model differs form the one of Reguant (2014) mainly in the fact that complex bids are submitted on a quarterly basis. This would requires us to compute an estimate of the

[15] This approach is based on the works by Gans and Wolak (2008) and Reguant (2014), who also pool similar days to construct the sample analogues of moment conditions.

firms' expected sequence of their profits for the following 90 days. Because the available data for complex bids is short, we cannot directly estimate the underlying joint distribution of this sequence. Hence, in the current version of the study we do not estimate the implicit start-up costs of thermal units. Moreover, although we are still able to estimate the expectation terms for the first order condition of simple bids and the parameters of the marginal cost function (24), we have to assume that firms submit truthful bids about their start-up costs in order to compute a counterfacutal experiment.

For the second point of the bootstrap algorithm described above, we define a sample of similar days to t, as follows. The similarity criteria between days is defined according to state variables we assume firms use to condition their expectations; namely, the price of Fuel No. 6, as well as to the aggregate water stock and river flow. Hence, we define a vector consisting of the three variables previously mentioned for day t, denoted as $\mathbf{x}_t$. Then we define from the sample the set of all days with the same day of the week as t as WD_t and build the sequence of vectors $\{\mathbf{x}_\tau\}_{\tau \in \mathrm{WD}_t}$. Given the sequence of approximately 74 days,[16] we measure the euclidean distance between $\mathbf{x}_t$ and $\mathbf{x}_\tau$ for every $\tau \in \mathrm{WD}_t$ and select the vectors associated with the N smallest computed distance. We set $N = 30$ which gives us samples for bootstrapping i's competitors strategies of size between 1200 and 1470. Notice that since we match similar days without conditioning of firm-specific features, we can use the same sample of bidding strategies for every firm i.

4.2 Approximation of derivatives

Once market outcomes are simulated, the challenge that remains is the computation of the derivative terms involved in the optimality conditions for both simple and complex bids. To address this problem, we follow an smoothing approach that has become frequent in the context of electricity auctions (Gans & Wolak, 2008; Wolak, 2007).

The approximate versions of the derivative terms used to construct the empirical moments are the following:

$$\widehat{\frac{\partial D_{iht}^{R,bs}}{\partial b_{ijt}}} = \frac{1}{\nu} \sum_{k \neq i} \sum_{(k,j) \in s^{bs}} g_{kjht} \mathcal{K}\left(\frac{b_{ljt} - p_{ht}^{bs}}{\nu} \right) \tag{18}$$

$$\widehat{\frac{\partial Q_{iht}^{bs}}{\partial b_{ijt}}} = \frac{1}{\nu} \sum_{(i,j) \in s^{bs}} g_{ijht} \mathcal{K}\left(\frac{b_{ijt} - p_{ht}^{bs}}{\nu} \right) \tag{19}$$

where $\mathcal{K}$ is a Kernel density weight and ν is a bandwith parameter. In particular, we set $\mathcal{K}$ as the normal density function and ν following the rule of thumb for every different firm.

4.3 Identification and econometric specification

Following equation (7), the econometric specification of the daily weighted average marginal costs for a thermal unit j is the following:

$$\bar{\zeta}_{jt}^{\text{therm}}(\gamma_{jt}) = \gamma_{jt1} + \gamma_{jt2} \sum_h \tilde{q}_{ijh} + \gamma_{ijt3} \sum_h (2q_{ijh} - q_{ijh-1} - q_{ijh+1}) + \epsilon_{jt}^{\text{therm}}, \tag{20}$$

where $\epsilon_{jt}^{\text{therm}}$ represents the econometric specification error. As suggested by Reguant (2014), this error term can also be interpreted as a shock on marginal costs known to the firm or as an optimization error. Notice that the cost parameters are both unit and day-specific. We allow

[16] That is, 53 weeks in 1.4 years.

the cost derivatives to vary over time in order to capture the effect of fuel prices on both variable and ramping costs. In particular we specify each parameter as follows:

$$\gamma_{jt1} = \gamma_{j1}^{\text{cons}} + \gamma_{j1}^{\text{Pfuel}} \ln \text{Pfuel}_{jt} + \gamma_{j1}^{\text{Foil6}} \ln \text{Foil6}_t + \gamma_{j1}^{\text{TRM}} \ln \text{TRM}_t + \gamma_{j1}^{\text{CERE}} \ln \text{CERE}_t + \gamma_{j1}^{\text{FAZN}} \ln \text{FAZN}_t, \tag{21}$$

$$\gamma_{jt2} = \gamma_{j2}^{\text{cons}} + \gamma_{j2}^{\text{Pfuel}} \ln \text{Pfuel}_{jt} + \gamma_{j2}^{\text{Foil6}} \ln \text{Foil6}_t + \gamma_{j2}^{\text{TRM}} \ln \text{TRM}_t, \tag{22}$$

$$\gamma_{jt3} = \gamma_{j3}^{\text{cons}} + \gamma_{j3}^{\text{Pfuel}} \ln \text{Pfuel}_{jt} + \gamma_{j3}^{\text{Foil6}} \ln \text{Foil6}_t + \gamma_{j3}^{\text{TRM}} \ln \text{TRM}_t, \tag{23}$$

where Pfuel_{jt} is the current price in day t for the fuel used by unit j; Foil6 is the price of the Fuel Oil No. 6, used by most thermal units as a substitute of their main fuel; TRM is the COP/USD daily average exchange rate; CERE and FAZN denote the taxes firms must pay for each KWh generated.

In this study we focus on estimating the marginal cost function for thermal units only. However, in the estimation process we do control for river flows and water stock in order to account for the dynamic incentives of hydroelectric units.

Marginal cost parameters, γ_{jt}, can be identified given the observed position of contract sales. The intuition is that variations in the contract sales positions affect markups but not costs. Therefore, since the quantity sold by the firm in bilateral contracts is not defined at the same time as the price bids, marginal production costs can be identified with enough variation in the contract sales position. This identification strategy has become standard in energy markets (see Hortacsu and Puller (2008) and Wolak (2007)).

4.4 Estimation method

In the current version of the study we restrict to estimate only the parameters of the marginal cost function (20). Consequently, we define the set of parameters to estimate for firm i as $\theta_i = \{\gamma_{it}\}_{t=1}^{T}$.

The empirical moment conditions implied by equation (12) are given by

$$\begin{aligned} m_{ijt}(\theta_i, \nu, M) = \frac{1}{M} \sum_{bs=1}^{M} \sum_{h=0}^{23} \mathbf{1}\{j \text{ in}\} \Bigg[\widehat{\frac{\partial p_{ht}^{bs}}{\partial b_{ijt}}} \Bigg(\big(b_{ijt} - \bar{\zeta}_{ijt}(\theta_i)\big) \widehat{\frac{\partial D_{iht}^{R,bs}}{\partial b_{ijt}}} + Q_{ih}^{bs} - \Bigg(1 + \widehat{\frac{\partial \Delta I_t^{bs}}{\partial b_{ijt}}}\Bigg) v_{ih} \Bigg) + \\ \sum_{l=1}^{J_i} \mathbf{1}\Big\{\mathcal{I}_{ilt}^{bs} = 1\Big\} \Bigg(\Bigg(\widehat{\frac{\partial \Delta I_t^{bf}}{\partial b_{ijt}}} q_{ilht}^{bs} + \widehat{\frac{\partial q_{ilht}^{bs}}{\partial b_{ijt}}} \Delta I_t^{bs} \Bigg) \hat{\phi}_{ilt}^{bs} + \Big(\Delta I_t^{bs} \times q_{ilht}^{bs}\Big) \widehat{\frac{\partial \phi_{ilt}^{bs}}{\partial b_{ijt}}} \Bigg) \Bigg]. \end{aligned} \tag{24}$$

Then, the Generalized Method of Moments (GMM) estimator for the parameters is defined as follows:

$$\theta_i^* = \underset{\theta_i}{\arg\min} \left[Z_t' m_{ijt}(\theta_i, \nu, M)\right]' \Phi \left[Z_t' m_{ijt}(\theta_i, \nu, M)\right] \tag{25}$$

where Z is a matrix of instruments assumed to be orthogonal to ϵ, and Φ is a weighting matrix. Among the instruments we include fixed effects for days of the week, months of the year, observed load demand, aggregate river flows, fuel prices, firms' net contract position in the forward market, the monthly average of contract prices as well as forecast probabilities for *El Niño* events. The weighting matrix is computed according to the two-stage estimator where the initial guess is the identity matrix.

It is worth mention that, even though, we do not estimate the cost parameters for hydro units, we do account for hydro units incentives when computing the moment conditions defined above since we control for water stock and river flows when building the bootstrap sample in the procedure of estimating firm i's expectation terms. Therefore, if the assumptions we have made so far hold, the solution to equation (25) should be a consistent estimator of a restricted version of the full parametric model.

5 Estimation results

In this section we show the estimated parameters of the marginal costs function for thermal units defined in (24) as well as the estimated series and the implied bid-cost mark-ups. The results presented below are generated using the bootstrap algorithm described in subsection 4.1 and the GMM estimator defined by equation (25) on auction data at the generator level from August 13th, 2011 to December 31st, 2012.

Tables 2-5 show the coefficient estimates of the cost function for units fueled by diesel, fuel oil, coal and natural gas, respectively.[17] It is worth to mention that these coefficients by themselves lack of economic interpretation since the constant and variable part of the marginal cost function, as well as the ramping cost part depend on the fuel prices.

Consequently, to ease interpretation, we turn to Table 6. Here we present weighted averages of the estimated values for γ_{jt1}, γ_{jt2} and γ_{jt3} across time and units, by fuel type. The average constant marginal cost, $\bar{\gamma}_1$, is positive and higher for those units using the most expensive fuels (i.e. diesel and fuel oil). For example, average constant marginal cost for coal-fueled units is about 172 COP/KWh, while for those using diesel the cost is about 258 COP/KWh. The variable part of the marginal cost, $\bar{\gamma}_2$, suggest a reduction on the total marginal cost for every KWh generated over the unit's minimum production level. In other words, on average, thermal units face a higher cost when generating during their soak or desynchronization phases. Moreover, this effect is higher in magnitude for coal units and substantially lower for gas units. In particular, results suggest that coal units face, on average, a marginal cost 18.54 COP/KWh lower for every KWh produced over their minimum level, whereas for gas units the respective reduction is 3.02 COP/KWh. Finally, the average ramping cost, denoted by $\bar{\gamma}_3$, are in most cases positive. This suggests that, except for fuel oil units, the average marginal cost of a thermal unit increases whenever the unit has to change its output in less than one hour. In particular, the extra cost of changing output is on average 0.14, 0.10 and 0.04 COP/KWh for coal, gas and diesel units, respectively.

Given the parameter estimates we are able to project an estimate of the average marginal cost for each unit across the sample period and compute the implied bid-cost markup. First, we compare our estimates of the average marginal costs with those computed by de Castro et al. (2014) using the following engineering expression:

$$mc^{\text{en}}_{jt} = \frac{\text{HR}_j}{\text{CP}_j} \times \text{Pfuel}_{jt} + \text{VOM}_t + \text{CERE}_t + \text{FAZN}_t, \tag{26}$$

where HR and CP denote the unit's heat rate and calorific power value, respectively, and VOM represents the variable operating and maintenance costs.

Figures 4-8 compare the evolution of the daily marginal costs implied by the estimated parameters and the observed production schedules with the engineering costs computed by de Castro et al. (2014) for the same period, by fuel type. On one hand, notice that is most cases the engineering approximation underestimates the firm's valuation for each KWh to be sold since it does not account for the non-convexities associated with each unit's technological restrictions, as well as unobserved opportunity costs associated with fuel prices and exchange rates. On the other hand, the estimated marginal costs implied by the model specification (7) allow us to identify whether peaks in the market spot price are caused by technical inflexibilities faced by the marginal thermal units or by unilateral market power exercised by the generating firms.

Then, we compare the estimated daily average of the marginal costs with the observed bids, by fuel type, in figures 9-13. In these figures we also show the observed price bid by the unit for

[17] Although the other units were taken into account in the bootstrap algorithm the parameter estimates for their marginal costs were not estimated in this version of the document.

the respective day. For some units the implied markup can be higher 5 times the mean spot price. Notice also that for several days, the implied markups are negative. Such events occur days when the firm's supply function at the respective unit's bid price is lower than its aggregate energy sold in the bilateral contract market, $S_i(p_{ij}) < v_i$. This is consistent with literature regarding the bilateral contracts in electricity markets (Hortacsu & Puller, 2008; Wolak, 2003).

In Table 7 we compute the weighted mean, weighted standard deviation, minimum and maximum of the implied bid-cost markups by fuel type. The results suggest that the highest markups are charged in average by fuel oil units, while coal units charge the lowest. At the same time, we observe more dispertion on other fuel type technologies than on those using fuel oil, as suggested by the standard deviation. That is, coal units not only charge the highest markup on average, but they also exercise such market power consistently along the sample period.

