A Model of Competitive Signaling*

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Abstract

Multiple candidates (senders) compete over an exogenous number of jobs. There are different tasks in which the candidates' unobservable ability determines their probability of success. We study a signaling game with multiple senders each choosing one task to perform, and one receiver who observes all task choices and performances (success or failure) and matches the senders to jobs. In order to analyze the effects of different levels of competition we consider two refinements of the concept of sequential equilibrium: (i) sequential equilibria that survive when varying the number of senders; (ii) sequential equilibria that are supported by out-of-the-equilibrium-path beliefs satisfying a monotonicity condition (implied by Banks and Sobel's divinity refinement). We show that the set of sequential equilibria includes simple pooling equilibria where all senders choose the same task, and these simple pooling equilibria are the only type of sequential equilibria that satisfies (i). The unique sequential equilibrium under both (i) and (ii) is a simple pooling equilibrium with every sender choosing the most informative task. If senders have a lower overall likelihood of success in more informative tasks, this unraveling towards conspicuousness is inefficient.

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1 Introduction

Why do scholars spend seemingly disproportionate amounts of time working in projects that systematically yield results that are elegant, technically sound and often beautiful, yet in the eyes of others have little use or not many applications? How do scientific research programs persist, often in the midst of significant internal and external questioning of their scientific promise? Why do academic research efforts to answer a given question often focus on a single or a small number of plausible routes instead of simultaneously examining several alternatives? Some sociological overtones in recent rounds of the debate that has surrounded string theory over the last two decades provide an interesting insight that may go a long way in simultaneously answering these three questions. Namely, the persistence in time of research agendas may have as much to do with signaling and screening within scientific communities as it does with the scientific validity and promise of the theories that they encompass.

String theory arose in the 1960's and early 1970's as a theory of a class of subatomic particles called hadrons. In 1974 Scherk and Schwarz published a paper[1] in which they provided a mathematical argument suggesting that the theory could be developed into a theory of gravity. The theory thus became one among just a few plausible avenues towards a Theory of Everything and turned into a significant area of attention for the theoretical particle physics community. Four decades later string theory is the only candidate Theory of Everything within mainstream physics, but nevertheless the spectrum of attitudes towards it is quite broad, ranging from the contention that due to its elegance and mathematical beauty it is bound to be true[2], to the skepticism from people Penrose (see Penrose (2007) [29]) affirm that it is a case of fashion in science with little experimental support. The current debate surrounding string theory is an example of the situation in which no significant contending alternatives emerge even though the only prevailing theory is in the midst of substantial external and internal questioning of its contributions.

The process whereby string theory rose into the mainstream has been the subject of scholarly papers (see for example Hedrich (2007) [22], Schroer (2008) [31], and Zapata (2009) [38]), popular science books[3], and several mass media articles[4]. Some of these also address the

[1] J. Scherk and J. H. Schwarz, "Dual Models for Nonhadrons", *Nuclear Physics B*, 81 (1974):118-144.

[2] "I can only speak for myself, though I suspect that most others working in this field would agree. I believe that we have found the unique mathematical structure that consistently combines quantum mechanics and general relativity. So it must almost certainly be correct." (Schwarz (1998) [32])

[3] See for example *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory* by Brian Greene and the controversial *Not Even Wrong: The Failure of String Theory and the Search for Unity in Physical Law* and *The Trouble with Physics* published in 2007 by Peter Woit and Lee Smolin respectively.

[4] See for example *Unstrung* an article by Jim Holt published in October 2006 in The New Yorker, or an

inexistence of mainstream competing alternatives and consider the social mechanisms that may contribute to explain it. One particular argument that has surfaced within this category is that because theoretical physics is specially competitive, young scholars are forced to specialize in string theory in order to signal their talent and skill.[5] Signaling models in economics are able to shed light onto several seemingly paradoxical behaviors by taking into account the information that choices can convey (or hide) about unobservable agent characteristics. Cast in these terms we can think of the behavior in this example as discussed by the skeptics as a form of inefficient pooling. There is however one key part of the argument that is not addressed by the standard signaling framework, which is the idea that the behavior is a consequence of the very high competitiveness of theoretical particle physics. In the standard signaling model, there is generally a large set of equilibria, and although there are many refinements which significantly reduce the size of this set, none of them are related to the competitiveness of the environment.

In this paper we study the possible role of high competitiveness in explaining this type of inefficient pooling (a pattern of behavior whereby agents pool in a single task). With that purpose in mind we formulate a signaling game where multiple senders can choose among different tasks. They can either succeed or fail at the task and their performance is correlated with their ability. These tasks are ordered by the informativeness of success, such that for a fixed sender a success in a task with a higher index implies a higher likelihood of him having a higher ability. We study the set of sequential equilibria of the game as the number of competing senders (the competitiveness of the environment) increases. We show that restricting attention to sequential equilibria that are supported by out-of-the-equilibrium-path beliefs that satisfy a monotonicity condition, the only sender behavior that is part of some equilibrium at all levels of competition entails every sender choosing the most informative task. For sufficiently high levels of competition, the monotonicity condition is implied by Banks and Sobel's divinity refinement of sequential equilibrium (Banks and Sobel (1987) [3]). If it is the case that agents have a lower overall likelihood of success in more informative tasks then this unraveling towards conspicuousness is inefficient.

interview with Roger Penrose published in the September 2009 issue of Discover Magazine.

[5] The following quotation, taken from a mass media article authored by a physicist, exemplifies this argument: *"One reason that only one new theory has blossomed is that graduate students, postdocs and untenured junior faculty interested in speculative areas of mathematical physics beyond the Standard Model are under tremendous pressures. For them, the idea of starting to work on an untested new idea that may very well fail looks a lot like a quick route to professional suicide. So some people who do not believe in string theory work on it anyway. They may be intimidated by the fact that certain leading string theorists are undeniably geniuses. Another motivation is the natural desire to maintain a job, get grants, go to conferences and generally have an intellectual community in which to participate. Hence, few stray very far from the main line of inquiry." -Peter Woit, American Scientist [36]*

Our model links intense competition to possibly socially undesirable outcomes in realms beyond the scientific world. One area in which it has many applications is conspicuous behavior emerging in competitions for social status. Some concrete examples include the lavish wedding ceremony traditions in rural India as discussed in Bloch et al. (2004) [5], the high expenditure on luxury items to signal wealth as documented in Charles et al. (2009) [9], and conspicuous generosity as the basis for extensive food sharing in small scale societies, as discussed by Smith and Bliege Bird (2005) [33]. The phenomenon of conspicuousness has two distinct levels of detail. One level is about separation: how differences in status can be conveyed through small variations within a given category of behavior. The other level is about pooling: how due to competitive pressure this process of fine differentiation may lead to very intense devotion to that specific category of behavior by the whole community. Standard signaling models address the separation at the level of finer detail, but they do not explain the pooling at the broader level, which is what we do in our model. One other interesting feature of these examples is that they suggest that this aggregate convergence may have negative consequences as in the case of the generalized expenditure in luxury items or the lavish weddings in rural India, or very positive consequences as in the case of the resulting informal insurance system in the example of Smith and Bliege Bird (2005) [33] about food sharing in small foraging societies.

Our research contributes to the literature in the following ways:

- We provide a simple model of the choice of research tasks in the early stages of academic careers, capturing the tension that academic institutions face between providing the appropriate incentives for the production of research of high social value in the present, and screening researchers in order to be able to better allocate them to maximize the value of future research endeavors.
- More generally our framework studies the robustness of sequential equilibria to competition in a general signaling game, applicable to other settings in which messages are noisy. The standard analysis of conspicuous behavior that builds upon formal signaling frameworks has the property that conspicuousness arises in a small subset of a large variety of equilibria. Our analysis shows that greater competition in conjunction with plausible restrictions on off-the-equilibrium-path beliefs, reduces the multiplicity of equilibria significantly to a simple pooling equilibrium where all senders choose the most informative task.

The paper is structured as follows. In Section 2 we introduce our main arguments using a very simple example with just two types of senders and two tasks. In Section 3 we consider a general environment allowing for a broader class of applications with multiple types of senders and tasks. Section 4 characterizes the equilibrium refinements we use and the equilibrium behavior that the model predicts. We also show that the restriction on out-of-the-equilibrium path beliefs behind our uniqueness result is implied by Banks and Sobel's divinity refinement for sufficiently high levels of competition. The last section concludes.

1.1 Related Literature

There is a growing philosophy of science and economics literature which uses methods from economic analysis to address various aspects of the pursuit of knowledge and the scientific enterprise, broadly encompassed by the label *Economics of Scientific Knowledge* (see Goldman and Shaked (1990) [21], Kitcher (1993) [23], David (1994) [11], Davis (1997) [14], David (1998a) [12], David (1998b) [13], Goldman (1999) [20], Brock and Durlauf (1999) [6] and Mandler (2009) [25]). Zamora Bonilla (2005) [37] offers a comprehensive survey and contextualizes the view of the scientific enterprise as being driven by fallible self-regarding participants, in contrast to the more traditional perspective of the philosophy of science in which the pursuit of knowledge is mainly conducted by truth-seekers.

