# Terms of trade in Small Open Economies: the role of monopolistic output markets*

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**Abstract**

This paper revisits the role of terms of trade using a small open economy (SOE) model in which imports are inputs in production, output markets are imperfectly competitive and firms are connected in an input-output network. Otherwise, the model nests the standard SOE model commonly used in quantitative macro. Using this framework, this paper delivers the following results: (i) terms of trade shocks affect TFP in the same way as in the data, (ii) terms of trade shocks increase the volatility of consumption relative to that of output, and (iii) input-output linkages amplify the influence of terms of trade on the real economy. The model is calibrated to Mexico. Numerical experiments show that terms of trade shocks alone account for about *half* of observed TFP volatility and approximately $45$ percent of the observed output volatility. With respect to the excess volatility of consumption, terms of trade shocks imply a volatility of consumption that is $54$ percent larger than the volatility of output. Plugging productivity and terms of trade shocks into the model generates a consumption volatility that is $5$ percent more than that of output, close to the actual ratio of volatilities in the data.

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# 1 Introduction

Do terms of trade have a real effect on small open economies? If they do, what is the mechanism behind? Several studies have noticed that the terms of trade (ToT) - defined as the ratio of import prices to export prices - have a negative correlation with output and TFP and that larger terms of trade shocks play an important role in explaining larger business cycles, e.g. Mendoza (1995), Kose (2002), Easterly et. al (1993), Becker and Mauro (2005), Izquierdo et. al. (2008), Kehoe and Ruhl (2008). However, recent research has challenged some of these empirical findings by showing that standard macro models predict that changes in the ToT have no first order effect on TFP if output (GDP) is measured using chain-weighted methods, e.g. Kehoe and Ruhl (2008).

This paper revisits the aforementioned questions using a small open economy (SOE) model in which imports are inputs in production, output markets are imperfectly competitive and firms are connected in an input-output network. Otherwise, the model nests the standard SOE model commonly used in quantitative macro, e.g. Mendoza (1991). Using this framework, this paper delivers the following results:

1. ToT shocks affect *measured* TFP (Solow residual) in the same way as in the data.

2. ToT shocks increase the volatility of consumption relative to that of output.

3. Input-output linkages amplify the influence of ToT shocks on the real economy.

The main finding is that ToT affect negatively the aggregate output in this economy (GDP) through a term that resembles the conventional measure of TFP. To understand this result, first note that in the presence of intermediate inputs, output is really the total quantity of goods produced by firms net of the real opportunity cost of intermediate inputs. This notion of output corresponds to real value added and it essentially requires subtracting *real cost of imports* - i.e. cost of imports at constant ToT - from the *gross output*. Now, suppose that there is an (infinitesimal) increase in the ToT that causes a decrease in the use of imported inputs. As a result, for each unit of less imports, gross output falls by an amount equal to the marginal product of imported inputs (increasing real value added) and the real cost of imports falls by an amount equal to the constant ToT (decreasing value added). The net effect depends on whether or not the marginal product is equal to ToT in equilibrium. Monopolistic behavior distorts this equalization, i.e. the marginal product of imports is higher than the ToT. It follows that, even for small changes in the ToT, real value added (output) falls. Importantly, *such drop in output would be recorded as a drop in TFP*. This result holds for different statistical indexes of real value added, e.g. Divisia index, chain-weighted ideal Fisher index, Laspeyres index and Paasche index.

In the model the influence of ToT on TFP is summarized by an elasticity. The absolute magnitude of the elasticity of TFP to ToT is proportional to the excess price over marginal cost or *markup*. This occurs because in monopolistic competitive equilibria markups create a constant positive wedge between the marginal product of imports and the ToT. Hence, the greater is the markup, or the lower the elasticity of substitution among competing products, the greater is the influence of the ToT on TFP. The intuition works as follows. With markups firms under-produce and thus use a sub-optimal low level of imports. A ToT improvement partially undoes this under-production yielding additional *quantity* of value added. When the monopolistic distortion disappears, i.e. zero markups, the elasticity of TFP to the ToT collapses to zero. This knife-edge case corresponds to perfect competition which is a common assumption in standard business cycles models. Under perfect competition, profit maximization also optimizes real value added, which kills the first order effects of ToT. In other words, as a direct consequence of the envelope theorem, in a perfectly competitive environment the ToT effects on TFP are confined to be second order.

Other elements of the model determine the magnitude of the elasticity of TFP to the ToT. One element is the degree of exposure to imported inputs as measured by the share of imports on GDP. The intuition here is that markups act like a tax on imports and the share of imports to GDP is the tax base. Hence, the share of imports on GDP re-scales the distortion (which affects all production) into value added units. The interplay between the markup and the share of imports on GDP determines the magnitude of the *direct* effect of the ToT on TFP. If firms engage in inputs transactions to one another, then these interconnections will enlarge the magnitude the elasticity of TFP to the ToT. Specifically, each firm is *indirectly* affected by the ToT via those firms it supplies inputs to and those firms it buys inputs from. These indirect effects set in motion a sequence of feedback loops among firms that amplify aggregate shocks.

The last qualitative result is that ToT increase the volatility of consumption relative to that of output. The general idea is that consumption is really driven by income and not by output. Now, while output measures the quantity of final goods produced by domestic factors of production, real domestic income measures the purchasing power of households' income generated by those factors of production. This difference implies that real domestic income is more elastic to the ToT than output. Consequently, ToT volatility increases income volatility (relative to the volatility of output), and through it, the volatility of consumption (relative to the volatility of output). Yet, access to international capital markets bound the volatility of consumption below the volatility of income.

I *quantify* to what extent the aforementioned results matter in the data. To that end, I calibrate the model to Mexico and perform a series of numerical simulations. These simulations show that ToT shocks alone account for about *half* of the volatility of the Mexican TFP (Solow residual). The model also performs quite well for other moments of the TFP. For example, the in-sample predictions for TFP, obtained after plugging the observed sequence of ToT into the model, have a correlation of $0.64$ with TFP in the data. Taking into account the endogenous response of labor and capital, ToT shocks explain approximately $45$ percent of the actual output volatility in Mexico. With respect to the excess volatility of consumption, ToT shocks imply that consumption is $54$ more volatile than output. In contrast, with productivity shocks, consumption is $12$ percent less volatile than output. Plugging both shocks into the model generates a consumption volatility that is 5 percent more than that of output, close to the actual ratio of volatilities in the Mexican data.

### 1.1 Related literature

This paper is connected with a strand of quantitative macro literature studying the role of ToT within SOE RBC framework. One of the first attempts is Mendoza (1995), who extends the basic competitive SOE RBC model to include a competitive exporting sector. In his model ToT fluctuations have consequences on the aggregate economy because: (i) factors of production reallocate from non-exporting to the exporting sector (or viceversa) and (ii) a ToT improvement today forecasts an increase in productivity tomorrow[1] and, hence, encourages factor accumulation. Clearly, in this paper, I abstract from those channels. Instead, I assume that imports are used in the production of a final tradable (exportable) good, ToT shocks are independent from productivity shocks and all firms are affected equally by ToT shocks. With this framework, I highlight a different channel: one that goes from ToT to TFP.

Other papers have studied ToT fluctuations in models where imports are intermediate inputs in production, e.g. Kohli (2004), Kehoe and Ruhl (2008), and Feenstra et al. (2009). The main message of these papers is that ToT do not have a direct effect on output if the latter is measured using a chain-weighted method. In consequence, as emphasized by Kehoe and Ruhl, there is a envelope condition that guarantees that ToT have no first-order effects on TFP. A common theme in these analyses is the assumption of perfectly competitive markets. In this paper I relax this assumption given the empirical evidence in favor non-competitive markets, e.g. Broda and Weinstein (2006) and Hendel and Nevo (2006). I show that, under monopolistic (and hence distorted) competitive equilibria, ToT have first-order effects on TFP. This occurs because the monopolistic behavior of firms introduces a wedge between the marginal product of imports and the ToT, thus, breaking the envelope condition Kehoe and Ruhl refer to.

There are other papers analyzing the consequences of imperfect competition for TFP, e.g. Hall (1990) and Basu and Fernald (2002) among others. In particular, there exist some noteworthy coincidences between Basu and Fernald and my work. Specifically these authors show that imperfect competition introduces several non-technological factors into industry Solow residual. One of these factors arises from aggregate industrial output (real value added), which is computed as the difference between gross output and intermediate inputs valued at their purchase price, not their marginal product. Importantly, for this result to hold, a fraction of intermediate inputs must be produced outside the industry. That is exactly the insight I emphasized in this paper, except that intermediates inputs must be produced by another country.

A recent group of papers analyze the role nonconvexities for the effects of ToT on aggregate

[1]Specifically, Mendoza (1995) includes ToT shocks and productivity shocks that are jointly normal with a variance-covariance matrix calibrated to match the correlation between ToT and TFP in the data. Another paper including similar mechanisms is Kose (2002).

outcomes, e.g. Alessandria et al. (2010) and Gopinath and Neiman (2012). This research shows that nonconvexities in imports have important consequences not only for the level of trade, but also for the response of industry aggregates after ToT shocks. In particular, in an independent work, Gopinath and Neiman build a monopolistic competitive model similar to the one considered here except that firms must pay a fixed cost for each input variety they decide to import.[2] There are important similarities and differences between their work and mine. Both papers are similar in the sense the impact of the ToT on TFP arises due to monopolistic behavior by firms. Furthermore, Gopinath and Neiman's model features a richer trade adjustment after ToT shock which create an additional mechanisms through which ToT affect TFP and at the same time allows them to match a set of new micro-facts.[3] In contrast, I consider a much simpler trade adjustment pattern while I expand the analysis to other macro consequences of ToT shocks such as the response of investment, consumption, labor and aggregate output.

This paper also contributes to the study of ToT effects on other macro variables. In the context of perfect competition, Kohli (2004) shows that real GDP tends to underestimate the increase (decrease) in real income and welfare when the ToT improve (deteriorate). For example, from a balance-trade position, an improvement in the ToT implies that the same amount of exports can produce more imports. As a consequence, real income and welfare rise directly from that effect. In contrast, the real GDP, which focuses on production, subtracts this direct price effect. I show that this also true for the case of imperfect competition, i.e. real income responds more forcefully than GDP. Moreover, I connect this result with the so-called *excess volatility of consumption puzzle*, which as a salient feature of SOE business cycles, specially in emerging economies, e.g. Neumeyer and Perri (2005). Aguiar and Gopinath (2007) provide an explanation to this phenomenon that is based permanent shocks to the growth rate of TFP. The key idea of their argument lies on the permanent income hypothesis, namely, consumption responds more to the permanent component of income than to the transitory one.[4] To some extent, ToT shocks induce a similar result with the difference that the excess volatility of consumption arises from a difference between income and output.

My work is connected with a literature studying the role of intermediate inputs in macroeconomics, e.g. Basu (1995), Jones (2011), Acemoglu et. al. (2011) among others. Two lessons are derived from this literature. First, input-output linkages amplify disturbances in the econ-

[2] Gopinath and Neiman (2012) show that the size of the fixed cost and the ToT shock determine the adjustment in several margins: (i) the number of imported varieties, (ii) the number of importing firms and (iii) who import and who not (selection effect). These margins are important part of the trade adjustment pattern observed in the aftermath of 2001 Argentine crisis.

[3] For instance, in their model firm level import shares affect the level of productivity. Moreover, because a ToT shocks change the number of imported varieties, statistical import price index may differ from the ideal price index. This mismeasurement introduces an artificial ToT effect on TFP, see Feenstra et al (2009).

[4] Other common explanations to the excess volatility of consumption involve essentially shocks to the real interest rates that not only affect the stochastic discount factor but also affect output through financial constraints, e.g. Neumeyer and Perri (2005).

omy, e.g. Jones (2011). Second, the architecture of the input-output matrix matters for aggregate volatility, e.g. Acemoglu et. al. (2011). Following Basu, in my model all domestically produced goods can serve either as final outputs or as inputs for the production of other goods. I modify this structure by adding imported intermediate inputs, which are supplied elastically (at a given price) by an external sector.[5] Moreover, I have assumed that the domestic technology of production depends equally on imported inputs. This implies that the external sector plays the role of a general-purpose technology. ToT shocks can be interpreted as shocks to this general-purpose technology and the amplification occurs downstream as all firms using imports are interconnected to each other.

The rest of the paper is organized as follows. Section 2 focuses on establishing the first result, i.e. when output markets are imperfectly competitive, variations in input prices have real effects in constant-price measures of economic activity. To that end, I first consider an axiomatic analysis with very slack restrictions on technology and preferences. Then I analyze a very simple static SOE model in which imports are used as inputs and firms sell their output subject to a CES demand system. To simplify the analysis, I assume that physical productivity is constant, the supply of domestic inputs is exogenous, production does not require intermediate domestic inputs, and there is balance trade. I use this basic framework to show the implications of imperfect competition for aggregate output and TFP. Section 3 outlines a more general version which relaxes all aforementioned assumptions. This section also discusses other consequences of ToT fluctuations and presents the quantitative results. Section 4 concludes.

## 2 Preliminary analysis

This section analyzes the effects of changes in input prices on constant-price based measures of economic activity. It shows that when firms are not price takers in their output markets, (small) changes in input prices affect measured economic activity in a particular way. First, I illustrate this result using an axiomatic analysis that shows that small changes in input prices do affect profits at constant prices under imperfect output markets. Specifically, a small increase (decrease) in the price of an input, decreases (increases) profits at constant prices. Then, I show how this result creates a connection between aggregate output and the ToT in a simple small open economy framework. In particular, output is a negative function of the ToT. Finally, I argue that the negative link between output and the ToT appears even after controlling for other factor of production and hence it manifests through measures of TFP like the Solow residual.

### 2.1 Axiomatic analysis

In this subsection I use an axiomatic analysis to quantify the effect of a change in input prices on profits measured at constant prices, henceforth *real* profits. I focus on two opposite cases, one in

[5] Input-output linkages are also included in Gopinath and Neiman (2012).

which the producer is price taker and another in which the producer has *monopolistic power* in the output markets. Under the first case, an envelope condition guarantees that changes in prices do not have any direct effect on real profits. In contrast, under monopolistic power, the same envelope condition does hold and this causes changes in input prices to have a direct effect on real profits.

The setup is quite simple. Consider a production process where the set of goods that can be output is *distinct* from the set of goods that can be inputs. I will restrict to two goods: $y$ and $m$, where the former is the output and the latter is the input. The production vector is $\mathcal{Z} = (m, y)' \in \mathbb{R}^2$. The production function is represented by continuous differentiable function $y = f(m)$ with $f' > 0$ and $f'' < 0$. The production set then is defined as $\Omega = \{(y, m) : y - f(m) \leq 0 \text{ and } m \geq 0\}$. For each good $j = \{y, m\}$ there is positive price $P_j$. Prices are summarized by a price vector $\mathcal{P} = (p_y, p_m)' \gg 0$.

For a given price vector $\mathcal{P} \gg 0$ and production vector $\mathcal{Z} \in \mathbb{R}^2$, profits are defined as

$$\pi = \mathcal{P}.\mathcal{Z} = p_y y - p_m m \; .$$

**Perfect competition** I begin with the case of perfect competition that implies that prices $(p_y, p_m)$ are independent of the production plan $(y, \; m)$.[6] Given the technological constraints and the prices $(p_y, p_m)$, profit maximization problem is then,

$$\max_{y,m} p_y y - p_m m \; ,$$

$$\text{s.t} : (y, \; m) \in \; \Omega \; .$$

Hence, for a production set $\Omega$, there is profit function $\Pi(p_y, p_m)$ defined for every price vector $(p_y, p_m)$:

$$\pi(p_y, p_m) = \max_{y,m} \{p_y y - p_m m : (y, \; m) \in \; \Omega \; \}$$

A profit maximizer production plan for every price vector $(p_y, p_m)$ is given by,

$$(y(p_y, p_m) \; , \; m(p_y, p_m)) = \arg\max_{y,m} \{p_y y - p_m m : (y, \; m) \in \; \Omega \; \} \; .$$

Optimality can be also characterized by the first order condition for $m$:

$$f'(m(p_y, p_m)) = \frac{p_m}{p_y},$$

which states that at the optimal production plan the marginal product equals the producer's price of inputs.

Suppose that the input price change from $p_m^0$ in date $0$ to $p_m^1$ in date $1$. Normalizing $p_y$ to one in both dates implies that change in $p_m$ corresponds to a change in the producer's *ToT*.

[6]See Kohli (2004) for a similar exposition.

First, note that, by the envelope theorem, an increase in the input price reduces the *value* of profits in units of output, i.e.

$$\frac{\partial \pi\left(p_y, p_m\right)}{\partial p_m}=-m<0, \tag{1}$$

which is not surprising since the producer can only produce less for the same amount of inputs.

What is not so obvious is whether or not real profits (*quantity* of profits) changes when the in the input prices. The difficulty arises because real profits are not defined as a physical object like output or inputs. This makes the functional form for real profits in general *unknown*. To address this problem I compute profits at constant prices, i.e. output and inputs are valued at prices of a specific reference date.