Table 2: Coefficient estimates of the marginal cost function for units using diesel

Unit	γ_1^{cons}	γ_1^{Pfuel}	γ_1^{Foil6}	γ_1^{TRM}	γ_1^{CERE}	γ_1^{FAZN}	γ_2^{cons}	γ_2^{Pfuel}	γ_2^{Foil6}	γ_2^{TRM}	γ_3^{cons}	γ_3^{Pfuel}	γ_3^{Foil6}	γ_3^{TRM}
TERMOCANDELARIA 1	−16076.01*	−766.00*	−167.51*	3421.85*	−310.90*	4233.60*	551.30	−36.82	14.38	−45.56	79.53	1.99	−9.06	−2.97
	(2976.14)	(163.41)	(85.63)	(289.02)	(101.16)	(1559.66)	(5027.92)	(505.18)	(27.17)	(190.94)	(1666.43)	(67.15)	(17.63)	(166.31)
TERMOCANDELARIA 2	−7781.39*	−903.72*	−10.87	2304.32*	−158.11	1506.57*	33521.69*	−1080.89*	538.97*	−3778.37*	252.73	−3.02	−4.77	−24.84
	(3459.67)	(132.75)	(79.11)	(316.29)	(87.59)	(776.78)	(58373.82)	(6349.19)	(139.01)	(986.55)	(182.81)	(7.01)	(3.40)	(19.25)
TERMOEMCALI 1	−37948.75*	5045.31*	−404.00*	176.44*	−983.22*	−7263.78*	−385.99*	14.17*	15.43*	17.53*	119.61	−3.45	−7.03	−4.04
	(838.75)	(57.93)	(21.74)	(63.81)	(43.10)	(106.12)	(72.58)	(3.87)	(1.84)	(10.27)	(172.02)	(8.61)	(4.60)	(17.44)
FLORES 1	3998.19*	−27.78	−239.22*	−234.69*	28.38*	550.32*	−12.50*	−0.80*	3.49*	−1.28*	0.41	0.69	−1.82*	1.17
	(634.76)	(74.16)	(12.93)	(21.72)	(13.62)	(115.07)	(6.45)	(0.33)	(0.22)	(0.43)	(19.34)	(0.69)	(0.55)	(1.55)
TERMOSIERRAB	−17745.33*	1670.95*	55.00*	449.65*	−163.43*	−1268.96*	148.14*	−10.40*	−1.74*	−5.53*	−44.42	2.96	0.38	2.00
	(973.39)	(77.59)	(14.54)	(37.31)	(23.12)	(66.47)	(12.44)	(0.84)	(0.20)	(0.77)	(40.84)	(2.57)	(0.49)	(2.39)
TERMOVALLE 1	3623.00*	220.30*	−67.04*	−685.73*	63.46*	544.84*	1.57	−7.59*	2.31*	6.28	−9.40	1.30	−0.60	0.36
	(290.72)	(13.70)	(3.48)	(25.56)	(10.86)	(41.14)	(40.04)	(1.60)	(0.47)	(5.65)	(34.45)	(2.44)	(0.54)	(3.41)

Bootstraped standard errors are in parentheses. * Significant at 5%.

Table 3: Coefficient estimates of the marginal cost function for units using fuel Oil

Unit	γ_1^{cons}	γ_1^{Pfuel}	γ_1^{Foil6}	γ_1^{TRM}	γ_1^{CERE}	γ_1^{FAZN}	γ_2^{cons}	γ_2^{Pfuel}	γ_2^{Foil6}	γ_2^{TRM}	γ_3^{cons}	γ_3^{Pfuel}	γ_3^{Foil6}	γ_3^{TRM}
CARTAGENA 1	9035.61*	−1865.81*	965.93*	−178.18	28.45	4905.86*	−7036.72*	358.68*	−23.62	543.47*	1983.32	−139.44	8.95	−110.76
	(3940.56)	(238.31)	(62.55)	(486.94)	(109.11)	(1024.69)	(2011.41)	(72.56)	(34.12)	(196.77)	(1415.19)	(82.69)	(33.32)	(76.01)
CARTAGENA 2	18859.23*	−1921.51*	596.07*	−511.19*	−1110.88*	5656.78*	−2890.88*	244.90*	−38.67*	140.78	−231.99	15.25	32.89	−23.47
	(2168.89)	(205.01)	(46.63)	(254.83)	(78.06)	(520.28)	(721.17)	(55.91)	(19.94)	(83.24)	(747.82)	(37.93)	(21.36)	(55.21)
CARTAGENA 3	824.78	−1177.34*	613.22*	875.00*	−831.64*	5542.06*	−979.77	87.35*	−24.55*	55.42	−11.28	−12.99	8.71	6.94
	(2200.24)	(222.40)	(44.48)	(253.99)	(74.19)	(545.86)	(590.67)	(29.41)	(12.30)	(51.07)	(428.21)	(20.07)	(14.78)	(37.29)
BARRANQUILLA 3	−43301.74*	5803.52*	−416.86*	45.28	−966.46*	−5433.90*	1998.20*	−101.74*	−79.26*	−59.12*	−231.18	13.23	9.28	5.01
	(980.10)	(92.73)	(25.13)	(85.33)	(33.36)	(205.06)	(204.28)	(10.94)	(6.32)	(12.93)	(212.49)	(11.35)	(7.28)	(13.71)
BARRANQUILLA 4	−44633.65*	5088.79*	−290.12*	792.55*	−797.75*	−3238.51*	2824.83*	−189.34*	−41.52*	−109.20*	−698.38*	48.26*	9.82	25.81
	(961.92)	(113.28)	(24.67)	(103.21)	(34.29)	(268.60)	(240.30)	(13.97)	(10.31)	(14.55)	(234.78)	(14.15)	(10.89)	(14.02)

Bootstraped standard errors are in parentheses. * Significant at 5%.

Table 4: Coefficient estimates of the marginal cost function for units using coal

Unit	γ_1^{cons}	γ_1^{Pfuel}	γ_1^{Foil6}	γ_1^{TRM}	γ_1^{CERE}	γ_1^{FAZN}	γ_2^{cons}	γ_2^{Pfuel}	γ_2^{Foil6}	γ_2^{TRM}	γ_3^{cons}	γ_3^{Pfuel}	γ_3^{Foil6}	γ_3^{TRM}
PAIPA 1	2253.99*	90.25	88.40*	−663.38*	347.02*	25.23	−720.49*	27.56*	10.16*	42.23*	147.17	−1.64	−6.62	−9.61
	(765.32)	(55.71)	(10.49)	(43.57)	(22.83)	(102.42)	(53.40)	(6.10)	(2.25)	(10.69)	(91.41)	(14.10)	(5.04)	(13.82)
PAIPA 2	−7446.12*	−112.48*	244.58*	576.78*	749.71*	−90.41	−39.79	15.95*	−6.24*	−12.27*	25.55	−3.85	1.25	1.09
	(557.89)	(40.05)	(10.45)	(31.52)	(17.66)	(71.25)	(35.89)	(3.32)	(0.80)	(4.73)	(53.10)	(6.09)	(2.00)	(6.34)
PAIPA 3	8778.63*	−1235.31*	312.08*	195.60*	488.06*	−2147.85*	−209.51*	44.32*	−6.22*	−32.22*	63.94	−9.33*	0.45	5.10
	(533.50)	(40.49)	(8.27)	(35.51)	(15.75)	(73.03)	(17.15)	(1.65)	(0.54)	(2.83)	(36.73)	(4.10)	(2.20)	(5.35)
PAIPA 4	−801.38	−630.66*	343.08*	467.87*	502.61*	28.15	0.60	3.66*	−1.80*	−3.67*	24.11	−3.18	1.37	0.11
	(522.47)	(37.68)	(9.93)	(29.42)	(10.59)	(75.03)	(7.29)	(0.44)	(0.16)	(0.74)	(24.59)	(1.81)	(1.13)	(3.00)
GUAJIRA 1	1035.91*	−70.75*	0.76	−0.37	−5.49	−264.51*	−25.99*	−2.99*	1.25*	6.57*	6.28	1.82*	−0.58*	−2.93*
	(140.83)	(11.25)	(2.50)	(8.51)	(5.16)	(22.81)	(3.10)	(0.34)	(0.14)	(0.52)	(4.18)	(0.48)	(0.22)	(0.62)
GUAJIRA 2	−308.36*	86.93*	−14.31*	−32.48*	−33.43*	−106.96*	−28.06*	−4.16*	1.40*	8.44*	8.38*	2.34*	−0.68*	−3.87*
	(162.92)	(13.51)	(2.58)	(10.18)	(4.97)	(24.20)	(3.13)	(0.46)	(0.10)	(0.59)	(3.92)	(0.58)	(0.16)	(0.71)
TASAJERO 1	1321.49*	−53.61*	0.40	−90.74*	47.07*	−218.27*	−17.39*	1.99*	−0.08*	−0.61*	10.72*	−1.24*	0.06	0.39
	(68.67)	(4.34)	(1.11)	(3.40)	(1.69)	(9.70)	(0.47)	(0.12)	(0.04)	(0.19)	(1.09)	(0.20)	(0.08)	(0.26)
ZIPAEMG 2	3581.06*	−180.07*	−21.26*	−145.07*	−54.77*	390.52*	−278.30*	−1.72	−5.87*	46.26*	185.94	3.00	2.65	−32.25
	(222.62)	(16.14)	(3.47)	(13.06)	(8.91)	(33.45)	(58.32)	(1.78)	(1.77)	(7.40)	(367.27)	(10.03)	(10.07)	(41.90)
ZIPAEMG 3	5352.35*	−383.41*	−112.76*	−23.76	69.31*	−94.56*	−0.47	−4.11*	9.40*	−4.15*	−6.57	2.78	−3.56	0.63
	(241.01)	(18.63)	(5.13)	(14.89)	(10.26)	(36.68)	(10.96)	(0.83)	(0.79)	(1.86)	(25.50)	(2.81)	(2.22)	(4.27)
ZIPAEMG 4	3747.15*	−308.08*	−86.69*	92.17*	−41.14*	151.65*	118.22*	−5.28*	−1.84*	−5.68*	−65.31*	2.13	2.24	2.98
	(254.13)	(18.69)	(4.93)	(15.65)	(10.43)	(38.12)	(6.99)	(0.60)	(0.49)	(0.99)	(28.57)	(3.11)	(1.83)	(4.00)
ZIPAEMG 5	6696.89*	−324.36*	−139.08*	−256.85*	73.03*	−453.92*	41.32*	−11.61*	1.85*	9.96*	−31.76	5.31	−0.56	−3.16
	(236.29)	(18.15)	(5.17)	(15.92)	(10.85)	(35.92)	(8.32)	(1.68)	(0.66)	(2.57)	(36.05)	(8.44)	(2.34)	(12.13)

Bootstraped standard errors are in parentheses. * Significant at 5%.

Table 5: Coefficient estimates of the marginal cost function for units using gas

Unit	γ_1^{cons}	γ_1^{Pfuel}	γ_1^{Foil6}	γ_1^{TRM}	γ_1^{CERE}	γ_1^{FAZN}	γ_2^{cons}	γ_2^{Pfuel}	γ_2^{Foil6}	γ_2^{TRM}	γ_3^{cons}	γ_3^{Pfuel}	γ_3^{Foil6}	γ_3^{TRM}
MERILECTRICA 1	−5616.88*	310.29*	268.05*	−169.77*	521.16*	3009.55*	120.78*	−4.66*	−18.42*	9.72*	23.60	0.96*	0.75	−5.07*
	(345.45)	(6.95)	(15.39)	(30.19)	(21.90)	(80.21)	(24.14)	(0.53)	(2.39)	(2.05)	(17.49)	(0.26)	(1.25)	(2.18)
PROELECTRICA 1	−724.61*	15.53*	10.28*	61.99*	63.64*	415.41*	−117.31*	−3.92*	−5.42*	26.31*	1.51	0.00	−0.04	−0.16
	(30.53)	(0.36)	(0.54)	(4.21)	(1.44)	(5.13)	(29.24)	(0.51)	(0.95)	(4.33)	(3.95)	(0.06)	(0.11)	(0.54)
PROELECTRICA 2	−843.14*	20.16*	14.20*	68.11*	59.98*	485.22*	−79.19	14.24*	20.91*	−28.80*	2.99	−0.08	−0.15	−0.14
	(31.40)	(0.29)	(0.54)	(4.23)	(1.42)	(5.18)	(40.54)	(1.32)	(2.20)	(6.45)	(2.47)	(0.06)	(0.11)	(0.42)
TEBSAB	−1129.40*	0.70	12.33*	164.30*	−9.02*	−37.31*	1.86*	−0.02*	0.13*	−0.37*	−1.41*	0.03*	−0.07*	0.23*
	(57.21)	(1.19)	(2.43)	(7.62)	(4.62)	(11.65)	(0.33)	(0.01)	(0.02)	(0.04)	(0.55)	(0.01)	(0.04)	(0.07)
FLORES 4B	−2402.80*	15.43*	210.39*	34.51	63.90*	1377.08*	−5.29*	0.37*	−0.89*	1.25*	3.44	−0.15*	0.40	−0.72
	(336.81)	(3.75)	(18.99)	(44.95)	(8.25)	(51.14)	(1.26)	(0.03)	(0.13)	(0.26)	(2.37)	(0.07)	(0.27)	(0.52)
TERMOCENTRO CC	−6139.71*	205.01*	521.58*	−232.72*	452.36*	2357.86*	−57.51*	−5.47*	−9.35*	24.25*	23.65*	1.48*	3.18*	−8.36*
	(618.47)	(14.66)	(37.47)	(27.33)	(19.00)	(119.28)	(4.38)	(0.30)	(0.58)	(1.02)	(8.33)	(0.22)	(0.50)	(1.24)
TERMOYOPAL 2	6503.54*	23.40*	29.71*	−988.54*	199.57*	−810.41*	−968.81*	−6.96*	−2.84*	139.84*	124.76*	2.21*	3.00*	−22.36*
	(157.86)	(2.74)	(6.08)	(20.27)	(6.04)	(33.71)	(26.41)	(0.45)	(0.87)	(3.38)	(46.88)	(0.57)	(1.04)	(6.99)

Bootstraped standard errors are in parentheses. * Significant at 5%.

