Size-dependent distortions and labor substitution: Labor outsourcing and missing gaps.

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This version: February 18, 2016[4]

ABSTRACT

I study, theoretically and empirically, the effects of substitution between direct and outsourced labor on firms' total labor demand as a response to an implicit tax on direct labor which only applies to firms above a given size, a size-dependent distortion. A key result from the model is that it predicts positive mass of firms in the total employment distribution at the threshold of compliance with regulation, which is consistent with the empirical evidence but contradicts the predictions of standard models with homogeneous labor. The model also provides useful insights about the effects of size-dependent distortions on the increased use of outsourced labor observed in developing countries. I test the model's predictions using Colombian manufacturing data and an exogenous change in the apprenticeship contract regulation in 2002 which implicitly taxed direct labor for firms hiring at least 15 workers. Intent to treat estimators suggest that firms affected by the change demanded less direct and total labor, compared to firms not subject to the regulation, while increasing their share of outsourced labor.

JEL classification: O40, J23, J48

Keywords: Size-dependent distortions, Labor Substitution, Labor Outsourcing.

[1] Carlos Ospino is an economics Ph.D student at Universidad de los Andes, his dissertation advisor is Marcela Eslava. All errors are my own.

[2] I appreciate helpful comments from Andrés Zambrano who is part of my dissertation committee.

[3] I thank the participants at: IDB/Cornell Conference on Size and Type Dependent Regulations, LACEA Labor Network's conference "Are labor markets in Latin America prepared for the new challenges?", Universidad de los Andes' doctoral workshop and Universidad del Norte's IEEC seminar, for their helpful comments and suggestions.

[4] Please download the most recent version at https://sites.google.com/site/spinmetrics/

1. Introduction

Total factor productivity (TFP) is the main factor explaining the large differences in income per capita between countries (Restuccia, 2013). There is a growing literature on how the size distribution of firms and the allocation of factors of production affect TFP (See Hopenhayn H. A., 2014; Restuccia & Rogerson, 2013). A group of authors have studied how specific regulations, including size-dependent policies and other correlated distortions, affect the allocation of factors of production and the firm size distribution. Correlated distortions are policies that change the relative prices of factors faced by firms which differ in their productivity (Restuccia & Rogerson, 2008). Guner, Ventura & Xu (2008) (GVX) coined the term size-dependent policies[5] which are distortions that affect factor demands of firms in the neighborhood of the employment distribution where the policy kicks in. They show that these are widely used around the world and affect output and productivity.

In this paper I study the profit maximization problem of firms facing a size-dependent distortion on the use of direct labor to analyze the role of labor substitution in explaining changes in labor demand, labor composition and the distribution of total employment around the threshold of compliance with this regulation. In attempting to develop a model that is consistent with the empirical evidence about the firm size distribution and the increased share in the use of outsourced labor, I study the effects of a size-dependent tax that is independent of wages but proportional to the demand of direct labor. This type of regulations is present in countries where apprentice's quotas are calculated based on one type of worker hired by the firm such as Colombia and Costa Rica. In particular, the model considers the imperfect substitution between directly hired and outsourced labor as a response to restrictions on the former. This margin of response to the regulation has not been explicitly modeled in the size-dependent policies literature, even though policies that can induce this type of substitution are widely used in developing countries (See Ramaswamy (2013) and Chaurey (2015) for the case of contract workers in India discussed below).

One of the results of size-dependent distortion models is that the firm size distribution exhibits a lack of mass (a gap) at the threshold where a policy that taxes all units of labor kicks in. However, several studies have found that there is no such gap in the overall employment distribution. Hsieh & Olken (2014) find very small changes in the firm size distribution at the thresholds where different policies kick-in in India, Indonesia and Mexico suggesting that size-dependent taxes if anything should generate some bunching in the firm size distribution but not a gap. Braguinsky, Branstetter & Regateiro (2011) conclude that the combination of different policies interacting around the same threshold may account for the lack of a "bulge" right before the policy threshold in Portugal. Others find that although some degree of bunching is observed in the empirical total employment distribution (See Garicano, LeLarge, & Van Reenen, 2013; Gourio & Roys, 2014), it does not translate into a gap in the distribution either. This generates the contradiction between theory and the data which I refer to as the "missing gap puzzle".

Some ideas have been suggested to solve this puzzle. Garicano, LeLarge & Van Reenen (2013) proposed measurement error as the most likely cause for the absence of a gap in the firm size

[5] Size dependent policies are in fact correlated distortions (Hopenhayn H. A., 2014).

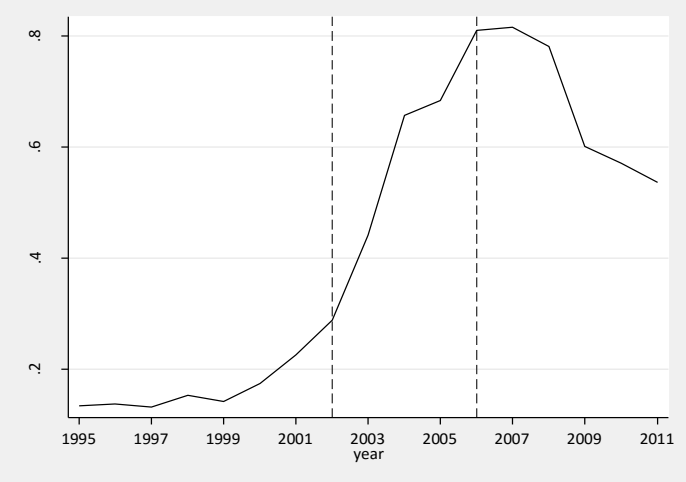
distribution in France. Gourio & Roys (2014) study the same regulation and proposed that a combination of a fixed cost tax and a small proportional tax (as well as some measurement error) can reproduce the firm size distribution around the 50 worker threshold where the regulation kicks in. These proposed solutions are to some degree ex-post ways to accommodate the model's implications with the data rather than optimal agent responses within these models that match what it's observed in the data. Labor substitution as proposed in this paper provides an alternative plausible answer to this puzzle.

Regarding labor substitution and its relationship to the increase of outsourcing, empirical evidence from India (Ramaswamy (2013); Chaurey (2015)) shows that the share of contract workers (those hired through contractors and not directly by the firm) increased by 12 percentage points between 1999 and 2009. Chaurey (2015) finds that firms in states with more strict labor laws demand more contract labor as a response to local demand shocks, relative to firms in more flexible labor markets. Further, Ramaswamy (2013) finds that share of contract workers is higher for the group of firms between 50 and 99 direct workers, compared to other size groups. Stricter labor protection regulation regarding layoff or contract termination applies to firms with 100 or more regular workers but does not apply to contract workers. The Indian evidence points to an increase both on the average share of outsourced labor and at specific thresholds as firms attempt to avoid complying with the labor regulation.

As in the Indian case, Figure 1 shows that the ratio of outsourced to direct labor has increased in Colombia since the end of the 90's. The largest increase in this ratio occurs starting in 2002 which coincides with the passing of labor law 789 of 2002[6]. This law imposed an implicit tax on the use of direct labor by defining this type of labor as the base to determine whether firms should be subject or not to the apprenticeship contract regulation. Further, this regulation applies only to firms that hire 15 or more regular workers, making it a size-dependent distortion. It's important to mention that law 789 of 2002 also introduced reforms such as reductions in the severance payments of workers with at least 10 years of tenure, reduced overtime costs and extended regular working hours until 10 p.m. All these measures would have the opposite effect of the apprenticeship contract regulation on outsourcing, since it reduces the relative costs of direct workers versus non permanent workers. This is reasuring since it implies that regulation change studied is the only one expected to increase the relative share of outsourced labor in total labor demand.

[6] The reduction of the ratio in 2006 is explained by the passing of legislation which restricted the use of outsourced labor contracts for permanent tasks at the firm.

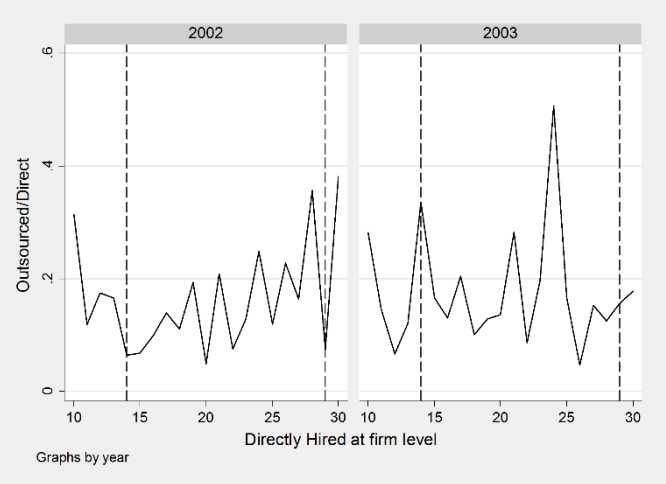
Figure 1 Average ratio of outsourced labor to direct labor.



Source: Own calculations using Colombia's Annual Manufacturing Survey.
Dashed lines at years 2002 and 2006.

The increase in the share of outsourced labor was also present at specific firm size levels and not just on average. Figure 2 shows that between 2002 and 2003 the average ratio of outsourced to directly hired labor increased importantly at 14, just before the threshold of compliance with the apprenticeship contract regulation. Firms with 15 direct works must hire one apprentice. This ratio also appears to have risen at 29 direct workes. By regulation, firms with 30 workers must hire a second apprentice. This rise in the ratio of outsourced labor is consistent with firms attempting to avoid complying with the regulation or attempting to avoing increasing its appretice share in total employment.

