Bank Loan Securitization: an alternative for banks to find liquidity

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Abstract

Asset backed securitization can convert illiquid assets, as bank loans, into liquidity. This paper analyzes securitization of loans as a mechanism for banks to find the liquidity needed to finance projects. I develop a model in which a bank can have an investment opportunity when having only illiquid assets such as loans. The bank can raise funds in two ways: by selling its loans on a secondary market (i.e. securitize), or by borrowing from investors using its loans as collateral. By assumption the project to be financed is socially beneficial, and the bank is the only one to know the quality of its loans. The social first best cannot be achieved in any funding alternative, and only when the cost of securitization is small, selling loans is the social best alternative. I finally show that, when the government intervenes by taxing the bank, the first best can be reached, but only when using securitization.

Keywords: Securitization, Liquidity.
JEL Codes: G21, G28.

1. Introduction

Securitization is a complex financing technique used to convert illiquid assets such as mortgages and car loans into tradeable securities. This technique was introduced in the 1970s to inject liquidity in the American mortgage market. The Government National Mortgage Association (Ginnie Mae) issued the first securities backed by a pool of its residential mortgages. Since Asset Backed Securities (ABS) were created, most of the issued ABS have

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been based on loans originated by financial intermediaries, mainly banks. The most common ABS are the mortgage backed securities (MBS).

In an asset backed securitization a bank sells loans that are converted into securities that are negotiated in a secondary market. Banks are highly interested on selling some of their loans for different reasons. When a bank sells some of its loans in a secondary market, it can take out those loans from its balance sheet and its capital ratio increases. Then securitization can be used by banks as a mechanism to correct their capital ratio requirement without issuing equity.

Securitization is an alternative for banks to correct their capital ratio and at the same time to release capital for more lending. In almost all the cases, the originator of the loan, the bank, continues to be the servicer of the loans, which means that originators continue to be in charge of collecting the principal and interest payments from borrowers. Banks are the best monitors of final debtors (Diamond (1984, 1991), Besanko and Kanatas (1993)), they specialize in a monitoring technology, and they have also interest on maintaining a business relationship with final debtors. In many cases, final debtors are not even aware their debts have been sold and converted into securities. As a result, with securitization, banks earn fees, and originate loans without permanently funding them.

The purpose of this paper is to analyze bank loan securitization as a mechanism for banks to raise funds. We develop a model in which a bank can have an investment opportunity when having only illiquid assets such as loans. Then the bank has two alternatives to raise funds to invest: to sell its loans, or part of them, in a secondary market (i.e. to securitize), or to borrow from investors using its assets (loans) as collateral. We are interested in comparing the level of liquidity the bank can raise using this two alternatives, taking into account that the bank privately knows the quality of its loans and its possible new investment opportunities. Therefore investors have no information about the quality of the loan when buying bank loan securities, or when lending to the bank.

Many papers analyze different sources of liquidity for banks (Rochet (2004), Freixas, Parigi and Rochet (2003), Gorton and Huang (2004), Diamond and Rajan (2005)), but those papers focus in cases in which the liquidity needs are caused by negative shocks generating a risk of failure for the bank. In this paper the liquidity need is generated by a positive shock: the bank needs liquidity to invest in a new investment opportunity. In any case the bank fails if it does not take advantage of this new opportunity. We are interested in the level of liquidity the bank can obtain because we assume the new investment technology is socially beneficial, so we want to maximize the amount invested in this new opportunity. Thus we want to see whether using bank loan securitization is socially better than using credit lines.

We have obtained that for a high cost of securitization, it is socially better to use a credit line to raise funds. In both cases the first best is not reached because only the bank has information about its loans. Then we study how the government can intervene to improve social welfare. We have found that the government can correct the information problem using taxes or subsidies, and therefore it can improve the amount invested by the bank when it has new opportunities,but this is only possible when the bank uses securitization to raise funds. The government intervention is useless when the bank borrows from investors.

This paper is organized as follows. On the next section an overview of bank loan securitization is presented. On the third and fourth sections the model and the different possible contracts between the bank and the investors are described. On the fifth section we present how the government can intervene to improve the social welfare. On the sixth section we finalize with some concluding remarks.

2. An Overview of Bank Loan Securitization

In a traditional securitization transaction, at least four parties are involved: borrowers, originators (the bank), buyers of assets, and investors in the ABS. The buyer is usually a Special Purpose Entity (SPE). A SPE is established solely to purchase assets and to issue securities against the assets (e.g. Fannie Mae in the U.S.).

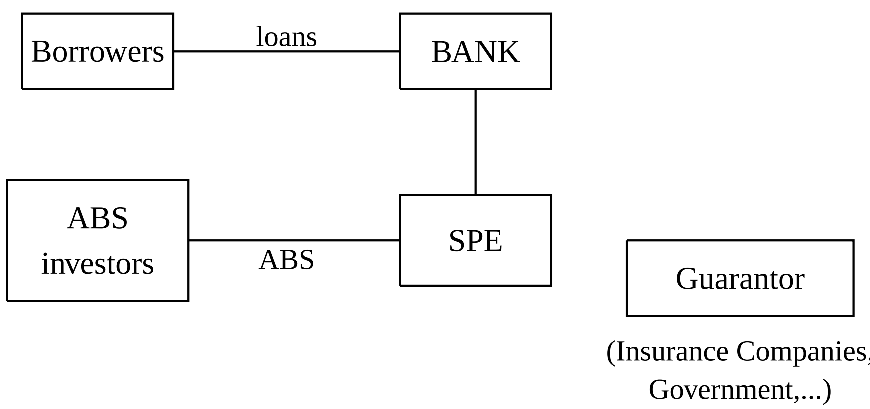

Figure 1: Participants in a bank loan securitization

In a securitization transaction, the originator will often do the transfer to the buyer so that it constitutes a "true sale", a sale that is sufficient under bankruptcy law to remove the assets from the originator’s bankruptcy estate. ABS can be structured as a pass-through or pay-through. Under a pay-through structure, the investors’ payments are routed through the SPE who does not strictly pay the investors only when the receivables are collected, but keeps paying on the stipulated dates irrespective of the collection dates. Under a pass-through structure the SPE makes the payments, or rather passes the payments to investors (after deducing fees and expenses) when they are collected from the original borrowers.

To guarantee on-time payments to security buyers, the SPE usually uses a tranching structure, as well as guaranteed investment contracts or credit enhancements or both[1]. In a tranching structure, the most senior tranche (often called "Class A") is the safest; the most subordinated tranche is the one that absorbs all initial risks, and it must usually be bought by originators to correct moral hazard problems. In addition to those tranches, the structures often contain several intermediary tranches (called Classes B, C, D, according to their subordination level).

As a complement to tranching structure, SPEs may arrange with a third party to provide credit enhancement. A typical credit enhancement for pay-through securities is a credit line guaranteed by a third party (another bank, an insurance company, an international agency, a government institution). In case of default the guarantor is obliged to repay the security buyers. Usually the securitized loans have good collateral, but recovering its value can take a long time, that is why a credible credit line is necessary.

Banks are motivated to securitize to capture the liquidity value of the loans. The liquidity value of an ABS depends on the credibility of the securitizer's guarantee. Securities receive a qualification from credit rating agencies depending on their characteristics, structure and mainly on their guarantees. Even if the quality of the assets in which securities are based is not very high, if the ABSs have a credible and a high credit enhancement, the qualification will be high. ABSs sold on the market have generally a high credit rating, therefore a low probability of default.

The SPEs that have explicit or implicit government backing (as the Government Sponsored Entities) can sell securities without the credit enhancements needed in the other securitizers. Passmore, Sparks and Ingpen (2002) study the transmission of the government subsidies to SPEs, to the mortgage interest rates. They have found that the interest rates of the mortgage loans securitized by a GSE are usually lower, specially when GSEs behave competitively. They can issue debt at lower interest rates than they could otherwise and they can securitize mortgages without credit enhancements. Then the government has the possibility to help on the development of a secondary market giving guarantee through credit lines. In this paper we focus on another mechanism that can be used by the government to improve the benefits of securitization: the use of taxes or subsidies to banks.

[1] For a paper in tranches see Plantin (2002).

3. The Model

We consider a model with three dates based on Rochet (2004) and Gorton and Huang (2004). At date 0 a risk neutral bank invests one unit in a loan technology that yields at date 2 a high value H with probability π and a low value L with probability $1-\pi$, thus we assume that H is greater than L.

Assumption 1 $\pi H + (1-\pi)L \geq 1$ *and* $1 \geq L \geq 0$

Assumption 1 means that the bank loan technology is profitable. This assumption implies that H must be strictly greater than 1.

At date 1 the bank privately observes the realization of the loan. In other words, at date 1 the bank knows if the loan technology is going to yield H or L at date 2. With probability θ the bank has access to a new investment technology at date 1. We suppose that only the bank has access to this new investment opportunity that is a constant returns to scale technology: 1 unit invested at date 1 generates R at date 2. This investment is supposed to be socially good, so it is socially optimal that the bank invests as much as it can in the date 1 investment technology.