Table 6: Average estimate of the marginal costs coefficient by fuel type

Fuel	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\gamma}_3$
Diesel	258.11	-10.05	0.04
Coal	171.86	-18.64	0.14
Fuel oil	250.37	-13.15	-0.12
Gas	177.09	-3.02	0.10

Table 7: Summary statistics of estimated mark ups by fuel tipe

Fuel	Mean	Std.Dev	Min	Max
Diesel	21.79	571.04	-11535.39	1248.20
Coal	2.97	33.04	-213.96	159.20
Fuel oil	386.64	154.73	-96.74	785.19
Gas	15.78	66.93	-89.78	1049.76

Figure 4: Estimated vs. engeneering marginal costs of diesel units

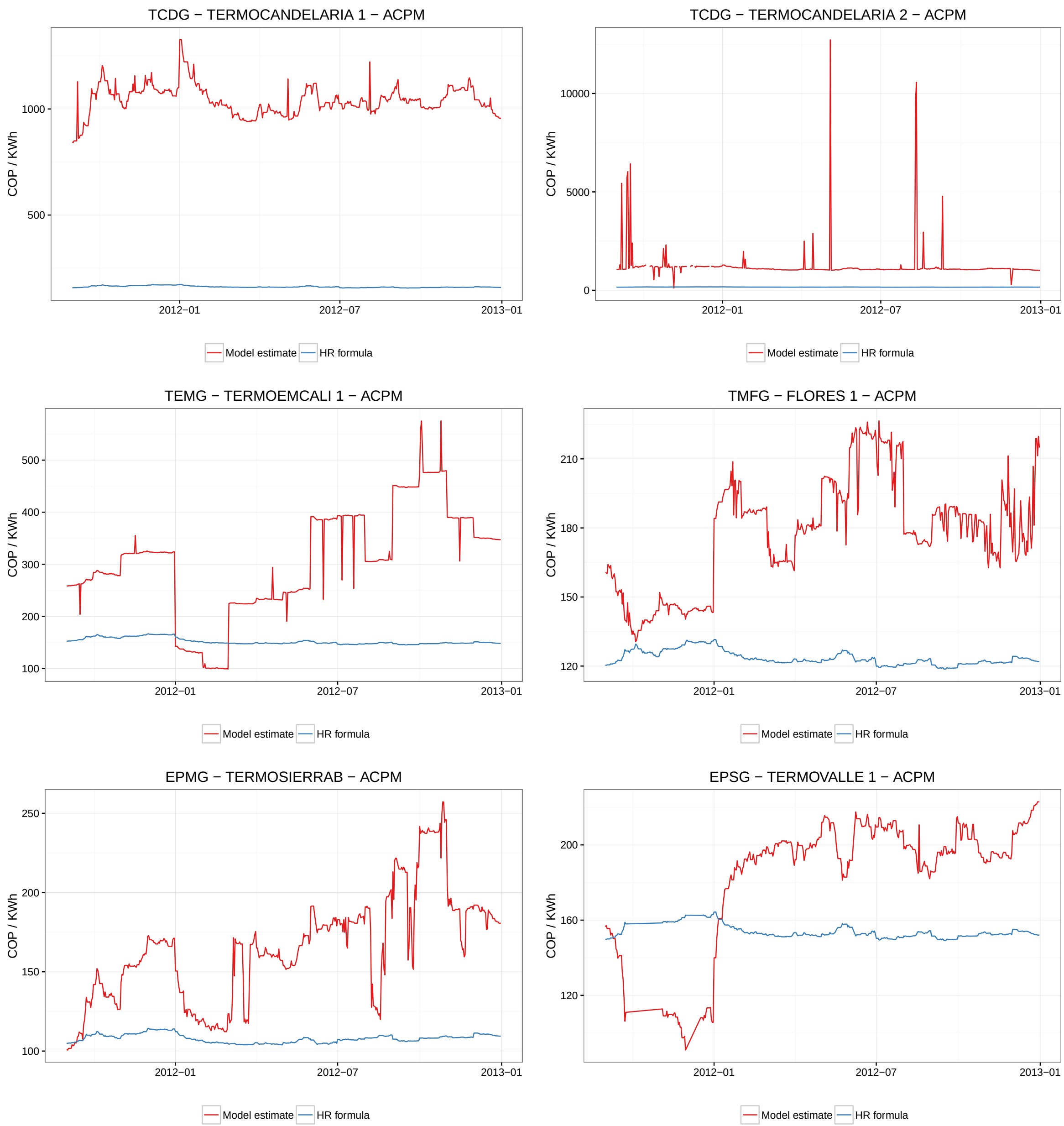


Figure 5: Estimated vs. engeneering marginal costs of coal units, 1 of 2

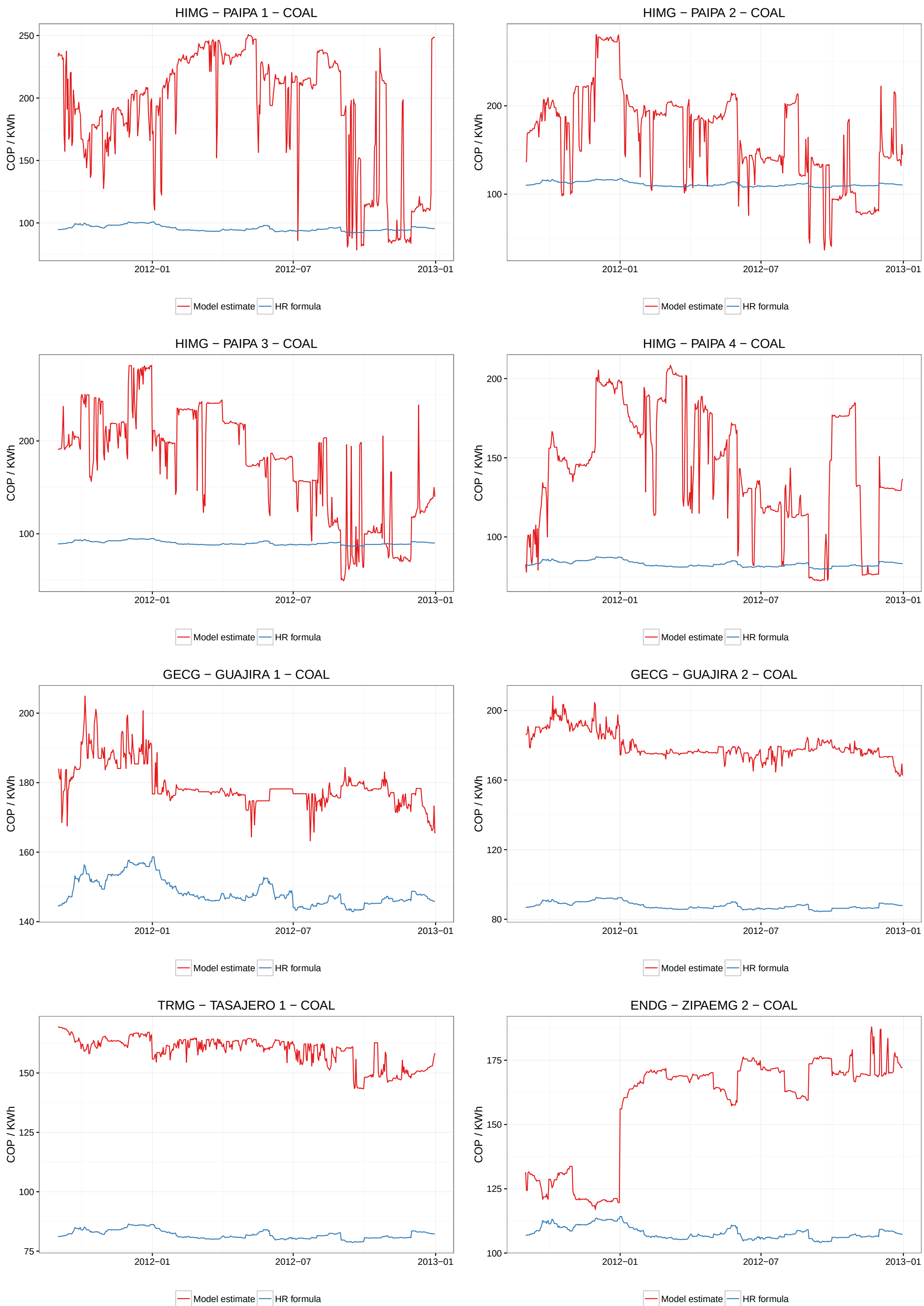


Figure 6: Estimated vs. engeneering marginal costs of coal units, 2 of 2

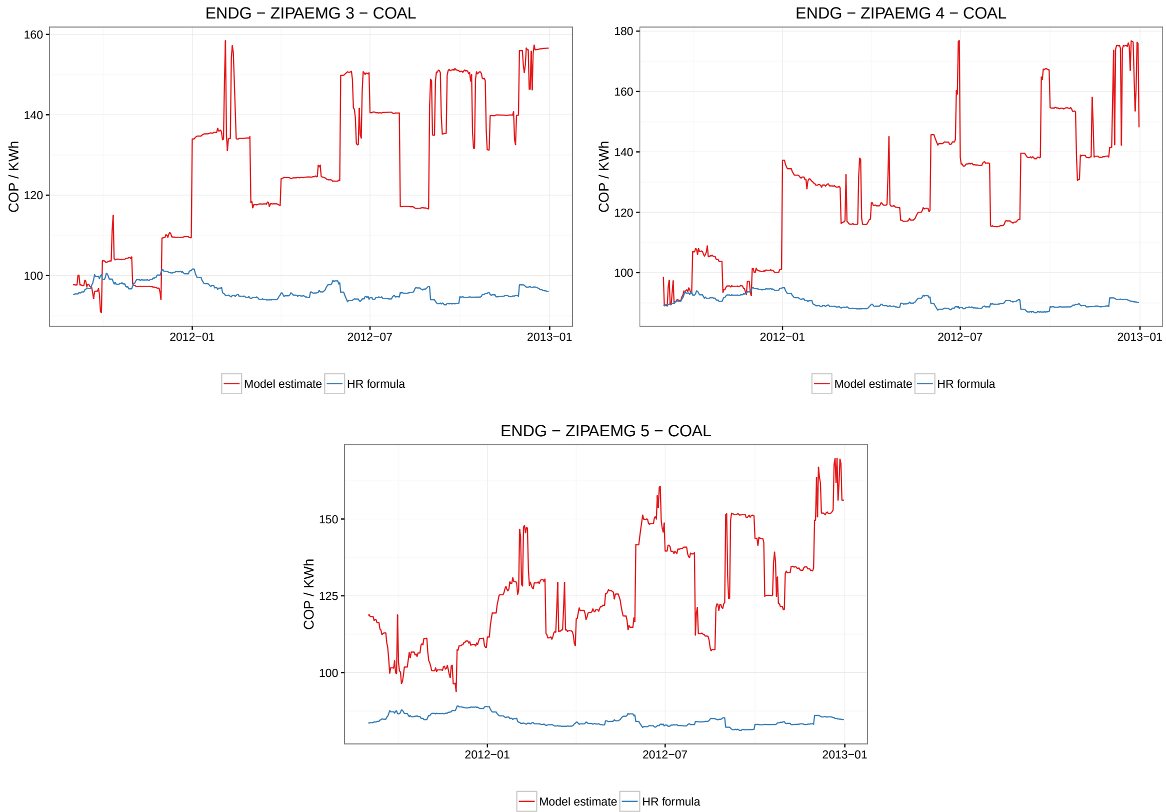