Figure 2 Ratio of outsourced to direct labor by direct labor demand. 2002-2003.

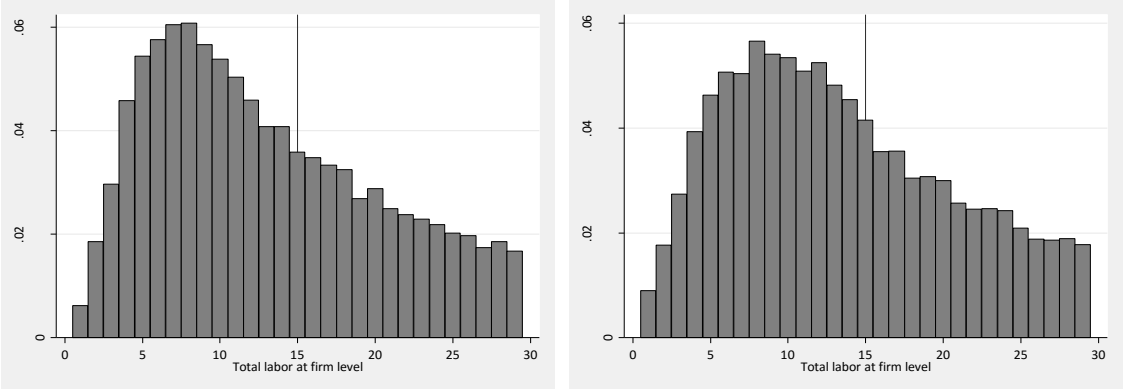


Source: Own calculations using Colombia's Annual Manufacturing Survey.
Vertical line at 14 and 29 direct workers.

Related to the previous point, since the current apprenticeship contract regulation has been in effect a disportionate increase in the mass of firms's total labor demand below the 15 worker

threshold has been observed. Figure 3 shows that firm's growth in total labor demand has been particularly active below the regulation threshold in the period that followed the reform and not as much before. Nevertheless, the distribution of total labor demand appears to be smooth around this threhshold of compliance with the apprenticeship contract regulation unlike the distribution of total labor demand in France which showed a sharp drop (Garicano, LeLarge, & Van Reenen, 2013; Gourio & Roys, 2014).

Figure 3 Distribution of total labor demand. 1995-2002 (Left panel). 2003-2011 (Right panel)



Source: Own calculations using Colombia's Annual Manufacturing Survey.
Vertical line shows the 15 worker threshold.

Results from the theoretical model show that a size-dependent policy which taxes the use of direct labor generates a substitution from the direct to outsourced labor, affecting the firm size distribution in the vicinity of the threshold in a way that is consistent with Figure 1 and Figure 3. In contrast to the case with a single contract when firms have no choice but to constraint themselves at a level that prevents them from being subject to the regulation (Guner, Ventura, & Xu, 2008; Garicano, LeLarge, & Van Reenen, 2013; Gourio & Roys, 2014), labor substitution allows these firms to increase their labor demand, output and profits. The patterns of substitution are consistent with the empirical evidence and gaps do not arise in the theoretical distribution of total employment. This alternative explanation for the "missing gap puzzle" is a key result, reconciling the theoretical implications of these models with the empirical evidence.

The model also sheds light about the effects of size-dependent distortions on labor composition. The response of firms to the policy changes the mix of contract types in the economy. It is constant for all firms in absence of distortions, but higher and increasing in idiosyncratic productivity once a size-dependent policy is introduced. This result is relevant from a policy perspective since these policies may inadvertently induce outsourcing or informality. Further, to discuss the implications of the model for unemployment I assume that wages are downward rigid. Wage rigidity under a size-dependent distortion forces the labor market to adjust via quantities, generating unemployment.

To test the model predictions regarding labor demand and outsourcing, I estimated "intent to treat" difference in difference models using Colombian manufacturing data. The difference in difference estimation showed that firms affected by the policy reduced total and direct labor demand, relative

to firms not subject to the regulation, in line with the model's prediction. Firms with at least 15 direct workers increased their ratio of outsourced to direct labor, relative to firms not subject to the regulation. While causality cannot be claimed, the correlations found in these estimations provide support for the theoretical model's predictions.

The paper is structured as follows. In addition to this introduction, in Section 2 I develop the theoretical framework to analyze the labor market effects of size-dependent regulations which are independent of wages. In Section 3 I assume that wages are downward rigid and discuss the implications of the model for unemployment and informality. In section 4 I empirically test the model's implication using Colombian manufacturing data. Finally, in section 5 I conclude with a discussion of my findings, their implications for public policy and suggest some ideas for future work.

2. A conceptual framework to analyze the labor demand effects of size dependent policies.

I consider an economy with heterogeneous production units in a partial equilibrium set up, building on a simplified version the producer's problem from GVX's version of Lucas (1978) span of control model. The main differences with GVX are the use of heterogeneous labor and a distortion which is independent of wages. In the model, heterogeneity in firm productivity allows the coexistence of production units of different sizes, measured by total labor demand. The production technology exhibits decreasing returns to the use of inputs. The production technology is given by $y(s) = s^{1-\gamma}n^{\gamma}$, where s is idiosyncratic productivity and n is total labor, which is divided between two types of workers that differ in the way in which they are hired by the firm and are imperfectly substitutable. γ is the parameter that governs the returns to scale on the use of labor, the "span of control" parameter.

The production technology requires the use of both types of labor, $n = (n_d)^{\sigma}(n_o)^{1-\sigma}$, where n_d denotes workers hired directly by the firm and n_o workers hired through outsourcing. Imperfect substitution between these factors will allow me to explicitly model a particular margin of adjustment to regulation mentioned by Garicano, LeLarge, & Van Reenen (2013), the use of outsourced labor. The motivation for making these two types of labor necessary for production lies both in the observation that in the data the use of outsourced labor has gained importance and the decision to rule out corner solutions to the labor demand problem[7].

The motivation for the margin of substitution modeled and then tested empirically comes in part from the Indian evidence discussed previously where only direct labor is implicitly taxed and from regulations that use the number of directly hired workers to determine the apprenticeship quotas in Colombia and Costa Rica. While the model's notation emphasizes the directly vs. outsourced worker margin of substitution, the model may apply more broadly to other distinctions between

[7] Notice that perfect substitution would, by construction, yield the result that the total employment distribution does not exhibit a gap at the threshold of compliance with the regulation, but would not generate bunching at the threshold.

worker types or contracts which may be better suited for each specific context and regulation (Permanent vs. Temporary, Formal vs. Informal, Skilled vs. Unskilled).

In this economy there is a representative household with a large number of members, who supply labor inelastically at the given market wages. Each household member is endowed with s units of idiosyncratic productivity, distributed with support $s = [0, \bar{s}]$, cdf=$F(s)$, and pdf=$dF(s)$. Household members can be either workers or managers. Those with ability above a certain threshold $\hat{s}$ are assigned by the household to be managers. They contribute their firm's profits, which is a function of their idiosyncratic productivity. If a household member becomes a worker, her idiosyncratic productivity level becomes one unit of labor services and her income is given by the current market wage, which workers take as given. Workers can be hired directly or via outsourcing. Workers are allowed to have different wages and I assume that wages for outsourced labor are lower than those for directly hired workers.

In what follows of the paper I focus only on the production problem since the interest lies in the response of labor demand to size-dependent policies that restrict direct labor use. I treat issues related to general equilibrium effects on wages, reallocation of labor between firms and the effects of the policy design on welfare in a different research project that looks at how different size-dependent distortions, which generate the same tax revenue, affect the overall performance of the economy in a calibrated static general equilibrium model. I will not model the use of the tax revenues and intend to capture only the direct costs of regulation in terms of labor demand and aggregate production, but not potential benefits such as investment in human capital or training[89]. I now discuss the production problem.

The production problem of a firm facing a Size-dependent distortion

I now state the problem firms face under of size-dependent distortions on direct labor use. Firms must choose the amount of directly hired and outsourced labor to maximize profits, taking wages and labor supply as given in a competitive labor market[10]. The size-dependent tax is proportional to the labor demand of directly hired workers. This distortion differs from those considered in the literature. Most models study restrictions on labor use modeling size-dependent taxes which are proportional to the total wage bill ($\tau * w * n_d$). Nevertheless, the qualitative implications of distortions that are proportional to the wage bill and those that are proportional to direct labor demand, as the one I model, are similar in terms of labor substitution when the model includes two

[8] I thank Ravi Kanbur for pointing out that these policies while distortive, are usually put in place because they are thought to have benefits for the workers or because tax revenues are needed to finance programs that have potential benefits.

[9] In a related research project I study the effects of restrictions on the use of apprenticeship contracts in a paper that builds a stochastic matching model of the labor market in the spirit of Pissarides (2000), but abstracts from tax distortions and total labor demand.

[10] A natural amplifying mechanism is allowing for labor market frictions which would introduce unemployment and misallocation via profitable matches that will not take place when firms face distortions (see Lagos, 2006). Section 3 introduces one such labor market distortion: downward wage rigidity.

labor types[11]. The choice of the distortion modeled in this paper is a closer match to the policy for which the implications of the model will be tested empirically without sacrificing generality relative to the more common set of distortions studied in models like GVX. However, what models with only one labor type miss is the ability to capture the labor substitution effect that size-dependent distortions generate as firms respond to the regulation and its implications for the distribution of total employment which the empirical evidence shows does not have gaps.