Assumption 2 $R > 1$

To have a new investment opportunity at date 1 has no connection with the return of the date 0 loan investments, that is

Assumption 3 π *and* θ *are independent*

To make interesting the model we suppose (as in Diamond and Rajan 2001) that the bank cannot find liquidity borrowing against the realization of its new investment. At date 1 there are risk neutral investors with liquidity. Thus at the intermediate date the bank can meet liquidity need in two ways: by selling its loan investment through securitization, or by using a credit line. Unsecured credit is not available to the bank, that means the bank cannot borrow without pledging collateral. So when borrowing from investors the bank has to use its date 0 loan investment as collateral.

Because there are investors with liquidity at date 1, the bank has no reason to hold liquid reserves in order to invest in the new investment technology. Nevertheless, at date 1 the bank is the only one to know the quality of its loan asset, thus there is an information problem in the negotiation between the bank and the investors. If the the type of the bank loan is H, the bank has no access to all its capacity of liquidity because investors are not going to believe the bank when it says that the quality of its loan is H. However, when the loan is L, the bank has not this problem if it says the true about the quality of its loan. So if the bank has a H loan it is not able to raise liquidity as much as it could.

The bank and the investors sign a contract that specify the amount transferred between them at date 1 and date 2. The contract can be a credit or a securitization contract. In both cases the transfers depend on a message sent by the bank at date 1 that specifies its state. At date 1 there are four possible states for the bank, we call $s = (s_1, s_2)$ the state of the bank at date 1. The first component, s_1, indicates the realization of the loan investment, so s_1 is equal to H or L. The other component, s_2, is equal to R if the bank has a new investment opportunity different from storage, otherwise it is equal to 1. The set of all the possible states of the bank is called S. Thus, at date 1 the bank send a message $m = (m_1, m_2)$ where m is in S. Without loss of generality, we can restrict ourselves to contracts such that telling the true (i.e. m equal to s) is always a dominant strategy for the bank.

The contract specifies the transfers made by the investor to the bank at date 1, and the payment made by the bank to the investor at date 2. Those transfers depend on the message m. A contract has the form $C = \{(q_{(H,R)}, t_{(H,R)}); (q_{(L,R)}, t_{(L,R)}); (q_{(H,1)}, t_{(H,1)}); (q_{(L,1)}, t_{(L,1)})\}$, where $t(m)$ is the transfer made by the investors to the bank at date 1, and $q(m)m_1$ is the amount the investors receive at date 2 when the bank has sent the true message. More precisely when there is securitization, $q(m)$ is the fraction of the loan that is sold, and $t(m)$ is the corresponding payment. Also, when the contract is a credit contract, $t(m)$ is the amount lend to the bank and $q(m)m_1$ the corresponding face value. The sequence of events is summarized in the following figure.

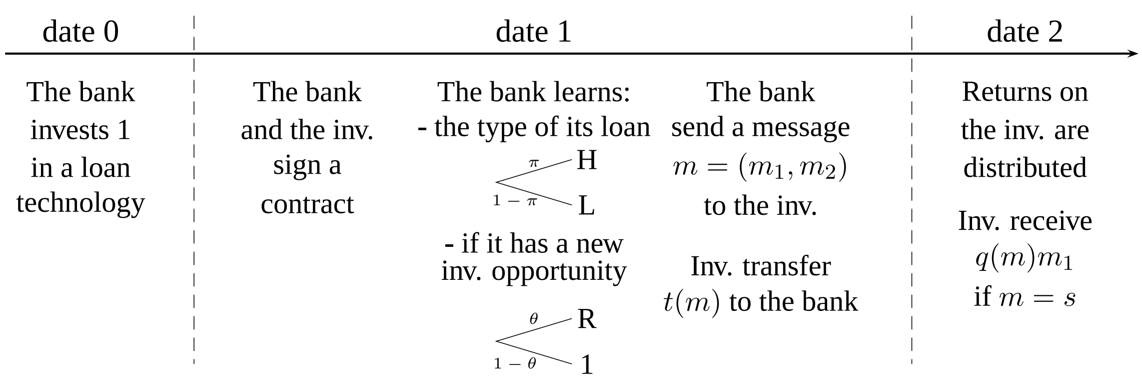

Figure 2: Time line

When there is securitization, transfering a fraction q of the loan from the bank to the investors has a positive cost $q\gamma$ per unit of loan. Who pays this cost doesn't change our results, thus we suppose it is paid by the investors.

Assumption 4 $\gamma \leq \gamma_m \equiv (1-\frac{1}{R})L$.

This assumption can be rewritten as $R(L-\gamma) \geq L$. It means that it is good for the bank to securitize the loans with low value to invest in the outside opportunity, even taking into account the cost of the assets' transfer to the security buyers. In this model, the only reason to assume that L is lower than 1 is to ensure that γ is lower than 1. We could have γ lower than 1 when L is greater than 1, but our results do not change when assuming that.

Before finding the terms of the different possible contracts between the investors and the bank, we may wish to find the "social" first best. In order to find it, we have to maximize the total social welfare subject to the participation constraints of the bank and the investors for each state, and the restrictions for the fractions $q(s)$. That is,

$$\max_{q(\cdot),t(\cdot)} E_s[s_1 + t(s)(s_2-1) - \widehat{\gamma}q(s) - 1] \quad s \in S \tag{1}$$

subject to

$$t(s)s_2 + (1-q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \widehat{\gamma}q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{2}$$

where $\widehat{\gamma}$ is equal to γ when there is securitization and to zero when the bank borrows against its assets to invest at date 1. Solving the problem we obtain that the first best is then $C^* = \{(1, H-\widehat{\gamma}); (1, L-\widehat{\gamma}); (0,0); (0,0)\}$. So when there is no information problem the maximum the bank can invest in the outside opportunity is $H-\widehat{\gamma}$ when the date 0 loan is high, and $L-\widehat{\gamma}$ when it is low. Observe that when the bank has an outside opportunity, the bank passes all the return of the date 0 loan to the investors. We have also that no transfers occur when there is no outside opportunity. Remember we are supposing that at date 1 the bank can only use its loan assets to raise liquidity, that is why the transfers $t(m)$ are limited to the return of the loan. On the next sections we study the different contracts and we compare them with the first best we just have found.

4. The Contracts

4.1 Securitization - when the bank proposes the contract

When there is bank loan securitization $q(m)$ represents the fraction of the loan sold to the investors, and $t(m)$ is the corresponding payment made at date 1 by the investors.

Observe that the terms of the contract depend on the message m sent at date 1 by the bank to the investors. Remember we are interested in a truthful revelation mechanism, that is $m = s$. At date 1, the investors buy the fraction $q(m)$ of the loan and they pay $t(m)$ to the bank. Then at date 2 the investors receive $q(m)s_1$, even if the bank has lied about the quality of the loan, that is m_1 different from s_1.

When the bank sells its loan in the secondary market, the security buyers can be small investors. On the contrary when the bank uses a credit line, the investors must have the technology necessary to force the bank to repay the corresponding face value. In the case of securitization, we can have two possibilities: the bank issues the securities, then the bank proposes the securitization contract; or the bank transfers the loan to a SPE who issues the securities. In the second case, the SPE proposes the contract as long as it is independent from the bank. Notice it can occur that the SPE is a sort of branch of the bank, or that the bank is shareholder of the SPE. Thus, on that cases, the bank proposes the terms of the negotiation. On this section we present the securitization contract when the bank has the bargaining power. On the next section we present the other securitization contract.

The bank solves the following problem $\mathcal{P}_1$ in order to find the terms of the contract $C = \{(q_{(H,R)}, t_{(H,R)}); (q_{(L,R)}, t_{(L,R)}); (q_{(H,1)}, t_{(H,1)}); (q_{(L,1)}, t_{(L,1)})\}$.

$$(\mathcal{P}_1) \qquad \max_{q(\cdot),t(\cdot)} [t(s)s_2 + (1 - q(s))s_1 - 1] \quad s \in S \tag{3}$$

subject to

$$s = \arg\max_{m \in S} \{t(m)s_2 + (1 - q(m))s_1 - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1 - q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{4}$$

In the problem $\mathcal{P}_1$, the bank maximizes its expected profit (3) subject to the incentive compatibility constraint (IC$_b$), the participation constraints (PC$_b$) and (PC$_I$) of the bank and the investors for each state, and the constraint (4) for the fractions $q(\cdot)$. The incentive compatibility constraint (IC$_b$) ensures the bank sends the true message.

The first best is not implementable because when being in the state (L, R) the bank has incentives to lie and to send a message equal to (H, R) (i.e. the (IC)$_{LR}$ is not satisfied with the first best).

