Debt: Deleveraging or Default*

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December 2, 2015

Abstract

Private information in credit markets may be resolved through deleveraging or default, depending on the volatility and the evolution of collateral value. We develop a dynamic model in which all borrowers have collateral subject to systematic uncertainty, but only good borrowers have additional income that is unobservable. When the volatility of collateral is low, good borrowers are able to fully separate by deleveraging, that is, raising debt and subsequently paying it down with unobservable income. For higher volatility, the amount of debt that is necessary for full separation may force bad borrowers to default, so that good borrowers must trade off the benefit of separation against an adverse selection cost of higher debt. For sufficiently high volatility, only partial separation is achieved because the cost of higher debt outweighs the benefit of separation. (*JEL* D14, G32)

Keywords: Adverse selection, Collateral, Leverage, Reputation, Signaling, Unsecured debt

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*We thank conference participants at the 2013 Society of Economic Dynamics meetings in Seoul, the Fall 2013 meetings of the Midwest Macroeconomics Conference in Minneapolis and the 2014 North American Summer Meeting of the Econometric Society, as well as seminar participants at Wharton, Federal Reserve Board and McGill University.

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1. Introduction

In the presence of private information and costly information acquisition, lenders infer the quality of a borrower based on repayment history. For example, repayment history is the most important of five factors that determine the FICO Score, which a leading measure of reputation in consumer credit markets (Fair Isaac Corporation 2011). Similarly, repayment history is the only factor that determines the PAYDEX Score, which is a leading measure of reputation for small business credit (Dun and Bradstreet Credibility Corporation 2015). In this paper, we study how reputation evolves through repayment and default in a dynamic model of credit markets with private information. In particular, we show the conditions under which repayment history resolves asymmetric information costlessly.

In our baseline model, borrowers live for three periods and roll over one-period debt in each period. There are two types of borrowers, good and bad. Both types of borrowers have a pledgable asset that can be used as collateral, whose income is volatile. Only good borrowers have a non-pledgable asset that cannot be used as collateral but generates unobservable income. Borrowers maximize net worth, as perceived by the lender or outside investors, which is increasing in reputation (i.e., the probability that the borrower is good). The lender is risk neutral and prices debt to break even, conditional on reputation. Reputation is updated through Bayes' rule, based on repayment versus default and the amount of new debt issued conditional on repayment.

Good borrowers have an incentive to signal their type through a strategic path of debt that reveals the presence of unobservable income. Bad borrowers have an incentive to mimic the path of debt, whenever feasible, to delay or prevent the revelation of their type. When the volatility of the collateral value is low, good borrowers are able to fully separate by raising debt and subsequently paying it down with unobservable income. Bad borrowers, who do not have unobservable income, must roll over higher debt in order to repay. Therefore, the ability to deleverage signals that a borrower is good. The revelation of private information through deleveraging is costless in the sense that there is no default in equilibrium.

When the volatility of collateral is higher, full separation is no longer possible through deleveraging alone. The amount of debt that is necessary for full separation may force bad borrowers to default if the collateral value falls. Although good borrowers do not default, they still bear an ex-ante cost for the possibility that bad borrowers default, through a higher interest rate that arises from adverse selection. This interest cost from a higher default probability is a direct cost that arises from private information only, since deadweight costs of default are absent in our baseline model. In choosing the optimal amount of debt, good borrowers must trade off the benefits of revealing their type against the interest cost of issuing more debt. The nature of the tradeoff depends critically on the volatility of collateral values because the benefit of separation is constant, whereas the cost in terms of higher interest rates rises with collateral volatility.

When the volatility of collateral is intermediate, the benefit of separation outweighs the interest cost of higher debt, so that there is full separation in equilibrium, sometimes involving default. Good borrowers raise sufficiently high debt so that, if the collateral value subsequently falls enough, bad borrowers are forced to default. If the collateral value rises instead, good borrowers can fully separate by deleveraging.

When the volatility of collateral is high, the interest cost of higher debt outweighs the benefit of separation, so that full separation is no longer optimal. Good borrowers raise relatively low debt so that, if the collateral value subsequently falls, they can fully separate by deleveraging. If the collateral value rises instead, the low level of debt rules out separation because even bad borrowers can repay by rolling over little debt.

In summary, how much it costs to reduce asymmetric information frictions rises with the uncertainty about collateral values. When the volatility of collateral is low, deleveraging is a costless way for good borrowers to reveal their type, and asymmetric information is always solved. When the volatility of collateral is intermediate, the revelation of private information entails default by bad borrowers if the collateral value falls. Even though asymmetric information is always solved, it is costly for good borrowers to convey such information. When the volatility of collateral is high enough, the cost of revealing private information is high enough so that the asymmetric information problem is not solved in all states of the world.

This paper shows then that the possibilities to use credit to relax information frictions over time critically depends on the characteristics of the assets used as collateral. In particular, the less volatile is the collateral, the easier and cheaper it is for borrowers to use the dynamics of credit to signal their type.

Ours is not the first model in the literature to argue that managing credit can be useful to reveal information about borrowers' private information. However, to the best of our knowledge, this is the first model that argues that it is not the level of credit, but rather the change in credit what reveals private information, and that the evolution of collateral assets is critical for the cost and feasibility of information revelation. Furthermore, we also show that the data strongly supports the testable predictions of our dynamic model in comparison to the other models suggested by the literature.

Static models of credit markets with private information, pioneered by Ross (1977) and Leland and Pyle (1977), predict that good borrowers use high leverage to reveal their type.[1]

[1]This literature was extended by Heinkel (1982) to study managers with different objective functions, Blazenko (1987) and John (1987) to study projects with different mean returns and Brick, Frierman and

Moreover, these static models predict that good borrowers default with higher probability because the higher deadweight cost of default is precisely what they use to signal their type.

Our work complements recent effort to extend models of credit markets with private information to a dynamic setting, such as Hennessy, Livdan and Miranda (2010) and Morellec and Schürhoff (2011). These previous papers essentially reduce the optimal choice of debt to a static problem, by assuming that private information is short-lived or that debt is a one-time choice in a real options framework. In contrast, our work allows for a richer set of signaling strategies through an optimal path of debt, which are ruled out by assumption in this literature. On the other hand, these previous papers model optimal investment choice, which we abstract from in order to isolate the effect of credit on solving asymmetric information problems. Indeed, in our setting there is no benefit from raising debt (other than signaling firms' types) and there is no deadweight cost of default (other than the higher interest rate good borrowers have to face due to adverse selection).

The rest of the paper is organized as follows. In Section 2, we set up a dynamic model of credit markets with private information and uncertainty about the collateral value. In Section 3, we characterize some important properties of the equilibrium that will be used to prove our main results. In Section 4, we present our main results on how the dynamics of collateral value shape the revelation of private information through deleveraging or default. Section 5 concludes.

2. A Dynamic Model of Credit Markets

In this section, we extend the classic model of credit markets with private information (Ross 1977) to a dynamic setting and also allow for uncertainty in collateral value. The dynamic model introduces a richer set of signaling strategies through the path of debt, which may reveal private information in equilibrium. Uncertainty affects the precision of these signaling strategies and, therefore, whether information revelation happens or not, and if it happens whether it entails default in equilibrium or not.

2.1. Pledgable and Non-pledgable Assets

There are two types of assets, pledgable and non-pledgable, which generate stochastic income streams. The pledgable asset can be used as collateral in credit transactions, whereas the non-pledgable asset cannot be used as collateral. Loosely speaking, we think about the pledgable

Kim (1998) to study projects with different variance of returns. For an excellent survey of this literature see Harris and Raviv (1991).

asset as tangible or verifiable (e.g., land, plant, or equipment) and the non-pledgable asset as intangible or non-verifiable (e.g., innovative ability, managerial skill, or organizational structure).

The pledgable asset generates an observable income X_t in each period t, which follows a martingale:

$$\mathbf{E}_t[X_{t+s}] = X_t. \tag{1}$$

Let R denote the gross riskless interest rate in the credit market, which satisfies $R^2(R-1) < 1$. The value of the pledgable asset is the present value of its income:

$$V_t = \frac{\mathbf{E}_t[X_{t+1} + V_{t+1}]}{R} = \sum_{s=1}^{\infty} \frac{\mathbf{E}_t[X_{t+s}]}{R^s} = \frac{X_t}{R-1}. \tag{2}$$

The non-pledgable asset generates unobservable income Y_t in each period t, which also follows a martingale:

$$\mathbf{E}_t[Y_{t+s}] = Y_t. \tag{3}$$

We assume that all income is perishable and must be immediately consumed or used to repay debt. By assuming away hidden savings, we simplify the analysis without affecting the substance of our results.

2.2. Borrowers with Private Information

There are two types of borrowers, "good" and "bad". Both types of borrowers are endowed with a unit of the pledgable asset. Only good borrowers are also endowed with a unit of the non-pledgable asset.[2] Whether a given borrower is good or bad is private information to the borrower, which arises from the unobservability of whether the borrower owns the non-pledgable asset that generates unobservable income.

Each borrower is in the credit market for at most three periods, which we denote as $t \in \{1, 2, 3\}$. Let F_0 denote the face value of existing debt that is due in period 1 and $\pi_0 \in (0, 1)$ denote the lender's prior that a borrower is good in period 1. We use the term "reputation" to refer to the lender's subjective probability that a borrower is good.[3]

[2] The assumption that bad borrowers do not have the non-pledgable asset is just normalization. The key assumption is that bad borrowers have non-pledgable assets that are poorer in quality than non-pledgable assets of good borrowers (e.g., poor innovative ability, poor managerial skill, or poor organizational structure).

[3] Here we simply assume the prior π_0 is exogenous, but it can be endogenized as in Atkeson, Hellwig and Ordonez (2014), capturing the mass of borrowers who choose to invest in non-pledgable assets and

Suppose a borrower enters period $t \in \{1, 2\}$ with maturing debt F_{t-1} and reputation π_{t-1}. The borrower receives income $X_t + Y_t$ if good or X_t if bad. The borrower can either repay the face value of maturing debt or default. Let $D_{i,t}$ denote the default boundary such that it is feasible and optimal for a borrower of type $i = \{g, b\}$ (i.e., good or bad) to repay if $F_{t-1} \leq D_{i,t}$ and to default otherwise. To repay its maturing debt, the borrower can use its income as well as the proceeds from issuing new one-period debt with face value $F_{i,t}$ at the equilibrium price P_t. Conditional on repayment, the lender updates reputation to π_t. Note that not only repayment, but also the face value of new debt, can serve as signals for the updating of reputation. Conditional on default, the lender takes possession of the collateral (i.e., the pledgable asset and its income in period t) and updates reputation to $\widehat{\pi}_t$.

The borrower essentially faces the same problem in period 3. The only difference is that instead of issuing new debt, it can sell the pledgable asset at market value to repay its maturing debt. Therefore, only repayment can serve as a signal for the updating of reputation in the terminal period.

Following Ross (1977), we assume that the borrower maximizes net worth, as perceived by the lender (or outside investors with knowledge of only reputation). The value of the non-pledgable asset in period 3 is

$$W_3 = \frac{\pi_3 Y_3}{R - 1} \tag{4}$$

in the case of repayment and

$$\widehat{W}_3 = \frac{\widehat{\pi}_3 Y_3}{R - 1} \tag{5}$$

in the case of default. That is, the value of the non-pledgable asset is equal to the probability that the borrower owns the non-pledgable asset times its value conditional on ownership. Let $\mathbf{1}_g(i)$ be an indicator function that is equal to one if the borrower is good and zero otherwise. The net worth for a borrower of type i in period 3 is

$$J_{i,3} = \begin{cases} X_3 + \mathbf{1}_g(i) Y_3 + V_3 + W_3 - F_2 & \text{if } D_{i,3} \geq F_2 \\ \mathbf{1}_g(i) Y_3 + \widehat{W}_3 & \text{if } D_{i,3} < F_2 \end{cases}. \tag{6}$$

In the case of repayment, net worth is income plus the terminal value of both types of assets minus the face value of maturing debt. In the case of default, net worth is the terminal value of the non-pledgable asset and (for a good borrower) its income.

participate in credit markets.