I begin by defining a profit function for every price vector $(p_y, p_m)$ and a feasible production plan $(y,\ m)$ :

$$\pi'\left(y, m; p_y, p_m\right)=\left\{\ p_y y-p_m m:(y,\ m) \in\ \Omega\ \right\}\ .$$

Clearly, $\pi'\left(y, m; p_y, p_m\right) \leq \pi\left(p_y, p_m\right)$ with equality if, and only if, $y=y\left(p_y, p_m\right)$ and $m=m\left(p_y, p_m\right)$. In other words, since the value of profits is maximized at $y\left(p_y, p_m\right)$ and $m\left(p_y, p_m\right)$, any other feasible production plan delivers lower profits. Given $p_m^0$ in date $0$ and $p_m^1$ in date $1$, there are two optimal production plans, one for each input price. Under date $0$ prices, the optimal production plan is: $y^0\ \equiv\ y\left(1, p_m^0\right)$ and $m^0\ \equiv\ m\left(1, p_m^0\right)$. Under date $1$ prices the optimal production plan is: $y^1 \equiv y\left(1, p_m^1\right)$ and $m^1 \equiv m\left(1, p_m^1\right)$.

The first approximation is profits valued at date $0$ prices. For reasons that will be clear later, I will refer to these profits as *Laspeyres* profits. Profit maximization at date $0$ implies,

$$y^1-p_m^0 m^1 \leq y^0-p_m^0 m^0, \tag{2}$$

where the left hand side are the Laspeyres profits at date 1 and the right hand side of the inequality are the profits at date 0. Alternatively, one can write (2) as:

$$\pi'\left(y^1, m^1; p_m^0\right) \leq \pi\left(p_m^0\right),$$

which means that Laspeyres profits decline after a fall in input prices, which is exactly the opposite what happens with the value of profits, see equation 1. The logic is quite simple. Compared to the previous date profits, date $1$ production plan is always suboptimal at date $0$ prices. Note that this result implies that the Laspeyres quantity index of real profits is at most one,

$$\mathbb{I}_L\left(p_m^0\right) \equiv \frac{y^1-p_m^0 m^1}{y^0-p_m^0 m^0} \leq 1.$$

What if use a different reference date like date 1? I will refer to real profits at date 1 prices as *Paasche* profits. Next I show that in this case real profits increase. Profit maximization at date $1$ implies,

$$\pi'\left(y^0, m^0; p_m^1\right) \leq \pi\left(p_m^1\right), \tag{3}$$

i.e. change in Paasche profits is positive. Again the logic is quite simple: date 1 production plan is always better than any other feasible plan, e.g. date 0 plan. This implies that the Paasche quantity index of real profits is at least one,

$$\mathbb{I}_P\left(p_m^1\right) \equiv \frac{y^1 - p_m^1 m^1}{y^0 - p_m^1 m^0} \geq 1,$$

Figure 1 illustrates what I have just discussed. Output is measured in the vertical positive axis and inputs in the horizontal axis. The production set $\Omega$ is the shaded area. At date $0$, prices are given by $p_m^0$ and $p_y^0 = 1$. The line $\ell_0$ has a slope equal to $p_m^0$. The tangency point between $\ell_0$ and $\Omega$ determine the optimal production plan at date $0$ prices, denoted by the point $\mathcal{Z}^0 = \left(m^0, y^0\right)$. This tangency is the point in which the slope of the production function, i.e. marginal product of $m$, equals $p_m^0$. The intercept of line $\ell_0$ with the vertical axis are the maximized profits, $\pi_0$. Now, suppose that the input price decreases from $p_m^0$ in date $0$ to $p_m^1$ in date $1$. At date $1$, prices are given by $p_m^1$, which is represented by the slope of line $\ell_1$. In this case, the optimal production plan is $\mathcal{Z}^1$ and profits are $\pi_1$. Comparing these two situations, it is clear then that profits at current prices increase when the input price falls. If profits are evaluated at constant prices this comparison changes. Date 1 Laspeyres profits are given by the vertical intercept of $\ell_0'$, denoted by $\pi_1'$.[7] Note that $\pi_1' < \pi_0$, i.e. real profits at date $0$ prices decline. Also, the Laspeyres index, the ratio of $\pi_0$ to $\pi_1'$, is less than one. Applying the same idea, date $0$ Paasche profits are given by $\pi_0' < \pi_1$, i.e. real profits at date $1$ prices augment. The Paasche index, the ratio of $\pi_1$ to $\pi_0'$, is greater than one.

The above analysis indicates that in general profits at constant prices, Laspeyres or Paasche, are affected by changes in the input price. This occurs only because current prices differ from prices at the reference date. Proposition 1 shows that, at the reference date prices, infinitesimal changes in input prices have no effect on profits at constant prices. This follows from a envelope condition. At reference date, the only effect that an infinitesimal change in the input price has on profits is through changes in output and inputs. These changes cancel each other out if, and only if, prices are exactly equal to the reference date price.

**Proposition 1** *Suppose, $p_m^1 < p_m^0$, then the derivative of Laspeyres profits with respect to $p_m$ at these two prices is:*

$$\begin{aligned} \frac{d\pi^0}{dp_m}\Big|_{p_m = p_m^0} &= 0, \\ \frac{d\pi^0}{dp_m}\Big|_{p_m = p_m^1} &= \frac{dm}{dp_m}\Big|_{p_m = p_m^1} \times \left(p_m^1 - p_m^0\right) < 0, \end{aligned}$$

[7] Line $\ell_0'$ is parallel line to $\ell_0$. Note that $\ell_0'$ must cross the production set at $\mathcal{Z}^1$. This means that date $0$ prices, date $1$ optimal plan is suboptimal.

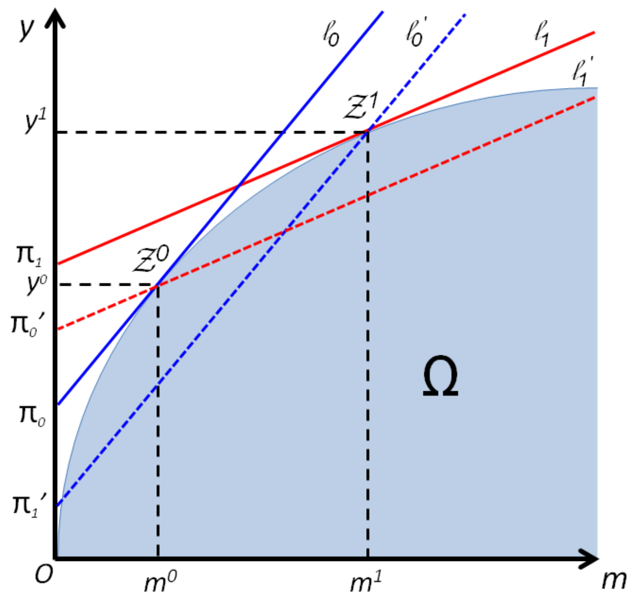


Figure 1: Effect of a decline in price of inputs $p_m$ under perfect competition.

*and the derivative of Paasche profits with respect to $p_m$ at these two prices is:*

$$\begin{aligned} \frac{d\pi^1}{dp_m}|_{p_m=p_m^1} &= 0, \\ \frac{d\pi^1}{dp_m}|_{p_m=p_m^0} &= \frac{dm}{dp_m}|_{p_m=p_m^0} \times \left(p_m - p_m^1\right) > 0. \end{aligned}$$

*where,*

$$\frac{dm}{dp_m} = \frac{1}{f''(m)} < 0$$

**Proof.** See Appendix. ■

In general, Laspeyres or Paasche indexes provide conflicting answers if current prices are different from those at the reference date. This biases can be reduced if one uses a superlative index such as the Fisher index, defined as geometric weighted average of the Laspeyres and Paasche indexes, i.e. $\mathbb{I}_F\left(p_m^0, p_m^1\right) = \sqrt{\mathbb{I}_L\left(p_m^0\right)\mathbb{I}_P\left(p_m^1\right)}$. The next proposition shows that infinitesimal changes in input prices have no effect on the real profits derived from a Fisher formula, denoted by $\pi^f$. This is a direct consequence of Proposition 1 and the fact that the Fisher Index lies between Laspeyres and Paasche Index. It follows that if the reference date is continuously updated (chain-weighted), $\pi^f$ will be invariant to infinitesimal changes in input prices.

**Proposition 2** *As $p_m^1 \to p_m^0$, the derivative of Fisher profits with respect to $p_m$ is zero.*

$$\frac{d\pi^f}{dp_m} = 0 \,.$$

**Proof.** See Appendix. ■

**Imperfect competition** Suppose the producer has monopolistic power in the output markets. In particular, assume a downward sloping inverse demand function: $p_y = \mathcal{D}(y)$, with $\mathcal{D}'(\cdot) < 0$. There is perfect competition in the input markets, i.e. $p_m$ is taken as given.

Given the technological constraints, *the demand function* $\mathcal{D}(y)$, and the price $p_m$, profit maximization problem is then,

$$\max_{y,m} \mathcal{D}(y)\, y - p_m m \,,$$
$$\text{s.t} : (y,\ m) \in\ \Omega \,.$$

The profit function $\pi(p_m)$ defined for every price $p_m$:

$$\pi(p_m) = \max_{y,m} \{\mathcal{D}(y)\, y - p_m m : (y,\ m) \in\ \Omega\ \} \ .$$

A profit maximizer production plan for every price $p_m$ is given by,

$$(y(p_m)\,,\ m(p_m)) = \arg\max_{y,m} \{\mathcal{D}(y)\, y - p_m m : (y,\ m) \in\ \Omega\ \}\,,$$

and the optimal price plan is $p_y(y(p_m)) = \mathcal{D}(y(p_m))$. The optimality condition can be summarized in the first order condition for $m$ :

$$f'(m) = \frac{|\varepsilon|}{|\varepsilon| - 1} \frac{p_m}{p_y} \text{ with } |\varepsilon| \equiv -\frac{\mathcal{D}(y)}{y} \frac{1}{\mathcal{D}'(y)}, \tag{4}$$

where $|\varepsilon|$ is the absolute value of the demand elasticity. Note that the f.o.c is distorted relative to the f.o.c under perfect competition. In particular, the marginal product of inputs is higher than the producer's input price. This distortion disappears when the demand becomes infinitely elastic, $|\varepsilon| \to +\infty$.

In addition, define a profit function for every price $(p_m)$ and a feasible production plan $(y,\ m)$ :

$$\pi'(y, m; p_m) = \{\mathcal{D}(y)\, y - p_m m : (y,\ m) \in\ \Omega\} \ .$$

Clearly, $\pi'(y, m; p_m) \leq \pi(p_m)$ with equality if, and only if, $y = y(p_m)$ and $m = m(p_m)$. In other words, since the value of profits is maximized at $y(p_m)$ and $m(p_m)$, any other production plan delivers lower profits.

Now suppose that the input price change from $p_m^0$ in date $0$ to $p_m^1$ in date $1$. At date $0$, the optimal plan is given by $y^0 \equiv y\left(p_m^0\right)$ and $m^0 \equiv m\left(p_m^0\right)$. Similarly, at date $1$, the optimal plan is given by $y^1 \equiv y\left(p_m^1\right)$ and $m^1 \equiv m\left(p_m^1\right)$. At date $0$ prices, profit maximization implies,

$$p\left(y^1\right) y^1 - p_m^0 m^1 \leq p\left(y^0\right) y^0 - p_m^0 m^0 \,.$$

Similarly, profit maximization at date 1 implies,

$$p\left(y^0\right)y^0 - p_m^1 m^0 \leq p\left(y^1\right)y^1 - p_m^1 m^1 \ .$$

Note that, in contrast with the perfect competition case, profit maximization does not tell directly the direction of change in profits at constant prices. This is because the price of output is no longer independent of the production plan. Let me illustrate this point with an example in which profits at constant prices, Laspeyres or Paasche, increase after a reduction in the price of the input. Recall that under perfect competition, Laspeyres profits do not increase while Paasche profits do not decrease.

Figure 2 plots what happens when the input price decreases from $p_m^0$ in date 0 to $p_m^1$. The line $\ell_0$ has a slope equal to $p_m^0$. At this price, the optimal plan is $\mathcal{Z}^0 = \left(m^0, y^0\right)$. Note that at $\mathcal{Z}^0$, the marginal product of inputs, the slope of the production function, is higher than the slope of $\ell_0$, i.e. $\ell_0$ crosses the production set. This implies that at constant prices, there are other production plans better than $\mathcal{Z}^0$. The intercept of line $\ell_0$ with the vertical axis are the maximum profits at date 0, $\pi_0$. When the price of the input decreases to $p_m^1$, the producer maximizes profits at point $\mathcal{Z}^1$, which is also a point in which the marginal product of the input is higher than its price. Date 1 profits are $\pi_1$ clearly greater than $\pi_0$. Now, let's analyze what happens if profits are evaluated at constant prices. Begin with the case of Laspeyres profits. To calculate date 1 Laspeyres profits, one only needs to find a parallel line to $\ell_0$ that crosses $\mathcal{Z}^1$. In this example, I assumed that at $\mathcal{Z}^1$ the line $\ell_0'$ is exactly tangent to the production set, i.e. at date 0 prices there is no other production plan better than $\mathcal{Z}^1$. Hence, Laspeyres profits increase from $\pi_0$ to $\pi_1'$. On the other hand, date 0 Paasche profits are given by $\pi_0' < \pi_1$, i.e. Paasche profits also increase.

Hence, different from the case of perfect competition, the previous example shows that a decrease in the input price increases both Laspeyres or Paasche profits. It follows that Fisher profits will also increase. The question now is whether this effect persists for infinitesimal changes in the input price. Proposition 3 shows that, *at the reference date*, infinitesimal changes in input prices have an impact on profits at constant prices, Laspeyres, Paasche or Fisher.

**Proposition 3** *At $p_m = p_m^0$, the derivative of Laspeyres profits with respect to $p_m$ is.*

$$\frac{d\pi^0}{dp_m}|_{p_m=p_m^0} = \frac{dm}{dp_m}|_{p_m=p_m^0} \times \frac{1}{|\varepsilon|-1}p_m^0 < 0 \ .$$

*At $p_m = p_m^1$, the derivative of Paasche profits with respect to $p_m$ is,*

$$\frac{d\pi^1}{dp_m}|_{p_m=p_m^1} = \frac{dm}{dp_m}|_{p_m=p_m^1} \times \frac{1}{|\varepsilon|-1}p_m^1 < 0 \ .$$

*As $p_m^1 \to p_m^0$, the derivative of Fisher profits with respect to $p_m$ is,*

$$\frac{d\pi^f}{dp_m} = \frac{dm}{dp_m} \times \frac{1}{|\varepsilon|-1}p_m < 0 \ .$$

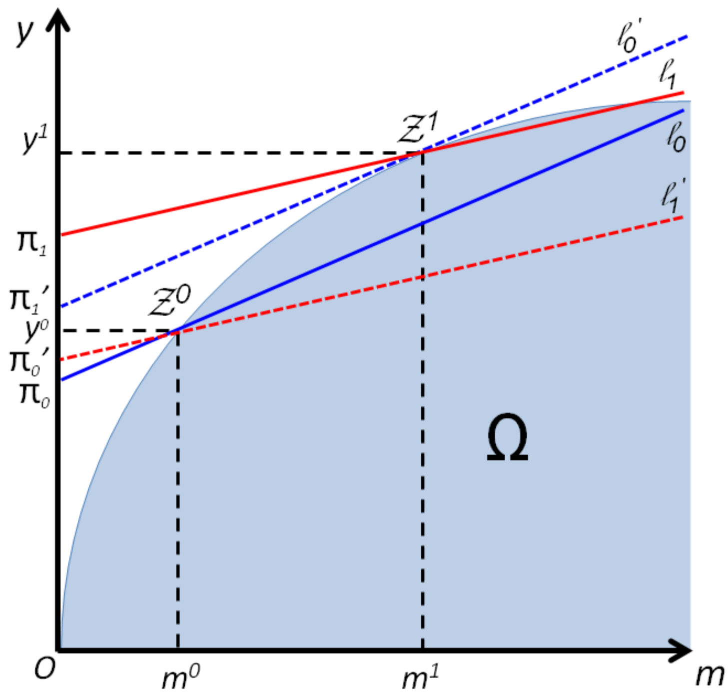


Figure 2: Effect of a decline in price of inputs $p_m$ under imperfect competition.

**Proof.** See Appendix. ■

What explains the results of shown in Proposition 3? The answer to this question is in equation $(4)$, which says that the marginal product of inputs is above the producer's input price under imperfect competition, i.e. $|\varepsilon| < +\infty$. Hence, small variations in the input price have greater effects on output than on the cost of intermediate inputs at constant prices. This gap implies that changes in the input price have an impact over real profits. Now, as the monopolistic distortion vanishes, i.e. $|\varepsilon| \to +\infty$, the gap in the first order condition disappears as well as effect of input prices on real profits.