Figure 7: Estimated vs. engeneering marginal costs of fuel oil units

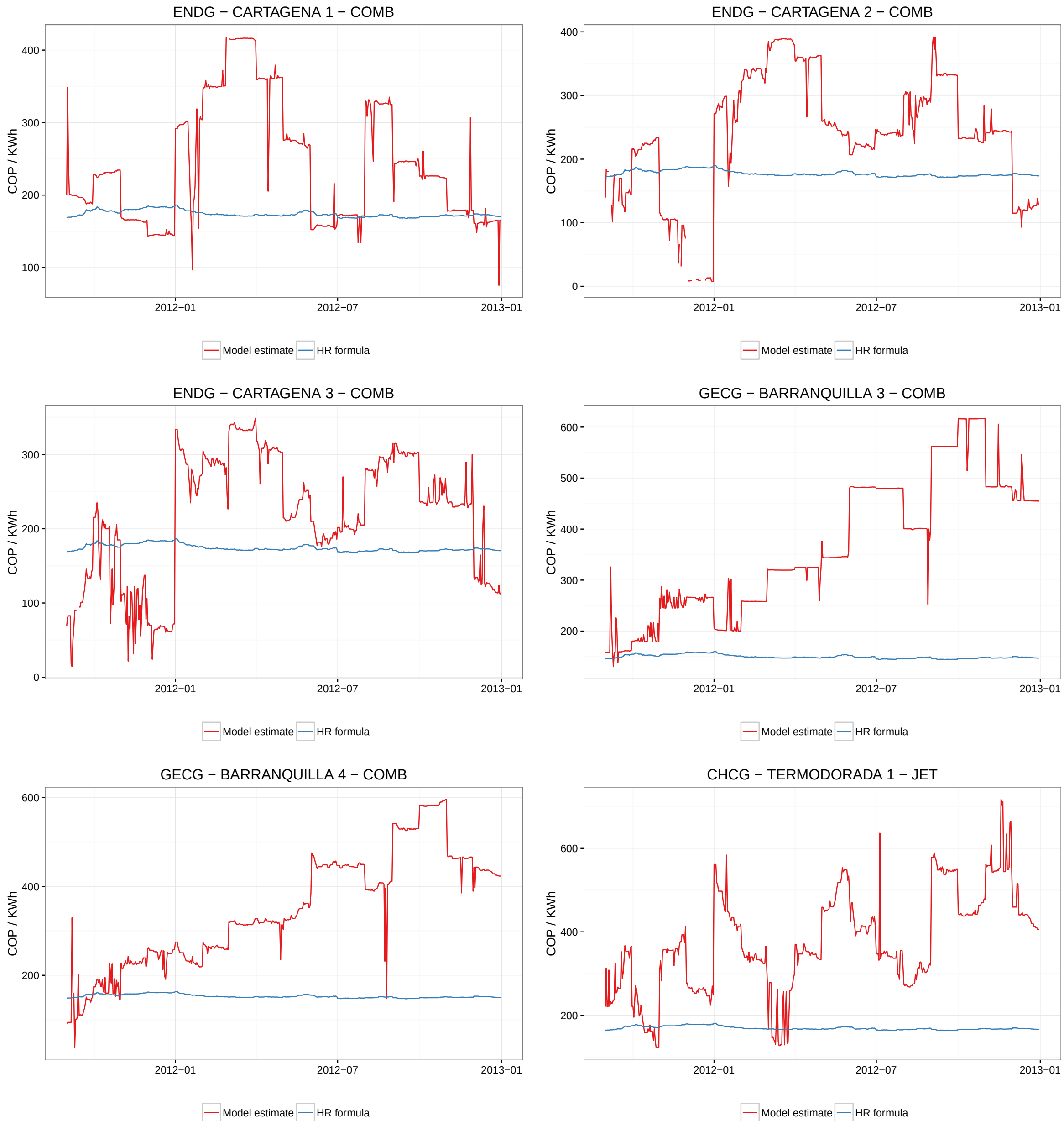


Figure 8: Estimated vs. engeneering marginal costs of natural gas units

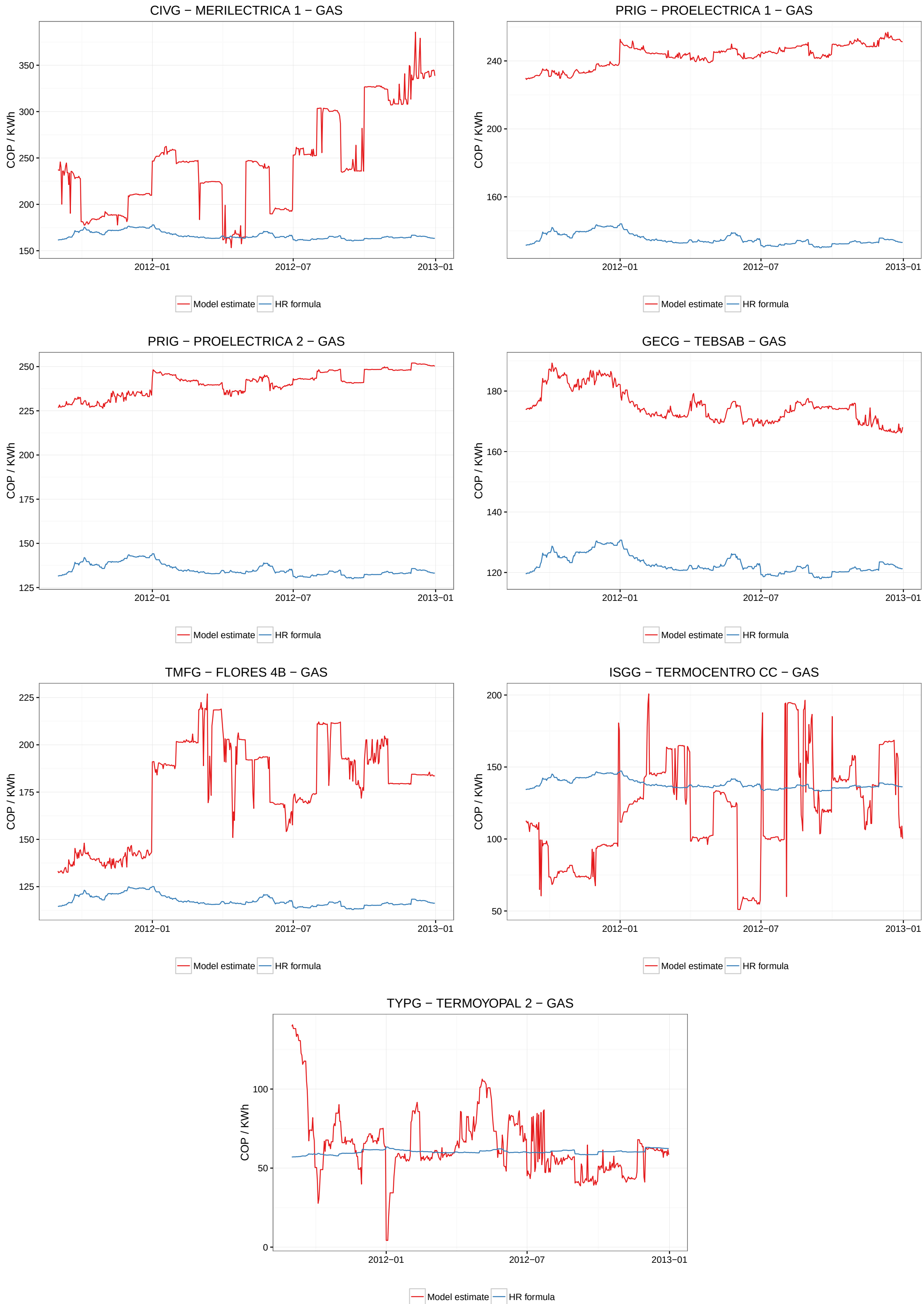


Figure 9: Price bids and marginal costs of diesel units

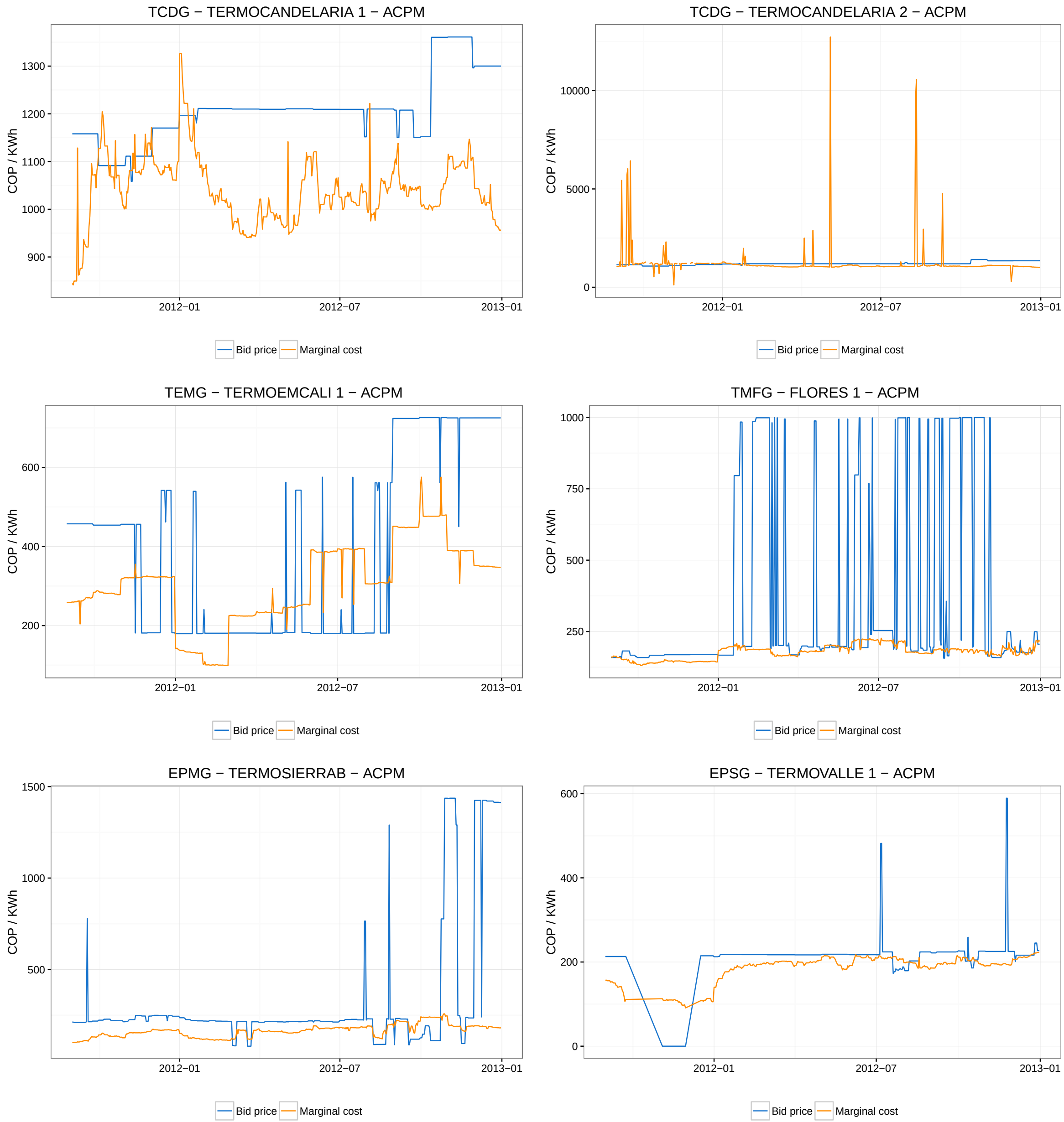


Figure 10: Price bids and marginal costs of coal units, 1 of 2

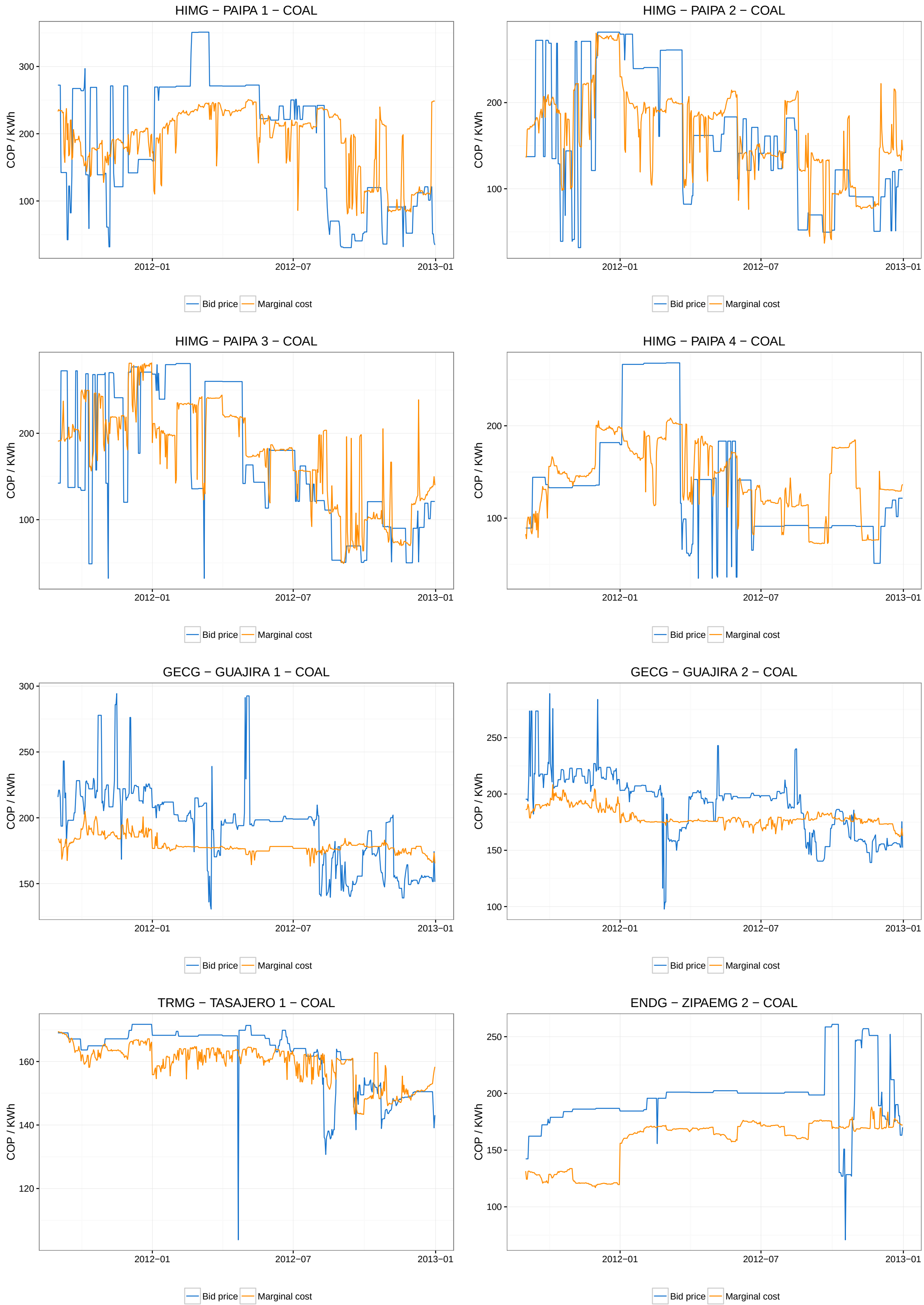


Figure 11: Price bids and marginal costs of coal units, 2 of 2

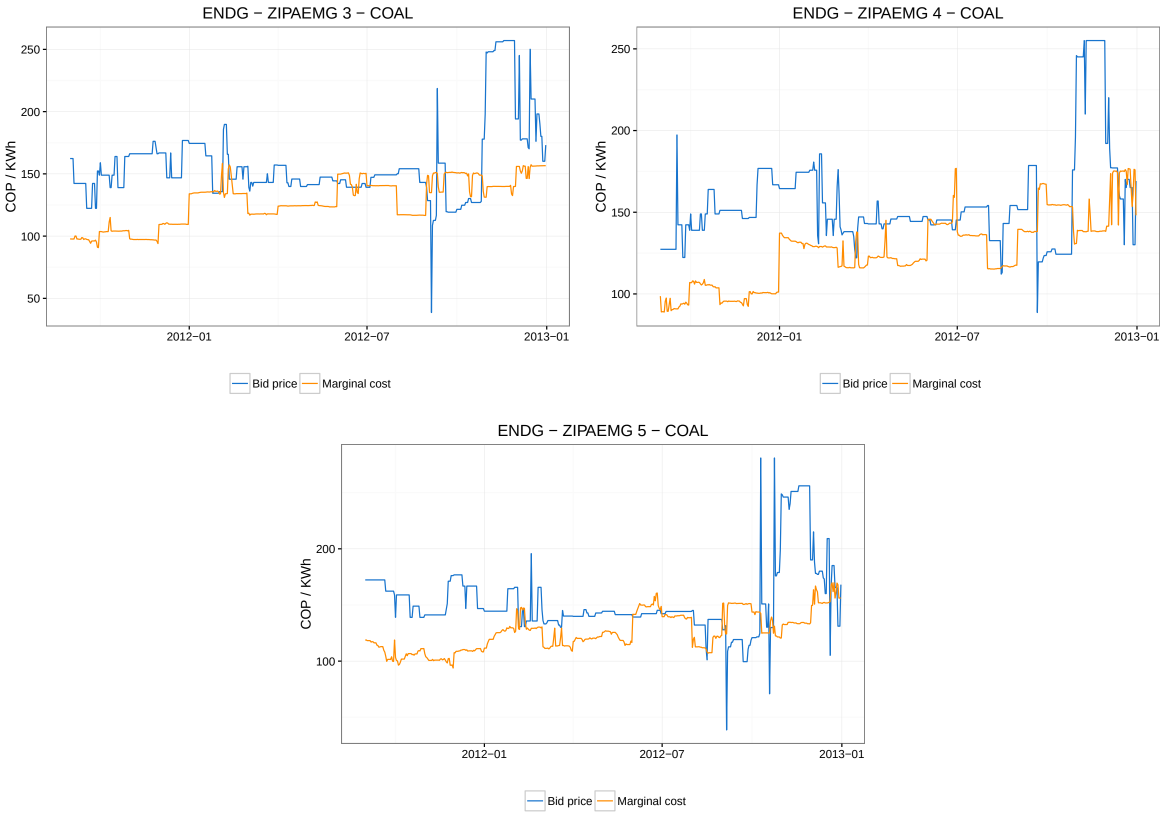


Figure 12: Price bids and marginal costs of fuel oil units

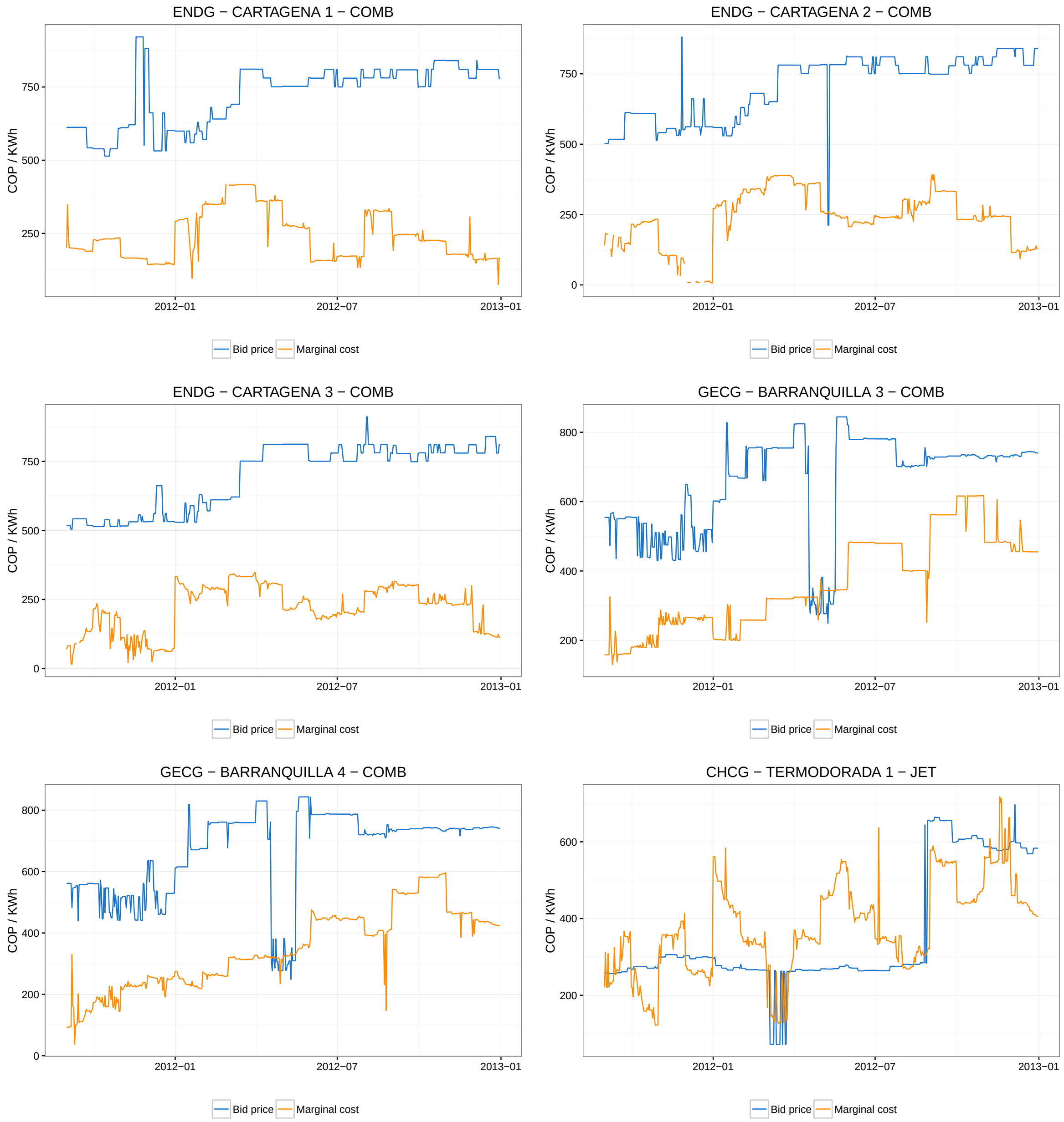
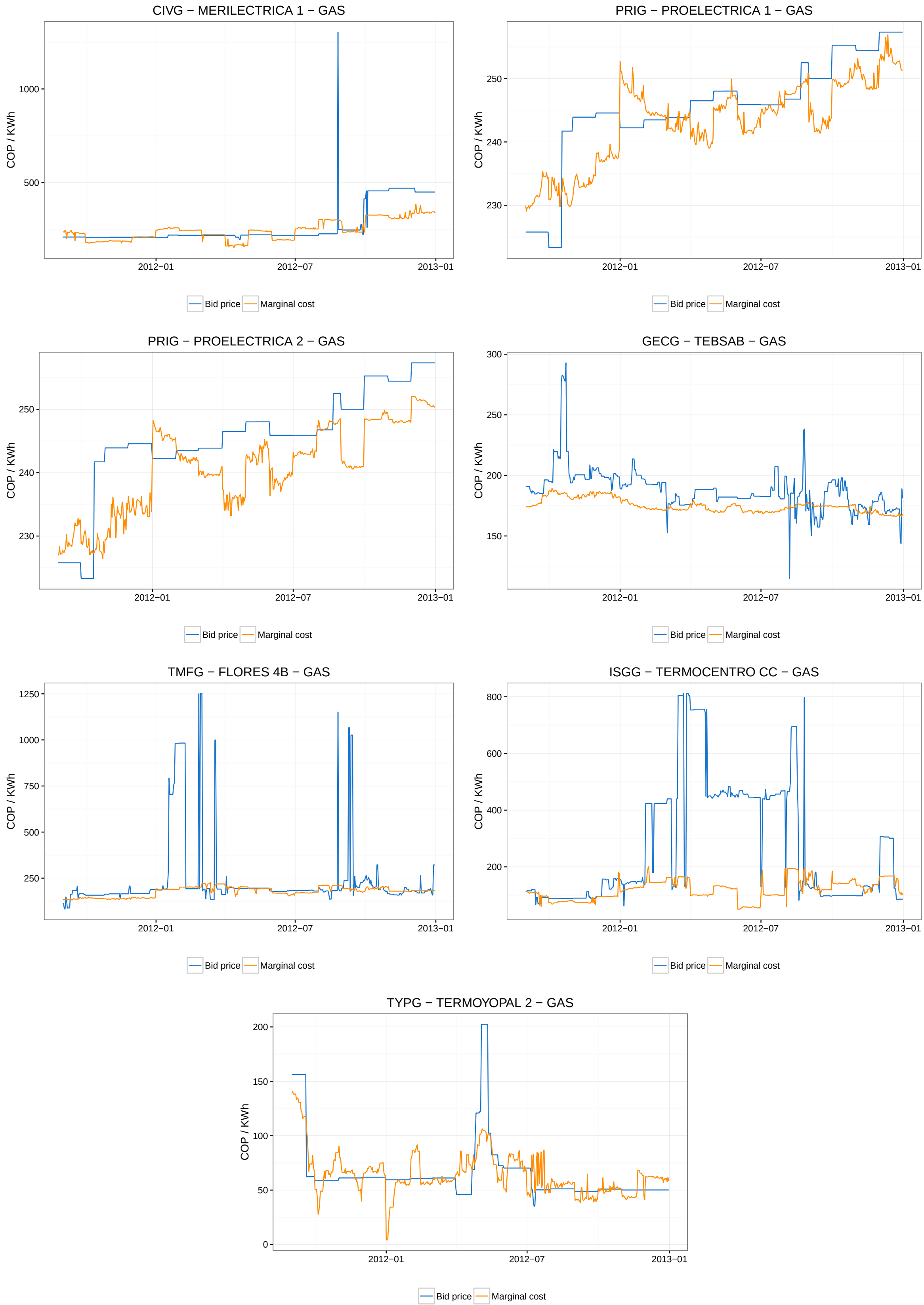


Figure 13: Price bids and marginal costs of natural gas units



6 Counterfactual experiment

The purpose of this study is to answer the question about whether the current dispatch mechanism for generation in the Colombian electricity market led to a reduction in the energy cost for the consumers. In particular, we propose a counterfactual experiment that allow us to compare the realized cost of the energy sold between August, 2011 and December, 2012 with the cost that the system would have faced if the dispatch mechanism was the previous self unit commitment mechanism used in Colombia before Resolution 051 was introduced, for the same period. The results of such experiment allow us to conclude that the aggregate cost of energy sold would was lower during the perior August, 2011 and December, 2012 with the current centralized unit commitment mechanism in cpmparasson to the conuterfactual, self unit commitment mechanism.