We define the borrower's net worth in period $t \in \{1,2\}$ recursively as

$$J_{i,t} = \begin{cases} X_t + \mathbf{1}_g(i)Y_t + P_tF_t - F_{t-1} + \frac{\mathbf{E}_t[J_{i,t+1}]}{R} & \text{if } D_{i,t} \geq F_{t-1} \\ \mathbf{1}_g(i)Y_t + \widehat{W}_{i,t} & \text{if } D_{i,t} < F_{t-1} \end{cases}, \tag{7}$$

where

$$\widehat{W}_{i,t} = \sum_{s=1}^{3-t} \frac{\mathbf{1}_g(i)\mathbf{E}_t[Y_{t+s}]}{R^s} + \frac{\mathbf{E}_t[\widehat{\pi}_3 Y_3]}{R^{3-t}(R-1)} = \frac{((R^{3-t}-1)\mathbf{1}_g(i) + \widehat{\pi}_t)Y_t}{R^{3-t}(R-1)}. \tag{8}$$

In the case of repayment, net worth is income plus the net proceeds from rolling over debt plus the borrower's continuation value. In the case of default, net worth is the terminal value of the non-pledgable asset and (for a good borrower) its income through period 3. Note that once a borrower defaults in period 2, there is no further updating of reputation so that $\widehat{\pi}_3 = \widehat{\pi}_2$.

2.3. Lenders

The representative lender is risk neutral and earns an expected return R on each loan. The lender does not know whether a given borrower is good or bad. However, the lender updates reputation based on repayment versus default and the amount of new debt conditional on repayment.

We assume throughout that $F_0 < X_1$ such that there is never default in period 1. Similarly, the assumption of a terminal period implies there is no refinancing in the last period. This implies that lenders update reputation based on the level of debt issued in period 1, based on both the level of debt issued and default decisions in period 2, and only based on default decisions in period 3.

More specifically, conditional on repayment in period $t \in \{1,2\}$, the lender updates reputation through Bayes rule:

$$\pi_t = \left[1 + \frac{(1-\pi_{t-1})\Pr(\{D_{b,t} \geq F_{t-1}\} \cap \{F_{b,t} = F_t\})}{\pi_{t-1}\Pr(\{D_{g,t} \geq F_{t-1}\} \cap \{F_{g,t} = F_t\})}\right]^{-1}. \tag{9}$$

This formula accounts for the fact that not only repayment, but also the face value of new debt F_t, potentially reveals private information about the borrower type. Conditional on repayment in period 3, the terminal reputation is

$$\pi_3 = \left[1 + \frac{(1-\pi_2)\Pr(D_{b,3} \geq F_2)}{\pi_2 \Pr(D_{g,3} \geq F_2)}\right]^{-1}. \tag{10}$$

That is, the lender updates reputation based on repayment alone because there is no choice of new debt in the terminal period. Conditional on default in period t, the terminal reputation is

$$\widehat{\pi}_t = \left[1 + \frac{(1-\pi_{t-1})\Pr(D_{b,t} < F_{t-1})}{\pi_{t-1}\Pr(D_{g,t} < F_{t-1})}\right]^{-1}. \tag{11}$$

In order to characterize the equilibrium, we must make auxiliary assumptions about beliefs off the equilibrium path. We make a natural assumption that repayment is assumed to be good if all borrowers default in equilibrium. Similarly, default is assumed to be bad if all borrowers repay in equilibrium.[4] We state our assumptions more formally as follows.

Assumption 1. *The lender's off-equilibrium beliefs are given by*

$$\pi_t = 1 \text{ if } \Pr(D_{g,t} \geq F_{t-1}) = \Pr(D_{b,t} \geq F_{t-1}) = 0,$$
$$\widehat{\pi}_t = 0 \text{ if } \Pr(D_{g,t} < F_{t-1}) = \Pr(D_{b,t} < F_{t-1}) = 0.$$

In period $t \in \{1, 2\}$, the lender's break-even condition determines the equilibrium price of debt P_t, given face value F_t and reputation π_t:

$$P_t F_t = \pi_t C_{g,t} + (1-\pi_t) C_{b,t}, \tag{12}$$

where

$$C_{i,t} = \frac{\Pr(D_{i,t+1} \geq F_t)F_t + \Pr(D_{i,t+1} < F_t)\mathbf{E}_t[X_{t+1} + V_{t+1} | D_{i,t+1} < F_t]}{R}. \tag{13}$$

That is, the lender breaks even if the value of debt is equal to the expected repayment discounted at R. The expected repayment is equal to the probability that the borrower is good multiplied by good borrowers' expected repayment levels plus the probability that the borrower is bad multiplied by bad borrowers' expected repayment levels .

2.4. Timing and Summary of the Model

The borrower can signal through the amount of new debt in periods 1 and 2 and through repayment in periods 2 and 3. We summarize the model as follows.

Period 1. The borrower starts with face value of debt $F_0 \leq X_1$ and reputation π_0.

(a) The borrower receives income $X_1 + Y_1$ if good and X_1 if bad.

[4] These restrictions on off-equilibrium beliefs arise naturally from the application of the intuitive criterion.

(b) The borrower issues new debt with face value F_1 at the equilibrium price P_1. The lender updates reputation to π_1.

Period 2. The borrower enters with face value of debt F_1 and reputation π_1.

(a) The borrower receives income $X_2 + Y_2$ if good and X_2 if bad.

(b) The borrower decides whether or not to repay F_1.

- In the case of repayment, the borrower issues new debt with face value F_2 at the equilibrium price P_2. The lender updates reputation to π_2.
- In the case of default, the lender takes possession of the pledgable asset (i.e., $X_2 + V_2$) and updates reputation to $\widehat{\pi}_2$. The borrower's terminal value is the non-pledgable asset and its income (i.e., $\mathbf{1}_g(i)Y_2 + \widehat{W}_{i,2}$).

Period 3. In the case of repayment in period 2, the borrower enters with face value of debt F_2 and reputation π_2.

(a) The borrower receives income $X_3 + Y_3$ if good and X_3 if bad.

(b) The borrower decides whether or not to repay F_2.

- In the case of repayment, the lender updates reputation to π_3.
- In the case of default, the lender takes possession of the pledgable asset (i.e., $X_3 + V_3$) and updates reputation to $\widehat{\pi}_3$. The borrower's terminal value is the non-pledgable asset and its income (i.e., $\mathbf{1}_g(i)Y_3 + \widehat{W}_3$).

3. Properties of the Equilibrium

In this section, we characterize some important properties of the equilibrium that do not depend on additional distributional assumptions. We will use the lemmas in this section to prove our main results in Section 4.

3.1. Borrowers' Maximization Problem

In period 3, a borrower of type i can repay its maturing debt if

$$X_3 + \mathbf{1}_g(i)Y_3 + V_3 \geq F_2. \tag{14}$$

That is, the borrower can repay if its income plus the value of the pledgable asset exceeds the face value of maturing debt. Moreover, equation (6) implies that it is optimal for the borrower to repay if

$$X_3 + V_3 + W_3 - \widehat{W}_3 \geq F_2. \tag{15}$$

Combining both feasibility and optimality, the default boundary in period 3 is

$$D_{i,3} = X_3 + V_3 + \min\left\{\mathbf{1}_g(i)Y_3, W_3 - \widehat{W}_3\right\}. \tag{16}$$

In period $t \in \{1, 2\}$, a borrower of type i can repay its maturing debt if

$$X_t + \mathbf{1}_g(i)Y_t + \max_{F_t} P_t F_t \geq F_{t-1}. \tag{17}$$

That is, the borrower can repay if its income plus the maximum amount of new debt that it can issue exceeds the face value of maturing debt. The following lemma establishes the condition under which repayment is optimal, which implies the default boundary when combined with feasibility.

Lemma 1. *In period $t \in \{1, 2\}$, the borrower's net worth is*

$$J_{i,t} = \begin{cases} X_t + \mathbf{1}_g(i)Y_t + V_t + W_{i,t} - F_{t-1} & \text{if } D_{i,t} \geq F_{t-1} \\ \mathbf{1}_g(i)Y_t + \widehat{W}_{i,t} & \text{if } D_{i,t} < F_{t-1} \end{cases}, \tag{18}$$

where the value of the non-pledgable asset conditional on repayment is

$$\begin{aligned} W_{i,t} = &- (\mathbf{1}_g(i) - \pi_t)(C_{g,t} - C_{b,t}) + \frac{\mathbf{1}_g(i)Y_t + \Pr(D_{i,t+1} \geq F_t)\mathbf{E}_t[W_{i,t+1}|D_{i,t+1} \geq F_t]}{R} \\ &+ \frac{\Pr(D_{i,t+1} < F_t)\mathbf{E}_t\left[\widehat{W}_{i,t+1}|D_{i,t+1} < F_t\right]}{R}. \end{aligned} \tag{19}$$

The default boundary is

$$D_{i,t} = X_t + V_t + \min\left\{\mathbf{1}_g(i)Y_t + \max_{F_t} P_t F_t - V_t, W_{i,t} - \widehat{W}_{i,t}\right\}, \tag{20}$$

where

$$W_{i,t} - \widehat{W}_{i,t} = -(\mathbf{1}_g(i) - \pi_t)(C_{g,t} - C_{b,t}) + \frac{\mathbf{E}_t[\widehat{\pi}_{t+1}Y_{t+1}] - \widehat{\pi}_t Y_t}{R^{3-t}(R-1)}$$
$$+ \frac{\Pr(D_{i,t+1} \geq F_t)\mathbf{E}_t\left[W_{i,t+1} - \widehat{W}_{i,t+1}|D_{i,t+1} \geq F_t\right]}{R}. \tag{21}$$

Proof. We show that equations (7) and (18) are equivalent by induction. Suppose equation (18) holds for period $t+1$. Then

$$\frac{\mathbf{E}_t[J_{i,t+1}]}{R} = \frac{\Pr(D_{i,t+1} \geq F_t)\mathbf{E}_t[X_{t+1} + \mathbf{1}_g(i)Y_{t+1} + V_{t+1} + W_{i,t+1} - F_t|D_{i,t+1} \geq F_t]}{R}$$
$$+ \frac{\Pr(D_{i,t+1} < F_t)\mathbf{E}_t\left[\mathbf{1}_g(i)Y_{t+1} + \widehat{W}_{i,t+1}|D_{i,t+1} < F_t\right]}{R}$$
$$= V_t - C_{i,t} + \frac{\Pr(D_{i,t+1} \geq F_t)\mathbf{E}_t[\mathbf{1}_g(i)Y_{t+1} + W_{i,t+1}|D_{i,t+1} \geq F_t]}{R}$$
$$+ \frac{\Pr(D_{i,t+1} < F_t)\mathbf{E}_t\left[\mathbf{1}_g(i)Y_{t+1} + \widehat{W}_{i,t+1}|D_{i,t+1} < F_t\right]}{R}. \tag{22}$$

Substituting equations (12) and (22) into equation (7), equation (18) holds for period t. Equations (17) and (18) imply equation (20). □

In the case of repayment in period $t \in \{1, 2\}$, the borrower chooses F_t to maximize its net worth (18). However, all the components of net worth are pre-determined, except for the value of the non-pledgable asset. Therefore, the borrower's maximization problem reduces to

$$\max_{F_t} W_{i,t} \text{ subject to } X_t + \mathbf{1}_g(i)Y_t + P_t F_t \geq F_{t-1}. \tag{23}$$

3.2. Benchmark without Private Information

The only friction in our model is private information about whether or not the borrower owns the non-pledgable asset. The benchmark without private information is a special case of our model where reputation has converged to $\pi_{t-1} \in \{0, 1\}$. In this special case, we recover the standard result that debt (or leverage) is indeterminate.

