

Fighting For The Best, Losing With The Rest: A Case for Restricting Credit to Business Start-ups*

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September 5, 2017

Abstract

The Jumpstart Our Business Startups (JOBS) act of 2012 aims at increasing funding access for young firms by easing securities regulation. Motivated by this, we ask if there is a role for the regulation of the market of funds for firms that lack collateral and have a large uncertainty about their ability to generate profits. To answer that we characterize optimal financial contracts in a competitive environment with risk, adverse selection and limited liability. We find that competition among financial intermediaries always forces them to fund projects with negative expected returns both from a private and from a social perspective. Intermediaries use steep payoff schedules to screen entrepreneurs, but limited liability implies this can only be done by giving more to all entrepreneurs. In equilibrium, competition for the best entrepreneurs forces intermediaries to offer better terms to all customers, there is cross subsidization among entrepreneurs and intermediation profits are nil. The three main features of our framework (competition, adverse selection and limited liability) are necessary in order to get the inefficient *laissez-faire* outcome and a role for barriers to entry into financial intermediation. Our result remains robust when firms can collateralize some portion of the credit as long as there is still an unsecured fraction.

JEL Codes: D82, G14, G28

Keywords: Adverse Selection, Market Inefficiency, Bank Regulation, Deregulation, Incentive Compatible Mechanism

*We thank Hal Cole, Guillermo Ordoñez, Dirk Krueger and Ali Sourideh for their guidance and useful comments. We also want to thank Alberto Ciancio, Michael Chirico, Matt Darst, and participants to European Winter Meeting of the Econometric Society 2015, LACEA-LAMES meeting 2014 and the University of Pennsylvania Macroeconomics Club. The usual waiver of liabilities applies.

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1 Introduction

Innovation has been widely recognized as the key source of economic growth, at least going back to the work of Schumpeter (1934) and startups are the backbone of technological progress. (Arrow, 1962) argued that potential entrants have stronger incentives to undertake innovation than incumbent monopolists, and indeed, an important fraction of new projects and products come from start-up firms. However, in many situations the firm or entrepreneur having access to a potential project lacks the financial resources needed to undertake it and has to rely on external lenders or investors.

Financial markets for small R&D intensive start-ups feature several frictions that generically lead to inefficient outcomes. First, the profitability of a particular endeavor is not guaranteed, the research project or new product may or may not succeed and this uncertainty can only be resolved after the investment is made. In that sense, projects are *risky*. Moreover, since innovations require specific and sophisticated knowledge, it is likely that the entrepreneur will know more about the project's prospects than investors do. This introduces a second friction, *asymmetric information*. Third, by the nature of the project, its most important assets are the knowledge, time and effort, devoted by its team of workers. If the project fails, the salvage value is close to zero. As a consequence, there is a severe restriction emanating from the *limited liability* clause.

The traditional view on government regulation of the financial intermediation services for startups has stressed the role of barriers to entry in reducing investment, innovation and growth: see King and Levine (1993) for the seminal contribution and Levine (2005) for a more recent survey on the literature. Following that prescription, legislation has intended to deregulate the market for start-ups financing: The Jumpstart Our Business Startups (JOBS) act of 2012 eased securities regulation making it easier for companies to both go public, and raise capital privately. This motivates us to ask if complete deregulation is desirable in a market featuring risk, asymmetric information and limited liability as described above.

Given the new deregulated environment and the recent developments of new ways to fund startups¹ we approach the question from an unrestricted contract space, that is, we characterize the contracts offered in equilibrium by a competitive financial sector to entrepreneurs facing risk and limited liability, in an environment with adverse selection as we aim to capture the market response to a deregulated environment. We then proceed to describe the welfare implications of the resulting equilibrium. Our main result is that the optimal contract delivers an inefficient outcome. This is in contrast with the screening literature in environments with linear types (such as ours): when utility is linear in the type, optimal contracts achieve first best allocations. In our environment, the presence of limited liability and an outside option for entrepreneurs do not allow to achieve the first best outcome.

The key characteristic is the intrinsic quality of the project, which is known to the entrepreneurs but not to the financial intermediaries. Mechanism design and contract theory

¹The JOBS act reduced requirements for entrepreneurs to advertise their projects to specialized investors and even the general public, which gave birth to dedicated websites that listed potential investors like AngelList, and crowdfunding approaches like Indiegogo and Kickstarter. Under the old regulations, those were considered private share offerings and thus subject to all the securities regulations.

have developed ways to get around adverse selection problems like this one. However in our environment, when both limited liability and competition in the financial markets are present, the contracting equilibrium is not efficient. Interestingly, if financial intermediaries were able to collude or entrepreneurs were not protected by the limited liability clause, the first best outcome would be achieved. Our main result shows the inefficiency can be corrected using simple policy tools such as a fixed cost per financial contract, which resembles the red-tape requirements brought down by the JOBS act. This is also in contrast with the traditional view of Levine (2005) who argue that such barriers to entry diminish output.

In our model, there is a continuum of entrepreneurs, each one having access to a risky project. The entrepreneurs are heterogeneous on the probability with which their project will succeed. Also, they typically won't have enough resources to fund their projects and will need to rely on financial intermediaries. Financial intermediaries supply financial contracts in a competitive way, aiming to maximize their profits, but do not observe the ex ante probabilities of success. Beyond the information friction and limited liability, the set of contracts to be offered is completely unrestricted. Naturally, all intermediaries would like to attract the best entrepreneurs, and competition will force them to offer good terms on borrowing for the best types. They also will need to provide incentives in the right way to distinguish good entrepreneurs from bad entrepreneurs which implies a (very specific) gap between the contracts offered to good and bad types. However, limited liability imposes a lower bound on the terms of a contract. Hence, in order to sustain those incentives, bad types must also receive good terms. In equilibrium, intermediaries will break even, but they will fund both projects with positive expected profits (from good types) and projects with negative expected profits (with bad types). The inefficiency arise because entrepreneurs have an outside option (or equivalently a utility cost). We show that if an intermediary is only funding socially efficient projects (i.e: projects with expected profit higher than the cost of capital plus the opportunity cost of the entrepreneur), she is making positive profits. As a consequence, her competitor is willing to improve borrowing terms for good types, even though some low types with inefficient projects will take the contract. The inefficiency can be corrected with simple tax instruments.

In the basic model, entrepreneurs cannot collateralize their assets. We extend the model to the case in which entrepreneurs have some collateral. We show that as long as loans cannot be fully collateralized, the inefficiency is reduced but not removed. We then proceed to study how the inefficiency changes when the parameters of the model change. The effect of the entrepreneur's outside option is non-monotonic: the inefficiency is zero if there is no outside option (all projects are socially efficient), but it is also zero if the outside option is so high that nobody want to undertake risky projects (no project is efficient). Further, the inefficiency (relative to the net economic surplus) increases with the cost of capital faced by financial intermediaries and with the relative density of lower types. When we allow for collateralizable assets, the inefficiency decreases with the mean of the asset distribution.

Starting from Stiglitz and Weiss (1981), and extensive body of literature has studied asymmetric information in financial markets. The main message of Stiglitz and Weiss (1981) is that the interest rate cannot clear the credit market because of a standard "lemons" problem, and as a result, there is credit rationing. Subsequent papers allow the financial intermediaries (banks) to use different tools, other than the interest rate, to screen borrowers' types. A first strand

of papers allow intermediaries to use collateral, on top of the interest rate, to screen types. Bester (1985) turns down the credit rationing result, by allowing intermediaries to offer interest rate/collateral contract pairs. By using collateral in addition to the interest rate, banks can screen borrowers: risky borrowers will accept to pay higher interest rates in order to benefit from a lower collateral requirement. However, in Bester (1985)'s economy, there is no limit to the amount of collateral that borrowers can provide. The question of limits to collateral is studied by Besanko (1987). The environment is similar to Bester (1985)'s, and safer types will prefer loans with low interest rate and high collateral. Nonetheless, it may be that the borrower has not enough wealth to provide the required collateral. In that case, the collateral/interest rate pair cannot achieve the sufficient spread in payoffs necessary to separate types. Besanko (1987) solve this issue by allowing the contracts to additionally depend on the probability of approval. To achieve the necessary spread of utilities, low interest - high collateral credits will be denied with positive probability. An interesting point in Besanko (1987) that relates to our result is that the paper compares welfare when the financial intermediation sector is competitive or a monopoly, and finds that monopoly may lead to a higher welfare, depending on parameter values.

Another strand of papers have departed from the Stiglitz and Weiss (1981) result by allowing intermediaries to screen borrowers using the size of the loan. A contract is hence a pair interest rate - loan size. In Milde and Riley (1988), borrowers are entrepreneurs with access to a project with risky returns. The return on the project depends on both the borrower's type and the size of the loan. The interaction between type and loan size in the project's payoff allows to separate types using interest rate - loan size menus. The outcome, however, depends strongly on the specific function mapping the type and loan size to the return of the project. In general good types take bigger loans accompanied by higher interest rates. However, Milde and Riley (1988) provide examples of production functions for which the opposite happens: good types take smaller loans and pay lower interests. A point to keep in mind from Milde and Riley (1988) is that projects won't be funded to its optimal, full-information size.

More recently, Martin (2009) uses a similar framework to study the relation between entrepreneurial wealth and aggregate investment. In his model, intermediaries can use both collateral and the size of the loan to screen types. He shows that when entrepreneurial wealth is high, collateral can be used to separate types. When entrepreneurial wealth is low, screening is mainly done by restricting the level of investment, and becomes more costly. As a result, in the later case, a pooling equilibrium is more likely. However, Martin (2009) restricts the interest rate to be un-contingent. We show that, when transfers contingent of success of the project are allowed (say by a contingent interest rate or an equity-like contract), the intermediaries never distort the level of investment to screen types. Instead they find optimal to use the contingent transfer.

Although close in terms of topic, all the papers cited above impose ad-hoc restrictions to the space of contracts potentially offered by the financial sector. In contrast, in our model, the set of contracts is only restricted by the features of the environment.

Lester et al. (2015) develop a model where screening contracts are unrestricted. Their environment features adverse selection between informed sellers and uninformed variables. Beyond asymmetric information, they introduce imperfect information coming from search theoretic

frictions. The later feature allows them to do comparative statics on the degree of imperfect competition and how it interacts with the severity of adverse selection. As in our environment, they find that increasing competition may reduce welfare when markets are competitive.

In our model, the financial intermediation is competitive. Intermediaries fund projects in which they expect to lose, but are socially efficient because the payoff of the entrepreneur compensates the intermediary's losses. They also fund projects that are socially inefficient in the sense that they generate a dead-weight loss. Our environment is close to Rothschild and Stiglitz (1992), in which adverse selection is introduced to a competitive market. In their environment the equilibrium (when it exists), is separating. The limited liability constraint in our model prevents the separating outcome. In our model, the inefficiency results from the interaction of several forces: first, there is asymmetric information that introduces a "lemons" problem; second, limited liability puts a bound on the screening that can be done by financial intermediaries; third, competition among intermediaries introduces profitable deviations from the efficient outcome (that would be reached by a monopolist lender).

The rest of the paper is organized as follows: In the next section we describe the model which is the core of the paper. Then, in section 3 we extend the model to allow for a distribution of assets among entrepreneurs. In section 4 we introduce numerical example, and show how the deadweight loss changes with the parameters of the model. Concluding remarks are made in section 5.

2 Basic Model

2.1 Environment

In this section we describe the main mechanism of the paper in a partial equilibrium static economy. The economy is populated by a continuum of agents with mass 1 indexed by their heterogeneous ability $\theta \in [0, 1]$. Each agent can work for a wage w or undertake a project with a risky outcome, that depends both on the ability of the agent and the capital invested. If an agent decides to start his own project, he or she will have to borrow funds from a financial intermediary².

The financial intermediaries have access to capital at the (gross) risk free rate R . They cannot observe the entrepreneurs ability and will have to provide incentives in order to get that information. Intermediaries can observe investment in the project, i.e. agents cannot divert funds from their projects without being caught. Both agents and intermediaries are risk neutral. However, if a project fails the intermediaries cannot exert any claims on the entrepreneurs. In that sense, projects in this economy are subject to limited liability.

The ability of each agent will determine the probability that an entrepreneurial venture succeeds. We denote $G(\theta)$ the cumulative distribution of abilities. More specifically, if the agent decides to become an entrepreneur and invests an amount k of capital in the project, the project will succeed with probability $p(\theta, k)$

²This w can also be interpreted as the opportunity cost, pecuniary or not, of running the project for the entrepreneur.

Assumption: $p(\theta, k)$ is multiplicatively separable: i.e. $p(\theta, K) = g(\theta)f(k)$. Where f is continuous and $f' > 0$, $f'' < 0$, $f(0) = 0$ and $\lim_{k \rightarrow \infty} f(k) = 1$. Without loss of generality, we can set $g(\theta) = \theta$ since we are just renaming the unobservable types. . Although we are interpreting θ as entrepreneurial ability, notice that it could be anything that is known by the entrepreneur, but not by the intermediary, and that increases the probability that the project succeeds.

The assumption of multiplicative separability is important because it allows us to abstract from Riley-style signaling distortions in investment and highlight our main mechanism where competition in the intermediation sector generates overinvestment. It also helps to keep the model tractable and allows a better characterization of the optimal contract.³

In line with the endogenous growth literature, we interpret a successful project as the arrival of a new innovation, which allows the entrepreneur to create or “steal” some market. We denote π the payoff of a successful project which, in turn, can be interpreted as the value of the innovation. As a result, the expected surplus of an entrepreneurial venture is given by,

$$\theta f(k)\pi - Rk - w$$

As we will show later, in equilibrium, the surplus will be shared between the entrepreneur and the financial intermediary who lends the funds.

In principle, a contract is a triple, (k, x_1, x_0) where the k is the size of the loan, x_1 the repayment in case of success and x_0 the repayment if the project fails. However, for convenience we make a linear transformation of the contract that will allow a more straightforward relation with the mechanism design literature. Set $x = -x_0$ and $z = \pi - (x_1 - x_0)$ From the point of view of the agent the contract (k, x, z) prescribes a fixed pay for the agent x , an additional payment contingent on success z and an investment amount k that determines the probability of the contingent payment happening.

The intermediaries’ objective is to offer a profit maximizing contract schedule, taking into account that agents would choose the better option available to them, and also the competition from other entrepreneurs.

2.2 Contract menus and entrepreneur choices

2.2.1 The game

In this subsection we formally define the game. For simplicity we assume there are only two financial intermediaries, indexed by $i \in \{1, 2\}$, competing *a la* Bertrand for entrepreneurs.⁴

³Without multiplicative separability the optimal contract is harder to characterize, however the main message of the paper remains: the optimal contract yields an inefficient outcome. When the cross-partial derivatives of $\ln(p(\theta, k))$ are not zero, the entrepreneurs will use k to signal type as in Riley (????). As a result, the project size k will be distorted which will lead to another source of inefficiency. Relative to our results, this will lead to less extensive inefficiency (fewer *ex-ante* suboptimal projects are started) but more intensive efficiency (all projects will be run at a suboptimal scale). This is in contrast with the equilibrium of the linear environment we present, in which k is always the full information optimal level.