It's not uncommon for labor regulation to apply to only one type of worker. For example, employment protection regulation only applies to regular or permanent workers, limiting the ability of firms to adjust this workforce costlessly (Bentolila, Cahuc, Dolado & Le Barbanchon, 2012; Hopenhayn & Rogerson, 1993). For the case of temporary contracts Cappellari, Dell'Aringa & Leonardi (2012) evaluated a reform in Italy that relaxed the restrictions to the use of apprenticeship contracts for older workers and no longer required firms to certify the qualifications obtained by apprentices. The authors found that the reform induced substitution between different types of temporary contracts among which apprenticeship contracts are one type, and not just between these and open ended contracts. The framework proposed in this paper provides a way to model labor substitution induced by size-dependent taxes that restrict the demand of directly hired labor.

Under a proportional size-dependent tax on labor demand the production problem is given by **PP**.

$$\begin{gathered}\underset{\{n_d, n_o\}}{max.}\ \pi(s,n,w_i) = s^{1-\gamma}[(n_d)^{\sigma}(n_o)^{1-\sigma}]^{\gamma} - w_d n_d - w_o n_o - \tau(n_d) \\ where, \qquad \tau(n_d) = \begin{cases} 0 & n_d \le \hat{n}_d \\ \alpha n_d & n_d > \hat{n}_d \end{cases} \\ where\ \gamma \in (0,1);\ \sigma \in (0,1); \alpha > 0, i = d, o\end{gathered} \qquad \textbf{(PP)}$$

This type of tax implies that labor costs differ for firms at $\hat{n}_d$, since firms pay taxes on all units of directly hired labor, not just marginal units. α is the per worker value of the distortion, therefore $w_d + \alpha$ is the marginal cost of labor that firms with demand for directly hired workers above $\hat{n}_d$ must pay. The marginal cost for outsourced labor demanded for firms below this threshold is w_o, which further increases the relative difference in labor costs between direct and outsourced labor.

The first order condition with respect to n_d is given by equation (1), while the one with respect to n_o is given by equation (2).

$$\begin{gathered}s^{1-\gamma}\gamma\sigma[(n_d)^{\sigma}(n_o)^{1-\sigma}]^{\gamma-1}(n_d)^{\sigma-1}(n_o)^{1-\sigma} = w_d + \alpha \\ where, \qquad \alpha = \begin{cases} 0 & n_d \le \hat{n}_d \\ \alpha & n_d > \hat{n}_d \end{cases}\end{gathered} \qquad \textbf{(1)}$$

$$s^{1-\gamma}\gamma(1-\sigma)[(n_d)^{\sigma}(n_o)^{1-\sigma}]^{\gamma-1}(n_d)^{\sigma}(n_o)^{-\sigma} = w_o \qquad \textbf{(2)}$$

[11] This is a result in a related research project which explores the effects of labor substitution on aggregate productivity and factor allocation in the presence of different types of size-dependent regulations.

Combining (1) and (2) I find an expression for the ratio of outsourced to directly hired workers, which measures the composition of labor in the labor market. The first key result after introducing a proportional size-dependent tax is that the distorted ratio of labor types, R_α (equation (3) below), is higher than the one present in the undistorted economy, which is found by setting $\alpha = 0$. It is an increasing function of the tax and the relative costs of direct to outsourced labor. The ratio is constant for firms and proportional to the contribution of each factor in the production technology and to relative wages. Differences in relative labor demand in absence of distortions will be only explained by technology requirements and relative labor costs. This is a result of the homogeneity assumption of the production function (Guner, Ventura, & Xu, 2008)[12].In contrast to previous literature which only consider one type of labor ((Guner, Ventura, & Xu, 2008), (Garicano, LeLarge, & Van Reenen, 2013), (Gourio & Roys, 2014)), labor substitution as a response to regulation changes the composition of labor demand in the economy. Thus this model links the effects of labor market distortions in workers' relative labor costs to changes in the composition of labor contracts.

$$\frac{n_0}{n_d} = \left(\frac{w_d + \alpha}{w_o}\right)\left(\frac{1-\sigma}{\sigma}\right) \equiv R_\alpha \qquad \text{and } \alpha = \begin{cases} 0 & n_d \leq \hat{n}_d \\ \alpha & n_d > \hat{n}_d \end{cases} \qquad \textbf{(3)}$$

Using equations (1), (2) and (3), labor demands for directly hired and outsourced workers under a proportional size-dependent tax are given by (4) and (5). These demands are a function of the distortion captured by R_α. Demands are linear in idiosyncratic productivity, thus the ablest managers will have larger firms and labor demands are decreasing function of the respective wages. Linearity in s is the result of the production function used, which treats idiosyncratic productivity as a fixed input and greatly simplifies the analysis[13].

$$n_d^{\alpha*}(s, w_i, \alpha) = \left[\frac{\gamma\sigma R_\alpha^{\gamma(1-\sigma)}}{w_d}\right]^{\frac{1}{1-\gamma}} * s \equiv \left[\frac{\Omega_\alpha}{w_d}\right]^{\frac{1}{1-\gamma}} * s \qquad \textbf{(4)}$$

$$n_o^{\alpha*}(s, w_i, \alpha) = \left[\frac{\gamma(1-\sigma)}{w_o R_\alpha^{\sigma\gamma}}\right]^{\frac{1}{1-\gamma}} * s \equiv \left[\frac{\Phi_\alpha}{w_o}\right]^{\frac{1}{1-\gamma}} * s \qquad \textbf{(5)}$$

Definition: ***Undistorted firms*** are those whose direct labor demand is at most $\hat{n}_d$ and are not subject to the size-dependent regulation. ***Distorted firms*** are those whose labor demand is greater than $\hat{n}_d$ and are therefore subject to the size-dependent regulation.

[12] A CES production function yields a ratio that is a function of the elasticity of substitution between labor types, R^ϵ, where ϵ is the elasticity of substitution between labor types. Heterogeneity in the elasticity of substitution is not a margin I exploit in this paper therefore the simpler Cobb-Douglas function is appropriate.

[13] Many models assume a linear effect of idiosyncratic productivity which results in nonlinear labor demand functions. See Restuccia, 2013; Garicano, LeLarge, & Van Reenen, 2013; Braguinsky, Branstetter, & Regateiro, 2011; Hopenhayn H. A., 2014.

Comparing the labor demands for undistorted and distorted firms provides a measure of the costs in terms of labor demand of a size-dependent distortion. Since undistorted firms do not have to pay taxes ($\alpha = 0$), as was stated in the producer problem **PP**, these firms will demand exactly the same amount of labor as they would before the size-dependent tax was introduced. Taking the ratio of labor demand equation (4) for distorted firms and undistorted firms it gives $\frac{n_d^{\alpha*}(\alpha>0)}{n_d^{\alpha*}(\alpha=0)} = \left[\frac{w_d+\alpha}{w_d}\right]^{\frac{\gamma(1-\sigma)-1}{1-\gamma}}$ (which takes a value lower than one), shows that the demand for direct labor is lower in the presence of a size-dependent distortion relative to the undistorted economy for the same level of idiosyncratic productivity.

Turning to outsourced labor demand, dividing equation (5) for distorted firms by those of undistorted firms provides the result that the demand of outsourced labor is also lower relative to the undistorted economy for distorted firms in the presence of a size-dependent tax for the same level of productivity, $\frac{n_o^{\alpha*}(\alpha>0)}{n_o^{\alpha*}(\alpha=0)} = \left[\frac{w_d+\alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}}$. Undistorted firms, however, will demand as much outsourced labor as they did in absence of the tax.

By comparting the factor by which direct labor demand falls under a size-dependent tax, it's clear that this value greater than the factor by which outsourced labor falls for distorted firms when a size-dependent distortion is present, which explains why $R_\alpha(\alpha > 0) > R\_\alpha(\alpha = 0)$. Considering that the demand for both types of labor falls for these firms when a size-dependent distortion is introduced, total labor demand for the economy, relative to the undistorted economy, will be lower even though undistorted firms do not reduce their total labor demand.

Using the labor demands, it can be determined that both output and profits for distorted firms will be lower than in the undistorted economy by a factor of $\left[\frac{w_d+\alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}}$, which is the same factor by which outsourced labor drops in the presence of a size-dependent tax. Thus a key result is that one of the costs of size-dependent distortions on labor use is a reduction in total output due to the contraction in total labor demand relative to a tax-free economy[14]. However, since output and profit drops are determined by the factor of the reduction in outsourced labor demand and not by the one of direct labor, it is clear that labor substitution is a channel that reduces output costs in the economy. If all labor were taxed, the reduction in output would be larger.

$$y(s, w_i, \alpha) = \left[\frac{w_d + \alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}} \left[\left[\frac{\Omega}{w_d}\right]^{\frac{\sigma\gamma}{1-\gamma}} \left[\frac{\Phi}{w_o}\right]^{\frac{(1-\sigma)\gamma}{1-\gamma}}\right] s \qquad \textbf{(6)}$$

[14] This is the result of the misallocation effect in correlated distortion models which is driven by the fact that undistorted firms hire the workers that are not demanded by distorted firms. I thank Andrés Zambrano for pointing this out.

$$\pi(s, w_i, \alpha) = \left[\frac{w_d + \alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}} \left\{ \left[\frac{\Omega}{w_d}\right]^{\frac{\sigma\gamma}{1-\gamma}} \left[\frac{\Phi}{w_o}\right]^{\frac{(1-\sigma)\gamma}{1-\gamma}} - \left[\frac{\Omega}{w_d^{\gamma}}\right]^{\frac{1}{1-\gamma}} - \left[\frac{\Phi}{w_o^{\gamma}}\right]^{\frac{1}{1-\gamma}} \right\} s \equiv \left[\frac{w_d + \alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}} \Theta s \quad \textbf{(7)}$$

So far I have discussed the labor demand of distorted and undistorted firms. The empirical evidence from India and France show that some firms constrain themselves at a level of labor demand which makes them not subject to size-dependent regulations. From a theoretical perspective, GVX show that these firms will legally fix labor demand at $\hat{n}_d$ to avoid being subject to the regulation. In the current model however, firms can increase total labor demand by substituting direct for outsourced labor and still avoid being subject to the regulation. This allows them to have higher profits through labor substitution than in absence of these mechanism even though they are constrained in the use of direct labor.