Lemma 2 (Modigliani and Miller (1958))**.** *If $F_{t-1} \leq X_t$ and $\pi_{t-1} \in \{0, 1\}$, borrowers are indifferent between any amount of debt such that $P_t F_t \leq V_t$. The equilibrium interest rate is $P_1^{-1} = R$.*

Proof. When there is no further updating of reputation, the value of the non-pledgable asset is $W_{i,t} = \widehat{W}_{i,t}$. That is, the borrower's objective function is independent of F_t. Therefore, the borrower is indifferent between any amount of debt such that repayment is feasible (i.e., $P_tF_t \geq F_{t-1} - X_t - \mathbf{1}_g(i)Y_t$). Moreover, the maximum amount of new debt is

$$\begin{aligned} P_tF_t =& \frac{\Pr(X_{t+1}+V_{t+1} \geq F_t)F_t + \Pr(X_{t+1}+V_{t+1} < F_t)\mathbf{E}_t[X_{t+1}+V_{t+1}|X_{t+1}+V_{t+1} < F_t]}{R} \\ \leq& \frac{\Pr(X_{t+1}+V_{t+1} \geq F_t)\mathbf{E}_t[X_{t+1}+V_{t+1}|X_{t+1}+V_{t+1} \geq F_t]}{R} \\ &+ \frac{\Pr(X_{t+1}+V_{t+1} < F_t)\mathbf{E}_t[X_{t+1}+V_{t+1}|X_{t+1}+V_{t+1} < F_t]}{R} \\ =& V_t, \end{aligned} \tag{24}$$

with equality when $F_t = \mathbf{E}_t[X_{t+1}+V_{t+1}|X_{t+1}+V_{t+1} \geq F_t]$. That is, debt is fully collateralized and riskless. □

3.3. Signaling under Private Information

In the presence of private information, good borrowers have an incentive to signal through repayment versus default and the amount of new debt conditional on repayment. Bad borrowers have an incentive to mimic the good in order to delay or avoid the revelation of private information.[5] This incentive arises from two sources. The first source is that bad borrowers pay interest that is lower than under perfect information, given that they are more likely to default in the future. This incentive is captured by the first term, $\pi_t(C_{g,t} - C_{b,t}) \geq 0$, in equation (19). The second source is a higher terminal value of the non-pledgable asset, if bad borrowers can altogether avoid revealing their type. This incentive is captured by the last two terms in equation (19).

The following lemma formally establishes that bad borrowers are more likely to default than the good.

Lemma 3. *The default boundary for good borrowers is higher than that for bad borrowers:*

$$X_t + V_t \leq D_{b,t} \leq D_{g,t} \leq X_t + Y_t + V_t. \tag{25}$$

If the event of full separation in period t, the first and third inequalities are equalities, and the second inequality is strict.

Proof. The proof is in Appendix A. □

[5] To simplify the statement of our results, we follow the convention that bad borrowers mimic good borrowers in knife-edge cases of indifference.

In our model, reputation either remains the same or updates fully to one or zero for good and bad borrowers, respectively. Since collateral shocks are the only source of uncertainty in this model, the borrower's action is either fully revealing or not at all, conditional on the realized shock.[6] The following lemmas establish whether or not there is full separation, depending on the face value of maturing debt and the realized collateral value.

Lemma 4. *Suppose $F_{t-1} \leq X_t$ in period $t \in \{1, 2\}$, or $F_2 \leq D_{b,3}$ in period 3. All borrowers repay, and private information is not revealed in equilibrium.*

Proof. If $F_{t-1} \leq X_t$, both types of borrowers can repay when borrowing $P_t F_t \geq F_{t-1} - X_t$. □

Lemma 5. *Suppose $F_{t-1} \in (X_t, D_{b,t}]$ in period $t \in \{1, 2\}$. Borrower type is fully revealed in equilibrium. Good borrowers repay maturing debt by borrowing*

$$R \max\{0, F_{t-1} - X_t - Y_t\} \leq F_t < R(F_{t-1} - X_t).$$

Bad borrowers repay maturing debt by borrowing

$$R(F_{t-1} - X_t) \leq F_t \leq RV_t.$$

Proof. In this region, it is optimal for all borrowers to repay. Good borrowers can repay by borrowing at least $P_t F_t \geq \max\{0, F_{t-1} - X_t - Y_t\}$. Bad borrowers can repay by borrowing at least $P_t F_t \geq F_{t-1} - X_t$. Therefore, good borrowers can separate by borrowing at most $P_t F_t < F_{t-1} - X_t$. Lemma 2 implies that the equilibrium interest rate is $P_t^{-1} = R$. □

Lemma 6. *Suppose $F_{t-1} \in (D_{b,t}, D_{g,t}]$ in any period t. Only the bad borrowers default, so that borrower type is fully revealed in equilibrium. In period $t \in \{1, 2\}$, good borrowers repay maturing debt by borrowing*

$$R \max\{0, F_{t-1} - X_t - Y_t\} \leq F_t \leq RV_t.$$

Proof. In this region, bad borrowers are forced to default. In period $t \in \{1, 2\}$, good borrowers can repay by borrowing at least $P_t F_t \geq \max\{0, F_{t-1} - X_t - Y_t\}$. Lemma 2 implies that the equilibrium interest rate is $P_t^{-1} = R$. □

Lemma 7. *Suppose $F_{t-1} > D_{g,t}$ in any period t. All borrowers default, and private information is not revealed in equilibrium.*

[6] It is straightforward, but complicates the exposition, to also assume shocks to the non-pledgable asset, or shocks to borrowers' types, such that reputation is updated gradually and there is never full revelation of the borrower's type.

These lemmas combined imply that there are potentially two ways in which the borrower's type is fully revealed in period 2. If maturing debt is low as in Lemma 5, good borrowers can reduce their debt by repaying with unobservable income. Bad borrowers, who do not have unobservable income, must roll over higher debt in order to repay. Therefore, rolling over debt by issuing a lower level of new debt signals that a borrower is good since only borrowers with non observable income are able to follow such strategy. If maturing debt is higher as in Lemma 6, good borrowers can repay with their unobservable income, while bad borrowers are forced to default. In this case, then, repayment signals that a borrower is good.

Previous lemmas take as given F_1. However F_1 is an endogenous choice for borrowers in period 1. Good borrowers can issue relatively low debt to prepare for signaling by *deleveraging* or issue higher debt to prepare for signaling by *forcing default*. If the volatility of collateral is sufficiently low, good borrowers prefer to issue relatively low debt in period 1 to fully separate by deleveraging in period 2. This is because equation (19) implies that the value of the non-pledgable asset is $W_{1,t} = \frac{Y_1}{R-1}$ under deleveraging and

$$W_{1,t} = -(1-\pi_t)(C_{g,t} - C_{b,t}) + \frac{Y_1}{R-1} \tag{26}$$

under forcing default. Forcing default is costly because good borrowers have to pay higher interest in period 1, captured by the first term in equation (26), due to adverse selection.

If the volatility of collateral is sufficiently high, it is not possible for good borrowers to fully separate by deleveraging. Intuitively, when the volatility is high, good borrowers have to raise large amounts of debt in period 1. Otherwise, when the value of collateral rises, bad borrowers would be also able to repay just using collateral, avoiding information revelation. However, raising large amounts of debt introduces the risk that in case the value of collateral drops, good borrowers have to default. In that case, good borrowers must trade off the benefit of separation against the interest cost of higher debt because the probability of default. We analyze how this tradeoff depends on the volatility of collateral in the next section.

4. Signaling through Deleveraging or Default

In this section we analyze the tradeoff between the benefit of separation and the interest cost of higher debt. For expositional convenience we now make additional parametric assumptions. First, we assume that unobservable income is a constant proportion of observable income. Moreover, unobservable income is less than the collateral value, so that signaling through repayment from unobservable income is non-trivial.

Assumption 2. *Unobservable income is a constant proportion y of observable income. Moreover, unobservable income is less than the collateral value. That is,*

$$y = \frac{Y_t}{X_t} < \frac{X_t + V_t}{X_t} = \frac{R}{R-1}.$$

Second, we assume that observable income follows a binomial version of the geometric random walk. This assumption allows focusing the analysis to a manageable number of cases.

Assumption 3. *The growth rate of observable income is distributed as*

$$x_t = \frac{X_t}{X_{t-1}} = \begin{cases} \overline{x} & \text{with probability } 1-p \\ \underline{x} & \text{with probability } p \end{cases},$$

where $\overline{x} \geq \underline{x}$ and $(1-p)\overline{x} + p\underline{x} = 1$.

There are only two free parameters between $\overline{x}$, $\underline{x}$, and p because of the normalization that the mean growth rate is one. In characterizing the equilibrium, it is convenient to divide the parameter space into regions along $\frac{\overline{x}}{\underline{x}}$ and $(1-p)\overline{x}$, as illustrated in Figure 1. We analyze the region $(1-p)\overline{x} \geq 0.5$ in this section and leave its complement $(1-p)\overline{x} < 0.5$ to Appendix C.

4.1. Low Volatility of Collateral

When the volatility of collateral is low, the equilibrium can be described as full separation through deleveraging. We state the result formally in Proposition 1.

Proposition 1. *Suppose the volatility of collateral is low. That is, $\frac{\overline{x}}{\underline{x}} < \frac{R}{R-1}$. In period 1, all borrowers borrow $X_1\overline{x} < F_1 \leq RV_1\underline{x}$ at the interest rate $P_1^{-1} = R$.*

In period 2, borrower type is fully revealed. Good borrowers repay maturing debt by borrowing

$$R \max\{0, F_1 - X_2 - Y_2\} \leq F_2 < R(F_1 - X_2).$$

Bad borrowers repay maturing debt by borrowing

$$R(F_1 - X_2) \leq F_2 \leq RV_2.$$

Proof. Lemma 9 in Appendix B implies the equilibrium in period 1. Lemma 5 implies the equilibrium in period 2. □

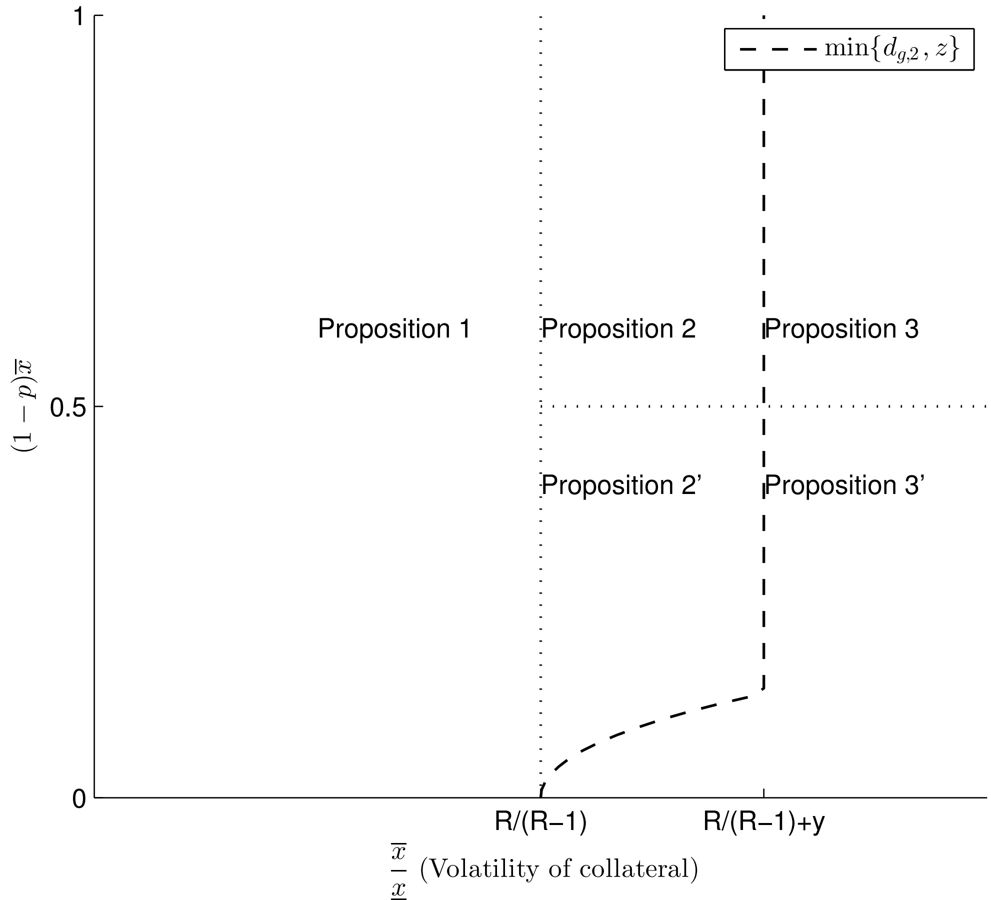

Figure 1: Regions of the Parameter Space.

