

Dynamic Moral Hazard, Risk-Shifting, and Optimal Capital Structure*

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Abstract

I develop an analytically tractable model that integrates the risk-shifting problem between bondholders and shareholders with the moral hazard problem between shareholders and the manager. The presence of managerial moral hazard exacerbates the risk-shifting problem. An optimal contract binds shareholders and the manager. The flexibility of this contract allows shareholders to relax the incentive constraint of the manager when a good profitability shock is drawn. Hence, the optimal contract amplifies the upside thereby increasing shareholder appetite for risk-shifting. Moreover, some empirical studies find a positive relation between risk-shifting and leverage, while others studies find a negative relation. The model predicts a non-monotonic relation between risk-shifting and leverage and has the potential to reconcile this empirical evidence. Implications for capital structure, business cycles and executive compensation are also considered.

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Keywords: risk-shifting, moral hazard, principal-agent problem, capital structure.

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1 Introduction

Shareholders may have incentives to undertake risky projects with negative net present value because they benefit from a positive outcome if things go well, leaving bondholders to face the losses if things go poorly. This is the risk-shifting problem studied in Jensen and Meckling (1976). At the same time, managers have the incentive to shirk (moral hazard) when their effort is not observable. These two problems have been studied separately. In this paper, I construct a model in which I jointly model the risk-shifting problem between shareholders and bondholders, and the moral hazard problem between shareholders and the manager.

Using this unified framework I explore the following questions: i) How the presence of managerial moral hazard affects the risk-shifting problem, and ii) How optimal leverage and managerial compensation change when the two problems are considered jointly.

I consider three types of players in the model: shareholders, bondholders, and a manager (agent), all of which are risk-neutral. The manager is impatient compared to the shareholders and the bondholders. The cash-flows of the project depend on the manager's effort and the profitability of the project. The profitability of the project is time varying, random, and its variance depends on the amount of risk-shifting chosen. The amount of risk-shifting is observable and shareholders specify it in the manager's contract.

Once debt is in place, shareholders have an incentive to increase the riskiness of the firm's cash-flows because of limited liability. If things don't go well shareholders exercise their option to default and walk away from their debt obligations. Bondholders have to face the costs of bankruptcy. Moreover, they have rational expectations and correctly anticipate the instances in which the shareholders will default. Therefore, the optimal capital structure of the firm will trade-off the tax advantage of debt with the costs of bankruptcy that results from risk-shifting.

At the same time, shareholders need to compensate the manager for her work. In the case in which effort is observable, shareholders pay the manager her outside option immediately (since the manager is impatient), and she exerts effort until liquidation. I solve this model in closed-form and obtain the optimal amount of risk-shifting, total firm value, and leverage. This will be the benchmark model without moral hazard which will serve as a reference point of comparison once I introduce the agency problem.

When effort is not observable, shareholders need to provide incentives for the manager to work. Thus, shareholders design a contract that specifies required effort, deferred compensation, amount of risk-shifting, and termination as a function of the observed history of

output. The firm's output history determines the manager's current expected utility, which I refer to as the manager's continuation value. The continuation value of the manager and the profitability of the project are the two state variables which encode the contract-relevant history of the firm.

The contract exhibits deferred compensation, in which the manager is only paid after a sufficiently good history of output. Deferred compensation optimally trades off the cost of delaying payments to an impatient manager with the benefit of postponing her compensation, thereby reducing the probability of costly termination of the contract. The contract is terminated after the firm experiences low cashflows and the continuation value of the manager hits zero. At this point it is not possible to provide incentives for the manager to work, and it is necessary to replace the manager. Since replacing the manager is costly, it is natural to interpret the continuation value of the manager as a proxy for financial slack. Moreover, because equity is more sensitive to financial distress than debt, financial distress and leverage move in the same direction.

The model yields the following results. First, the optimal amount of risk-shifting in the presence of managerial moral hazard is larger than in the benchmark case without moral hazard. There are two effects causing this result, which I call: *i) leverage effect* and *ii) internal hedging effect*. The leverage effect states that highly levered firms (i.e. firms closer to default) have a greater incentive to engage in risk-shifting activities. Since equity can be viewed as a call option on the firm's assets with strike price zero, firms that are closer to default benefit more from the convexity of the call option if the risk increases. Thus, holding the amount of debt constant, moral hazard creates a deadweight loss which reduces the total value of the firm. Consequently, leverage and risk-shifting increase.