Proposition 1 *When there is bank loan securitization at date 1, the bank proposes the following contract C_1 to the investors:*

$$C_1 = \{(q_{1(H,R)}, t_{1(H,R)}); (1, L - \gamma); (0, 0); (0, 0)\}, \quad \textit{where}$$

$$q_{1(H,R)} = \begin{cases} \frac{RL-R\gamma-L}{(R-1)L} & \textit{if } \gamma \leq H-L \textit{ and } RL \geq H \\ \frac{RL-R\gamma-L}{RH-R\gamma-L} & \textit{if } \gamma > H-L \textit{ and } RL \geq H \\ 0 & \textit{if } RL < H \end{cases}$$

$$\textit{and} \quad t_{1(H,R)} = \begin{cases} \frac{RL-R\gamma-L}{R-1} & \textit{if } \gamma \leq H-L \textit{ and } RL \geq H \\ \frac{(H-\gamma)(RL-R\gamma-L)}{RH-R\gamma-L} & \textit{if } \gamma > H-L \textit{ and } RL \geq H \\ 0 & \textit{if } RL < H \end{cases}$$

Proof. Annexe.

Observe the terms of contract C_1 of proposition 1 is independent of the probabilities π and θ. This is because the participation constraints of the bank and the investors are not expected but real. Neither the bank, nor the investors, loose money in any state of the nature. For this reason when the bank has not the outside opportunity, that is s_2 is equal to one, there is no transaction between the bank and the investors, that is $q_{1(H,1)}$, $t_{1(H,1)}$, $q_{1(L,1)}$ and $t_{1(L,1)}$ are equal to zero. Notice that when the transaction is in expected values, then the bank or the investor can renegotiate in the case they will loose money, and then it will be difficult for the principal to be sure the agent will carry out the contract. It is clear that the first best is not reached.

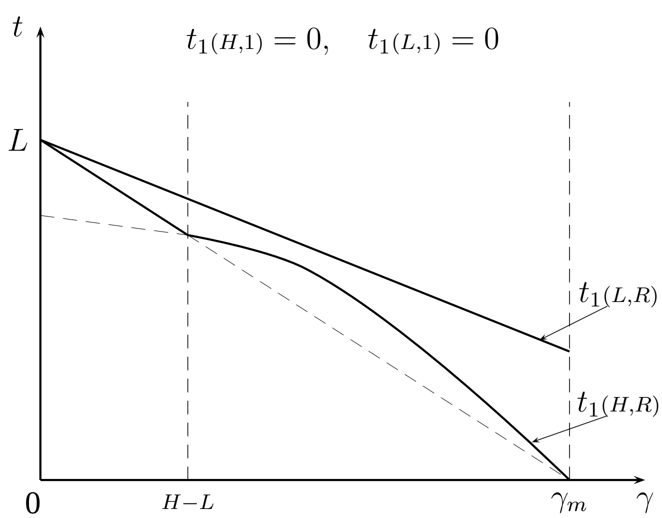

Figure 3: Security buyers' payments when $RL \geq H$ and $\gamma_m \geq H - L$

In the figure 3 the transfers are represented. We observe that when having high loans the bank invest less in the outside opportunity than when it has low loans. It sounds paradoxical, but banks with L loans can raise more funds than banks with H loans. In compensation, when having a high loan the bank keeps a fraction of its loan. This can be interpreted as a signal send by the bank to the investors, when the loan is high. Instead, when the loan is L the bank sells all the loan without keeping a fraction of it.

4.2 Securitization - the case of a SPE

Now we present the contract in which the SPE has the bargaining power. Even though the security buyers can be many small investors, when the SPE participates in the transaction, and it is independent of the bank, it can be the one that proposes the terms of the contract to the bank. In this case the SPE, that here plays the role of the investor with available liquidity, solves the following problem ($\mathcal{P}_2$) to find the securitization contract $C_2 = \{(q_{2(H,R)}, t_{2(H,R)}); (q_{2(L,R)}, t_{2(L,R)}); (q_{2(H,1)}, t_{2(H,1)}); (q_{2(L,1)}, t_{2(L,1)})\}$:

$$(\mathcal{P}_2) \qquad \max_{q(\cdot),t(\cdot)} E_s[q(s)s_1 - t(s) - \gamma q(s)] \quad s \in S \tag{5}$$

subject to

$$s = \arg\max_{m \in S}\{t(m)s_2 + (1 - q(m))s_1 - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1 - q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{6}$$

Observe the problem of the SPE only differs from the problem solved on the previous section in the objective function. Because now the principal of the contract is the investor and not the bank, we maximize the expected profit (5) of the investor subject to the incentive compatibility constraint (IC$_b$), the participation constraints (PC$_b$) and (PC$_I$) for each state, and the constraint (6) for the fractions $q(\cdot)$. The following proposition presents the solution of the problem.

Proposition 2 *When at date 1 there is an investor who buys a fraction of the bank loan, it proposes the following contract C_2 to the bank:*

$$C_2 = \{(q_{2(H,R)}, t_{2(H,R)}); (1, q_{2(L,R)}); (0, 0); (0, 0)\}$$

where,

(i) when $H - L \geq \pi(RH - R\gamma - L)$, but also when $RL < H$ then $q_{2(H,R)} = t_{2(H,R)} = 0$ and $q_{2(L,R)} = L/R$;

(ii) when $\pi(RH - R\gamma - L) \geq H - L$ and $R(L - \gamma) < H \leq RL$ then $q_{2(H,R)} = \frac{RL - R\gamma - L}{H - L}$, $t_{2(H,R)} = \frac{(RL - R\gamma - L)H}{R(H - L)}$ and $t_{2(L,R)} = L - \gamma$;

(iii) when $\pi(RH - R\gamma - L) \geq H - L$ *and* $R(L-\gamma) \geq H$ *then* $q_{2(H,R)} = 1$ *and* $t_{2(H,R)} = t_{2(L,R)} = H/R$.

Proof. Annexe

As in proposition 1, the terms of the contract are independent of the probability θ. However, the transfers, as the fractions, depend on the probability π of having a high loan. The participation constraints (PC_b) and (PC_I) continue to be expressed in real terms, and not in expected values, but the term π, that is in the objective function, is not eliminated as in the previous case. The following figure shows the terms of the contract C_2 for the different combinations of γ and π when RL is greater than H.

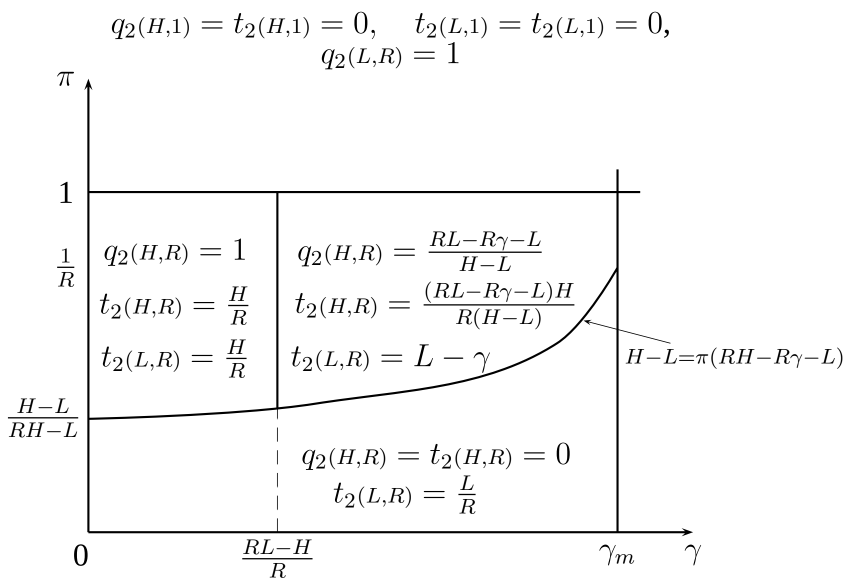

Figure 4: Security contract when $RL \geq H$

It is not surprising there are no transactions between the investor and the bank when there is not an outside opportunity. Notice that when the probability of having a high loan is small and the loan is high, there is no transaction between the bank and the investor. The following figure presents the transfers for high values of π. Observe the bank can invest the same amount H/R in the new investment technology when the cost of securitization is small. However, as for the contract C_1, for the other values of the cost γ, the bank can raise more less funds when having a low loan, than when having a high loan. Finally, for high values of γ and π, the bank retains a fraction of its loan when the loan is high. As we have said, this can be interpreted as a signal of having a high loan.