Figure 2 is a graphical illustration of Proposition 1. The figure shows the equilibrium range of debt, normalized by collateral value to be in units of leverage (i.e., $\frac{P_t F_t}{V_t}$), for both types of borrowers. In period 1, all borrowers start with the same amount of debt. Note that the equilibrium range of debt is narrower than in the benchmark without private information (Modigliani and Miller 1958), where leverage is indeterminate between zero and one. In period 2, good borrowers fully separate by reducing debt, while bad borrowers are forced to roll over higher debt.

The intuition for Proposition 1 is straightforward. In period 2, good borrowers can fully separate by deleveraging if the face value of maturing debt F_1 is greater than observable income X_2. If that condition is satisfied, good borrowers can repay maturing debt from their observable income, part of their unobservable income, and the remainder from issuing new debt. Bad borrowers, who do not have unobservable income, must issue a higher amount of new debt. In period 1, good borrowers must borrow at least $F_1 > X_1\overline{x} \geq X_2$ in order to ensure separation in period 2, even if the realized observable income is high.

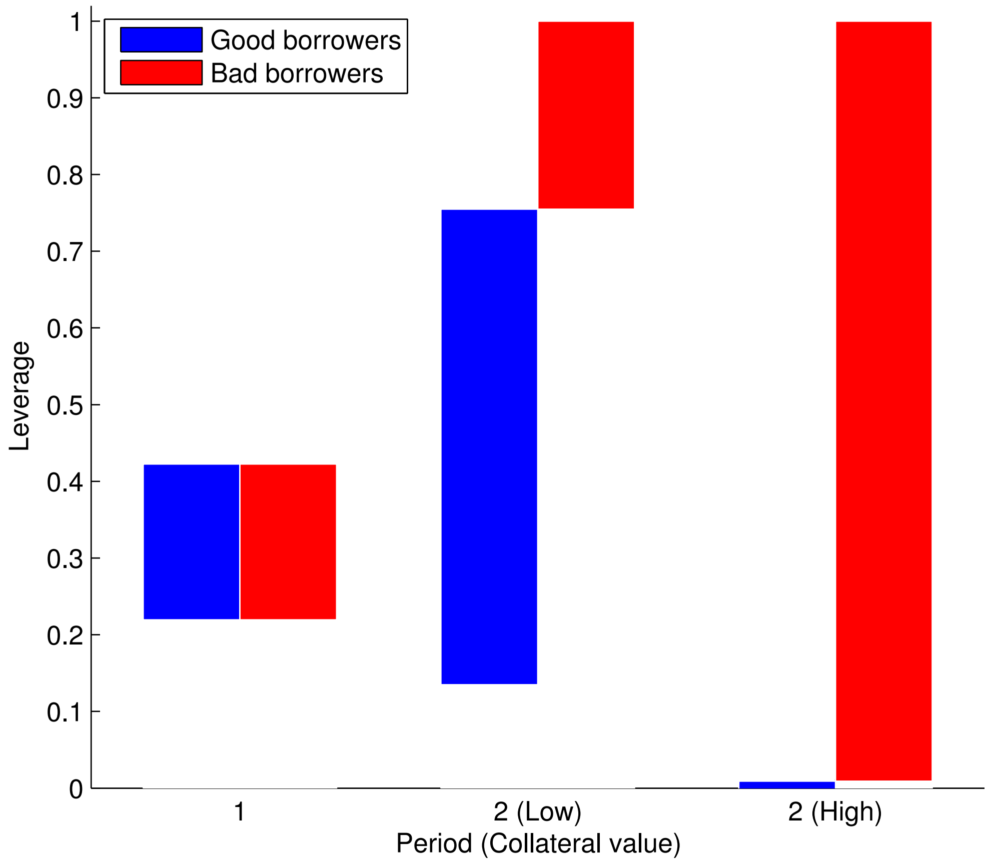

Figure 2: Equilibrium Debt for Low Volatility of Collateral. The range of leverage in period 2 is conditional on choosing the median within the range in period 1.

Bad borrowers repay in period 2 as long as the face value of maturing debt F_1 is less than the collateral value $X_2 + V_2$. Therefore, good borrowers cannot borrow more than $F_1 \leq RV_1\underline{x} \leq X_2 + V_2$ to ensure that bad borrowers do not default, even if the collateral value falls. As discussed in Section 3, good borrowers prefer not to force default since they bear an adverse selection cost through higher interest in period 1.

The range of equilibrium debt in period 1 shrinks as the volatility of collateral rises. This is because the lower bound on debt must rise to ensure full separation, even if the collateral rises in period 2. At the same time, the upper bound on debt must fall to prevent bad borrowers from defaulting, even if the collateral value falls in period 2. The range of equilibrium debt shrinks until it becomes a point at which $X_1\overline{x} = RV_1\underline{x}$, which is equivalent to $\frac{\overline{x}}{\underline{x}} = \frac{R}{R-1}$. Denote such point $\bar{F}_1 \equiv X_1\overline{x} = RV_1\underline{x}$. If the borrower takes debt for more than $\bar{F}_1$ it would force default by bad borrowers in case the value of collateral falls in the subsequent period. If the borrower takes debt for less than $\bar{F}_1$ bad borrowers would be also able to repay without raisin new debt in case the value of collateral increases in the subsequent period.

Therefore, full separation through deleveraging alone is possible only as long as the volatility of collateral is sufficiently low (i.e., $\frac{\overline{x}}{\underline{x}} < \frac{R}{R-1}$). In the next subsection, we examine the case where the volatility of collateral is higher.

Recall at this point that private information is the only friction in our baseline model. In particular, we do not have deadweight costs of default, which is the key friction that allows good borrowers to signal through higher debt in the static model of Ross (1977). Deadweight costs would not alter our conclusions in Proposition 1 because full separation is achieved without default. Put differently, leveraging followed by deleveraging is a superior way of signaling in a dynamic setting, which is ruled out by construction in a static model.

4.2. Higher Volatility of Collateral

When the volatility of collateral is intermediate, the equilibrium can be described as full separation through deleveraging or default. We state the result formally in Proposition 2.

Proposition 2. *Suppose the volatility of collateral is intermediate. That is,* $\frac{R}{R-1} \leq \frac{\overline{x}}{\underline{x}} < \min\{d_{g,2}, z\}$*, where*

$$\begin{aligned} d_{g,2} =& \frac{R}{R-1} + y, \\ z =& \frac{R}{R-1} + \frac{(1-p)\overline{x}y}{R(R-1)}. \end{aligned}$$

In period 1, all borrowers borrow $F_1 > X_1\overline{x}$ *at an interest rate* $P_1^{-1} > R$ *that satisfies*

$$P_1F_1 = \frac{(1-(1-\pi_0)p)F_1}{R} + \frac{(1-\pi_0)pX_1\underline{x}}{R-1}.$$

If the collateral value falls in period 2 (i.e., $x_2 = \underline{x}$*), only the bad borrowers default, so that borrower type is fully revealed. Good borrowers repay maturing debt by borrowing*

$$R\max\{0, F_1 - X_2 - Y_2\} \leq F_2 \leq RV_2.$$

If the collateral value rises in period 2 (i.e., $x_2 = \overline{x}$*), borrower type is fully revealed. Good borrowers repay maturing debt by borrowing* $F_2 = 0$*. Bad borrowers repay maturing debt by borrowing* $0 < F_2 \leq RV_2$*.*

Proof. Lemma 9 in Appendix B implies the equilibrium in period 1. Lemma 6 implies the equilibrium if the collateral value falls in period 2, and Lemma 5 implies the equilibrium if the collateral value rises instead. □

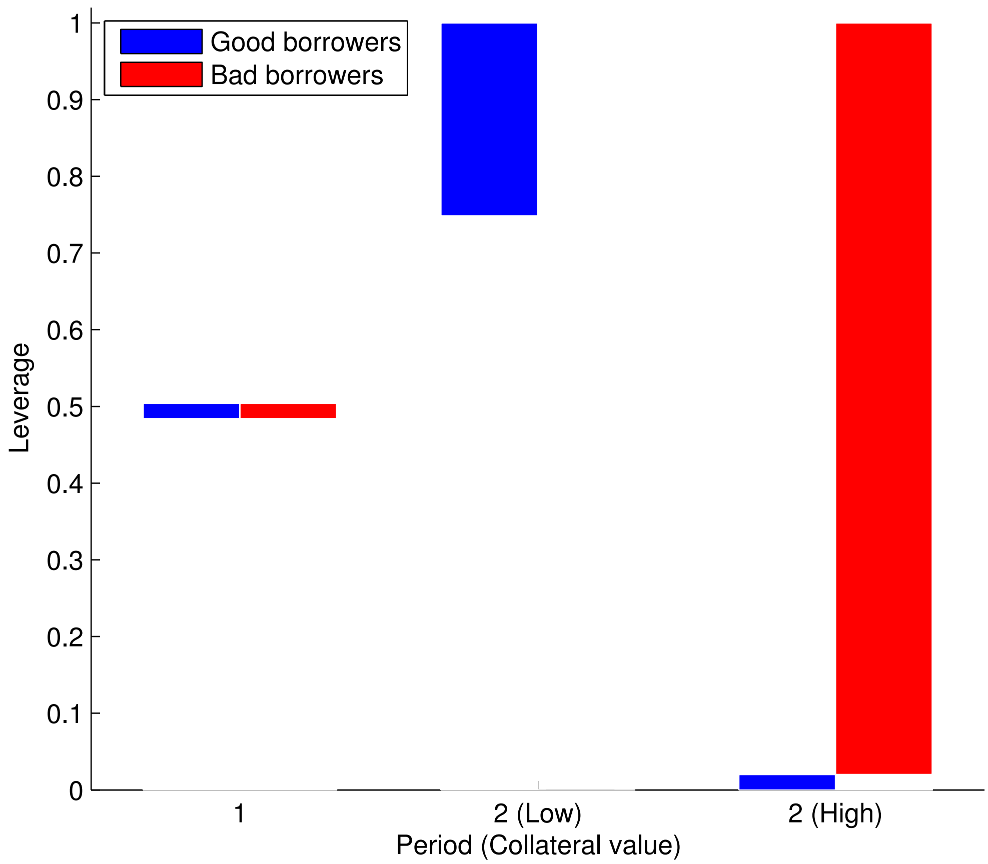

Figure 3: Equilibrium Debt for Intermediate Volatility of Collateral. The range of leverage in period 2 is conditional on choosing the median within the range in period 1.

Figure 3 is a graphical illustration of Proposition 2. In period 1, all borrowers start with the same amount of debt. If the collateral value falls in period 2, good borrowers fully separate by repaying, while bad borrowers default. If the collateral value rises in period 2, good borrowers fully separate by reducing debt, while bad borrowers are forced to roll over higher debt.