Definition: ***Direct labor constrained firms***: These are firms who fix the demand of direct labor at $\hat{n}_d$ in order to avoid being subject to the regulation.

These firms face the decision of fixing their direct labor demand at $\hat{n}_d$ and not being subject to the regulation or demanding more direct labor and paying the tax. In order to decide which of these strategies to choose they must compare the profits of both decisions. To explicitly study this firm problem, I now state and solve the profit maximization problem if all firms were direct labor constrained, but from now on I will just call them constrained firms.

$$\max_{\{n_o\}} \pi(s, n_o, \hat{n}_d, w_i) = s^{1-\gamma}[(\hat{n}_d)^{\sigma}(n_o)^{1-\sigma}]^{\gamma} - w_d\hat{n}_d - w_o n_o \quad \textbf{(PP2)}$$
$$where\ \gamma \in (0,1);\ \sigma \in (0,1), i = d, o$$

For constrained firms the optimal demand for outsourced labor, is given by the solution to **PP2**. Firms choose n_o to maximize profits, and the FOC to this problem is:

$$s^{1-\gamma}\gamma(1-\sigma)[(\hat{n}_d)^{\sigma}(n_o)^{1-\sigma}]^{\gamma-1}(\hat{n}_d)^{\sigma}(n_o)^{-\sigma} = w_o \quad \textbf{(8)}$$

Solving for n_o yields the outsourced labor demand of distorted firms.

$$n_o^*(\hat{n}_d, s, w_i) = \left[\frac{\gamma(1-\sigma)}{w_o}(\hat{n}_d)^{\sigma\gamma}\right]^{\frac{1}{1-\gamma+\sigma\gamma}} * s^{\frac{1-\gamma}{1-\gamma+\sigma\gamma}} \quad \textbf{(9)}$$

Notice that the demand for outsourced labor is no longer linear on idiosyncratic productivity for constrained firms, but rather concave and increasing in s. This can be checked by differentiating (9) with respect to s.

A second result when a size-dependent proportional tax on labor is introduced, is that firms' decisions about direct labor demand will determine whether they are subject to the regulation or not. In equilibrium there will be a pair of idiosyncratic productivity levels that separates the three groups of firms (undistorted, constrained and distorted) in an economy with a size-dependent tax on direct labor. These idiosyncratic productivity levels will be determined once the expressions for firm profits are derived. Equation (10) represents directly hired labor demands by the three types of firms.

$$n_d^*(s, w_i, \alpha) = \begin{cases} n_d^{\alpha}(s, w_i, 0) & \forall \text{ Undistorted firms} \\ \hat{n}_d & \forall \text{ Constrained firms} \\ n_d^{\alpha}(s, w_i, \alpha) & \forall \text{ Distorted firms} \end{cases} \quad \textbf{(10)}$$

To derive an expression of the profit function for constrained firms, substitute $\hat{n}_d$ and the demand for constrained firms (equation (9)) in the profit function, which yields equation (11)

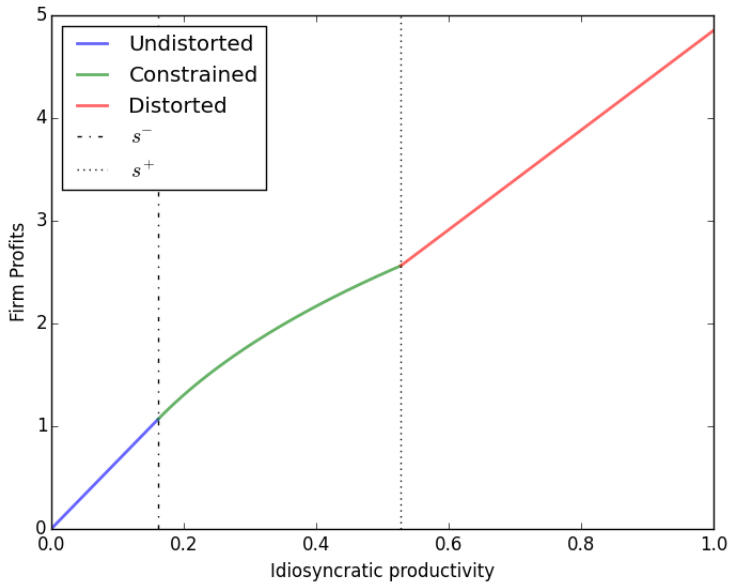
$$\pi(s, \hat{n}_d, w_i) = \left[\frac{s^{1-\gamma}{\hat{n}_d}^{\sigma\gamma}}{w_o^{\gamma(1-\sigma)}}\right]^{\frac{1}{1-\gamma+\sigma\gamma}} \left[[\gamma(1-\sigma)]^{\frac{\gamma(1-\sigma)}{1-\gamma+\sigma\gamma}} - [\gamma(1-\sigma)]^{\frac{1}{1-\gamma+\sigma\gamma}}\right] - w_d\hat{n}_d \quad \textbf{(11)}$$

Now that the profit functions have been defined for the three types of firms, a formal characterization in terms of idiosyncratic productivity can be made. Let s^- and s^+ be the lowest and highest level of idiosyncratic productivity that gives constrained firms the same level of profits as undistorted and distorted firms, respectively. To determine s^-notice that since constrained firms demand $\hat{n}_d$ there must be a productivity level where these firms obtain the same profit as the undistorted firm which demands $\hat{n}_d$. The level of idiosyncratic productivity where this occurs is the level where $\pi(s, \hat{n}_d, w_i)$=$\pi(s, w_i)$. To determine s^+ a similar reasoning follows. Firms must be indifferent between being constrained at $\hat{n}_d$ and increasing direct labor demand and complying with the regulation ($\pi(s, \hat{n}_d, w_i)$=$\pi(s, w_i, \alpha)$). The solution to equation (12) provides the values for s^- (When $\alpha = 0$) and s^+.

$$\left[\frac{s^{1-\gamma}{\hat{n}_d}^{\sigma\gamma}}{w_o^{\gamma(1-\sigma)}}\right]^{\frac{1}{1-\gamma+\sigma\gamma}} \left[[\gamma(1-\sigma)]^{\gamma(1-\sigma)} - [\gamma(1-\sigma)]\right]^{\frac{1}{1-\gamma+\sigma\gamma}} - w_d\hat{n}_d = \left[\frac{w_d + \alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}} \Theta s \quad \textbf{(12)}$$

Implicit differentiation of equation (12) yields that s^- and s^+ are increasing in $\hat{n}_d$. These results imply that increasing the regulation threshold should increase the range of the set of constrained firms. On the other hand increases in α will only increase s^+, which will have the unambiguous effect of increasing the range of the set of constrained firms, since α does not change s^-.

Figure 4 Profit function



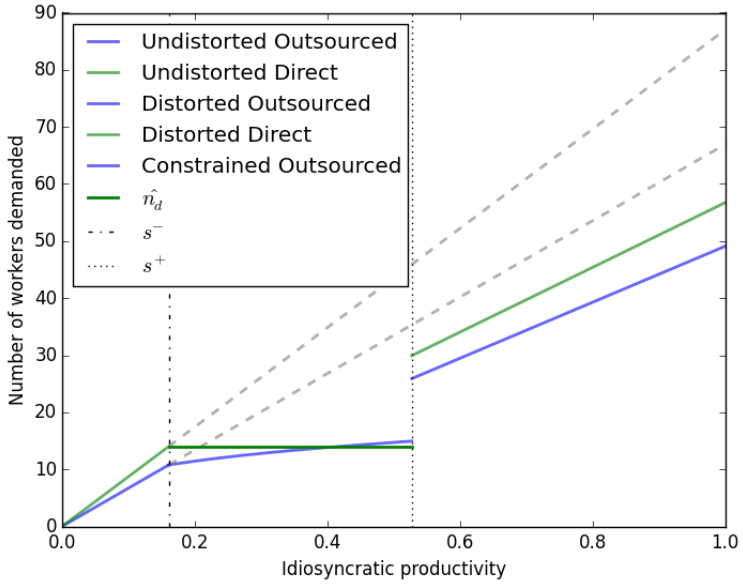
Note: Functions simulated using a log normal distribution and the following parameter values: $\gamma = 0.802, \sigma = 0.65, \alpha = 0.025, W_o = 0.14, W_d = 0.2, \hat{n}_d = 14.\ s^- = 0.16, s^+ = 0.527$