⁴As long as the entrepreneurs observe all offered contracts, the outcome will be the same with more intermediaries, although the optimal strategies of each one may differ. Hence this is just a notational simplification for free entry in the intermediaries sector.

- **Players:** 2 intermediaries, 1 entrepreneur. The intermediaries are identical. The entrepreneur has a private type θ drawn from a distribution $G(\theta)$.
- **Timing:** Intermediaries move simultaneously, posting arbitrary sets of contracts. Then the entrepreneur chooses among the available contracts an her outside option.
- **Strategies:** For intermediaries the strategy space contains any subset of contracts of the form (k, x, z) . The strategy space is hence the power set of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denoted $\mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})$. We denote the strategy of intermediary i (or his contract menu) by \mathcal{C}_i . For the entrepreneur a strategy is a probability distribution $s : \Theta \times \mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})^2 \rightarrow \Delta\mathbb{R}^3$ such that $Supp\{s(\theta, \mathcal{C}_1, \mathcal{C}_2)\} \subseteq \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{(0, w, 0)\}$. Above, $\Delta\mathbb{R}^3$ denotes the set of probability measures over \mathbb{R}^3 , $Supp\{f\}$ denotes the support of f and $(0, w, 0)$ is the outside option. Abusing notation, we let $s(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z]$ be the cumulative density function of s evaluated at (k, x, z) .

Note that we allowed for mixed strategies for entrepreneurs. They will be able to randomize over any subset of the offered contracts, including the outside option. We focus on the case in which intermediaries play pure strategies.

- **Payoffs:** All players are risk neutral and care only about expected payoff. For an entrepreneur of type θ the expected payoff of signing a contract (k, x, z) is

$$u(\theta, (k, x, z)) = \theta f(k)z + x.$$

in particular, if the entrepreneur take his outside option, his payoff is $u(\theta, (0, w, 0)) = w$. The entrepreneur's expected payoff is,

$$U(\theta, s, \mathcal{C}_1, \mathcal{C}_2) = \int_{(k,x,z) \in \mathbb{R}^3} (\theta f(k)z + x) ds(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z]$$

Conditional on an entrepreneur of type θ signing a contract (k, x, z) with intermediary i her expected payoff is: $\theta f(k)(\pi - z) - x - Rk$.

The total expected payoff of intermediary i can be written as:

$$v_i(s, \mathcal{C}_1, \mathcal{C}_2) = \int_0^1 \int_{(k,x,z) \in \mathbb{R}^3} [\theta f(k) \cdot (\pi - z) - x - R \cdot k] ds(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z] dG(\theta),$$

Given s, \mathcal{C}_1 and \mathcal{C}_2 it is useful to define the set of types *strictly* willing to take a contract from intermediary i ,

$$A_i(s, \mathcal{C}_1, \mathcal{C}_2) = \{\theta : Supp(s(\theta, \mathcal{C}_1, \mathcal{C}_2)) \subseteq \mathcal{C}_i \setminus \mathcal{C}_{-i}\},$$

the set of types indifferent between the two intermediaries,

$$B(s, \mathcal{C}_1, \mathcal{C}_2) = \{\theta : Supp(s(\theta, \mathcal{C}_1, \mathcal{C}_2)) \subseteq \mathcal{C}_i \cap \mathcal{C}_{-i}\}$$

and the set of types willing to sign a contract, rather than taking the outside option,

$$A = A_1 \cup A_2 \cup B$$

Notice that we are using the maintained assumption that when indifferent between being an entrepreneur or a worker, agents prefer entrepreneurship.

2.2.2 Equilibrium definition

The equilibrium concept applicable to this framework is the Bayes-Nash Equilibrium.

Definition A strategy profile (C_1^*, C_2^*, s^*) is a *Bayes-Nash Equilibrium* if:

1. For all $\theta \in \Theta$

$$\begin{aligned} \text{Supp}(s^*(\theta, C_1^*, C_2^*)) \subseteq & \arg \max_{(k,x,z)} \theta f(k)z + x \\ & \text{s.t. } (k, x, z) \in C_1^* \cup C_2^* \cup \{(0, w, 0)\} \end{aligned}$$

2. For each intermediary $i \in \{1, 2\}$, given the entrepreneur strategy s^* and the competitor's contract menu C_{-i}^* her own contract menu C_i^* maximizes her expected utility

$$\begin{aligned} C_i^* \in \arg \max_{C_i} v_i(s^*, C_i, C_{-i}^*) \\ \text{s.t. } \quad \forall (k, x, z) \in C_i \subset \mathbb{R}^3 \\ k \geq 0, \quad x \geq 0, \quad x + z \geq 0. \end{aligned}$$

The conditions $x \geq 0$ and $x + z \geq 0$ make sure that limited liability is satisfied: if the project fails, the entrepreneur cannot make any payment to the intermediary, but the intermediary could potentially make a transfer to the entrepreneur.

2.3 The Equilibrium Contract

In this section we state a sequence of claims leading to the characterization of the equilibrium contract. As will be shown, intermediaries make zero profits in any equilibrium and the entrepreneur's payoff is linear in their type. Although the equilibrium is by no means unique, all equilibria are payoff equivalent.

2.3.1 The Payoff of Entrepreneurs

Let $U(\theta; C_1, C_2)$ be the *potencial* payoff that a θ -type agent could get, conditional on becoming an entrepreneur, when intermediaries play C_1 and C_2 . That is,

$$U(\theta; C_1, C_2) = \max_{(k,z,x) \in C_1 \cup C_2} \theta f(k)z + x$$

Let $(k(\theta), z(\theta), x(\theta))$ be a representative of the (equivalence class of) maximizers. Under the assumptions for θ and $f(k)$, the Spence-Mirrless conditions (single crossing) hold and local incentive compatibility conditions are equivalent to the global incentive compatibility conditions. Hence, Myerson's lemma can be applied:

Lemma 1. *If C_1^* and C_2^* are part of an equilibrium:*

$$U(\theta; C_1^*, C_2^*) = U_i(0) + \int_0^\theta f(k(s))z(s)ds, \tag{1a}$$

$$f(k(\theta))z(\theta) \text{ is non-decreasing.} \tag{1b}$$

$$x(\theta) = U(0) + \int_0^\theta f(k(s))z(s)ds - \theta f(k(\theta))z(\theta), \tag{1c}$$

We refer to $(k(\theta), z(\theta), x(\theta))$ as an *incentive compatible* contract menu.

The equilibrium payoff of a θ -type agent is $\max\{w, U(\theta)\}$.

2.3.2 Project/Loan Size

We now aim to characterize the amount of capital lent to each entrepreneur in equilibrium. First, define $k^*(\theta)$ as the full information optimal investment in a project of type θ .

$$k^*(\theta) = \arg \max_k \{\theta f(k)\pi - Rk\}$$

And let $S(\theta)$ be maximum gross surplus generated by an entrepreneur of type θ ,

$$\begin{aligned} S(\theta) &= \max_k \{\theta f(k)\pi - Rk\} \\ &= \theta f(k^*(\theta))\pi - R \cdot k^*(\theta). \end{aligned}$$

$S(\theta)$ is a gross surplus because it doesn't include the opportunity cost of forgoing the outside option w . Note that under the assumptions for $f(k)$, the optimal project size $k^*(\theta)$ is a continuous and strictly increasing function of θ .

The payoff of the entrepreneur only depends on k through the product $f(k)z$. Because of the multiplicative separability, given k^0, z^0 , the intermediary can offer $k^*(\theta)$ and adjust z to keep $f(k)z$ constant.

Claim 1 formalizes the argument above. All proofs are in the appendix.

Claim 1. *Contracts signed in equilibrium have $k(\theta) = k^*(\theta)$ for (almost) every $\theta \in A$.*

2.3.3 The Payoff of Intermediaries

The market structure resembles Bertrand competition, and it is natural to conjecture that if an intermediary were to make profits, his competitor could offer contracts slightly more generous and steal the market.

Claim 2 (Zero Profit Condition). *In any equilibrium, the profits for intermediaries is zero.*

2.3.4 Fighting for the Best

We just showed that entrepreneurs make zero profits. In some sense, entrepreneurs have all the bargaining power and one could suspect that intermediaries will make zero profits type by type and each entrepreneur would receive all the economic surplus she produces. That suspicion would be correct in a similar framework without limited liability, or without asymmetric information. However, the interaction between the two frictions does not allow for that to happen in our economy. On the contrary, the expected surplus of the project tends to grow much faster than incentives can be provided: whenever expected profits are positive, locally expected revenues increase faster than expected costs. In a nutshell, intermediaries want to invest more in more able types, but cannot increase rewards too fast to keep incentives.

Claim 3. *Suppose $S(\hat{\theta}) \geq U(\hat{\theta})$ for some $\hat{\theta} > 0$. Then, $S(\theta') > U(\theta')$ for almost every $\theta' > \hat{\theta}$.*

Because both intermediaries are making zero profits, and U and S are continuous, there is a θ such that $U(\theta) = S(\theta)$. The claim above implies that such θ is necessarily unique: although intermediaries make zero profits on average, they make strict profits with the best types and strict losses with other types.

The result above is in *sharp contrast* with Rothschild and Stiglitz (1992). In such an environment, if there were profits to be made with a particular type, the market will “cream skim” it, until profits are zero type by type. We will show that the limited liability constraint binds and prevents that from happening. Before, we need to characterize the set A of types choosing to be entrepreneurs.

The next claim states that A takes the form of an interval, and the lowest type willing to take one of the offered contracts rather than the outside option is well defined. We will refer to such a type as θ_L .

Claim 4. *The set A of all types that are willing to accept at least one of the offered contracts is an interval of the form $A = [\theta_L, 1]$ for some $\theta_L \in [0, 1]$. Moreover, if $\theta_L > 0$, $U(\theta_L) = w$.*

When entrepreneurs are risk neutral, it is natural to expect that if a project fails, intermediaries won't pay anything to entrepreneurs, and hence $x(\theta) = 0$. If that was not the case, intermediaries could “cream skim” the market. That is, intermediary i could deviate to a contract serving all the profitable types, and leave all the unprofitable types to his competitor.

Claim 5. *Competition in the intermediation market and the limited liability constraint imply that any contract offered and signed in equilibrium has $x(\theta) = 0$ for all $\theta > \theta_L$.*

It is useful to note that the result above would not hold, absent limited liability. In that case, the optimal mechanism specifies that entrepreneurs get all the profits from the project and pay $R \cdot k^*(\theta)$ no matter if the project succeeds or fail. Limited liability puts a bound to the separation of types. As a result, high types are more profitable for intermediaries.

2.3.5 Limited Incentive Provision

Given the above results, incentives can only be provided using z . Claim 5 implies that $U(\theta) = \theta f(k^*(\theta))z(\theta)$ for all θ . Hence $U(0) < w$ and then $\theta_L > 0$. Also, the set of maximizers of $\theta f(k)z$ is independent of θ . It follows that $f(k(\theta))z(\theta)$ cannot depend on θ . Since, $\theta_L f(k(\theta_L))z(\theta_L) = w$, we get from the condition above:

$$f(k(\theta))z(\theta) = \frac{w}{\theta_L}. \quad (2)$$

This together with claims 1 and 5, fully characterize the equilibrium optimal contract.

Proposition 1. *In equilibrium, the contract signed by almost every θ is:*

$$k^*(\theta) = \arg \max_k \{\theta f(k)\pi - Rk\}, \quad z^*(\theta) = \frac{w}{\theta_L f(k^*(\theta))}, \quad x^*(\theta) = 0.$$

where θ_L , the lowest type who accepts the contract, is such that the financial intermediaries make zero profits.

2.3.6 Some Implications

The expected payoff for the entrepreneur of type θ is $\frac{\theta}{\theta_L}w$, the gross rate of return is $\frac{\theta}{\theta_L k^*(\theta)}w$. Thus the expected return may be increasing or decreasing in θ , depending on f .

The expected payoff for the intermediary on a θ -type project is $\theta f(k(\theta))\pi - \frac{w\theta}{\theta_L} - Rk(\theta) = S(\theta) - U(\theta)$. It follows from claim 3 that this payoff can be zero only for one particular θ and will be positive for higher types. Moreover $S(\theta) - U(\theta)$ is a convex function of θ .⁵ The good types are very valuable for intermediaries: the profits coming from them will compensate for the losses from bad types.

Interestingly, *firms cannot fight for segments of the market (i.e. try to steal only a subset of $\Theta = [0, 1]$)*. The expected return for entrepreneurs is independent of k and z . If a firm offers a more attractive contract for some θ , it must be because θ_L is lower, and hence the contract is better for every θ . In other words, in a world where the limited liability constraint is binding, intermediaries are not able to “skim the cream” up to the point of making zero profits type by type. This is in contrast with Rothschild and Stiglitz (1992).

The following picture illustrates the equilibrium described by proposition 1. The figure shows several monetary payments as functions of θ . We plot the entrepreneurs’ expected payoff $U(\theta) = \frac{\theta}{\theta_L}w$ and their outside option w (which is assumed to be independent of θ); the expected surplus of the project, net of financial costs, $S(\theta) = \theta f(k^*(\theta))\pi - Rk^*(\theta)$.⁶ The area shaded in gray represents the profits of an intermediary. From the figure it is clear that changes in θ_L change the slope of the entrepreneurs expected payoff. These changes, in turn modify the intermediaries’ payoffs and hence θ_L can be adjusted so that intermediaries make zero profits.

Figure 2.3.6 illustrates the key force in this economy: to provide the correct incentives, the payoff to entrepreneurs cannot grow as fast as the total surplus does. That means that intermediaries always make higher profits with the highest types. Since high types are so profitable, intermediaries are willing to lose money with low types to offer more attractive contracts to (profitable) high types and maintain incentives.

2.4 Welfare and optimal policy

The expected social surplus of a project is $\theta f(k^*(\theta))\pi - Rk^*(\theta) - w$. Hence a social planner sets $\theta f'(k^*(\theta))\pi = R$ as before, but only for those θ such that the expected social benefit is not negative. As the surplus is increasing in θ by the envelope theorem, there is a lower bound θ_P , for those socially valuable projects. Then it is worth for the society to devote resources to all projects with $\theta \geq \theta_P$ where $\theta f(k^*(\theta_P))\pi - Rk^*(\theta_P) = w$.