When the volatility of collateral is high, the equilibrium can be described as partial separation through deleveraging. We state the result formally in Proposition 3.

Proposition 3. *Suppose the volatility of collateral is high. That is,* $\frac{\overline{x}}{\underline{x}} \geq \min\{d_{g,2}, z\}$. *In period 1, all borrowers borrow* $X_1\underline{x} < F_1 \leq RV_1\underline{x}$ *at the interest rate* $P_1^{-1} = R$.

If the collateral value falls in period 2 (i.e., $x_2 = \underline{x}$*), borrower type is fully revealed. Good* *borrowers repay maturing debt by borrowing*

$$R \max\{0, F_1 - X_2 - Y_2\} \leq F_2 < R(F_1 - X_2).$$

Bad borrowers repay maturing debt by borrowing

$$R(F_1 - X_2) \leq F_2 \leq RV_2.$$

If the collateral value rises in period 2 (i.e., $x_2 = \overline{x}$), private information is not revealed. All borrowers repay maturing debt by borrowing $F_2 > RV_2\overline{x}$ at an interest rate $P_2^{-1} > R$ that satisfies

$$P_2 F_2 = V_2 + \pi_0(1 - p)\left(\frac{F_2}{R} - V_2\overline{x}\right).$$

Note that debt is unsecured since $P_2F_2 > V_2$. Subsequently, if the collateral value falls in period 3 (i.e., $x_3 = \underline{x}$), all borrowers default, so that private information is not revealed. If the collateral value rises instead (i.e., $x_3 = \overline{x}$), only the bad borrowers default, so that borrower type is fully revealed.

Proof. Lemma 9 in Appendix B implies the equilibrium in period 1. Lemma 5 implies the equilibrium if the collateral value falls in period 2, and Lemma 8 in Appendix B implies the equilibrium if the collateral value rises instead. Lemma 7 implies the equilibrium if the collateral value subsequently falls in period 3, and Lemma 6 implies the equilibrium if the collateral value rises instead. □

Figure 4 is a graphical illustration of Proposition 3. In period 1, all borrowers start with the same amount of debt. If the collateral value falls in period 2, good borrowers fully separate by rolling over lower debt than the bad borrowers. If the collateral value rises in period 2, all borrowers raise the same amount of debt that is higher than the collateral value.

We now discuss the intuition for Propositions 2 and 3, by sketching the essential elements of the formal proofs in the appendix. As discussed in the last subsection, full separation through deleveraging is not possible when $\frac{\overline{x}}{\underline{x}} \geq \frac{R}{R-1}$. Good borrowers then face a tradeoff between *costly full separation* and *partial separation*.

Costly full separation occurs if good borrowers issue $F_1 > X_1\overline{x}$, as in Proposition 2. The value of the non-pledgable asset under this strategy is

$$W_{g,1} = -(1 - \pi_0)p\left(\frac{F_1}{R} - \frac{X_1\underline{x}}{R - 1}\right) + \frac{Y_1}{R - 1}. \quad (27)$$

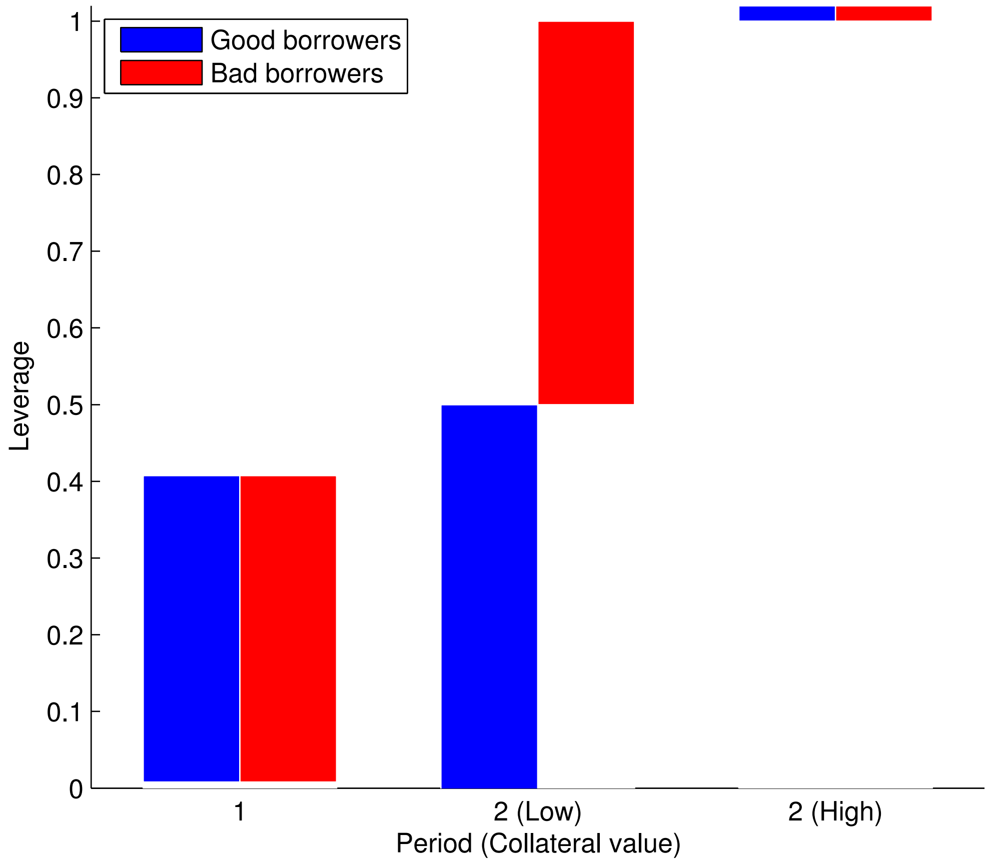

Figure 4: Equilibrium Debt for High Volatility of Collateral. The range of leverage in period 2 is conditional on choosing the median within the range in period 1.

The first term is the interest cost of debt in period 1, which arises from the default of bad borrowers in period 2 if the collateral value falls with probability p. To minimize this cost, good borrowers optimally choose F_1 that is arbitrarily close to $X_1\overline{x}$. The second term is the benefit of full separation. Note that the benefit is constant, whereas interest cost rises in the volatility of collateral.

Partial separation occurs if good borrowers issue $F_1 \leq RV_1\underline{x}$, as in Proposition 3. Under this strategy, good borrowers essentially avoid the interest cost of higher debt, at the cost of not being able to fully separate in all future states. The value of the non-pledgable asset under this strategy is essentially

$$W_{g,1} = \frac{Y_1}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}Y_1}{R^2(R-1)}. \tag{28}$$

The first term is the benefit of full separation. The second term accounts for the fact that good borrowers cannot separate if the collateral value rises in period 2 with probability $1-p$, then falls in period 3 with probability p.

Comparing equations (27) and (28), good borrowers prefer costly full separation to partial separation when

$$-(1-\pi_0)p\left(\frac{X_1\overline{x}}{R}-\frac{X_1\underline{x}}{R-1}\right) > -\frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}Y_1}{R^2(R-1)}, \tag{29}$$

which is equivalent to

$$\frac{\overline{x}}{\underline{x}} < \frac{R}{R-1}+\frac{(1-p)\overline{x}y}{R(R-1)} = z.$$

That is, good borrowers prefer costly full separation when the volatility of collateral is sufficiently low. As the interest cost of debt rises in volatility, the optimal strategy switches to partial separation at the point z. Note that this point does not depend on the mass of good borrowers π_0, which drops out of inequality (29). Even if costly full separation is preferred, it may not be feasible if the face value of debt must be so high that even good borrowers would want to default. Therefore, the boundary between Propositions 2 and 3 is the minimum of z (i.e., the point at which partial separation becomes preferred) and $d_{g,2}$ (i.e., the point at which full separation becomes infeasible).

5. Conclusion

Borrowers who hold assets generating cash flows that are not easily observable by others, can use debt dynamics to signal their holding of those assets. In particular borrowers would like to raise debt and then pay it back using the unobservable income, and in this way signal their higher repayment possibilities. We have developed a dynamic model of signaling through the use of debt, extending the related literature dominated by static models. We find that the effectiveness of using debt to signal non-observable assets critically hinges on the dynamic characteristics of observable assets that are used as collateral.

The model delivers testable implications that contrast with implications from static models. Using data of firms leverage (as a proxy of debt) and credit ratings (as a proxy of reputation) we show empirically that the data supports our predictions. First, deleveraging firms experience an increase in credit ratings. While these firms also experience a reduction in credit ratings when they leverage, this effect is not symmetric, with deleveraging having a stronger effect in the change of credit ratings. Second, firms with lower ratings are those that experience larger increases of credit ratings when deleveraging. Finally, when the volatility of a firm's collateral is low, then deleveraging induces a larger increase in credit ratings.

Our model has relevant implications for aggregate dynamics once we consider the evolution of systematic components in the value of assets used as collateral. During the financial crisis, we witnessed considerable deleveraging for both households and firms alike. A leading hypothesis is that the supply of bank credit dried up during the financial crisis, which caused households and firms to become financially constrained. This supply-driven channel is undoubtedly important, but the empirical evidence suggests that it might not be the entire story. An important part of deleveraging seems to be explained by the decline in the demand for debt by both households and firms alike (Brown, Haughwout, Lee and van der Klaauw 2013, Kahle and Stulz 2012).

A leading hypothesis for the decline in the demand for debt is precautionary saving. That is, households and firms effectively became more risk averse due to higher macroeconomic uncertainty. In this paper, we highlight another possibility: leverage cycles may play an important role in resolving adverse selection in credit markets. In a sense, deleveraging in our setting has the benefit of improving information flows in credit markets. During downturns private information about credit quality is revealed in equilibrium, as bad borrowers cannot pay down debt or are forced into delinquencies or default. We have shown that the revelation of information is likely to be more effective during periods in which collateral values decline, such as in periods of declines in housing prices, or in economic downturns accompanied by a decline in stock prices.

We show empirically, however, that these predictions are counterfactual (which is also shown using different approaches by Rajan and Zingales (1995), Fama and French (2002) and others).[7]

Our work offers an alternative theory of credit or leverage cycles in the aggregate economy – a period of aggregate leveraging followed by aggregate deleveraging or a wave of defaults – if we consider collateral assets with values that experience aggregate fluctuations (such as housing prices). Leverage cycles have been studied by Kiyotaki and Moore (1997) and Geanakoplos (2009) in very different settings. In Kiyotaki and Moore (1997), a bad productivity shock leads to deleveraging through the dynamic interaction between asset prices, collateral constraints, and investment. In Geanakoplos (2009), bad news lead to deleveraging through the dynamic interaction between asset prices, collateral constraints, and heterogeneous beliefs. In contrast to these previous papers, deleveraging in our model is not a consequence of binding collateral constraints. Instead, deleveraging is a consequence of optimal debt choice in an effort to resolve asymmetric information in credit markets. Moreover, the previous papers portray deleveraging as a negative feature of credit markets because it depresses asset prices and output. In contrast, our work highlights a positive side

[7]A complete survey of this empirical evidence is Klein, O'Brien and Peters (2002)

of deleveraging, through its role in resolving adverse selection frictions and speeding up the revelation of useful information about borrowers' quality.

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Appendix A. Proof of Lemma 3

We first show that inequality (25) holds in period 3. By equation (16), it suffices to show that $0 \leq \pi_3 - \widehat{\pi}_3 \leq 1$, which would imply that

$$0 \leq W_3 - \widehat{W}_3 \leq \frac{Y_3}{R-1}. \tag{A1}$$

If $F_2 \leq \min\{D_{b,3}, D_{g,3}\}$, all borrowers repay their debt. Therefore, equation (10) and Assumption 1 imply that $\pi_3 = \pi_2$ and $\widehat{\pi}_3 = 0$. If $F_2 > \max\{D_{b,3}, D_{g,3}\}$, all borrowers default. Therefore, equation (11) and Assumption 1 imply that $\pi_3 = 1$ and $\widehat{\pi}_3 = \pi_2$.

