Dynamic Scoring in a Romer-style Economy

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Abstract

This paper analyzes how changes in tax rates affect government revenue in a Romer-style endogenous growth model. Lower tax rates on financial income (returns to physical capital and intellectual property) do increase the tax base in this environment, but not enough to expand revenue from this tax alone. Lower financial income taxes stimulate innovation and enhance labor productivity in the long run. However, for plausible parameter values, a smaller fraction of a financial income tax cut is self-financing in the Romer-style model than in the Ramsey model. The analysis reveals the dynamics of the economy to be very sluggish and for some variables non-monotonic. Half-lives for many variables are on the order of decades, rather than years.

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1. Introduction

Policymakers who want to change the tax rates that apply to capital income and wages must consider the effects that this will have on overall revenues. Traditional approaches to estimating the revenue effects of tax rate changes focused on static behavioral responses – essentially the short-run response of labor and capital income to a change in the tax rate.[1] Fullerton (1982) discusses whether labor income tax reductions might induce expansions of tax revenue, arguing that such Laffer curve effects (higher government revenue at lower tax rates) are unlikely due to low labor supply elasticities. Malcomson (1986) studies the same question and emphasizes the relevance of general equilibrium effects.

In spite of skepticism about the short-run revenue enhancing effects of tax cuts in the 1980s, recent studies have considered the possibility that the long-run effect of a tax cut is to expand the government's tax collection. Economic research has generally been skeptical of large short-run behavioral responses to tax rate changes. By contrast, more economists believe that long-run responses of labor supply and especially of capital supply may be large, potentially justifying lower tax rates.[2] As Mankiw and Weinzierl (2006) point out in the context of the Ramsey model, the accumulation of capital means that a lower tax rate on capital will ultimately increase the tax base, limiting the long-run reduction in revenues from a tax rate reduction. Auerbach (1996) gives a general presentation of issues related to dynamic scoring. See also Auerbach and Kotlikoff (1987) who study a wide range of issues related to dynamic aspects of fiscal policy.

A number of other studies have considered the dynamic effects of taxes on government revenue in endogenous growth models. Those who have used AK models to explore the effects of taxes include Stokey and Rebelo (1995); Agell and Persson (2001); Ireland (1994); Bruce and Turnovsky (1999). The results of the AK model are fairly straightforward to develop. Production is proportional to capital, $Y = AK$, though individuals may perceive this production function

[1] This issue gained prominence in the early 1980s when some claimed that the United States had tax rates so high that lower tax rates would *increase* tax revenue. This hypothetical situation was known as being on the wrong side of the Laffer Curve.

[2] A notable counterexample is Goolsbee (2000) who argues that a reduction in high-income tax rates had large short-run effects but small long-run effects.

to be $Y = \tilde{A}K^{\alpha}L^{1-\alpha}$. Absent depreciation, the real interest rate is $r = \alpha A$. The growth rate of the economy is determined from the consumption Euler equation: $\dot{C}/C = \sigma((1-\tau_v)\alpha A - \rho)$, where σ is the intertemporal elasticity of substitution, ρ is the discount rate, and τ_v is the tax rate on capital income. As the tax rate on capital income τ_v falls the steady-state growth rate of the economy increases. Tax cuts therefore tradeoff current revenue losses for future revenue gains.

In the AK model above, the optimal growth rate exceeds the growth rate in the decentralized allocation. As such, the appropriate policy is to subsidize capital income, rather than taxing it. It is natural to think that lower taxes (at least when such taxes are positive) would expand the tax base.

Others have used models in which growth is driven by the accumulation of human capital, as in Lucas (1988). Examples include Novales and Ruiz (2002); Pecorino (1995); De Hek (2006); Kim (1998); Milesi-Ferretti and Roubini (1998a,b); Hendricks (1999). While some find that lower tax rates provide extensive stimulus to the economy, others report more modest responses. For example, Hendricks (1999) presents a life-cycle model with human capital accumulation. In his model human capital accumulation drives the economy in the long run, but lower tax rates do not generate large increases in the scale of the economy.

Another strand of the literature on the effects of taxation discusses the uses of government revenue. For example, Jones et al. (1993, 1997) modify the classic Chamley (1986) and Judd (1985) result that the optimal tax rate on capital income is zero. In their model the government uses tax revenue to provide productive public goods. Cutting taxes means having to cut public services, which may reduce output. Ferede (2008) follows a similar approach. In these papers, a reduction in the tax rate may not be followed by large increases in the tax base since the government has to reduce its investments in public infrastructure. For example, while Mankiw and Weinzierl find that 50% of a tax cut on capital income is self-financing, Ferede concludes that only 6% of the tax cut would be self-financing if the tax cut meant the government had to cut back on productive spending.

The AK model as described above relies on large spillovers from using capital to generate endogenous growth through capital accumulation as well as being consistent with facts about the share of income paid to capital. Furthermore, empirical evidence on the effect of the size of government on the economy's growth rate argues against such strong scale effects (Easterly and Rebelo (1993); Mendoza et al. (1997); Jones (1995b)). While this may be consistent with appropriately parametrized AK models, as in Stokey and Rebelo (1995), it is also consistent with the model I present in which tax rates do not affect the steady-state growth rate of the economy, but may affect the steady-state level of activity.

This paper presents new results on the short-run and long-run effects of taxes in an endogenous growth model. The model is a version of Romer (1990), with growth driven by the production of new designs for capital goods.[3] The model is designed to be consistent with long-run balanced growth when the population is also growing. It is a second-generation endogenous growth model in which scale effects are present in the level of activity rather than the growth rate of activity, as emphasized by Jones (1995b). Government policies, such as the tax rate on capital income, do not affect the growth rate, but do affect the level of output and tax revenue. I focus on the issue of how a tax rate reduction affects government revenue. The taxation of returns to accumulated factors does not distinguish between physical capital and knowledge.

This paper adds to the growing literature on the dynamic revenue response to tax rate changes by embedding the policymaker in an endogenously growing economy as modelled by Romer (1990) and Jones (1995a). In this kind of model, the long-run growth rate and level of economic activity are determined in part by the deliberate actions of entrepreneurs and engineers who develop new products and techniques. The extent of innovation is driven by the returns to innovation, and these may be influenced by the supply of capital and labor. Therefore, taxes on these factors may have additional effects on economic activity through this innovation channel.

I will argue that lower tax rates on financial income do not generate large increases in government revenue. In the short run, lower tax rates for financial income reduce government revenue, and the financial income tax base only ex-

[3]Peretto (2011) analyzes more specific changes in tax policy in a quality ladder model without physical capital. Peretto's results suggest the lower tax rates enacted in the Job Growth and Taxpayer Relief Reconciliation Act 2003 substantially reduce the economy's steady-state growth rate and cause massive reductions in welfare due to changes in the composition of research and development activity. A lot of this result depends on the strong scale effects present in Peretto's model, but absent in this paper.

pands modestly even in the long run. Since the innovative sector develops new intermediate product varieties, labor productivity and labor income tax revenue eventually increase. This increase in revenue means that around one-third of the tax cut is self-financing. The analysis draws attention to general equilibrium effects and also to the distinction between short-run and long-run outcomes. The dynamics of the model are so sluggish as to call into question whether the long run effects could be relevant for policy or detected empirically.

The paper proceeds as follows. Section 2 presents the Romer-style model with taxes on capital income and discusses the steady-state and transition dynamics in this model. Section 3 presents comparative dynamic responses of tax revenue to tax rates. Section 4 concludes.