## 2.2 Simple open economy

This subsection discusses the effects of the ToT in an simple small open economy model. I show that when output markets are monopolistic competitive *aggregate output is a negative function of the ToT*. The logic for this result is the same as the one discussed in the previous section. I also discuss how ToT fluctuations drive the wedge between output and real domestic income, or the purchasing power generated by domestic production. Finally, I argue that the negative effect of ToT fluctuations in output manifests through changes in conventional measures of TFP.

The model represents a simple small open economy in which production of a continuum of imperfect substitutable goods requires both domestic inputs and imported inputs. Each firm uses this technology to produce one variety which is sold in a monopolistically competitive market. Factor markets are perfect. To simplify the analysis, assume that physical productivity is constant, the supply of domestic inputs is exogenous, production does not require intermediate domestic inputs, and there is balance trade. Except for the assumption of monopolistic competitive markets, this model is the same as the one analyzed by Kehoe and Ruhl (2008). The next section outlines a more general version which relaxes all aforementioned assumptions.

Suppose there is a continuum of imperfect substitutable goods indexed by $i \in [0, 1]$. There is a representative firm which uses all imperfect substitutable goods as inputs to assemble a tradable final good $Y$ using the following constant returns to scale technology:

$$Y = \left( \int_0^1 Y_i^{\theta} di \right)^{\frac{1}{\theta}} ,$$

where $\theta$ controls the elasticity of substitution among inputs, i.e. $1/(1-\theta)$. Equilibrium requires this elasticity to be greater than one, i.e. $\theta < 1$.

The final good producer maximize profits taking prices as given.

$$\max_{Y_i} PY - \int_0^1 P_i Y_i di$$

The first order condition for good $i$:

$$Y_i = \left( \frac{P}{P_i} \right)^{\frac{1}{1-\theta}} Y, \tag{5}$$

where

$$P = \left( \int_0^1 P_i^{\frac{\theta}{\theta-1}} di \right)^{\frac{\theta-1}{\theta}}$$

Each firm produces one variety $i$. All firms have the same production function $F$ which depends on domestic inputs and imported inputs,

$$Y_i = F(V_i, M_i), \tag{6}$$

where $V_i$ is composite of domestic inputs (e.g. capital and labor), and $M_i$ are imports. Imports have a specific meaning in this model. These are goods produced outside the economy and combined with domestically produced inputs in order to produce a domestic tradable final good. This capture the idea that imports really consist of raw materials, intermediate goods, or finished products which have to be mixed with domestic factors before they meet their final demand.

I assume that the production function $F(\cdot)$ is concave, continuously differentiable, and linear homogenous (*constant returns to scale)*. By linear homogeneity,

$$Y = F_V V + F_M M, \tag{7}$$

and:

$$\frac{\mathrm{d}Y_i}{Y_i} = \frac{F_V V_i}{F}\frac{\mathrm{d}V_i}{V_i} + \frac{F_M M_i}{F}\frac{\mathrm{d}M_i}{M_i}. \tag{8}$$

In addition, I assume that $F(\cdot)$ is *weak separable* on imports, see Sims (1969) and Sato (1976). This assumption ensures that the marginal rate of technical substitution between any pair of domestic inputs is independent of imported inputs $M$.

A monopolistically competitive firm chooses $M_i$ and $V_i$ to solve:

$$\Pi_i = \max_{M_i, V_i} P_i F(V_i, M_i) - P_V V_i - P_M M_i, \text{ s.t : } (5) \text{ and } (6),$$

where the price of imports is $P_M$ and the price of domestic inputs is $P_V$.

Before presenting the first order conditions of this problem, let me explain how monopolistic competition distorts firms' decisions. Plugging $(5)$, the firm problem can be rewritten as:

$$\Pi_i = \max_{M_i, V_i} PY^{1-\theta} Y_i^{\theta} - P_V V_i - P_M M_i, \text{ s.t : } (6).$$

Note that under monopolistic competition firms act as if there were subject to decreasing returns to scale, i.e. $\theta < 1$, even when their technology has constant returns to scale. Notice also that the output decisions of all other firms affect firm $i$ profits through its demand, $Y$. These features have clear consequences for production. In particular, in the monopolistically competitive equilibrium, each producer, given others' output decisions, has no incentive to increase its output.[8] Obviously, underproduction translates into factor demands. To see this, consider the f.o.c,

$$F_M(V_i, M_i) = \frac{1}{\theta}\frac{P_M}{P_i} \tag{9}$$

$$F_V(V_i, M_i) = \frac{1}{\theta}\frac{P_V}{P_i} \tag{10}$$

Note that, since $\theta < 1$, at the optimum the marginal product of all inputs is greater than their relative prices. This implies that, given prices, firms operate with too little of inputs.

The rest of the economy is completed as follows. The final good can be either consumed $(C)$ or exported $(X)$:

$$Y = C + X, \tag{11}$$

The production of imported inputs is determined by balanced trade (financial autarky):

$$P_M \int_0^1 M_i di = PX,$$

where $P_M$ is the price of imports and $X$ are exports.

By the small open economy assumption, the price of imports is given exogenously. I assume that the total supply of domestic inputs denoted by $V$ is exogenous well. Next, I analyze this economy at its symmetric equilibrium, i.e. $Y_i = Y$, $P_i = P$, $V_i = V$ and $M_i = M$. I normalize the price of the final good, i.e. $P = 1$. Thus, $P_M$ represents the ToT.

[8] Underproduction has consequences for welfare. Suppose that for some reason all producers increase their output simultaneously; this increases the aggregate demand of everyone, i.e. $Y$. The increase in output reduces the initial distortion of underproduction and hence increases welfare.

**Terms of trade and output** Here I discuss the relationship between the ToT and output. The main finding is that output is a negative function of the ToT if, and only if, firms have monopolistic power.

In general, the value output is not exactly equal to aggregate production since one has to subtract the real opportunity cost of intermediate inputs. The difference between aggregate production and cost of intermediate inputs is known as *value added*. Next I show that when value added is aggregated over the entire economy, it is identically equal to total expenditure and total income.

Single-deflated value added, $VA^{SD}$, is defined as:

$$VA^{SD} = Y - P_M M \ , \tag{12}$$

where $Y$ is *gross* output (real revenues), $M$ are imports and $P_M$ are the ToT. In other words, $VA^{SD}$ is the difference of revenues and the cost of intermediate inputs, deflated by the price of the final good. Note that plugging $(11)$ into $(12)$ yields the standard expenditure identity:

$$VA^{SD} = C + X - P_M M \ .$$

Moreover, from the definition of profits, $(12)$ can be rewritten as real gross domestic income (profits plus domestic factor services deflated by the price of the final good):

$$VA^{SD} = \Pi + P_V V \ .$$

Thus, $VA^{SD}$ measures the purchasing power of households' income generated by domestic production. For this reason, hereafter I also refer to $VA^{SD}$ as real domestic income. It is instructive to consider the effects of a change in the ToT on $VA^{SD}$. From the envelope condition,

$$\frac{dVA^{SD}}{dP_M} = -M < 0.$$

It is straightforward to show that, starting from a balanced trade position, consumption falls by the same amount.

Now I turn to the relationship between the ToT and the *quantity* of output in this economy. The quantity of output is given real GDP, which is measured by *real value added*. From the value added approach I have:

$$P^{GDP} GDP = Y - P_M M \tag{13}$$

where $P^{GDP}$ is the GDP deflator. Note that real value added subtracts the real costs of imports valued their purchase price, not their marginal product.[9] After some algebra, differentiation of $(13)$ at constant prices yields:

$$\frac{dGDP}{GDP} = \frac{1}{1 - S_M}\frac{dY}{Y} - \frac{S_M}{1 - S_M}\frac{dM}{M} \tag{14}$$

[9] This is the insight used in Basu and Fernald (2002) to show that industry TFP reflect non-technological factors.

here $S_M$ is the revenue share of imports,

$$S_M = \frac{P_M M}{Y}.$$

Equation $(14)$ is the Divisia Index of real value added, which is an quantity index continuously "chained", Sims (1969). Using the first order condition $(9)$ and the properties from the linear homogeneity, $(7)$ and $(8)$ , equation $(14)$ can be re-expressed as:

$$\frac{dGDP}{GDP} = \frac{dV}{V} + \frac{1-\theta}{\theta}\frac{P_M M}{P_{GDP}GDP}\left(\frac{dM}{M} - \frac{dV}{V}\right). \tag{15}$$

This formula brings about the following result: output per worker would growth at a slower rate than capital per worker if imports are growing at a lower rate than capital and labor. In other words, imports affect output very much like a productivity shock.[10] This occurs because goods are priced over their marginal costs. As this distortion disappears, $\theta \to 1$, the excess price over marginal cost disappears and the effect of imports on output vanishes. This limiting case is equivalent to perfect competition analyzed by Kohli (2004) and Kehoe and Ruhl (2008).

The next proposition characterizes the response of output to a ToT shock. Output growth responds negatively to a ToT deterioration. The effect of the ToT on output is determined by: (i) the excess price over marginal cost, i.e. $(1-\theta)/\theta$, (ii) the share of nominal imports on nominal output, and (iii) the elasticity of imports to the ToT, i.e. dlog $M/$dlog $P_M < 0$.

**Proposition 4** *Aggregate output is a negative function of the terms of trade. In particular,*

$$\frac{d\log GDP}{d\log P_M} = \frac{1-\theta}{\theta}\frac{P_M M}{P_{GDP}GDP}\frac{d\log M}{d\log P_M} < 0, \tag{16}$$

**Proof.** See Appendix. ■

The explanation for this result is similar to the one highlighted in section 2.1. First, at the equilibrium optimum, firms operate at marginal product of all inputs higher than their purchase price. Second, output is computed as the difference between gross output and intermediate inputs, valued at constant prices. Now, to understand the interplay between these two consider the following example. Suppose that labor and capital are following a exogenous process and there is an (infinitesimal) increase in the ToT that causes a (infinitesimal) decrease in the use of imports. As a result, gross output (at constant prices) falls by an amount equal to the marginal product of imports and imports fall by an amount equal to the price at which they are valued. Under imperfect competition, the wedge between the marginal product of imports and the ToT implies that the latter has a first order effect on output. Moreover, with monopolistic competitive output markets, a ToT deterioration has a greater negative impact on gross output than on imports, rendering an output function that depends negatively on the ToT.

[10] I will be more explicit about this next.

**Terms of trade and measured output** In practice national statistical agencies do not perform the kind of continuous chain weighting required by the Divisia Index. Some statistical agencies calculate GDP using constant base year prices, i.e. the base prices are updated after several years. Other statistical agencies apply chain-weighting methods at discrete periods (e.g. chained weighted quarterly GDP). Here I show that these two measures of GDP are sensitive to the ToT.[11]

GDP at constant base year prices:

$$GDP_t = Y_t - P_{M,0} M_t, \tag{17}$$

where $P_{M,0}$ are the ToT in the base year. Notice that because base year prices differ from current prices, the ToT have an artificial effect on output.

For the above mentioned reason, some statistical agencies use chain–weighted methods. Here I consider a chain-weighted method based on Fisher Index, which is used in the BEA's NIPA and recommended by several multilateral organizations.[12] This method consists on deflating nominal GDP by a Fisher price index,

$$GDP_{t+1} = \frac{Y_{t+1} - P_{M,t+1} M_{t+1}}{P_{t+1}^{Fisher}}, \tag{18}$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and Laspeyres indices between the current period and the previous period:

$$P_{t+1}^{Fisher} = \left( \frac{Y_{t+1} - P_{M,t+1} M_{t+1}}{Y_{t+1} - P_{M,t} M_{t+1}} \right)^{\frac{1}{2}} \left( \frac{Y_t - P_{M,t+1} M_t}{Y_t - P_{M,t} M_t} \right)^{\frac{1}{2}} P_t^{Fisher}.$$

The next proposition summarizes the effects of the ToT on base year GDP and chain-weighted GDP up to first order approximation. The main result is that the effect presented in Proposition 4 carries over these two ways of computing the quantity of output. This is not surprising since both measures try to approximate the Divisia Index.

**Proposition 5** *The effect of terms of trade on GDP growth measured at base year prices, equation* (17) , *is given by:*

$$\frac{d \log GDP_t}{d \log P_{M,t}} = \frac{P_{M,t} - P_{M,0}}{P_{M,t}} \left( \frac{P_{M,t} M_t}{GDP_t} \right) \left( \frac{d \log M_t}{d \log P_{M,t}} \right) + \frac{1-\theta}{\theta} \left( \frac{P_{M,t} M_t}{GDP_t} \right) \left( \frac{d \log M_t}{d \log P_{M,t}} \right) \tag{19}$$

*The effect of terms of trade on GDP growth under chain-weighting, equation* (18) , *is given by:*

$$\frac{d \log GDP_t}{d \log P_{M,t}} = \frac{1-\theta}{\theta} \frac{P_{M,t} M_t}{P_t^{Fisher} GDP_t} \left( \frac{d \log M_t}{d \log P_{M,t}} \right) < 0 \tag{20}$$

[11]See, Kehoe and Ruhl (2008) for the perfect competition case.

[12]See: "System of National Accounts 2008," published jointly by European Commission, IMF, OECD, United Nations and World Bank, 2009.

**Proof.** See Appendix. ■

Both elasticities are negative. The common term was explained before. The additional term in the case of GDP at base year prices arises from the differences between base year prices and current ones.

**Terms of trade and measured TFP** The Solow residual is often used to measure TFP, see for example Bergoing et. al. (2002). The standard formula of the Solow Residual is:

$$TFP = \frac{GDP}{K^{\alpha}L^{1-\alpha}}.$$

Suppose that in the model $V \equiv K^{\alpha}L^{1-\alpha}$. Using $(15)$ , TFP growth is,

$$\frac{dTFP}{TFP} = \frac{1-\theta}{\theta}\frac{P_M M}{P_{GDP}GDP}\left(\frac{dM}{M} - \frac{dV}{V}\right).$$

Then, measured TFP is a negative function of the ToT,

$$\frac{d\log TFP}{d\log P_M} = \frac{1-\theta}{\theta}\frac{P_M M}{P_{GDP}GDP}\frac{d\log M}{d\log P_M} < 0,$$

The above is true if output is measured using either chain-weighted or base year methods. If output is measured using base year prices, the additional term capturing the discrepancy between current and base year prices will also affect measured TFP.

## 3 Model

This section outlines a small open economy model along the lines of Mendoza (1991). The domestic production sector of this economy is modified to include intermediate inputs in the context of monopolistic competitive output markets as in Basu (1995). Following Kohli (2004) and Kehoe and Ruhl (2008) production also requires imported intermediate inputs, purchased from world markets at an exogenous given price. The economy is subject to productivity and ToT shocks.

The economy is populated by 3 types of agents- a representative household, a final goods producer and a continuum of intermediate producers (henceforth firms). The representative household consumes, invest on physical capital, supplies labor and holds a risk-less non-contingent bond. The final good producer assembles a tradable good using the intermediate inputs produced by the continuum of intermediate producers. The final tradable good is consumed, used in the formation of new physical capital or exported. Firms produce one intermediate input using a technology that requires labor, capital, other domestic intermediate inputs and imported inputs. Firms sell their output in monopolistic competitive markets. All other transactions in the model happen in perfectly competitive markets.

**Household:** The problem of the household is standard. The household maximizes expected discounted utility

$$\max_{\{C_t,I_t,K_t,B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^{\nu}\left(1-L_t\right)^{1-\nu}\right)^{1-\sigma}-1}{1-\sigma},$$

subject to a budget constraint and capital accumulation equation:

$$C_t + I_t + B_t + \frac{\kappa}{2}(B_t - \bar{B})^2 = w_t L_t + r_t K_{t-1} + \Pi_t + R_{t-1} B_{t-1},$$

$$\begin{aligned} K_t &= (1-\delta) K_{t-1} + I_t + \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}}-1\right)^2 K_{t-1}, \\ & K_{-1}, B_{-1} \qquad \text{given} \end{aligned}$$

The household buys consumption $C_t$, invest in new physical capital $I_t$, or saves in a one-period non-contingent riskless bond which $B_t$ which pays a exogenous real (gross) interest rate $R_t$ next period. In addition, she pays a quadratic cost for holding a stock of bonds different than the steady bond holdings $\bar{B}$.[13] The household supplies a fraction $L_t$ of her time at wage rate $w_t$ and gets a rental rent $r_t$ for every unit of physical capital supplied to the market. She also receives the sum of profits across production units $\Pi_t$ as a lump sum transfer. The capital accumulation includes a capital adjustment cost which is used in the numerical simulations to modulate investment volatility. Parameter $\beta \in (0,1)$ is the discount factor, $\sigma > 0$ is the coefficient of risk aversion and $\nu \in (0,1)$ is a preference share parameter, $\delta \in (0,1)$ is the depreciation rate, $\kappa$ and $\phi$ are positive real numbers. I assume that the real interest rate on bond holdings is constant and equal to the inverse of the discount factor.