The internal hedging effect emerges as a result of the optimal contract's adjustments to the manager's continuation value in response to the realized profitability. Intuitively, the optimal contract attempts to minimize the probability of liquidation when the firm has a high profitability. Therefore, the continuation value of the manager increases when the firm draws a high profitability, thereby relaxing the incentive constraint. Conversely, the continuation value decreases when a low profitability is drawn. When a low profitability is drawn there is no need for the firm to have an incentivized manager, since the firm finds it optimal to default. Hence, the internal hedging effect is a result of the dynamic nature of the optimal contract that allows shareholders to relax the incentive constraint precisely when they need it the most. Consequently, the benefits from the upside are amplified and shareholders find it desirable to engage in more risky activities.

Second, the model predicts a non-monotonic relation between risk-shifting and leverage.

This result has the potential to reconcile seemingly contradictory empirical evidence relating risk-shifting and financial distress. Eisdorfer (2008) shows that 1) financially distressed firms increase their investment in response to a raise in uncertainty, 2) the investment undertaken by financially distressed firms has negative NPV. Together, he interprets these findings as evidence of a positive relation between risk-shifting and financial distress. In contrast, Rauh (2009) compares the asset allocation of pension funds across firms. He finds that firms with poorly funded pension plans and low credit ratings invest a greater share of their portfolios in safer securities such as government bonds and cash, while firms with well-funded plans and high credit ratings allocate a larger proportion to riskier assets such as stocks. Therefore, risk-shifting seems to be negatively related to financial distress.

In this model risk-shifting is initially increasing in leverage as documented by Eisdorfer (2008), but decreases for high levels of leverage as in Rauh (2009). Intuitively, when the continuation value of the agent is low, there is little room to punish the agent in response to a bad realization. This limits the scope to compensate the agent in response to a good realization.¹ As a consequence, the internal hedging effect is weak when the continuation value of the manager is low, making risk-shifting lower. Similarly, when the continuation value of the agent is large, there is no need to provide rewards in response to a good shock, since the probability of inefficient liquidation is already low. Hence, the internal hedging effect is not active for large values of the continuation value. However, for intermediate values of the continuation value, the contract inflicts large punishments to the manager in bad states, and provides high rewards for the manager in good states. The internal hedging effect is stronger for firms with intermediate levels of financial distress. Hence, this mechanism induces a non-monotonic relation between risk-shifting and financial distress. This model's prediction implies that in the presence of managerial moral hazard standard linear models relating risk-shifting to measures of financial distress are misspecified, and a non-linear relation should be estimated instead.

Third, firms in which there is greater concern for moral hazard issue less debt, and choose lower levels of leverage. As discussed above, moral hazard increases risk-shifting thereby increasing the expected costs of bankruptcy. In anticipation of these bankruptcy costs bondholders reduce their demand for debt, making debt financing more costly for the firm. Thus, the firm finds it optimal to reduce the risk-shifting incentives of the shareholders by issuing less debt, thereby lowering initial leverage.

Fourth, the model illustrates a potential amplification mechanism of output shocks via

¹In order to satisfy the promise keeping constraint, rewards in some states have to be balanced out with punishments in other states.

counter-cyclical risk-shifting. Since the optimal contract exhibits deferred compensation for the manager, her continuation value on average tends to increase. This brings the firm away from financial distress. When firms are not financially distressed leverage and risk-shifting are low. However, a sufficiently bad sequence of output shocks erodes the continuation value of the manager and brings the firm into financial distress. As a consequence, the firm increases its amount of risk shifting, making the probability of filing for bankruptcy all the more likely. Thus, the initial negative shock is amplified by the aggregate deadweight cost of bankruptcy.

Finally, the model implies a compensation structure for the manager in which she is rewarded for events outside of her control. While the manager has no control over the outcome resulting from the risk-shifting action, her continuation value is increased in response to a good draw, and decreased in response to a bad one. In this sense, my model is consistent with the empirical evidence of Bertrand and Mullainathan (2001) documenting that managerial wealth responds to “lucky shocks” such as increments in oil prices, or changes in exchange rates. The model implies that, on average, the manager will be rewarded in response to shocks after a bad recent history of output, and punished after a good recent history of output. Thus, the model implies rewarding managers for luck in periods of financial distress such as recessions. I show that this “reward for luck” exhibited by the contract is what induces a non-monotonicity between risk-shifting and leverage. In particular, when shareholders are not allowed to reward the manager for luck, the relation between risk-shifting and leverage becomes monotonic.