2. The Romer Model with Income Taxes

2.1. The Economic Environment and Agents

The economic environment consists of three production sectors. Final goods are produced using durable intermediate goods and labor. The intermediate goods are produced using final output (in the form of capital) and designs. These designs come from the research and development sector, which uses labor and previously developed designs in production, though existing designs used in the R&D sector are not compensated in the decentralized allocation considered here.

The production of new designs used for making intermediate goods proceeds according to

$$\dot{A}_t = \nu A_t^{\phi} L_{At}^{\lambda}, \quad \phi < 1, \lambda > 0, A_0 > 0, \nu > 0 \tag{1}$$

The intermediate goods sector uses capital together with designs to produce differentiated intermediate inputs. One unit of capital produces one unit of the intermediate good. Each intermediate goods producer owns the design used in production. The measure of designs is A_t. Total production of intermediate goods is determined by the size of the capital stock:

$$\int_0^{A_t} x_{it} di = K_t \tag{2}$$

Final output, which can be consumed or transformed into capital, is produced with intermediate inputs and labor

$$Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} \tag{3}$$

The decentralized equilibrium in this economy features solutions to the following problems.

Household Problem. The household problem is to choose time paths of c_t (consumption) and v_t (financial assets) that maximize

$$\int_0^{\infty} e^{-(\rho-n)t} \frac{c_t^{1-1/\sigma} - 1}{1-1/\sigma} dt$$

taking the full time series of prices and taxes as given, and subject to the following constraints

$$\dot{v}_t = ((1-\tau_v)r_t - n)v_t + (1-\tau_w)w_t - c_t + tr_t, \quad v_0 > 0 \tag{4}$$

$$\lim_{t\to\infty} v_t \exp\left\{ -\int_0^t ((1-\tau_v)r_s - n)ds \right\} \geq 0 \quad NPG \tag{5}$$

where v is assets per person, c is consumption per person, w is the wage rate, r is the pre-tax return on assets, ρ discounts future utility, n is the growth rate of population, τ_w is the labor income tax rate, and τ_v is the tax rate for asset income.[4]

Final Goods Problem. The final goods sector is perfectly competitive. At each point in time, firms demand labor and intermediate goods, taking wages and intermediate goods prices as given, to maximize

$$\left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} - w_t L_{Yt} - \int_0^{A_t} p_{it} x_{it} di \tag{6}$$

Intermediate Goods Problem. Patent-holding firms in the intermediate goods

[4]I abstract from a menu of taxes that includes taxes on consumption. Without a labor-leisure choice or home production consumption taxes do not distort allocations.

sector choose a price p_{it} and quantity to produce to maximize profits

$$x(p_{it})(p_{it} - r_t - \delta) \tag{7}$$

Research and Development Problem. Firms in the R&D sector produce new designs that intermediate goods firms use to produce new intermediate inputs. There is free entry in this sector, but there are externalities. Firms perceive a constant returns to scale production function, ignoring diminishing returns to labor at the aggregate level in this sector. Increases in activity (L_A) generate something akin to congestion effects, lowering the marginal product of labor. Firms sell their patented designs for price P_{At}. They demand labor, paid at the economy-wide wage rate w_t, maximizing profits

$$P_{At}\bar{\nu}_t L_{At} - w_t L_{At} \tag{8}$$

where $\bar{\nu} = A^{\phi} L_A^{\lambda-1}$.

Government Budget. The government simply collects taxes and returns them to households lump sum. In per capita terms, the government budget constraint is:

$$tr_t = \tau_v r_t v_t + \tau_w w_t \tag{9}$$

The government uses two tax instruments: a linear tax on labor income and a linear tax on non-labor income which is derived from two forms of assets, physical capital and intellectual capital. In practice, it may be hard to separate corporate income into a contribution from physical capital and a contribution from intellectual property, and the model's tax code is consistent with this notion. In this model there is neither government consumption nor public goods provision. (See Barro (1990); Jones et al. (1993); Ferede (2008) and others for models where the government can provide productive public goods.) Households are Ricardian, so the timing of tax rebates is irrelevant to the households' decisions. Assuming the government rebates all revenues immediately means that we do not have to keep track of the government's asset position.

2.2. Definition of Equilibrium

The *decentralized equilibrium* in this Romer economy with taxes is a time path for quantities $\{c_t, L_{Yt}, L_{At}, L_t, A_t, K_t, Y_t, v_t, \{\pi_{it}\}_{i=0}^{A_t}, \{x_{it}\}_{i=0}^{A_t}, \bar{\nu}_t, tr_t\}_{t=0}^{\infty}$ and prices $\{P_{At}, \{p_{it}\}_{i=0}^{A_t}, w_t, r_t\}_{t=0}^{\infty}$ such that for all t:

1. c_t, v_t solve the household problem
2. $\{x_{it}\}_{i=0}^{A_t}$ and L_{Yt} solve the final goods firm problem
3. $\{p_{it}\}_{i=0}^{A_t}$ and $\{\pi_{it}\}_{i=0}^{A_t}$ solve the intermediate goods firm problem
4. L_{At} solves the research and development firm problem
5. $Y_t = \left(\int_0^{A_t} x_{it}^{\theta} di\right)^{\alpha/\theta} L_{Yt}^{1-\alpha}$
6. A_t follows from equation (1)
7. K_t satisfies $\int_0^{A_t} x_{it} di = K_t$
8. $\bar{\nu}_t$ satisfies the ideas production function: $\bar{\nu}_t = A_t^{\phi} L_{At}^{\lambda-1}$
9. Asset arbitrage: $r_t = \frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}}$
10. r_t clears the capital market: $v_t L_t = K_t + P_{At} A_t$
11. w_t clears the labor market: $L_{Yt} + L_{At} = L_t$
12. $L_t = L_0 e^{nt}$
13. tr_t satisfies the government budget constraint: $tr_t = \tau_v r_t v_t + \tau_w w_t$

Note that households are taxed on their financial income, derived from either physical capital (K) or intellectual property (A). Asset arbitrage implies that the returns to investing a dollar in each asset class be the same. This is condition (9) in the definition of equilibrium above. In the presence of taxes, this condition implies that capital gains from appreciating prices of intellectual property are taxed. If only profits were taxed, the arbitrage equation would be:

$$(1-\tau_v) r_t = (1-\tau_v)\frac{\pi_{it}}{P_{At}} + \frac{\dot{P}_{At}}{P_{At}},$$

and this would have different implications for the steady-state price of patented ideas. In fact, it is possible to show that in the absence of depreciation, or if income from physical capital is taxed without allowing for depreciation, a policy that taxes only dividend payments and not capital gains of patented technologies makes the composition of the capital stock (i.e., the share of the overall capital stock that is physical capital distinct from intellectual property) invariant to the capital income tax rate.

2.3. Balanced Growth Path

This section presents some properties of the balanced growth path for the economy. Consider first static aspects of the equilibrium allocation. Each intermediate goods producer faces the same problem, so they will produce the same quantity x and sell it for the same price p. The profit π for each patent holder will be the same and all patents will trade at the same price P_A. Since the entire capital stock is divided among the intermediate goods producers, we find that

$$x_{it} = x_t = \frac{K_t}{A_t}$$

and the price charged is a markup over marginal cost

$$p_{it} = p_t = \frac{1}{\theta}(r_t + \delta)$$

so that the profit for each firm is

$$\pi_{it} = \pi_t = \frac{1-\theta}{\theta}(r_t + \delta)\frac{K_t}{A_t} = \alpha(1-\theta)\frac{Y_t}{A_t}.$$

The share of final output Y paid out as pure profits is $\alpha(1-\theta)$. If profits actually represent 3% of final output (Basu and Fernald, 1997) and α is one-third, then the appropriate value for θ would be about 0.9. The gross markup set by intermediate goods producers is $1/\theta$. So in order to match net markups of 10%, θ should be around 0.9.