As shown in equation (13), the slope of the profit function in Figure 4 for distorted firms is lower than for undistorted ones. There are two distinct values of idiosyncratic productivity, s^- and s^+ where the profits of the constrained firm problem equal the profits of undistorted firms and distorted firms, respectively. Notice that in the segment given by $[s^-, s^+]$ the constrained profit function is increasing but not linear in s. This occurs because directly hired labor is being held constant, while outsourced labor is increasing, and the function exhibits a positive but decreasing return to variable inputs and idiosyncratic productivity. These two values separate the three types of firms in the economy: Undistorted firms will have idiosyncratic productivity level $s \in [0, s^-]$, constrained firms will have idiosyncratic productivity level $s \in [s^-, s^+]$, and distorted firms will have idiosyncratic productivity level $s \in [s^+, \bar{s}]$. Therefore, the profit function is a piecewise continuous function given by:

$$\Pi(s, w_i, \alpha) = \begin{cases} \pi(s, w_i, 0) & \forall\ s \in [0, s^-] \\ \pi(s, \hat{n}_d, w_i, 0) & \forall\ s \in [s^-, s^+] \\ \pi(s, w_i, \alpha) & \forall\ s \in [s^+, \bar{s}] \end{cases} \quad \textbf{(13)}$$

Figure 5 plots labor demand functions for the three types of firms. Firm size, measured as total labor demand, while lower than in the undistorted economy (Shown in dashed lines) is still strictly increasing in idiosyncratic productivity, in contrast to what models with just one worker type find. Notice that at s^+ labor demands for direct and outsourced labor jumps from $\hat{n}_d$ to $n_d^{\alpha*}(s, w_i, \alpha)$ and from $n_o^*(\hat{n}_d, s, w_i)$ to $n_o^{\alpha*}(s, w_i, \alpha)$, respectively. This implies that the absence of a gap in the total employment distribution is not guaranteed.

Figure 5 Labor demand functions



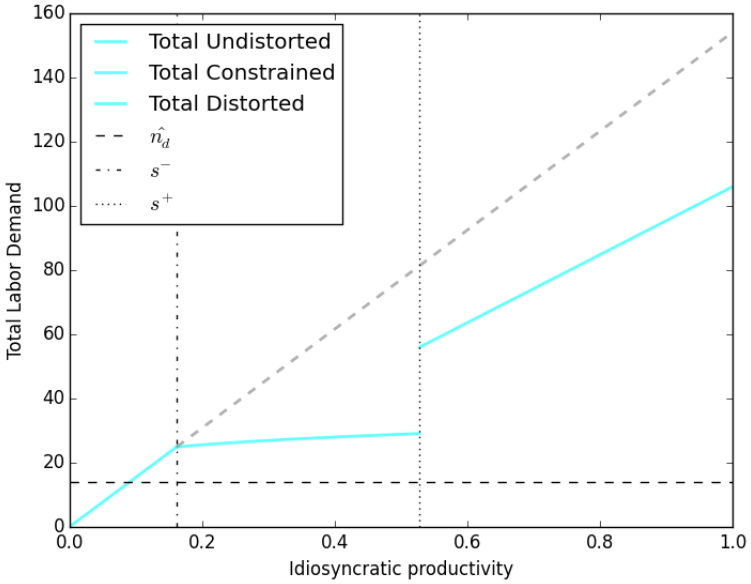
Note: Functions simulated using a log normal distribution and the following parameter values: $\gamma = 0.802, \sigma = 0.65, \alpha = 0.025, W_o = 0.14, W_d = 0.2, \hat{n}_d = 14.\ s^- = 0.16, s^+ = 0.527$

Equation (14), n_T, plotted in Figure 6 represents total labor demand for the three types of firms.

$$n_T(s, w_i, \alpha) = \begin{cases} n_d^*(s, w_i) + n_o^*(s, w_i) & \forall\ s \in [0, s^-] \\ \hat{n}_d + n_o^*(\hat{n}_d, s, w_i) & \forall\ s \in [s^-, s^+] \\ n_d^{\alpha*}(s, w_i, \alpha) + n_o^{\alpha*}(s, w_i, \alpha) & \forall\ s \in (s^+, \bar{s}] \end{cases} \quad \textbf{(14)}$$

The jump in labor demands at s^+ has implications for the existence or lack thereof a gap in the employment distribution. GVX find that the set of taxed inputs $n \in [n^-, n^+]$, which represent total labor demand at $s \in [s^-, s^+]$, will not be demanded inducing the existence of a gap in the total employment distribution of firms at the threshold of compliance with the regulation. In the current model when firms are faced with a restriction to hire direct labor they will be able to adjust their total labor demand by hiring outsourced labor. However, whether the amount of labor that is substituted is able to eliminate the gap that is generated by the jump in direct labor demand at the regulation threshold depends on whether $n_o^*(\hat{n}_d, s^-, w_i) > 0$. A sufficient condition for this is $R > 0$. Imperfect labor substitution guarantees that there is positive (outsourced) labor demand by constrained firms over the range of direct labor that is not demanded ($n_d \in (\hat{n}_d, n_d^{\alpha*}(s^+, w_i, \alpha))$. All that is required to eliminate the gap in total demand distribution in size-dependent models is that constrained firms can substitute towards the untaxed labor. Therefore, labor substitution is an answer to the missing gap puzzle.

Figure 6 Total labor demand.



Note: Functions simulated using a log normal distribution and the following parameter values: $\gamma = 0.802, \sigma = 0.65, \alpha = 0.025, W_o = 0.14, W_d = 0.2, \hat{n}_d = 14.\ s^- = 0.16, s^+ = 0.527$

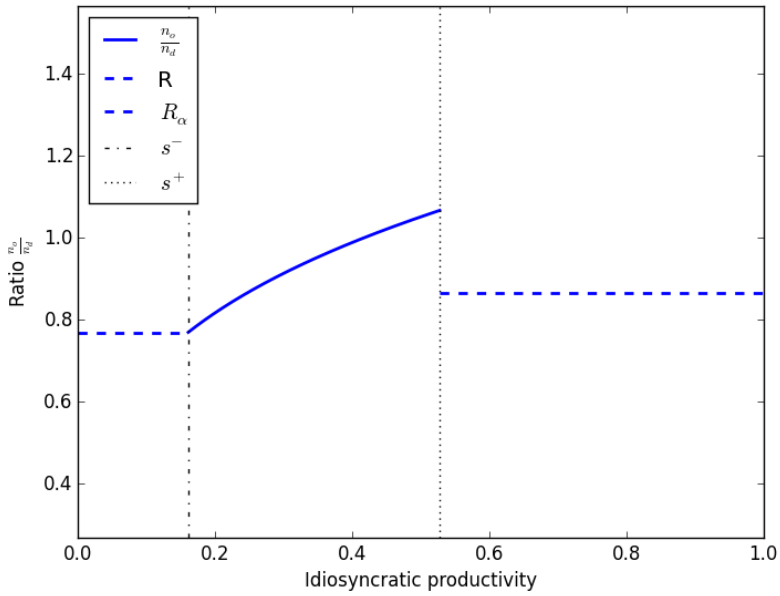
To obtain the previous result substitute s^- in equation (9) which produces the left hand side of the inequality $n_o^*(\hat{n}_d, s^-, w_i) > 0$. Solving this condition provides the result that when $\hat{n}_d R > 0$ there is no employment gap at the threshold. This condition does not rule out, however, the existence of gaps in the distribution at higher levels of employment since both labor demands jump at s^+, as can be seen in Figure 6[15]. $\hat{n}_d R > 0$ is always satisfied in technologies with imperfect substitution and is trivially satisfied under perfect substitution. The implication of this result is that to produce total employment distributions which are consistent with the data, size-dependent models should explicitly model the substitution of labor that takes place when firms are faced with the decision to become subject to the regulation.

Garicano, LeLarge & Van Reenen (2013) propose measurement error as the most likely cause of observing positive mass in the firm size distribution after the threshold where firms have to comply with the regulation. Using this approach, they can match the data, and argue that measurement error is a better answer than alternatives such as adjustment costs, employment shocks, bounded rationality, fixed proportions factors, or sunk costs of regulation. Alternatively, Gourio & Roys (2014) match the French data using a combination of fixed and proportional labor costs as well as some measurement error, and test for the presence of sunk costs. The answer I propose, by allowing substitution towards an untaxed labor, directly generates this empirical implication, and is perhaps more intuitive than measurement error.

[15] The jump at s^+ generates a gap further along the distribution which is consistent with the missing middle hypothesis. See Hsieh & Olken (2014).

The final implication of the model is an increase both in the level and dispersion of the ratio of outsourced to direct labor in an economy with a size-dependent distortion, relative to a distortion-free economy. Figure 7 plots these ratios for the three types of firms. Given that optimal outsourced labor demand for constrained firms, $n_o^*(\hat{n}_d, s, w)$, is increasing in idiosyncratic productivity in $s \in [s^-, s^+]$ and demand for directly hired labor is fixed at $\hat{n}_d$, $\frac{n_o}{n_d}$ is increasing in s as can be checked in equation (15). Further, at s^+ this ratio jumps down to the level R_α which is constant for all distorted firms.

Figure 7 Ratio of outsourced to direct labor demand.