To perform such experiment we need to simulate an equilibrium firms play their optimal strategies according with an environment where state variables are the ones observed during our period of study but the game corresponds to the previous auction format used in the Colombian market. There are many reasons why the computation of such equilibrium is complicated. First, we cannot discard multiplicity of equilibria in the game described by previous auction format, which makes it hard to pin down one particular counterfactual equilibrium or even to put bounds on the set of plausible equilibria. Second, even if we focus on a particular type of equilibria, we need to struggle with the challenge of computing the distribution of firms' expectations over a combination of market structure and state space that is not observed in the data.

These complexities are usually referred to as the main reasons why previous studies in electricity markets have not been able to perform counterfactual experiments in which firms play a game other than a Vickrey-equivalent mechanism. Among the few empirical works on energy markets that are able to estimate parameters of the marginal costs, most limit their analyses to description of costs statistics, measuring market power through price-cost markups and to verify theoretical predictions. For example, Hortacsu and Puller (2008) use their estimations on contract positions to verify how well the optimal bidding predicted by their model fits the observed bids. Martin (2015) focus on the analysis of the water opportunity costs to identify if peaks in prices during shortage periods where associated with strategic behavior or as a result of an increase in the dynamic incentives to store water. Even in the work by Reguant (2014), the estimates of marginal and start-up costs parameters are only used to measure market power and identify difference in productive efficiency across fuel types. To our knowledge, there are no studies that are able to perform counterfactual experiments that are able to compare the equilibria between different market structures.

In this study, we contribute to the empirical literature on energy markets by overcoming these difficulties by making two major assumptions. Specifically, we assume that firms always play the same symmetric equilibrium for the respective game that we observe in the data for each of the two dispatch mechanisms (i.e. the previous acution format and the current dispatch with complex bids used in Colombia). In other words, we assume that, outside of the estimation sample, the conditional distribution of expectations can be extrapolated as a game-specific function of the observed state variables.

Formally, let T_a and T_b be the set of auctions observed between January, 2007 and December, 2008 when the dispatch mechanism was similar to a uniform-price auction and between August, 2011 and December, 2012, when the dispatch mechanism was the one described by the model presented in section 3, respectively. We define $S_i^{(0)} : S_{-i}^{(0)} \times \Omega \to \mathbb{R}_+$ be firm i's optimal strategy profile that describes the simple bids submitted by firm i for the set of equilibria observed in T_a, and $S_i^{(1)} : S_{-i}^{(1)} \times \Omega \to \mathbb{R}_+$ as firm i's optimal strategy profile describing the simple bids submitted by firm i observed in T_b. We assume that, for any realization of $\boldsymbol{\omega}$, firms always play the same symmetric equilibrium described by the strategic profiles $\{S_i^{(0)}(\cdot,\boldsymbol{\omega})\}_{i=1}^N$ or $\{S_i^{(1)}(\cdot,\boldsymbol{\omega})\}_{i=1}^N$ if the dispatch mechanism is, whether a the previous auction format with simple bids or the complex-bid auction, respectively. Therefore, firm i's optimal strategy for the counterfactual equilibrium we propose, denoted $\tilde{s}_i$, can be computed as follows:

$$\tilde{s}_i = S_i^{(0)}(\tilde{s}_{-i}, \boldsymbol{\omega}_{T_b}). \tag{27}$$

We develop this method by following the approach described by Carranza, Houde, and Clark (2011). To the best of our knowledge, this strategy has never been used to compute counterfactual experiments in the field of energy markets.

In our experiment we simulate an equilibrium in which firms play their optimal strategies according with an environment where the game corresponds to the previous auction format with simple bids between August, 2011 and December, 2012. Below we describe the procedure using the available data. First we define firm i's optimal strategy for simple bids, $S_i^{(0)}(\cdot)$, by the well known first-order conditions of the uniform-price auction game:

$$\sum_{h=1}^{24} b_{ijt} - MC_{ijth} + \frac{E[Q_{ith} - v_{ith} \mid p_{ith} = b_{ijt}]}{\partial E[Q_{ith} - v_{ith} \mid p_{ith} = b_{ijt}] / \partial b_{ijt}} = 0. \tag{28}$$

Then, to compute equation (28) for the period of study we need to simulate two unobserved terms: marginal costs and the expectation terms. For thermal units, we use our parameter estimates from section 5 to simulate the marginal costs that firms would face under different generation schedules. As for the expectation terms, we rely on our assumption that any firm's expected payoffs at the time of bidding can be expressed as a function of the state variables, the active units and the dispatch mechanism. In particular, we apply the bootstrapping algorithm on observed data between January, 2007 and December, 2008, to estimate a function of expectations for each firm which is associated to the previous auction format with simple bids and which can be conditioned on the state variables observed during our period of study. Formally, the estimation of the expectation terms implied in (28) is as follows:

1. For any day t in the original sample period (i.e. from August, 2011 to December, 2012), define the vector of state variables, $\mathbf{x}_t$, including: price of fuel oil No. 6, aggregate water stock and river flows.
2. In the outside sample (i.e. from January, 2007 to December, 2008) select a subsample of the same days of the week as t and with same active units.
3. For each day τ in the subsample built in step 2, define the vector of state variables as specified in step 1, $\mathbf{x}_\tau$.
4. Compute the Euclidean distance between $\mathbf{x}_t$ and $\mathbf{x}_\tau$ for every τ.
5. Select the days of the subsample built in step 2 associated with the lowest 30 distances computed between the state variable vectors.
6. Use the bids of those 30 days to perform a bootstrapping algorithm to estimate firm i's expectations in the previous dispatch mechanism setting.

A second challenge that remains, however, is the projection of hydro units' optimal bids. This occurs because we do not directly estimate the parameters of the cost function for hydro units we need. Hence, for hydro units, we do not compute the optimal bidding strategy following the the structural first-order condition defined in (28). Instead, we follow the reduced-form function of the optimal bidding proposed by Balat et al. (2015), which accounts for the strategic and dynamic incentives of these units.

Given the optimal bidding functions, the expectations and marginal cost estimates, we are able to compute the equilibrium for an hypothetical scenario in which generating firms in Colombia play the previous auction setting with simple bids between August 1st, 2011 and December 31st, 2012. Hence, the underlying consumer cost difference between the observed equilibrium for the current dispatch and the simulated one for the previous auction format will serve to answer which mechanism was more efficient when accounting for strategic behavior during a period of normal hydrologic conditions.

Below, we present the results of our counterfactual experiment. These results allows us to conclude that the aggregate cost of energy sold between August, 2011 and December, 2012 would have been lower if the dispatch mechanism was under the previous auction setting, instead of the current dispatch with complex bids.

Figure 14 shows the comparison between the daily average of the observed and the simulated spot price. Moreover, we present the percentage difference between the two series in Figure 14. The results show that the current dispatch mechanism is associated with higher prices in average. In other words, before taking into account start-up costs, we find that the aggregate cost of energy sold is systematically higher under the current dispatch mechanism. Such conclusion is expected *a priori* as the dispatch defined by (1a) does not only accounts for the variable cost of energy but also the start up costs of thermal units but for the start-up costs as well. In mathematical terms, this is equivalent to compare the minima between a constrained (current dispatch) and an unconstrained (previous dispatch) optimization problem.

Figure 14: Evolution the daily average of observed and simulated spot price between August 1st, 2011 and December 31st, 2012.

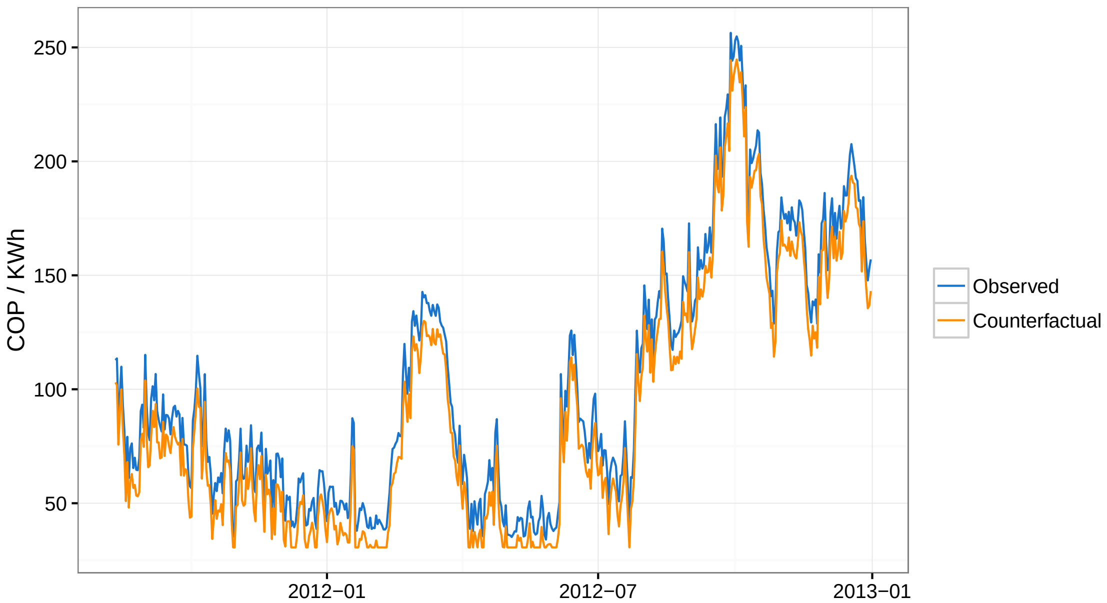
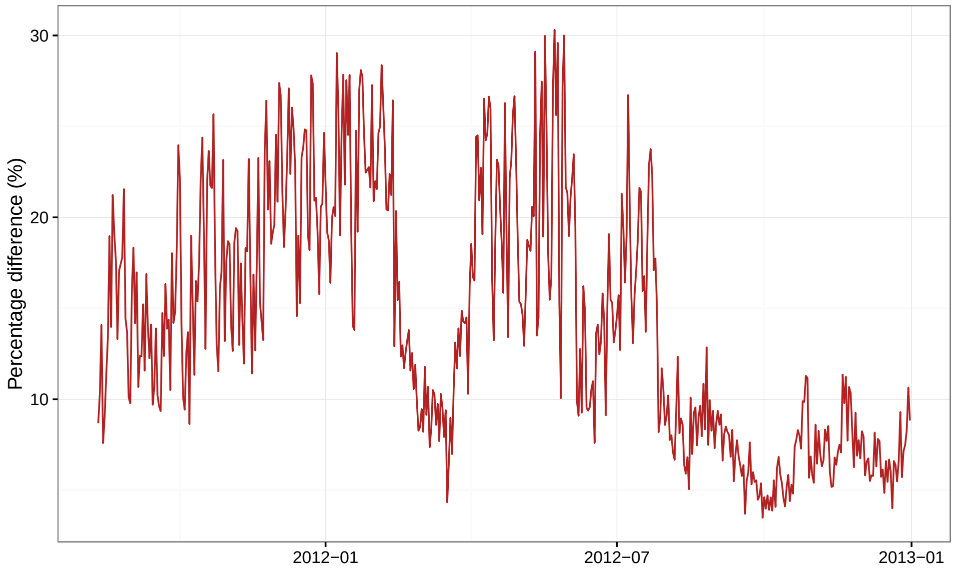


Figure 15: Evolution the percentage difference between the daily average of the observed and simulated spot price, between August 1st, 2011 and December 31st, 2012.



However, we also find that spot prices increase not only because the marginal bidder submits a higher price. As we present in Table 8, submitted bids under the observed equilibrium are systematically higher as well. In other words, the actual submitted bid prices are much higher in average than those which firms would have bid in the previous auction environment. This result is consistent with the economic intuition. Specifically, since firms' expect higher spot prices in the equilibrium of the current dispatch mechanism, they find optimal to increase their bid prices.