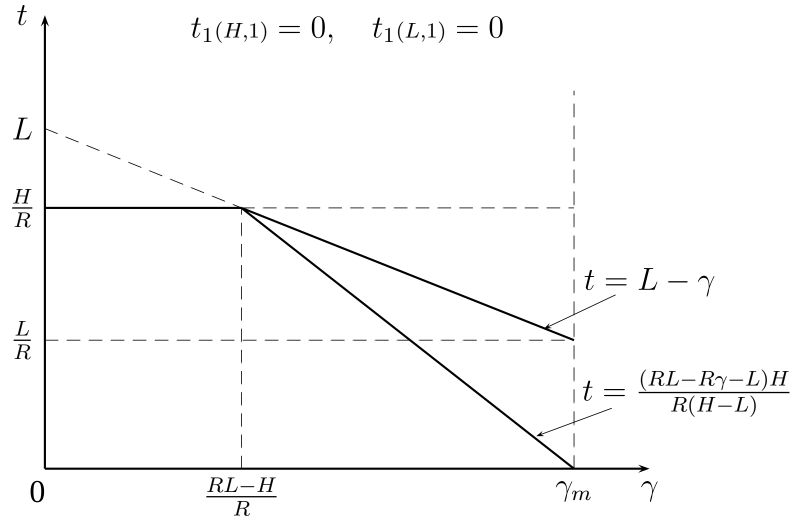

Figure 5: Security buyers' payments when $RL \geq H$ and $\gamma_m \geq H - L$

4.3 Credit Contract

For the case when the bank borrows from investors, we only study the case where is one investor who proposes the contract to the bank. That is because when investors have liquidity to lend to the bank, they usually have the bargaining power, and we suppose they do not compete to lend liquidity to the bank.

When the investors grant a credit to the bank of an amount $t(m)$, the corresponding face value is $q(m)m_1$. In consequence, when the bank lies about the quality of its loan investment, that is m_1 is different from s_1, the bank pays to the investor the minimum between the face value $q(m)m_1$ and the real value of the loan, s_1. This is because the investor lends against the date 0 loan investment of the bank. Therefore, the bank does not pay more than the date 0 loan return. In case it is possible for the investor to lend to the bank against the new investment opportunity that yields R per unit invested, the investor would lend to the bank as much as it can, we suppose this is not the case.

A representative investor solves the following problem $\mathcal{P}_3$ to find the credit contract. Notice that the incentive compatibility constraint changes with respect to the problems $\mathcal{P}_1$ and $\mathcal{P}_2$ presented in the previous sections. That is because the investors do not buy the loan so the bank at date 2 has to pay a face value that is equal to $q(m)m_1$, no matter the value of m_1. When there is securitization, the bank transfers $q(m)s_1$ to the investors, even when m_1 is different from s_1.

$$(\mathcal{P}_3) \qquad \max_{q(\cdot),t(\cdot)} E_s[q(s)s_1 - t(s)] \quad s \in S \tag{7}$$

subject to

$$s = \arg\max_{m \in S}\{t(m)s_2 + [s_1 - q(m)m_1]_+ - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1 - q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{8}$$

The investor maximizes his expected profit (7) subject to the incentive compatibility constraint (IC$_b$), the participation constraints of the bank (PC$_b$) and the investors (PC$_I$), and the constraint (8) for the fractions $q(\cdot)$. The following proposition presents the solution of the problem $\mathcal{P}_3$.

Proposition 3 *When at date 1 an investor lends liquidity to the bank, it proposes the contract C_3 to the bank where $C_3 = \{(L/H, L/R); (1, L/R); (0,0); (0,0)\}$.*

Proof. Annexe

The investor lends to the bank the same level of liquidity no matters the quality of the bank loan. On the same way the face value is the same when the bank receives a credit. The investor drains the return of the outside opportunity, lending to the bank gives a return R per unit lent.

As a consequence, the profit of the bank does not change when there is an outside opportunity. We can say the bank is then indifferent between raising or not funds at date 1. However, banks are always interested in investing in non risky opportunities, even when they do not give benefits, because they can keep clients or they can attract new ones.

Observe the transfers are independent of the probabilities θ and π. We have also, as for C_1 and C_2, that there is no transactions when the bank has not a new investment opportunity at date 1. Because the credit is against the date 0 loan, and the investor does not want to loose in any state of the bank, the face value $q(m)m_1$ is limited to the lowest return of the date 0 loan, that is L.

We finally compare the amounts the bank can raise at date 1 using a credit line or securitizing. When the bank has a low loan, to securitize is always socially better. However, only for small values of the cost γ, securitization gives more funds to the bank, when the date 0 loan is high. Figures 3 and 5 show this clearly.

5. Government intervention

At date 1, the social first best is not reached neither when using securitization, nor when using a credit. The banks with H loans are not getting all the value of their assets, then the date 1 investment is not maximized. In this section we focus in how the government can intervene to improve the social welfare.

We proposes that at date 2 the bank pays to the government an amount that depends on the message sent by the bank at date 1. That payment, called $p(m)$, can be positive or negative. When it is positive it can be considered as a tax, and when it is negative as a subsidy. Then, at date 2 the government taxes or subsidies the bank, depending on the message it has sent.

We introduce the payment $p(m)$ to change the incentive constraints of the bank, to give it the possibility to raise more funds, then to invest more when there is an outside opportunity. Firstly we analyze how the results change when the government intervenes and there is securitization at date 1. We focus in the case in which the bank issues the securities. The bank solves the following problem $\mathcal{P}_4$.

$$(\mathcal{P}_4) \qquad \max_{q(\cdot),t(\cdot)} E_s[t(s)s_2 + (1-q(s))s_1 - p(s) - 1] \quad s \in S \tag{9}$$

subject to

$$s = \arg\max_{m \in S}\{t(m)s_2 + (1-q(m))s_1 - p(m) - 1\} \tag{IC^s}$$

$$t(s)s_2 + (1-q(s))s_1 - p(s) - 1 \geq s_1 - p(s) - 1 \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \tag{10}$$

The idea now is to look for the conditions needed to reach the first best. The following proposition shows those conditions.

Proposition 4 *When the bank issues securities, the first best is reached for* $RL - H \geq (R-1)\gamma$*, when* $p(H,1) = p(L,1)$ *and*

$$p(L,R) = p(H,R) - R(H-L) \tag{11}$$

$$RH - R\gamma - H \geq p(H,R) - p(H,1) \tag{12}$$

$$p(H,R) - p(H,1) \geq R(H-L) - \gamma \tag{13}$$

$$R(H-\gamma) \geq p(H,R) \tag{LL$^b_{HLR}$}$$

$$L \geq p(H,1) \tag{LL$^b_{L1}$}$$

$$\theta R(H-\gamma) + (1-\theta)[\pi H + (1-\pi)L] - 1 \geq \theta p(H,R) + (1-\theta)p(H,1) \tag{PC$^b_0$}$$

Proof. Annexe

The first three conditions of the propositions are obtained forcing the solution $\mathcal{P}_4$ to be the social first best. We have included limit liability conditions for the bank at date 2, because when p is a tax, we must be sure it is lower than the return of the bank. We finally include a

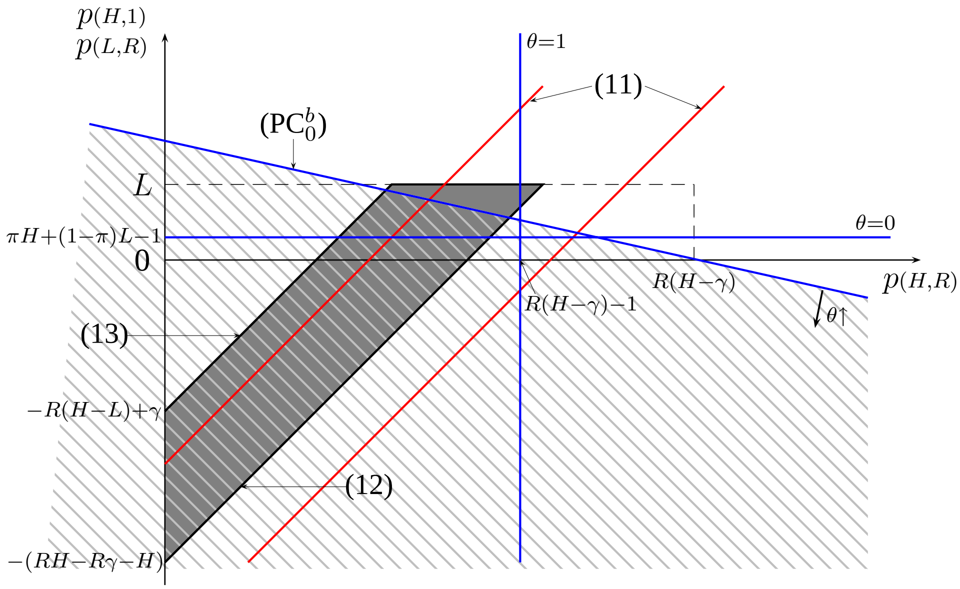

Figure 6: Conditions to reach the first best when $RL - H \geq (R-1)\gamma$

date 0 participation constraint for the bank, to be sure for the bank is still profitable to invest at date 0. The figure 6 gives an idea of the possible values for the payments in each state.

The intersection of the gray and the striped region represents the possible combinations for $p_{(H,1)}$ and $p_{(H,R)}$. The red lines, those representing the condition (11), indicate the possible values of $p_{(L,R)}$ for a corresponding value of $p_{(H,R)}$. Notice that the condition (PC$_0^b$), represented in blue, changes with θ. We deduce the maximum value for $p_{(H,R)}$ is $R(H-\gamma)-1$. Therefore the maximum value for $p_{(L,R)}$ is $R(L-\gamma)-1$.