**Final good producer:** The tradable final good is produced using a continuum $[0,1]$ of intermediate inputs $G_i$. The production technology is a Dixit-Stiglizt aggregator with constant returns to scale,

$$G_t = \left(\int_0^1 G_{it}^{\theta} di\right)^{\frac{1}{\theta}} \quad \text{with } 0 < \theta \leq 1 \tag{21}$$

where $\theta$ controls the elasticity of substitution among intermediate goods, i.e. $\frac{1}{1-\theta}$. As $\theta \to 1$, intermediate inputs are perfect substitutes.

The final good producer takes its price (normalized to one) and the price of all the inputs as given. The final good producer problem is:

$$\max G_t - \int_0^1 P_{it} G_{it} di$$

subject to $(21)$.

**Intermediate producers:** There is a continuum of intermediate good producers indexed $i \in [0,1]$. These firms produce differentiated goods using primary inputs (capital and labor) and

[13] This guarantees that the stock of bonds is stationary. See Uribe and Schmidtt-Grohe (2003).

intermediate inputs. Each differentiated good can be used as in the production of the final good or in the production of other differentiated goods. I assume these producers have monopolistic power in output markets.[14]

The production function of firm is,

$$Y_{it} = A_{it} \left( K_{it}^{\alpha} L_{it}^{1-\alpha} \right)^{1-\mu} \left( D_{it}^{\gamma} M_{it}^{1-\gamma} \right)^{\mu} \tag{22}$$

where $A_i$ is firm's $i$ productivity, $Y_i$ is gross output, $K_i$ is capital, $L_i$ is labor, $D_i$ is a composite of domestic inputs and $M_i$ are imported inputs. Domestic intermediate input is an Dixit-Stiglitz aggregator of all differentiated goods produced within the economy

$$D_{it} = \left( \int_0^1 D_{ijt}^{\theta} dj \right)^{\frac{1}{\theta}} \qquad \text{with } 0 < \theta < 1, \tag{23}$$

where $D_{ij}$ is the domestic intermediate used by firm $i$ and produced by firm $j \in [0,1]$ . Parameter $\theta$ controls the elasticity of substitution among intermediate goods, i.e. $\frac{1}{1-\theta}$. This is the same elasticity that governs the elasticity of substitution competing products in the final good market. This assumption simplifies the analysis as firms face a demand with a unique elasticity of substitution, see Basu (1995).

Firm $i$ production equalizes its demand:

$$Y_{it} = G_{it} + \int_0^1 D_{jit} dj, \tag{24}$$

where $G_i$ is the demand of firm $i$ output used in the production of the final good and $D_{ji}$ is the aggregate demand of firm $i$ output used in the production of good $j$.

Note that this structure implies an input-output matrix with particular characteristics. First, each firm has the same number of downstream interconnections, i.e. it supplies inputs to every other firm. Second, given that technology is symmetric across firms, each firm relies equally on other firms' inputs. Hence, no firm plays a dominant role as supplier in this input-output network. Yet, all firms' production depends equally on imported inputs. This implies that the external sector plays the role of a general-purpose technology. In this sense, ToT shocks can be interpreted shocks to this general-purpose technology and the amplification occurs downstream as all firms using imports are interconnected to each other.

Firm $i$ problem is:

$$\Pi_{it} = \max_{\{P_{it}, K_{it}, L_{it}, D_{ijt}, M_{it}, Y_{it}\}} P_{it} Y_{it} - r_t K_{it} - w_t L_{it} - \int_0^1 P_{jt} D_{ijt} dj - P_{Mt} M_{it} \tag{25}$$

subject to $(22)$ , $(23)$ and $(24)$ . $P_{Mt}$ are the ToT, i.e. price of imports relative to the price of the final tradable good.

[14] A similar structure is used in Gopinath and Neiman (2012). In addition these authors assume that firms have access to a continuum of imperfectly substitutable varieties of imported inputs and must pay a fixed cost for each variety.

**Shocks:** Firm $i$ production function is affected by $A_{it}$, firm $i$ physical productivity. I assume that $A_{it}$ has a time variant aggregate component and firm idiosyncratic static component:

$$A_{it} = \bar{A}_i A_t$$

where the aggregate component follows an AR(1), in logs:

$$A_{t+1} = A_t^{\rho_a} \exp\left(\epsilon_{a,t+1}\right) \qquad \text{with } \epsilon_{a,t+1} \,\tilde{}\, N\left(0, \sigma_a^2\right) \text{ and } 0 < \rho_a < 1$$

In addition, this economy is affected by aggregate ToT shocks which follow an AR(1), in logs,

$$P_{Mt+1} = \left(P_{Mt}\right)^{\rho_m} \exp\left(\epsilon_{m,t+1}\right) \qquad \text{with } \epsilon_{m,t+1} \,\tilde{}\, N\left(0, \sigma_m^2\right) \text{ and } 0 < \rho_m < 1$$

Hence, ToT shocks are assumed to be exogenous.

### 3.1 Equilibrium

Given the sequences of aggregate productivity shocks and ToT shocks, an equilibrium is:

(i) a sequence of allocations $C_t$, $K_t$, $B_t$, $L_t$ for the household,
(ii) a sequence of allocations $G_t$, $G_{it}$ for the final good producers,
(iii) a sequence of allocations $K_{it}$, $L_{it}$, $D_{ijt}$, $M_{it}$, $Y_{it}$ and prices $P_{it}$ for all $i$,
(iv) a sequence of real wages $w_t$ and capital rental rates $r_t$,
such that:
(a) given (iv), (i) solves the problem of the household,
(b) given $P_i$, (ii) solves the problem of the final good producer,
(c) given (iv), (ii) solves the problem of the intermediate good producer $i$,
(d) markets clear:

$$\begin{aligned} L_t &= \int_0^1 L_{it} di, & (26)\\ K_{t-1} &= \int_0^1 K_{it} di, & (27)\\ Y_{it} &= G_{it} + \int_0^1 D_{jit} dj \qquad \forall i, & (28)\\ G_t &= C_t + I_t + X_t & (29) \end{aligned}$$

where $X_t$ are exports.

### 3.2 Terms of trade, income, output and TFP

In this section I analyze two measures of the real value added at the firm level: (i) single-deflated value added and (ii) double-deflated value added. I show three things about these two measures. The first is an aggregation result. The aggregation of single-deflated value added across firms leads to real domestic income and the aggregation of double-deflated value added across firms leads to aggregate output, or GDP. Second, as shown in section 2.2, income and output are both negative functions of the ToT, but the former is more elastic to ToT shocks than the latter. The input-output linkages are key in amplifying the magnitude of both elasticities while the size of the excess price over marginal cost is key only for the elasticity of output to the ToT. Third, the effect of the ToT on income or output manifests through a term that can be classified as total factor productivity, i.e. the part that is *unexplained* by capital and labor utilization.

**Single deflated value added** From the definition of nominal value added at the firm level,

$$P_{it} VA_{it}^{SD} = P_{it} Y_{it} - \int_0^1 P_{jt} D_{ijt} dj - P_{Mt} M_{it} \tag{30}$$

where $VA_{it}^{SD}$ is *single-deflated value added* (SDVA), i.e. nominal value added divided by firm $i$ price. SDVA has an inherent problem: it is affected by trading gains or losses. For example, when imports become relatively cheaper, the firm gains in value added measured in units of the final good.

Aggregate SDVA is defined as:

$$VA_t^{SD} = \int_0^1 P_{it} VA_{it}^{SD} di \; . \tag{31}$$

It is straightforward to show, using the definition of firm's profits in equation (25), that aggregate *SDVA equals real domestic income*,

$$VA_t^{SD} = \Pi_t + r_t K_{t-1} + w_t L_t \; . \tag{32}$$

Thus, as in the simple model outlined in section 2.2, $VA^{SD}$ measures the purchasing power of households' income generated by domestic production. Moreover, using the definition of firm's demand (24) and the market clearing condition (29), aggregate SDVA can be written as the total real expenditure in final goods:

$$VA_t^{SD} = C_t + I_t + X_t - P_{Mt} M_t, \tag{33}$$

where $M_t = \int_0^1 M_{it} di$.

The next proposition characterizes the formula of aggregate SDVA. Hereafter, I refer to aggregate SDVA as real income.

**Proposition 6** *Real income is given by:*

$$VA_t^{SD} = TFP_t^{SD} \times K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{34}$$

*with $TFP_t^{SD}$ given by:*

$$TFP_t^{SD} = \Omega \, A_t^{\frac{1}{1-\mu}} \, P_{Mt}^{\psi^{SD}}, \tag{35}$$

*where:*

$$\psi^{SD} \equiv -\frac{\mu(1-\gamma)}{1-\mu} < 0$$
$$\Omega \equiv \left((1-\mu\theta)^{1-\mu}(\mu\theta)^{\mu}\right)^{\frac{1}{1-\mu}}\left(\gamma^{\gamma}(1-\gamma)^{1-\gamma}\right)^{\frac{\mu}{1-\mu}}\widetilde{A}^{\frac{1}{1-\mu}}$$

**Proof.** See Appendix. ■

Note that $TFP^{SD}$ moves with aggregate productivity and the ToT. The elasticity of $TFP^{SD}$ with respect to aggregate productivity is $1/(1-\mu)$. This is the input multiplier which arises from input-output linkages. Jones (2011) shows the role of intermediate input linkages in determining this multiplier. The broad idea is that problems in one firm output propagate downstream through the input-output linkages. For example, if firm $i$ increases output, this can increase the output in a range of firms, e.g. firm $j$, but this in turn feeds back and further increases the output of firm $i$. More specifically, this multiplier works as follows: higher output leads to more intermediate goods, which raises output, and so on. The elasticity of output to intermediate inputs is $\mu$. Hence, the overall multiplier is $1+\mu+\mu^2+...=1/(1-\mu)$.

The elasticity of $TFP^{SD}$ with respect to ToT is given by $\psi^{SD}$. The first term is the aforementioned input multiplier. Intuitively, ToT deteriorations reduce gross output for all firms, as a negative productivity shock, and this feeds back through a lower supply of intermediate inputs. The multiplier is scaled down by a second term, which is the elasticity of gross output with respect to imported inputs. Note that the degree of competition in the economy plays no role in this elasticity.

**Double deflated value added** National accounts use double-deflated value added (hereafter DDVA) which is difference between real output and real inputs. Aggregated over the entire economy, DDVA equals aggregate output.

I begin with the quantity Divisia Index of value added. To simplify the exposition, I used the formulation used Appendix in which value added in (30) can be re-expressed as:

$$P_{it}VA_{it} = P_{it}Y_{it} - P_{Zt}Z_{it} \tag{36}$$

where:

$$Z_{it} = D_{it}^{\gamma}M_{it}^{1-\gamma}$$
$$P_{Zt} = \frac{P_{Dt}^{\gamma}P_{M_t}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

$P_{Zt}$ is the implicit price deflator of $Z_{it}$ and $P_{Dt}$ is the price deflator of the domestic intermediate input composite $D_{it}$.

Total differentiation of (68) at constant prices delivers,

$$P_{it}dVA_{it}^{DD} = P_{it}dY_{it} - P_{Zt}dZ_{it},$$

which can be written in growth rates as:

$$\frac{dVA_{it}^{DD}}{VA_{it}^{DD}}=\frac{1}{1-\mu\theta}\frac{dY_{it}}{Y_{it}}-\frac{\mu\theta}{1-\mu\theta}\frac{dZ_{it}}{Z_{it}}.$$

where the revenue share of intermediate inputs, i.e. $P_{Zt}Z_{it}/P_{it}Y_{it}$, equals $\mu\theta$. It follows that $P_{it}VA_{it}/P_{it}Y_{it}=1-\mu\theta$. See Appendix for details.

Aggregate DDVA is a the quantity Divisia index computed from equation $(31)$:

$$\frac{dVA_{t}^{DD}}{VA_{t}^{DD}}=\int_{0}^{1}\omega_{it}\frac{dVA_{it}^{DD}}{VA_{it}^{DD}}di, \tag{37}$$

where $\omega_{it}\equiv P_{it}VA_{it}/VA_{t}^{SD}$ is the weight for firm $i$ in value added.

Proposition 7 presents the aggregate DDVA and the associated TFP. It shows that double-deflation does not eliminate all ToT effects from value added if there is monopolistic competition, i.e. $\theta<1$. The *residual* ToT effects manifest through the TFP term. Aggregate DDVA corresponds to output or GDP computed from the value added approach. Hereafter, I refer to aggregate DDVA as output.

**Proposition 7** *Aggregate output is given by,*

$$VA_{t}^{DD}=\Phi_{0}TFP_{t}^{DD}K_{t-1}^{\alpha}L_{t}^{1-\alpha}, \tag{38}$$

*with $TFP_{t}^{DD}$ given by:*

$$TFP_{t}^{DD}=\Phi_{1}A_{t}^{\frac{1}{1-\mu}}P_{Mt}^{\psi^{DD}} \tag{39}$$

*where:*

$$\begin{aligned}\psi^{DD} &= -\frac{1}{1-\mu}\frac{1-\theta}{\theta}\frac{P_{Mt}M_{t}}{VA_{t}^{SD}}<0\\ \Phi_{0},\Phi_{1} &> 0\end{aligned}$$

**Proof.** See Appendix. ■

The elasticity of TFP to the ToT is given by the coefficient $\psi^{DD}<0$. The intuition is the same as in section 2, namely, the excess price over marginal costs, i.e. $(1-\theta)/\theta$, causes the marginal product of imports to be above its purchase price. In other words, the complete contribution of imports to the production process is not reflected on prices. As this distortion disappears, $\theta\rightarrow 1$, the effect of ToT on output vanishes, $\psi^{DD}\rightarrow 0$. Besides the excess price over marginal cost, other elements determine the magnitude of $\psi^{DD}$. One of these elements is the input multiplier: $1/(1-\mu)$. This is because the contribution of imported inputs is magnified through the existing input-output linkages. The second element is the share of imports on value added, which in the model a constant: $\mu\theta(1-\gamma)/(1-\mu\theta)$. Note that the elasticity of income with respect to ToT shocks is larger in magnitude than the one of output, i.e. $\left|\psi^{DD}\right|<\left|\psi^{SD}\right|$. In contrast, the elasticity of output and income with respect to productivity shocks is the same.

### 3.3 Other effects of the terms of trade

ToT have other effects in the model that go beyond the ones highlighted previously. First, ToT shock affect household's wealth and welfare. Second, ToT shocks also affect consumption-leisure margin as well as the capital accumulation margin. I highlight that these margins are affected by the real income and not by output, which makes perfect sense since the household compares the marginal value of supplying one more unit of labor or capital to the market in terms of marginal utility of consumption.

I begin with the formula real gross domestic income,

$$VA_t^{SD} = \Pi_t + r_t K_{t-1} + w_t L_t \ .$$

Note that household problem can be reformulated as a quasi-planner problem:

$$\mathcal{W} = \max_{\{C_t, I_t, K_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t^{\nu}\left(1-L_t\right)^{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma} \ ,$$

subject to:

$$C_t + I_t + B_t + \frac{\kappa}{2}(B_t - \bar{B})^2 = VA_t^{SD} + R_{t-1}B_{t-1}, \tag{40}$$

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + I_t + \left(\frac{\phi}{2}\right)\left(\frac{K_t}{K_{t-1}} - 1\right)^2 K_{t-1}, \\ & K_0, B_0 \qquad \text{given} \end{aligned}$$

The first order conditions are:

$$\frac{(1-\nu)}{\nu}\frac{C_t}{1-L_t} = (1-\alpha)\frac{VA_t^{SD}}{L_t} \tag{41}$$

$$1 + \phi\left(\frac{K_t}{K_{t-1}} - 1\right) = \beta\mathbb{E}_t \frac{U_C(t+1)}{U_C(t)}\left(\alpha\frac{VA_{t+1}^{SD}}{K_t} + (1-\delta) - \frac{\phi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2 - 1\right)\right) \tag{42}$$

$$1 + \kappa\left(B_t - \bar{B}\right) = \beta\mathbb{E}_t \frac{U_C(t+1)}{U_C(t)} R_t \tag{43}$$

where,

$$U_C(t) \equiv \nu C_t^{\nu(1-\sigma)-1}\left(1-L_t\right)^{(1-\nu)(1-\sigma)} .$$

Cast in this way, ToT deterioration affect the economy just like a negative productivity shock. These shocks have three effects. The first one is a wealth effect in $(40)$ , which affects the level of household welfare. Following Basu and Fernald (2002), invoking the envelope theorem, the effect of an exogenous unexpected change in income, i.e. $TFP^{SD}$, on the household's welfare can be written as,[15]

$$\frac{\partial \mathcal{W}}{\partial TFP_t^{SD}} = \frac{d\mathcal{W}}{dTFP_t^{SD}} = \beta^t \mathbb{E}_0\left(U_C(t)\frac{VA_t^{SD}}{TFP_t^{SD}}\right) > 0$$

[15] Assuming that productivity and terms of trade shocks exhibit no auto-correlation.