Goldman and Shaked (1990) [21] analyzes a model in which a scholar chooses among various possible scientific activities in order to maximize the credit that he gets from his peers, which depends on the extent that his actions succeed in modifying other scientists' beliefs regarding the true state of the world. It shows that under certain assumptions on the nature of the *scientific acts* that are available to the scholar, the credit-seeking motive fosters socially efficient truth acquisition. The main difference between Goldman and Shaked's paper and ours is that in our model the key element in society's impression of a scholar is what it can infer about his unobservable research ability and in their paper what matters is the extent to which his acts are able to shape other scientists' beliefs about the object of scientific interest. Thus we predict a possibly inefficient pooling equilibrium instead of efficient truth acquisition. In a similar vein to ours, Brock and Durlauf (1999) [6] tries to understand the entrenchment and resilience of long-standing scientific theories and the nonlinear dynamics observed in processes of assessment and adoption of newer ones. It shows that these dynamics can be explained by scientists' willingness to conform to predominant views of the scientific community. Mandler (2009) [25] in turn addresses the underinvestment of scientists in new fields of research. In his framework, this underinvestment is due to the fact that new avenues are riskier and tend to yield low returns to pioneers, but have positive externalities as they can give rise to very valuable follow-up work. In this paper, we show that focusing on the meanings conveyed by behaviors, underinvestment may be caused by rising competition which entails all senders pooling in the most informative research task.

A second strand of the literature which addresses issues related to our questions is the one

comprised by articles in economics and sociology attempting to understand the institution of tenure and measure its effects over academic output in different stages of scholars' careers in various disciplines. This literature offers several explanations for the structure of academic careers: The first one is that tenure provides the right amount of job security that scholars need in order to take the risks that high quality research often entails. The second family of explanations exemplified by Charmichael (1988) [7] focuses on tenure's role in allowing senior faculty to make optimal hiring decisions, by shielding them from otherwise undesirable competition. Faria and Monteiro (2008) [15] argue that the probationary period in academics' careers serves the crucial role of building good habits, as seen by the fact that many academics continue to be productive once they obtain tenure. It is also seen as mainly providing an extended observation period that serves to screen agents for a variety of unobservable traits (see for example McPherson and Winston (1983) [27] and McKenzie (1996) [26]). Finally Carter et al. (2009) [8] contends that tenure is within the class of optimal contracts under the assumption that the value added to a university by a researcher results from his cumulative scientific contributions rather than from his spot contributions. Under this assumption it is optimal for universities to formulate contracts with strong incentives for the accumulation of research achievements in the early stages of scholars' careers and with weaker incentives close to retirement, once the future period over which the benefits from research successes can be reaped is short.
To our knowledge ours is the first paper to specifically focus on the effects of lengthy probationary periods in academic careers over the choice of research tasks. The question of whether these effects should be seen as a side-product of a mechanism mainly concerned with other objectives, or as pertaining the probation periods' main role is beyond the scope of this paper.

Finally, our paper is related to the broad literature on social status and conspicuous behavior. This literature explains a variety of behaviors as resulting from the human drive to gain better standing in hierarchies. Beginning with Veblen (1899) [35] this theme has surfaced over the past century by way different concepts and applications: Veblen effects, snob effects, positional goods and status goods to name a few. More recently Frank (1987, 1995, 2005) [16] [18] [17] and Basu (1989) [4] provide general frameworks which allow them to offer explanations for a large variety of socioeconomic phenomena including wage dispersion, initiation rituals, wastefulness and individual patterns of consumption in time, in terms of conspicuousness. While most of the recent theoretical and empirical papers in this literature are related to consumption (see for example Bagwell and Bernheim (1996), [2] Corneo and Jeanne (1997) [10], Ravina (2007) [30] and Charles et al. (2009) [9]) there are also a number of articles motivated by other applications. Glazer and Konrad (1996) [19] presents a model philanthropy and Bloch et al. (2004) [5] apply the idea to explain wedding celebrations in

India. In general all this literature implicitly builds upon the arguments of Spence (1973) [34] and Akerlof (1976) [1] that show how information can be conveyed in equilibrium and make explicit the necessary correspondence between the underlying costs of actions and the meanings that these can convey. One issue however is that in standard signaling models conspicuousness emerges as one among many possible patterns of stable behavior. Our paper shows that under a plausible restriction on off-the-equilibrium-path beliefs, conspicuous equilibria are the only ones that are robust to competition.[6]

2 A Simple Example

There are two agents, indexed by $i \in \{1, 2\}$. The types of the agents are independently drawn from $\{t_L, t_H\}$ where $t_L < t_H$, and represent their unobservable research abilities. The type of an agent is t_L with probability $p(t_L)$ and t_H with probability $p(t_H) = 1 - p(t_L)$. After agents observe their own types, they simultaneously choose one of two possible research tasks, m_1 and m_2. The outcomes of the tasks are random variables which depend on the agent's type and the task chosen as follows. An agent of type t_H has success with probability $\mathbf{s}(m_1, t_H)$ if he chooses to work on task m_1 and $\mathbf{s}(m_2, t_H)$ if he chooses task m_2. An agent of type t_L has success with probability $\mathbf{s}(m_1, t_L)$ in m_1 and 0 in m_2. We further assume that $0 < \mathbf{s}(m_1, t_L) < \mathbf{s}(m_1, t_H)$ and that $\mathbf{s}(m_2, t_H) > 0$, such that high types have greater probability of success in both tasks. After observing the tasks chosen by the agents and their performance, society praises the agent who it deems more likely to have a higher research ability. Agents do not value their research output directly and only care about social recognition. We normalize the utility that an agent derives from being praised to 1, and assume that an agent gets 0 in case that he is not recognized. Finally, we assume that the agents' labels contain no additional information allowing society to make a decision, and thus society chooses the recipient of its praise uniformly at random if it believes that either agent is equally likely to be the one with higher research ability. In what follows, we

[6] Mijford (2012) [28] sets forth a model with similar features to study information revelation in job interviews in which a principal has the ability to noisily verify the information submitted by the candidates (they state whether they are high types or low types). In this model pooling onto stating that they are high types is the only equilibrium that survives as competition rises. A formal analogy can be drawn between research tasks in our model that differ in the quality that success in them provides about senders' underlying abilities and the noisy verification process in Mijford (2012). One crucial assumption of their model that is appropriate in the context of telling the truth or lying in job interviews but which does not fit the applications of our paper is that high types always pass the verification process. This assumption implies that in all sequential equilibria the beliefs of the principal after observing a failed verification test must be that the candidate in question is a low type. This implication is important for their subsequent analysis and determines the techniques that they use. Given the applications in the scope of our paper, our model allows the beliefs of the receiver after observing successes or failures in any task to be non-degenerate.

characterize the set of pure strategy symmetric sequential equilibria of the game.[7]

Claim 1 *There are no separating equilibria.*

Proof of Claim 1: There are no separating equilibria since rather than revealing himself, the low type t_L could do strictly better by imitating the high type t_H. This results from the fact that the outcomes are probabilistic and that society treats the agents equally when it cannot distinguish them. ∎

Claim 2 *In all pure strategy pooling equilibria of the game described above, agents pool at m_2 if*

$$\mathbf{s}(m_2, t_H) > \frac{1}{2} + p(t_L)\left(\frac{\mathbf{s}(m_1, t_H) - \mathbf{s}(m_1, t_L)}{2}\right) \tag{1}$$

Proof of Claim 2:

Suppose there existed a sequential equilibrium in which agents pooled on m_1. In such an equilibrium, the expected payoff of an agent of type t_H would be given by (after simplification):

$$\frac{1}{2} + p(t_L)\left(\frac{\mathbf{s}(m_1, t_H) - \mathbf{s}(m_1, t_L)}{2}\right)$$

Since agents of type t_L have no probability of succeeding in m_2, upon observing a success in m_2 society must conclude that the agent is a high type for sure. As a result, the minimum expected payoff that the agent gets from deviating to m_2 is $\mathbf{s}(m_2, t_H)$. Therefore, provided that (1) is true, there does not exist a sequential equilibrium in which the agents pool on m_1.

To finish the proof we just note that there exist sequential equilibria in which both agent types select task m_2 with probability 1. Specifically, the strategy profile according to which both types of agents select m_2 for sure can be sustained as a sequential equilibrium of the game under the belief that an agent deviating to m_1 is of type t_L. ∎

We therefore conclude that under condition (1), all sequential pure strategy equilibria of the game entail both players choosing m_2. This example represents the following arguments: (i) A pattern of behavior in which researchers with different abilities behave differently (in pure strategies) can't be stable because the low types would reveal themselves. Instead they could imitate the high types, in which the worst case that could happen is that society detected them, and more so, with positive probability they would get misclassified to their advantage.

[7] That is, sequential equilibria in pure strategies in which the players' strategies are independent of their labels.

(ii) On the other hand if society's expectation is that both types are to work on m_1, by deviating to m_2 and in case of success, the high type could force the correct conclusion that he is indeed a high type. Note that in this game and in sustaining a pooling equilibrium in m_1, the concept of sequential equilibrium allows society to rely on any posterior beliefs at the information sets in which it observes an agent choose m_2 and fail. In particular it could then automatically brand the player in question as a low type. However the posterior beliefs after observing some agent succeed in m_2 cannot be defined *conventionally* as due to the laws of nature ($\mathbf{s}(m_2, t_L) = 0$) it is impossible that the agent in question has low research ability. In this sense we speak of m_2 as being the more conspicuous task. If in addition, the high type's success probability in m_2 is sufficiently high, then he finds it strictly profitable to deviate. It is worth noting that his success probability can nevertheless be much lower than in m_1, since when both types select the same task, their ability to distinguish themselves gets trebled down by competition.