If $F_2 \in (\min\{D_{b,3}, D_{g,3}\}, \max\{D_{b,3}, D_{g,3}\}]$, we show that $D_{b,3} \leq D_{g,3}$ by contradiction. Suppose $D_{b,3} > D_{g,3}$. Equations (10) and (11) imply that $\pi_3 = 0$ and $\widehat{\pi}_3 = 1$. Equation (16) then implies that $D_{b,3} = D_{g,3} = X_3 + V_3 - \frac{Y_3}{R-1}$, which contradicts $D_{b,3} > D_{g,3}$. Therefore, $D_{b,3} \leq D_{g,3}$. In the event of full separation in period 3, $\pi_3 = 1$, $\widehat{\pi}_3 = 0$, and $W_3 - \widehat{W}_3 = \frac{Y_3}{R-1}$. Therefore, the default boundary (16) simplifies to

$$D_{i,3} = X_3 + V_3 + \mathbf{1}_g(i) Y_3. \tag{A2}$$

We now show that inequality (25) holds in period $t \in \{1, 2\}$. By equation (20), it suffices to show that

$$\max_{F_t} P_t F_t \geq V_t \tag{A3}$$

and

$$0 \leq W_{b,t} - \widehat{W}_{b,t} \leq W_{g,t} - \widehat{W}_{g,t} \leq \frac{Y_t}{R^{3-t}(R-1)}. \tag{A4}$$

Suppose inequalities (25) and (A4) hold in period $t+1$. The proof is by induction.

Equation (13) implies that

$$C_{g,t} - C_{b,t} = \Pr(F_t \in (D_{b,t+1}, D_{g,t+1}]) \mathbf{E}_t \left[\frac{F_t}{R} - \frac{X_{t+1}}{R-1} | F_t \in (D_{b,t+1}, D_{g,t+1}] \right] \geq 0, \tag{A5}$$

where the inequality follows from

$$\frac{F_t}{R} - \frac{X_{t+1}}{R-1} \geq 0 \iff F_t \geq \frac{R X_{t+1}}{R-1} = X_{t+1} + V_{t+1} \tag{A6}$$

and the induction hypothesis. Inequality (A3) then follows from $\max_{F_t} C_{b,t} = V_t$.

We now prove the first part of inequality (A4). Inequality (A5) implies that the first term in equation (21) is weakly positive for bad borrowers. The third term in equation (21) is weakly positive by the induction hypothesis. The numerator in the second term of equation

(21) can be rewritten as

$$\mathbf{E}_t[\widehat{\pi}_{t+1}Y_{t+1}] - \widehat{\pi}_t Y_t = \begin{cases} \pi_t \Pr(D_{g,t+1} < F_t)\mathbf{E}_t[Y_{t+1}|D_{g,t+1} < F_t] & \text{if } F_{t-1} \leq \min\{D_{b,t}, D_{g,t}\} \\ \Pr(D_{g,t+1} < F_t)\mathbf{E}_t[Y_{t+1}|D_{g,t+1} < F_t] & \text{if } F_{t-1} \in (D_{b,t}, D_{g,t}] \\ -Y_t & \text{if } F_{t-1} \in (D_{g,t}, D_{b,t}] \\ 0 & \text{if } F_{t-1} > \max\{D_{b,t}, D_{t,t}\} \end{cases}, \tag{A7}$$

which is weakly positive unless $D_{g,t} < F_{t-1} \leq D_{b,t}$. In this case, we show that $D_{b,t} \leq D_{g,t}$ by contradiction. Suppose $D_{b,t} > D_{g,t}$. Equations (9) and (11) imply that $\pi_t = 0$ and $\widehat{\pi}_t = 1$. Moreover, Lemma 2 implies that $D_{b,t+1} = D_{g,t+1}$ and $W_{i,t+1} = \widehat{W}_{i,t+1}$. Equations (20) and (A3) then imply that $D_{b,t} = D_{g,t} = X_t + V_t - \frac{Y_t}{R^{3-t}(R-1)}$, which contradicts $D_{b,t} > D_{g,t}$. Therefore, $D_{b,t} \leq D_{g,t}$.

We now prove the third part of inequality (A4). Inequality (A5) implies that the first term in equation (21) is weakly negative for good borrowers. If $F_{t-1} \leq D_{b,t}$, the sum of the second and third terms of equation (21) is less than or equal to

$$\begin{aligned} &\frac{\pi_t \Pr(D_{g,t+1} < F_t)\mathbf{E}_t[Y_{t+1}|D_{g,t+1} < F_t]}{R^{3-t}(R-1)} + \frac{\Pr(D_{g,t+1} \geq F_t)\mathbf{E}_t[Y_{t+1}|D_{g,t+1} \geq F_t]}{R^{3-t}(R-1)} \\ &< \frac{Y_t}{R^{3-t}(R-1)} \end{aligned} \tag{A8}$$

by equation (A7) and the induction hypothesis. If $F_{t-1} \in (D_{b,t}, D_{g,t}]$, the second term of equation (21) is less than or equal to $\frac{Y_t}{R^{3-t}(R-1)}$ by equation (A7), and the third term is zero by Lemma 2.

We now prove the second part of inequality (A4). Equations (21) and (A5) imply that

$$\begin{aligned} &W_{g,t} - \widehat{W}_{g,t} - \left(W_{b,t} - \widehat{W}_{b,t}\right) = \\ &\Pr(F_t \in (D_{b,t+1}, D_{g,t+1}])\mathbf{E}_t\left[\frac{Y_{t+1}}{R^{3-t}(R-1)} - \frac{F_t}{R} + \frac{X_{t+1}}{R-1}|F_t \in (D_{b,t+1}, D_{g,t+1}]\right] \\ &+ \frac{\Pr(D_{b,t+1} \geq F_t)\mathbf{E}_t\left[W_{g,t+1} - \widehat{W}_{g,t+1} - \left(W_{b,t+1} - \widehat{W}_{b,t+1}\right)|D_{b,t+1} \geq F_t\right]}{R}. \end{aligned} \tag{A9}$$

The first term is positive if

$$\frac{Y_{t+1}}{R^{3-t}(R-1)} - \frac{F_t}{R} + \frac{X_{t+1}}{R-1} \geq 0 \iff F_t \leq X_{t+1} + V_{t+1} + \frac{Y_{t+1}}{R^{2-t}(R-1)}, \tag{A10}$$

which holds by the induction hypothesis. The second term is also positive by the induction hypothesis.

In the event of full separation in period t, $\max_{F_t} P_t F_t = \max_{F_t} C_{i,t} = V_t$ and $W_{i,t} - \widehat{W}_{i,t} = \frac{\mathbf{1}_g(i)Y_t}{R^{3-t}(R-1)}$. Therefore, the default boundary (20) simplifies to

$$D_{i,t} = X_t + V_t + \mathbf{1}_g(i)Y_t. \tag{A11}$$

Appendix B. Lemmas Used to Prove Propositions 1 to 3

We let lowercase letters denote the corresponding variables divided by X_t. That is, $f_t = \frac{F_t}{X_t}$, $d_{i,t} = \frac{D_{i,t}}{X_t}$, $w_{i,t} = \frac{W_{i,t}}{X_t}$, and $c_{i,t} = \frac{C_{i,t}}{X_t}$.

Lemma 8. *If $f_1 \leq x_2$ in period 2, all borrowers borrow $f_2 = d_{b,3}\overline{x} + \varepsilon_2$, for arbitrarily small $\varepsilon_2 > 0$, at an interest rate $P_2^{-1} < R$ that satisfies*

$$P_2 f_2 = \begin{cases} \frac{1}{R-1} + \pi_1\left(\frac{\overline{x}-1}{R-1} + \frac{\varepsilon_2}{R}\right) & \text{if } \frac{\overline{x}}{\underline{x}} < \frac{d_{g,3}}{d_{b,3}} \\ \frac{1}{R-1} + \frac{\pi_1(1-p)\varepsilon_2}{R} & \text{if } \frac{\overline{x}}{\underline{x}} \geq \frac{d_{g,3}}{d_{b,3}} \end{cases}. \tag{B1}$$

The value of non-pledgable assets for good borrowers is

$$w_{g,2} = \begin{cases} -(1-\pi_1)\left(\frac{\overline{x}-1}{R-1} + \frac{\varepsilon_2}{R}\right) + \frac{y}{R-1} & \text{if } \frac{\overline{x}}{\underline{x}} < \frac{d_{g,3}}{d_{b,3}} \\ -\frac{(1-\pi_1)(1-p)\varepsilon_2}{R} + \frac{y}{R-1} - \frac{(1-\pi_1)p\underline{x}y}{R(R-1)} & \text{if } \frac{\overline{x}}{\underline{x}} \geq \frac{d_{g,3}}{d_{b,3}} \end{cases}. \tag{B2}$$

Proof. If $f_1 \leq x_2$, Lemma 4 implies no updating of reputation in period 2, so that $\pi_2 = \pi_1$. Good borrowers choose f_2 that maximizes $w_{g,2}$, and bad borrowers mimic the good.

In period $t \in \{1, 2\}$,

$$c_{i,t} = \begin{cases} \frac{1}{R-1} & \text{if } f_t > d_{i,t+1}\overline{x} \\ \frac{(1-p)f_t}{R} + \frac{p\underline{x}}{R-1} & \text{if } f_t \in (d_{i,t+1}\underline{x}, d_{i,t+1}\overline{x}] \\ \frac{f_t}{R} & \text{if } f_t \leq d_{i,t+1}\underline{x}. \end{cases}. \tag{B3}$$

If $d_{b,t+1}\overline{x} < d_{g,t+1}\underline{x}$,

$$c_{g,t} - c_{b,t} = \begin{cases} 0 & \text{if } f_t > d_{g,t+1}\overline{x} \\ (1-p)\left(\frac{f_t}{R} - \frac{\overline{x}}{R-1}\right) & \text{if } f_t \in (d_{g,t+1}\underline{x}, d_{g,t+1}\overline{x}] \\ \frac{f_t}{R} - \frac{1}{R-1} & \text{if } f_t \in (d_{b,t+1}\overline{x}, d_{g,t+1}\underline{x}] \\ p\left(\frac{f_t}{R} - \frac{\underline{x}}{R-1}\right) & \text{if } f_t \in (d_{b,t+1}\underline{x}, d_{b,t+1}\overline{x}] \\ 0 & \text{if } f_t \leq d_{b,t+1}\underline{x}. \end{cases}. \tag{B4}$$

Otherwise, if $d_{b,t+1}\overline{x} \geq d_{g,t+1}\underline{x}$,

$$c_{g,t} - c_{b,t} = \begin{cases} 0 & \text{if } f_t > d_{g,t+1}\overline{x} \\ (1-p)\left(\frac{f_t}{R} - \frac{\overline{x}}{R-1}\right) & \text{if } f_t \in (d_{b,t+1}\overline{x}, d_{g,t+1}\overline{x}] \\ 0 & \text{if } f_t \in (d_{g,t+1}\underline{x}, d_{b,t+1}\overline{x}] \\ p\left(\frac{f_t}{R} - \frac{\underline{x}}{R-1}\right) & \text{if } f_t \in (d_{b,t+1}\underline{x}, d_{g,t+1}\underline{x}] \\ 0 & \text{if } f_t \leq d_{b,t+1}\underline{x}. \end{cases}. \tag{B5}$$

We obtain equation (B1) by substituting $\pi_2 = \pi_1$ as well as equations (B4) and (B5) into equation (12).