1.1 Related Literature

This paper belongs to the growing literature on dynamic moral hazard that uses recursive techniques to characterize optimal dynamic contracts. I rely on the martingale techniques developed in Sannikov (2008) to deal with the principal-agent problem in a continuous time environment in which output follows a diffusion process, and on the extension to point processes developed by Piskorski and Tchisty (2010) in the context of optimal mortgage design. This paper is most closely related to the seminal work of DeMarzo and Sannikov (2006) and Bias, Mariotti, Plantin and Rochet (2007) in which the agent is risk-neutral but has limited liability. The main contribution of this paper is to integrate the risk-shifting problem into their framework and use it to explore the interaction between moral hazard and risk-shifting, the implications for capital structure, executive compensation, and business cycle amplification.

DeMarzo, Fishman, He and Wang (2010) and Bias, Mariotti, Rochet, and Villanueva (2010) embed investment with adjustment costs into the principal-agent problem. They find

that financially constrained firms have lower investment rates, and that investment is below the first best benchmark when moral hazard is absent. The key difference with my paper is that they consider risk-less investment while I focus on risky investments which can be desirable for shareholders protected by limited liability in the presence of debt commitments. He (2011) studies the optimal capital structure of the firm when the manager influences the growth rate of the firm. He find that the debt-overhang effect on the managerial incentives lowers optimal leverage.

In the context of dynamic models of risk-shifting, Leland (1998) finds that the costs of the risk-shifting problem are small when compared to the tax advantage of debt, and should not affect the leverage choice significantly. Ericsson (2000) reaches the opposite conclusion and shows that risk-shifting can lower the firm's optimal leverage up to 20%. Both of these papers suppose that managers behave in shareholder's interest hence assuming away the moral hazard problem.

This paper is also related to the literature that studies how managerial compensation can mitigate the risk-shifting problem. John and John (1993) in a three period model show that reducing the pay-to-shareholder wealth sensitivity of the manager in response to higher debt can help her internalize the cost of bankruptcy, thus reducing the incentive to take risks. Subramanian (2003) in the context of Leland (1998) shows that the managers optimal compensation is proportional to the firm's cash-flows, but subject to a ceiling and a floor.

Finally, the model is consistent with the empirical findings of Eisdorfer (2008), and Panousi and Papanikolaou (2012) who show that firms in which the interests of shareholders and managers are more closely aligned engage in more risk-shifting. In particular, Eisdorfer (2008) finds that firms in which managers hold a greater share of the firm's total equity engage in more risky investment. Panousi and Papanikolaou (2012) show that during the great recession investment declined significantly as a result of the rise in uncertainty. However, they showed that firms in which managers are compensated with options the reduction in investment was substantially smaller.

The paper is organized as follows. Section 2 presents the model. Section 3 formulates our benchmark case in the absence of moral hazard. Section 4 explores the moral hazard case, and characterizes the optimal contract. Section 5 presents the results of the model, and compares the results to the benchmark case without moral hazard. Section 6 concludes. Proofs are relegated to the appendices.

2 The Model

In this section I lay out the model. I first present the players preferences, the timing of events, and the firm's technology. Then, I introduce the risk-shifting problem between bondholders and shareholders. Finally, I describe the moral hazard problem between shareholders and the manager and formulate the optimal contract.

2.1 Preferences, Timing, and Technology

Time is continuous and infinite. There are three types of players: bondholders, shareholders and a manager (agent). Everyone is risk neutral and has rational expectations about the future. Bondholder and shareholders discount the future at rate r , while the manager is more impatient and discounts the future at rate $\gamma > r$.

The initial shareholders of the firm have access to a project with a stream of cumulative cash-flows Y_t that evolves according to:

$$dY_t = a_t \mu_t dt + \sigma dB_t, \quad (2.1)$$

where $a_t \in \{0, 1\}$ denotes the amount of effort that the manager exerts, μ_t is the profitability of the firm, B_t is a standard brownian motion process with respect to the filtration Σ_t , and σ is the volatility. I interpret μ_t as the underlying profitability of the project, which has initial value μ_0 . Importantly, the profitability μ_t is time varying. The manager can choose from a continuum of risky investments $i \in [0, I]$. By selecting investment i the profitability is subject to a Poisson shock with arrival rate α_i . Upon arrival of the shock, the profitability of the project will jump to a new value that is drawn independently from a uniform distribution with support $[\mu_0 - \delta, \mu_0 + \delta]$ where $\mu_0 - \delta > 0$. Moreover, the choice of i implies a flow cost $c(\alpha_i)dt$ which satisfies $c'(\alpha) > 0$ and $c''(\alpha) > 0$. The idea is that shareholders who want to engage in risk-shifting will have to choose investments with a lower return.² In other words, by assuming a negative relationship between the investment's riskiness and its net present value I get rid of risk-return tradeoff. Thus, I focus exclusively on the risk-shifting motive as the sole driver of investment choices. Formally, μ_t satisfies:

$$d\mu_t = (\hat{\mu} - \mu_0)dJ_t, \quad (2.2)$$

²Even though managers are the ones who ultimately choose the amount of risk-shifting α , because this action is observable and contractible, it is equivalent to think of α as being chosen directly by the shareholders.

where $\hat{\mu} \sim U[\mu_0 - \delta, \mu_0 + \delta]$, $J = \{J_t, F_t; 0 \leq t < \tau_S\}$ is a standard compound Poisson process with intensity α_t , and τ_S denotes the arrival time of the first (and only) Poisson shock.

At time $t = 0$ the initial shareholders choose the amount of debt issuance. Debt is issued once and for all at time 0. I assume that debt takes the form of a perpetuity that makes coupon payments C per period and pays $(1 - \phi)\mu/r$ upon the firm's default, where μ/r is the first best value of the unlevered firm. I interpret ϕ as the fraction of firm value that is lost as a result of bankruptcy. I assume that debt is subject to a tax-shield ψ . Thus, the optimal amount of debt will have to trade-off costly bankruptcy with the tax advantage of debt. Once debt is in place, the firm is entirely controlled by the remaining shareholders.

The bondholders purchase this debt at fair value. The shareholders have limited liability and default once the value of the firm is equal to zero. Once debt is in place shareholders do not internalize the cost of bankruptcy imposed on the bondholders. Hence, bondholders anticipate the instances in which the shareholders will endogenously default, and will price in these expectations in their demand for debt. Thus, the value of debt D_0 will be given by

$$D_0 = E \left[\int_0^\tau e^{-rt} C dt + e^{-r\tau} (1 - \phi) \frac{\mu_\tau}{r} \right], \quad (2.3)$$

where τ is the endogenous time of default chosen by the shareholders.

2.2 The Risk-Shifting Problem

Shareholders value the stochastic cumulative cash-flows from the firm net of coupon payments and payments to the manager:

$$E \left\{ \int_0^{\min\{\tau_S, \tau_T\}} e^{-rt} (dY_t - c(\alpha)dt - (1 - \psi)C_t dt - dP_t) + 1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S} \int_{\tau_S}^{\tau_T} e^{-r(t-\tau_S)} (dY_t - (1 - \psi)C_t dt - dP_t) + e^{-r\tau_T} \bar{F}(W_{\tau_T}) \right\},$$

where P_t are the cumulative payments made to the manager, $\bar{F}(W_{\tau_T})$ represents the payoff the shareholders will receive upon termination of the contract,³ and $c(\alpha)$ reflects the decreasing returns of engaging in more risky investments. Once debt is in place shareholders cannot

³In particular, if the manager is not replaced the shareholders will default and get zero. However, if the manager is replaced $\bar{F}(W_{\tau_S})$ will represent the expected profits from the new contract net of the cost of replacing the manager.

commit to internalize the cost of bankruptcy incurred by the bondholders. Thus, because shareholders have limited liability, they have an incentive to choose risky investments. A larger value of α implies that the shock will occur earlier. Since the new profitability is drawn from a mean-preserving distribution, a higher arrival rate α implies a higher variance of future cash-flows.⁴

The managers will be instructed by the shareholders to optimally choose the amount of risk-shifting as to benefit from the option to default when the realized cash-flows are low.⁵ Because of this conflict of interests between bondholders and shareholders, at time 0 the optimal capital structure will have to trade-off the tax advantage of debt with the expected costs of bankruptcy resulting from the risk-shifting behavior of the shareholders.