By contrast, in the decentralized equilibrium, θ_L yields zero profit for intermediaries:

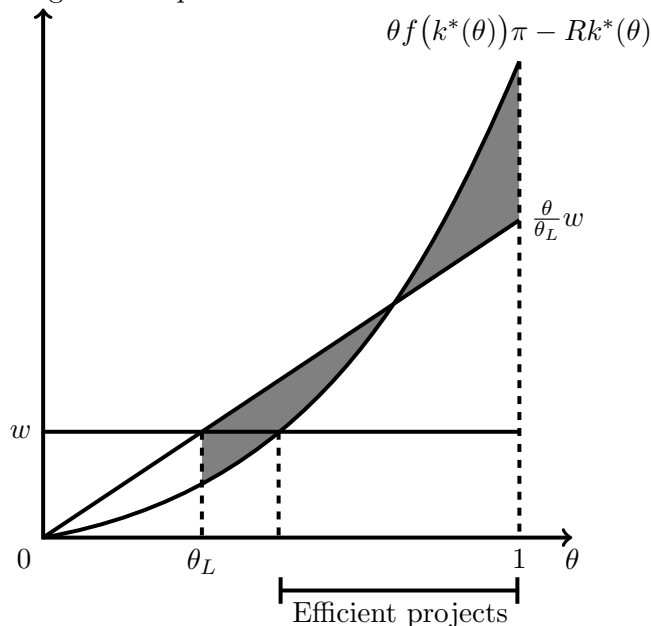
$$\int_{\theta_L}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_L}^1 \theta \frac{w}{\theta_L} dG(\theta) = 0$$

Proposition 2. *In the decentralized solution, socially inefficient projects are enacted. That is $\theta_L < \theta_P$. Moreover, efficiency will be restored if any of the following becomes true:*

⁵By the envelope theorem, the derivative is $\pi f(k^*(\theta)) - \frac{w}{\theta_L}$

⁶Note that the expected surplus is similar to the profit function of a competitive firm facing an output price of θ . It is well known that profit functions are convex in prices and hence the surplus is convex in θ , as displayed.

Figure 1: Equilibrium Contract and Zero Profits



- *No adverse selection: Types become public information*
- *No limited liability: Intermediaries become able to recover any contracted amount.*
- *No competition: There is only one intermediary or intermediaries collude.*

The intuition behind it is as follows: if only efficient projects were financed, the intermediaries would make profits in all the projects. But then, positive profits attract new intermediaries who can “steal” the market by offering more generous contracts. More generous contracts involve some cross-subsidization, and as a result socially inefficient projects will be active.

If types are public information but all other conditions remain the same, intermediaries will break even on each type. This implies that all signed offer is such that $U(\theta) = S(\theta)$, and only types $\theta > \theta_P$ sign contracts. Contracts will not be completely determined since many combinations of $x(\theta)$ and $z(\theta)$ yield $U(\theta) = S(\theta)$, but $k(\theta) = k^*(\theta)$. The equilibrium payoff of type θ is $U(\theta) = \max\{w, S(\theta)\}$.

If there is no limited liability but all other conditions hold, the only IC contract schedule in equilibrium is $k(\theta) = k^*(\theta)$, $x(\theta) = -R \cdot k^*(\theta)$ and $z(\theta) = \pi$, which implies $U(\theta) = S(\theta)$. This is just the risk free debt contract. As it is well understood, when the entrepreneur is risk neutral, transferring all the risk to her solves the incentive problem.

If there is only one intermediary, but adverse selection and limited liability still hold, the only equilibrium is as follows: the contract that maximizes profits for the intermediary is $k(\theta) = k^*(\theta)$, $x = w$, $z = 0$ and entrepreneurs with types $\theta \geq \theta_L$ take it, all others reject. The intermediary will take all the surplus and her profits would be $\int_{\theta_P}^1 (S(\theta) - w) dG(\theta)$.

2.4.1 Optimal policy

In this section we consider two possibilities for taxation. A fixed sum tax per contract to financial intermediaries ϕ and a tax rate τ on entrepreneurs profits⁷. In both cases the full information optimal allocation can be achieved.

Claim 6. *Let ϕ^* be a fixed tax per contract defined by:*

$$\phi^* \equiv \left[\int_{\theta_P}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_P}^1 \theta \frac{w}{\theta_P} dG(\theta) \right] \left(\int_{\theta_P}^1 \frac{\theta w}{\theta_P} dG(\theta) \right)^{-1}$$

Then only efficient projects (and all of them) are funded.

This tax can also be seen as fixed subsidy on w , or any instrument that increases the outside option of every entrepreneur. However the revenue implications for the tax authority would be different. Since we do not model the nature of the wage or the outside option, we stick to the contract fee tax interpretation.

Now suppose there is a profit or dividend tax. Intermediaries make zero profit, so they wont be affected by such tax. However entrepreneurs do make profits if the project is successful. Hence a tax rate τ on profits would make any contract (k, x, z) look to the entrepreneurs like $(k, x, (1 - \tau)z)$

Claim 7. *Let $\tau^* = \frac{\phi^*}{w + \phi^*}$ be the tax rate on profits, where ϕ^* is the fixed tax rate found in claim 6. Then only efficient projects (and all of them) are funded.*

Taxing R would be troublesome since it will distort $k^*(\theta, R)$ reducing the total surplus, which is the standard result on capital income taxation. In principle this would be at odds with claim 7, but this is because we assume taxation on capital income is made at the individual level. Hence as long as the intermediaries can aggregate profits on investments before taxes, or claim back taxes on dividends from entrepreneurs firms in which they have a stake, they wont be affected by the taxation, only the entrepreneurs. So far we have avoided to make any interpretation of the legal stance the contracts will have. We can think of the contracts as debt contracts with limited liability clauses or as equity stakes in a firm, those details are irrelevant for our discussion above but become important once we face a complex tax law. Of course the discussion of that topic is far away from the scope of this paper.

3 Capital Holdings

As the previous section shows, for θ high enough, the return of the project for the entrepreneur would be higher if he had access to the risk free rate. Hence skilled entrepreneurs who own capital are willing to use it on their project. The same will happen if the asset owned is not liquid but is pledgeable: the entrepreneurs are willing to pledge their assets if doing so gets them better loan terms. We allow for that possibility in the current section.

⁷A tax to entrepreneurs could be considered but it would require them to have some external funds that the government can seize even in case of failure. In that case:

$$\phi^* \equiv \theta_P \left[\int_{\theta_P}^1 \theta dG(\theta) \right]^{-1} \left[\int_{\theta_P}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_P}^1 \theta \frac{w}{\theta_P} dG(\theta) \right]$$

3.1 Observable Capital Holdings

Assume entrepreneurs have assets $a \in \mathcal{A}$, distributed according to $G(\theta, a)$. Two interpretations are possible, both will yield the same results : for a project of size k the entrepreneur provides a as capital and the intermediary finances $k - a$ or the intermediary finances k collateralized by Ra from the entrepreneur. In what follows the latter will be used.

The asset holdings from entrepreneurs serve two purposes, they relax the limited liability constraint for the intermediaries and increase the outside option for the entrepreneurs.

Claim 8. *The outside option $O(\theta, a)$ of an entrepreneur of type θ that holds assets a is characterized as follows:*

$$O(\theta, a) = \max \left\{ w + Ra, \theta F(\min\{a, k^*(\theta)\})\pi + R \cdot \max\{0, a - k^*(\theta)\} \right\}$$

The expression for the outside option results from the fact that an entrepreneur of type θ holding a units of capital can always produce using his skills and own capital, or work and lend his capital.

With observable assets, contract schedules are contingent on a . A strategy for an intermediary specifies a subset of \mathbb{R}^3 for each $a \in \mathcal{A}$.

To facilitate interpretation, let the equilibrium payoff of a type θ entrepreneur with assets a be $U(\theta, a) + Ra$.

Myerson's lemma and claims A.1 to 2 will still hold for each a , since nothing in their proofs depends on the limited liability condition being exactly zero. With minor changes to the proof we can state:

Proposition 3. *In equilibrium with observable assets, the contract offered by all financial intermediaries to almost every (θ, a) is:*

$$k^*(\theta, a) = k^*(\theta) = \arg \max_k \{ \theta f(k)\pi - Rk \}, \quad z^*(\theta, a) = \frac{O(\theta_L(a), a)}{\theta_L(a)f(k^*(\theta))}, \quad x(\theta) = -Ra.$$

where $\theta_L(a)$, the lowest θ among those entrepreneurs with a assets who accepts the contract. For each a , it must be the case that:

$$\int_{\theta_L(a)}^1 \left\{ \theta f(k^*(\theta))\pi - \theta \frac{O(\theta_L(a), a)}{\theta_L} - R(k^*(\theta) - a) \right\} dG(\theta|a) = 0$$

The following figure illustrates the equilibrium,

As long as entrepreneur's capital holdings are observable several loan markets will be active, one for each asset level a . Those markets will be described by proposition 3. In the next subsection we deal with the case of unobservable asset holdings.

3.2 Unobservable Capital Holdings

In principle, an entrepreneur with wealth a could hide part of his own wealth and take a contract designed for a lower a if it is more profitable. This is akin to an entrepreneur setting up a corporation but only investing a fraction of his wealth. We ruled out this possibility by assuming that a was known (observable) by the intermediary. In this subsection we drop that assumption.

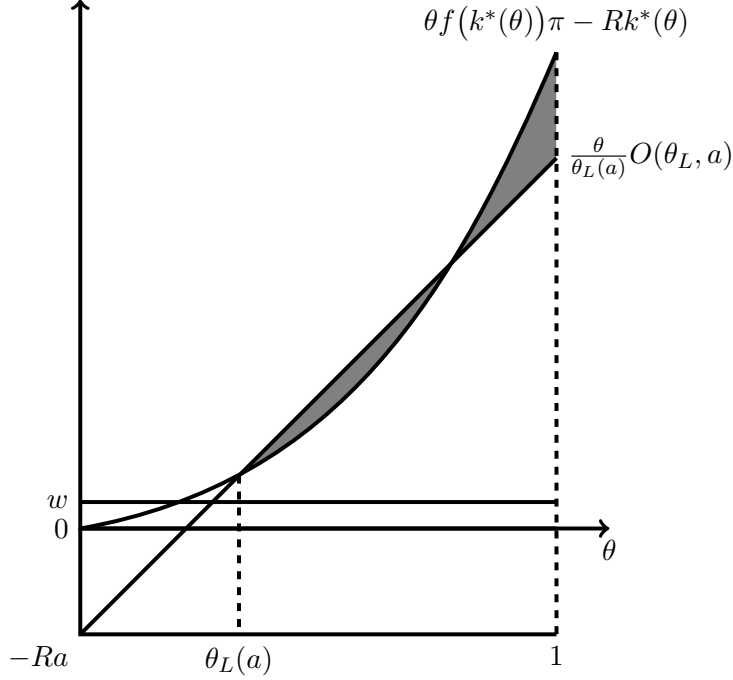


Figure 2: Equilibrium Contract with Observable Asset Level a

Just as in the previous section, the strategy of intermediary i , \mathcal{C}_i specifies a subsets of \mathbb{R}^3 for each $a \in \mathcal{A}$. But now, an entrepreneur with type (θ, a) solves,

$$U(\theta, a; \mathcal{C}_1, \mathcal{C}_2) = \max_{(k, z, x) \in \mathcal{C}(a)} \theta f(k)z + x$$

where $\mathcal{C}(a) \equiv \bigcup_{a' \leq a} \mathcal{C}_1(a') \cup \bigcup_{a' \leq a} \mathcal{C}_2(a')$

Let $(k(\theta, a), z(\theta, a), x(\theta, a))$ denote a representative solution of the above problem. Incentive compatibility across assets only requires $U(\theta, a)$ to be nondecreasing in a . This because an entrepreneur cannot pledge more collateral or invest more capital than the amount he owns, which implies he can only lie by hiding some assets. For the incentive compatibility over θ , Myerson lemma holding fixed each asset level is enough. Limited liability in this environment still requires $x(\theta, a) \geq -Ra$.

It can be shown that, as before, $x(\theta, a)$ is non increasing in θ , $k(\theta, a) = k^*(\theta)$ for (almost) all (θ, a) , $U(\theta, a)$ will be nondecreasing in θ . If we define the set $A_i(a)$ as those θ such that types (θ, a) are willing to take the contract from intermediary i it is still the case that $\bigcup_i A_i(a) = A(a) = [\theta_L(a), 1]$. Also there would be a zero profit condition but in an aggregate sense across assets. Hence proofs depending on a zero profit condition per asset level need to be updated.

The unobservability of the assets may (will) imply limited liability is not binding for some cases. However we still can state an updated version of claim 5.

Claim 9. *Competition in the intermediation market and the limited liability constraint imply that any equilibrium contract schedule satisfies $x(\theta, a) = -Ra$ for all (θ, a) such that $S(\theta) \geq U(\theta, a) > w$.*

This implies that limited liability will bind for those types for which the intermediaries expect to make some profits. Now, for each a define $\theta_e(a)$ as the solution to $U(\theta, a) = S(\theta)$ which means the intermediary expects to break even with this type. Above those $\theta_e(a)$ claim 9 implies $x = -Ra$ and therefore for all $\theta > \theta_e(a)$:

$$U(\theta, a) = \theta \frac{S(\theta_e(a)) + Ra}{\theta_e(a)} - Ra \quad (3)$$

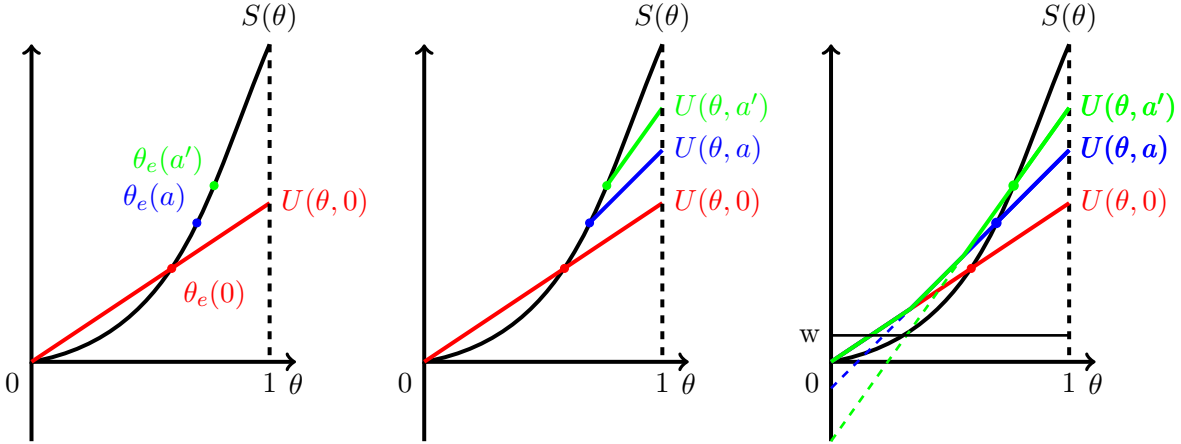
Now, fix the non decreasing function $\theta_e(a)$. That completely determines the expected earnings for intermediaries among the profitable contracts. For types below $\theta_e(a)$ losses are incurred in expectation so the intermediary wants to offer as little as possible. However, the IC constraint across assets may bind. Equation (3) is then a lower bound for the surplus given to those types. If the IC across assets binds for a type (θ, a) is because there exists some $0 \leq a' < a$ such that:

$$\theta \frac{S(\theta_e(a')) + Ra'}{\theta_e(a')} - Ra' > \theta \frac{S(\theta_e(a)) + Ra}{\theta_e(a)} - Ra.$$

After fixing $\theta_e(a)$, a profit maximizing intermediary has no way to improve. The contracts she is profiting with are fully determined, and for those types she is expected to lose she has a lower bound on the utility she has to deliver. Hence for $\theta < \theta_e(a)$ a profit maximizing intermediary sets:

$$U_i(\theta, a) = \sup_{0 \leq a' \leq a} \theta \frac{S(\theta_e(a')) + Ra'}{\theta_e(a')} - Ra' \quad (4)$$

Figure 3: Characterization of the profit maximizing contract given $\theta_e(a)$



Equations (3) and (4) fully characterize the contracts given $\theta_e(a)$. Figure 3.2 illustrates this feature. The zero profit condition is not enough to pin down that function, since zero profits must hold on aggregate across types. To pin down that function consider the following:

Claim 10. *In any equilibrium, contract schedules must offer the same utility to (almost) all types willing to sign at least one of the offered contracts.*

The intuition rests on the fact that the average of two IC contracts is IC because the expected utility for entrepreneurs is linear in the varying elements of the contract (x, z) .