As in Jones (1995a), using g_z for the growth rate of variable z, the steady-state growth rate of A is given by

$$g_A = \frac{\lambda n}{1-\phi}.$$

Since output is equal to

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_{Yt}^{1-\alpha}$$

the growth rate of output in steady state is given by

$$g_Y = g_K = n + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta} g_A = \left(1 + \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\lambda}{1-\phi}\right) n$$

so that the growth rate of output and capital in steady state depends only on structural parameters, not on investment rates or tax rates.[5] From the capital accumulation equation ($\dot{K}_t = Y_t - C_t - \delta K_t$) we know that consumption grows at the same rate as output and capital in steady state. Therefore, the consumption Euler equation determines the steady-state interest rate: from the household problem, the growth rate of consumption is

$$\dot{c}_t/c_t = \sigma((1-\tau_v)r_t - \rho) \tag{10}$$

$$\rightarrow g_Y - n \tag{11}$$

$$\Rightarrow r^* = \frac{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{g_A}{\sigma} + \rho}{1-\tau_v} \tag{12}$$

Financial income taxes do not affect the long-run growth rate of consumption. Higher tax rates raise the steady-state return to assets the household owns. Since the marginal product of capital is decreasing in the amount of capital, this means that the steady-state capital stock is lower. The stock of knowledge is also lower in a steady state with higher capital income taxes.

The fraction of labor allocated to the research and development sector is consistent with integrated labor markets. The wage paid to researchers is equal to

[5] This is the distinguishing feature of semi-endogenous growth models. By contrast, first generation endogenous growth models (Romer (1990); Grossman and Helpman (1991); Aghion and Howitt (1992)) have strong scale effects so that the growth rate may be influenced by tax rates. See Jones (1999) and Jones (2005) for more on this point.

the wage received by laborers producing final output. Therefore,

$$P_{At}\frac{\dot{A}_t}{L_{At}} = (1-\alpha)\frac{Y_t}{L_{Yt}} \tag{13}$$

which implies that

$$\frac{s_{At}}{1-s_{At}} = \frac{P_{At}\dot{A}_t}{(1-\alpha)Y_t}. \tag{14}$$

On the balanced growth path, asset arbitrage requires $P_{At} = \frac{\pi_t}{r^* - g_{P_A}}$, where $g_{P_A} = g_\pi = g_Y - g_A$. Consequently

$$\frac{s_A^*}{1-s_A^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A))} \equiv \psi^*. \tag{15}$$

The steady-state share of labor allocated to research and development is

$$s_A^* = \frac{\psi^*}{1+\psi^*} = \frac{\alpha(1-\theta)g_A}{(1-\alpha)(r^* - (g_Y - g_A)) + \alpha(1-\theta)g_A}. \tag{16}$$

Of the terms in this equation, only the steady-state interest rate depends on the tax rate applied to capital income. Intuitively, since higher interest rates reduce the present value of future profits resulting from innovation, they reduce the price of a patented idea. Lower prices for patents discourage the research and development required to develop new ideas. Alternatively, higher tax rates cause less capital to be accumulated, raising its marginal product and therefore the interest rate. It follows that the share of labor working in R&D is lower the higher is the tax rate τ_v. As a result, the stock of knowledge is affected by τ_v.

The production function for new ideas shows that on the balanced growth path

$$A_t^* = \left[\frac{\nu L_t^\lambda s_A^{*\lambda}}{g_A}\right]^{\frac{1}{1-\phi}}. \tag{17}$$

Higher financial income taxes raise the interest rate and lower the fraction of workers producing new ideas. Therefore the balanced growth path stock of ideas is lower when tax rates are higher.

Along a balanced growth path, capital and output are equal to

$$\left(\frac{K}{Y}\right)^* = \frac{\alpha\theta}{r^* + \delta} \tag{18}$$

$$Y_t^* = A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{K}{Y}\right)^{*\frac{\alpha}{1-\alpha}} (1 - s_A^*)L_t \tag{19}$$

$$K_t^* = \left(\frac{K}{Y}\right)^* Y_t^* \tag{20}$$

$$= A_t^{*\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}} \left(\frac{K}{Y}\right)^{*\frac{1}{1-\alpha}} (1 - s_A^*)L_t. \tag{21}$$

Increases in the financial income tax rate lower A and K/Y but increase the fraction of workers producing physical output, so there are competing effects of τ_v on output. This mirrors the relationship between the optimal and equilibrium allocations in the Romer model. For some parametrizations the equilibrium involves overinvestment in R&D, while in others there is too little R&D (s_A is too small).[6] If their effect through labor market channels is strong enough, higher financial income taxes could actually increase the size of the stock of physical capital, though this requires unlikely parameter values.

The stock of assets includes both physical capital and patented ideas. The total value of these assets on the balanced growth path is

$$V_t^* = K_t^* + P_{At}^* A_t^* \tag{22}$$

$$= \left(\frac{\alpha\theta}{r^* + \delta} + \frac{\alpha(1-\theta)}{r^* - g_{P_A}}\right) Y_t^* \tag{23}$$

so that the share of assets in the form of physical capital (versus patented ideas) depends on the steady state interest rate, which in turn depends on the tax rate on capital income.[7]

[6]For more, see Jones and Williams (1998, 2000) for a discussion of the social returns to R&D. Those papers discuss a related model that also includes a creative destruction distortion. Jones (2005) shows how the socially optimal rates of investment relate to the decentralized allocation's rates of investment in a model that does not have the creative destruction distortion.

[7]If $\delta = 0$ and $\alpha = \theta$, then $V_t^* = \alpha\frac{Y_t^*}{r^*}\left(\frac{r^* - \alpha n}{r^* - n}\right)$. In that case the asset structure of the economy depends on the growth rate of population and on the steady-state interest rate, which may respond to τ_v. Assuming there is no population growth, the share of assets that are physical capital is α, independent of τ_v. More generally, the effect of population growth on the composition of

Since the value of innovations in the R&D sector are paid out to researchers as wages, changes in the allocation of labor and of the price of new ideas can affect the labor share of income. Note that total income in this model is $Y + P_A \dot{A}$. Payments to labor are $wL = (1-\alpha)Y + P_A \dot{A}$. A reduction in the tax rate on capital income lowers the real interest rate and raises the value of output in the R&D sector relative to the final goods sector. This in turn means the the labor share of income rises.

Tax revenue for the government is $\tau_v r^* V^* + \tau_w w^* L$. Since there are two taxes, there are two tax bases. The financial income tax base is $r^* V^*$ while the labor income tax base is $w^* L$. The Laffer conjecture is that a reduction in the tax rate will cause the tax base to increase so much that the product of the two (revenue) increases. Here we have two tax bases. The analysis reveals that the financial income tax base does rise in response to a lower tax rate, but not enough to generate revenue gains. Having a lower financial income tax rate raises the labor income tax base too, but this effect is too small to generate an overall increase in revenue at lower financial income tax rates.