In order to further highlight this last point we now examine an environment like the one just analyzed, but with an arbitrary number of agents, $n \geq 2$, competing to be selected by society. As above, society only selects one agent, and we impose the restriction that it chooses uniformly at random in case its beliefs make it indifferent among a subset of the agents. We first note that there are no separating equilibria and this can be seen in the same way as in the case of $n = 2$.

CLAIM **3** *All pure-strategy pooling equilibria involve all agents choosing m_2, if*

$$\mathbf{s}(m_2, t_H) > \frac{1}{n} + \frac{\mathbf{s}(m_1, t_H) - \mathbf{s}(m_1, t_L)}{n} \left(\frac{1 - (1 - \mathbf{s}(m_1, t_L))^{n-1}}{\mathbf{s}(m_1, t_L)} \right)$$

Proof of Claim 3: Suppose that there existed a sequential equilibrium in which both types of agents chose task m_1. The expected utility to an agent of type t_H in such an equilibrium would be bounded above by her expected utility conditional on the remaining $n - 1$ agents being of type t_L. We can compute this upper bound as follows.

$$\mathbf{s}(m_1, t_H) \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\mathbf{s}(m_1, t_L)^k (1 - \mathbf{s}(m_1, t_L))^{n-1-k}}{k+1} + \frac{(1 - \mathbf{s}(m_1, t_H))(1 - \mathbf{s}(m_1, t_L))^{n-1}}{n}$$
$$= \frac{1}{n} + \left(\frac{\mathbf{s}(m_1, t_H) - \mathbf{s}(m_1, t_L)}{n} \right) \left(\frac{1 - (1 - \mathbf{s}(m_1, t_L))^{n-1}}{\mathbf{s}(m_1, t_L)} \right)$$

Since in case of success in t_2 a high type player gets immediately identified by society as such, his expected payoff from deviating from the equilibrium strategy profile would be bounded from below by $\mathbf{s}(m_2, t_H)$. The existence of a pooling sequential equilibrium in which both players choose m_1 therefore requires that $\mathbf{s}(m_2, t_H)$ does not exceed the upper bound that we have just computed. ▮

When pooling in m_1, the excess probability that the high type has of being selected by society due to a greater research ability vanishes as n grows, just as the mean probability, $\frac{1}{n}$, that a player has of being selected *before learning his type*. Therefore as long as $\mathbf{s}(m_2, t_H) > 0$ and regardless of the parameter values governing the success probabilities in m_1, the unique pure strategy sequential equilibrium involves all players selecting m_2, when the level of competitiveness, as indexed by the number of players in the game, is high enough.

2.1 Productive Efficiency and Screening Efficiency

There are two key considerations in assessing the social efficiency of behavior in our context:

1. *Productive efficiency:* The direct social value of the successful research projects.

2. *Screening efficiency:* The future social value of the information about the unobservable research abilities that society extracts from observing the agents' choices and their performances.

The problem is interesting to the extent that there is a tension between the screening and productive efficiencies of behavior. The crucial aspect of the example is that the nonexistence of a sequential equilibrium in which both agents choose m_1 does not depend on the relative productive efficiency of research in m_1 and m_2.

In order to fix ideas, suppose that the direct value of a successful research endeavor of each kind[8] is v. One plausible interpretation for m_1 and m_2 is the following:

- m_1 represents relatively new tasks in which not too much research has been carried out. Reaching fertile grounds at the forefront of the literature in one of these tasks may therefore require a moderate amount of time, and the probability of producing a significant contribution is high.

- m_2 encompasses well established research tasks that have been addressed by scholars for a long time, and which have thereby acquired technical maturity. Reaching the forefront of the literature in these tasks in general takes more time than for tasks in m_1, and the probability of making a valuable contribution is lower. Research in m_2 is more *difficult* than research in m_1 in the sense that success is less likely for

[8] Since there may be a variety of research tasks of the same kind with potentially different values, this can be seen as a distributional assumption on the values of these different tasks, and the preferences of researchers over tasks (which lie outside the scope of this project) within each kind.

agents of any research ability. The parametric restriction $\mathbf{s}(m_2, t_L) = 0 < \mathbf{s}(m_1, t_L)$ guarantees this to be the case for the low type agents, and $\mathbf{s}(m_1, t_L) < \mathbf{s}(m_1, t_H)$ already captures the greater research ability of t_H. In order to complete the picture we just add $\mathbf{s}(m_2, t_H) < \mathbf{s}(m_1, t_H)$.

- The crucial assumption in the example is that m_2 is more discriminating (or conspicuous), in the sense that $\mathbf{s}(m_2, t_L) = 0 < \mathbf{s}(m_2, t_H)$.[9] Note that this feature is independent of the overall difficulty, discussed in the previous point.

Under this interpretation of m_1 and m_2, the most conspicuous research topics are also the ones with the lowest yields. Specifically, the expected value of the contribution of an agent working in m_1 is $v(p(t_L)\mathbf{s}(m_1, t_L) + p(t_H)\mathbf{s}(m_1, t_H))$, whereas the expected contribution of an agent working in a topic in m_2 is $v(p(t_H)\mathbf{s}(m_2, t_H))$. The pooling equilibrium in m_2 is therefore the worst possible allocation of scholars to research topics in terms of productive efficiency. As discussed in Claim 1, there are no separating equilibria, and provided that $\mathbf{s}(m_2, t_H)$ is not too low, the most efficient pooling equilibrium in terms of information revelation is the one in which both agents choose m_2. The problem is that in this setting there is a trade-off between the screening efficiency and productive efficiency of behaviors, and society is unable to choose the optimal level of screening in order to balance both considerations. As we show in the rest of the paper, the sets of equilibria of a broad class of status mechanisms shift towards the more conspicuous tasks with rising competition, thus favoring screening efficiency and in detriment of productive efficiency.

3 The General Model

In this section we describe our general model of competitive signaling.

- **Stage 0: Senders**
  There are n senders $\{S_1, ..., S_n\}$, each one having some type t_i from a finite set $T = \{t^0 < t^1 < ... < t^k\} \subset [0, 1]$. The types are independently drawn and type t^m occurs with probability $p_\alpha(t^m)$, where $\sum_{m=1}^{k} p_\alpha(t^m) = 1$. To be clear, we reserve subindices to refer to the type of a specific sender or a given subset of senders. Specifically we let t_i denote the type of sender i and t_{-i} denote the vector of types of all senders other than i. $\overrightarrow{t}$ denotes the complete vector of types $(t_1, t_2, ..., t_n)$.

[9] More difficult tasks (in the sense of success probabilities) need not be more discriminating of research abilities than easy tasks. It seems to be a plausible assumption regarding research, and the proposed properties of m_1 and m_2.

- **Stage 1: Tasks**
  After observing his own type, S_i selects a task[10] m_i that he wants to perform from a finite set $M = \{m^0 < m^1 < m^2 < ... < m^z\} \subset [0,1]$. The sender can succeed or fail at the task. A sender of type t succeeds at task m with probability $\mathbf{s}(m,t) \in (0,1)$.

- **Stage 2: Receiver**
  After observing the tasks chosen by all senders and whether they succeed or fail, the receiver (R) takes an action $x \in X$ whereby

  $$X = \{f : \{1,2,3,...,n\} \to \{1,2,3,...,n\}, f \text{ is one to one }\}$$

  allocating the senders to $r \leq n$ jobs available, where $f(i) > r$ means that sender S_i remains unemployed.[11]

It is important to keep track of the beliefs of the receiver about the types of senders at three stages of the game.

1) τ_α denotes the random variable with pmf $p_\alpha(t)$ representing any sender's type in view of the receiver, before the sender chooses a task.

2) For each $m \in M$, $\tau_\mu(m)$ denotes the random variable with pmf $p_\mu(t|m)$ representing the type of a sender in view of the receiver after the sender chooses a task m.

3) For each $m \in M$ and $s \in \{0,1\}$, $\tau_\omega(m,s)$ denotes the random variable with pmf $p_\omega(t|m,s)$ representing the type of a sender in view of the receiver after the sender chooses a task m and yields performance $s \in \{0,1\}$ ($s = 1$ denotes success, and $s = 0$ denotes failure). [12]

Strategies
A (mixed) strategy for a sender S_i is a function $\sigma_i : T \to \Delta_M$, and we let $\sigma_i(m|t) \in [0,1]$ denote the probability that he chooses task m when his type is t, such that $\sum_{j=1}^{z} \sigma_i(m^j|t) = 1$. In the same fashion, a strategy for the receiver (R) is a function $\rho : (M \times \{0,1\})^n \to \Delta_X$ and

[10] The standard terminology involves *sending a message* rather than *performing a task*, but the fact that the sender can succeed or fail at it makes the second expression less awkward than the first.

[11] It is convenient to express the job assignment as a function onto $\{1,2,3,...,n\}$ as it reflects the idea that the receiver (R) is ranking the senders, which will become apparent once the utility functions are specified below.

[12] The only one of these random variables which is already fully specified is τ_α, distributed according to pmf p_α. The allowable distributions of the $\tau_\mu(m)$ and $\tau_\omega(m,s)$ will depend on the the strategies of the players and on the equilibrium concept. For simplicity the notation does not explicitly reflect these dependencies, but it is hopefully not at the expense of clarity as we only use it once the particular belief generation process in question is clear.

we let $\rho(x|\overrightarrow{(m,s)}) \in [0,1]$ denote the probability that R takes action x after observing the vector of task choices and successes/failures $\overrightarrow{(m,s)}$. We denote the set of all strategies of the receiver by Σ_R.