An intermediary can decide not to offer any contract with $x(\theta, a) = R\bar{b}$ for some asset level $\bar{b} > 0$. By doing so she gives up the expected profit of types with $\theta \geq \theta_e(\bar{b})$ and assets \bar{b} , she

avoids the losses with those types $\theta < \theta_e(\bar{b})$ with assets \bar{b} . But also there could have been some types (θ, a) with $\theta \leq \theta_e(a)$ and $a > \bar{b}$ but such that:

$$U(\theta, a) = \theta \frac{S(\theta_e(\bar{b})) + R\bar{b}}{\theta_e(\bar{b})} - R\bar{b}.$$

If that was the case, half of those types were taking the contract with intermediary i before she dropped those contracts. To quantify the effect of that action define

$$A(a|b) = \left\{ \theta : x(\theta, b) = -Ra \right\}$$

Incentive compatibility implies that if two types are offered the same $x(\theta, a)$ they should be offered the same $f(k^*(\theta))z(\theta, a)$. Hence for all $\theta \in A(a|b)$ it must be the case that:

$$U(\theta, b) = \theta \frac{S(\theta_e(a)) + Ra}{\theta_e(a)} - Ra$$

Definition Let $P(a) = \frac{S(\theta_e(a)) + Ra}{\theta_e(a)}$ be the slope of all contracts such that $x(\theta, b) = -Ra$.

Claim 11. *The sets $A(a|b)$ are intervals. For all $b \leq c$ $A(a|c) \subset A(a|b)$.*

Given that, we can now write a closed form for intermediary's profits given some non decreasing function $\theta_e(a)$.

$$\Pi = 0.5 \int_{a=0}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2 G(\theta, b) da \quad (5)$$

Now define $\Pi(c)$ as the profits brought by contracts with $x(\theta, a) \leq -Rc$:

$$\Pi(c) = 0.5 \int_{a=c}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2 G(\theta, b) da$$

Claim 12. *For all c profits brought by contracts with $x(\theta, a) \leq -Rc$ are zero.*

Proof. If $\Pi(c)$ is negative, the intermediary can always drop all those contracts and increase profits. If $\Pi(c)$ is positive, analogously to the zero profit condition, the intermediary can give some $\varepsilon > 0$ more to all types with assets equal or higher than c , stealing the half of the market serviced by the other intermediary and getting $2\Pi(c) - \varepsilon$. \square

Taking derivative of $\Pi(c)$ we obtain that for all a

$$0 = \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2 G(\theta, b) \quad (6)$$

which implies that for each a the intermediaries should break even on those contracts such that $x(\theta, b) = -Ra$. This result is analogous to Rothschild and Stiglitz (1992), where zero profits should be made contract by contract. In our setting all contracts with $x(\theta, b) = -Ra$ are equivalent for the entrepreneurs because $f(k^*(\theta))z(\theta, a)$ has to be constant across all those contracts, but different to the intermediary because of the different expected surplus. Next the following proposition fully characterizes the equilibrium.

Proposition 4. *In equilibrium, the contract offered by all financial intermediaries to almost every θ and every a is:*

$$\begin{aligned} k^*(\theta, a) &= \arg \max_k \theta f(k)\pi - Rk, \\ U(\theta, a) &= \max_{0 \leq a' \leq a} \theta P(a') - Ra' \\ x(\theta, a) &= -R \left[\arg \max_{0 \leq a' \leq a} \theta P(a') - Ra' \right]. \end{aligned}$$

where $\theta_e(\cdot)$ is such that for all a the financial intermediaries make zero profits over all entrepreneurs taking a contract with $x = -Ra$. That is,

$$\int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2 G(\theta, b) = 0.$$

4 A Numerical Example

We have established that in the environment described above, competition among financial intermediaries yields to an inefficient outcome. The result raises a natural question: in which markets, industries or countries will be the inefficiency more pronounced? In the current section we use a numerical example to illustrate the changes in the deadweight loss when various parameters of the model change.

4.1 Parameterization

For the numerical example we set the entrepreneurs' outside option $w = 15$; the output of the project in case of success, $\pi = 100$; and the gross interest rate is $R = 1.02$.

Recall that a project succeeds with probability $p(\theta, k) = \theta f(k)$. We let $f(k) = 1 - \exp(-\beta k^\alpha)$ for $\beta > 0$ and $\alpha \in (0, 1)$. This functional form has several properties. First, it is continuous and strictly increasing and strictly concave on \mathbb{R}_+ . Second, $f(0) = 0$ and $\lim_{k \rightarrow \infty} f(k) = 1$. Third, $\lim_{k \rightarrow 0} f'(k) = \infty$. The last condition ensures that for every $\theta > 0$ there is a scale such that the θ -type project is profitable (Inada condition). A way to interpret the above functional form is that the $p(\theta, k)$ is the product of θ and the probability that an exponential random variable is lower than k^α ⁸. As exponential variables are usually employed for waiting times for a poisson process, it can be interpreted as the waiting time until the arrival of a new innovation (success), an amount k of capital allows the entrepreneur to run the project for k^α periods and hence the probability of a good idea arriving would be $f(k)$.

We set $\beta = 0.1$, implying that the latent exponential random variable would have a mean of 10. Under the above interpretation increasing k increases the probability that the random falls below k^α at a decreasing rate. We set $\alpha = 0.1$.

Last but not least, the joint distribution of assets and types will be key to compute intermediaries profits. We focus on the conditional distribution of θ given a , and let $g(\theta|a) = \frac{a+1}{\eta} \theta^{\frac{a+1}{\eta}-1}$

⁸Equivalently, $p(\theta, k)$ is the product of θ and the probability that a Weibul random variable is lower to k . In that case, the waiting time interpretation would be that the longer it takes for a project to succeed the less likely it will succeed in the future, Jovanovic and Szentes (2013) use a similar approach hence our functional forms may be regarded as a reduced form of their results

be its density. The exponent $\frac{a+1}{\eta} - 1$ controls the participation of high types on the conditional distribution. The higher is the exponent, the higher will be the density of types higher than a fixed value θ (Saffie and Ates (2013)). We let the exponent to be increasing on the asset level a , meaning that assets are positively correlated with the types. In addition, we assume the unconditional distribution of assets has density $\lambda \exp(-\lambda a)$. Hence an average entrepreneur has an asset level of 2. All the parameters and functional forms used in the numerical example are summarized in Table 1.

Table 1: Benchmark Parameters

w	15
π	100
R	1.02
$f(k)$	$1 - \exp(-\beta k^\alpha)$
α	0.7
β	0.1
$g(\theta a)$	$\frac{a+1}{\eta} \theta^{\frac{a+1}{\eta} - 1}$
η	1
$g(a)$	$\lambda \exp(-\lambda a)$
λ	0.5

4.2 Benchmark Results

We begin by describing the case where entrepreneurs do not have assets. In order to facilitate comparability with the upcoming section where assets are introduced, we use the marginal distribution of types that is consistent with the joint distribution of assets and types that will be used next (i.e: intermediaries will face the same population in both cases).

As defined before, an equilibrium is fully described by the triple $(k(\theta), x(\theta), z(\theta))$. We established that $x(\theta) = 0, \forall \theta$. The functions $k(\cdot)$ and $z(\cdot)$ are plotted in the Figure 4 below. An important equilibrium object is the lowest type taking the contract, θ_L . For this example, the value of θ_L is 0.49, which implies that 74% of the entrepreneurs take the contract. Compare θ_L with the lowest type that would be funded by a social planner $\theta_P = 0.62$.

In the Figure 4.2 we plot $S(\theta) = \pi \theta f(k^*(\theta)) - Rk^*(\theta)$ as well as the expected payoff of entrepreneurs $U(\theta) = w\theta/\theta_L$. In addition we plot the the wage (horizontal line) to allow the reader to picture the equilibrium deadweight loss. In what follows we will use the relative inefficiency, defined as the ratio of the deadweight loss to the net economic surplus,

$$\mathcal{I} = \frac{\int_{\theta_L}^{\theta_P} (w - S(\theta)) dG(\theta)}{\int_{\theta_P}^{\theta_L} (S(\theta) - w) dG(\theta)}$$

For our benchmark parameterization, the relative inefficiency is $\mathcal{I} = 5.03\%$. In the next section we study how this inefficiency responds to changes in the parameters of the model.

4.3 The Relative Inefficiency: Comparative Statics

In the current section we change, one by one, all the parameters of the model, holding the other parameters at their respective benchmark values.

The outside option of entrepreneurs, w , is very important because, absent this cost, the economy would not be inefficient. In fact, when $w = 0$, all projects are socially profitable - by assumption -, and they are all funded at the optimal scale. An increase in w directly increases the deadweight loss and decreases the net economic surplus (everything else equal). It also induces an endogenous adjustment of θ_L . A higher w tends to increase θ_L , but this increment is dampen because the pool of types is increased and competition induces intermediaries to increase the terms of the contract. However, increases in θ_L reduce the deadweight loss. As illustrated in Figure 6, when the wage is very high, the force is strong enough to actually decrease the inefficiency. Note that is the wage is high enough, all the entrepreneurs take the outside option and the deadweight loss disappears. The relative inefficiency is maximized when $w = 31$, attaining 6.8% of the net surplus.

The gross interest rate, R , is similar to the wage, in the sense it represents the outside option, or opportunity cost of the capital in hands of financial intermediaries. A higher gross interest rate not only decreases the payoff of intermediaries, but it also decreases the optimal scale of the projects, and hence their expected return. The condition $\theta\pi f'(k^*) = R$ implies that the function $S(\theta; R) = \pi\theta f(k^*) - RK^*$ is linearly homogenous in (θ, R) . Since $\theta \in [0, 1]$, an increase in R can be interpreted graphically as a downward shift of the curve $S(\theta)$ as shown in the right panel of Figure 7. This force increases both the net surplus and the deadweight loss everything else equal. However, θ_L will increase to satisfy the zero profit condition. As shown in the left panel of Figure 7, when we let R vary between 1 and 1.8, the relative inefficiency increases up to 11%, at $R = 1.8$.

We move to describing how the inefficiency responds to changes in the shape of function

Figure 4: Optimal Contract

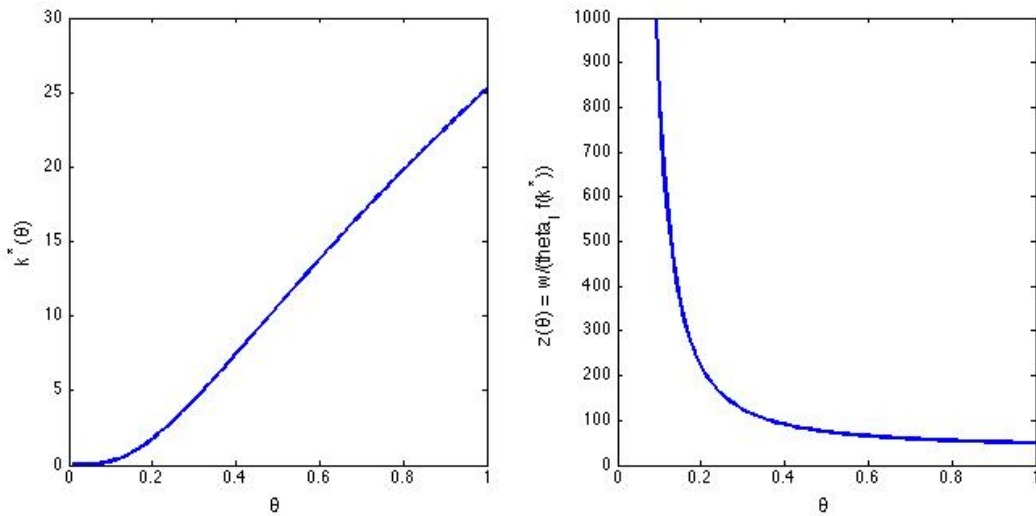


Figure 5: Surplus, Payoff, Inefficiency

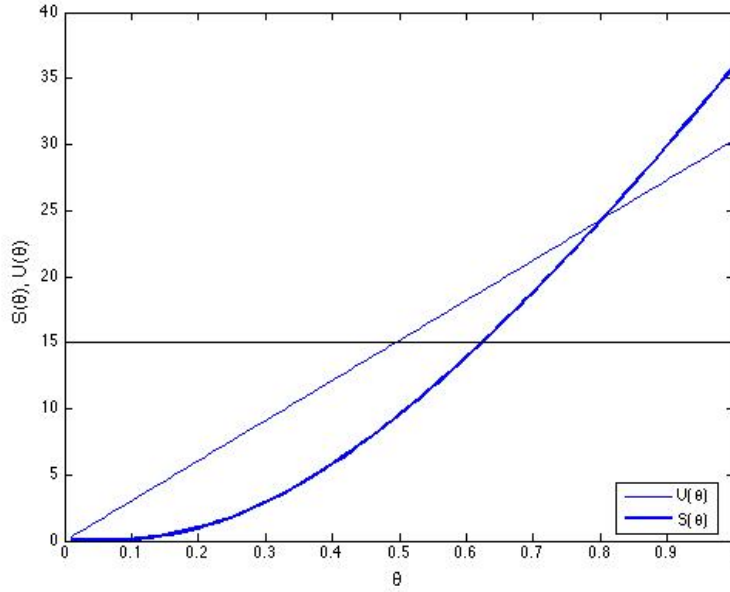
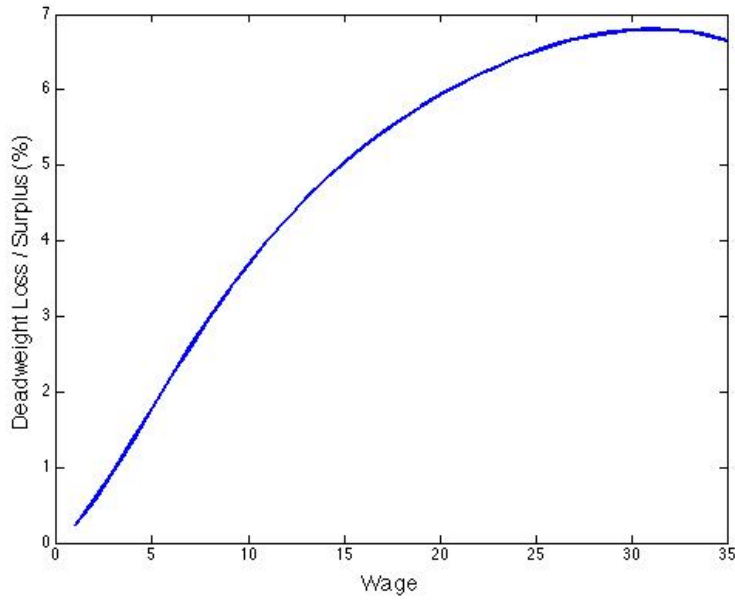