2.4. Transition Dynamics

This section discusses transition dynamics for a log-linearized version of the economy. This forms the basis for the comparative dynamics exercises in section 3.. More details are provided in the appendix. I log-linearize the model as

assets depends on other parameters in the model. If $\alpha = \theta$, then higher n causes the growth rate of the price of an idea to be higher. This lowers the current price of a new idea and means more of the stock of assets will be physical capital. If $\alpha < \theta < 1$ and $\lambda > 1 - \phi$, entirely plausible values, it is possible for this effect to be reversed. For some such combinations of parameters higher population growth lowers the growth rate of the price of an idea, increasing its current price and the extent of investment in R&D.

follows. Define the vector γ as

$$\gamma_t = \begin{pmatrix} \gamma_{1t} \\ \gamma_{2t} \\ \gamma_{3t} \\ \gamma_{4t} \end{pmatrix} \equiv \begin{pmatrix} \log(C_t/K_t) \\ \log(\bar{Y}_t/K_t) \\ \log(s_{At}) \\ \log(\bar{\dot{A}}_t/A_t) \end{pmatrix} \tag{24}$$

where $\bar{Y}$ is the maximum output that could be obtained at a point in time, based on setting s_A equal to zero, and $\bar{\dot{A}}$ is the maximum rate of change of A that is possible at a point in time, based on setting s_A equal to one.[8] Therefore,

$$Y_t = A_t^{\alpha\frac{1-\theta}{\theta}} K_t^{\alpha} L_t^{1-\alpha} (1-s_{At})^{1-\alpha} = \bar{Y}_t (1-s_{At})^{1-\alpha} \tag{25}$$

and

$$\bar{\dot{A}}_t = \nu A_t^{\phi} L_t^{\lambda} = \dot{A}_t s_{At}^{-\lambda} \tag{26}$$

The limiting values of these variables are determined as follows. Equation (16) determines the steady-state value γ_3^*. Then γ_4^* is equal to $\log(g_A (s_A^*)^{-\lambda})$. The steady-state interest rate in equation (12) determines the steady-state capital output ratio, which combined with s_A^* determines γ_2^*. Finally, $C/K = Y/K - \dot{K}/K - \delta$ which determines γ_1^*.

It is convenient to work with these four variables since they are each constant on a balanced growth path. Two correspond roughly to the state variables in the model (γ_4 relates to the stock of knowledge, γ_2 to the capital stock), and do not jump. By contrast, the other two correspond to control variables (γ_1 to the investment rate, and γ_3 to the intensity of research and development efforts) and can jump. The dynamics of the four variables are determined by two initial condi-

[8]Arnold (2006) analyzes the dynamics of this model with $\lambda = 1$. He reduces the model to a similar set of variables as I do here. The main differences are that one of his variables corresponds roughly to P_A rather than s_A and the variable that represents the output-capital ratio in his paper uses actual output rather than maximum output. I prefer to use maximum output so that this variable is a genuine state variable and is unable to jump. With my set-up there are two obvious state variables and two control variables that correspond to the two key allocation decisions in the model: to consume or invest, and to produce final output or to produce ideas. See also Schmidt (2003) for another discussion of transition dynamics in the Romer model.

tions (K_0 and A_0) and two endpoint conditions (the limiting behavior of C and s_A).

The rate of change of γ is given by

$$\dot{\gamma}_t = \begin{pmatrix} \frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t} \\ \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \\ \frac{\dot{s_{At}}}{s_{At}} \\ \lambda \frac{\dot{L}_t}{L_t} - (1-\phi)\frac{\dot{A}_t}{A_t} \end{pmatrix} \tag{27}$$

Equations for three elements of this vector are straightforward. The growth rate of consumption is given by the household's Euler equation. The growth rate of the capital stock comes from the capital accumulation equation. The growth rate of the maximum growth rate of A is determined by the growth rate of A and of population. The growth rate of s_{At} is rather more complicated, and is based on the dynamics of the labor market equilibrium condition in equation (14). This equation implies that the rate of change of s_A is influenced by the rate of change of P_A, A, K, and L. If the price of patented ideas is rising over time then, all else equal, the fraction of labor allocated to R&D will also be rising. The derivation of each equation is covered in the appendix.

For all the calibrations I applied, the log-linearized system of equations was characterized by two negative and two positive eigenvalues. This is consistent with there being two state variables (K and A) and two jump variables (C and s_A). Arnold (2006) shows that a slightly simpler version of this model without taxes must have two negative and two positive eigenvalues.

3. Comparative Dynamics: Response to τ_v Changes

This section discusses the response of the economy in general and tax revenues in particular when there is a change in the financial income tax rate. It shows the long-run response of tax revenues to tax rates as well as transition paths for a range of variables when there is a change in τ_v.

3.1. Static Response of Tax Revenue

As a benchmark consider the Ramsey model where the interest rate at a point in time is determined by the capital stock. Factor supplies are inelastic in the short-run, so the marginal product of capital is a given. The short-run elasticity of tax revenue with respect to the capital income tax rate is equal to one in the Ramsey model (Mankiw and Weinzierl (2006)). This is because the marginal product of capital, which determines the real interest rate, is pinned down by the capital stock, and the capital stock cannot jump.

In the Romer model, the marginal product of capital depends on the allocation of labor between the two sectors. And even if the real interest rate were not to jump in the R&D model, if the price of a patented idea jumps, then the stock of assets whose income streams are taxed also jumps so that the elasticity of tax revenue with respect to changes in the tax rate need not be one. For some parametrizations, tax revenue jumps less than the percentage of the change in the tax rate, while in other parametrizations it jumps more. In spite of these possibilities, for my calibrations, the elasticity of financial income tax revenue with respect to τ_v is approximately one in the short run.

3.2. Long-run Response of Tax Revenue

When thinking about the long-run response of tax revenues to changes in the financial income tax rate, it potentially matters whether we consider the effect on financial income tax revenues or overall tax revenues. Since changes in τ_v affect incentives to innovate, they have long-run effects on labor productivity and wages. So a tax cut for financial income can stimulate labor income tax revenue eventually.

First consider the effect on revenue from the financial income tax. In the long run, a small number of key parameters affect the response of tax revenue to the tax rate on financial income. First, θ, which governs the substitutability in production of different kinds of capital goods, has a particularly important role. For low values of θ, low tax rates can generate high tax revenues, so that a Laffer curve effect is salient. Evidence on the share of income received as pure profits (returns to patents) and on markups suggest that such values of θ are implausible. For higher values of θ (closer to 0.9 so that markups are around 10%) suggest that tax revenues are maximized at tax rates closer to 85%.[9]

Figure 1a shows the (log of) steady-state tax revenue as a function of the tax rate for three different values of θ.[10] For high values of θ the long-run elasticity of output with respect to A is low. Therefore, lower tax rates, while they increase the stock of knowledge and hence output, do not raise government revenue except when starting from extremely high tax rates. For lower values of θ (such as 0.5, which would imply markups of 100% and pure profit shares of 16%), tax revenue peaks as a function of the tax rate at relatively moderate tax rates. Such low values of θ imply that a larger share of income is accrued as pure profits than findings of Basu and Fernald (1997) and Broda and Weinstein (2006) would imply.

Total tax revenue derives from financial income taxes and labor income taxes. Since higher financial income taxes lower productivity, by reducing the extent of innovation and capital accumulation, they lower wages and labor income tax revenue. So even if higher financial income tax rates increase financial income tax receipts, they might lower overall tax revenue through the effect on labor markets, and conversely. Some pertinent results are summarized in Figure 1b, which shows that the tendency for total revenues to be higher at low tax rates is most pronounced for implausibly low values of θ. However, even for $\theta = 0.7$ lower tax rates tend to increase total revenue.

The subsequent figures, 2a and 2b, show steady-state output and consumption as functions of the tax rate, again for different values of θ. Since taxing asset income does not correct the underlying distortions in this economy, higher taxes are always associated with lower output and consumption in the model.