Preferences

The preferences of the senders are represented by any collection of functions $\{u_i\}_{i=1}^n$ (Senders) and v (Receiver) with the following properties.

<u>Senders</u>

- They strictly prefer being assigned to jobs with lower indexes:
  $u_i(x) > u_i(x') \Leftrightarrow x(i) < x'(i)$ and $x(i) < r$.
- They are only concerned with their own assignment:
  $u_i(x) = u_i(x')$ if $x(i) = x'(i)$.
- There do not exist different kinds of unemployment:
  If $x(i) > r$ and $x'(i) > r$ then $u_i(x) = u_i(x')$.

In what follows we assume without loss of generality that all senders have the same utility function u.[13]

Let $U(m_i, m_{-i}, t_i, t_{-i}, \rho)$ denote the utility that sender i of type t_i would expect from choosing task m_i, knowing that the vector of types of all other senders was t_{-i}, and expecting them to choose tasks m_{-i} and the receiver to behave according to ρ. Let $\zeta(m_{-i}, t_{-i}, s_{-i}) = \prod_{k\neq i: s_k=1} \mathbf{s}(m_k, t_k) \prod_{j\neq i: s_j=0} (1 - \mathbf{s}(m_j, t_j))$ denote the probability of performance vector s_{-i}, given that senders other than i having types t_{-i} choose tasks m_{-i}. Then:

$$U(m_i, m_{-i}, t_i, t_{-i}, \rho) =$$
$$\sum_{x\in X} u(x) \sum_{s_{-i}} \zeta(m_{-i}, t_{-i}, s_{-i}) \left[\mathbf{s}(m_i, t_i)\rho(x|, m_{-i}, m_i, s_{-i}, 1) + (1 - \mathbf{s}(m_i, t_i))\rho(x|, m_{-i}, m_i, s_{-i}, 0)\right]$$

If rather than expecting senders to choose tasks m_{-i}, i expects them to play according to strategy σ_{-i}, then based on the above description we have:

$$U(m_i, \sigma_{-i}, \rho, t_i, t_{-i}) = \sum_{m_{-i}} \left(\prod_{j\neq i} \sigma_j(m_j|t_j)\right) U(m_i, m_{-i}, \rho, t_i, t_{-i})$$

We denote by $U(m_i, \sigma_{-i}, \rho, t_i)$ the utility that a sender of type t_i expects from choosing task m_i, given that he expects the other senders to follow strategy σ_{-i} and the receiver to follow strategy ρ. Then:

[13] As an example of a function satisfying these requirements, consider $u_i(x) = r + 1 - x(i)$ if $x(i) \leq r$ and $u_i(x) = 0$ if $x(i) > r$.

$$U(m_i,\sigma_{-i},\rho,t_i) = \sum_{t_{-i}\in T^{(n-1)}} \left(\prod_{k\neq i} p_\alpha(t_k)\right) U(m_i,\sigma_{-i},\rho,t_i,t_{-i})$$

Finally, the expected payoff to player i when using strategy σ_i is:

$$U(\sigma_i,\sigma_{-i},\rho,t_i) = \sum_{m_i} \sigma_i(m_i|t_i) U(m_i,\sigma_{-i},\rho,t_i)$$

<u>Receiver</u>

The receiver (R) in our game is a *representative agent* that stands for a matching process of the senders to r jobs, that works as follows:

There are r principals, each seeking to hire one sender in order to maximize the expected value $E[g(t)]$, where g is a strictly increasing function of t and which is identical for all principals. After observing the task and the performance of each sender (m_i, s_i), principals rank the senders by the expectation $E[g(t_i)|\overrightarrow{(m,s)}]$ using the distribution $P(\overrightarrow{t}|\overrightarrow{(m,s)})$ and make job offers. One implication of the fact that g is strictly increasing and which we use repeatedly throughout what follows is:

(F1) If τ' and τ are random variables representing beliefs over of types, where τ' first order stochastically dominates τ (which we denote τ' FOSD τ) then $E_{\tau'}[g(t)] > E_\tau[g(t)]$.

We can think of this stage as a serial process whereby the principal of job 1 offers the job to her most preferred candidate, who automatically accepts the offer and leaves the pool of senders, then the principal of job 2 hires her most preferred candidate from the remaining senders, and so forth. We assume that principals cannot discriminate among senders by relying on labels, such that if a principal has identical beliefs over the types of several candidates (the most preferred), she must make offers to them uniformly at random.[14]

To simplify language we represent this kind of more elaborate matching process by a single receiver who makes an assignment of the senders ($x \in X$) with preferences over job assignments $v : X \times T^n \longrightarrow \mathbb{R}_+$ that are consistent with the underlying matching process. The receiver picks her strategy ρ to maximize:

$$V(\rho, P(\overrightarrow{t}|\overrightarrow{(m,s)}), \overrightarrow{(m,s)}) = \sum_{x\in X} \rho(x|\overrightarrow{(m,s)}) \left(\sum_{\overrightarrow{t}\in T^n} P(\overrightarrow{t}|\overrightarrow{(m,s)}) v(x,\overrightarrow{t}) \right)$$

We will make reference to the preferences of the principals at various points of our analysis in order to make use of the restrictions that they imply on the preferences of the receiver.

[14] As the senders agree on their preferences regarding the various jobs, the unique Nash equilibrium of a variety of games whereby the principals make proposals and then the senders accept or reject, involves the principal of the preferred job getting her most preferred candidate, the principal of the 2^{nd} most preferred job getting her most preferred candidate after excluding the candidate chosen by principal 1 and so on.

Given two random variables τ and τ', taking values on T, we denote $E_{\tau'}[g(t)] \geq E_\tau[g(t)]$ by $\tau' \succeq_p \tau$.

Our analysis focuses on what happens to equilibria when the amount of competition in the environment as given by n increases. Given a prior type distribution, sender and receiver preferences, and success function $\mathbf{s} : M \times T \longrightarrow (0,1)$, we denote the game described above on n players by $\mathcal{G}_n$.

Main Parametric Assumptions

Three assumptions about the success probabilities at the various tasks characterize our environment:

(A1) (Meaning of t)

Higher types are more likely to succeed in all tasks:

$$\forall m, t' > t, \quad \mathbf{s}(m,t') > \mathbf{s}(m,t)$$

(A2) (Higher tasks are more revealing)

The success likelihood ratio is increasing in m:

$$\frac{\mathbf{s}(m',t')}{\mathbf{s}(m',t)} > \frac{\mathbf{s}(m,t')}{\mathbf{s}(m,t)} \text{ whenever } m' > m \text{ and } t' > t$$

(A3) (Difficulty)

The probability of success is lower for all types in higher tasks:

$$\mathbf{s}(m,t) > \mathbf{s}(m',t), \forall t \text{ whenever } m' > m$$

3.1 Sequential Equilibria

We focus on strategies that are label independent, so despite having multiple senders we will always have a single sender strategy σ. We are interested in receiver behaviors that reflect the underlying principals' rankings of candidates given their inferences of their research abilities. We therefore restrict attention to receiver strategies which treat senders identically when the principals in the underlying matching process are indifferent.

A sequential equilibrium is a tuple $\langle \sigma, \rho, \{P(\overrightarrow{t}|\overrightarrow{(m,s)})\}\rangle$, where σ is a sender strategy, ρ is a receiver strategy, and for each $\overrightarrow{(m,s)} \in (M \times \{0,1\})^n$, $P(\cdot|\overrightarrow{(m,s)}) : T^n \to [0,1]$ is a *pmf* describing the beliefs of the receiver, with the following three properties:

(P1) **The receiver's strategy is sequentially rational given her beliefs:**

$$\forall \overrightarrow{(m,s)} \;\; \rho \in argmax_{g \in \Delta_X} V(g, P(\overrightarrow{t}|\overrightarrow{(m,s)}), \overrightarrow{(m,s)})$$

(P2) **The senders' strategy is sequentially rational for every sender type given the receiver's strategy:**

$$\forall t_i \in T, \;\; \sigma \in argmax_{\sigma_i \in \Delta_M} U(\sigma_i, \sigma, t_i, \rho)$$

(P3) **The beliefs of the receiver are consistent**

The belief consistency requirements of sequential equilibrium restrict the possible beliefs $\{P(\overrightarrow{t}|\overrightarrow{(m,s)})\}$ in three ways:[15]

(BC1) The beliefs of R regarding the type of a given sender must be independent of her beliefs regarding any other sender. This follows from the belief consistency requirement in the definition of sequential equilibrium, the assumption that senders' types are independently drawn and the assumption that a sender has no information about other senders' types when choosing his tasks. That is:

$$P(t_1, t_2, t_3, ..., t_n|\overrightarrow{(m,s)}) = \prod_i p_\omega(t_i|m_i, s_i)$$

(BC2) On the equilibrium path the beliefs are fully determined by Bayes rule and the senders' strategy.

$$\forall m \text{ such that } \sigma(m|t) > 0 \text{ for some } t, \quad p_\mu(t|m) = \frac{p_\alpha(t)\sigma(m|t)}{\sum\limits_{t' \in T} p_\alpha(t')\sigma(m|t')}$$

(BC3) The beliefs of R after observing an sender's performance are completely determined by Bayes rule and her beliefs after observing the sender's task choice. In particular:

$$p_\omega(t|m,1) = \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t' \in T} p_\mu(t'|m)\mathbf{s}(m,t')} \quad \text{and} \quad p_\omega(t|m,0) = \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t' \in T} p_\mu(t'|m)(1-\mathbf{s}(m,t'))}$$

[15] In fact, these are necessary and sufficient conditions for the beliefs to be consistent in the sense of Kreps and Wilson (1982) [24]. More specifically by adjusting the rates of convergence of the strictly positive strategies in the modified game to the equilibrium strategies we can obtain any system of beliefs $\{p_\mu(t|m)\}$ as long as it satisfies these conditions (along with making the senders' strategies sequentially rational).