Figure 6: Relative Inefficiency and Wage

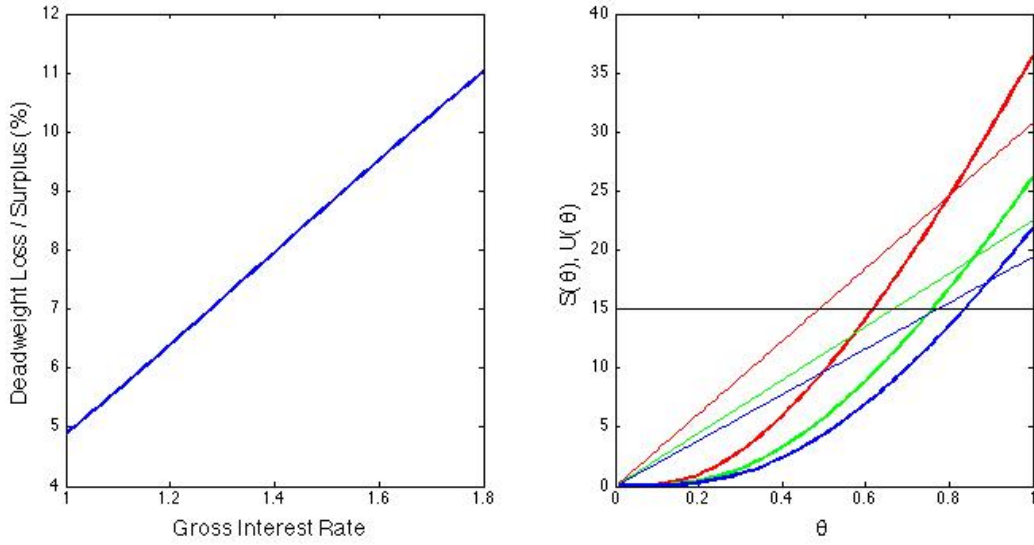


$f(k)$. Figure 4.2 above suggests that the size of the inefficiency is very related to the concavity of the function $S(\theta)$. This concavity only depends on the shape of f . In fact,

$$S''(\theta) = f(k^*(\theta)) - \frac{(f'(k^*))^2}{f''(k^*)}$$

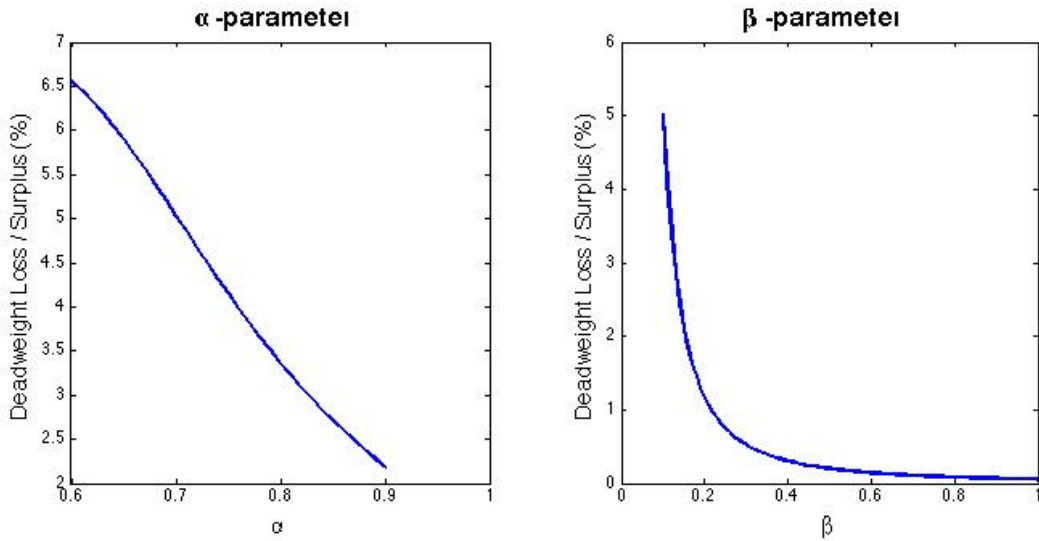
Not surprisingly, the relative inefficiency is quite sensitive to the parameters α and β , that

Figure 7: Relative Inefficiency and Interest Rate



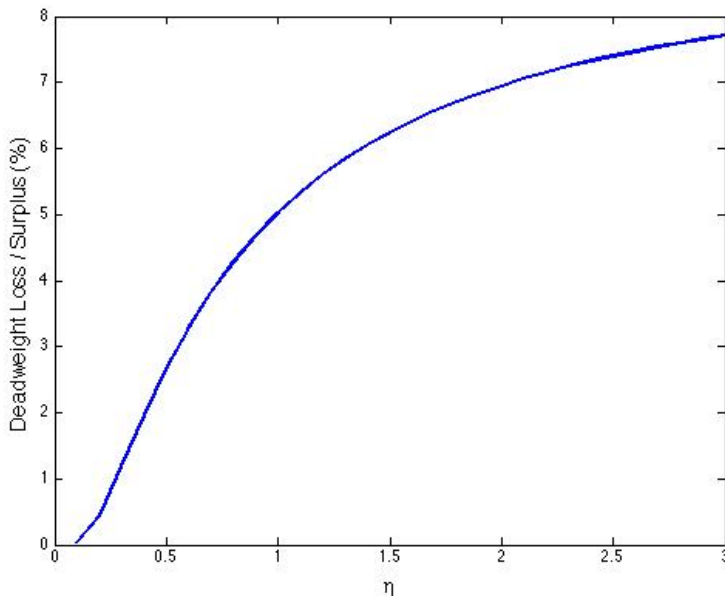
govern the shape of f in the current example. The results are displayed in Figure 8. The relative inefficiency is decreasing in both α and β . It decreases particularly fast as β increases, getting to 0.05% when $\beta = 1$.

Figure 8: Relative Inefficiency and the Shape of $f(k)$



Last, the distribution of types importantly affects the size of the inefficiency. The parameter η governs the shape of the distribution of θ . More precisely, as η increases, the density is shifted toward lower types. Holding the outside option fixed, an increase in η decreases the net surplus, because some density will be shifted from socially profitable projects to unprofitable

Figure 9: Relative Inefficiency and the Distribution of Types



projects. However, when good types are scarcer, intermediaries will offer less generous contracts, increasing θ_L . The last force tends to decrease the inefficiency. Figure 9 shows that the relative externality actually increases, reaching 7.7% when $\eta = 3$. On the other hand, for values of η close to zero, the relative inefficiency gets close to zero.

4.4 Assets

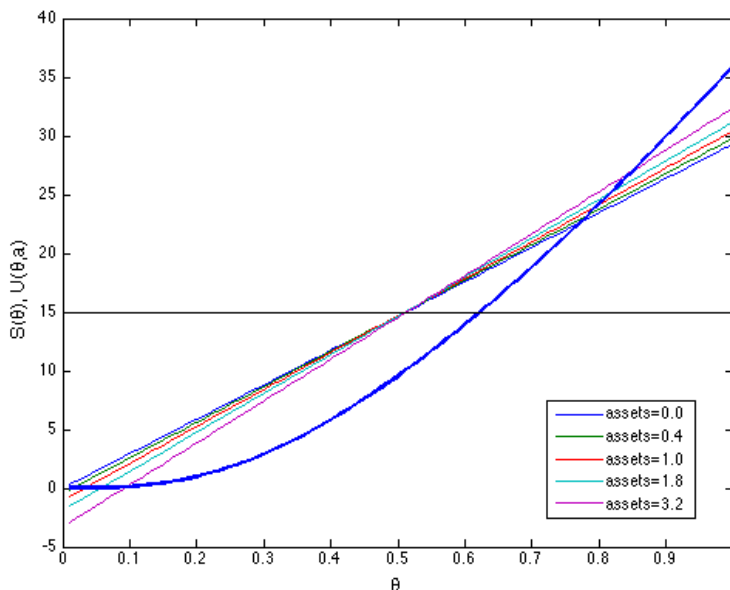
For simplicity, we discretize the space of assets, using the quantiles of the of the marginal distribution described in section 4.1, $g(a) = \lambda \exp(-\lambda a)$. We use five asset levels, 0, 0.45, 1.02, 1.83 and 3.22; there will be of 20% of the population holding each of the levels of assets. To put this numbers in perspective, a entrepreneur of type $\theta = 1$ will optimally invests $k^*(1) = 25$, while the for the lowest type taking the contract in the absence of assets, $k^*(\theta_L) = 10.3$ (see Figure 4). Recall that assets and types are correlated, and $g(\theta|a) = \frac{a+1}{\eta} \theta^{\frac{a+1}{\eta}-1}$.

The equilibrium is summarized in Table 2 and Figure 10. It will prove useful to introduce some further notation and let $\theta_F = \inf A(a|a)$ be the lowest type with asset level a , taking the contract with slope $P(a)$. By contrast, $\theta_L(a)$, the lowest type with assets a taking any contract. Hence, $\theta_L(a) = \min_{\hat{a} \leq a} \theta_F(\hat{a})$. Then, the relative inefficiency becomes,

$$\mathcal{I} = \frac{\sum_{a=1}^5 \int_{\theta_L(a)}^{\theta_P} (w - S(\theta)) dG(\theta) Pr(a)}{\int_{\theta_P}^{\theta_L(a)} (S(\theta) - w) dG(\theta)}$$

The lowest type taking the 0 asset contract is type $\theta = 0.5123$. Next, the lowest type taking the contract that requires to advance $a = 0.45$ is $\theta_F(0.45) = 0.5108$. This implies that the contract that requires $a = 0.45$ is more generous than the one that doesn't require assets. By contrast, the lowest type taking the contract requiring $a = 3.22$ is 0.5478. When θ is between

Figure 10: Optimal Contract Under Asset Holdings



0.5136 and 0.5478, an entrepreneur with $a = 3.22$ prefers to take the contract that only requires a collateral of 1.8.

In this setting, the relative inefficiency is 3.94%. Hence, allowing intermediaries to condition their contracts on collateral decreases the inefficiency by more than a percentage point (from 5.03 %) in this example. There are two reasons why the inefficiency is reduced when the contracts depend on assets. First, the introduction of assets relaxes the limited liability constraint. Second, because the collateral is more likely to be held by higher types, conditioning contracts on collateral allows the intermediaries to do some sort of screening.

In order to disentangle the two effects, we compute the equilibrium for an economy with the same marginal distributions of assets and types, but where assets and types are independent. When assets and types are independent, the relative inefficiency is 4.62%. We also consider the case in which intermediaries observe a signal, that is correlated with types in the exact same way that assets were, but (of course) does not affect the limited liability constraint. In that case, the economy has a relative inefficiency of 4.65%.

The results are presented in Table 2. The upper panel displays the optimal contract for entrepreneurs holding assets that are independent of types, and the second panel shows the results when there is an observable signal correlated with the types. In the first case, entrepreneurs with higher asset holdings receive more generous contracts than their peers with lower assets. To understand this, recall that limited liability prevents “cream-skimming” in this environment once $x = 0$. For entrepreneurs with higher assets, the limited liability is not binding until it hits $x = -Ra$. Hence, intermediaries have incentive to skim the cream until they hit the constraint. Under this situation, although the contracts are more generous, they are taken by a lower number of types. In the second case, entrepreneurs with higher signals also get more generous contracts, but simply because they are better on average, and hence intermediaries break

even offering higher terms to entrepreneurs. In this case, the lowest type taking the contract will be decreasing on the signal.

Table 2: Separate Asset Effects

<i>Independent Types</i>					
Asset Level	0	0.45	1.02	1.83	3.22
θ_F	0.5004	0.5248	0.5498	0.5766	0.6378
Expected Payoff	30.8θ	$31.6\theta - 0.5$	$32.5\theta - 1.0$	$33.7\theta - 1.9$	$35.4\theta - 3.3$
<i>Informative Signals</i>					
Asset Level	0	0.45	1.02	1.83	3.22
θ_F	0.5119	0.5065	0.4998	0.4906	0.4769
Expected Payoff	29.3θ	29.6θ	30.0θ	30.6θ	31.5θ

We end this section presenting some comparative statics of the inefficiency to the distribution of assets. In particular, we consider changes in the parameter λ of the exponential distribution of assets. We let λ vary between 0.5 and 50, which implies that the mean of the asset distribution varies approximately between 2 and $1/50$ ⁹. If the mean of the asset distribution is high enough, the inefficiency can be arbitrarily reduced, because most of the entrepreneurs can fund their own projects. On the other extreme, as $\lambda \rightarrow \infty$, the economy converges to one in which all entrepreneurs have zero assets, and the inefficiency converges to 7.5% of the surplus. It is important to note that the limit economy where all entrepreneurs have zero assets is not the same as the benchmark economy described in section 4.2. In the benchmark economy, the distribution of types, is the marginal distribution implied by joint distribution of types and assets used throughout the section. I.e, in the benchmark economy $g(\theta) = \int_0^\infty g(\theta, a) da$. When $\lambda \rightarrow \infty$, the distribution of types faced intermediaries is $g(\theta|0)$. Because of the positive correlation between assets and types, “lemons” are more abundant in the later economy.