Corollary 1 *When $RL - H \geq R\gamma$ we can choose $p_{(L,R)} = p_{(H,1)} = 0$ and $p_{(H,R)} = R(H-L)$.*

Therefore, when the cost of securitization is small, taxing the bank when it has a high loan and an outside investment opportunity is sufficient to reach the first best.

Corollary 2 *For $R\gamma > RL - 1$ the government has to subsidy the bank when it has low loans and an outside investment opportunity, but to tax it in the other states.*

While the government intervention can make possible to reach the first best when securitizing, this is not the case when the bank borrows from investors. When securitizing, without government intervention the bank has to retain a fraction of its loan when the loan is high. With the government participation, instead of keeping part of its loan, the bank can accept to pay a tax when its loan is high. This cannot be used when there is a credit contract between the bank and the investors because the corresponding face value of the debt must be lower than the minimum return of the date 0 loan. Observe the incentive constraint of the bank,

$$s = \arg\max_{m \in S}\{t(m)s_2 + [s_1 - q(m)m_1]_+ - p(m) - 1\} \qquad (\text{IC}_b)$$

More precisely for $s = (H, R)$ and $s = (L, R)$ we have,

$$t_{HR}R - q_{HR}H - p_{HR} \geq t_{LR}R - q_{LR}L - p_{LR} \qquad (\text{IC}_{HR})$$

$$t_{LR}R + L - q_{LR}L - p_{LR} \geq t_{HR}R + [L - q_{HR}H]_+ - p_{HR} \qquad (\text{IC}_{LR})$$

Then $t_{HR}R - q_{HR}H - p_{HR} \geq t_{HR}R + [L - q_{HR}H]_+ - p_{HR} - L$ and $L - q_{HR}H \geq [L - q_{HR}H]_+ \geq 0$. Thus q_{HR} must be lower than L/H which is strictly lower than 1. By the participation constraint of the investor in the state (H, R), we have that t_{HR} has to be lower than $q_{HR}H$ that cannot be greater than L. Therefore the first best cannot be reached.

6. Concluding Remarks

Securitization is a mechanism that can be used by banks to raise funds. Depending on the cost of securitization, the level of liquidity that can be recovered by a bank is higher or lower than the amount it can borrow using its assets as collateral. In any of the two cases, the total value of the bank assets, that is the first best, cannot be reached. However, bank loan securitization is the best social alternative when the government intervenes.

In the model presented in this paper, we have assumed when the bank has a new investment opportunity, it is good (R is greater than one). It can be interesting to analyze what can occur when the new investment opportunity of the bank is not necessarily good. It can occur that the bank uses securitization to raise funds to invest in new bad investments. With securitization banks could sell good loans and replace them by bad loans. A good banking supervision is maybe necessary when banks have access to a secondary market.

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Annexe

Proof of Proposition 1.

$$\max_{q(\cdot),t(\cdot)} E_s[t(s)s_2 + (1-q(s))s_1 - 1] \quad s \in S \tag{14}$$

$$\text{s.t.} \quad s = \arg\max_{m\in S}\{t(m)s_2 + (1-q(m))s_1 - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1-q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{15}$$

For $s = (H, 1)$ the participation constrains for the investors and the bank are $q_{H1}(H - \gamma) \geq t_{H1} \geq q_{H1}H$ then necessarily $\underline{q_{H1} = t_{H1} = 0}$. On the same way for $s = (L, 1)$ the participation constrains are $q_{L1}(L - \gamma) \geq t_{L1} \geq q_{L1}L$, then $\underline{q_{L1} = t_{L1} = 0}$. The problem of the bank is then,

$$\max_{q(\cdot),t(\cdot)} \theta\pi[t_{HR}R + (1-q_{HR})H] + \theta(1-\pi)[t_{LR} + (1-q_{LR})L]$$

$$\text{s.t.} \quad t_{HR}R - q_{HR}H \geq t_{LR}R - q_{LR}H \tag{IC$_{HR-LR}$}$$

$$t_{HR}R \geq q_{HR}H \tag{IC$_{HR-HL1}$PC$^b_{HR}$}$$

$$t_{LR}R - q_{LR}L \geq t_{HR}R - q_{HR}L \tag{IC$_{LR-HR}$}$$

$$t_{LR}R \geq q_{LR}L \tag{IC$_{LR-HL1}$PC$^b_{LR}$}$$

$$q_{HR}H \geq t_{HR} \tag{IC$_{H1-HR}$}$$

$$q_{LR}H \geq t_{LR} \tag{IC$_{H1-LR}$}$$

$$q_{HR}L \geq t_{HR} \tag{IC$_{L1-HR}$}$$

$$q_{LR}L \geq t_{LR} \tag{IC$_{L1-LR}$}$$

$$q_{HR}(H-\gamma) \geq t_{HR} \tag{PC$^I_{HR}$}$$

$$q_{LR}(L-\gamma) \geq t_{LR} \tag{PC$^I_{LR}$}$$

$$1 \geq q_{HR} \geq 0 \tag{16}$$

$$1 \geq q_{LR} \geq 0 \tag{17}$$

The constraints IC_{H1-HR}, IC_{H1-LR} and IC_{L1-LR} can be ignored because they are respectively implied by the constraints IC_{L1-HR}, IC_{L1-LR} and PC^I_{LR}. With the constraints IC_{HR-LR} and IC_{LR-HR} we deduce $q_{LR} \geq q_{HR}$ so we can ignore the constraints $1 \geq q_{HR}$ and $q_{LR} \geq 0$. Finally with the constraints IC_{LR-HR} and PC^b_{HR} we can deduce PC^b_{LR}. Then the problem can be simplified to (notice the Greek letters on the right correspond to the K-T multipliers),

$$\max_{q(\cdot),t(\cdot)} \pi[t_{HR}R - q_{HR}H] + (1-\pi)[t_{LR}R - q_{LR}L]$$

$$t_{HR}R \geq q_{HR}H \tag{λ_1}$$

$$q_{HR}L \geq t_{HR} \tag{λ_2}$$

$$q_{HR}(H-\gamma) \geq t_{HR} \tag{λ_3}$$

$$q_{LR}(L-\gamma) \geq t_{LR} \tag{λ_4}$$

$$(q_{LR} - q_{HR})H \geq (t_{LR} - t_{HR})R \tag{λ_5}$$

$$(t_{LR} - t_{HR})R \geq (q_{LR} - q_{HR})L \tag{λ_6}$$

$$q_{HR} \geq 0 \tag{β_0}$$

$$1 \geq q_{LR} \tag{β_1}$$

The K-T conditions are:

$$q_{HR}: \quad -\pi H = \lambda_1 H - \lambda_2 L - \lambda_3(H-\gamma) + \lambda_5 H - \lambda_6 L - \beta_0 \tag{18}$$

$$t_{HR}: \quad \pi R = -\lambda_1 R + \lambda_2 + \lambda_3 - \lambda_5 R + \lambda_6 R \tag{19}$$

$$q_{LR}: \quad -(1-\pi)L = -\lambda_4(L-\gamma) - \lambda_5 H + \lambda_6 L + \beta_1 \tag{20}$$

$$t_{LR}: \quad (1-\pi)R = \lambda_4 + \lambda_5 R - \lambda_6 R \tag{21}$$

When adding (20) multiply by R with (21) multiply by L we obtain

$$q_{LR}R + t_{LR}L: \quad 0 = -\lambda_4[R(L-\gamma) - L] + \beta_1 R - \lambda_5 R(H-L) \tag{22}$$

We deduce $\beta_1 \neq 0$ because by the condition (21) we have $\lambda_4 + \lambda_5 \neq 0$, so $\underline{q_{LR} = 1}$.

When $RL < H$, $q_{HR} = t_{HR} = 0$, $q_{LR} = 1$ and $t_{LR} = L - \gamma$. For $RL \geq H$ we analyze the following three different cases:

- If $q_{HR} = 1$ then $\beta_0 = \lambda_2 = \lambda_3 = 0$ and $t_{HR} = t_{HR}$. Adding (18) multiply by R with (19) multiply H we deduce $\lambda_6 = 0$ but by (19) $\lambda_6 \neq 0$, then $q_{HR} \neq 1$.

- If $q_{HR} = 0$ then $t_{HR} = 0$. By the condition (21) we have $\lambda_4 + \lambda_5 \neq 0$, so $\lambda_6 = 0$.

  Adding (18) multiply by R with (19) multiply H we obtain $\beta_0 R = -\lambda_2(RL - H) - \lambda_3(RH - R\gamma - H)$ which is a contradiction, because by the condition (19) we have $\lambda_2 + \lambda_3 \neq 0$. So $q_{HR} \neq 0$ when $RL \geq H$.

- If $\underline{q_{HR} \in (0,1)}$ then $\beta_0 = 0$.

  Firstly suppose that $\lambda_1 \neq 0$, then $\lambda_2 = \lambda_3 = 0$ because $q_{HR} \neq 0$. To have the K-T condition (19) satisfied we need $\lambda_6 \neq 0$ so $\lambda_5 = 0$ because $q_{HR} \neq 1$. By the K-T conditions (18) and (19) we obtain $\lambda_1 = -\pi$ which is not possible. So necessarily $\lambda_1 = 0$.