Table 8: Summary statistics of actual vs. simulated bids between August 1st, 2011 and December 31st, 2012 (COP/KWh)

Mechanism	Mean	Std.Dev.	Min	Max
Current dispatch	270.01	295.48	30.62	2113.91
Previous dispatch	258.62	300.47	30.62	2550.14

Nevertheless, it is important to compare also the total costs and revenue, which include start up costs. For both scenarios we compute, on every day:

$$Cost = \sum_{h=0}^{23} \sum_{i=1}^{N} \sum_{j=1}^{J_i} \hat{mc}_{ij} q_{ijh} + A_{ij} \mathbf{1}_{ijh}^{\text{start}}, \tag{29}$$

as the total cost of energy produced, and

$$Revenue = \sum_{h=0}^{23} \sum_{i=1}^{N} \sum_{j=1}^{J_i} b_{ij} q_{ijh} + A_{ij} \mathbf{1}_{ijh}^{\text{start}}, \tag{30}$$

as the total revenue of energy sold on the respective day.

The aggregate cost and revenue for both regimes between August 1st, 2011 and December 31st, 2012 are presented in Table 9. The results show that the old auction format is associated with the higher aggregate costs of generation but also with lower aggregate revenue of energy sold for the firms. This suggests that, although the current dispatch mechanism is designed to reduce the total cost of the daily energy dispatch, the underlying incentives of the firms to increase bid markups is such that the efficiency gains of the new dispatch benefits firms over consumers. In other words, if the assumptions of our model and counterfactual exercise hold, Colombian consumers would have saved about 786 billions of COP between 2011 and 2012 if the dispatch mechanism was the previous simple-bid auction as used to be before 2009.

Table 9: Aggregate cost measures of dispatch between August 1st, 2011 and December 31st, 2012 (billions of COP)

	Current mechanism	Previous dispatch
Cost of energy produced	8,671.95	9,214.84
Revenue of energy sold	13,760.84	12,974.52

7 Final remarks

In this study we introduce a structural model of bidding behavior that accounts for the presence of complex bids and the dynamic incentives of both hydro and thermal generators. The purpose of this study is to answer the question about whether the current dispatch mechanism for generation in the Colombian electricity market led to a reduction in the energy cost for the consumers.

Using observed auction data and bilateral contract sales position, we are able to identify the unobserved distribution of expectations and marginal production costs of thermal generators, which are used in turn to perform a counterfactual experiment that allows us to compare the realized cost of the energy sold between August, 2011 and December, 2012 with the cost the system would have faced if the dispatch mechanism was the previous simple-bid auction format as before Resolution 051 (2009), for the same period.

The estimation algorithm relies on the bootstrapping strategy, standard in the literature for energy markets, by using computational model introduced by Camelo et al. (2016) to compute the bootstrapped moments. Average estimates of the marginal costs parameters are in average consistent with intuition about fuel type and the non-convexities of thermal generation technology. In other words, marginal costs are higher for fuel oil units than for units that use natural gas. At the same time, units fueled by coal show a higher scale efficiency. That is, marginal costs decrease faster as output increases over the minimum output for coal units than for other fuel types. As for ramping costs, we find that the increase in marginal costs is not statistically significant as thermal units need to change their output between one hour range.

Following Carranza et al. (2011), we develop an approach to perform our computational experiment using observed auction data for each dispatch mechanism to compute the counterfactual equilibrium in which firms play the previous simple-bid auction game over a period when the current dispatch mechanism with complex bids was in course. We consider the introduction of this method our main contribution to the literature of structural econometrics in energy markets.

According to the results, we conclude that the aggregate cost of energy sold during our period of study would have been higher if the dispatch mechanism was under the previous auction setting, instead of the current dispatch with complex bids. However, at the same time, we find that the aggregate revenue for all the energy sold during the evaluation period is substantantially higher under the current dispatch as firms have more incentives to increase their markups. In other words, after accounting for strategic behavior, the current dispatch mechanism used in the Colombian electricity market between August, 2011 and December, 2012, worked in benefiting the generating firms over consumers.

References

Balat, J., Carranza, J. E., & Martin, J. D. (2015). *Dynamic and strategic behavior in hydropower-dominated electricity markets: Empirical evidence for colombia.* (Working paper. Borradores de Economía No. 886. Banco de La República.)

Camelo, S., de Castro, L., Papavasiliou, A., Riascos, A., & Oren, S. (2016). *A structural model to evaluate the transition from self-commitment to centralized unit commitment.* (Working paper. Borradores de Economía No. 992. Banco de La República.)

Carranza, J. E., Houde, J.-F., & Clark, R. (2011). *Dynamic entry and firm reconfiguration in the canadian gasoline markets.* (Working paper.)

Carranza, J. E., Riascos, A. J., Morán, J. J., & Bermeo, L. (in press). *Mecanismos de mercado e instituciones del sector eléctrico colombiano.* (Documento de trabajo, Banco de La República Colombia)

Ciarreta, A., & Espinosa, M. (2010). Market power in the spanish electricity auction. *Journal of Regulatory Economics*, *37*, 42–69.

de Castro, L., Oren, S., Riascos, A., & Bernal, M. (2014). *Transition to centralized unit commitment: An econometric analysis of Colombia's experience.* (Working paper. Borradores de Economía No. 830. Banco de La República.)

Gans, J. S., & Wolak, F. A. (2008). *A comparison of ex ante versus ex post vertical market power: Evidence from the electricity supply industry.* (SSRN Working Paper)

Garcia, A., Campos, E., & Reitzes, J. (2005). Dynamic pricing and learning in electricity markets. *Operations Research*, *55*(2), 231-241.

Garcia, A., Reitzes, J. D., & Stacchetti, E. (2001). Strategic pricing when electricity is storable. *Journal of Regulatory Economics*, *20*(3), 223-247.

Hortacsu, A., & McAdams, D. (2010). Mechanism choice and strategic bidding in divisible good auctions: an empirical analysis of the Turkish treasury auction market. *Journal of Political Economy*, *118*(5), 833–865.

Hortacsu, A., & Puller, S. L. (2008). Understanding strategic bidding in multi-unit auctions: A case study of the Texas electricity spot market. *RAND Journal of Economics*, *39*(1), 86-114.

Kastl, J. (2011). Discrete bids and empirical inference in divisible good auctions. *Review of Economic Studies*, *78*(3), 974–1014.

Martin, J. D. (2015). *Identification of water opportunity costs in hydropower dominated electricity markets.* (M.S. Thesis. Icesi University.)

Reguant, M. (2014). Complementary bidding mechanisms and startup costs in electricity markets. *Review of Economic Studies*, *81*(4), 1708-1742.

Riascos, A., Bernal, M., de Castro, L., & Oren, S. (2016). Transition to centralized unit commitment: An econometric analysis of colombia's experience. *The Energy Journal*, *37*(3).

Sioshansi, R., Oren, S., & O'Neill, R. (2008a). The cost of anarchy in self-commitmen based electricity markets. In F. Sioshansi (Ed.), *Competitive electricity markets: Design, implementation & performance.* Elsevier. US Energy Information Administration.

Sioshansi, R., Oren, S., & O'Neill, R. (2008b). Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets. *IEEE Transactions on Power Systems*, *23*, 344-352.

Sioshansi, R., Oren, S., & O'Neill, R. (2010). Three-part auctions versus self-commitment in day-ahead electricity markets. *Utilities Policy*, *18*, 165-173.

Stacchetti, E. (1999). *Auction design for the Colombian electricity market.* (Working paper, University of Michigan)

Vegard Hansen, P. (2009). Inflow uncertainty in hydropower markets. *Scandinavian Journal of Economics*, *111*(1), 189–207.

Wolak, F. A. (2000). An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market. *International Economic Journal*, *14*(2), 1–39.

Wolak, F. A. (2003). Identification and estimation of cost functions using observed bid data: An application to electricity markets. In M. Dewatripont, L. P. Hansen, & S. J. Turnovsky (Eds.), *Advances in economics and econometrics* (pp. 133–169). Cambridge University Press.

Wolak, F. A. (2007). Quantifying the supply-side benefits from forward contracting in wholesale electricity markets. *Journal of Applied Econometrics*, *22*(7), 1179–1209.

Appendix

A The computational model for the dispatch

As mentioned in the previous sections, we use the dispatch model introduced by Camelo et al. (2016) in order to estimate each firm's expectations about its competitors' strategies. In this section we provide a detailed description of our model of ideal dispatch. The model is cast as a mixed integer linear program. We also highlight the main differences with the ISO ideal dispatch model.

A.1 Model setup

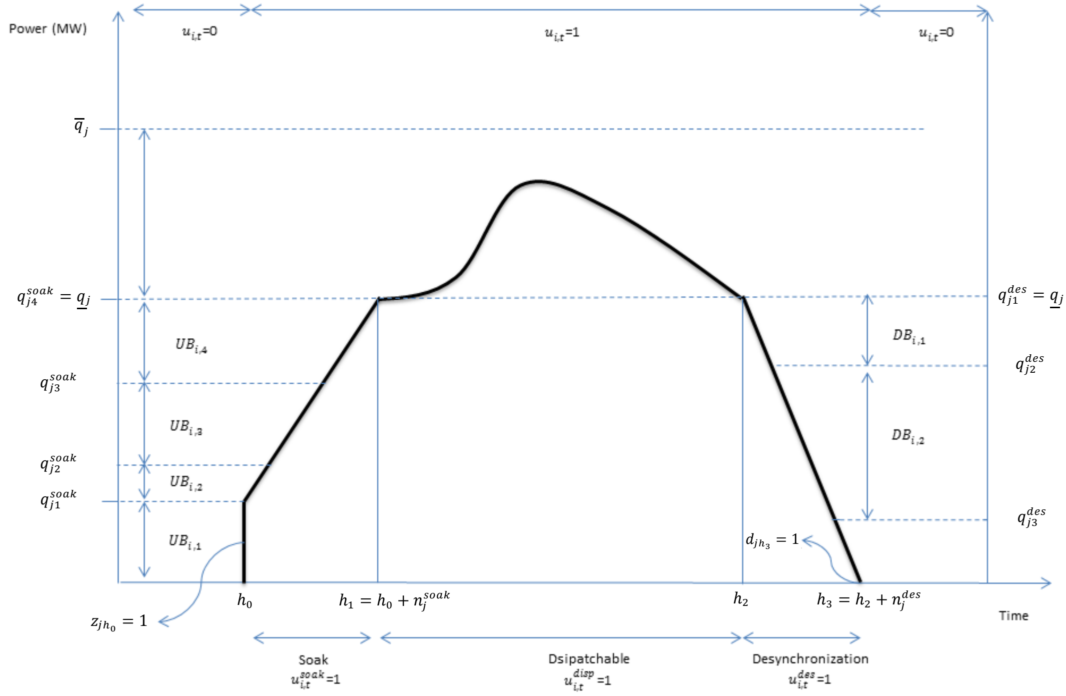
A.1.1 Additional Nomenclature

- q_{jh}^{soak} is the power provided by unit j during hour h and start-up phase.
- q_{jh}^{des} is the power provided by unit j during hour h and de-synchronization phase.
- UB_{jk} is the k-th ramp up blocks of unit j.
- DB_{jk} is the k-th ramp down blocks of unit j.
- u_{jh} is a binary variable indicating if unit j is up in period h.
- z_{jh} is a binary variable indicating if unit j is started in period h.
- d_{jh} is a binary variable indicating if unit j is stopped in period h.
- u_{jh}^{soal} is a binary variable indicating if unit j is in the start-up phase.
- u_{jh}^{dis} is a binary variable indicating if unit j is in the dispatch phase.
- u_{jh}^{des} is a binary variable indicating if unit j is in the shut-down phase.
- n_{jh}^{soak} represents the number of hours during the start-up phase (since start-up until output is at the technical minimum).
- n_{jh}^{des} represents the number of hours during shut-down phase (from a technical minimum to shut-down).
- n_{jh} is the minimum up-time of unit j.
- l_{jh} is the minimum down-time of unit j.

A.1.2 Ramp model

The ramp model is similar to Simoglou et al (2010). We assume that thermal units follow three consecutive phases of operation: (i) soak or start-up phase (from zero to technical minimum), (ii) dispatchable (when output is between the technical minimum and maximum feasible power output) and (iii) de-synchronization phase (when output is below the technical minimum and just before shut-down). In the soak phase, the power output follows a block model.

Figure 16: Ramp model of a thermal unit



Source: Camelo et al. (2016)

In the dispatchable phase we assume an affine model for power. In the de-synchronization phase we assume a block model. Figure 16 shows an example of the assumed ramp model for a thermal unit with a ramp of $K = 4$ blocks.

A.1.3 Optimization problem

The ideal dispatch is the solution to the following optimization problem. It is a mixed integer linear program.