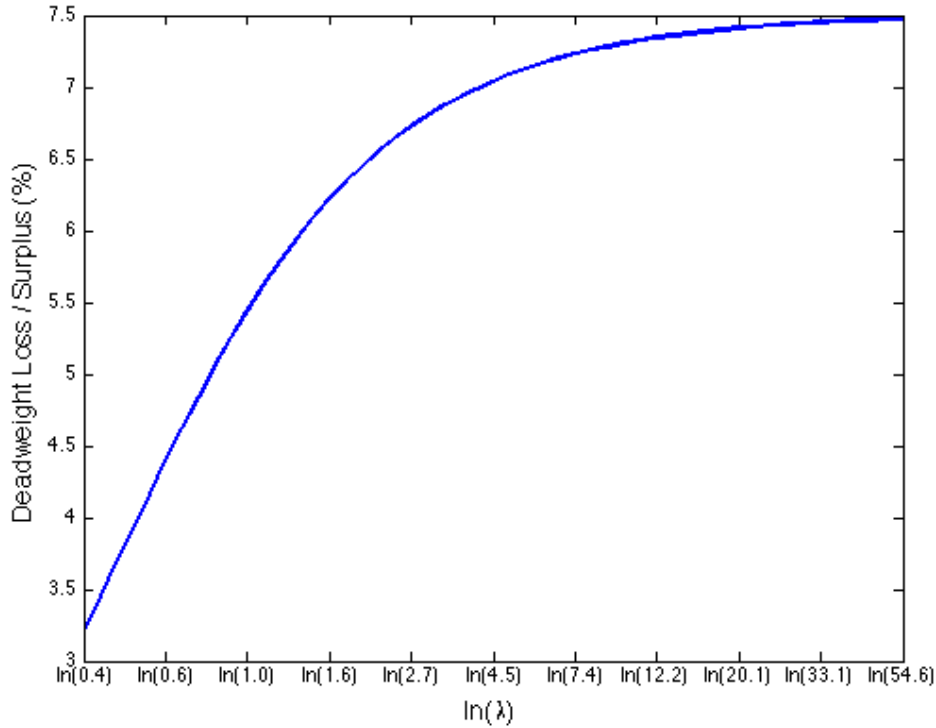
5 Conclusion

We characterized financial contracts in a competitive environment with risk, adverse selection and limited liability. We find that, under the optimal contract the highest types are rewarded below the expected value of their project, and hence financial intermediaries make profits with high types. They also make losses with lower types. Our main result is that the optimal contract generates an inefficient outcome: projects that wouldn’t be funded by a social planner are funded in equilibrium. We show that asymmetric information, limited liability and competition are all necessary to generate the result. We also show both analytically and numerically that the ability to use some assets as collateral mitigates but does not eliminate the inefficient outcome.

An implication of our results is that competition among financial intermediaries is not desirable. Note, however, that a monopolist financial intermediary would get all the economic surplus. In fact, the monopolist attains the first best because he has the ability to set loan

⁹The mean of the exponential distribution is $1/\lambda$. In our case, this is, however, only an approximation because we discretize the distribution.

Figure 11: Inefficiency and Asset Distribution



sizes and interests rates, that is, has the ability to price discriminate. Needless to say, being the monopolist of financial intermediation in the economy would be extremely profitable. Who, if anyone, should get this rents? Even if the government gets the rents, a lot of political economy consideration would arise, that may challenge the efficiency of the result.

The results of this paper suggest at least two directions for future research. On the theoretical front, allowing for dynamics would generate an incentive for firms/entrepreneurs to retain earnings in order to escape from the inefficiency, and can be related to the literature on firm dynamics. On the empirical front, the degree to which working capital can be collateralized varies exogenously across industries. This opens the door to test the implications of the model.

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A Proofs

A.1 Zero assets proofs

The following is a useful consequence of lemma 1.

Lemma A.1. *In any IC contract schedule, $x(\theta)$ is non-increasing.*

A.1.1 Proof of lemma A.1

Proof. Let $\theta' > \theta$ from equation (1c) it follows that:

$$x(\theta) = U(0) + \int_0^\theta [f(k(s))z(s) - f(k(\theta))z(\theta)] ds \quad \text{then,}$$

$$x_i(\theta') - x(\theta) = (\theta' - \theta) [f(k(\theta))z(\theta) - f(k(\theta'))z(\theta')] + \int_\theta^{\theta'} [f(k(s))z(s) - f(k(\theta'))z(\theta')] ds$$

And both terms in the last equation are negative or zero because of equation (1b) hence the claim holds. \square

A.1.2 Proof of claim 1

Proof. Start with an equilibrium in which an entrepreneur of type θ receives payoff $U^0(\theta)$. Let the equilibrium incentive compatible contract schedule be $(k^0(\theta), x^0(\theta), z^0(\theta))$. Finally let A^0 denote the set of types taking one of the contracts rather than the outside option. Suppose there is a set $C \subseteq A^0$ such that $k^*(\theta) \neq k^0(\theta)$ for all $\theta \in C$. We will show that contract schedule is profit maximizer for intermediary 1 only if C has measure zero.

Construct C' as the set $\{(k'(\theta), x'(\theta), z'(\theta)) | \theta \in A^0\}$, where,

$$k'(\theta) = k^*(\theta) \quad z'(\theta) = \frac{f(k^0(\theta))z^0(\theta) + \delta\theta}{f(k^*(\theta))} \quad x'(\theta) = x^0(\theta) + \frac{\delta}{2}(1 - \theta^2) \quad (7)$$

If an entrepreneur of type θ takes the contract $(k'(\hat{\theta}), z'(\hat{\theta}), x'(\hat{\theta})) \in C'$, her payoff will be,

$$\theta f(k^0(\hat{\theta}))z^0(\hat{\theta}) + x^0(\hat{\theta}) + \delta\hat{\theta} + \frac{\delta}{2} - \frac{\delta\hat{\theta}^2}{2}$$

The above payoff is maximized at $\hat{\theta} = \theta$ and the term $\delta\hat{\theta} - \delta\hat{\theta}^2/2$ ensures the maximizer is *unique*. The resulting payoff is $U'(\theta) \equiv U^0(\theta) + \frac{\delta}{2}(1 + \theta^2) > U^0(\theta)$. Thus every entrepreneur $\theta \in A^0$ signs a contract with $k^*(\theta)$.

Next, we will show that offering C' constitutes a profitable deviation for intermediary 1. Let v_i^0 , $i = 1, 2$ be the intermediaries payoff in the original equilibrium. Note that, because intermediaries have always the option of offering empty sets of contracts, $v_i^0 \geq 0$.

The following equation follows from the definition of payoffs,

$$\int_{A^0} \left\{ \pi f(k^0(\theta)) - Rk^0(\theta) \right\} dG(\theta) = v_1^0 + v_2^0 + \int_{A^0} U^0(\theta) dG(\theta) \quad (8)$$

Let,

$$M \equiv \int_{A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) \right\} dG(\theta) - \int_{A^0} \left\{ \pi f(k^0(\theta)) - Rk^0(\theta) \right\} dG(\theta)$$

Since C has positive measure, the definition of $k^*(\theta)$ implies, $M > 0$.

Now, let v_1' be the payoff of intermediary 1 when she deviates to C' and A' the set of types signing a contract after the deviation.

$$\begin{aligned} v_1' &= \int_{A'} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) \\ &= \int_{A' \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) + \int_{A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) \\ &= \int_{A' \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) + M + v_1^0 + v_2^0 - \int_{A^0} \left\{ U(\theta) - U^0(\theta) \right\} dG(\theta) \end{aligned}$$

The last equality follows from equation 8.

The measure of A^0 is obviously bounded by one and $U'(\theta) - U^0(\theta) \leq \frac{\delta}{2}(1 + \theta^2) \leq \delta$. Hence,

$$\int_{A^0} \left\{ U(\theta) - U^0(\theta) \right\} dG(\theta) \leq \delta$$

Moreover, $\pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta)$ is bounded and it follows that, as the measure of $A \setminus A^0$ goes to zero,

$$\int_{A \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) \rightarrow 0$$

Now,

$$A \setminus A^0 = \{\theta : U^0(\theta) < w \leq U'(\theta)\} \subset \{\theta : w - \delta < U^0(\theta) < w\}$$

And it is clear that the measure of $A \setminus A^0$ goes to zero as δ goes to zero.

As a result of the previous observations, there exists a value for δ , low enough, such that,

$$M > - \int_{A \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) + \int_{A^0} \{U(\theta) - U^0(\theta)\} dG(\theta)$$

For such a δ ,

$$v'_1 - v_1^0 > 0$$

Since the intermediary payoff is strictly higher, if the measure of C is strictly positive the alternative schedule C' will achieve strictly higher profits for intermediary 1. \square

A.1.3 Proof of claim 4

Proof. We begin by showing that $U(\theta)$ is non-decreasing in θ . Once that is established, it will be straightforward to see that A must be an interval.

By lemma 1 it suffices to show $f(k(\theta))z(\theta) \geq 0$ for all $\theta > 0$. Suppose there is an equilibrium with IC contract schedule $(k(\theta), x(\theta), z(\theta))$, and there exists $\theta_o > 0$ such that $f(k(\theta_o))z(\theta_o) < 0$. By claim 1, there exists a sequence $\{\theta^n\} \in [0, \theta_o]$ converging to zero such that $k(\theta^n) = k^*(\theta^n)$ and this implies $\{f(k(\theta^n))\}$ also converges to zero.

Equation 1b guarantees that $f(k(\theta^n))z(\theta^n) \leq f(k(\theta_o))z(\theta_o) < 0$ for all n . Taking limit on both sides implies $\lim_{n \rightarrow \infty} z(\theta^n) = -\infty$. But limited liability implies $x(\theta) \geq 0$ and $x(\theta) + z(\theta) \geq 0$. As $x(\theta)$ is non-increasing by claim A.1, $x(0)$ is an upper bound for all $x(\theta)$ and thus $z(\theta)$ has to be bounded below, contradicting $\lim_{n \rightarrow \infty} z(\theta^n) = -\infty$.

Hence, any contract offered in equilibrium satisfies $U(\theta)$ is nondecreasing.

Now, because $U(\theta)$ is nondecreasing $\theta \in A$ implies $\theta' \in A$ for all $\theta' > \theta$ (when indifferent between being a worker and an entrepreneur, agents choose the later by assumption). The continuity of U implies that if A is nonempty then $A = [\theta_L, 1]$ for some $\theta_L \in [0, 1]$. Also, if $\theta_L > 0$ then $U(\theta_L) = w$. \square

A.1.4 Proof of zero profit condition (Claim 2)

Proof. By contradiction suppose intermediary 1 is making profit. We will show intermediary 2 is not choosing an optimal contract schedule.

Let the contract schedules be $(k(\theta), x(\theta), z(\theta))$ for $i \in \{1, 2\}$ and $U(\theta)$ the corresponding expected utilities for entrepreneurs. Suppose the profit for intermediary 1 is $M > 0$. Consider the alternative schedule for intermediary 2 in which increases all entrepreneurs' utility by a small amount ϵ . More precisely, this alternative schedule is defined by $C'_2 = \{(k'(\theta), x'(\theta), z'(\theta)) | \theta \in A\}$, where,

$$k'(\theta) = k(\theta) \quad z'(\theta) = \frac{f(k(\theta))z(\theta) + \delta\theta}{f(k(\theta))} \quad x'(\theta) = x(\theta) + \frac{\delta}{2}(1 - \theta^2) \quad (9)$$

As shown in the proof of claim 1, the payoff $U'(\theta)$ resulting from optimally choosing a contract in $C_1 \cup C'_2$ is strictly greater than the original payoff: $U'(\theta) > U(\theta)$. Hence by deviating

to \mathcal{C}_2 , intermediary 2 will steal half the market intermediary 1 was servicing alone (A_1). The revenue of intermediary 2 increases by at least M . Naturally the costs also increase because entrepreneurs get more generous contracts, but also because the more generous contracts induce an entry of new entrepreneurs. However, as shown in the proof of claim 1 δ can be chosen so that the extra cost are smaller than M . For such a δ , \mathcal{C}'_2 is a profitable deviation for intermediary 2. \square

A.1.5 Proof of claim 3

Proof. Suppose $U(\theta') \geq S(\theta')$ for some $\theta' > \hat{\theta}$ such that $k(\theta') = k^*(\theta')$ are not positive. By the envelope theorem $S'(\theta) = f(k^*(\theta))\pi$, which is increasing in θ because f is increasing and concave. Hence $S(\theta)$ is convex. Remembering that $S(0) = 0 \leq x(\theta')$, by incentive compatibility:

$$U(\hat{\theta}) \geq \hat{\theta}f(k^*(\theta'))z(\theta') + x(\theta') = \frac{\hat{\theta}}{\theta'}U(\theta') + \frac{\theta' - \hat{\theta}}{\theta'}x(\theta') \geq \frac{\hat{\theta}}{\theta'}S(\theta') + \frac{\theta' - \hat{\theta}}{\theta'}S(0) > S(\hat{\theta})$$

contradicting $S(\hat{\theta}) > U(\hat{\theta})$. As $k(\theta') = k^*(\theta')$ for almost all θ' , the lemma follows. \square

A.1.6 Proof of claim 5

Proof. Let $\mathcal{C}_1^0, \mathcal{C}_2^0, s^0$ be an equilibrium with corresponding payoffs $v_1^0, v_2^0, U^0(\theta)$. Denote the incentive compatible schedule by $k^0(\theta), z^0(\theta), x^0(\theta)$ and the set of types taking the contract by A^0 .

Suppose there there is $\tilde{\theta} \in A^0$ such that $x(\tilde{\theta}) > 0$. Without loss of generality, the associated contract is offered by intermediary 1.

Among the contracts in \mathcal{C}_1 , there could be some that are dominated by another contract in \mathcal{C}_1 , for every entrepreneurial type. We focus on the contracts such that this is not the case:

$$\{k_1(\theta), x_1(\theta), z_1(\theta) : \theta \in [0, 1]\}$$

such that $(k_1(\theta), x_1(\theta), z_1(\theta))$ is a maximizer of,

$$U_1(\theta) \equiv \max_{(k, x, z \in \mathcal{C}_1)} \theta f(k)z + x \quad (10)$$

We will construct a strategy for intermediary 2 allowing him to “cream skim” the market. That is, intermediary 2 will serve all the profitable types, leaving the unprofitable types to intermediary 1.

Because no intermediary would make loses in equilibrium, if a positive measure among the contracts in \mathcal{C}_1 are actually signed by entrepreneurs, there must be an positive measure subset over which \mathcal{C}_1 yields non-negative profits. Hence there is a $\hat{\theta} > 0$ (in that set) such that $k_1(\hat{\theta}) = k^*(\hat{\theta})$ and with whom the intermediary makes non-negative profits. By claim 3, $S(\theta) > U(\theta)$ for all $\theta > \hat{\theta}$.