Note that given properties $(BC1)$ and $(BC3)$, we can describe a sequential equilibrium as a triple $\langle\sigma, \rho, \{p_\mu(t|m)\}\rangle$ in which ρ is sequentially rational given the terminal beliefs $\{p_\omega(t|m,s)\}$ induced by $\{p_\mu(t|m)\}$, σ is sequentially rational given ρ, and for all m such that $\sigma(m|t) > 0$ for some t, $p_\mu(t|m)$ is determined by Bayes rule, σ, and the prior type distribution $p_\alpha(t)$. As above we use τ_α, $\tau_\mu(m)$ and $\tau_\omega(m,s)$ to denote random variables distributed according to $p_\alpha(\cdot)$, $p_\mu(\cdot|m)$ and $p_\omega(\cdot|m,s)$. Note that in the matching process underlying the receiver's behavior, the beliefs about sender types induce a preference relation for the principals over the random variables induced by different task-performance profiles. In what follows $\tau_\omega(m,s) \succeq_P \tau_\omega(m',s')$ $(\tau_\omega(m,s) \succ_P \tau_\omega(m',s'))$ denotes that the principals weakly (strictly) prefer the random variable induced by $(m's')$ to the random variable induced by (m,s).

4 Equilibrium Characterization

The following lemma summarizes some properties that are met by any belief system in a sequential equilibrium, and which we use repeatedly in the proofs of our main results. Intuitively, it shows that the receiver must believe that (1) a successful sender in any given task (on or off the equilibrium path), is more likely[16] to be a higher type than an unsuccessful sender with the same task; (2) a successful sender is also more likely to be a higher type than a sender with unknown performance when they choose the same task; (3) Given two tasks, chosen by the same distribution of senders, a successful sender in the more informative task is more likely to be a higher type than a successful sender in the less informative one; (4) senders with a higher type have a higher utility in equilibrium. The proof can be found in the appendix.

Lemma 1 *Let $\langle\sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ denote a sequential equilibrium.*

1. *If $(A1)$ holds then $\tau_\omega(m,1)$ FOSD $\tau_\omega(m,0)$ for all tasks m such that $\tau_\mu(m)$ is non-degenerate.*

2. *If $(A1)$ holds then $\tau_\omega(m,1)$ FOSD $\tau_\mu(m)$ for all tasks m such that $\tau_\mu(m)$ is non-degenerate.*

3. *If $(A2)$ holds, $m' > m$ and $\tau_\mu(m') = \tau_\mu(m)$ then $\tau_\omega(m',1)$ FOSD $\tau_\omega(m,1)$.*

4. *If $(A1)$ holds then $t' > t \Rightarrow U(\sigma^*, \rho^*, t') > U(\sigma^*, \rho^*, t)$.*

[16] In a stochastic sense, the type distribution of the succeeding sender FOSD the one of the unsuccessful one.

First we show that in our game there do not exist sequential equilibria in which a type gets fully separated from all others. Suppose some type t is fully separated from all others in some equilibrium. If it is the lowest type, this full separation would fully reveal this to be the case and the sender would be given the lowest priority in the matching. If it is not the lowest type, the lowest type could choose some task to mimic the behavior of type t and earn a higher utility by the last point of Lemma 1. So full separation is not a possible equilibrium outcome.

PROPOSITION **1 (No Separation)** *If $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ is a sequential equilibrium, then there do not exist m and t, such that $\sigma(m|t) > 0$ and $\sigma(m|t') = 0$ $\forall t' \neq t$.*

Proof of Proposition 1:

Suppose to the contrary that there exist m and t such that $\sigma(m|t) > 0$ and $\sigma(m|t') = 0$ $\forall t' \neq t$.

Case (I) $\exists\, t'' < t$

Consider some $t'' < t$. By Lemma 1 $U(\sigma^*, \rho^*, t) > U(\sigma^*, \rho^*, t'')$. Moreover as t is the only type for which $\sigma(m|t) > 0$, the payoff $\tau_\omega(m, 0) = \tau_\omega(m, 0)$ and therefore the payoffs under success or failure are the same. Type t'' can therefore do strictly better by playing t'', and this contradicts the sequential rationality of σ^* for t''.

Case (II) $\nexists\, t'' < t$

Then $\tau_\omega(m', 0)$ FOSD $\tau_\omega(m, 1)$ for any m' such that $\sigma(m'|t'') > 0$ for some $t'' \neq t$. By optimality of ρ^* we must therefore have $U(m', \sigma^*, \rho^*, t) > U(\sigma^*, \rho^*, t)$, contradicting the sequential rationality of σ^* for t. ∎

Proposition 1 shows that in our setting any task on the equilibrium path is always populated by at least two types. The next proposition shows that not surprisingly, for any n, there is always an equilibrium in $\mathcal{G}_n$ in which all sender types pick the same task.

PROPOSITION **2 (Simple Pooling Equilibria)** *For each $m^* \in M$, $\mathcal{G}_n$ has a sequential equilibrium $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ such that $\sigma^*(m^*|t) = 1$ $\forall t$.*

Proof of Proposition 2:

Let the beliefs on the equilibrium path $\{p_\mu(t|m^*)\}$ be derived using Bayes rule from the prior distribution $p_\alpha(t)$. For any $m \neq m^*$ let $p_\mu(t^0|m) = 1$ and $p_\mu(t|m) = 0$ $\forall t \neq t^0$. These beliefs can be seen to be the limit of the beliefs obtained by Bayesian updating along the sequence of perturbed strategies converging to σ^* given by $\sigma^*_n(m^*|t) = 1 - \left(\frac{1}{n^2}\right)$ for $t \neq t^0$, $\sigma^*(m^*|t^0) = 1 - \left(\frac{1}{n}\right)$, and for each task $m \neq m^*$ $\sigma^*_n(m|t) = \left(\frac{1}{(z-1)n^2}\right)$ for $t \neq t^0$, and $\sigma^*_n(m|t^0) = \left(\frac{1}{(z-1)n}\right)$. Given these beliefs $\tau_\omega(m^*, 0)$ FOSD $\tau_\omega(m, 1)$ for all tasks m,which by $(F1)$ implies that the payoff to any sender i from deviating from σ^* is 0. Moreover, the payoff to any sender type from σ^* is strictly positive as can be seen from the fact that the event whereby all other senders but sender i of type t fail in task m^* occurs with positive probability. In this event sender i of type t would be assigned to the best job. ▮

For a given number of players n there are many other possible equilibria in the game $\mathcal{G}_n$. In what follows we show that any given pattern of sender behavior, other than simple pooling, can only be part of equilibria of games $\mathcal{G}_n$ up to a given level of competition, as given by the number of players n. We also examine an equilibrium refinement which is implied by Banks and Sobel' s divinity refinement of sequential equilibrium applied to $\mathcal{G}_n$ for sufficiently high n.

4.1 Lack of Robustness to Competition

As in the case of the example in Section 2 we are interested in characterizing the collection of sender behaviors that are robust to competition.

DEFINITION **1 (Robustness to Competition)** *A sender strategy σ^* is robust to competition if for each game $\mathcal{G}_n$ it is part of some sequential equilibrium $\langle \sigma^*, \rho, \{p_\mu(t|m)\}\rangle$.*

Intuitively this refinement selects equilibria which are preserved as the number of senders changes. The main motivation for this refinement is that in many settings a group of competing senders may only have a very rough idea of the number of competitors, and this number may frequently vary in time. On the other hand we are attempting to understand patterns of behavior, which characterize institutions to the point that they become observable regularities.[17]

If we restrict our attention to equilibria which are robust to competition, then the remaining equilibria are simple pooling equilibria.

PROPOSITION **3** *The only sender strategies that are robust to competition are given by $\sigma(m|t) = 1\ \forall t \in T$, for some $m \in M$.*

The proof of Proposition 3 relies on the following lemma, which we also refer to in later results. For clarity we introduce some notation:

- Tasks on the equilibrium path: $e(\sigma) = \{m : \sigma(m|t) > 0 \text{ for some } t \in T\}$
- Set of tasks such that a success in any one of them is weakly preferred by the principals to success in any other task: $Pf(\sigma) = \{m : \tau_\omega(m,1) \geq_p \tau_\omega(m',1) \text{ for all } m'\}$[18].

[17]See the application of the model to theoretical particle physics or to conspicuous consumption.
[18]This set depends on the equilibrium sender strategy σ through the beliefs $\tau_\omega(m,1)$.

The Lemma says that when the number of senders in the market is large enough, the tasks chosen on the equilibrium path must be among the receiver's most preferred (as defined above). That is, given the receiver's preferences over task-performance profiles induced by her beliefs, for sufficiently high n the difference in success probabilities across different tasks becomes a second order consideration for all sender types.