Objective function

$$\min_{q_{jh},q_{jh}^{\text{soak}},q_{jh}^{\text{disp}},q_{jh}^{\text{des}},z_{jh},d_{jh},u_{jh},u_{jh}^{\text{soak}},u_{jh}^{\text{disp}},u_{jh}^{\text{des}}} \sum_{h=0}^{23} \sum_{j} b_j q_{jh} + A_j z_{jh} \tag{31}$$

Restrictions

Feasible output:

$$\tilde{D}_h \leq \sum_j q_{jh}, \quad \forall h \in \{0, \ldots, 23\} \tag{32}$$

Soak phase starts immediately following start-up:[18]

$$\sum_{\tau=h-n_j^{\text{soak}}+1}^{h} z_{i\tau} = u_{jh}^{\text{soak}} \tag{33}$$

[18]We make two simplifications with respect to the Colombian ISO ideal dispatch model. We only consider one type of start-up (as opposed to a cold, warm, or hot, start-up) and we only consider one type of configuration per plant (i.e., a fixed ramp per plant). Not sure what ramp has to do with configuration.

Now let Q^{soak}_{jr} be the power provided by unit j during period r following start-up:

$$Q^{\text{soak}}_{jr} = \sum_{k=1}^{r} \text{UB}_{jk}.$$

Then, during soak phase, the power output of the unit is constrained by:

$$\sum_{\tau=h-n_j^{\text{soak}}+1}^{h} z_{i\tau} Q^{\text{soak}}_{jh-\tau+1} = q^{\text{soak}}_{jh} \tag{34}$$

Dispatch phase: We simplify the current model by assuming linear up and down ramp constraints.

$$q_{jh} \leq \frac{UR + b \times q_{jh-1}}{a} + N(u^{\text{soak}}_{jh} + u^{\text{des}}_{jh}) \tag{35}$$

$$q_{jh} \geq \frac{-DR + c \times q_{jh-1}}{d} - N(u^{\text{soak}}_{jh} + u^{\text{des}}_{jh} + d_{jh}), \tag{36}$$

where N is a sufficiently large parameter.[19]

The de-synchronization phase starts before shut-down:

$$\sum_{\tau=h+1}^{h+n_j^{\text{des}}} d_{j\tau} = u^{\text{des}}_{jh}. \tag{37}$$

Now let the power provided by plant j, for s periods after de-synchronization is started be defined as:

$$Q^{\text{des}}_{jH^{\text{des}}-r+1} = \sum_{k=1}^{r} \text{DB}_{jk}$$

Then, during the de-synchronization phase the power output of a unit is constrained by:[20]

$$\sum_{\tau=h+1}^{h+n_j^{\text{des}}} d_{j\tau} Q^{\text{des}}_{jh+1-\tau+n_j^{\text{des}}} = q^{\text{des}}_{jh} \tag{38}$$

Minimum up time. Units are constrained to be up for n_j^{up} periods after they are started up:

$$\sum_{\tau=h+n_j^{up}+1}^{h} z_{j\tau} \leq u_{jh} \tag{39}$$

Minimum down time. Units are constrained to be down for n_j^{down} periods after they are shut down:

$$\sum_{\tau=h+n_j^{down}+1}^{h} d_{j\tau} \leq 1 - u_{jh} \tag{40}$$

[19] We have approximated the ISO model for the dispatchable region. The ISO model is based on maximum and minimum power variations depending on the level of outputs (segments model called Model number 2 by ISO). Our model for the dispatchable region is a special case of ISO's model number 3 used by some plants as an alternative to model 2. This discussion is esoteric and should probably be removed.

[20] This is a simplification of the current Colombian dispatch model on two dimensions. We do not consider an alternative shut down ramp whenever output is not at the technical minimum.

Power Output Constraints:

$$q_{jh} \geq q_{jh}^{\text{soak}} + q_{jh}^{\text{des}} + q_{jh}^{\text{soak}} + \underline{q}_j u_{jh}^{\text{disp}} \tag{41}$$

$$q_{jh} \leq q_{jh}^{\text{soak}} + q_{jh}^{\text{des}} + q_{jh}^{\text{soak}} + \overline{q}_j u_{jh}^{\text{disp}} \tag{42}$$

$$q_{jh} \leq q_{jh}^{\text{soak}} + q_{jh}^{\text{des}} + q_{jh}^{\text{soak}} + \overline{q}_j u_{jh}^{\text{disp}} + (\underline{q}_j - \overline{q}_j) z_{jh+n_j^{\text{des}}} \tag{43}$$

Logical status of commitment. The following are restrictions required for the transition of the binary variables:

$$u_{jh} = u_{jh}^{\text{soak}} + u_{jh}^{\text{disp}} + u_{jh}^{\text{des}} \tag{44}$$

$$z_{jh} - d_{jh} = u_{jh} - u_{jh-1} \tag{45}$$

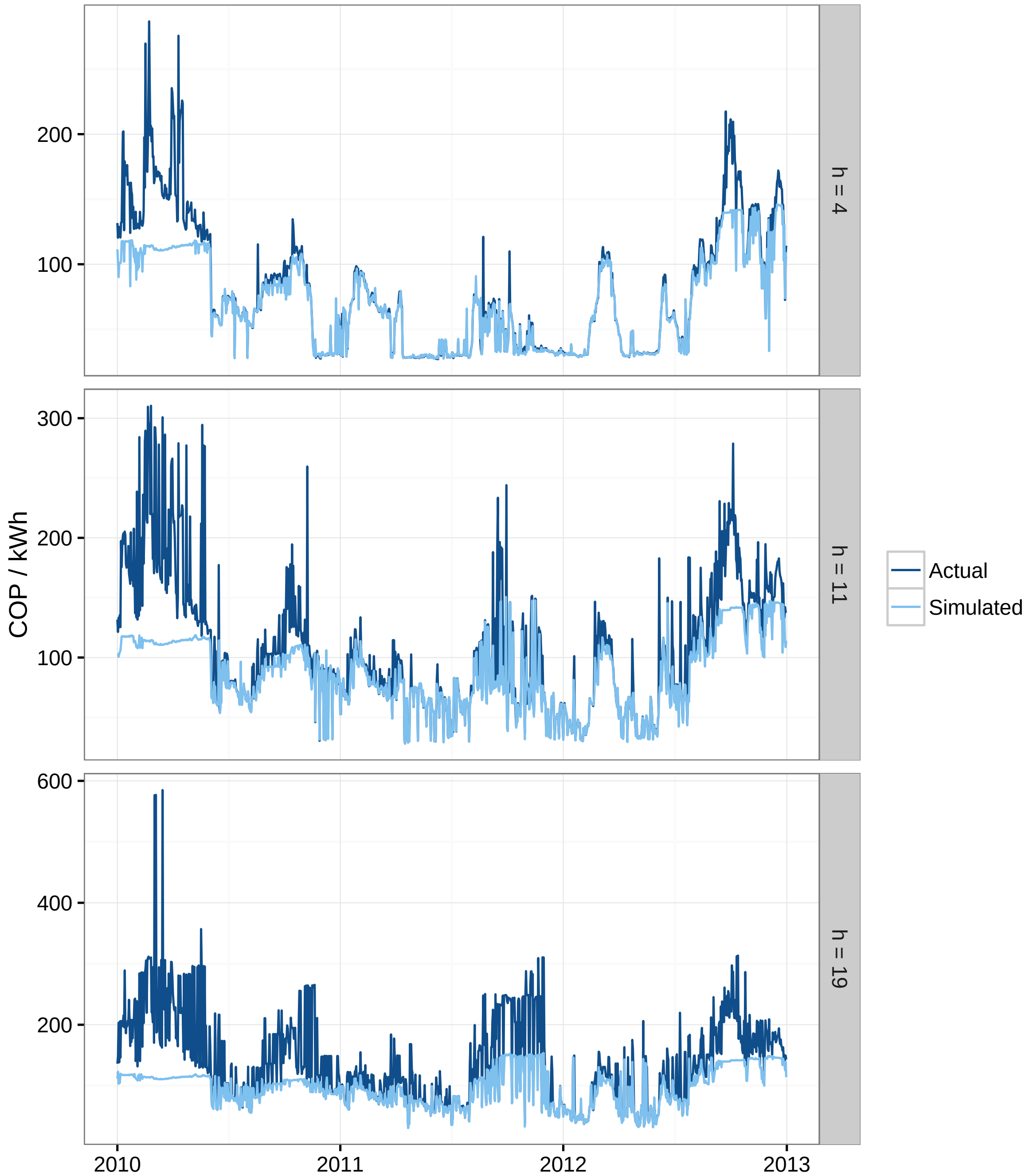
$$z_{jh} + d_{jh} \leq 1 \tag{46}$$

A.2 Simulation exercise

In this subsection we perform a small simulation exercise to illustrate the goodness of fit of this dispatch model compared with the observed ideal dispatch computed by XM.

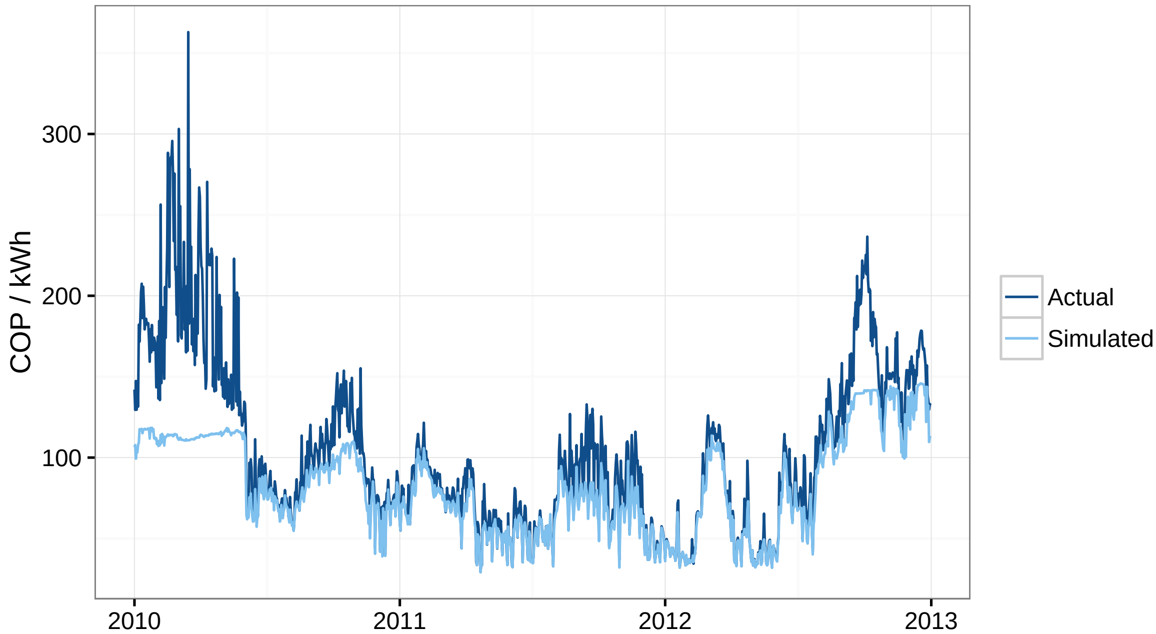
To test the validity of our model, we simulate the period from June 2010 to October 2012 using real start-up costs and bids. Then, we compare the simulated market price with the real one, as reported by XM. Below, Figure 17 shows the actual versus the sumulated market prices for three hourly periods, including the peak hour $h = 19$. On the other hand, Figure 18 shows the same comparison but between the daily averages of the real versus the simulated market price.

Figure 17: Comparison between actual and simulated market prices by hourly periods



Source: Own calculations based on data from XM.

Figure 18: Comparison between actual and simulated average daily market prices



Source: Own calculations based on data from XM.

As the plots show, except for the highest peaks, the model has a good fit for the market price. In Table 10 we present a comparison between some summary statistics between real and simulated market prices.

Table 10: Summary statistics for actual and simulated hourly market prices

	Actual	Simulated
Min	26.97	27.54
Quantile 5	30.71	30.73
Quantile 10	33.32	32.72
Quantile 20	52.21	51.58
Quantile 50	87.73	79.17
Quantile 80	148.74	113.72
Quantile 90	181.76	123.85
Quantile 95	215.79	141.49
Max	1889.15	153.44

Notice that our computational model underestimates actual market prices, specially for peak hours. One of the reasons for this discrepancy could be that in the actual dispatch performed by the exchange there are a number of complex rules which exclude generators deemed inflexible from participation in the price setting.

In hour theoretical model we assume that firms cannot fully anticipate the meeting of such additional restrictions and that the implied differences in the market outcomes can be summarized in the random component of the demand ε_{ht}.

This computational model for the dispatch is used to generate the bootstrapped moments that allows us to approximate each firm's expectation terms that are implied in the optimality conditions of the model. In other words we use the optimization algorithm to simulate several equilibrium to get a distribution of different market outcomes that will empirically approximate firm i's beliefs about the realization of its own profits, which will subsequently used to construct the empirical moment conditions defined in (24).