Note: Functions simulated using a log normal distribution and the following parameter values: $\gamma = 0.802, \sigma = 0.65, \alpha = 0.025, W_o = 0.14, W_d = 0.2, \hat{n}_d = 14. s^- = 0.16, s^+ = 0.527$

$$\frac{n_o^*(\hat{n}_d, s, w_i)}{\hat{n}_d} = \left[\frac{\gamma(1-\sigma)}{w_o}(\hat{n}_d)^{1-\gamma} s^{1-\gamma}\right]^{\frac{1}{1-\gamma+\sigma\gamma}} \quad \forall\, s \in [s^-, s^+] \qquad \textbf{(15)}$$

Changes in the composition of labor contracts can be assessed in an intuitive manner. In absence of distortions the average ratio of contracts type is given by R regardless of firm productivity. However, the distorted economy has an average ratio that is higher and is given by equation (16). The second term in this equation is the ratio of outsourced labor to direct labor for constrained firms, which as was discussed before is increasing in idiosyncratic productivity[16]. From a policy perspective is important to note that size-dependent policies will affect the composition of labor where the untaxed labor will increase its share in the contract mix, and may have unintended consequences

[16] The increased dispersion in the ratio of outsourced to direct labor can also affect aggregate productivity and is an example of a positive correlated distortion, where more productive firms have a higher share of the untaxed labor. The effects of a higher share of outsourced labor should be further explored as a source of misallocation and productivity losses.

like the increase in contract labor observed in India or the increased share of informal labor observed in many Latin American countries.

$$\bar{R} = R\int_0^{s^-} s\,dF(s) + \int_{s^-}^{s^+}\left[\frac{\gamma(1-\sigma)}{w_o}\hat{n}_d^{1-\gamma}s^{1-\gamma}\right]^{\frac{1}{1-\gamma(1-\sigma)}} dF(s) + R_\alpha\int_{s^+}^{\bar{s}} s\,dF(s) \quad \textbf{(16)}$$

The model proposed in this section predicts a reduction in total employment demand and an increase in the share of outsourced labor in total employment by distorted firms due to higher labor costs of direct labor. Another prediction is that constrained firms will also increase their share for outsourced labor through labor substitution to legally avoid being subject to the regulation. The increase in demand by constrained firms will not be enough to compensate total labor demand and employment will be lower if wages can't adjust. These responses imply that the share of outsourced labor in total labor demand will be an increasing function of firm's idiosyncratic productivity. Finally, labor substitution may be able to generate a distribution of total employment without gaps at the regulation threshold consistent with the empirical data from developed and developing countries. This requires only the imperfect substitution of labor in the production technology. I now examine the implications of these results for employment losses in the presence of wage rigidities.

3. Employment losses

Although the framework developed in this paper is a partial equilibrium one, it can be used to discuss the implications for unemployment of size-dependent distortions that restrict the use of specific types of labor and generates labor substitution. Let's assume that wages are downward rigid so that in the current framework we're able to abstract from changes in wages that would reallocate labor across firms eliminating unemployment as in most size-dependent distortions models. In countries where the minimum wage is high relative to average wages, this is not a farfetched assumption.

From Figure 5 one can construct measures of employment losses due to size-dependent regulations. Since undistorted firms will not change their labor demand relative to a distortion-free economy, losses can be evaluated by examining only distorted and constrained firms. Equation (17) is the difference in total labor demand by distorted firms relative to a distortion-free world. It shows the measure of employment losses due to the reduction in labor demand, where it's clear that in absence of distortions ($\alpha = 0$) there would not be any employment losses.

Employment losses for constrained firms is given by equation (18). The last term of this equation shows that the availability of outsourced labor prevents higher employment losses. Given that outsourced employment increases with idiosyncratic productivity for constrained firms, the difference in employment relative to the distortion-free economy is lower than if it was constant, as is the case with direct labor for these firms. This is an example of the positive effects for employment growth of having alternative forms of contracting, since it may help reduce unemployment.

$$\left\{\left[1-\left[\frac{w_d+\alpha}{w_d}\right]^{\frac{-\gamma\sigma}{1-\gamma}}\right]\left[\frac{\Phi}{w_o}\right]^{\frac{1}{1-\gamma}}+\left[1-\left[\frac{w_d+\alpha}{w_d}\right]^{\frac{\gamma(1-\sigma)-1}{1-\gamma}}\right]\left[\frac{\Omega}{w_d}\right]^{\frac{1}{1-\gamma}}\right\}\int_{s^+}^{\bar{s}} s\,dF(s) \quad \textbf{(17)}$$

$$\left\{\left[\frac{\Phi}{w_o}\right]^{\frac{1}{1-\gamma}}+\left[\frac{\Omega}{w_d}\right]^{\frac{1}{1-\gamma}}\right\}\int_{s^-}^{s^+} s\,dF(s)-\hat{n}_d-\left[\frac{\gamma(1-\sigma)}{w_o}(\hat{n}_d)^{\sigma\gamma}\right]^{\frac{1}{1-\gamma+\sigma\gamma}}*\int_{s^-}^{s^+} s^{\frac{1-\gamma}{1-\gamma+\sigma\gamma}}\,dF(s) \quad \textbf{(18)}$$

In this section I've shown that under wage rigidity the effects of a size-dependent policy which restricts the use of labor will generate a lower employment demand relative to the undistorted economy. This is particularly important for economies where minimum wages are binding and the margin of adjustment is the extensive one. Therefore, the availability of flexible forms of contracting can help reduce employment losses when firms face size-dependent distortions.

4. Empirical test of model implications

The Colombian apprenticeship contract offers an opportunity to test for the model's implications The 2002 reform to the apprenticeship contract regulation which has been in effect since 1960, changed both the threshold of compliance and the type of workers used to determine the mandatory quota for apprenticeship contracts. It changed from 20 skilled workers in 2002 to 15 directly hired workers starting in 2003.[17] The 15 worker threshold applies only to this particular regulation reducing the chance that multiple regulations are interacting to explain the absence of a gap in the distribution at the threshold, a problem faced by Braguinsky, Branstetter & Regateiro (2011). Finally, the data for this exercises comes from Annual Manufacturing Survey (EAM) which captures measures of direct and outsourced employment. It also allows for the estimation of total factor productivity and other firm outcomes such as output and revenue for all manufacturing firms in Colombia with at least 10 workers or production values of at least 35.000 USD. I use data for 2002 and 2003, but some robustness checks are carried out using data from 1995 to 2011 as well. The data is proprietary data by DANE, the Colombian statistics institution, and must be accessed on-site with previous formal request for the use of this information.

The change in the determination of the quota is so remarkable that it can be seen as the introduction of an entirely new regulation. The regulation in 2002 considered skilled workers which were mainly non-production workers, therefore the policy applied for the most part to larger firms only. Large firms are more likely the ones that have a large enough number of non-production workers to be subject to the regulation. However, the reform that took place starting in 2003 implied that the apprenticeship quota was to be calculated using the total number of directly hired workers, independently of whether they were production or administrative workers[18]. The implication is that

[17] The regulation sets a quota at a rate of 1 apprentice for every 20 workers, but firms between 15 and 29 are required to hire only one apprentice. Further fractions of apprenticeship contracts are rounded up to the nearest integer which implies the existence of many thresholds in Colombia. Given that the model studies regulations with only one threshold I focus on the first threshold of compliance, meaning firms which are required to hire just one apprentice. The analysis of heterogeneous effects of the regulation for the other thresholds is left as part of a broader research agenda.

[18] Messengers, security guards, drivers and janitors are not considered to calculate the quota.

a larger number of firms are now subject to the regulation than before 2003, particularly those from the lower tail of the firm size distribution, which was reflected in the number of apprenticeship contracts that were signed per year between 2002 and 2003. Saavedra & Medina (2012) report an increase from 33.3 thousand to 72.1 thousand apprenticeship contracts using SENA administrative data.

From section 2 we learned that the introduction of a size-dependent regulation, as the one considered here, relative to a distortion-free world should: 1-Reduce direct, outsourced and total labor demand by constrained and distorted firms and 2-Increase the ratio of outsourced to direct labor for constrained and distorted firms. To empirically test these two implications, I estimate equation (19), which exploits the exogenous variation in the policy between 2002 and 2003 to capture the effects of the policy introduction in labor demand using a fixed effects estimation.

To link the model's implication with the empirical test, it is necessary to accommodate the fact that the model implications are defined in terms of idiosyncratic productivity and this variable is not directly observable, but can be estimated. In the theoretical framework developed, in absence of distortions, there is a one to one relationship between idiosyncratic productivity and labor demand. Therefore, with information from a distortion free period a mapping from idiosyncratic productivity to direct labor demand can be made.

$$y_{it} = \alpha_i + \beta_1 T_t + \beta_2 D_{2i} * T_t + \beta_3 D_{3i} * T_t + \beta_4 w_{it} + \beta_5 s_{it} + \beta_4 R_{it} + \beta_5 TFP_{it} + \mu_{it} \quad \textbf{(19)}$$

In this specification, $y_{it} = n_o, n_d, n_T, R$, where these variables are, respectively, outsourced labor, direct labor, total employment and the ratio of outsourced to direct labor. $T_t = 0$ if t=2002 and $T_t = 1$ if t=2003. w_{it} is the corresponding average payroll for each labor demand, where for total employment the total average payroll is used. s_{it} is the inverse share of non-production workers which is a proxy for managerial ability[19]. R_{it} is the ratio of both labor demands and TFP_{it} is total factor productivity for firms which was estimated non parametrically using factor cost shares. The treatment dummies are defined in the year 2002 as follows: $D_{1i} = 1[\, n_d \leq 13]$, $D_{2i} = 1[n_d = 14]$, $D_{3i} = 1[n_d \geq 15]$. These dummies characterize each of the three types of firms from the model in terms of direct labor demand. D_{1i} is the omitted category since this group should not change its labor demands as a response to the introduction of the policy. These time invariant dummy variables are not part of equation (19) since they would be dropped in the fixed effects transformation. The parameters of interest are β_2 and β_3 which capture the response to policy of constrained and distorted firms, relative to firms that should be unaffected by the policy. The inclusion of s_{it}, w_{it} and R_{it} as controls is motivated by total labor demand equation (10), however to check for robustness an estimation with and without these controls was carried out[20].