  Suppose now that $\lambda_5 \neq 0$, then $\mu_1 = 0$ because $q_{HR} \neq 1$. Adding (18) with (19) multiply by $(H-\gamma)$ we obtain $(\pi+\lambda_5)(RH-R\gamma-H) = \lambda_2(H-L-\gamma)$. Adding (18) with (19) multiply by L we obtain $(\pi+\lambda_2)(RL-H) = -(H-L-\gamma)\lambda_3$. Remember all the K-T multipliers are positive, we have then a contradiction so $\lambda_5 = 0$.

  The K-T conditions are then,

  $$-\pi H = -\lambda_2 L - \lambda_3(H-\gamma) - \lambda_6 L \tag{23}$$

  $$\pi R = \lambda_2 + \lambda_3 + \lambda_6 R \tag{24}$$

  $$-(1-\pi)L = -\lambda_4(L-\gamma) + \lambda_6 L + \beta_1 \tag{25}$$

  $$(1-\pi)R = \lambda_4 - \lambda_6 R \tag{26}$$

  We deduce $\lambda_4 \neq 0$, $\lambda_6 \neq 0$ and $\lambda_2 \neq 0$ or $\lambda_3 \neq 0$. So $q_{LR} = 1$, $t_{LR} = L - \gamma$, $q_{HR} = \frac{RL-R\gamma-L}{\min\{(R-1)L;(RH-R\gamma-L)\}}$ and $t_{HR} = L - \gamma - (1-q_{HR})L/R$.$\square$

Proof of Proposition 2

$$\max_{q(\cdot),t(\cdot)} E_s[q(s)s_1 - t(s) - \gamma q(s)] \quad s \in S \tag{27}$$

$$\text{s.t.} \quad s = \arg\max_{m \in S}\{t(m)s_2 + (1-q(m))s_1 - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1-q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{28}$$

Observe this problem differs from the problem of proposition 1 only in the objective function. We have then $\underline{q_{H1} = t_{H1} = 0}$, $\underline{q_{L1} = t_{L1} = 0}$ and the problem can be simplified to,

$$\max_{q(\cdot),t(\cdot)} \pi[q_{HR}(H-\gamma) - t_{HR}] + (1-\pi)[q_{LR}(L-\gamma) - t_{LR}]$$

$$t_{HR}R \geq q_{HR}H \tag{λ_1}$$

$$q_{HR}L \geq t_{HR} \tag{λ_2}$$

$$q_{HR}(H-\gamma) \geq t_{HR} \tag{λ_3}$$

$$q_{LR}(L-\gamma) \geq t_{LR} \tag{λ_4}$$

$$(q_{LR} - q_{HR})H \geq (t_{LR} - t_{HR})R \tag{λ_5}$$

$$(t_{LR} - t_{HR})R \geq (q_{LR} - q_{HR})L \tag{λ_6}$$

$$q_{HR} \geq 0 \tag{β_0}$$

$$1 \geq q_{LR} \tag{β_1}$$

The K-T conditions are:

$$q_{HR}: \quad \pi(H-\gamma) = \lambda_1 H - \lambda_2 L - \lambda_3(H-\gamma) + \lambda_5 H - \lambda_6 L - \beta_0 \tag{29}$$

$$t_{HR}: \quad -\pi = -\lambda_1 R + \lambda_2 + \lambda_3 - \lambda_5 R + \lambda_6 R \tag{30}$$

$$q_{LR}: \quad (1-\pi)(L-\gamma) = -\lambda_4(L-\gamma) - \lambda_5 H + \lambda_6 L + \beta_1 \tag{31}$$

$$t_{LR}: \quad -(1-\pi) = \lambda_4 + \lambda_5 R - \lambda_6 R \tag{32}$$

Adding (31) multiply by R with (32) multiply by L we obtain $(1-\pi)(RL - R\gamma - L) = -\lambda_4(RL - R\gamma - L) - \lambda_5 R(H-L) + \beta_1 R$. Then $\beta_1 \neq 0$ so $\underline{q_{LR} = 1}$. By the condition (32) we have $\lambda_6 \neq 0$, then $(t_{LR} - t_{HR})R = (q_{LR} - q_{HR})L$.

When $RL < H$, $q_{HR} = t_{HR} = 0$, $q_{LR} = 1$ and $t_{LR} = L/R$. For $RL \geq H$ we analyze the following three different cases:

- If $q_{HR} = 0$ then $t_{HR} = 0$, $t_{LR} = L/R$ and $\lambda_4 = \lambda_5 = 0$. Replacing in the K-T conditions we find $\lambda_6 R = (1-\pi)$, $\beta_3 R = (1-\pi)(RL - R\gamma - L)$, $\lambda_3 = \lambda_1 R - \lambda_2 - 1$ and $\beta_0 R = (1-\pi)(RH - R\gamma - L) + \lambda_2 R(H - \gamma - L) - \lambda_1 R(RH - R\gamma - H)$. Because the K-T multipliers must be positive we need,

  $$(1-\pi)(RH-R\gamma-L)+\lambda_2 R(H-\gamma-L) \geq \lambda_1 R(RH-R\gamma-H) \geq (\lambda_2+1)(RH-R\gamma-H)$$

  Rearranging we deduce we need $(H-L) \geq \pi(RH - R\gamma - L)$.

- If $q_{HR} = 1$ then $\beta_0 = 0$ and by conditions λ_5 and λ_6 we have $t_{HR} = t_{LR}$ and $\lambda_2 = \lambda_3 = 0$. Adding the conditions (30) with (32) we obtain $-1 = -\lambda_1 R + \lambda_4$ so $\lambda_1 \neq 0$ and $t_{HR} = t_{LR} = H/R$. The constraint corresponding to the multiplier λ_4 is satisfied only if $R(L - \gamma) \geq H$.

  We have $\lambda_4 = 0$, $\lambda_1 = 1/R$, $\beta_1 R = \pi(RH - R\gamma - H) - \lambda_6 R(H - L)$ and $\lambda_5 R = \lambda_6 R - (1 - \pi)$. To be sure β_1 and λ_5 are positive we need $\pi(RH - R\gamma - L) \geq (H - L)$.

- If $q_{HR} \in (0, 1)$ then $\beta_0 = 0$ and $\lambda_5 = 0$ because $q_{HR} \neq 1$ and $\lambda_6 \neq 0$. By condition (30) we deduce $\lambda_1 \neq 0$ then $t_{HR} R = q_{HR} H$ and $t_{LR} R = L + q_{HR}(H - L)$. So $\lambda_2 = 0$ because $q_{HR} \neq 1$ and $\lambda_3 = 0$ because $q_{HR} \neq 0$. The other K-T multipliers are always positive except λ_4 for which it is necessary to have $\underline{\pi(RH - R\gamma - L) \geq (H - L)}$. Finally because $\lambda_4 \neq 0$ we deduce $\underline{t_{LR} = L - \gamma}$, $\underline{q_{HR} = (RL - R\gamma - L)/(H - L)}$ and $\underline{t_{HR} = (RL - R\gamma - L)H/[R(H - L)]}$. To be sure that q_{HR} is strictly lower than 1 we need $\underline{(H - L) > (RL - R\gamma - L)}$. $\square$

Proof of Proposition 3

$$\max_{q(\cdot), t(\cdot)} E_s[q(s)s_1 - t(s)] \quad s \in S \tag{33}$$

$$\text{s.t. } s = \arg\max_{m \in S} \{t(m)s_2 + [s_1 - q(m)m_1]_+ - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1 - q(s))s_1 \geq s_1 \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{34}$$