LEMMA **2** *For sufficiently large n, if $\langle \sigma, \rho, \{p_\mu(t|m)\}\rangle$ is a sequential equilibrium of $\mathcal{G}_n$ it must be the case that $e(\sigma) \subseteq Pf(\sigma)$.*

The idea of the proof is as follows. When n is large enough, there are many senders with each type t_i. If some most preferred task ($m \in Pf(\sigma)$) occurs on the equilibrium path, it is almost certain that there are more senders succeeding in this task than the number of jobs such that each job almost certainly gets assigned to senders who choose from the set of most preferred tasks. So it cannot be optimal for any sender type to choose a task m' such that $(m', 1)$ is not among the preferred tasks.[19] On the other hand, if none of the most preferred tasks can occur on the equilibrium path, then by deviating to one of the most preferred tasks a sender would for sure obtain the best job upon success. And regardless of how low this success probability is, the expected payoff from deviation would eventually exceed the payoff on the equilibrium path which converges to 0 when increasing the number of senders.

Proof of Lemma 2

Suppose it is not the case that $e(\sigma) \subseteq Pf(\sigma)$. We split the argument into two cases: (I) $e(\sigma) \cap Pf(\sigma) \neq \varnothing$ and (II) $e(\sigma) \cap Pf(\sigma) = \varnothing$.

(I) $e(\sigma) \cap Pf(\sigma) \neq \varnothing$

Consider any $m' \in e(\sigma)$ such that $m' \notin Pf(\sigma)$. A necessary condition for a sender to get a job by choosing $m' \in e(\sigma)$ is that one of the r principals is forced to resort to his less preferred options (call this event E_1). E_1 the complement of the event (E_1^c) that each principal is able to assign her respective job to some candidate with one of her most preferred tasks. A sufficient condition for E_1^c is that at least r senders are successful in some task $m \in e(\sigma) \cap Pf(\sigma)$. Let $q = min\{\sigma(m|t) : t \in T\}$. We can bound the probability of E_1^c with the probability of the less likely event that r senders succeed in m with the lowest success probability.[20] So we have that:

[19] For low values of n this is not the case as when the principal gets to choose, it is possible that there is no sender left with the principal's most preferred tasks so it is likely that he needs to continue going down the list to his second most preferred tasks or even further.

[20] For the purpose of establishing this bound we think of the event of succeeding in a specific task as two independent events in a sequence: (1) choosing the task,which happens with probability at least q and (2) succeeding at it, which happens with probability at least $\mathbf{s}(m, t^0)$.

$$
\begin{aligned}
P(E_1) &= 1 - P(E_1^c) \\
&\leq 1 - \sum_{i=r}^{n} \binom{n}{i} q\mathbf{s}(m,t^0)^i (1 - q\mathbf{s}(m,t^0))^{n-i} \\
&= \sum_{i=0}^{r-1} \binom{n}{i} q\mathbf{s}(m,t^0)^i (1 - q\mathbf{s}(m,t^0))^{n-i} \\
&\leq \frac{1}{2} e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}
\end{aligned}
$$

where the last inequality is Hoeffding's bound for the cumulative distribution function of binomially distributed random variables. So the payoff to a sender from choosing m' can be bounded above for large enough n by $u(1)\frac{1}{2}e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}$

On the other hand, the probability that a sender gets a job from choosing $m \in e(\sigma) \cap Pf(\sigma)$ is bounded below by $\frac{\mathbf{s}(m,t^0)}{n}$, which is the probability that the sender succeeds at m and is then chosen uniformly at random from among all other succeeding senders, which are less n. Thus the payoff can be bounded from below by $u(r)\frac{\mathbf{s}(m,t^0)}{n}$. We therefore have that:

$$
\begin{aligned}
\frac{U(m',\sigma_{-i},\rho,t)}{U(m,\sigma_{-i},\rho,t)} &\leq \frac{u(1)\frac{1}{2}e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}}{u(r)\frac{\mathbf{s}(m,t^0)}{n}} \\
&= \frac{u(1)}{2u(r)\mathbf{s}(m,t^0)} n e^{-2(nq^2\mathbf{s}(m,t^0)^2 - 2(r-1)q\mathbf{s}(m,t^0) + \frac{(r-1)^2}{n})} \\
&\longrightarrow 0 \text{ as } n \text{ goes to } \infty
\end{aligned}
$$

The convergence to 0 holds because ne^{-cn} converges to 0 as n goes to infinity for any constant $c > 0$. Thus it is not optimal to choosing m', which contradicts the assumption $m' \in e(\sigma)$. We can conclude that $e(\sigma) = Pf(\sigma)$.

(II) $e(\sigma) \cap Pf(\sigma) = \varnothing$

The payoff that the sender would get from choosing some task $m \in Pf(\sigma)$ is bounded from below by $u(r)\mathbf{s}(m,t^0)$, whereas his payoff from choosing any task $m \in e(\sigma)$ converges to 0 as $n \longrightarrow \infty$. This is because for any large enough $w > r$, the probability that at least w other senders choose m and succeed converges to 1 as n goes to infinity. Upon such a tie, the r jobs are assigned uniformly at random among these $w+1$ senders, so the probability that any given sender gets a job converges to 0. ∎

Proof of Proposition 3:

By Lemma 2 for sufficiently large n it is true that for all tasks m chosen on the equilibrium path $\tau_\omega(m,1) \geq_p \tau_\omega(m',1)$ where m' is any other task. The proposition now follows from noting that for sufficiently large n if $e(\sigma) \subseteq Pf(\sigma)$ then $e(\sigma)$ must be a singleton. The reason is that for sufficiently large n all r jobs almost certainly get assigned uniformly at random among the succeeding senders in the tasks in $e(\sigma)$[21] and as a result the part of any sender's expected payoff that corresponds to the event of being assigned a job after failing converges exponentially fast to 0, as in the proof of Lemma 2. As a result it becomes strictly better for each sender to pick the task in $e(\sigma)$ at which he is most likely to succeed, which by assumption (A3) is the one in $e(\sigma)$ with the lowest index, for all sender types. So unless this is the unique element of $e(\sigma)$, σ can't possibly be part of a sequential equilibrium $\langle\sigma^*, \rho, \{p_\mu(t|m)\}\rangle$. ∎

The argument in the proof of Proposition 3 is similar to the one behind the competitive unraveling in our example in Section 2. Underneath any equilibrium there are some tasks which are most preferred by the principals (inside or outside the equilibrium path) ($Pf(\sigma)$ in our notation). Although at low levels of competition, senders may trade off attempting one of the principal-preferred tasks $m \in Pf(\sigma)$ for a higher chance of success at some task $m' \notin Pf(\sigma)$ which is still likely to be rewarded with a job, as n rises, the cost of not competing for one of the employer-preferred tasks comes to exceed the benefits of the higher success probability at some other task.

Formally speaking equilibria of $\mathcal{G}_n$ and $\mathcal{G}_{n'}$ when $n \neq n'$ are necessarily different as the number of players is a primitive of the game and thus to begin with the spaces of receiver strategies are formally different in both games. However, as can be seen throughout the discussion, the only components of the receiver strategy that play a role in satisfying the definition of sequential equilibrium in our model are her strategies on the equilibrium path, and on information sets that are reached by deviations by a single sender from the equilibrium path. Given a pooling strategy of the senders on any given m, there are equivalent equilibria sustaining it in the corresponding games $\mathcal{G}_n$ for all values of n in the sense that that they involve exactly the same receiver beliefs about each individual sender under any possible behavior on and off the equilibrium path.

Proposition 3 established that the only sender strategies that are *robust to competition* are those which prescribe that every type chooses a given task with probability 1. The fact that for all n, in the game $\mathcal{G}_n$ there is a sequential equilibrium $\langle\sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ for any such pooling sender strategy σ^* depends crucially on the large margin for discretion in setting the out of the equilibrium path beliefs left by the definition of belief consistency in sequential equilibria. In particular the principals must believe that any sender undertaking a task out of the equilibrium path is of a sufficiently low type regardless of his performance. In the following section we discuss an argument which rules out these beliefs for all pooling

[21] Keeping in mind that the principals choose uniformly at random among the senders among which they are indifferent.

equilibria but the one in the most discriminating task m^z.

4.2 Out-of-Equilibrium-Path Beliefs in Our Framework

We begin this section with a simple observation:

(Obs1) Assumption (A2) implies that the posterior distribution of the type of a fixed sender, inferred via Bayes' rule after observing him succeed in a task m' would first order stochastically dominate the posterior computed after observing him succeed in any task $m < m'$.

This observation motivates the following out-of-the-equilibrium path beliefs restrictions

Definition **2 (Success Monotonicity of Beliefs)** *Given a sequential equilibrium* $\langle \sigma^*, \rho, \{p_\mu(t|m)\}\rangle$, *beliefs* $\{p_\mu(t|m)\}$ *are said to be* ***success monotonic*** *if* $\tau_\omega(m', 1)$ *FOSD* $\tau_\omega(m^{min}, 1)$ *whenever* $m' > max\{m : m \in e(\sigma)\}$ *and* $m^{min} \in \{m : m \in e(\sigma)$ *and* $\nexists\, m' \in e(\sigma), m' \neq m$ *such that* $\tau_\omega(m', 1) \preceq_p \tau_\omega(m, 1)\}$.