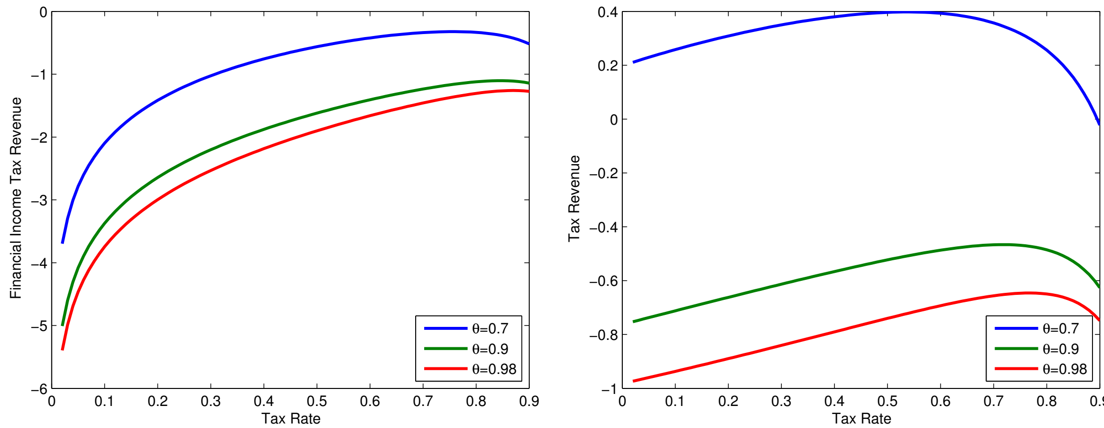
Figures 3a and 3b show how steady-state tax revenue responds to the tax rate for different values of ϕ and λ. Since there is less direct evidence that can be brought to bear on appropriate values of ϕ and λ, the figures show a wide range of possibilities. The central calibration sets $\lambda = 0.5$ which implies that a doubling of the number of workers in R&D leads R&D output to increase by about 40%, so

[9]In the Ramsey model, tax revenues are maximized when $\tau_v = (1-\alpha)(1-\tau_w)$ where α is the elasticity of ouptut with respect to capital and τ_w is the tax rate on labor income.

[10]These are for a particular point in time. On the balanced growth path tax revenue grows at the same rate regardless of the tax rate, so choosing a different point in time would amount to shifting the curves in this graph up or down by some constant amount.

Figure 1: Tax Revenue as a Function of τ_v

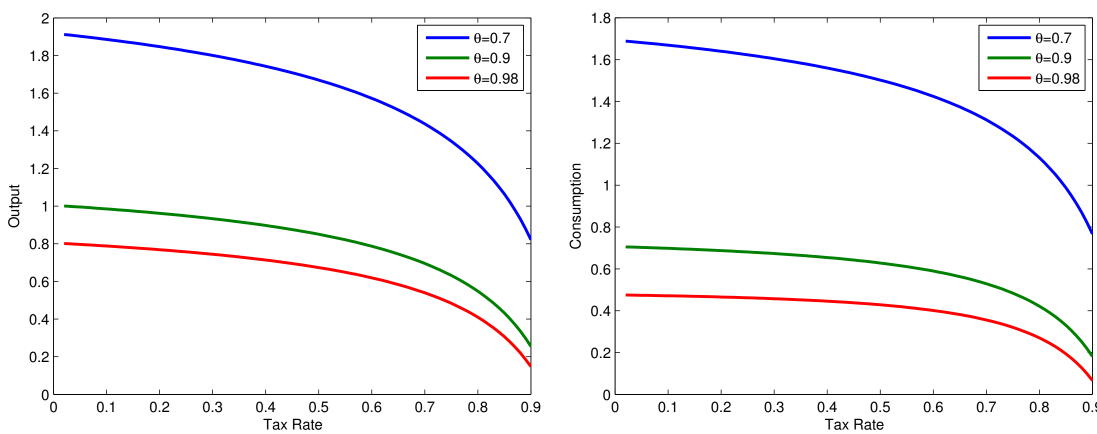
(a) Financial Income Tax Revenue. (b) Total Tax Revenue.



Notes: log of financial income tax revenue (first panel, $\tau_v rV$) and total tax revenue (second panel, $\tau_v rV + \tau_w wL$) along balanced growth path as a function of τ_v. Parameter values in Table 1.

Figure 2: Steady-State Output and Consumption as Functions of τ_v

(a) Output. (b) Consumption.



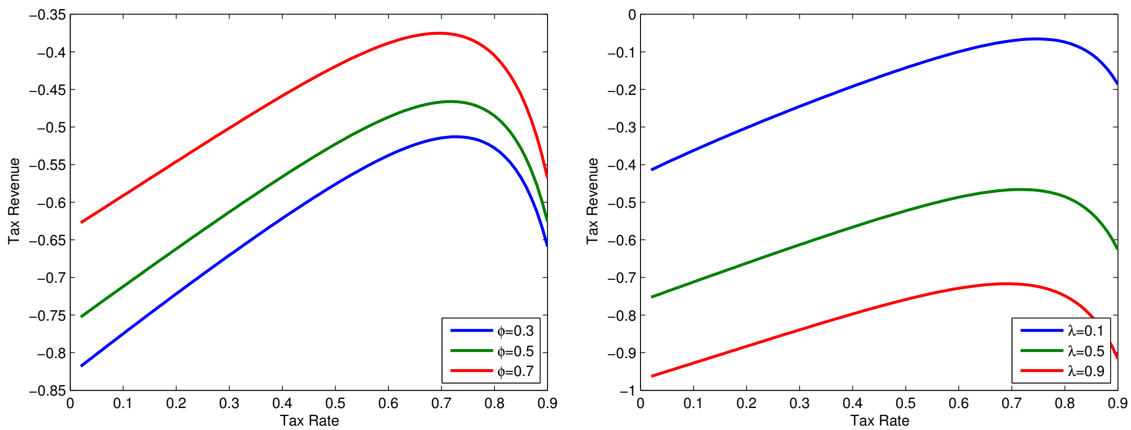
Notes: output and consumption, in logs, at a point in time along the balanced-growth path; any other time on the balanced-growth path shifts these lines up or down in parallel.

Table 1: Calibrated Values of Model Parameters

Parameter	Value	Comment
α	1/3	Elasticity of Y w.r.t. K
θ	0.9	Related to Elasticity of Substitution between Varieties of Capital
δ	0.05	Depreciation rate for physical capital
ν	1	Productivity in R&D
λ	0.5	Elasticity of $\dot{A}$ w.r.t. L_A
ϕ	0.5	Elasticity of $\dot{A}$ w.r.t. A
ρ	0.02	Discount rate
σ	1	Intertemporal elasticity of substitution
n	0.01	Population growth rate
τ_w	0.25	Labor income tax rate

Figure 3: Steady-State Total Tax Revenue as a Function of τ_v

(a) For values of ϕ (b) For values of λ



Notes: total tax revenues, in logs, at a point in time along the balanced-growth path; any other time on the balanced-growth path shifts these lines up or down in parallel.

that there are significant stepping-on-toes problems. At other extremes shown, these problems are severe when $\lambda = 0.1$ and minor when $\lambda = 0.9$. For values of ϕ displayed, the central calibration has $\phi = 0.5$ so that a doubling of the existing stock of knowledge causes innovation to be about 40% greater at present, holding constant the number of workers in the sector. These spillovers from past discoveries could be stronger ($\phi = 0.7$) or weaker ($\phi = 0.3$). The combination of ϕ and λ in the central calibration are consistent with slow steady-state growth, as argued in Jones (2002). Variation in ϕ and λ do have some effect on the shape of the tax revenue-tax rate curve. However, for each value of ϕ graphed, the revenue-maximizing tax rate is between 60% and 80%. For different values of λ the revenue-maximizing tax rates again exceed 50%.