2.3 The Moral Hazard Problem

In this section I introduce an agency conflict resulting from the unobservability of managerial effort. Recall from (2.1) how manager's effort a_t influences cash-flows Y_t . However, the amount of effort the manager exerts is her private information, and shareholders need to infer effort from the realized path of cash-flows. Moreover, when the manager exerts effort $a_t \in \{0, 1\}$ she enjoys private benefits at the rate $\lambda(1 - a_t)\mu_t$ where $0 \leq \lambda \leq 1$. I say that the manager *works* if $a_t = 1$ and *shirks* if $a_t = 0$. Alternatively, $1 - a_t$ can be interpreted as the fraction of cash that is diverted by the manager for her private benefit, with $1 - \lambda$ being the the fraction lost by the diversion. In either case, λ captures the magnitude of the agency problem, and it pins down the incentives required to motivate the manager to work. I also assume that the manager controls the amount of risk-shifting α . However, the amount of risk-shifting is observable, and that it is costless for the manager to choose an arbitrary α . While the effort the manager exerts is not observable, it is realistic to assume that the type of investment chosen is public information. For example, it is public information whether a pharmaceutical company has decided to open a new R&D laboratory, or if a clothing retailer is launching a new product line.

I assume that the firm's cash-flows Y_t , the profitability μ_t , and the amount of risk shifting α_t are observable and contractible. The shareholders design a contract (α, P, τ_T) that specifies the firm's investment choice α , the cumulative compensation to the manager P , and the termination of the contract τ_T ,⁶ all of which depend on the realized history of output Y_t , and

⁴When $\hat{\mu}$ is drawn from a normal distribution this property can be shown in closed-form solution. In the case of the uniform distribution I use numerical simulations to verify this intuition.

⁵Through out the paper I assume that managers are only responsible to shareholders (Allen, Brealey, and Myers (2006))

⁶When the manager is not replaced termination is equivalent to liquidation $\tau_T = \tau$.

of the profitability μ_t . Limited liability by the manager requires that $dP_t \geq 0$. Moreover, if the manager's saving interest rate is lower than the principal's discount rate DeMarzo and Sannikov (2006) show that there is an optimal zero savings contract. Under this condition, it is without loss of generality that I equate the manager's cumulative consumption with P_t . Henceforth, denote an arbitrary contract by $\Gamma = \Gamma(\alpha, P, \tau_T)$ and relegate further regularity conditions to the Appendix. I assume the shareholders and the manager can commit to such a contract. Moreover, I assume that the manager can be replaced and that the cost of replacing the manager is a linear function of the project's profitability i.e. $M = \kappa\mu$.

Fix an arbitrary contract Γ , the manager chooses an effort process a as to maximize her expected utility at time $t = 0$:

$$W(\Gamma) = \max_{a \in A} E^a \left\{ \int_0^{\tau_T} e^{-\gamma t} (\mu_t \lambda (1 - a_t) dt + dP_t) + e^{-\gamma \tau_T} R \right\},$$

where $A = \{a_t \in \{0, 1\} : 0 \leq t < \tau\}$ is the set of effort process that are measurable with respect to F_t , and the manager receives utility R from her outside option if the contract is terminated, irrespective of whether the shock takes place or not. For simplicity, I assume that the outside option of the manager $R = 0$ for the rest of the paper.⁷ For the remainder of the paper, I focus on the case in which it is optimal for shareholders to make the manager work $a_t = 1$ at all times. Intuitively this is true when the private benefit the manager derives from shirking is small compared to the gain shareholders derive from a manager that works. In propositions (3) and (7) below I provide a sufficient condition for the optimality of implementing work. For the remaining of the paper I use the expectations operator $E(\cdot)$ to denote the expectation induced under $a_t = 1$ at all times. I say that a contract $\Gamma = \Gamma(\alpha, P, \tau_T)$ is *incentive compatible* if the manager's expected utility is maximized by choosing work.

The shareholders problem (upon debt issuance) is to solve the following maximization problem:

$$\begin{aligned} \max_{\Gamma} E \left\{ \int_0^{\min\{\tau_S, \tau_T\}} e^{-rt} (dY_t - c(\alpha)dt - (1 - \psi)C_t dt - dP_t) \right. \\ \left. + 1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S} \int_{\tau_S}^{\tau_T} e^{-r(t-\tau_S)} (dY_t - (1 - \psi)C_t dt - dP_t) + e^{-r\tau_T} \bar{F}(W_{\tau_T}) \right\}, \end{aligned} \quad (2.4)$$

⁷Relaxing this assumption is straight forward but does not contribute much to the analysis.

s.t Γ is incentive compatible and $W(\Gamma) = W_0 \geq 0$,

where W_0 is the initial expected utility to the manager.⁸ Shareholders maximize the expected present value of firm's cash-flows net of the flow cost associated with the risky investment, the coupon payments made to the bondholders, and the payments made to the manager. For simplicity, shareholders have the full bargaining power when choosing the initial expected utility of the manager W_0 . Shareholders will choose $W_0 \geq 0$ as to maximize their expected profits.