Parameters β_2 and $\beta_3$are intent to treat estimators since treatment is defined in 2002 before the policy reform took place. However, if treatment were defined in 2003, when firms have had time to

[19] A lower share of administrative workers denotes a higher ability to manage more workers, a higher span of control. This measure however, is inversely related to firm size, which is why I use its inverse.
[20] When the dependent variable is R_{it} structural controls will not include this variable.

react to policy, it would be endogenous possibly making these estimators biased. It is not straight forward to state the direction of this bias. On the one hand, firms trying to avoid the regulation will demand less direct labor overstating the reduction in direct labor demand, while firms who find the policy to their advantage may increase their direct labor demand biasing the result downward. Finally, to take into account the fact that firms may have experienced idiosyncratic shocks that affected their labor demand between both years, a set of sector indicators[21] interacted with the time dummy will be included. Ideally one would control for individual time-varying effects but this would greatly reduce the degrees of freedom. The sample of plants for the estimation of equation (19) will be those hiring between 10 and 29 directly hired workers.

Results

Table 1 Shows summary statistics for all variables included in the empirical exercise for the years 2002 and 2003.

Table 1 Summary statistics. Dependent and independent variables.

Year	2002		2003	
Variable	Mean	Std. Dev.	Mean	Std. Dev.
Log(Total Employment)	2,857	0,452	2,880	0,453
Log(Directly Hired)	2,789	0,347	2,798	0,337
Log(Outsourced)	0,253	0,858	0,315	0,929
Outsourced/Directly	0,155	0,990	0,179	1,025
1(Nd<=13)	0,325	0,468	0,341	0,474
1(Nd==14)	0,067	0,250	0,069	0,253
1(Nd>=15)	0,608	0,488	0,590	0,492
TFP	3,647	2,320	3,716	2,368
Span of control	4,899	4,072	4,951	4,138
Payroll per worker	5472,6	2793,2	5717,0	3250,9
Observations	2416		2595	

Source: Own calculations using Colombia's Annual Manufacturing Survey. Table shows summary statistics for firms hiring between 10 and 29 directly hired workers in each year.

Table 2 shows the results of estimating equation (19). Panel A shows the effect of introducing the regulation on the total labor demand of distorted and constrained firms. The effect for distorted firms is robust to the inclusion of controls and shows reductions in their total labor demand between 8.3 and 9 log points as a result of the regulation. Constrained firms also showed a reduction in total labor demand, but of a smaller magnitude, between 4.4 and 5.9 log points. Panel B shows the effect on direct labor demand for both firms. The magnitude of these effects are similar to those of total labor demand. Panel C shows the effects of regulation on outsourced labor demand[22]. Results show negative effects for constrained firms and positive effects for distorted firms. The fact that the

[21] I thank Gabriel Ulyssea for suggesting the analysis of heterogeneous effects by industry sector and worker's skills.

[22] Outsourced labor displays a high number of zero values in the data which become missing when log transformed. For this reason, the variable was modified by adding 1 and should be interpreted as log(Outsourced +1) number of workers.

response in outsourced labor is lower than the response in direct labor demand is consistent with the model and may suggest that these firms increased the ratio of outsourced to direct labor. The increase in outsourced labor by distorted firms is consistent with a labor substitution effect due to the policy. Distorted firms increased outsourced labor by about 4.5 log points. This effect is not robust however to the inclusion of structural controls. The increase in labor demand of outsourced labor by distorted firms was unexpected as the theory developed points to reductions in labor demand. Possible explanations are general equilibrium effects that reduced the cost of outsourced labor perhaps due to a higher labor supply of this labor type.

As a robustness check to the results in equation 19, Table 4 in the appendix shows the result of estimating a single treatment where now constrained and distorted firms are part of the treatment group and the control group are firms with 13 or less workers. These results confirm that most effects are driven by distorted firms which make up the largest share of firms affected by the policy.

Table 2. Effects of apprenticeship contract regulation on labor demand

VARIABLES	(1)	(2)	(3)	(4)
	Panel A: Log(Total Employment)			
T	0.059***	0.057***	-0.344***	-0.028
	[0.008]	[0.007]	[0.010]	[0.031]
1.D_2#1.T	-0.044**	-0.047***	-0.054***	-0.059***
	[0.018]	[0.016]	[0.017]	[0.016]
1.D_3#1.T	-0.083***	-0.087***	-0.086***	-0.090***
	[0.010]	[0.009]	[0.010]	[0.010]
Constant	2.864***	2.878***	2.869***	2.884***
	[0.017]	[0.029]	[0.016]	[0.029]
Observations	4,316	4,142	4,270	4,098
Number of nordest	2,459	2,372	2,431	2,345
Structural controls	NO	YES	NO	YES
Sector controls	NO	NO	YES	YES
model	fe	fe	fe	fe
	Panel B: Log(Directly Hired)			
T	0.055***	0.053***	-0.328***	-0.019
	[0.007]	[0.007]	[0.010]	[0.030]
1.D_2#1.T	-0.040**	-0.042***	-0.051***	-0.054***
	[0.017]	[0.016]	[0.017]	[0.016]
1.D_3#1.T	-0.098***	-0.087***	-0.099***	-0.090***
	[0.009]	[0.009]	[0.009]	[0.009]
Constant	2.785***	2.859***	2.785***	2.863***
	[0.014]	[0.028]	[0.013]	[0.028]
Observations	4,316	4,142	4,270	4,098
Number of nordest	2,459	2,372	2,431	2,345
Structural controls	NO	YES	NO	YES
Sector controls	NO	NO	YES	YES
model	fe	fe	fe	fe
	Panel C: Log(Outsourced)			
T	0.027**	0.027**	-0.050***	-0.027
	[0.013]	[0.011]	[0.019]	[0.049]
1.D_2#1.T	-0.035	-0.037*	-0.030	-0.034*
	[0.023]	[0.020]	[0.024]	[0.021]
1.D_3#1.T	0.045**	0.008	0.044**	0.009
	[0.018]	[0.016]	[0.019]	[0.017]
Constant	0.281***	0.127*	0.293***	0.131*
	[0.061]	[0.069]	[0.060]	[0.070]
Observations	4,316	4,142	4,270	4,098
Number of nordest	2,459	2,372	2,431	2,345
Structural controls	NO	YES	NO	YES
Sector controls	NO	NO	YES	YES
model	fe	fe	fe	fe

Robust standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

Table shows the results of estimating equation 19 for each variable in Panels A through C. Data restricted to the years 2002 and 2003. Structural controls are: Ratio of outsourced to direct labor, inverse share of managers in total employment and average wage bill. Sector controls are interacted with period dummy (T=I(year=2003)). All regressions control for firm TFP which was estimated non parametrically using factor cost shares. The estimation sample includes only firms hiring between 10 and 29 directly hired workers.

Table 3 shows the effect of the regulation on the ratio of outsourced to direct labor demand. Results show that this ratio increased 4 percentage points for distorted firms but was not different from zero for constrained ones. This result provides evidence that the ratio of labor types is higher in the presence of a size-dependent distortion for distorted firms consistent with implication 2.

Table 3 Effect of apprenticeship contract regulation on labor ratio

	(1)	(2)
VARIABLES	No/Nd Ratio	No/Nd Ratio
T	-0.001	-0.045***
	[0.008]	[0.012]
1.D_2#1.T	0.002	0.006
	[0.010]	[0.011]
1.D_3#1.T	0.039***	0.040***
	[0.011]	[0.012]
Constant	0.168***	0.177***
	[0.028]	[0.027]
Observations	4,316	4,270
Number of nordest	2,459	2,431
Structural controls	NO	NO
Sector controls	NO	YES
model	fe	fe

Robust standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

Table shows the results of estimating equation 19 for the ratio of labor demads. Data restricted to the years 2002 and 2003. Sector controls are interacted with period dummy (T=I(year=2003)). All regressions control for firm TFP which was estimated non parametrically using factor cost shares. The estimation sample includes only firms hiring between 10 and 29 directly hired workers.

The identification of the causal effect of the policy on outcomes of interest rests on the common trend hypothesis being true. Figure 8 trough Figure 11 provide graphical evidence of the trends in the average value of labor demands and labor ratios from 1995 to 2011. Only the contrast of outsourced labor demand by constrained firms relative to undistorted ones reflects the two groups as having a common trend previous to the onset of the regulation. For this reason, the estimations in Table 2 and Table 3 should be interpreted as showing correlations that support the model's predictions and not as the causal effect of the policy on firm's labor demands.