3.3. Dynamic Response of Tax Revenue

This section illustrates several points about the response of the economy to a reduction in financial income tax rates from 40% to 30% while labor income taxes are maintained at 25%.[11] All tax revenue is immediately returned in lump-sum fashion to households. The economy starts on its balanced growth path, then faces a new, permanently lower tax rate. The dynamic response of the economy is computed using the log-linearized version of the model.

Figures 4a and 4b show the responses of the two key allocation choices in the economy, (log) consumption and the allocation of labor between the two sectors. Initially consumption drops around 7%, but after a twelve periods rises to be above the previous balanced growth path, eventually converging to the new steady-state with consumption around 2% higher than it would have been without the tax reform. This cut in current consumption is a response to the suddenly higher after-tax returns available. Consumers are willing to reduce current consumption because, more than at the higher tax rate, this generates a build-up in the capital stock that advances output.

As for all other variables in this model, the convergence of consumption does not occur at a constant rate. Since there are two state variables, there are two eigenvalues that govern the speed of convergence to the steady state. Initially, consumption converges swiftly, with half the gap to the new balanced growth path closed in about five years. Each subsequent five year period sees the gap between actual and steady-state consumption close by a smaller and smaller percentage. There is a long period of very gradual convergence to the new balanced growth path.

Initially the share working in R&D jumps up toward the new steady state, but after the jump the share gradually falls before eventually converging to the new steady-state value.[12] Mechanically, the non-monotonic convergence is due to the dynamics of the system being governed by two eigenvalues. For s_A the signs of the coefficients on the corresponding eigenvectors are opposite, hence the initial drift away from the steady state before convergence.

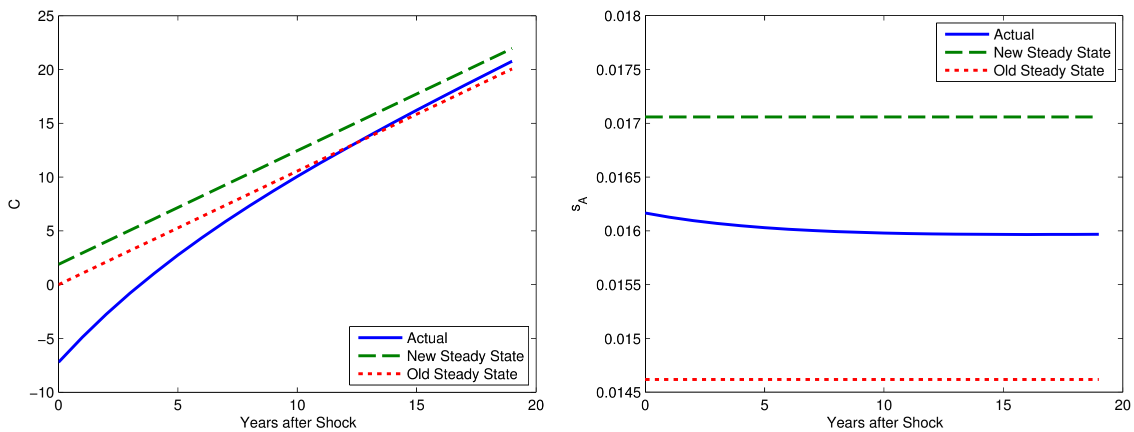
More intuitively, the dynamics of the labor share are determined by the relative values of the marginal product of labor in final output and in R&D. At first,

[11]OECD (2011) reports a corporate income tax rate of 39.2% for the United States (see Table II.1). Exemptions mean that firms typically do not pay a marginal tax rate of 39.2% to the U.S. government, but the tax does affect their incentives, and there are additional taxes assessed to non-wage income, such as dividend taxation, that should be thought of as included in τ_v.

[12]This is confirmed in Figure B.1.

Figure 4: Response of Allocations to a Lower Tax Rate

(a) Consumption. (b) Labor Allocation.



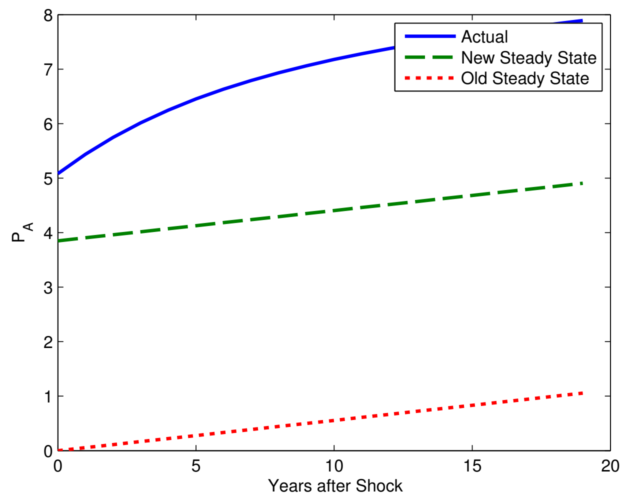
Notes: log of the consumption (left panel, C) and the share of workers employed in R&D (right panel, s_A) in response to a date 0 reduction in τ_v from 40% to 30%. Parameter values in Table 1.

when the financial income tax falls, the value of producing new designs increases, so there is a jump in s_A toward the R&D sector. After this jump, there is an expansion of A, which increases the incentives to invest in physical capital, which in turn increases the marginal product of labor in final output. Therefore labor migrates back toward the final output sector. The incentives to invest in K slow quickly, but the ongoing accumulation of A gradually increases the perceived marginal product of labor in the R&D sector, drawing workers back toward that sector.

Figure 5 shows the price of patented inventions jumping up from its initial balanced growth path, overshooting its new balanced growth path. This graph suggests how difficult it could be to infer the effects of a policy change from the data. The asset price jumps up initially and then continues to grow, showing the same nonmonotonic convergence as s_A. After twenty years, an observer could reasonably, but wrongly, guess that the policy raised asset prices around 7% rather than the actual 4%. The glacial speed of convergence in the model makes it hard to detect the effects of tax policy.

Figure 6a shows the financial income tax revenue generated for the govern-

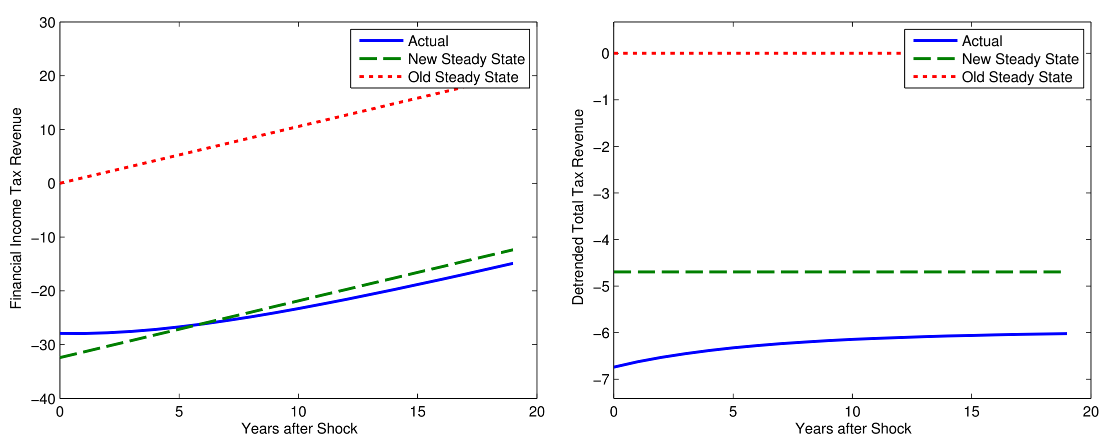
Figure 5: Response of the Price of a Patent to a Lower Tax Rate



Notes: log of the price of a patented idea (P_A) in response to a date 0 reduction in τ_v from 40% to 30%. Parameter values in Table 1.