If $d_{b,3}\overline{x} < d_{g,3}\underline{x}$, equations (8), (19) and (B4) imply that

$$w_{g,2} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_2)y}{R(R-1)} & \text{if (1) } f_2 > d_{g,3}\overline{x} \\ -(1-\pi_2)(1-p)\left(\frac{f_2}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_2)p\underline{x}y}{R(R-1)} & \text{if (2) } f_2 \in (d_{g,3}\underline{x}, d_{g,3}\overline{x}] \\ -(1-\pi_2)\left(\frac{f_2}{R} - \frac{1}{R-1}\right) + \frac{y}{R-1} & \text{if (3) } f_2 \in (d_{b,3}\overline{x}, d_{g,3}\underline{x}] \\ -(1-\pi_2)p\left(\frac{f_2}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_2)(1-p)\overline{x}y}{R(R-1)} & \text{if (4) } f_2 \in (d_{b,3}\underline{x}, d_{b,3}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_2)y}{R(R-1)} & \text{if (5) } f_2 \le d_{b,3}\underline{x}. \end{cases} \quad \text{(B6)}$$

Note that $w_{g,2}$ is decreasing in f_2 in regions (2), (3) and (4). In the other regions, $w_{g,2}$ is independent of f_2. Let $w_{g,2}(n)$ denote the maximized value of $w_{g,2}$ in region (n). $w_{g,2}(3)$ is greater than $w_{g,2}(2)$. $w_{g,2}(4)$ is greater than $w_{g,2}(1)$ and $w_{g,2}(5)$. $w_{g,2}(3)$ is greater than $w_{g,2}(4)$ when $(1-p)\overline{x} \ge 0.5$. Therefore, $w_{g,2}$ is maximized in region (3) when $f_2 = d_{b,3}\overline{x} + \varepsilon_2$.

If $d_{b,3}\overline{x} \ge d_{g,3}\underline{x}$, equations (8), (19) and (B5) imply that

$$w_{g,2} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_2)y}{R(R-1)} & \text{if (1) } f_2 > d_{g,3}\overline{x} \\ -(1-\pi_2)(1-p)\left(\frac{f_2}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_2)p\underline{x}y}{R(R-1)} & \text{if (2) } f_2 \in (d_{b,3}\overline{x}, d_{g,3}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_2)y}{R(R-1)} & \text{if (3) } f_2 \in (d_{g,3}\underline{x}, d_{b,3}\overline{x}] \\ -(1-\pi_2)p\left(\frac{f_2}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_2)(1-p)\overline{x}y}{R(R-1)} & \text{if (4) } f_2 \in (d_{b,3}\underline{x}, d_{g,3}\underline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_2)y}{R(R-1)} & \text{if (5) } f_2 \le d_{b,3}\underline{x}. \end{cases} \quad \text{(B7)}$$

Note that $w_{g,2}$ is decreasing in f_2 in regions (2) and (4). In the other regions, $w_{g,2}$ is independent of f_2. $w_{g,2}(4)$ is greater than $w_{g,2}(1)$, $w_{g,2}(3)$, and $w_{g,2}(5)$. $w_{g,2}(2)$ is greater than $w_{g,2}(4)$ when $(1-p)\overline{x} \ge 0.5$. Therefore, $w_{g,2}$ is maximized in region (2) when $f_2 = d_{b,3}\overline{x} + \varepsilon_2$. □

Lemma 9. *If $f_0 \le x_1$ in period 1, all borrowers borrow*

$$f_1 \in \begin{cases} (\overline{x}, d_{b,2}\underline{x}] & \text{if } \frac{\overline{x}}{\underline{x}} \in [1, d_{b,2}) \\ \overline{x} + \varepsilon_1 & \text{if } \frac{\overline{x}}{\underline{x}} \in [d_{b,2}, \min\{d_{g,2}, z\}) \\ (\underline{x}, d_{b,2}\underline{x}] & \text{if } \frac{\overline{x}}{\underline{x}} \ge \min\{d_{g,2}, z\} \end{cases},$$

for arbitrarily small $\varepsilon_1 > 0$, at an interest rate $P_1^{-1} < R$ that satisfies

$$P_1 f_1 = \begin{cases} \frac{f_1}{R} & \text{if } \frac{\overline{x}}{\underline{x}} \in [1, d_{b,2}) \\ \frac{(1-(1-\pi_0)p)f_1}{R} + \frac{(1-\pi_0)p\underline{x}}{R-1} & \text{if } \frac{\overline{x}}{\underline{x}} \in [d_{b,2}, \min\{d_{g,2}, z\}) \\ \frac{f_1}{R} & \text{if } \frac{\overline{x}}{\underline{x}} \ge \min\{d_{g,2}, z\}, \end{cases} . \quad \text{(B8)}$$

Proof. If $f_0 < x_1$, Lemma 4 implies no updating of reputation in period 1, so that $\pi_1 = \pi_0$. Good borrowers choose f_1 that maximizes $w_{g,1}$, and bad borrowers mimic the good.

We obtain equation (B8) by substituting $\pi_2 = \pi_1$ as well as equations (B4) and (B5) into equation (12).

In period $t \in \{1, 2\}$, Lemma 3 and Assumption 2 imply that the default boundaries satisfy the inequality

$$\frac{d_{g,t}}{d_{b,t}} \leq \frac{\frac{R}{R-1} + \frac{y}{R^{3-t}(R-1)}}{\frac{R}{R-1}} \leq \frac{R+y}{R} < \frac{R}{R-1} \leq d_{b,2}. \tag{B9}$$

In addition, Lemmas 5, 6, and 8 imply that

$$w_{g,2} = \begin{cases} \frac{y}{R-1} & \text{if } f_1 \in (x_2, d_{g,2}x_2] \\ w_{g,2}(2,3) < \frac{y}{R-1} & \text{if } f_1 \leq x_2 \end{cases}, \tag{B10}$$

where $w_{g,2}(2,3)$ denotes equation (B2).

If $\frac{\overline{x}}{\underline{x}} < \frac{d_{g,2}}{d_{b,2}}$, equations (8), (19), (B4), and (B10) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{g,2}\underline{x}, d_{g,2}\overline{x}] \\ -(1-\pi_0)\left(\frac{f_1}{R} - \frac{1}{R-1}\right) + \frac{y}{R-1} & \text{if (3) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\underline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} & \text{if (4) } f_1 \in (d_{b,2}\underline{x}, d_{b,2}\overline{x}] \\ \frac{y}{R-1} & \text{if (5) } f_1 \in (\overline{x}, d_{b,2}\underline{x}] \\ \frac{y}{R-1} - \frac{(1-p)\overline{x}}{R}\left(\frac{y}{R-1} - w_{g,2}(2,3)\right) & \text{if (6) } f_1 \in (\underline{x}, \overline{x}] \\ \frac{y}{R-1} - \frac{1}{R}\left(\frac{y}{R-1} - w_{g,2}(2,3)\right) & \text{if (7) } f_1 \leq \underline{x}. \end{cases} \tag{B11}$$

Note that $w_{g,1}$ is maximized in region (5) for any $f_1 \in (\overline{x}, d_{b,2}\underline{x}]$.

If $\frac{\overline{x}}{\underline{x}} \in [\frac{d_{g,2}}{d_{b,2}}, d_{b,2})$, equations (8), (19), (B5), and (B10) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (3) } f_1 \in (d_{g,2}\underline{x}, d_{b,2}\overline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} & \text{if (4) } f_1 \in (d_{b,2}\underline{x}, d_{g,2}\underline{x}] \\ \frac{y}{R-1} & \text{if (5) } f_1 \in (\overline{x}, d_{b,2}\underline{x}] \\ \frac{y}{R-1} - \frac{(1-p)\overline{x}}{R}\left(\frac{y}{R-1} - w_{g,2}(2,3)\right) & \text{if (6) } f_1 \in (\underline{x}, \overline{x}] \\ \frac{y}{R-1} - \frac{1}{R}\left(\frac{y}{R-1} - w_{g,2}(2,3)\right) & \text{if (7) } f_1 \leq \underline{x}. \end{cases} \tag{B12}$$

Note that $w_{g,1}$ is maximized in region (5) for any $f_1 \in (\overline{x}, d_{b,2}\underline{x}]$.

If $\frac{\overline{x}}{\underline{x}} \in [d_{b,2}, d_{g,2})$, equations (8), (19), (B5), and (B10) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (3) } f_1 \in (d_{g,2}\underline{x}, d_{b,2}\overline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} & \text{if (4) } f_1 \in (\overline{x}, d_{g,2}\underline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) & \\ -\frac{(1-\pi_0)(1-p)^2\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}y}{R^2(R-1)} & \text{if (5) } f_1 \in (d_{b,2}\underline{x}, \overline{x}] \\ -\frac{(1-\pi_0)(1-p)^2\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}y}{R^2(R-1)} & \text{if (6) } f_1 \in (\underline{x}, d_{b,2}\underline{x}] \\ -\frac{(1-\pi_0)(1-p)\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (7) } f_1 \leq \underline{x}. \end{cases} \quad \text{(B13)}$$

Note that $w_{g,1}$ is decreasing in f_1 in regions (2), (4) and (5). In the other regions, $w_{g,1}$ is independent of f_1. Let $w_{g,1}(n)$ denote the maximized value of $w_{g,1}$ in region (n). $w_{g,1}(3)$ is greater than $w_{g,1}(1)$ and $w_{g,1}(2)$. $w_{g,1}(6)$ is greater than $w_{g,1}(3)$, $w_{g,1}(5)$, and $w_{g,1}(7)$. Moreover, $w_{g,1}(4)$ is greater than $w_{g,1}(6)$ if and only if $\frac{\overline{x}}{\underline{x}} < z$. Therefore, $w_{g,1}$ is maximized in region (4) for $f_1 = \overline{x} + \varepsilon_1$ if $\frac{\overline{x}}{\underline{x}} < z$. Otherwise, $w_{g,1}$ is maximized in region (6) for any $f_1 \in (\underline{x}, d_{b,2}\underline{x}]$.

If $\frac{\overline{x}}{\underline{x}} \geq d_{g,2}$, equations (8), (19), (B5), and (B10) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (3) } f_1 \in (\overline{x}, d_{b,2}\overline{x}] \\ -\frac{(1-\pi_0)(1-p)^2\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1+(1-p)\overline{x})p\underline{x}y}{R^2(R-1)} & \text{if (4) } f_1 \in (d_{g,2}\underline{x}, \overline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) & \\ -\frac{(1-\pi_0)(1-p)^2\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}y}{R^2(R-1)} & \text{if (5) } f_1 \in (d_{b,2}\underline{x}, d_{g,2}\underline{x}] \\ -\frac{(1-\pi_0)(1-p)^2\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}p\underline{x}y}{R^2(R-1)} & \text{if (6) } f_1 \in (\underline{x}, d_{b,2}\underline{x}] \\ -\frac{(1-\pi_0)(1-p)\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (7) } f_1 \leq \underline{x}. \end{cases} \quad \text{(B14)}$$

Note that $w_{g,1}$ is decreasing in f_1 in regions (2) and (5). In the other regions, $w_{g,1}$ is independent of f_1. $w_{g,1}(3)$ is greater than $w_{g,1}(1)$ and $w_{g,1}(2)$. $w_{g,1}(6)$ is greater than $w_{g,1}(3)$, $w_{g,1}(4)$, $w_{g,1}(5)$, and $w_{g,1}(7)$. Therefore, $w_{g,1}$ is maximized in region (6) for any $f_1 \in (\underline{x}, d_{b,2}\underline{x}]$. □

Appendix C. Equilibrium for $(1-p)\overline{x} < 0.5$

When $(1-p)\overline{x} < 0.5$, the conclusions are essentially the same, except for two small differences. First, the point z at which partial separation becomes preferred to full separation takes a different expression. Second, in Proposition 3′, the optimal amount of debt when the collateral value falls in period 2 is lower than in Proposition 3. Therefore, if the collateral value falls in period 3, only the bad borrowers default, so that borrower type is fully revealed. If the collateral value rises instead, all borrowers repay their debt, so that private information is not revealed.