Intuitively *success monotonicity of beliefs* requires the following. When a sender deviates to a more difficult and conspicuous task m' (in the sense of (A1) and (A2)) than any task chosen on the equilibrium path, the receiver prefers a success in m' to a success in the least preferred on-equilibrium-path task m^{min} (least in the sense of FOSD). This is consistent with our observation (Obs1). Lemma 2 has a straightforward corollary that applies when we restrict attention to sequential equilibria sustained by beliefs that are success monotonic.

Corollary **1** *If we restrict attention to sequential equilibria* $\langle \sigma^*, \rho, \{p_\mu(t|m)\}\rangle$ *sustained by beliefs that are success monotonic then the only sender behavior that is robust to competition is* $\sigma(m^z|t) = 1\ \forall t$.

Proposition 2 together with the fact that as there are no tasks greater than m^z, imply that there exist success monotonic beliefs that can sustain $\sigma(m^z|t) = 1\ \forall t$ as part of a sequential equilibrium. Note that for low values of n other pure strategy pooling equilibria (at less informative research tasks than m^z) are not affected by this belief restriction as the additional benefits from succeeding at a more conspicuous task may not compensate the lower success probability and the possible worse outcomes upon failure.

We end this section with two propositions. The first one shows that in our game a relatively weak[22] refinement of sequential equilibrium, Banks and Sobel's divinity, implies success

[22] It is the weakest among the Divinity, D1, D2 and Universal Divinity and stability, which along with the intuitive criterion are some of the most common refinements of this kind.

monotonicity of beliefs when n is sufficiently large, for all pooling equilibria. The second proposition shows that the sequential equilibrium that involves all types choosing the most conspicuous task is also divine.

PROPOSITION 4 *For large enough n if a pooling sequential equilibrium of $\mathcal{G}_n$, $\langle \sigma^*, \rho, \{p_\mu(t|m)\}\rangle$ is divine, then the beliefs $\{p_\mu(t|m)\}$ are success monotonic.*

PROPOSITION 5 *For large enough n there exists a divine equilibrium of $\mathcal{G}_n$ in which the senders' strategy is $\sigma(m^z|t) = 1\ \forall t$.*
Moreover:

- *The only sender behavior that is part of a divine equilibrium (pooling, separating or mixed) for each game $\mathcal{G}_n$ is $\sigma(m^z|t) = 1\ \forall t$.*
- *For large enough n, $\sigma(m^z|t) = 1\ \forall t$ is the senders' strategy in all divine equilibria in pure strategies.*

The proof of proposition 4 involves showing that for large enough n the only strategies of the principals that justify a deviation from the equilibrium behavior by some type also justify a deviation by all other types. This implies that for large enough n the only candidate divine beliefs $p_\mu(m')$ for any $m' \notin e(\sigma^*)$ coincide with the prior distribution p_α. Part (3) of Lemma 1 then implies that when m' is more difficult than the tasks chosen on the equilibrium path, $\tau_\omega(m', 1)$ FOSD $\tau_\omega(m, 1)\ \forall m \in e(\sigma)$. This stochastic dominance relation is exactly the requirement for success monotonicity of beliefs. The first part of Proposition 5 follows from the fact when the beliefs are success monotonic and n is large enough, a deviation from the simple pooling equilibrium where $\sigma(m^z|t) = 1\ \forall t$ to choosing $m < m^z$, cannot be beneficial because the jobs are filled with successful senders in task m^z almost certainly. The second part in turn follows from Proposition 3, Proposition 4 and Corollary 1. We relegate these two formal proofs to the appendix as they do not shed much more light on the proposition than this argument, yet are technically cumbersome due to the requirements of the definition of divinity.

5 Conclusion

We study in detail a signaling model that captures scholars' choice of research tasks as they seek to make progress in the early stages of their academic careers. The model fits other settings of on-the-job screening, and more generally to situations in which senders are competing for prizes by sending noisy messages about unobservable characteristics. The key feature of the setting[23] is that the more difficult tasks are more conspicuous in the sense that they distinguish more clearly among different underlying research abilities. As it is often the case in settings with asymmetric information, in our model there are multiple sequential equilibria, reflecting a variety of conventions which society can uphold by relying on appropriate off-the-equilibrium-path beliefs. Different equilibria can be distinguished by two features which are essential in our context: their direct *productive efficiency* and their *screening efficiency*. We focus on the subset of sequential equilibria in which beliefs about the underlying research ability induced by a success in any more difficult and out-of-equilibrium task first order stochastically dominates the belief induced by a success in the least preferred on-equlilirium-path task. This selection criterion is implied by Banks and Sobel's divinity for a sufficiently high level of competition. We show that under this refinement all sequential equilibria vanish for sufficiently high competition, with the exception of the one in which all senders choose the most difficult/most conspicuous task.

While this tendency towards *conspicuousness* may be seen as detrimental in certain contexts, it can also serve the crucial role of providing the kind of continuity and persistence required for certain scientific breakthroughs. In fact, as can be seen throughout the history of science, the apparent fertility or barrenness of a scientific agenda at a given moment in time is often a bad predictor of its future.

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Appendix: Omitted Proofs

Proof of Lemma 1

Part 1

By $(BC3)$ we have that:

$$P(t \geq t^l|m,1) = \sum_{t\geq t^l} p_\omega(t|m,1) = \sum_{t\geq t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t\in T} p_\mu(t|m)\mathbf{s}(m,t)} \quad \text{and}$$

$$P(t \geq t^l|m,0) = \sum_{t\geq t^l} p_\omega(t|m,0) = \sum_{t\geq t^l} \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t\in T} p_\mu(t|m)(1-\mathbf{s}(m,t))}$$

So we have that:

$P(t \geq t^l|m,1) \geq P(t \geq t^l|m,0)$

$$\iff \sum_{t\geq t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t\in T} p_\mu(t|m)\mathbf{s}(m,t)} \geq \sum_{t\geq t^l} \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t\in T} p_\mu(t|m)(1-\mathbf{s}(m,t))}$$

$$\iff \sum_{t\in T} p_\mu(t|m) \sum_{t\geq t^l} p_\mu(t|m)\mathbf{s}(m,t) \geq \sum_{t\geq t^l} p_\mu(t|m) \sum_{t\in T} p_\mu(t|m)\mathbf{s}(m,t)$$

$$\iff \left(\sum_{t<t^l} p_\mu(t|m) + \sum_{t\geq t^l} p_\mu(t|m)\right) \sum_{t\geq t^l} p_\mu(t|m)\mathbf{s}(m,t) \geq$$

$$\sum_{t\geq t^l} p_\mu(t|m) \left(\sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t) + \sum_{t\geq t^l} p_\mu(t|m)\mathbf{s}(m,t)\right)$$

$$\iff \sum_{t<t^l} p_\mu(t|m) \sum_{t\geq t^l} p_\mu(t|m)\mathbf{s}(m,t) \geq \sum_{t\geq t^l} p_\mu(t|m) \sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t)$$

$$\iff \frac{\sum\limits_{t \geq t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t \geq t^l} p_\mu(t|m)} \geq \frac{\sum\limits_{t < t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t < t^l} p_\mu(t|m)}$$

which is necessarily true since $\mathbf{s}(m,t)$ is strictly increasing in t for all tasks m (A1):

$$\frac{\sum\limits_{t \geq t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t \geq t^l} p_\mu(t|m)} \geq \mathbf{s}(m,t^l) \geq \frac{\sum\limits_{t < t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t < t^l} p_\mu(t|m)}$$

Note that if $\tau_\mu(m)$ is non-degenerate then at least one of these two inequalities becomes strict under $(A1)$. ∎

Part 2

From Part 1, we know $\tau_\omega(m,1)$ FOSD $\tau_\omega(m,0)$. As $P(t \geq t^l|m) = P(s_i = 1|m)P(t \geq t^l|m,1) + P(s_i = 0|m)P(t \geq t^l|m,0)$ this implies $\tau_\omega(m,1)$ FOSD $\tau_\mu(m)$. ∎

Part 3

$P(t \geq t^l|m',1) \geq P(t \geq t^l|m,1)$

$$\iff \sum_{t \geq t^l} \frac{p_\mu(t|m')\mathbf{s}(m',t)}{\sum\limits_{t \in T} p_\mu(t|m')\mathbf{s}(m',t)} \geq \sum_{t \geq t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t \in T} p_\mu(t|m)\mathbf{s}(m,t)}$$

$$\iff \sum_{t \geq t^l} \frac{p_\mu(t|m')\mathbf{s}(m',t)}{\sum\limits_{t \in T} p_\mu(t|m')\mathbf{s}(m',t)} \geq \sum_{t \geq t^l} \frac{p_\mu(t|m')\mathbf{s}(m,t)}{\sum\limits_{t \in T} p_\mu(t|m')\mathbf{s}(m,t)}$$

$$\iff \sum_{t \in T} p_\mu(t|m')\mathbf{s}(m,t) \sum_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m',t) \geq \sum_{t \in T} p_\mu(t|m')\mathbf{s}(m',t) \sum_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m,t)$$

$$\iff \sum_{t < t^l} p_\mu(t|m')\mathbf{s}(m,t) \sum_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m',t) \geq \sum_{t < t^l} p_\mu(t|m')\mathbf{s}(m',t) \sum_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m,t)$$

$$\iff \frac{\sum\limits_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m',t)}{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m',t)} \geq \frac{\sum\limits_{t \geq t^l} p_\mu(t|m')\mathbf{s}(m,t)}{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m,t)}$$

Note that for each $t^* \geq t^l$ such that $p_\mu(t^*|m') > 0$, we have that:

$$\frac{p_\mu(t^*|m')\mathbf{s}(m',t^*)}{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m',t)} > \frac{p_\mu(t^*|m')\mathbf{s}(m,t^*)}{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m,t)}$$

$$\iff \frac{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m,t)}{p_\mu(t^*|m')\mathbf{s}(m,t^*)} > \frac{\sum\limits_{t < t^l} p_\mu(t|m')\mathbf{s}(m',t)}{p_\mu(t^*|m')\mathbf{s}(m',t^*)}$$

which is true due to $(A2)$. Note that if $p_\mu(t^*|m') = 0$ then the two expressions are equal. Note that in particular when $t^l = t^0$ and provided that $\tau_\mu(m) = \tau_\mu(m')$ is non-degenerate the inequality involving the cumulative distributions is strict. We therefore have the first order stochastic dominance relationship stated in the Lemma. ▮

Part 4:

First assume that t does not fully separate itself from other types. That is, there exists m such that $\sigma(m|t) > 0$ and $\sigma(m|t'') > 0$ for some $t'' \neq t$. From part 1 we have that $\tau_\omega(m, 1)$ FOSD $\tau_\omega(m, 0)$ and by (F1) the receiver strictly prefers success to failure. Moreover since $t' > t$ and $\mathbf{s}(m, t') > \mathbf{s}(m, t)$, type t' is more likely to succeed in all tasks. So if type t' mimics types t it would get a strictly higher payoff.