Figure 6: Dynamic Response of Tax Revenue to a Lower Tax Rate

(a) Financial Income Tax Revenue. (b) Total Income Tax Revenue (detrended).



Notes: log of financial income tax revenue (first panel, $\tau_v rV$) and detrended total tax revenue (second panel, $\tau_v rV + \tau_w wL$) in response to a date 0 reduction in τ_v from 40% to 30%. Parameter values in Table 1.

ment falls initially, since the tax rate is reduced. It continues to grow more slowly than its steady-state growth rate, dipping below the new balanced growth path to which it eventually converges. Since the financial income tax base ($r_t V_t$) barely responds in the short run, the elasticity of financial income tax revenue with respect to the tax rate is approximately one. Moreover, the base moves very little even in the long run, with the expansion of assets being offset by the lower returns achieved by these assets.

Accounting for dynamic general equilibrium effects in which the expanded innovation and capital accumulation increase wages shows that the overall tax revenue falls around 7% when the tax rate is cut, but around 5% in the long run. About 30% of the inital drop in revenue is recovered in the long run due to greater labor income tax revenue.[13] While dynamic general equilibrium considerations suggest that some of a financial income tax cut is self-financing, these effects emerge very slowly, so slowly that it is unlikely they could be detected in the data.

Remarkably, the amount of a financial income tax rate that is self-financing is smaller in this Romer-style model than in Mankiw and Weinzierl's Ramsey model. One might think that, since productivity responds to the financial income tax in the Romer model, financial income tax cuts would be more self-financing in the Romer model. For a different calibration, in particular with a lower value of θ, a financial income tax rate would be self-financing, so some of the result is due to the calibration. In addition, the interpretation of financial income in the Romer model differs from the Ramsey model. In the Ramsey model, this income is derived solely from physical capital, while in the Romer model it is due to physical capital and to intellectual property. Since the income derived from intellectual property is less responsive than income due to physical capital, financial income tax cuts are less self-financing in the Romer model.

[13] About two-thirds of the tax cut is self-financing if we increase knowledge spillovers ($\phi = 0.9$) and reduce congestion in R&D ($\lambda = 0.9$). If, instead, $theta$ is reduced to 0.6, about 90% of the tax cut is self-financing.

4. Conclusion

This paper has investigated the dynamic response of tax revenue to changes in the tax rate applied to capital income in a model of endogenous growth through research and development. The model modifies Romer (1990) and Jones (1995a) to incorporate a tax on financial income, which is income derived as returns to physical capital and intellectual property, and a tax on labor income. The model is log-linearized and the dynamic response of the economy to a tax cut is presented.

For some parameter values, especially for low values of θ, very strong dynamic Laffer curve effects are present. When the model is calibrated to be consistent with evidence on markups and profit shares, about one-third of a reduction in the financial income tax rate self-financing, less than in the Ramsey model even though there is an additional margin of adjustment. The analysis of a reduction in τ_v from 40% to 30% shows that most of the self-financing comes from wage growth. That is, dynamic general equilibrium effects are important: the extra research and development activity stimulates labor productivity and wages, increasing labor income tax revenue in the long run, though not by enough to offset the initial revenue losses.

The dynamic analysis reveals that the response of the economy to a change in the financial income tax rate is slow, occurs at variable speeds, and potentially non-monotonically. The half-lives of some variables are on the order of decades, rather than years. For variables that converge more quickly at first, such as consumption, the later convergence slows significantly. This aspect of the model's dynamics makes it plausible that the true effects of such a policy might never be uncovered empirically.

A crucial caveat to interpreting these results is the nature of international spillovers from research and development. The model presented here is a closed-economy model. It may contain useful insights for a world economy in which fiscal austerity is commonplace. However, what are the effects for a single country raising or lowering financial income taxes when others are not? Answering such questions requires a different model that explicitly captures the fact that only some R& D occurs domestically and carefully considers how foreign activity affects the domestic economy. An analysis along these lines would clearly come to different quantitative conclusions, and possibly to different qualitative conclusions too, but would build on the insights in this paper regarding the incentives of innovators and the sluggishness of the responses to tax rate changes.

A Appendix: Log-Linearizing the Model

A1. Rate of Change

From the household's Euler Equation, we know that

$$\frac{\dot{C}_t}{C_t} = \sigma((1-\tau_v)r_t - \rho) + n$$

where

$$\begin{aligned} r_t &= \alpha\theta\frac{Y_t}{K_t} - \delta \\ &= \alpha\theta\frac{\bar{Y}_t}{K_t}(1-s_{At})^{1-\alpha} - \delta \\ &= \alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta \end{aligned}$$

The capital accumulation equation is standard and gives

$$\begin{aligned} \frac{\dot{K}_t}{K_t} &= \frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta \\ &= e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - e^{\gamma_{1t}} - \delta. \end{aligned}$$

Therefore,

$$\dot{\gamma_{1t}} = \sigma((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta) - \rho) + n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta$$

The second element of γ changes according to the growth rates of $\bar{Y}$ and K. Note that maximum output can be written as

$$\bar{Y}_t = A_t^{\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}}\left(\frac{K_t}{\bar{Y}_t}\right)^{\frac{\alpha}{1-\alpha}} L_t$$

so the growth rate of $\bar{Y}$ is

$$\frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t} + \frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t} - \frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right) + n$$

This implies that

$$\begin{aligned}
\dot{\gamma_{2t}} &= \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}\frac{\dot{A}_t}{A_t} - \frac{\alpha}{1-\alpha}\left(\frac{\dot{K}_t}{K_t} - \frac{\dot{\bar{Y}}_t}{\bar{Y}_t}\right) + n - \frac{Y_t}{K_t} + \frac{C_t}{K_t} + \delta \\
&= \frac{\alpha}{1-\alpha}\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} - \frac{\alpha}{1-\alpha}\dot{\gamma_{2t}} + n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta \\
&= \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} + (1-\alpha)(n - e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} + e^{\gamma_{1t}} + \delta)
\end{aligned}$$

The rate of change of γ_4 is straightforward also.

$$\begin{aligned}
\dot{\gamma_{4t}} &= (\phi-1)\frac{\dot{A}_t}{A_t} + \lambda\frac{\dot{L}_t}{L_t} \\
&= -(1-\phi)e^{\lambda\gamma_{3t}+\gamma_{4t}} + \lambda n
\end{aligned}$$

The rate of change of γ_{3t} is equal to

$$\begin{aligned}
\dot{\gamma_{3t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}((1-\tau_v)(\alpha\theta e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - \delta) - (1-\alpha-\lambda)n \\
&\quad -\alpha(e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} - e^{\gamma_{1t}} - \delta) \\
&\quad +e^{\lambda\gamma_{3t}+\gamma_{4t}}(\phi - \alpha\frac{1-\theta}{\theta} + (1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}}))
\end{aligned}$$

A2. Linearization

Linearize the transition equations above. Evaluate the Jacobian at the steady-state values.