Proposition 2′. *Suppose* $(1-p)\overline{x} < 0.5$ *and the volatility of collateral is intermediate. That is,* $\frac{R}{R-1} \leq \frac{\overline{x}}{\underline{x}} < \min\{d_{g,2}, z'\}$*, where*

$$d_{g,2} = \frac{R}{R-1} + y, \tag{C1}$$

$$z' = \frac{R}{R-1} + \frac{((1-p)\overline{x})^2 y}{R(R-1)(1-(1-p)\overline{x})}. \tag{C2}$$

In period 1, all borrowers borrow $F_1 > X_1\overline{x}$ *at an interest rate* $P_1^{-1} > R$ *that satisfies*

$$P_1 F_1 = \frac{(1-(1-\pi_0)p)F_1}{R} + \frac{(1-\pi_0)pX_1\underline{x}}{R-1}. \tag{C3}$$

If the collateral value falls in period 2 (i.e., $x_2 = \underline{x}$*), only the bad borrowers default, so that borrower type is fully revealed. Good borrowers repay maturing debt by borrowing*

$$R\max\{0, F_1 - X_2 - Y_2\} \leq F_2 \leq RV_2.$$

If the collateral value rises in period 2 (i.e., $x_2 = \overline{x}$*), borrower type is fully revealed. Good borrowers repay maturing debt by borrowing* $F_2 = 0$*. Bad borrowers repay maturing debt by borrowing* $0 < F_2 \leq RV_2$*.*

Proof. Lemma 9′ below implies the equilibrium in period 1. Lemma 6 implies the equilibrium if the collateral value falls in period 2. Lemma 5 implies the equilibrium if the collateral value rises in period 2. □

Proposition 3′. *Suppose* $(1-p)\overline{x} < 0.5$ *and the volatility of collateral is high. That is,* $\frac{\overline{x}}{\underline{x}} \geq \min\{d_{g,2}, z'\}$*. In period 1, all borrowers borrow* $X_1\underline{x} < F_1 \leq RV_1\underline{x}$ *at the interest rate* $P_1^{-1} = R$*.*

If the collateral value falls in period 2 (i.e., $x_2 = \underline{x}$*), borrower type is fully revealed. Good borrowers repay maturing debt by borrowing*

$$R\max\{0, F_1 - X_2 - Y_2\} \leq F_2 < R(F_1 - X_2).$$

Bad borrowers repay maturing debt by borrowing

$$R(F_1 - X_2) \leq F_2 \leq RV_2.$$

If the collateral value rises in period 2 (i.e., $x_2 = \overline{x}$*), private information is not revealed. All borrowers repay maturing debt by borrowing* $F_2 > RV_2\underline{x}$ *at an interest rate* $P_2^{-1} > R$ *that satisfies*

$$P_2 F_2 = \frac{(1-(1-\pi_0)p)F_2}{R} + \frac{(1-\pi_0)pX_2\underline{x}}{R-1}. \tag{C4}$$

Subsequently, if the collateral value falls in period 3 (i.e., $x_3 = \underline{x}$*), only the bad borrowers* *default, so that borrower type is fully revealed. If the collateral value rises instead (i.e., $x_3 = \overline{x}$), all borrowers repay their debt, so that private information is not revealed.*

Proof. Lemma 9′ below implies the equilibrium in period 1. Lemma 5 implies the equilibrium if the collateral value falls in period 2. Lemma 8′ below implies the equilibrium if the collateral value rises in period 2. Lemma 6 implies the equilibrium if the collateral value subsequently falls in period 3. Lemma 4 implies the equilibrium if the collateral value rises instead in period 3. □

Lemma 8′. *Suppose $(1-p)\overline{x} < 0.5$ and $\frac{\overline{x}}{\underline{x}} \geq \frac{d_{g,3}}{d_{b,3}}$. If $f_1 \leq x_2$ in period 2, all borrowers borrow $f_2 = d_{b,3}\underline{x} + \varepsilon_2$, for arbitrarily small $\varepsilon_2 > 0$, at an interest rate $P_2^{-1} < R$ that satisfies*

$$P_2 f_2 = \frac{(1-(1-\pi_1)p)f_2}{R} + \frac{(1-\pi_1)p\underline{x}}{R-1}. \tag{C5}$$

The value of non-pledgable assets for good borrowers is

$$w_{g,2} = -\frac{(1-\pi_1)p\varepsilon_2}{R} + \frac{y}{R-1} - \frac{(1-\pi_1)(1-p)\overline{x}y}{R(R-1)}. \tag{C6}$$

Proof. The proof essentially follows that for Lemma 8. The only difference is that $w_{g,2}(4)$ is greater than $w_{g,2}(2)$ when $(1-p)\overline{x} < 0.5$. Therefore, $w_{g,2}$ is maximized in region (4) when $f_2 = d_{b,3}\underline{x} + \varepsilon_2$. □

Lemma 9′. *Suppose $(1-p)\overline{x} < 0.5$. If $f_0 < x_1$ in period 1, all borrowers borrow*

$$f_1 \in \begin{cases} (\overline{x}, d_{b,2}\underline{x}] & \text{if } \frac{\overline{x}}{\underline{x}} \in [1, d_{b,2}) \\ \overline{x} + \varepsilon_1 & \text{if } \frac{\overline{x}}{\underline{x}} \in [d_{b,2}, \min\{d_{g,2}, z'\}) \\ (\underline{x}, d_{b,2}\underline{x}] & \text{if } \frac{\overline{x}}{\underline{x}} \geq \min\{d_{g,2}, z'\} \end{cases},$$

for arbitrarily small $\varepsilon_1 > 0$, at an interest rate $P_1^{-1} < R$ that satisfies

$$P_1 f_1 = \begin{cases} \frac{f_1}{R} & \text{if } \frac{\overline{x}}{\underline{x}} \in [1, d_{b,2}) \\ \frac{(1-(1-\pi_0)p)f_1}{R} + \frac{(1-\pi_0)p\underline{x}}{R-1} & \text{if } \frac{\overline{x}}{\underline{x}} \in [d_{b,2}, \min\{d_{g,2}, z'\}) \\ \frac{f_1}{R} & \text{if } \frac{\overline{x}}{\underline{x}} \geq \min\{d_{g,2}, z'\} \end{cases}. \tag{C7}$$

Proof. The proof essentially follows that for Lemma 9.

Lemmas 5, 6, and 8′ imply that

$$w_{g,2} = \begin{cases} \frac{y}{R-1} & \text{if } f_1 \in (x_2, d_{g,2}x_2] \\ w_{g,2}(4) < \frac{y}{R-1} & \text{if } f_1 \leq x_2 \end{cases}, \tag{C8}$$

where $w_{g,2}(4)$ denotes equation (C6).

If $\frac{\overline{x}}{\underline{x}} < \frac{d_{g,2}}{d_{b,2}}$, $w_{g,1}$ is given by equation (B11) with $w_{g,2}(2,3)$ replaced by $w_{g,2}(4)$. Similarly, if $\frac{\overline{x}}{\underline{x}} \in [\frac{d_{g,2}}{d_{b,2}}, d_{b,2})$, $w_{g,1}$ is given by equation (B12) with $w_{g,2}(2,3)$ replaced by $w_{g,2}(4)$. In both cases, $w_{g,1}$ is maximized in region (5) for any $f_1 \in (\overline{x}, d_{b,2}\underline{x}]$.

If $\frac{\overline{x}}{\underline{x}} \in [d_{b,2}, d_{g,2})$, equations (8), (19), (B5), and (C8) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (3) } f_1 \in (d_{g,2}\underline{x}, d_{b,2}\overline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) + \frac{y}{R-1} & \text{if (4) } f_1 \in (\overline{x}, d_{g,2}\underline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) & \\ -\frac{(1-\pi_0)p(1-p)\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)^2\overline{x}^2 y}{R^2(R-1)} & \text{if (5) } f_1 \in (d_{b,2}\underline{x}, \overline{x}] \\ -\frac{(1-\pi_0)p(1-p)\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)^2\overline{x}^2 y}{R^2(R-1)} & \text{if (6) } f_1 \in (\underline{x}, d_{b,2}\underline{x}] \\ -\frac{(1-\pi_0)p\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}y}{R^2(R-1)} & \text{if (7) } f_1 \le \underline{x}. \end{cases} \quad . \tag{C9}$$

Note that $w_{g,1}$ is decreasing in f_1 in regions (2), (4) and (5). In the other regions, $w_{g,1}$ is independent of f_1. Let $w_{g,1}(n)$ denote the maximized value of $w_{g,1}$ in region (n). $w_{g,1}(3)$ is greater than $w_{g,1}(1)$ and $w_{g,1}(2)$. $w_{g,1}(6)$ is greater than $w_{g,1}(3)$, $w_{g,1}(5)$, and $w_{g,1}(7)$. $w_{g,1}(4)$ is greater than $w_{g,1}(6)$ if and only $\frac{\overline{x}}{\underline{x}} < z'$, where z' is given by equation (C2). Therefore, $w_{g,1}$ is maximized in region (4) for $f_1 = \overline{x} + \varepsilon_1$ if $\frac{\overline{x}}{\underline{x}} < z'$. Otherwise, $w_{g,1}$ is maximized in region (6) for any $f_1 \in (\underline{x}, d_{b,2}\underline{x}]$.

If $\frac{\overline{x}}{\underline{x}} \ge d_{g,2}$, equations (8), (19), (B5), and (C8) imply that

$$w_{g,1} = \begin{cases} \frac{y}{R-1} - \frac{(1-\pi_0)y}{R^2(R-1)} & \text{if (1) } f_1 > d_{g,2}\overline{x} \\ -(1-\pi_0)(1-p)\left(\frac{f_1}{R} - \frac{\overline{x}}{R-1}\right) + \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (2) } f_1 \in (d_{b,2}\overline{x}, d_{g,2}\overline{x}] \\ \frac{y}{R-1} - \frac{(1-\pi_0)p\underline{x}y}{R^2(R-1)} & \text{if (3) } f_1 \in (\overline{x}, d_{b,2}\overline{x}] \\ -\frac{(1-\pi_0)p(1-p)\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(p\underline{x}+(1-p)^2\overline{x}^2)y}{R^2(R-1)} & \text{if (4) } f_1 \in (d_{g,2}\underline{x}, \overline{x}] \\ -(1-\pi_0)p\left(\frac{f_1}{R} - \frac{\underline{x}}{R-1}\right) & \\ -\frac{(1-\pi_0)p(1-p)\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)^2\overline{x}^2 y}{R^2(R-1)} & \text{if (5) } f_1 \in (d_{b,2}\underline{x}, d_{g,2}\underline{x}] \\ -\frac{(1-\pi_0)p(1-p)\overline{x}\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)^2\overline{x}^2 y}{R^2(R-1)} & \text{if (6) } f_1 \in (\underline{x}, d_{b,2}\underline{x}] \\ -\frac{(1-\pi_0)p\varepsilon_2}{R^2} + \frac{y}{R-1} - \frac{(1-\pi_0)(1-p)\overline{x}y}{R^2(R-1)} & \text{if (7) } f_1 \le \underline{x} \end{cases} \quad . \tag{C10}$$

Note that $w_{g,1}$ is decreasing in f_1 in regions (2) and (5). In the other regions, $w_{g,1}$ is independent of f_1. $w_{g,1}(3)$ is greater than $w_{g,1}(1)$ and $w_{g,1}(2)$. $w_{g,1}(6)$ is greater than $w_{g,1}(3)$, $w_{g,1}(4)$, $w_{g,1}(5)$, and $w_{g,1}(7)$. Therefore, $w_{g,1}$ is maximized in region (6) for any $f_1 \in (\underline{x}, d_{b,2}\underline{x}]$.
□