5. Concluding remarks

In this paper I studied the role of labor substitution in explaining the observed increases in outsourced (contract) labor and the absence of gaps in the total employment distribution in size-dependent policies models. The model proposed in this paper predicts a reduction in total employment demand and an increase in the share of outsourced labor in total employment by distorted firms due to higher labor costs of direct labor under a size-dependent regulation. Another

prediction is that constrained firms will also increase their share for outsourced labor through labor substitution to legally avoid being subject to the regulation. However, the increase in demand by constrained firms will not be enough to compensate total labor demand and aggregate employment will be lower if wages can't adjust. These responses imply that the share of outsourced labor in total labor demand will be an increasing function of firm's idiosyncratic productivity. Finally, labor substitution is able to generate a distribution of total employment without gaps which is consistent with the empirical data from developed and developing countries. This requires only imperfect substitution of labor types in the production technology.

To discuss the implications of the model for employment I focused on the case when wages are downward rigid. I derived an expression to measure employment losses under wage rigidity relative to the undistorted economy. These losses are particularly important for economies where minimum wages are binding and the margin of adjustment is the extensive one. Therefore, the availability of flexible forms of contracting can help reduce employment losses when firms are faced with size-dependent distortions.

Estimating "intent to treat" difference in difference and regression discontinuity designs models I tested three model predictions using Colombian manufacturing data taking advantage of the regulation design and a natural experiment in 2002. The difference in difference estimation showed that distorted and constrained firms reduced total labor demand in line with the model's prediction. Constrained firms reduced their demand for outsourced labor, while distorted firms increased it, which was unexpected and may point to the existence of general equilibrium effects regarding the cost of outsourced labor. I provided evidence of firms switching treatment status in a non-trivial manner, as some firms reduced the demand of direct labor while others increased it. To get around the identification problems that treatment switchers pose for identification of treatment effects I estimate the model using treatment status in the year 2002, before the regulation took place.

From a policy perspective this paper's results shows that the design of size-dependent policies must consider that firms will react to the regulation in perhaps non-intendent ways. In the case considered for the empirical analysis, the apprenticeship contract reduced total labor demand in the manufacturing Colombian sector. Further the policy induced substitution from direct contracts to outsourcing, which may be an inferior form of contract from the worker's perspective and may reduce the incentives to invest in human capital as outsourced contracts are in general fixed-term contracts. The substitution towards outsourced labor may not only generate fiscal problems as employers attempt to avoid their obligations by turning to this contract types, but may also increase labor market dualism. Since recent discussions about a reform to the apprenticeship contract regulation have risen this paper's result provide a starting point to understanding the effects on firm's labor demand of size-dependent policies and how it can affect firm's performance and the composition of labor types in the economy.

As questions arising for future research, understanding the dynamic implications of labor substitution on growth, including the accumulation of capital, investments on managerial and worker ability is a promising research agenda. Also, the substitution of labor for capital can capture

other mechanisms that firms could use as a response to size-dependent taxes on labor that can affect productivity and growth. These are issues that now constitute additional research questions worth exploring in my research agenda.

6. References

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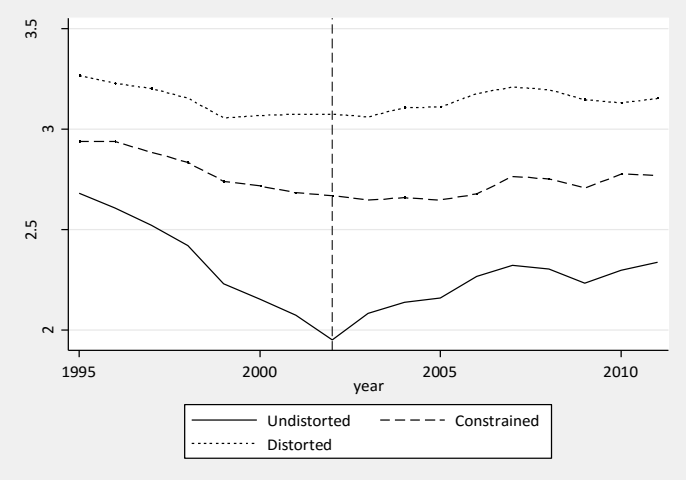
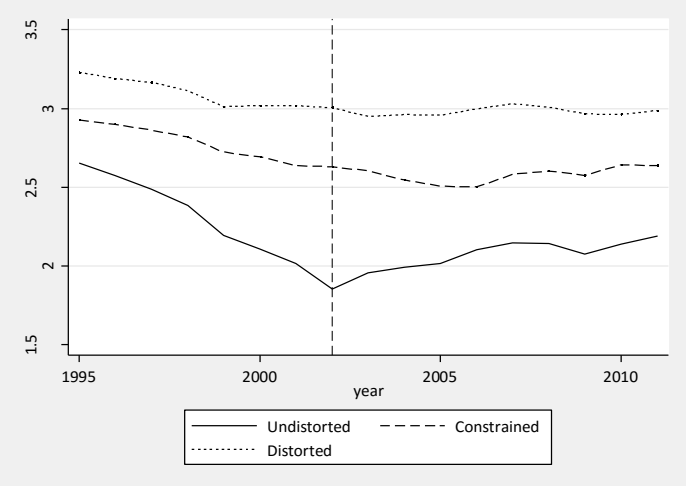
Appendix

Figure 8. Common trends test. Total employment.



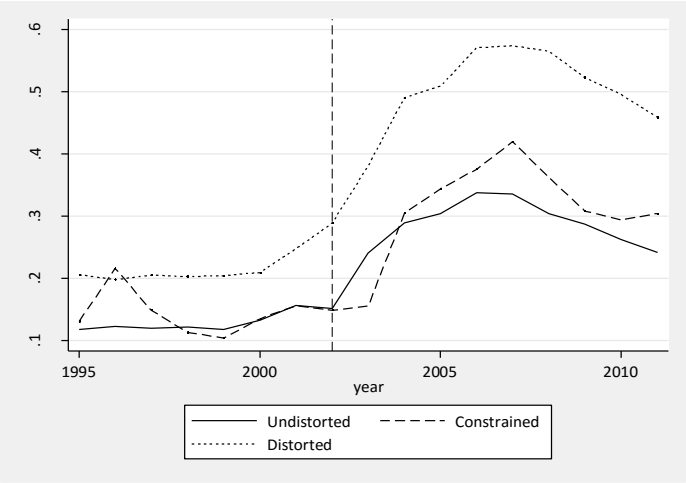
Graph plots the average total labor demand from 1995 to 2011 for the three groups of firms according to their 2002 labor demand.

Figure 9. Common trends test direct labor demand.



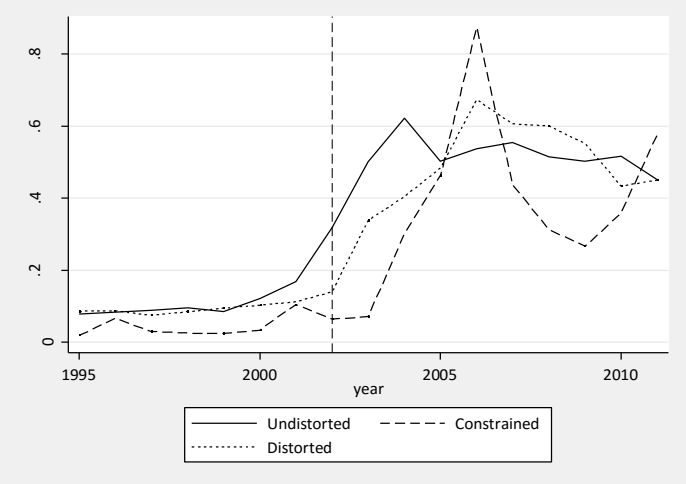
Graph plots the average direct labor demand from 1995 to 2011 for the three groups of firms according to their 2002 labor demand.

Figure 10. Common trends test. Outsourced labor demand.



Graph plots the average outsourced labor demand from 1995 to 2011 for the three groups of firms according to their 2002 labor demand.

Figure 11. Common trends test. Ratio of outsourced to direct labor demand.



Graph plots the average ratio of labor types from 1995 to 2011 for the three groups of firms according to their 2002 labor demand.

Table 4 Effect of policy on labor demand.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
VARIABLES	Log(Total Employment)				Log(Directly Hired)				Log(Outsourced labor)			
T	0.059***	0.057***	0.002	-0.032	0.055***	0.053***	0.000	-0.023	0.027**	0.027**	0.005	-0.022
	[0.008]	[0.007]	[0.002]	[0.031]	[0.007]	[0.007]	[0.001]	[0.030]	[0.013]	[0.011]	[0.007]	[0.048]
1.D14#1.T	-0.079***	-0.083***	-0.082***	-0.086***	-0.092***	-0.082***	-0.094***	-0.086***	0.037**	0.003	0.037**	0.004
	[0.010]	[0.009]	[0.010]	[0.009]	[0.009]	[0.009]	[0.009]	[0.009]	[0.017]	[0.015]	[0.018]	[0.016]
Constant	2.883***	2.896***	2.889***	2.903***	2.798***	2.876***	2.798***	2.880***	0.307***	0.139**	0.319***	0.143**
	[0.017]	[0.030]	[0.016]	[0.030]	[0.014]	[0.028]	[0.013]	[0.028]	[0.060]	[0.071]	[0.060]	[0.071]
Observations	4,499	4,314	4,453	4,270	4,499	4,314	4,453	4,270	4,499	4,314	4,453	4,270
Number of nordest	2,642	2,544	2,614	2,517	2,642	2,544	2,614	2,517	2,642	2,544	2,614	2,517
Structural controls	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES	NO	YES
Sector controls	NO	NO	YES	YES	NO	NO	YES	YES	NO	NO	YES	YES
model	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe	fe

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Treatment is defined as having 14 or more workers, which includes distorted and constrained firms.