Thus, in the model welfare is being affected by two shocks: productivity and ToT shocks. Specifically,

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial A_t} &= \frac{d\mathcal{W}}{dA_t} = \frac{1}{1-\mu}\,\beta^t \mathbb{E}_0 \left( U_C(t) \frac{VA_t^{SD}}{A_t} \right) > 0 \\ \frac{\partial \mathcal{W}}{\partial P_{Mt}} &= \frac{d\mathcal{W}}{dP_{Mt}} = \psi^{SD}\,\beta^t \mathbb{E}_0 \left( U_C(t) \frac{VA_t^{SD}}{P_{Mt}} \right) < 0\end{aligned}$$

The other effects appear on the consumption-leisure margin, i.e. equation $(41)$, and investment margin, i.e. $(42)$. Thus, in contrast to the standard RBC small open economy framework, in this economy household decisions about how much to consume, invest or work are determined by income and output. As it will be highlighted next, this feature has important consequences for the transmission of ToT shocks. For example, since income responds more forcefully to a ToT shock than what output does, this economy can exhibit *excessive* volatility in consumption relative to that of output.

### 3.4 Quantitative analysis

The objective of this subsection is to evaluate the role of the ToT in driving the business cycles of small open economies. I put particular emphasis on contrasting this role with that of standard productivity shocks. With this in mind, I first calibrate the model to the Mexican economy. Then I perform three quantitative exercises using first-order log-linear approximation of the model. The first exercise shows the impulse response functions to productivity and ToT shocks. In the second exercise I use the policy functions to simulate the paths of the main macroeconomic aggregates given the observed sequence of ToT shocks in Mexico. In compare these sequences to that of the actual data. In third exercise, I compute the standard business cycles second moments from the model and compare them with the actual data.

**Calibration** Some parameters and steady state conditions are set beforehand. The coefficient of relative risk aversion, $\sigma$, is set to 2, which is the typical value in the literature, e.g. Aguiar and Gopinath (2007). I fix the depreciation rate, $\delta$, at $0.13$ and the exponent of capital in the production function, $\alpha$, at $0.3$ to be consistent with the TFP series from Isaksson (2007).[16]. I normalize the ToT $P_M$ to 1 in steady state. Note that with constant ToT, output is the same as income, see equation $(34)$. I assume that all firms have the same level of static productivity, i.e. $\bar{A}_i = \bar{A}$, and choose $\bar{A}$ such that $TFP^{SD}$ is one in the steady state. As in Neumeyer and Perri (2005), parameter $\kappa$ in the bond holding quadratic cost function is set to the minimum value that guarantees that the equilibrium solution is stationary.

A key parameter in the calibration is $\theta$, which controls the elasticity of substitution across competing products, i.e. $1/(1-\theta)$. Given the monopolistic competitive assumption it also determines

[16]Bergoing et al. (2002), who analyze Mexico, use $\alpha = 0.3$ and $\delta = 0.05$.

the markup over marginal cost in the model, i.e. $1/\theta$. Thus, one alternative is to choose $\theta$ to match the level of markups in Mexico. Recent literature locates the level of markups between $2.25$ and $5.05$, see Hoekman et. al. (2001) and Kandilov and Leblebicioğlu (2011).[17] One potential issue with these estimates is that they probably reflect artificial market power created by trade barriers or other policy restrictions to competition, see Tybout and Westbrook (1995). To avoid this mismeasurement problem I consider a value of $\theta$ that generates plausible levels substitutability in a relatively competitive economy. As a benchmark I take the estimates of the elasticity of substitution obtained for the U.S. economy, which the trade and industrial organization literatures locate between $3$ to $10$, e.g. Broda and Weinstein (2006) and Hendel and Nevo (2006). Following this literature I set $\theta$ to $0.75$, which generates an elasticity of substitution of $4$ and a markup of $1.33$.

A subset of parameters is calibrated to match 1980-2000 averages of the Mexican economy.[18] The discount factor $\beta$ is chosen to match an average real interest rate in Mexico of $10$ percent, see Neumeyer and Perri (2005). I calibrate the share parameter in preferences $\nu$ to generate a share of time spent working of $0.4$ in steady state.[19] The steady state bond holdings $\bar{B}$ is calibrated to match ratio of trade balance to GDP is $0.72$ percent, which corresponds to the average net exports to GDP ratio in Mexico (source: World Bank's WDI). I set the elasticity of gross output to intermediate goods, $\mu$, to $0.56$ to generate a revenue share of intermediate goods, $\mu\theta$, of $0.42$, which corresponds to the average intermediate purchases to gross output in Mexico (source: United Nations Database). This value for $\mu$ implies an input multiplier, i.e. $1/\left(1-\mu\right)$, of $2.27$. Given the values for $\mu$, $\theta$ and $\alpha$ the implied labor income share is $0.40$, which is close to the Mexican labor income share of $0.42$, see Bergoing et al. (2002). Parameter $\gamma$ is chosen to generate an import to GDP ratio of $20$ percent, which is the average imports to GDP ratio in Mexico (source: World Bank's WDI). Recalling that the ratio of imports to GDP in the model is $\mu\theta\left(1-\gamma\right)/\left(1-\mu\theta\right)$, the required value of $\gamma$ is $0.7238$. All together, these parameters imply an elasticity of output with respect to the ToT, $\psi^{DD}$, of $-0.15$ and an elasticity of income with respect to the ToT, $\psi^{SD}$, of $-0.35$. Importantly, *these elasticities would be halved without the input-output linkages.*

The remaining block of parameters, $\phi$, $\rho_a$, $\sigma_a$, $\rho_m$ and $\sigma_m$, is calibrated to target specific moments of the Mexican data between 1972-2000.[20] The parameter controlling the capital stock ad-

[17] Hoekman et. al. (2001) estimates the markups using industrial data from UNIDO. Their estimate for Mexico is $5.05$. Kandilov and Leblebicioğlu (2011) use plant-level data from Mexico's Annual Industrial Survey (1984-1990). They proxy markups as ratio of sales to costs of goods sold (sum of wage bill and intermediate costs). The authors report an average markup of $2.25$.

[18] I restrict the sample to 1980-2000 period since some of the series referred to next are not available for earlier years.

[19] OECD database reports two statistics regarding hours. One statistic is annual hours per worker. For Mexico the average 1991-2000 is 1868 hours per worker. Assuming 44 weeks actually worked per year, 7 days of work per week and 16 hours per day, delivers a share of time spent on working of 0.38. Another statistic reported by OECD is the average usual weekly hours worked. For Mexico the average 1995-2000 is 44 hours per week. Assuming 7 days of work per week and 16 hours per day, delivers a share of time spent on working of 0.40.

[20] I restrict the sample to 1972-2000 period since some of the series referred to next are not available for earlier years. Annual Mexican data was logged, except net exports over GDP, and HP detrended using the usual smoothing parameter of 100.

Table 1: Baseline Calibration

| Shocks | Name | Value |
|---|---|---|
| $\rho_a$ | Productivity shock persistence | 0.3797 |
| $\sigma_a$ | Volatility productivity shock | 0.0124 |
| $\rho_m$ | Terms of trade shock persistence | 0.3322 |
| $\sigma_m$ | Volatility of terms of trade shock | 0.0718 |
| Utility parameters | Name | Value |
| $\beta$ | Discount factor | 0.9091 |
| $\sigma$ | Relative risk aversion | 2.0000 |
| $\nu$ | Utility share parameter | 0.4395 |
| Technology parameters | Name | Value |
| $1/(1-\theta)$ | Elasticity of substitution among goods | 4.0000 |
| $\mu$ | Exponent of production function (intermediate goods) | 0.5600 |
| $\alpha$ | Exponent of production function (capital) | 0.3000 |
| $\delta$ | Depreciation rate | 0.1300 |
| $\kappa$ | Bond holding cost | $10^{-5}$ |
| $\phi$ | Capital adjustment cost | 0.0776 |
| Steady state exogenous variables | Name | Value |
| $\bar{B}$ | Bond holdings | -0.0323 |
| $P_M$ | Terms of trade | 1.0000 |
| $\bar{A}$ | Static physical productivity | 2.8734 |

justment costs, $\phi$, is fixed to target the volatility of investment relative to that of real GDP in the Mexican data. Parameters $\rho_m$ and $\sigma_m^2$ were obtained by estimating an AR(1) process on the ToT. To calibrate parameters $\rho_a$ and $\sigma_a^2$ I take the following route. First I recover the sequence of productivity shocks that is consistent with the structure of the model. Specifically, I construct a sequence of productivity shocks as: $a_t = (1-\mu)\,(tfp_t - \psi_{DD}.p_{M,t})$, where $tfp_t$ and $p_{Mt}$ are the HP detrended series of TFP and ToT, respectively. Then, I estimate $\rho_a$ by fitting AR(1) process on $a_t$. Parameter $\sigma_a$ is calibrated to match the standard deviation of real GDP in the data.[21]

**Impulse responses** Here I illustrate the transmission mechanism of a ToT deterioration vis-à-vis a productivity drop using the impulse response functions (IRF) of the log-linear version of the model. For aforementioned reasons both shocks manifest in the economy through a fall in TFP and move all other variables in the same direction. Yet, ToT shocks have some distinctive characteristics with respect to productivity shocks. To facilitate this comparison, I normalize the response of real GDP to $-1$ percent upon shock. To generate this, the ToT shock has to 59 percent the calibrated value for $\sigma_m$ while the productivity shock has to be -29 percent of $\sigma_a$.

[21] An alternative is to estimate $\sigma_a$ as well from the recovered sequence of productivity shocks $a_t$. The OLS estimate of $\sigma_a$ is 0.0073.

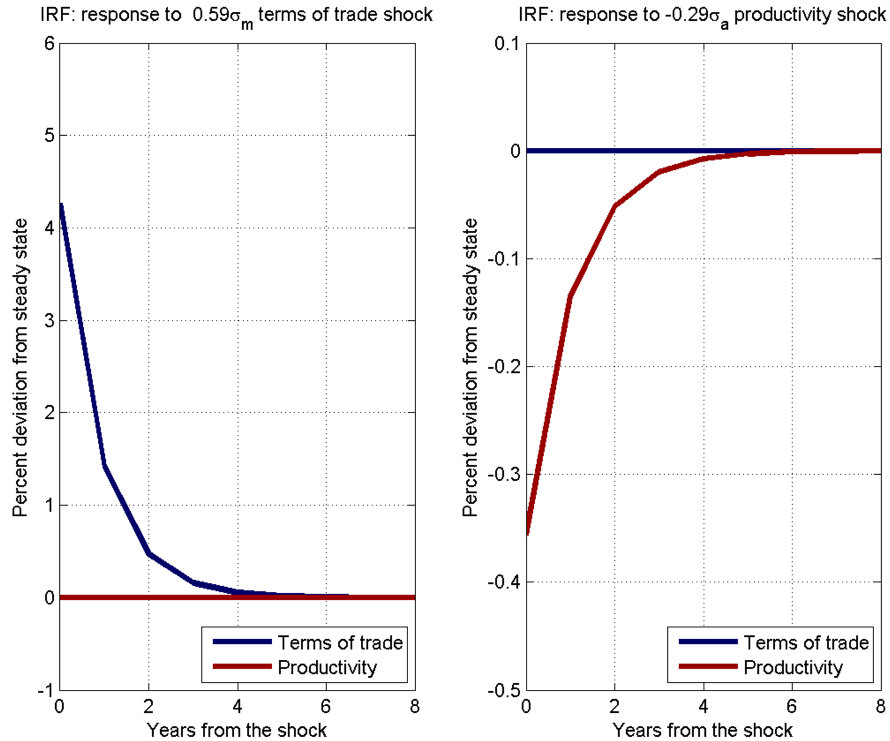


Figure 3: Impulse responses of exogenous variables after a ToT shock (left panel) and after a productivity shock (right panel).

Figure 3 depicts the IRF of the exogenous variables as percentage deviations from the steady state after a ToT shock (left panel) and after a productivity shock (right panel). Recall that both shocks are orthogonal to each other. Figure 4 depicts the IRF of the endogenous variables. The top row plot the response of capital, labor along with output and TFP. Qualitatively, both shocks induce a drop in TFP and in the utilization of factors of production. Quantitatively, TFP explains less of the evolution of output under a ToT shock than under productivity shock.

The graphs at the bottom row of figure 4 plot the response of consumption, investment, real net exports over output, output and income. Again, under both shocks the direction of change of all variables is the same: consumption, investment and income all fall while net exports increase. Quantitatively there are some important differences. First, upon a ToT shock income falls twice as much as output. The reason is that the former measure accounts for the household's purchasing power and hence it is more sensitive to trading losses. This is not case with productivity shocks because these shocks move output and income in equal proportions. Second, consumption responds more than output when the economy is hit by a ToT shock. This is because household's incentives to consume are dictated by income and output. Yet, note that households still smooth

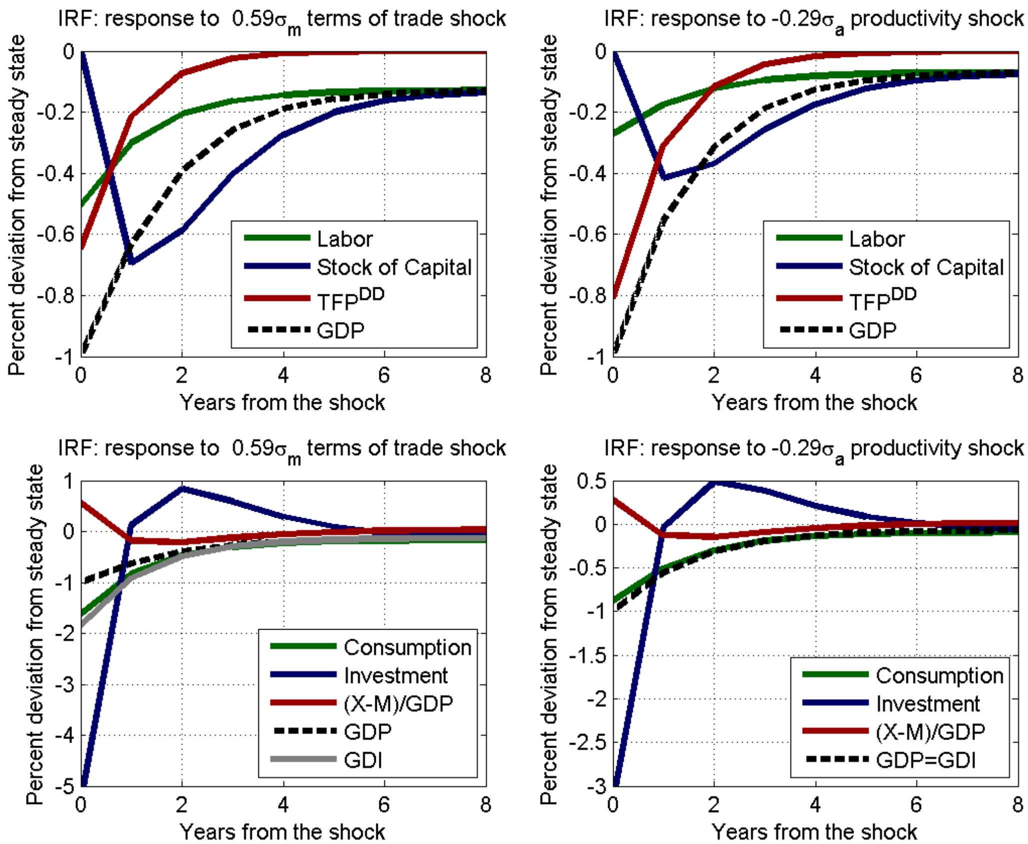


Figure 4: Impulse responses of endogenous variables after a ToT shock (left panel) and after a productivity shock (right panel).

their consumption out relative to their income. In contrast, when the economy is hit by shocks to physical productivity, consumption falls less than output, which equals income in this case.

**Accounting for Business Cycles** Here I compare the in-sample predictions of the model against the actual (HP detrended) time series in Mexico between 1975 and 2000. The main lessons learned from this exercise are: (i) ToT alone account for a sizable fraction of the business cycle fluctuations in Mexico, (ii) the distinction between ToT and productivity shocks as sources of TFP fluctuations matters specially for consumption.