$$\begin{aligned}
\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{1t}} &= e^{\gamma_{1t}} \\
\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{2t}} &= e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right) \\
\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{3t}} &= -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\left(\alpha\theta\sigma(1-\tau_v)-1\right)\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \\
\frac{\partial\dot{\gamma_{1t}}}{\partial\gamma_{4t}} &= 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial\dot{\gamma_{2t}}}{\partial\gamma_{1t}} &= (1-\alpha)e^{\gamma_{1t}} \\
\frac{\partial\dot{\gamma_{2t}}}{\partial\gamma_{2t}} &= -(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha} \\
\frac{\partial\dot{\gamma_{2t}}}{\partial\gamma_{3t}} &= \alpha\lambda\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}} + (1-\alpha)^2 e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}} \\
\frac{\partial\dot{\gamma_{2t}}}{\partial\gamma_{4t}} &= \alpha\frac{1-\theta}{\theta}e^{\lambda\gamma_{3t}+\gamma_{4t}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\dot{\gamma_{3t}}}{\partial\gamma_{1t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{1t}} \\
\frac{\partial\dot{\gamma_{3t}}}{\partial\gamma_{2t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\alpha e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}((1-\tau_v)\theta-1) \\
\frac{\partial\dot{\gamma_{3t}}}{\partial\gamma_{3t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}(\quad \alpha(1-\alpha)e^{\gamma_{2t}}(1-e^{\gamma_{3t}})^{1-\alpha}\frac{e^{\gamma_{3t}}}{1-e^{\gamma}_{3t}}(1-(1-\tau_v)\theta) \\
&\quad + e^{\lambda\gamma_{3t}+\gamma_{4t}}\lambda(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma_{3t}}}) - e^{\lambda\gamma_{3t}+\gamma_{4t}}\frac{\alpha}{1-\alpha}\frac{1-\theta}{e^{\gamma_{3t}}} \quad) \\
&\quad - \frac{\alpha}{1-e^{\gamma_{3t}}}\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}\frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}\dot{\gamma_{3t}} \\
\frac{\partial\dot{\gamma_{3t}}}{\partial\gamma_{4t}} &= \frac{1}{1-\lambda+\alpha\frac{e^{\gamma_{3t}}}{1-e^{\gamma_{3t}}}}e^{\lambda\gamma_{3t}+\gamma_{4t}}\left(\phi-\alpha\frac{1-\theta}{\theta}+(1-\theta)\frac{\alpha}{1-\alpha}\frac{1-e^{\gamma_{3t}}}{e^{\gamma^{3t}}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\dot{\gamma_{4t}}}{\partial\gamma_{1t}} &= 0 \\
\frac{\partial\dot{\gamma_{4t}}}{\partial\gamma_{2t}} &= 0 \\
\frac{\partial\dot{\gamma_{4t}}}{\partial\gamma_{3t}} &= -(1-\phi)\lambda e^{\lambda\gamma_{3t}+\gamma_{4t}} \\
\frac{\partial\dot{\gamma_{4t}}}{\partial\gamma_{4t}} &= -(1-\phi)e^{\lambda\gamma_{3t}+\gamma_{4t}}
\end{aligned}
$$

We can write the linearized system as

$$
(\gamma_t \dot{-} \gamma^*) \approx \Gamma(\gamma_t-\gamma^*)
$$

$$\Gamma = \begin{pmatrix} \frac{r^*+\delta}{\alpha\theta} - g - \delta & \frac{r^*+\delta}{\alpha\theta}(\alpha\theta\sigma(1-\tau_v)-1) & -\frac{r^*+\delta}{\theta}(1-\theta)\frac{\lambda n}{1-\phi}\frac{\alpha\theta\sigma(1-\tau_v)-1}{\rho+\frac{g-n}{\sigma}-(g-g_A)} & 0 \\ (1-\alpha)(\frac{r^*+\delta}{\alpha\theta}-g-\delta) & -(1-\alpha)\frac{r^*+\delta}{\alpha\theta} & \frac{\lambda n}{1-\phi}\frac{1-\theta}{\theta}(\alpha\lambda+(1-\alpha)(r^*+\delta)) & \alpha\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi} \\ x\alpha(\frac{r^*+\delta}{\alpha\theta}-g-\delta) & x\alpha\frac{r^*+\delta}{\alpha\theta}((1-\tau_v)\theta-1) & \Gamma_{3,3} & \Gamma_{4,4} \\ 0 & 0 & -\lambda^2 n & -\lambda n \end{pmatrix}$$

where r^* is the steady-state interest rate from equation (12), and g is the steady-state growth rate of output, capital and consumption; $x = (1-\lambda+\frac{\alpha^2}{1-\alpha}(1-\theta)\frac{\lambda n}{1-\phi}\frac{1}{(1-\tau_v)r^*-(g-g_A)})^{-1}$, and

$$\begin{aligned} \Gamma_{3,3} \quad = \quad & x(\alpha(r^*+\delta)\frac{1-\theta}{\theta}\frac{\lambda n}{1-\phi}\frac{1-(1-\tau_v)\theta}{(1-\tau_v)r^*-(g-g_A)} + \frac{\lambda^2\phi n}{1-\phi} - \\ & \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^*-(g-g_A))) \end{aligned}$$

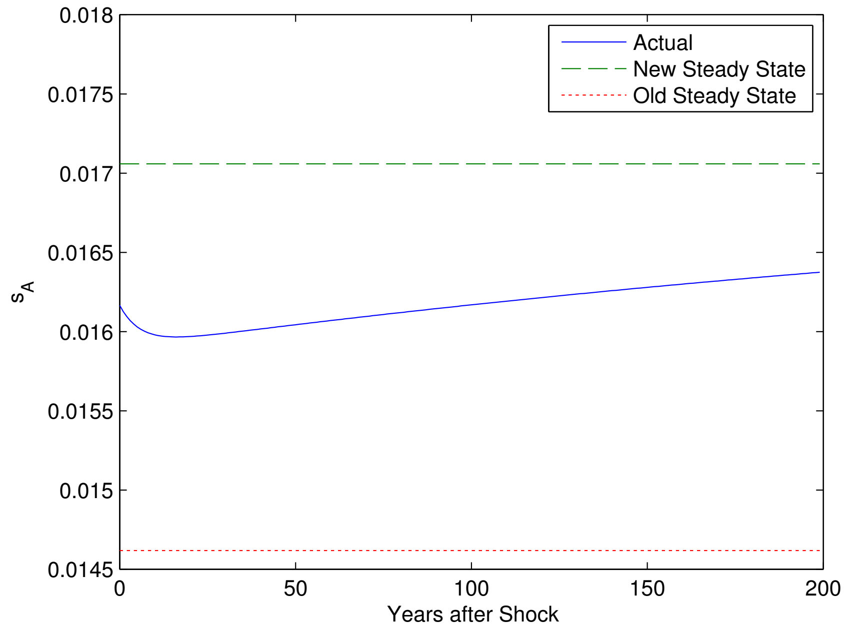
and

$$\Gamma_{4,4} = x\left(\frac{\lambda^2\phi n}{1-\phi} - \frac{\alpha\lambda^2 n}{1-\phi}\frac{1-\theta}{\theta} + \lambda((1-\tau_v)r^*-(g-g_A))\right)$$

A3. Solutions of the Linearized System

The linearized system of equations is solved using the standard eigenvalue decomposition. Initial conditions for K and A generate the required boundary conditions to obtain the particular solution.

Figure B.1: Labor Allocation Response to a Lower Tax Rate



B Appendix: Longer Horizon Responses to a Tax Change

Figure B.1 confirms that s_A eventually converges to the new steady-state value. The share of labor working in the R&D sector converges slowly and non-monotonically. When the tax rate falls, s_A initially jumps up toward the new steady-state value. For several periods after than, as capital accumulates pushing up the marginal product of labor in final output, labor migrates back toward the final output sector. With the passage of time, the advance of the stock of designs increases (perceived) R&D productivity so that workers are drawn back toward the R&D sector.

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