So we just need to rule out that t fully separates itself from all other types, which is when performance does not provide the receiver with additional information. Suppose that t fully separates itself. We claim that this implies that type t^0 is not acting optimally . If $t = t^0$ then due to full separation the receiver knows for sure that it is t^0 and she strictly prefers any sender choosing any task different from those chosen by t on the equilibrium path regardless of success or failure. So a type $t = t^0$ sender is not acting optimally as he would do strictly better by imitating any other type. Now suppose that $t > t^0$ and notice that by the discussion above t^0 does not use a fully separating strategy, which in turn implies that $U(\sigma^*, \rho^*, t) > U(\sigma^*, \rho^*, t^0)$. Moreover, if t^0 mimics t, then t^0 would earn utility $U(\sigma^*, \rho^*, t)$ since t fully separates itself and therefore performance is irrelevant.

It is therefore never the case that a sender type ever fully separates itself, and we can conclude from above that $U(\sigma^*, \rho^*, t') > U(\sigma^*, \rho^*, t)$. ▮

Definition of Divinity

What follows is the definition of divine equilibrium as introduced by Banks and Sobel (1987) [3]. The differences in notation simply reflect the greater complexity of the receiver's action space in our framework given that there are multiple senders, and the fact that in our analysis we need to refer to games $\mathcal{G}_n$ for different values of n.

Given some sequential equilibrium $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ of game $\mathcal{G}_n$, some $\rho \in \Sigma_R$ identical to ρ^* on $e(\sigma^*)$ and $m \notin e(\sigma^*)$, let:

- $\overline{\mu}(t, \rho) = \begin{cases} 1 & \text{if } U(m, \sigma^*, \rho, t) > U(\sigma^*, \rho^*, t) \\ [0,1] & \text{if } U(m, \sigma^*, \rho, t) = U(\sigma^*, \rho^*, t) \\ 0 & \text{if } U(m, \sigma^*, \rho, t) < U(\sigma^*, \rho^*, t) \end{cases}$
- $\Gamma(\rho) = \{\gamma \in \Delta_T : \exists c > 0 \text{ such that } \gamma(t) = c\overline{\mu}(t, \rho) p_\alpha(t), \forall t \in T\}$
- For any $A \subseteq \Sigma_R$ let $\overline{\Gamma}(A) = ConvexHull\left[\bigcup_{\rho \in A} \Gamma(\rho)\right]$

Given the above let:

- $\Sigma_R(m) = \{\rho \in \Sigma_R : \rho(\cdot|m,s) = \rho^*(\cdot|m,s)\ \forall s \text{ and } \forall m \in e(\sigma^*)\}$
- $\Gamma_0^n = \Delta_T,\ A_0^n = \Sigma_R(\rho^*)$
- For $k \geq 0$, $\Gamma_k^n = \begin{cases} \overline{\Gamma}(A_{k-1}^n) & \text{if } \overline{\Gamma}(A_{k-1}^n) \neq \varnothing, \\ \Gamma_{k-1}^n & \text{if } \overline{\Gamma}(A_{k-1}^n) = \varnothing, \end{cases}$
- $A_k^n = BR(\Gamma_k^n, (\sigma, m)), \quad \Gamma^{n,*} = \bigcap_k \Gamma_k^n, \quad A^{n,*} = \bigcap_k A_k^n$

  where $BR(\Gamma_k^n, (\sigma, m))$ denotes the subset $\Sigma_R(\rho^*)$ such that $\rho((m,s),(m',s')_{-i})$ is a best response for all $s \in \{0,1\}$ and all $(m',s')_{-i}$ on the equilibrium path, for some beliefs $\gamma \in \Gamma_k^n$,.

A *divine equilibrium* is a sequential equilibrium $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ such that for each task $m \notin e(\sigma)$ we have that $p_\mu(m) \in \Gamma^{n,*}$.

Proof of Proposition 4: Let m be the task on which all sender types pool and let $m' > m$. Following the adaptation to our framework of Banks and Sobel's (1987) [3] definition of divine equilibrium, we will show that for sufficiently large n, $p_\alpha \in \Gamma_k^n(m')\ \forall k$, and that there does not exist any other element of Δ_T belonging to $\Gamma_2^n(m')$. This will allow us to conclude that for sufficiently large n, $\Gamma^{n,*}(m') = \bigcap_k \Gamma_k^n(m') = \{p_\alpha\}$. That is, for sufficiently large n, divine beliefs require that $\tau_\mu(m')$ be distributed according to p_α. Assumption $(A2)$ then implies that $\tau_\omega(m',1)$ FOSD $\tau_\omega(m,1)$.

We first show that $p_\alpha \in \Gamma_k^n(m')\ \forall k$ for sufficiently large n. $\Gamma_0^n = \Delta_T$ and therefore $p_\alpha \in \Gamma_0^n$ for all n. Now consider the receiver strategy $\rho' \in A_0^n = \Sigma_R(\rho)$ which is identical to ρ on the equilibrium path and which assigns J_1 to any sender that deviates to m' and succeeds. This implies that $U(m', \sigma_{-i}, \rho, t) \geq u(1)\mathbf{s}(m', t_0)$. On the other hand, $U(\sigma, \rho, t)$ converges to 0 as $n \longrightarrow \infty$. So for sufficiently large n $U(\sigma, \rho, t) < U(m', \sigma_{-i}, \rho, t)$ for all t and therefore $\overline{\mu}^n(t, r) = 1\ \forall t$, so $p_\alpha \in \Gamma_1^n$. By (A2) and the fact that $\sigma^*(m|t) = 1\ \forall t$ and therefore $p_\mu(m) = p_\alpha$ we have that $\tau_\omega(m',1)$ FOSD $\tau_\omega(m,1)$ by part 3 of Lemma 1. The unique best response for the receiver is ρ'' which assigns J_1 to a sender that succeeds in m' which implies that $\rho \in A_1 = BR(\Gamma_1^n, m')$. Now we can iterate this argument to show that $p_\alpha \in \Gamma_k^n$ and $\rho \in A_k^n$ for all k and that therefore $p_\alpha \in \Gamma^{n,*}(m')$ as required.

We now show that $p_\alpha = \Gamma_2^n(m')$ for sufficiently large n. $A_1 = BR(\Gamma_1^n, m')$ and therefore if $\rho' \in A_q$ it can treat a successful deviating senders in one three ways: 1) It assigns him J_1, reflecting $\tau_\omega(m',1) \succ_P \tau_\omega(m,1)$; 2) It includes him in the random assignment of each of the r jobs among the senders that succeed[24] in task m; 3) It considers him for one of the job only

[24]Formally this category also contemplates the possibility that the number of succeeding senders in task m is less that r, but the probability of this event converges exponentially fast to 0 as n goes to infinity.

in the event that less than r senders succeed in m. Note that if ρ is of types 2) or 3) then $\overline{\mu}(t,\rho) = 0$ for sufficiently large n, which means that the only elements $\Gamma_2^n(m') = \overline{\Gamma}_2^n(A_1)$ are beliefs justified by receiver strategies in the first category. As in the first part of this proof this implies that $\Gamma_2^n(m') = p_\alpha$.

By Lemma 1 $\tau_\omega(m',1)$ FOSD $\tau_\omega(m,1)$ which implies that for sufficiently large n the only beliefs in $\Gamma^{n,*}(m')$ are success monotonic. ∎

Proof of Proposition 5: Note that as all the arguments in the proof of proposition 4 also apply to the pooling equilibrium on m^z, and to deviations to any task $m' < m^z$, so the only beliefs in $\Gamma^{n,*}(m')$ are also p_α. However $(A2)$ implies Lemma 1 that $\tau_\omega(m^z,1)$ FOSD $\tau_\omega(m',1)$, such that the receiver can only consider a successful sender in m' for one of the jobs if there are not enough successful senders in m^z. The probability of this event converges exponentially fast to 0 as n goes to infinity, so as in the proof of Lemma 2 all types of senders strictly prefer to choose task m^z than any task $m' < m^z$.

The second part in turn follows from Proposition 3, Proposition 4 and Corollary 1. ∎