To construct the in-sample predictions of the model I proceed as follows. With respect to the shocks, I consider three experiment. In the first one I assume that ToT shocks $\epsilon_m$ are the sole source of aggregate uncertainty (Exp. 1). The sequence of shocks $\epsilon_m$ is such that the simulated and actual time-series of ToT coincide. The second experiment is similar to the first except that both ToT and productivity shocks are included (Exp. 2). Since productivity shocks are unobservable,

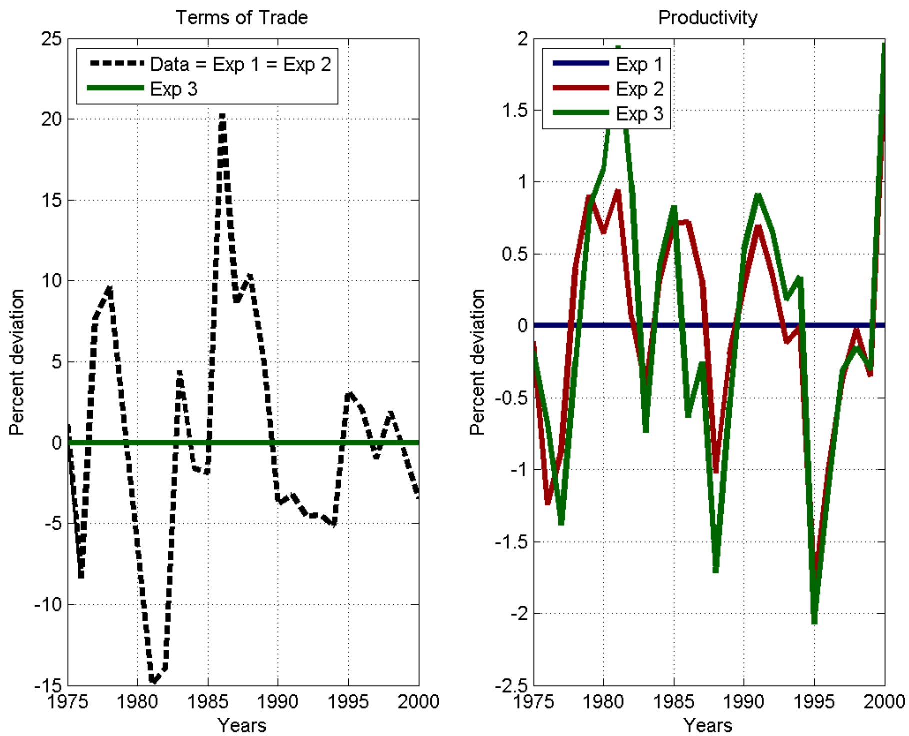


Figure 5: Data and simulated paths of exogenous variables in the three experiments. Data is HP detrended with smoothing parameter 100.

I recover from the residual between measured TFP in the data and the predicted TFP from the first experiment. The third experiment assumes that productivity shocks are the only source of aggregate uncertainty (Exp. 3). Accordingly, I recover these shocks from measured TFP in the data. This last scenario corresponds to what the standard RBC SOE model would predict. The second and third scenarios are similar in the sense that both consider shocks that completely account for measured TFP in the data. Then, conditional on the sequences of shocks, each exercise simulates the paths of the endogenous variables using the policy functions of the log-linearized version of the model. As initial conditions I assume that in 1972 the economy was in its steady state.

Let me begin with the first scenario, which shows the novel mechanism of the model, namely, the connection between ToT and TFP. Figure 5 plots the sequences of shocks observed ToT (HP detrended) for Mexico between 1975 and 2000.[22] Over the sample Mexico experienced periods of favorable ToT, around the beginning of 1980s and 1990s, followed by deteriorations of the ToT, around 1985 and 1994 (so-called Tequila crisis). Note also that ToT fluctuations are more volatile

[22]Recall that the first and second experiments recover the shocks that explain the sequence of terms of trade in the data.

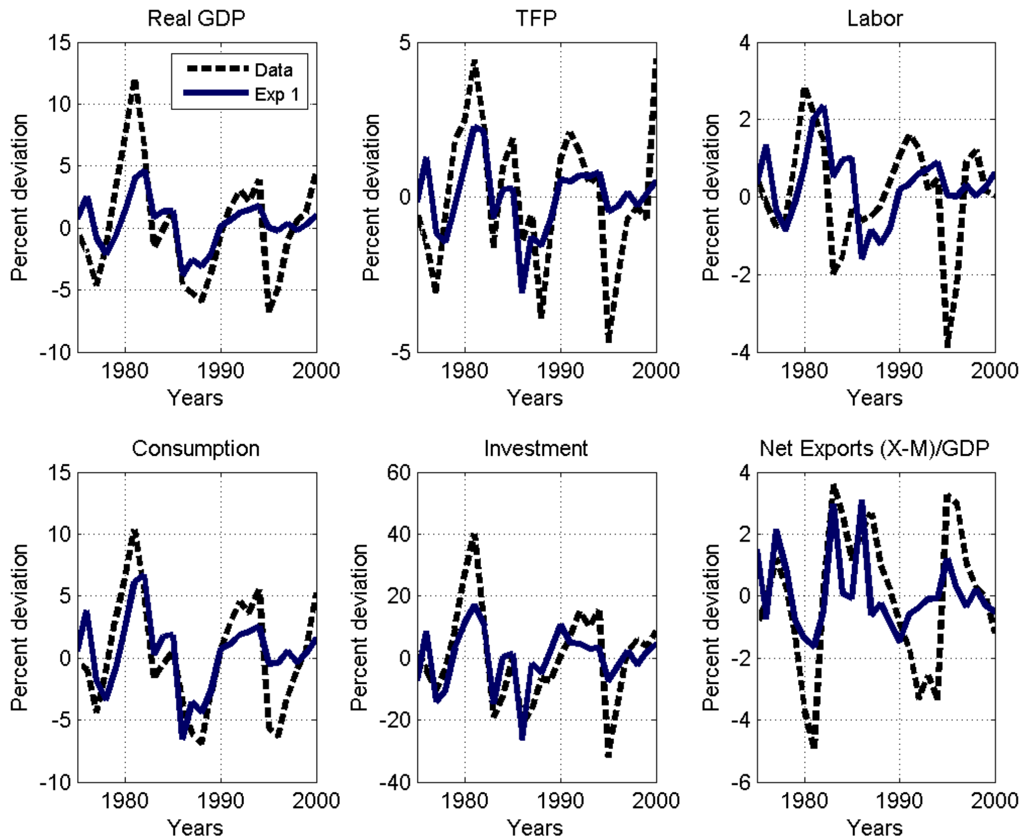


Figure 6: Data and simulated paths of endogenous variables in Exp 1. Data is HP detrended with smoothing parameter 100.

in the first 15 years of the sample.

Figure 6 plots the time series in the data against the path predicted by the model under the first experiment. Notice that the cyclical properties of predicted output and TFP are similar to the data. This similarity is more accentuated before 1994. In particular, the model predicts the expansion phase at the end of the 1970s and subsequent recession (mid 1980s) and recovery phases (beginning of 1990s), but misses the 1994 episode. Overall, the predicted and actual data comove. For example, the correlation between predicted and actual series of output is 0.73 while for TFP the correlation is 0.64. Similar levels of correlations are registered for other variables.[23]

Figure 7 plots the time series in the data against the path predicted by the model under the second and third experiments. Recall that both scenarios account completely for TFP in the data but differ in the sources of TFP fluctuations. As expected, under both scenarios the model has a

[23]This is not the case for labor since simulated and actual series display a low degree comovement.

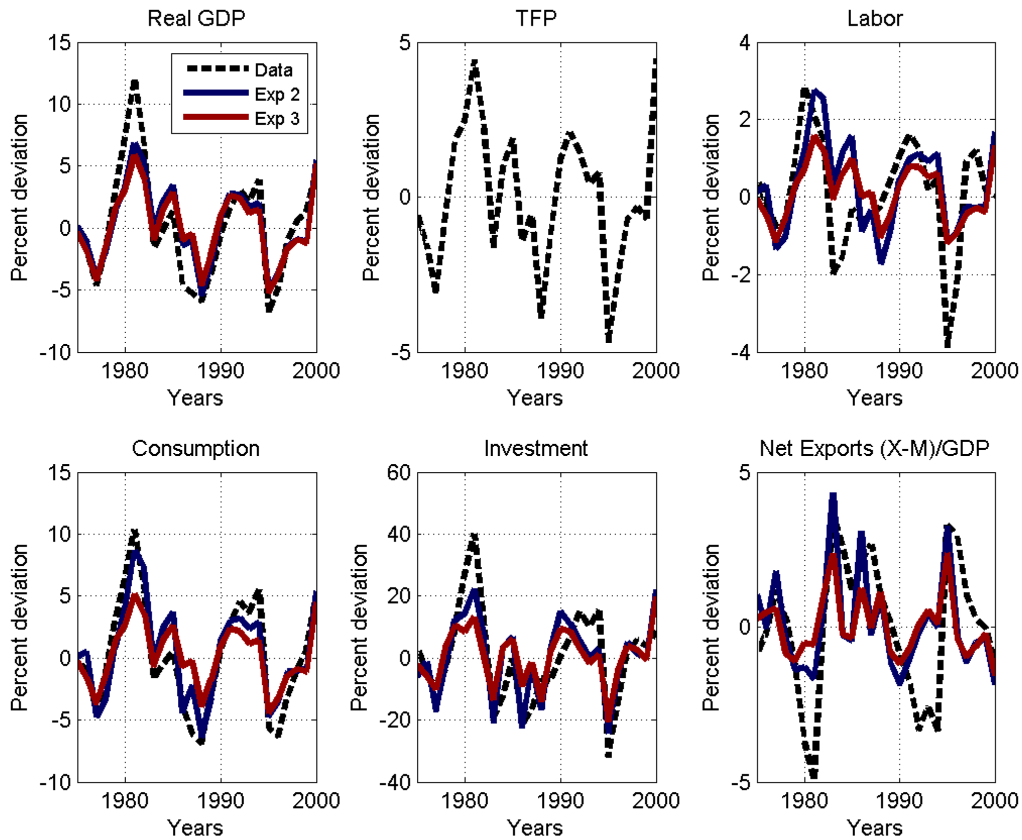


Figure 7: Data and simulated paths of endogenous variables in Exp. 2 and Exp. 3. Data is HP detrended with smoothing parameter 100.

better account of output fluctuations than the first scenario, specially for the 1994 episode. Despite having almost the same predictions for output, both scenarios differ in terms of consumption, investment and net exports. In particular, the scenario in which productivity shocks alone drive TFP predicts paths that are too smooth in comparison to the data. In contrast, in the scenario where TFP is a combination of ToT and productivity shocks predicts paths that are closer to those observed in the data, specially in periods in which ToT fluctuations were important.

**Business Cycle Moments** Table 2 reports second moments, i.e. standard deviations and cross correlations, from the actual data along with their simulated analogues. In order to assess the role of ToT shocks, I report two set of simulated moments, one which was obtained with only ToT shocks and another where both ToT and productivity are turned on. As a reference, I also report the simulated moments for the standard model, i.e. business cycles are driven by shocks to physical productivity.

Table 2: Simulated and actual moments

| | Data[a] | Model[b] | | Standard Model[c] |
|---|---|---|---|---|
| | | $P_{Mt}$ shocks only | $P_{Mt}$ and $A_t$ shocks | |
| **Std (percent)** | | | | |
| GDP | 4.31 | 1.95 | 4.31 | 4.31 |
| | (0.75) | | | |
| **Std relative to Std of GDP** | | | | |
| TFP | 0.51 | 0.59 | 0.74 | 0.76 |
| | (0.06) | | | |
| Labor | 0.32 | 0.50 | 0.34 | 0.28 |
| | (0.05) | | | |
| Consumption | 0.99 | 1.54 | 1.05 | 0.88 |
| | (0.06) | | | |
| Investment | 3.46 | 4.88 | 3.46 | 3.46 |
| | (0.22) | | | |
| Net Exports over GDP | 0.51 | 0.61 | 0.43 | 0.47 |
| | (0.06) | | | |
| Terms of Trade | 1.79 | 3.91 | 1.79 | - |
| | (0.28) | | | |
| **Correlation with GDP** | | | | |
| TFP | 0.88 | 0.93 | 0.96 | 0.97 |
| | (0.03) | | | |
| Labor | 0.72 | 0.98 | 0.93 | 0.95 |
| | (0.08) | | | |
| Consumption | 0.97 | 0.99 | 0.96 | 1.00 |
| | (0.01) | | | |
| Investment | 0.92 | 0.71 | 0.74 | 0.71 |
| | (0.04) | | | |
| Net Exports over GDP | -0.79 | -0.43 | -0.36 | -0.31 |
| | (0.08) | | | |
| Terms of Trade | -0.75 | -0.93 | -0.44 | - |
| | (0.09) | | | |

[a] Estimated using GMM. Standard errors in parentheses

[b] Moments are the average across 100 simulations.

[c] Parameters: $\phi = 0.0550$ , $\sigma_a = 0.0139$ and $\rho_a = 0.3951$. Averages across 100 simulations.

One key prediction of the model is that the business cycles volatility, measured by the standard deviation of output, is partly driven by ToT fluctuations. To quantify the contribution of terms of fluctuations to business cycles volatility I first compare the moments from the actual data with those of the model under ToT shocks (second column). The model generates a standard deviation of output of $1.95$ percent. Hence, ToT trade shocks alone account about 45 percent of the actual output volatility in Mexico.[24] Moreover, TFP is approximately $0.60$ as volatile as output in the simulation while in the actual data that number is $0.50$. This means that ToT shocks explain about half of the volatility of the actual TFP series. Similar conclusions are obtained for other variables as well.

As it was pointed out previously, ToT shocks generate more volatility in consumption because income is more sensitive to these shocks than output. In the data, consumption is as much as volatile as output. This level of volatility in consumption is puzzling for the standard model (last column), which generates a ratio of consumption volatile to that output of 0.88. Adding ToT shock improve the fit of the model along this dimension. ToT shocks alone (second column) induce a consumption volatility that is $54$ percent more than the one output. When ToT shocks and productivity shocks enter in the model (third column), the excess volatility of consumption falls to $5$ percent.

Importantly, the same excess volatility in consumption has been found as a salient feature of the business cycles in emerging economies data. The literature has provided alternative explanations to the excess volatility phenomenon. One explanation is that TFP is partly driven by shocks to the trend, see Aguiar and Gopinath (2007). The core of their argument lies on the permanent income hypothesis, namely, consumption responds more to the permanent component of real GDP than to the transitory one. To some extent, ToT shocks in the model induce a similar phenomenon with the different that the excess volatility of consumption in this case arises from a departure between output and income.

# 4 Summary

This paper revisited the connection between the terms of trade (ToT) and the real economy. I built a small open economy (SOE) model in which imports are inputs in production, output markets are imperfectly competitive and firms are connected in an input-output network. The model has stark predictions for the ToT/TFP connection. Just like in the data, in the model a ToT deterioration (improvement) leads to a drop (increase) in TFP. In addition, the model predicts that ToT fluctuations increase the volatility of real consumption relative to that of the output. Hence, the model offers an alternative explanation for the observed excess consumption volatility puzzle found in the data. Finally, input-output linkages amplify ToT shocks in this economy.

[24]Similar numbers have been reported in the literature. For example, Mendoza (1995) concludes that terms-of-trade shocks account for 45 to 60 percent of the observed variability of GDP.

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# Appendix A Proofs

**Proof of Proposition 1** This proof uses the first and second order conditions from the profit maximization problem,

$$\begin{aligned} f'(m) &= p_m, \\ f''(m) &< 0. \end{aligned}$$

From the first order condition, the implicit function theorem delivers,

$$\frac{dm}{dp_m} = \frac{1}{f''(m)} < 0.$$

Consider two dates: $0$ and $1$. The price of input at date $0$ is given $p_m^0$. At date $1$ the price is $p_m^1 = p_m^0 + \delta$ with $\delta \geq 0$. The change in Laspeyres profits between date $0$ and date $1$ is:

$$\Delta\pi^0 = f\left(m\left(p_m^0+\delta\right)\right) - p_m^0\, m\left(p_m^0+\delta\right) - f\left(m\left(p_m^0\right)\right) + p_m^0\, m\left(p_m^0\right).$$

The slope of Laspeyres profits is:

$$\frac{\Delta\pi^0}{\Delta p_m} = \frac{f\left(m\left(p_m^0+\delta\right)\right) - p_m^0\, m\left(p_m^0+\delta\right) - f\left(m\left(p_m^0\right)\right) + p_m^0\, m\left(p_m^0\right)}{\delta}.$$

By definition, $d\pi^0/dp_m$ at $p_m^0$ is given by:

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m} = \lim_{\delta\to 0}\frac{f\left(m\left(p_m^0+\delta\right)\right) - p_m^0\, m\left(p_m^0+\delta\right) - f\left(m\left(p_m^0\right)\right) + p_m^0\, m\left(p_m^0\right)}{\delta}$$

Rearranging,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m} = \lim_{\delta\to 0}\left\{\frac{m\left(p_m^0+\delta\right) - m\left(p_m^0\right)}{\delta}\times\left(\frac{f\left(m\left(p_m^0+\delta\right)\right) - f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right) - m\left(p_m^0\right)} - p_m^0\right)\right\}$$

Note that:

$$\lim_{\delta\to 0}\frac{m\left(p_m^0+\delta\right) - m\left(p_m^0\right)}{\delta} = \frac{dm}{dp_m} = \frac{1}{f''} < 0$$

Hence,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\lim_{\delta\to 0}\left(\frac{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}{\delta}\right)\times\lim_{\delta\to 0}\left(\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^0\right)$$

Note that,

$$\lim_{\delta\to 0}\left(\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^0\right)=f\,'\left(m_0\right)-p_m^0=0$$

Thus,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\frac{d\pi^0}{dp_m}|_{p_m=p_m^0}=0$$

Note that the change Laspeyres profits can be rewritten as:

$$\Delta\pi^0=f\left(m\left(p_m^1\right)\right)-p_m^0\,m\left(p_m^1\right)-f\left(m\left(p_m^1-\delta\right)\right)+p_m^0\,m\left(p_m^1-\delta\right).$$

Hence, the slope of Laspeyres profits at $p_m=p_m^1$ is given by:

$$\begin{aligned}\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m} &= \lim_{\delta\to 0}\left(\frac{m\left(p_m^1\right)-m\left(p_m^1-\delta\right)}{\delta}\right)\times\lim_{\delta\to 0}\left(\frac{f\left(m\left(p_m^1\right)\right)-f\left(m\left(p_m^1-\delta\right)\right)}{m\left(p_m^1\right)-m\left(p_m^1-\delta\right)}-p_m^0\right)\\ &= \frac{dm}{dp_m}|_{p_m=p_m^1}\times\left(p_m^1-p_m^0\right)<0\end{aligned}$$

Outside $p_m=p_m^0$, the derivative of Laspeyres profits is negative.