The investors and bank participation constrains for $s = (H, 1)$ and $s = (L, 1)$ imply $\underline{t_{H1} = q_{H1} H}$ and $\underline{t_{L1} = q_{L1} L}$. The problem of the bank is then simplify to

$$\max_{q(\cdot), t(\cdot)} \pi[q_{HR} H - t_{HR}] + (1 - \pi)[q_{LR} L - t_{LR}]$$

$$\text{s.t.} \quad t_{HR} R - q_{HR} H \geq t_{LR} R - q_{LR} L \tag{IC$_{HR-LR}$}$$

$$t_{HR} R - q_{HR} H \geq q_{H1}(R - 1)H \tag{IC$_{HR-H1}$}$$

$$t_{HR} R - q_{HR} H \geq q_{L1}(R - 1)L \tag{IC$_{HR-L1}$}$$

$$t_{LR} R + L - q_{LR} L \geq t_{HR} R + [L - q_{HR} H]_+ \tag{IC$_{LR-HR}$}$$

$$t_{LR} R + L - q_{LR} L \geq q_{H1} RH + [L - q_{H1} H]_+ \tag{IC$_{LR-H1}$}$$

$$t_{LR} R - q_{LR} L \geq q_{L1}(R - 1)L \tag{IC$_{LR-L1}$}$$

$$q_{HR}H \geq t_{HR} \tag{IC$_{H1-HR}$PC$^I_{HR}$}$$

$$q_{LR}L \geq t_{LR} \tag{IC$_{HL1-LR}$PC$^I_{LR}$}$$

$$L \geq t_{HR} + [L - q_{HR}H]_+ \tag{IC$_{L1-HR}$}$$

$$L \geq q_{H1}H + [L - q_{H1}H]_+ \tag{IC$_{L1-H1}$}$$

$$t_{HR}R \geq q_{HR}H \tag{PC$^b_{HR}$}$$

$$t_{LR}R \geq q_{LR}L \tag{PC$^b_{LR}$}$$

$$1 \geq q_{HR} \geq 0 \tag{35}$$

$$1 \geq q_{LR} \geq 0 \tag{36}$$

$$1 \geq q_{H1} \geq 0 \tag{37}$$

$$1 \geq q_{L1} \geq 0 \tag{38}$$

With (IC$_{HR-LR}$) and (IC$_{LR-HR}$) we have $t_{HR}R - q_{HR}H \geq t_{LR}R - q_{LR}L \geq t_{HR}R - L + [L - q_{HR}H]_+$. Then $L - q_{HR}H \geq [L - q_{HR}H]_+ \geq 0$. On the same way, with (IC$_{H1-LR}$) and (IC$_{LR-H1}$) we have $0 \geq t_{LR} - q_{LR}L \geq t_{LR}R - q_{LR}L \geq q_{H1}RH - L + [L - q_{H1}H]_+$. Then $L - q_{H1}H \geq L - q_{H1}RH \geq [L - q_{H1}H]_+ \geq 0$. The constraint PC$^b_{HR}$ can be ignored because it is deduced by IC$_{HR-H1}$ or IC$_{HR-L1}$.

By (IC$_{HR-LR}$) and (IC$_{LR-HR}$) we have $t_{LR}R = t_{HR}R - q_{HR}H + q_{LR}L$, replacing it on the problem and reorganizing we obtain,

$$\max_{q(\cdot),t(\cdot)} (\pi R + 1 - \pi)q_{HR}H + (1-\pi)q_{LR}RL - t_{HR}R$$

$$t_{HR}R - q_{HR}H \geq q_{H1}(R-1)H \tag{ρ_1}$$

$$t_{HR}R - q_{HR}H \geq q_{L1}(R-1)L \tag{ρ_2}$$

$$q_{HR}H \geq t_{HR} \tag{ρ_3}$$

$$q_{HR}H + q_{LR}(R-1)L \geq t_{HR}R \tag{ρ_4}$$

$$L \geq q_{HR}H \tag{η_1}$$

$$L \geq q_{H1}H \tag{η_2}$$

$$q_{HR} \geq 0 \tag{α_1}$$

$$1 \geq q_{LR} \tag{α_2}$$

$$q_{H1} \geq 0 \tag{α_3}$$

$$1 \geq q_{L1} \geq 0 \tag{$\alpha_4\alpha_5$}$$

The K-T conditions are:

$$q_{HR}: \quad (\pi R + 1 - \pi)H = \rho_1 H + \rho_2 H - \rho_3 H - \rho_4 H + \eta_1 H - \alpha_1 \tag{39}$$

$$q_{LR}: \quad (1-\pi)RL = -\rho_4(R-1)L + \alpha_2 \tag{40}$$

$$q_{H1}: \quad 0 = \rho_1(R-1)H + \eta_2 H - \alpha_3 \tag{41}$$

$$q_{L1}: \quad 0 = \rho_2(R-1)L + \alpha_4 - \alpha_5 \tag{42}$$

$$t_{HR}: \quad -R = -\rho_1 R - \rho_2 R + \rho_3 + \rho_4 R \tag{43}$$

By the condition (40) we have $\alpha_2 \neq 0$, and $\underline{q_{LR} = 1}$. Adding (39) multiply by R with (43) multiply by H we have $\pi(R-1)RH = -\rho_3(R-1)H + \eta_1 RH - R\alpha_1$, we deduce $\eta_1 \neq 0$, $\underline{q_{HR} = L/H}$ and $\alpha_1 = 0$. We know $\rho_1 + \rho_2 \neq 0$ by condition (43). So $\alpha_3 \neq 0$ or $\alpha_5 \neq 0$, then $\underline{t_{HR} = t_{LR} = L/R}$ and $\underline{q_{H1} = q_{L1} = 0}$.

The K-T multipliers are $\rho_3 = \rho_4 = \eta_2 = \alpha_4 = 0$, $\alpha_2 = (1-\pi)RL$, $\rho_1 + \rho_2 = 1$, $\eta_1 = \pi(R-1)$, $\alpha_3 = \rho_1(R-1)H$ and $\alpha_5 = \rho_2(R-1)L$. $\square$

Proof of Proposition 4

To find the conditions each $p(m)$ (with m in S) has to verify to reach the first best, we have to solve the new bank problem trying to find as solution the values of the first best. The new problem of the bank is,

$$\max_{q(\cdot),t(\cdot)} E_s[t(s)s_2 + (1-q(s))s_1 - p(s) - 1] \quad s \in S \tag{44}$$

$$\text{s.t.} \quad s = \arg\max_{m \in S}\{t(m)s_2 + (1-q(m))s_1 - p(m) - 1\} \quad \forall s \in S \tag{IC$_b$}$$

$$t(s)s_2 + (1-q(s))s_1 - p(s) \geq s_1 - p(s) \quad \forall s \in S \tag{PC$_b$}$$

$$q(s)s_1 - t(s) - \gamma q(s) \geq 0 \quad \forall s \in S \tag{PC$_I$}$$

$$1 \geq q(s) \geq 0 \quad \forall s \in S \tag{45}$$

For $s = (H,1)$ the participation constrains for the investors and the bank are $q_{H1}(H-\gamma) \geq t_{H1} \geq q_{H1}H$ then necessarily $\underline{q_{H1} = 0}$ and $\underline{t_{H1} = 0}$. On the same way for $s = (L,1)$ the participation constrains are $q_{L1}(L-\gamma) \geq t_{L1} \geq q_{L1}L$, then $\underline{q_{L1} = 0}$ and $\underline{t_{L1} = 0}$. The problem of the bank is then,

$$\max_{q(\cdot),t(\cdot)} \theta\pi[t_{HR}R(1-q_{HR})H - p_{HR}] + (1-\pi)\theta[t_{LR}R(1-q_{LR})L - p_{LR}]$$

$$\text{s.t.} \quad t_{HR}R - q_{HR}H - p_{HR} \geq t_{LR}R - q_{LR}H - p_{LR} \tag{IC$_{HR-LR}$}$$

$$t_{HR}R - q_{HR}H - p_{HR} \geq -p_{H1} \tag{IC$_{HR-H1}$}$$

$$t_{HR}R - q_{HR}H - p_{HR} \geq -p_{L1} \tag{IC$_{HR-L1}$}$$

$$t_{LR}R - q_{LR}L - p_{LR} \geq t_{HR}R - q_{HR}L - p_{HR} \tag{IC$_{LR-HR}$}$$

$$t_{LR}R - q_{LR}L - p_{LR} \geq -p_{H1} \tag{IC$_{LR-H1}$}$$

$$t_{LR}R - q_{LR}L - p_{LR} \geq -p_{L1} \tag{IC$_{LR-L1}$}$$

$$-p_{H1} \geq t_{HR} - q_{HR}H - p_{HR} \tag{IC$_{H1-HR}$}$$

$$-p_{H1} \geq t_{LR} - q_{LR}H - p_{LR} \tag{IC$_{H1-LR}$}$$

$$-p_{H1} \geq -p_{L1} \tag{IC$_{H1-L1}$}$$

$$-p_{L1} \geq t_{HR} - q_{HR}L - p_{HR} \tag{IC$_{L1-HR}$}$$

$$-p_{L1} \geq t_{LR} - q_{LR}L - p_{LR} \tag{IC$_{L1-LR}$}$$

$$-p_{L1} \geq -p_{H1} \tag{IC$_{L1-H1}$}$$

$$t_{HR}R - q_{HR}H \geq 0 \tag{PC$^b_{HR}$}$$

$$t_{LR}R - q_{LR}L \geq 0 \tag{PC$^b_{LR}$}$$

$$q_{HR}(H - \gamma) - t_{HR} \geq 0 \tag{PC$^I_{HR}$}$$

$$q_{LR}(L - \gamma) - t_{LR} \geq 0 \tag{PC$^I_{LR}$}$$

$$1 \geq q_{HR} \geq 0 \tag{46}$$

$$1 \geq q_{LR} \geq 0 \tag{47}$$

Observe we need p_{H1} to be equal to p_{L1}. Let p_1 equal to p_{H1}. The constraints IC$_{LR-HR}$ and IC$_{HR-HL1}$ imply IC$_{LR-HL1}$. The constraints IC$_{H1-HR}$ and IC$_{H1-LR}$ can be ignored because they are implied by the constraints IC$_{L1-HR}$ and IC$_{L1-LR}$. We have also that the constraints IC$_{HR-LR}$ and IC$_{LR-HR}$ imply $q_{LR} \geq q_{HR}$, so we can ignore the constraints $q_{LR} \geq 0$ and $1 \geq q_{HR}$. The problem of the bank can be then be simplified to (the Greek letters on the right are the corresponding positive K-T multipliers),