If $x_1^0(\hat{\theta}) > 0$, construct

$\mathcal{C}'_2 = \{(k'_2(\theta), x'_2(\theta), z'_2(\theta)) | \theta \in A^0\}$, where,

$$k'_2(\theta) = k_1^0(\theta) \quad z'_2(\theta) = \frac{f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta - \hat{\theta})}{f(k_1^0(\theta))} \quad x'_2(\theta) = \frac{\delta}{2}(1 - \theta^2)$$

The payoff of entrepreneur θ among contracts $(k_2'(\theta'), x_2'(\theta'), z_2'(\theta'))$ in \mathcal{C}'_2 is

$$U_2'(\theta) = \max_{\theta' \in [0,1]} \theta (f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta' - \hat{\theta})) + \frac{\delta}{2}(1 - \theta'^2)$$

which is uniquely maximized at $\theta' = \theta$. That is,

$$U_2'(\theta) = \theta \left(f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} \right) + \frac{\delta}{2}(1 - \theta^2)$$

Next, we compare the payoffs of signing the contract with intermediary 1 or 2. We show that for $\theta > \hat{\theta}$, the later is better, while $\theta < \hat{\theta}$ prefers the former.

First consider $\theta < \hat{\theta}$.

By the envelope theorem applied to 10

$$\begin{aligned} U_1^0(\theta) &= U_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f(k_1^0(s))z_1^0(s)ds \\ &= f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f(k_1^0(s))z_1^0(s)ds \end{aligned}$$

and $f(k_1(s))z_1(s)$ is non-decreasing in s .

Notice that,

$$\begin{aligned} U_2'(\theta) &= U_2'(\hat{\theta}) + (\theta - \hat{\theta}) \left(f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} \right) + \frac{\delta}{2}(\hat{\theta}^2 - \theta^2) \\ &= U_2'(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} \left(f(k_1^0(s))z_1^0(s) + x_1^0(\hat{\theta})/\hat{\theta} \right) ds + \frac{\delta}{2}(\hat{\theta}^2 - \theta^2) \end{aligned}$$

Hence

$$U_2'(\theta) - U_1^0(\theta) = \int_{\hat{\theta}}^{\theta} \left\{ f(k_1^0(s))z_1^0(s) - f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) \right\} ds + \frac{\theta - \hat{\theta}}{\hat{\theta}}x_1^0(\hat{\theta}) + \frac{\delta}{2}(\hat{\theta}^2 - \theta^2)$$

Since $f(k(\cdot))z(\cdot)$ is non-decreasing, for $\theta < \hat{\theta}$, the first term is non-positive. The second term is strictly positive as $x_1^0(\hat{\theta}) > 0$. We chose δ such that $\frac{\theta - \hat{\theta}}{\hat{\theta}}x_1^0(\hat{\theta}) + \frac{\delta}{2}(\hat{\theta}^2 - \theta^2)$ is still positive. We conclude that for $\theta < \hat{\theta}$

$$U_2'(\theta) - U_1^0(\theta) > 0$$

Next, consider entrepreneurs with $\theta > \hat{\theta}$

$$\begin{aligned} U_1^0(\hat{\theta}) &= \hat{\theta} f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) \\ &\geq \hat{\theta} \left(f(k_1^0(\theta))z_1^0(\theta) + \frac{x_1^0(\theta)}{\theta} \right) \\ &= \frac{\hat{\theta}}{\theta} U_1^0(\theta) \end{aligned}$$

where the weak inequality in the second line comes from the envelope theorem and the monotonicity of $f(k(\cdot))z(\cdot)$: an argument similar to the one describe in more detail for $\theta < \hat{\theta}$.

Since $\hat{\theta} > 0$,

$$\frac{\theta}{\hat{\theta}} U_1^0(\hat{\theta}) \geq U_1^0(\theta)$$

By construction, $U_2'(\hat{\theta}) = U_1^0(\hat{\theta})$. Moreover,

$$\begin{aligned}\frac{\hat{\theta}}{\theta}U_2'(\theta) &= U_2'(\hat{\theta}) + \frac{\delta}{2} \left(\frac{\theta}{\hat{\theta}}(1 - \hat{\theta}^2) - (1 - \theta^2) \right) \\ &> U_2'(\hat{\theta}) + \frac{\delta}{2} (\theta^2 - \hat{\theta}^2) \\ &> U_2'(\hat{\theta})\end{aligned}$$

We conclude that,

$$U_2'(\theta) \geq \frac{\theta}{\hat{\theta}}U_2'(\hat{\theta}) = \frac{\theta}{\hat{\theta}}U_1^0(\hat{\theta}) \geq U_1^0(\theta)$$

It follows that entrepreneurs with type $\theta > \hat{\theta}$ are better of contract \mathcal{C}_2' than with contract \mathcal{C}_1^0 . Hence intermediary 2 makes at least half of the profits over the interval $[\hat{\theta}, 1]$, which are strictly positive by claim 3.

So far we have assumed that $x_1^0(\hat{\theta}) > 0$. If $x_1(\hat{\theta}) = 0$, then it must be the case that $\tilde{\theta} < \hat{\theta}$ (by lemma A.1).

In this case, intermediary 2 can deviate to the strategy, $\mathcal{C}_2'' = \{(k_2''(\theta), x_2''(\theta), z_2''(\theta)) | \theta \in A^0\}$, where,

$$k_2''(\theta) = k_1^0(\theta) \quad z_2''(\theta) = \frac{f(k_1^0(\tilde{\theta}))z_1^0(\tilde{\theta}) + x_1^0(\tilde{\theta})/\tilde{\theta} + \delta(\theta - \tilde{\theta})}{f(k_1^0(\theta))} \quad x_2''(\theta) = \frac{\delta}{2}(1 - \theta^2)$$

By the same argument as before, when intermediary 2 deviates to \mathcal{C}_2'' , Every entrepreneur with type $\theta < \tilde{\theta}$ takes a contract from \mathcal{C}_1^0 , and every type $\theta > \tilde{\theta}$ takes a contract from \mathcal{C}_2'' .

It is left to show that intermediary 1 was making loses over $[\theta_L, \tilde{\theta}]$ - and because she wouldn't make loses, intermediary 2 steals a strictly profitable fraction of the market-. If this this was not the case, he must be making profits over a positive measure set and we can redefine $\hat{\theta}$ as a point in that interval. \square

A.1.7 Proof of proposition 2

Proof. Suppose $\theta_L \geq \theta_P$, then $S(\theta_L) \geq S(\theta_P) = w = U(\theta_L)$ by the definition of θ_L and θ_P . But then the intermediary expects not to loose with the type θ_L and, by claim 3, expects strictly positive profits with all $\theta' \in (\theta_L, 1]$. That implies the intermediary is making profits strictly positive aggregate profits contradicting the zero profit condition.

If types are public, intermediaries must break even with each type, which implies $U(\theta) = S(\theta)$. For those $\theta < \theta_P$, we have $S(\theta) < w$ hence none of them will take any contract. All the rest will accept the contract offered for their type, hence $\theta_L = \theta_P$ and the inefficiency vanishes.

If limited liability is removed, but all other features remain the same, we will show the equilibrium incentive compatible contract schedule has to be $k(\theta) = k^*(\theta)$, $x(\theta) = -R \cdot k^*(\theta)$ and $z(\theta) = \pi$ for (almost) all $\theta \geq \theta_P$.

We start showing that the following strategy profile is indeed an equilibrium:

Each intermediary offers, $\mathcal{C}_i = \{(k_i(\theta), x_i(\theta), z_i(\theta)) | \theta \in [0, 1]\}$, where,

$$k_i(\theta) = k^*(\theta) \quad z_i(\theta) = \pi \quad x_i(\theta) = -R \cdot k^*(\theta) \quad (11)$$

and entrepreneur θ flips a coin before selecting between $(k_1(\theta), x_1(\theta), z_1(\theta))$ and $(k_2(\theta), x_2(\theta), z_2(\theta))$, - but strictly prefers any of the two compared to any $(k_i(\theta'), x_i(\theta'), z_i(\theta'))$ for $\theta' \neq \theta$ -. As $z_i(\theta)$

is constant, $f(k^*(\theta))z_i(\theta)$ is (strictly) increasing, hence the contract satisfies the conditions of lemma ?? and $(k_i(\theta), x_i(\theta), z_i(\theta))$ maximizes entrepreneur θ 's utility among the available options. Also $U_i(\theta) = S(\theta)$, and by definition on θ_P , type θ takes the contract if and only if $\theta \geq \theta_P$. If intermediary i is offering the above contract, intermediary j 's best response cannot yield her any profit, since she would get only those types such that $U_j(\theta) \geq U_i(\theta) = S(\theta)$, and hence offering the same contract is a best response.

To see all equilibrium are payoff equivalent, notice that claims 1 and 2 still must hold. Define $B_2 = A_2 \cup B$, and suppose that in an equilibrium, intermediary 2 offers a contract schedule such that on a positive measure subset of $B_2 \subset [\theta_P, 1]$, $S(\theta) \neq U_2(\theta)$ for all $\theta \in B_2$. Denote the equilibrium strategies by $\mathcal{C}_1, \mathcal{C}_2$ and s . Define the associated contract schedules $(k_1(\theta), x_1(\theta), z_1(\theta))$ and $(k_2(\theta), x_2(\theta), z_2(\theta))$ as in equation ?? above.

We will show that intermediary 1 can post a contract schedule that strictly increases her profits.

Define the deviation by, $\mathcal{C}'_1 = \{(k'_1(\theta), x'_1(\theta), z'_1(\theta)) | \theta \in \cdot\}$, where,

$$k'_1(\theta) = 0.5k^*(\theta) + 0.5k_2(\theta) \quad z'_1(\theta) = 0.5\pi + 0.5z_2(\theta) \quad x'_1(\theta) = -0.5R \cdot k^*(\theta) + 0.5x_2(\theta) \quad (12)$$

The new contract schedule to be offered by intermediary 1 is just the average of intermediary 2's and the prescribed equilibrium contracts. Note that the new contract skims the cream: there is a threshold level $\bar{\theta}$, such that every $\theta > \bar{\theta}$ takes the contract offers by intermediary 2 (if any), and lower θ s take the contract by intermediary 1.

Define $B_2^+ = \{\theta \in B_2 : U_2(\theta) > S(\theta)\}$ and analogously $B_2^- = B_2 \setminus B_2^+$. If B_2^- has positive measure, this average contract makes profits with all $\theta \in B_2^-$ because $S(\theta) > U'_1(\theta) > U_2(\theta)$; it cannot make loses in B_2^+ since there $U_2(\theta) > U'_1(\theta)$; and, it is irrelevant outside B_2 . Hence if B_2^- has positive measure, this contract yields positive profits to intermediary 2 contradicting the zero profit condition. If B_2^- has measure zero and B_2^+ has positive measure, this implies that intermediary 2 is not making profit with any type, since B_2^- has measure zero, but then for her to avoid losses it must be the case that no positive measure of her contracts in B_2^+ is taken in equilibrium, which implies that for all θ in a positive measure set $B_1^+ \subset B_2^+$ we must have $U_1(\theta) > U_2(\theta) > S(\theta)$ which implies intermediary 1 is making loses there. Since she cannot make profits with any positive measure of types, because 2 offers at least $S(\theta)$ to everybody but those in B_2^- , she must be making negative profits over all types.

Hence for all $\theta \in [\theta_P, 1]$ it must be the case that $U_1(\theta) = U_2(\theta) = S(\theta)$. The envelope theorem for $S(\theta)$ yields $S'(\theta) = f(k^*(\theta))\pi$ which implies $z_1(\theta) = z_2(\theta) = \pi$ for (almost) all those θ . That in turn implies $x_i(\theta) = -R \cdot k^*(\theta)$.

Last, if there is a unique intermediary facing limited liability and adverse selection, the unique equilibrium is $k(\theta) = k^*(\theta)$, $z(\theta) = 0$ and $x(\theta) = w$ and (almost) all entrepreneurs with $\theta \geq \theta_P$ take the contract. In this case an equilibrium should be a contract schedule and a decision rule for entrepreneurs such that the schedule maximizes profit for the intermediary and the decision rule maximizes return to the entrepreneur. In the proposed equilibrium the intermediary extracts all the surplus, hence it is profit maximizing, and entrepreneurs are always indifferent between accepting or rejecting the contract, hence they are also maximizing. Note that in any equilibrium a type θ entrepreneur must take any contract such that her payoff is higher than w and reject anyone otherwise. So potentially the acceptance rule for some θ' could

be to accept the contract offering $k^*(\theta)$ with $\theta \neq \theta'$.

To see that can only happen in measure zero sets, suppose there exists an equilibrium where the acceptance rule differs from the one prescribed above in a set $\Theta \in [0, 1]$ with positive measure. For all $\varepsilon > 0$ the contract with $k(\theta) = k^*(\theta)$, $z(\theta) = \frac{\varepsilon}{\theta_P}$ and $x(\theta) = w - \varepsilon$ is such that $\theta_L = \theta_P$ and the profits for the intermediary are the total surplus less $\varepsilon \int_{\theta_P}^1 (1 - G(\theta)) d\theta$, this implies that in any equilibrium profits should be equal to the total surplus, otherwise the intermediary could do better with the contract above for some ε . But to have profits equal to the total aggregate surplus, almost all $\theta \in [\theta_P, 1]$ must accept the contract and almost all $\theta < \theta_P$ must reject it. Hence the result follows. \square

A.1.8 Proof of claim 6

Proof. A lump sum tax does not change the intermediaries optimal decisions. Then the contract offered in equilibrium is still characterized by proposition 1.

Now the decision rule for entrepreneurs is as follows: they must accept the best contract offered to their type if $U(\theta) - \phi > w$ and reject if the inequality is reversed. In this sense the tax can also be seen as a subsidy on w . Hence, at θ_L we must have $U(\theta_L) = w + \phi$. Then, the zero profit condition for θ_L is:

$$\int_{\theta_L}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_L}^1 \theta \frac{(w + \phi)}{\theta_L} dG(\theta) = 0$$

Which has a unique solution θ_L since its derivative with respect to θ_L is strictly positive. By definition of ϕ , $\theta_L = \theta_P$ solves the equation and is thus the only solution. \square

A.1.9 Proof of claim 7

Proof. Suppose intermediaries compete with contracts of the form $(k, x, (1 - \tau)z)$. Then the contract offered in equilibrium is still characterized by proposition 1 but the θ_L now has to solve:

$$\int_{\theta_L}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_L}^1 \theta \frac{w}{(1 - \tau)\theta_L} dG(\theta) = 0,$$

because intermediaries have to pay $(1 - \tau)^{-1}U(\theta)$ if she is supposed to deliver $U(\theta)$ net of taxes to the entrepreneur. Plugging the value for τ^* we obtain:

$$\int_{\theta_L}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} dG(\theta) - \int_{\theta_L}^1 \theta \frac{(w + \phi)}{\theta_L} dG(\theta) = 0$$

Which by claim 6 has a unique solution $\theta_L = \theta_P$. \square

A.2 Asset Holdings Proofs

The proofs for claims A.1 to 2 are analogous to those for the zero assets case. Fixing the asset level a , the IC constraint across types θ is the same, but now the limited liability restriction is $\underline{x}_i(\theta, a) \geq -Ra$. Recall the expected utility of a type θ entrepreneur with assets a under contract i be $U_i(\theta, a) + Ra$. Then $\underline{x}_i(\theta, a)$ is decreasing in θ and in a competitive equilibrium $k_i(\theta, a) = k^*(\theta)$ for (almost) all (θ, a) and $U_i(\theta, a)$ is nondecreasing in θ

A.2.1 Proof of claim 8

Proof. If $k^*(\theta) \leq a$, the entrepreneur can self-finance the project up to the optimal scale and save the rest, with expected profit $\theta F(k^*(\theta))\pi + R(a - k^*(\theta))$, which is the best possible outcome for the entrepreneur outside the credit market. If $k^*(\theta) > a$ the project can still be started but at a scale lower than the optimal, concavity of F implies that the best option, conditional on starting the project, is to invest all the assets in it, which yields $\theta F(a)\pi$. In any case that has to be compared with the option of not doing the project and getting the return on the assets. \square

A.2.2 Proof of proposition 3

Claim 3 carries over. The proof has to change a little.