Now consider a case where the input price at date 1 is fixed at $p_m^1$. At date, the input price is $p_m^0=p_m^1-\delta$. The change in Paasche profits between date 0 and date 1 is:

$$\Delta\pi^1=f\left(m\left(p_m^1\right)\right)-p_m^1\,m\left(p_m^1\right)-f\left(m\left(p_m^1-\delta\right)\right)+p_m^1\,m\left(p_m^1-\delta\right).$$

Following the same steps as before,

$$\lim_{\delta\to 0}\frac{\Delta\pi^1}{\Delta p_m}=\frac{d\pi^1}{dp_m}|_{p_m=p_m^1}=0$$

The change Paasche profits can be rewritten as:

$$\Delta\pi^1=f\left(m\left(p_m^0+\delta\right)\right)-p_m^1\,m\left(p_m^1+\delta\right)-f\left(m\left(p_m^0\right)\right)+p_m^1\,m\left(p_m^0\right).$$

Hence, the slope of Paasche profits at $p_m=p_m^0$ is given by:

$$\begin{aligned}\lim_{\delta\to 0}\frac{\Delta\pi^1}{\Delta p_m} &= \lim_{\delta\to 0}\left(\frac{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}{\delta}\right)\times\lim_{\delta\to 0}\left(\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^1\right)\\ &= \frac{dm}{dp_m}|_{p_m=p_m^0}\times\left(p_m^0-p_m^1\right)>0\end{aligned}$$

Outside $p_m=p_m^1$, the derivative of Paasche profits is positive.

**Proof of Proposition 2** The Fisher index is defined as $\mathbb{I}_F\left(p_m^0, p_m^1\right) = \sqrt{\mathbb{I}_L\left(p_m^0\right)\mathbb{I}_P\left(p_m^1\right)}$. Hence, Laspeyres and Paasche indexes are lower and upper bounds for the Fisher index.

$$\mathbb{I}_L\left(p_m^0\right) \leq \mathbb{I}_F\left(p_m^0, p_m^1\right) \leq \mathbb{I}_P\left(p_m^1\right)$$

These inequalities can be rewritten as:

$$\mathbb{I}_L\left(p_m^0\right) - 1 \leq \mathbb{I}_F\left(p_m^0, p_m^1\right) - 1 \leq \mathbb{I}_P\left(p_m^1\right) - 1$$

which implies:

$$\frac{\Delta\pi^0}{\pi^0} \leq \frac{\Delta\pi^f}{\pi^f} \leq \frac{\Delta\pi^1}{\pi'\left(y^0, m^0; p_m^1\right)}$$

Dividing by $\Delta p_m$ :

$$\frac{\Delta\pi^0}{\Delta p_m}\frac{1}{\pi^0} \leq \frac{\Delta\pi^f}{\Delta p_m}\frac{1}{\pi^f} \leq \frac{\Delta\pi^1}{\Delta p_m}\frac{1}{\pi'\left(y^0, m^0; p_m^1\right)}$$

From the proof of Proposition 1 as $p_m \to p_m^0$ the above inequality is bounded from below by $0$. Similarly, as $p_m \to p_m^1$ the inequality is bounded from above by $0$. It follows then, by the Squeeze Theorem, as $p_m^0 \to p_m^1$,

$$\frac{\partial\pi^f}{\partial p_m} = 0$$

**Proof of Proposition 3** This proof uses the first and second order conditions for the profit maximization problem. The f.o.c,

$$f'(m) = \frac{|\varepsilon|}{|\varepsilon| - 1}\frac{p_m}{p_y} \text{ with } |\varepsilon| \equiv -\frac{\mathcal{D}(y)}{y}\frac{1}{\mathcal{D}'(y)},$$

where $|\varepsilon|$ is the absolute value of the demand elasticity. The s.o.c can be written as,

$$\frac{\widetilde{\varepsilon} - 1}{|\varepsilon|}\mathcal{D}'(y)\left[f'(m)\right]^2 + \mathcal{D}'(y)\left[f'(m)\right]^2 + \frac{|\varepsilon| - 1}{|\varepsilon|}\mathcal{D}(y) f''(m) < 0$$

where $\widetilde{\varepsilon}$ is the super-elasticity, i.e. $\widetilde{\varepsilon} \equiv \frac{\partial \ln|\varepsilon|}{\partial \ln p_y}$.

From the first order condition, the implicit function theorem delivers,

$$\frac{dm}{dp_m} = \frac{1}{\frac{\widetilde{\varepsilon}-1}{|\varepsilon|}\mathcal{D}'(y)\left[f'(m)\right]^2 + \mathcal{D}'(y)\left[f'(m)\right]^2 + \frac{|\varepsilon|-1}{|\varepsilon|}\mathcal{D}(y) f''(m)} < 0.$$

Under constant elasticity of substitution, $\widetilde{\varepsilon} \to 0$, then

$$\frac{dm}{dp_m} = \frac{|\varepsilon|}{|\varepsilon| - 1}\frac{1}{\mathcal{D}'(y)\left[f'(m)\right]^2 + \mathcal{D}(y) f''(m)} < 0$$

Consider two dates: $0$ and $1$. The price of input at date $0$ is given $p_m^0$. At date $1$ the price is $p_m^1 = p_m^0 + \delta$ with $\delta \geq 0$. The change in Laspeyres profits between date $0$ and date $1$ is:

$$\Delta\pi^0 = p_y\left(p_m^0\right) f\left(m\left(p_m^0 + \delta\right)\right) - p_m^0\, m\left(p_m^0 + \delta\right) - p_y\left(p_m^0\right) f\left(m\left(p_m^0\right)\right) + p_m^0\, m\left(p_m^0\right).$$

The slope of Laspeyres profits is:

$$\frac{\Delta\pi^0}{\Delta p_m} = \frac{p_y\left(p_m^0\right) f\left(m\left(p_m^0 + \delta\right)\right) - p_m^0\, m\left(p_m^0 + \delta\right) - p_y\left(p_m^0\right) f\left(m\left(p_m^0\right)\right) + p_m^0\, m\left(p_m^0\right)}{\delta}.$$

By definition, $d\pi^0/dp_m$ at $p_m^0$ is given by:

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\lim_{\delta\to 0}\frac{p_y\left(p_m^0\right)f\left(m\left(p_m^0+\delta\right)\right)-p_m^0\,m\left(p_m^0+\delta\right)-p_y\left(p_m^0\right)f\left(m\left(p_m^0\right)\right)+p_m^0\,m\left(p_m^0\right)}{\delta}$$

Rearranging,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\lim_{\delta\to 0}\left\{\frac{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}{\delta}\times\left(p_y\left(p_m^0\right)\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^0\right)\right\}$$

Note that:

$$\lim_{\delta\to 0}\frac{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}{\delta}=\frac{dm}{dp_m}<0$$

Hence,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\lim_{\delta\to 0}\left(\frac{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}{\delta}\right)\times\lim_{\delta\to 0}\left(p_y\left(p_m^0\right)\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^0\right)$$

Note that, the f.o.c implies,

$$\lim_{\delta\to 0}\left(p_y\left(p_m^0\right)\frac{f\left(m\left(p_m^0+\delta\right)\right)-f\left(m\left(p_m^0\right)\right)}{m\left(p_m^0+\delta\right)-m\left(p_m^0\right)}-p_m^0\right)=p_y\left(p_m^0\right)f'\left(m_0\right)-p_m^0=\frac{1}{|\varepsilon|-1}p_m^0$$

Combining these results,

$$\lim_{\delta\to 0}\frac{\Delta\pi^0}{\Delta p_m}=\frac{d\pi^0}{dp_m}|_{p_m=p_m^0}=\frac{dm}{dp_m}|_{p_m=p_m^0}\times\frac{1}{|\varepsilon|-1}p_m^0<0$$

Thus, at $p_m=p_m^0$, the slope of Laspeyres profits is negative.

Now consider a case where the input price at date 1 is fixed at $p_m^1$. At date, the input price is $p_m^0=p_m^1-\delta$. The change in Paasche profits between date 0 and date 1 is:

$$\Delta\pi^1=p_y\left(p_m^1\right)f\left(m\left(p_m^1\right)\right)-p_m^1\,m\left(p_m^1\right)-p_y\left(p_m^1\right)f\left(m\left(p_m^1-\delta\right)\right)+p_m^1\,m\left(p_m^1-\delta\right).$$

Following the same steps as before,

$$\lim_{\delta\to 0}\frac{\Delta\pi^1}{\Delta p_m}=\frac{d\pi^1}{dp_m}|_{p_m=p_m^1}=\frac{dm}{dp_m}|_{p_m=p_m^1}\times\frac{1}{|\varepsilon|-1}p_m^1<0$$

Thus, at $p_m=p_m^1$, the slope of Paasche profits is negative.

Using the Squeeze theorem as in Proposition 2, it follows that at $p_m^0\to p_m^1$ :

$$\frac{\partial\pi^f}{\partial p_m}=\frac{dm}{dp_m}\times\frac{1}{|\varepsilon|-1}p_m<0$$

**Proof of proposition 4** Equation (15) can be rewritten as:

$$d\log GDP=d\log V+\frac{1-\theta}{\theta}\frac{P_MM}{P_{GDP}GDP}\left(d\log M-d\log V\right).$$

Dividing through by dlog $P_M$ :

$$\frac{d\log GDP}{d\log P_M}=\frac{d\log V}{d\log P_M}+\frac{1-\theta}{\theta}\frac{P_MM}{P_{GDP}GDP}\left(\frac{d\log M}{d\log P_M}-\frac{d\log V}{d\log P_M}\right).$$

Since $V$ are exogenous, $dV/dP_M=0$. Thus,

$$\frac{d\log GDP}{d\log P_M}=\frac{1-\theta}{\theta}\frac{P_MM}{P_{GDP}GDP}\frac{d\log M}{d\log P_M}<0$$

To determine the sign, apply the implicit function theorem on (9).

**Proof of proposition 5** First consider *real GDP at constant base prices*:

$$GDP_t = C_t + X_t - P_{M,0}M_t = Y_t - P_{M,0}M_t$$

where $P_{M,0}$ are the ToT in the base year.

Now consider the effect of a deterioration of the ToT - i.e. an increase in $P_{Mt}$- on real GDP at constant prices:

$$\frac{dGDP_t}{dP_{M,t}} = F_M \frac{dM_t}{dP_{M,t}} - P_{M,0}\frac{dM_t}{dP_{M,t}} = \left(\frac{P_{M,t}}{\theta} - P_{M,0}\right)\frac{dM_t}{dP_{M,t}}, \tag{44}$$

which can be rewritten as:

$$\frac{dGDP_t}{dP_{M,t}} = \left(P_{M,t} - P_{M,0}\right)\frac{dM_t}{dP_{M,t}} + \frac{1-\theta}{\theta}P_{M,t}\frac{dM_t}{dP_{Mt}}. \tag{45}$$

The elasticity of the real GDP to the ToT:

$$\frac{d\log GDP_t}{d\log P_{M,t}} = \frac{P_{M,t} - P_{M,0}}{P_{M,t}}\left(\frac{P_{M,t}M_t}{GDP_t}\right)\left(\frac{d\log M_t}{d\log P_{M,t}}\right) + \frac{1-\theta}{\theta}\left(\frac{P_{M,t}M_t}{GDP_t}\right)\left(\frac{d\log M_t}{d\log P_{M,t}}\right). \tag{46}$$

Now consider the *chain weighted Fisher index* of real GDP. For simplicity, I assume that primary factors are constant. Under Fisher chain-weighted real GDP, I have:

$$GDP_{t+1} = \frac{Y_{t+1} - P_{M,t+1}M_{t+1}}{P_{t+1}^{Fisher}},$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and Laspeyres indices between the current period and the previous period:

$$P_{t+1}^{Fisher} = \left(\frac{Y_{t+1} - P_{M,t+1}M_{t+1}}{Y_{t+1} - P_{M,t}M_{t+1}}\right)^{\frac{1}{2}}\left(\frac{Y_t - P_{M,t+1}M_t}{Y_t - P_{M,t}M_t}\right)^{\frac{1}{2}}P_t^{Fisher}.$$

Kehoe and Ruhl (2008) show that this yields the Fisher chain-weighted quantity index:

$$GDP_{t+1} = \left(\frac{Y_{t+1} - P_{M,t+1}M_{t+1}}{Y_t - P_{M,t+1}M_t}\right)^{\frac{1}{2}}\left(\frac{Y_{t+1} - P_{M,t}M_{t+1}}{Y_t - P_{M,t}M_t}\right)^{\frac{1}{2}}GDP_t.$$

The first order change of the logarithm of chain-weighted GDP is approximated as:

$$\log GDP_{t+1} - \log GDP_t \approx \frac{dGDP_t}{dP_{M,t+1}}\left(P_{M,t+1} - P_{M,t}\right).$$

Differentiating the natural logarithm of chain-weighted real GDP:

$$\begin{aligned}\frac{d\log GDP_{t+1}}{dP_{M,t+1}} \quad &= \quad \frac{F_M\left(V, M_{t+1}\right)M\left(P_{M,t}\right)' - P_{M,t+1}M\left(P_{M,t+1}\right)' - M_{t+1}}{2\left(Y_{t+1} - P_{M,t+1}M_{t+1}\right)} \\ &\quad + \frac{M_t}{2\left(Y_t - P_{M,t+1}M_t\right)} + \frac{F_M\left(V, M_{t+1}\right)M\left(P_{M,t}\right)' - P_{M,t}M\left(P_{M,t+1}\right)'}{2\left(Y_{t+1} - P_{M,t}M_{t+1}\right)}.\end{aligned}$$

Since $F_M\left(L, M\right) = \theta^{-1}P_{M,t}$, equation (9), the above simplifies to:

$$\begin{aligned}\frac{d\log GDP_{t+1}}{dP_{M,t+1}} \quad &= \quad \frac{1-\theta}{\theta}\frac{P_{M,t+1}M\left(P_{M,t+1}\right)'}{\left(Y_{t+1} - P_{M,t+1}M_{t+1}\right)} - \frac{M_{t+1}}{2\left(Y_{t+1} - P_{M,t+1}M_{t+1}\right)} \\ &\quad + \frac{M_t}{2\left(Y_t - P_{M,t+1}M_t\right)} + \frac{\left(P_{M,t+1} - P_{M,t}\right)M\left(P_{M,t}\right)'}{2\left(Y_{t+1} - P_{M,t}M_{t+1}\right)}.\end{aligned} \tag{47}$$

The first term of the right hand side of $(47)$ captures the effect imperfect competition. Evaluating the above expression at $P_{M,t+1} = P_{M,t}$, this terms remains,

$$\frac{d \log GDP_t}{dP_{M,t}} = \frac{1-\theta}{\theta} \frac{P_{M,t} M \left(P_{M,t}\right)'}{\left(Y_t - P_{M,t} M_t\right)}.$$

I can rewrite this as:

$$\frac{d \log GDP_t}{d \log P_{M,t}} = \frac{1-\theta}{\theta} \frac{P_{M,t} M_t}{P_t^{Fisher} GDP_t} \left( \frac{d \log M_t}{d \log P_{M,t}} \right) < 0. \tag{48}$$

**Equilibrium characterization of section 3** **Final good producer:** The first order condition reads:

$$G_{it} = P_{it}^{\frac{1}{\theta-1}} G_t. \tag{49}$$

The zero profit condition implies the following pricing condition:

$$P_{Gt} = \left( \int_0^1 P_{it}^{\frac{\theta}{\theta-1}} di \right)^{\frac{\theta-1}{\theta}}, \tag{50}$$

which is normalized to $1$, i.e. $P_{Gt} = 1 \; \forall \; t$.