$$\max_{q(\cdot),t(\cdot)} \pi[t_{HR}R - q_{HR}H] + (1-\pi)[t_{LR}R - q_{LR}L]$$

$$\text{s.t.} \quad t_{HR}R - q_{HR}H - p_{HR} \geq t_{LR}R - q_{LR}H - p_{LR} \tag{λ_1}$$

$$t_{LR}R - q_{LR}L - p_{LR} \geq t_{HR}R - q_{HR}L - p_{HR} \tag{λ_2}$$

$$t_{HR}R - q_{HR}H - p_{HR} \geq -p_1 \tag{λ_3}$$

$$-p_1 \geq t_{HR} - q_{HR}L - p_{HR} \tag{λ_4}$$

$$-p_1 \geq t_{LR} - q_{LR}L - p_{LR} \tag{λ_5}$$

$$t_{HR}R \geq q_{HR}H \tag{λ_6}$$

$$t_{LR}R \geq q_{LR}L \tag{λ_7}$$

$$q_{HR}(H-\gamma) \geq t_{HR} \tag{λ_8}$$

$$q_{LR}(L-\gamma) \geq t_{LR} \tag{λ_9}$$

$$q_{HR} \geq 0 \tag{β_0}$$

$$1 \geq q_{LR} \tag{β_1}$$

The K-T conditions are:

$$q_{HR}: \quad -\pi H = \lambda_1 H - \lambda_2 L + \lambda_3 H - \lambda_4 L + \lambda_6 H - \lambda_8(H-\gamma) - \beta_0 \tag{48}$$

$$t_{HR}: \quad \pi R = -\lambda_1 R + \lambda_2 R - \lambda_3 R + \lambda_4 - \lambda_6 R + \lambda_8 \tag{49}$$

$$q_{LR}: \quad -(1-\pi)L = -\lambda_1 H + \lambda_2 L - \lambda_5 L + \lambda_7 L - \lambda_9(L-\gamma) + \beta_1 \tag{50}$$

$$t_{LR}: \quad (1-\pi)R = \lambda_1 R - \lambda_2 R + \lambda_5 - \lambda_7 R + \lambda_9 \tag{51}$$

Adding (50) multiply by R with (51) multiply by L we obtain $\beta_1 R = \lambda_1 R(H-L) + \lambda_5(R-1)L + \lambda_9(RL - R\gamma - L)$. By condition (51) we have $\lambda_1 + \lambda_5 + \lambda_9 \neq 0$ so $\beta_1 \neq 0$ and $\underline{q_{LR} = 1}$.

Now adding (48) multiply by R with (49) multiply by H we deduce $\lambda_2 R(H-L) = \lambda_4(RL - H) + \lambda_8(RH - R\gamma - H) + \beta_0 R$. Then $\lambda_2 \neq 0$ because by condition (49) we have $\lambda_2 + \lambda_4 + \lambda_8 \neq 0$. Following we analyze two different possibilities for q_{HR}.

- If $q_{HR} = 1$ then $\beta_0 = 0$ and $t_{HR}R - p_{HR} = t_{LR}R - p_{LR}$. Observe $\lambda_4 + \lambda_8 \neq 0$ then $t_{HR} = \min\{L + p_{HR} - p_1; H - \gamma\}$. We are interested in the first best, then we want to force t_{HR} to be equal to $(H-\gamma)$ and t_{LR} to be equal to $(L-\gamma)$. Replacing those values on the K-T conditions we obtain,

$$p_{HR} - p_{LR} = R(H-L) \tag{$\lambda_1\lambda_2$}$$

$$RH - R\gamma - H \geq p_{HR} - p_1 \tag{λ_3}$$

$$p_{HR} - p_1 \geq H - L - \gamma \tag{λ_4}$$

$$p_{HR} - p_1 \geq R(H-L) - \gamma \tag{λ_5}$$

Observe the multipliers λ_4, λ_6 and λ_7 must be equal to zero, and we can have λ_3 and λ_5 equal to zero.

When looking for the values of the p's we have to include limited liability constraints for the bank in each state s and a participation constraint at date 0, to be sure the bank invest in loan technology at date 0, that is

$$t(s)s_2 + (1-q(s))s_1 - p(s) \geq 0 \qquad (\text{LL}^b_s)$$

$$E_s[t(s)s_2 + (1-q(s))s_1 - p(s)] \geq 1 \qquad (\text{PC}^b_0)$$

Observe LL^b_{L1} imply LL^b_{H1}. Then to have the first best we need the following conditions:

$$p_{LR} = p_{HR} - R(H-L) \qquad (52)$$

$$RH - R\gamma - H \geq p_{HR} - p_1 \qquad (53)$$

$$p_{HR} - p_1 \geq R(H-L) - \gamma \qquad (54)$$

$$R(H-\gamma) \geq p_{HR} \qquad (\text{LL}^b_{HLR})$$

$$L \geq p_1 \qquad (\text{LL}^b_{L1})$$

$$\theta R(H-\gamma) + (1-\theta)[\pi H + (1-\pi)L] - 1 \geq \theta(p_{HR} - p_1) + p_1 \qquad (\text{PC}^b_0)$$

Those inequalities can be satisfied only if $RH - R\gamma - H \geq R(H-L) - \gamma$, that is $\underline{RL - H \geq (R-1)\gamma}$.

- If $q_{HR} \in (0,1)$ then $\beta_0 = 0$, $\lambda_2 \neq 0$, $\lambda_4 + \lambda_8 \neq 0$, $\lambda_1 = 0$ and $\lambda_5 + \lambda_9 \neq 0$. So $t_{HR} = \min\{q_{HR}L + p_{HR} - p_1; q_{HR}(H-\gamma)\}$ and $t_{LR} = \min\{L + p_{LR} - p_1; L - \gamma\}$. To have $t_{HR} = q_{HR}(H-\gamma)$ and $t_{LR} = L - \gamma$ we need $\lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$ and,

$$q_{HR}(RH - R\gamma - L) = RL - R\gamma - L - p_{LR} + p_{HR} \qquad (\lambda_2)$$

$$q_{HR}(RH - R\gamma - H) \geq p_{HR} - p_1 \qquad (\lambda_3)$$

$$p_{HR} - p_1 \geq q_{HR}(H - L - \gamma) \qquad (\lambda_4)$$

$$p_{LR} - p_1 \geq -\gamma \qquad (\lambda_5)$$

As before the limited liability constraints (LL^b_s) and the participation constraint (PC^b_0) must be satisfied. Notice (LL^b_{H1}) is implied by (LL^b_{L1}).

$$q_{HR}(RH - R\gamma - H) \geq p_{HR} - H \qquad (\text{LL}^b_{HR})$$

$$RL - R\gamma \geq p_{LR} \qquad (\text{LL}^b_{LR})$$

$$L \geq p_1 \qquad (\text{LL}^b_{L1})$$

$$\theta\pi[q_{HR}(RH-R\gamma-H)+H-p_{HR}]+\theta(1-\pi)[RL-R\gamma-p_{LR}]+(1-\theta)[\pi H+(1-\pi)L]-(1-\theta)p_1 \geq 1 \tag{PC$_0^b$}$$

Observe that if the condition λ_3 and (LL$_{L1}^b$) are verified, then (LL$_{HR}^b$) is also verified. The following conditions must be verified by the p's. We have included (55) and (56) that corresponds to $q_{HR} \in (0,1)$.

$$q_{HR}(RH - R\gamma - L) = RL - R\gamma - L - p_{LR} + p_{HR} \tag{λ_2}$$

$$RL - R\gamma - L > p_{LR} - p_{HR} \tag{55}$$

$$R(H - L) > p_{HR} - p_{LR} \tag{56}$$

$$q_{HR}(RH - R\gamma - H) \geq p_{HR} - p_1 \tag{λ_3}$$

$$p_{HR} - p_1 \geq q_{HR}(H - L - \gamma) \tag{λ_4}$$

$$p_{LR} - p_1 \geq -\gamma \tag{λ_5}$$

$$RL - R\gamma \geq p_{LR} \tag{LL$_{LR}^b$}$$

$$L \geq p_1 \tag{LL$_{L1}^b$}$$

$$\theta\pi[q_{HR}(RH-R\gamma-H)+H-p_{HR}]+\theta(1-\pi)[RL-R\gamma-p_{LR}]+(1-\theta)[\pi H+(1-\pi)L]-(1-\theta)p_1 \geq 1 \tag{PC$_0^b$}$$

Observe we need $RH-R\gamma-H \geq H-L-\gamma$, that is $(R-1)H-(H-L) \geq (R-1)\gamma$.□