Proof. Note that because of the outside option, $\hat{\theta}f(a)\pi < U_i(\hat{\theta}, a) + Ra$ must hold. Also for all θ we have $S(\theta) \geq \theta f(a)\pi - Ra$ with equality only for some θ_a such that $k^*(\theta_a) = a$. Note that no profits can be made with types $\theta < \theta_a$ as those can fully self finance, hence $\hat{\theta} > \theta_a$. Suppose the expected profits for some $\theta' > \hat{\theta} > \theta_a$ such that $k_i(\theta') = k^*(\theta')$ are not positive, that is $U_i(\theta') \geq S(\theta')$. $S(\theta)$ is still convex. Remembering that $S(\theta_a) = \theta_a f(a)\pi - Ra \leq U_i(\theta_a, a)$, by incentive compatibility:

$$\begin{aligned}
U_i(\hat{\theta}, a) &\geq \hat{\theta}f(k^*(\theta'))z_i(\theta', a) + \underline{x}_i(\theta', a) = \frac{\hat{\theta}}{\theta'}U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta'}\underline{x}_i(\theta', a) \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a}U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'}U_i(\theta', a) + \frac{\theta' - \theta_a}{\theta'}\underline{x}_i(\theta', a) \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a}U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'}(\theta'f(a)\pi - Ra) - \frac{\theta' - \theta_a}{\theta'}Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a}U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\theta_a f(a)\pi - \frac{\theta_a}{\theta'}Ra - \frac{\theta' - \theta_a}{\theta'}Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a}S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} [\theta_a f(a)\pi - Ra] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a}S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a}S(\theta_a) > S(\hat{\theta})
\end{aligned}$$

contradicting the expected profits for $\hat{\theta}$. As $k_i(\theta') = k^*(\theta')$ for almost all θ' the lemma follows. \square

Lemma ?? is still valid, but we need a somewhat updated version:

Lemma A.2. Let $\mathcal{C}(\theta) = (k(\theta, a), x(\theta, a), z(\theta, a))$ be an incentive compatible contract schedule with assets a . For any $\hat{\theta} \in [0, 1]$, define the contract schedule $\mathcal{C}_{\hat{\theta}}$ by,

$$\mathcal{C}_{\hat{\theta}}(\theta) = \left(k(\theta, a), -Ra, \frac{f(k(\hat{\theta}, a))z(\hat{\theta}, a) + x(\hat{\theta}, a)/\hat{\theta}}{f(k(\theta))} \right)$$

Then,

- $\mathcal{C}_{\hat{\theta}}$ is incentive compatible.
- For all $\theta < \hat{\theta}$, $U(\theta, \mathcal{C}) \geq U(\theta, \mathcal{C}_{\hat{\theta}})$

- $U(\theta, \mathcal{C}) > U(\theta, \mathcal{C}_{\hat{\theta}})$ if and only if $x(\hat{\theta}) > -Ra$ or $f(k(\hat{\theta}, a))z(\hat{\theta}, a) > f(k(\theta, a))z(\theta, a)$ for some $\theta < \hat{\theta}$.

The proof of this lemma is analogous to the old one. Moreover, once the two lemmas have been established, the proof of claim 5 carries over, completing the proof of proposition 3.

A.3 Proofs with Unobservable Assets

Then the IC constraint across assets is $U_i(\theta, a)$ is nondecreasing in a (recall an entrepreneur cannot lie and give more collateral than what he has).

A.3.1 Linear loss minimization

We define a procedure to minimize losses for a given set of profitable contracts. An (expected) profitable contract is one where $U_i(\theta, a) < S(\theta)$ where $S(\cdot)$ is the surplus function.

For a fixed asset level a , the promised extra utility $U_i(\theta, a)$ curve of an IC contract with limited liability can only cross the curve $S(\theta)$ once¹⁰. Define $\theta_e(a)$ as the solution of $U_i(\theta, a) = S(\theta)$. Also define $\widehat{fz}(a) = \inf \left\{ f(k^*(\theta))z(\theta, a) : \theta > \theta_e(a) \right\}$, this is the maximum slope the contract can have at $\theta_e(a)$.

For all $\theta < \theta_e(a)$ the intermediary is making losses. Hence it is in his best interest to reduce $U_i(\theta, a)$ for all such theta. However the IC constraint over θ implies that $U_i(\theta, a) \geq S(\theta_e(a)) - (\theta_e(a) - \theta)\widehat{fz}(a)$ and the intermediary will try to set that. Unfortunately, there is also the IC constraint over a , that requires $U_i(\theta, a)$ to be nondecreasing in a . Therefore the loss minimization given some contract $\theta_e(a)$ and $\widehat{fz}(a)$ is achieved by setting:

$$U_i(\theta, a) = \sup \left\{ S(\theta_e(\hat{a})) - (\theta_e(\hat{a}) - \theta)\widehat{fz}(\hat{a}) : 0 \leq \hat{a} \leq a \right\}$$

A.3.2 Zero Profits

Zero profits will still happen but the proof needs to be modified. Below are the steps.

1. As before if one intermediary is making profits π , the other can always set a new contract as the max of the two current offered contracts. After that she need to increase her offers by an ε small enough such that she would take over all the market and profits fall just slightly.
2. Because of continuity, the issue is going to be the new entrants. First we need that for every δ there exists some ε such that the change in $\theta_L(a)$ is less than δ for a lot of a (meaning we can make the mass of those not bounded as small as wanted) when we give ε more to everybody.
 - (a) The new entrants are determined by the slope of the contract on $\theta_L(a)$. We need a bound for that slope from below that works for every a and is strictly positive. Unfortunately that is not generally possible. However we can find a bound for a lot of values, such that the measure of those not bounded is very small relative to the profits. In what follows we assume the contract, after the max process has been optimized with the linear loss minimization described above.

¹⁰There may be another cut, but that has to be below or at the outside option curve $O(\theta, a)$

- i. There is no need to worry about those a such that $\theta_L(a) = 0$. In fact we can disregard all asset levels such that $\theta_L(a) < \delta$. Note that for the remaining a 's $\widehat{fz}(a) > 0$.
- ii. First we deal with those a with positive marginal mass. There can be only countable many of those. Let the combined mass of all those a_i be $M_1 \leq 1$. Then there exists finite number of those N_1 such that $\sum_{i=1}^{N_1} m(a_i) > M_1 - \frac{\pi}{5w}$. From now on we will forget about all other a_i with positive mass and take a loss no greater than 0.2π with all those types with capital a_i for $i > N_1$. Now take $0 < f_1 = \min\{\widehat{fz}(a_i) : 0 \leq i \leq N_1\}$
- iii. Take any \bar{a} such that $S(\theta_e(\bar{a})) > w$ Look at the slope of the contract at $\theta_L(\bar{a})$ after the linear minimization process. We claim that for all $a' > \bar{a}$ the slope of the contract at $\theta_L(a')$ is greater or equal than the minimum between the slope of the contract at $\theta_L(\bar{a})$ and:

$$\frac{S(\theta_e(\bar{a})) - w}{\theta_e(\bar{a}) - \delta}.$$

Call that minimum $f_2(\bar{a})$. If $\theta_L(a') = \theta_L(\bar{a})$ then, by IC over the assets, the slope of the contract at $\theta_L(a')$ has to be greater or equal than the one at $\theta_L(\bar{a})$. If $\theta_L(a') < \theta_L(\bar{a})$ then, by the envelope theorem, the slope at $\theta_L(a')$ will be $\widehat{fz}(\hat{a})$ for some $\bar{a} < \hat{a} \leq a'$, but all those are bounded below by the slope of line that goes through the points (δ, w) and $(\theta_e(\bar{a}), S(\theta_e(\bar{a})))$.

Now, if $S(\theta_e(0)) > w$ set $f_2 = f_2(0)$ and then the slopes are all (but a small measure) bounded by $\min\{f_1, f_2\}$. If not, since $\theta_e(a)$ is nondecreasing it is measurable, so we can pick an asset level \bar{a} such that the mass of assets \hat{a} such that $S(\theta_e(\hat{a})) \in (w, S(\theta_e(\bar{a})))$ is less than any positive constant we want.

We are going to allow a bigger loss on those \hat{a} types. For all of them the contract offered will be the same as the one for \bar{a} , before the ϵ increase. This would generate a loss less than $S(\theta_e(\bar{a}))$ per entrepreneur, as the contract for \bar{a} makes expected losses only on those θ types that get offered an utility level less than that. Hence we pick \bar{a} such that the mass of \hat{a} such that $S(\theta_e(\hat{a})) \in (w, S(\theta_e(\bar{a})))$ is less than:

$$\frac{\pi}{5S(\theta_e(\bar{a}))},$$

and set $f_2 = f_2(\bar{a})$.

- iv. Now, we have to deal with those \hat{a} such that $S(\theta_e(\hat{a})) = w$, let $\theta_P = S^{-1}(w)$ then we are talking about those asset levels such that $\theta_e(a) = \theta_P$ they form an interval (a_0, a_1) which may be closed or open at both ends. Notice for all those the slopes $\widehat{fz}(\hat{a})$ are increasing in \hat{a} because of incentive compatibility w.r.t. \hat{a} . Now, if $\inf\{\widehat{fz}(\hat{a})\} > 0$ then set f_3 equal to that infimum. Otherwise, as before, for each \bar{a}_2 such that $S(\theta_e(\bar{a}_2)) = w$ we can raise the contract of all those $a' \in [a_0, \bar{a}_2)$ to $U_i(\theta, \bar{a}_2)$. The loss for that change is the maximum difference between the original contracts and this new one, which is bounded by the difference between the line $u = w$ and the line with slope $\frac{Ra+w}{\theta_P}$. As the marginal mass may be distributed in any way, the bound will be the difference between the value of that

linear function at $\theta = 1$ and w . So we pick \bar{a}_2 such that the mass in $[a_0, \bar{a}_2)$ is less than $\frac{\pi}{5(U_i(1, \bar{a}_2) - w)}$ and let $f_4 = \widehat{fz}(\bar{a}_2)$.

v. Last, we deal with those asset levels (if any) for which $U_i(\theta_w, a) < w$. Again, there is a \bar{a}_3 such that the mass of those $a' > \bar{a}_3$ such that $U_i(\theta_w, a') < w$ is less than $\frac{\pi}{5(U_i(1, \bar{a}_2) - w)}$. Again we will lift all those $\bar{a}_3 < \hat{a} < \bar{a}_2$ to $U_i(\theta, \bar{a}_2)$. The loss for doing that is less than 0.2π and let $f_5 = \widehat{fz}(\bar{a}_3)$

vi. To finish take the minimum of the f_i 's. That is the lower bound for the slope and hence define $\varepsilon(\delta) = \delta * \min\{f_i : i \in \{1, 2, 3, 4, 5\}\}$. So far we lost 0.4π with the unbounded types.

(b) After that we need that for each μ there exist a δ such that the mass of new entrepreneurs is less than μ given a reduction no larger than δ in $\theta_L(a)$ for all a . Measurability of the contracts imply $U_i(\theta, a)$ and $\theta_L(a)$ are measurable. The new entrants are a subset of $U_i^{-1}([w - \varepsilon, w]) \subset [\theta_L(a) - \delta, \theta_L(a)]$. Now we want to bound the mass of the latter intervals. For each a there exists some $\delta(a) > 0$ such that

$$G^-(\theta_L(a)|a) - G^-(\theta_L(a) - \delta(a)|a) < 0.5\mu$$

If the lower bound of those $\delta(a)$ is positive that is our δ and we are done. Otherwise we find a bound for all but a measure no bigger than $\frac{\pi}{5w}$, on which we take losses of w with all the potential entrants, so total losses wont exceed 0.2π , for the rest take the lower bound which must be positive.

A.3.3 Proof of claim 10

Proof. Suppose that is not the case. Then there exist some positive measure set $B = B^1 \cup B^2$ such that all types in B^i strictly prefer the contract offered by intermediary i than the one offered by j . Notice that the point wise average of two IC contract schedules with $k_i(\theta, a) = k^*(\theta)$ is an IC contract schedule. This is because utility is linear in $z_i(\theta, a)$ and $\underline{z}_i(\theta, a)$ so the IC constraint will be inherited from the IC of the two original schedules. We will show this average IC schedule has to increase profits for one of the intermediaries. For intermediary i , the newly proposed contract reduces the utility offered to those in B^i because that is averaged with the one offered by j to those types, which was assumed to be strictly lower. However types in B^i still prefer intermediary i contract. Outside B nothing changes since offered utilities were equal there, in B^j types still prefer intermediary j . Hence intermediary i by offering the average schedule reduces the surplus given away in the set B^i keeping all other sources of income fixed. If B^i has a positive measure then her profits strictly increase. Hence the measure of B^I must be zero which implies the measure of B has to be zero.

□

A.3.4 Proof of claim 11

Proof. Sup $\theta < \theta' < \theta''$ are such that θ and θ'' are in $A(a|b)$. Let $x(\theta', b) = -Rc$ for some $c \leq b$, notice that $P(a)$ is strictly increasing in a as it is the slope of the line passing through $(0, -Ra)$ and $(\theta_e(a), S(\theta_e(a)))$. Now, as $\theta' \notin A(a|b)$ then $\theta'P(c) - Rc \geq \theta'P(a) - Ra$. If $c > a$,

then $P(c) > P(a)$ and as $\theta'' > \theta'$ it follows that $\theta''P(c) - Rc > \theta''P(a) - Ra$ contradicting $\theta'' \in A(a|b)$, analogously if $c < a$ implies $\theta \notin A(a|b)$ hence $c = a$ and then $\theta' \in A(a|b)$.

If $\theta \in A(a, c)$ for $c > b \geq a$ then,

$$U_i(\theta, c) = \theta P(a) - Ra = \sup_{0 \leq a' \leq c} \theta P(a') - Ra' \geq \sup_{0 \leq a' \leq b} \theta P(a') - Ra' \geq P(a) - Ra,$$

which implies $\theta \in A(a|b)$

□