**Intermediate producer:** Given the input-output linkages in the model, it is useful to think about firm $i$ problem in several stages. Define two aggregate inputs:

$$\begin{aligned} V_{it} &= K_{it}^{\alpha} L_{it}^{1-\alpha} \\ Z_{it} &= D_{it}^{\gamma} M_{it}^{1-\gamma} \end{aligned}$$

with prices $P_V$ and $P_Z$, respectively. Given these two aggregate inputs, the production function can be re-written as:

$$Y_{it} = A_{it} V_{it}^{1-\mu} Z_{it}^{\mu}$$

First, firm $i$ finds the factor mix, i.e. $V_i, Z_i$ that minimizes the total cost of production taking as given factor prices. That is,

$$\begin{aligned} mc_{it} Y_{it} \quad \equiv \quad & \min_{\{V_{it}, X_{it}\}} P_{Vt} V_{it} + P_{Zt} Z_{it}, \\ & \text{s.t.} : Y_{it} = A_{it} V_{it}^{1-\mu} Z_{it}^{\mu} \end{aligned} \tag{51}$$

where $mc_i$ is the marginal cost. The f.o.c's:

$$V_{it} = (1-\mu) \frac{mc_{it} Y_{it}}{P_{Vt}}, \tag{52}$$

$$Z_{it} = \mu \frac{mc_{it} Y_{it}}{P_{Zt}}, \tag{53}$$

Note that all firms face the same input prices which means that the marginal revenue products of all inputs are equalized across firms. This implies that the allocation of resources is efficient.

Plugging these f.o.c's into the production function:

$$mc_{it} = \frac{\widetilde{P}_t}{A_{it}}, \tag{54}$$

where

$$\widetilde{P}_t = \frac{P_{Vt}^{1-\mu} P_{Zt}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}}$$

Given $V_i$, firm $i$ solves for the optimal mix of $K_i$ and $L_i$ such that:

$$\begin{aligned} P_{Vt}V_{it} &\equiv \min_{\{K_{it},L_{it}\}} r_t K_{it} + w_t L_{it}, \\ & \text{s.t.: } V_{it} = K_{it}^{\alpha} L_{it}^{1-\alpha}. \end{aligned}$$

The f.o.c's:

$$K_{it} = \frac{\alpha P_{Vt} V_{it}}{r_t}, \tag{55}$$

$$L_{it} = \frac{(1-\alpha) P_{Vt} V_{it}}{w_t}. \tag{56}$$

Substituting $(55) - (56)$ in $V_i$ yields:

$$P_{Vt} = \frac{r_t^{\alpha} w_t^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}. \tag{57}$$

Likewise, given $Z_i$, firm $i$ chooses the optimal mix of intermediate goods $D_i$ and $M_i$ such that:

$$\begin{aligned} P_{Zt}Z_{it} &\equiv \min_{D_{it},F_{it}} P_{Dt} D_{it} + P_{Mt} M_{it}, \\ & \text{s.t.: } Z_{it} = D_{it}^{\gamma} M_{it}^{1-\gamma}. \end{aligned}$$

The f.o.c's:

$$D_{it} = \gamma \left( \frac{P_{Zt}}{P_{Dt}} \right) Z_{it}, \tag{58}$$

$$M_{it} = (1-\gamma) \left( \frac{P_{Zt}}{P_{Mt}} \right) Z_{it}, \tag{59}$$

Substituting $(58) - (59)$ in $Z_i$ yields:

$$P_{Zt} = \frac{P_{Dt}^{\gamma} P_{M_t}^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \tag{60}$$

Given $D_i$, firm $i$ chooses the optimal mix of intermediate goods $D_{ij}$ taking their given prices. Firm $i$ minimizes the cost of the domestic intermediate good composite:

$$\begin{aligned} P_{Dt}D_{it} &\equiv \min_{\{D_{ijt}\}_{j\in[0,1]}} \int_0^1 P_{jt} D_{ijt} dj, \\ & \text{s.t.: } D_{it} = \left( \int_0^1 D_{ijt}^{\theta} dj \right)^{\frac{1}{\theta}} \end{aligned}$$

The first order condition for $D_{ij}$ :

$$D_{ijt} = \left( \frac{P_{jt}}{P_{Dt}} \right)^{\frac{1}{\theta-1}} D_{it}. \tag{61}$$

Substituting $(61)$ in $D_i$ delivers the following equation:

$$P_{Dt} = \left( \int_0^1 P_{jt}^{\frac{\theta}{\theta-1}} dj \right)^{\frac{\theta-1}{\theta}}. \tag{62}$$

From $(50)$, it follows that $P_{Dt} = 1 \; \forall \; t$.

Finally, firm $i$ sells its output to the market at a price $P_i$ in a monopolistic competitive fashion. The firm maximizes profits:

$$\Pi_{it} = \max_{P_{it}} P_{it}Y_{it} - mc_{it}Y_{it},$$

subject to:

$$\begin{aligned} Y_{it} &= G_{it} + \int_0^1 D_{jit}dj, \\ G_{it} &= P_{it}^{\frac{1}{\theta-1}} G, \\ D_{jit} &= P_{it}^{\frac{1}{\theta-1}} D_{jt}. \end{aligned}$$

The first order condition yields a constant markup rule:

$$P_{it} = \frac{mc_{it}}{\theta}. \tag{63}$$

Now, I highlight some useful properties of this equilibrium.

Plugging $(63)$ and $(54)$ into $(50)$ implies that:

$$\widetilde{P}_t = \theta A_t \widetilde{A}$$

where

$$\widetilde{A} \equiv \left( \int_0^1 \bar{A}_i^{\frac{\theta}{1-\theta}} di \right)^{\frac{1-\theta}{\theta}}. \tag{64}$$

Plugging back into $(63)$,

$$P_{it} = \frac{\widetilde{A}}{\bar{A}_i} \tag{65}$$

Next, I characterized the distribution of capital and labor. From $(52)$ , it follows that:

$$V_{it} = S_{it}V_t$$

where $S_i$ is firm $i$ revenue share:

$$S_{it} = \frac{P_{it}Y_{it}}{\int P_{it}Y_{it}di}$$

Plugging this into $(55) - (56)$, the distribution of capital and labor are characterized by:

$$\begin{aligned} K_{it} &= S_{it}K_{t-1} \qquad (66) \\ L_{it} &= S_{it}L_t \qquad (67) \end{aligned}$$

Next, I show that $S_{it}$ is time invariant. Multiply by $P_i$ in equation $(24)$ and replace $(49)$ and $(61)$:

$$P_iY_{it} = P_i^{\frac{\theta}{\theta-1}} \left( G + \int_0^1 D_j dj \right)$$

Integrating across $i$ :

$$\int_0^1 P_iY_{it}di = \left( \int_0^1 P_i^{\frac{\theta}{\theta-1}} di \right) \left( G + \int_0^1 D_j dj \right)$$

It follows:

$$S_{it} = P_i^{\frac{\theta}{\theta-1}}$$

**Proof of 6** To simplify the exposition, I used the formulation used Appendix in which value added in $(30)$ can be equivalently defined as:

$$P_{it}VA_{it}^{SD} = P_{it}Y_{it} - P_{Zt}Z_{it} \tag{68}$$

where:

$$\begin{aligned} Z_{it} &= D_{it}^{\gamma}M_{it}^{1-\gamma} \\ P_{Zt} &= \frac{P_{Dt}^{\gamma}P_{M_t}^{1-\gamma}}{\gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}} \end{aligned}$$

$P_{Zt}$ is the implicit price deflator of $Z_{it}$ and $P_{Dt}$ is the price deflator of the domestic intermediate input composite $D_{it}$.

Single deflated value added (SDVA):

$$VA_{it}^{SD} \equiv Y_{it} - \frac{P_Z}{P_{it}}Z_{it} = \left(1 - \frac{P_{Zt}}{P_{it}}\frac{Z_{it}}{Y_{it}}\right)Y_{it} = \left(1 - S_{Zit}\right)Y_{it}. \tag{69}$$

Using $(53) - (63)$, $(69)$ can be rewritten as:

$$VA_{it}^{SD} = \left(1 - \mu\theta\right)Y_{it}. \tag{70}$$

Replacing $(53)$ into $Y_i$ :

$$Y_{it} = A_{it}^{\frac{1}{1-\mu}}K_{it}^{\alpha}L_{it}^{1-\alpha}\left(\mu\theta\frac{P_{it}}{P_{Zt}}\right)^{\frac{\mu}{1-\mu}}.$$

Combining this equation with $(70)$ :

$$VA_{it}^{SD} = \left(\left(1-\mu\theta\right)^{1-\mu}\left(\mu\theta\right)^{\mu}\right)^{\frac{1}{1-\mu}}\bar{A}_i^{\frac{1}{1-\mu}}A_t^{\frac{1}{1-\mu}}\left(\frac{P_{it}}{P_{Zt}}\right)^{\frac{\mu}{1-\mu}}K_{it}^{\alpha}L_{it}^{1-\alpha},$$

Replacing $(65)$ and $(60)$, SDVA can be rewritten as,

$$VA_{it}^{SD} = TFP_{it}^{SD} \times K_{it}^{\alpha}L_{it}^{1-\alpha}, \tag{71}$$

with $TFP_{it}^{SD}$ given by,

$$TFP_{it}^{SD} = \Omega_0\,\bar{A}_i\,A_t^{\frac{1}{1-\mu}}\,P_{Mt}^{-\frac{\mu(1-\gamma)}{1-\mu}}$$

where

$$\Omega_0 \equiv \left(\left(1-\mu\theta\right)^{1-\mu}\left(\mu\theta\right)^{\mu}\right)^{\frac{1}{1-\mu}}\left(\gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{\mu}{1-\mu}}\widetilde{A}^{\frac{\mu}{1-\mu}}$$

Aggregate SDVA:

$$\begin{aligned} VA_t^{SD} &= \int_0^1 P_iVA_{it}^{SD}di \\ VA_{it}^{SD} &= A_t^{\frac{1}{1-\mu}}\,P_{Mt}^{-\frac{\mu(1-\gamma)}{1-\mu}}\int_0^1 K_{it}^{\alpha}L_{it}^{1-\alpha}di \end{aligned}$$

Combining with $(65)$, the integral can be written as:

$$VA_t^{SD} = TFP_t^{SD}\int_0^1 K_{it}^{\alpha}L_{it}^{1-\alpha}di$$

with,

$$TFP_t^{SD} = \Omega\,A_t^{\frac{1}{1-\mu}}\,P_{Mt}^{-\frac{\mu(1-\gamma)}{1-\mu}}$$

where:

$$\Omega \equiv \left(\left(1-\mu\theta\right)^{1-\mu}\left(\mu\theta\right)^{\mu}\right)^{\frac{1}{1-\mu}}\left(\gamma^{\gamma}\left(1-\gamma\right)^{1-\gamma}\right)^{\frac{\mu}{1-\mu}}\widetilde{A}^{\frac{1}{1-\mu}}$$

Lastly, using $(66) - (67)$, aggregate VASD:

$$VA_t^{SD} = TFP_t^{SD} \times K_{t-1}^{\alpha}L_t^{1-\alpha}$$

**Proof of Proposition 7** Firm level double-deflated value added (DDVA) is given by the Divisia Index:

$$\frac{dVA_{it}^{DD}}{VA_{it}^{DD}} = \frac{1}{1-\mu\theta}\frac{dY_{it}}{Y_{it}} - \frac{\mu\theta}{1-\mu\theta}\frac{dZ_{it}}{Z_{it}}$$

Or:

$$\frac{dVA_{it}^{DD}}{VA_{it}^{DD}} = \frac{dY_{it}}{Y_{it}} - \frac{\mu\theta}{1-\mu\theta}\left(\frac{dZ_{it}}{Z_{it}} - \frac{dY_{it}}{Y_{it}}\right)$$

From SDVA,

$$\frac{dY_{it}}{Y_{it}} = \frac{dVA_{it}^{SD}}{VA_{it}^{SD}}$$

Thus,

$$\frac{dY_{it}}{Y_{it}} = \frac{dTFP_{it}^{SD}}{TFP_{it}^{SD}} + \alpha\frac{dK_{it}}{K_{it}} + (1-\alpha)\frac{dL_{it}}{L_{it}}$$

where:

$$\frac{dTFP_{it}^{SD}}{TFP_{it}^{SD}} = \frac{1}{1-\mu}\frac{dA_t}{A_t} - \frac{\mu(1-\gamma)}{1-\mu}\frac{dP_{Mt}}{P_{Mt}}$$

Using $(53)$ and the fact that price $P_i$ is time invariant - which implies that $mc_i$ (relative to the numéraire) is also constant across periods:

$$\frac{dZ_{it}}{Z_{it}} - \frac{dY_{it}}{Y_{it}} = -\frac{dP_{Zt}}{P_{Zt}}$$

From $(60)$ at $P_D = 1$,

$$\frac{dP_{Zt}}{P_{Zt}} = (1-\gamma)\frac{dP_{Mt}}{P_{Mt}}$$

Therefore:

$$\frac{dVA_{it}^{DD}}{VA_{it}^{DD}} = \frac{dTFP_{it}^{DD}}{TFP_{it}^{DD}} + \alpha\frac{dK_{it}}{K_{it}} + (1-\alpha)\frac{dL_{it}}{L_{it}}$$

where:

$$\frac{dTFP_{it}^{DD}}{TFP_{it}^{DD}} = \frac{1}{1-\mu}\frac{dA_t}{A_t} - \frac{1}{1-\mu}\frac{1-\theta}{\theta}\frac{\mu\theta}{1-\mu\theta}(1-\gamma)\frac{dP_{Mt}}{P_{Mt}}$$

From $(53)$, the revenue share of intermediate goods (domestic and imported) is $\mu\theta$. Hence, $\mu\theta/(1-\mu\theta)$ is share of intermediate goods to value added. From $(59)$, the share of imported inputs on the total cost of intermediate inputs is $(1-\gamma)$. It follows then that the ratio of imports to value added, $P_M M/VA^{SD}$, is $\mu\theta(1-\gamma)/(1-\mu\theta)$.

Aggregate DDVA is a quantity Divisia index,

$$\frac{dVA_t^{DD}}{VA_t^{DD}} = \int_0^1 \omega_{it}\frac{dVA_{it}^{DD}}{VA_{it}^{DD}}di,$$

where $\omega_{it} \equiv P_{it}VA_{it}/VA_t^{SD}$ is the weight for firm $i$ in value added.

Plugging firm level DDVA,

$$\frac{dVA_t^{DD}}{VA_t^{DD}} = \frac{dTFP_t^{DD}}{TFP_t^{DD}} + \alpha\int_0^1 \omega_{it}\frac{dK_{it}}{K_{it}}di + (1-\alpha)\int_0^1 \omega_{it}\frac{dL_{it}}{L_{it}}di,$$

where:

$$\frac{dTFP_t^{DD}}{TFP_t^{DD}} = \frac{1}{1-\mu}\frac{dA_t}{A_t} - \frac{1}{1-\mu}\frac{1-\theta}{\theta}\frac{\mu\theta}{1-\mu\theta}(1-\gamma)\frac{dP_{Mt}}{P_{Mt}}$$

Capital and labor distribution in the model, summarized in $(66)-(67)$, imply:

$$\begin{aligned}\frac{dK_{it}}{K_{it}} &= \frac{dK_{t-1}}{K_{t-1}} \\ \frac{dL_{it}}{L_{it}} &= \frac{dL_t}{L_t}\end{aligned}$$

Therefore,

$$\frac{dVA_t^{DD}}{VA_t^{DD}} = \frac{dTFP_t^{DD}}{TFP_t^{DD}} + \alpha\frac{dK_{t-1}}{K_{t-1}} + (1-\alpha)\frac{dL_t}{L_t}.$$

Given that $\alpha$ is a constant, I can solve for the DDVA function by integrating with respect to time in both sides of the equation:

$$VA_t^{DD} = \Phi_0 TFP_t^{DD} K_{t-1}^{\alpha} L_t^{1-\alpha}$$

where $\Phi_0$ is an integration constant at time $t$ :

$$\Phi_0 = \frac{VA_0^{DD}}{TFP_0^{DD} K_{-1}^{\alpha} L_0^{1-\alpha}}$$

Proceeding in the same way for double-deflated TFP,

$$TFP_t^{DD} = \Phi_1 A_t^{\frac{1}{1-\mu}} P_{Mt}^{b}$$

where $\Phi_1$ is an integration constant at time $t$ :

$$\Phi_1 = \frac{TFP_0^{DD}}{A_0^{\frac{1}{1-\mu}} P_{M0}^{b}}$$

# Appendix B Data

Real GDP is measured by PPP Converted GDP chain-weighted (per equivalent adult or percapita) at 2005 constant prices (RGDPEAQ, Source: Penn World Tables 7.0). Consumption, investment, real net exports and employment headcount also come from Penn World Tables 7.0. The ToT are defined as the ratio of price of imports to the price of exports. These prices are approximated by the implicit price deflators in the national accounts. That is, the price of imports (exports) is computed as the ratio of the value of imports (exports) to imports (exports) at constant prices. Imports at current and constant prices were obtained from World Bank's WDI database. The TFP series from the World Productivity Database-UNIDO, see Isaksson (2007). All series are logged (expect the ratio of real net exports to GDP) and HP detrended using a smoothing parameter of 100.