3 Solution without Moral Hazard

In this section I solve the case in which the manager's choice of effort is observable by the shareholders. This case will serve as a benchmark of the risk-shifting problem in the absence of moral hazard. First, I solve for the equilibrium outcomes after the shock. Then, I solve the problem prior to the shock, and characterize the optimal amount of risk-shifting in the absence of moral hazard. Finally, I solve the initial shareholders problem and solve for the optimal capital structure at time $t = 0$.

3.1 Solution After the Shock

Assume for the moment that debt with coupon payment C is already in place. Later I will find the optimal coupon payment chosen at the initial time. By assumption, the outside option of the manager is 0. Since there is no moral hazard it is optimal for the shareholders to pay nothing to the manager and implement effort $a_t = 1$ at all times. I denote with a hat the quantities after the shock. Recall that for simplicity I assume that once the shock occurs, the profitability μ stays permanently at that value. Thus, after that there is no longer a risk-shifting problem. Let $\hat{F}(\mu)$ be the shareholder's value function when the profitability is μ . $\hat{F}(\mu)$ satisfies:

$$\hat{F}(\mu) = \max \left\{ \frac{\mu - (1 - \psi)C}{r}, 0 \right\}.$$

The shareholders can choose to either receive the stream of cash-flows from the project net of the coupon payments, or default and get 0. Therefore, shareholders will continue to service their debt if their profitability is large enough, i.e. if $\mu \geq (1 - \psi)C$.

⁸For simplicity, I have assumed that the outside option of the manager is 0. It is straight forward to generalize this framework to the case in which the outside option of the manager is strictly positive.

3.2 Solution Before the Shock

Let us now solve the shareholders problem before the shock. The shareholder's value function $F(\mu_0)$ solves:

$$F(\mu_0) = \max_{\alpha} E \left[\int_0^{\tau_S} e^{-rt} (dY_t - (1 - \tau)C dt - c(\alpha) dt) + e^{-r\tau_S} \int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) \right].$$

The shareholders receive the projects cash-flows net of the debt payments and the operating costs of the selected investment until the shock occurs. After the shock, shareholders get the value function averaged out over the possible realizations of the shock, where $dU(\cdot)$ is the density of the uniform distribution with support $[\mu_0 - \delta, \mu_0 + \delta]$. For the remainder of this paper I assume the functional form $c(\alpha) = \frac{\theta\alpha^2}{2}$. The parameter θ describes the rate at which riskier investments become less profitable. Large θ implies that riskier projects have higher operating costs and tend to discourage the shareholders from selecting them.

I solve this problem recursively. The shareholders value function satisfies:

$$rF(\mu_0) = \max_{\alpha} \left\{ \mu_0 - (1 - \psi)C - \frac{1}{2}\theta\alpha^2 + \alpha \left[\int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) - F(\mu_0) \right] \right\}. \quad (3.1)$$

The flow value of equity for the shareholders equals the expected cash-flows from the project net of the coupon payments and the operating cost of investment, plus the expected capital gain upon arrival of the shock. The FOC with respect to the optimal amount of risk-shifting α is:

$$\alpha = \frac{1}{\theta} \left[\int_{\mathbb{R}} (\hat{F}(\hat{\mu}) - F(\mu_0)) dU(\hat{\mu}) \right]. \quad (3.2)$$

The optimal amount of risk-shifting is proportional to the expected capital gain, and is inversely proportional to θ . Plugging back (3.2) in (3.1) I solve for $F(\mu_0)$ in closed form:

$$F(\mu_0) = \int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) + \theta r - \sqrt{\theta^2 r^2 + 2\theta \left(r \int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) - (\mu_0 - (1 - \psi)C) \right)}. \quad (3.3)$$

Plugging this expression back in (3.2) yields:

$$\alpha^{SB} = \frac{1}{\theta} \left[-\theta r + \sqrt{\theta^2 r^2 + 2\theta \left(r \int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) - (\mu_0 - (1 - \psi)C) \right)} \right], \quad (3.4)$$

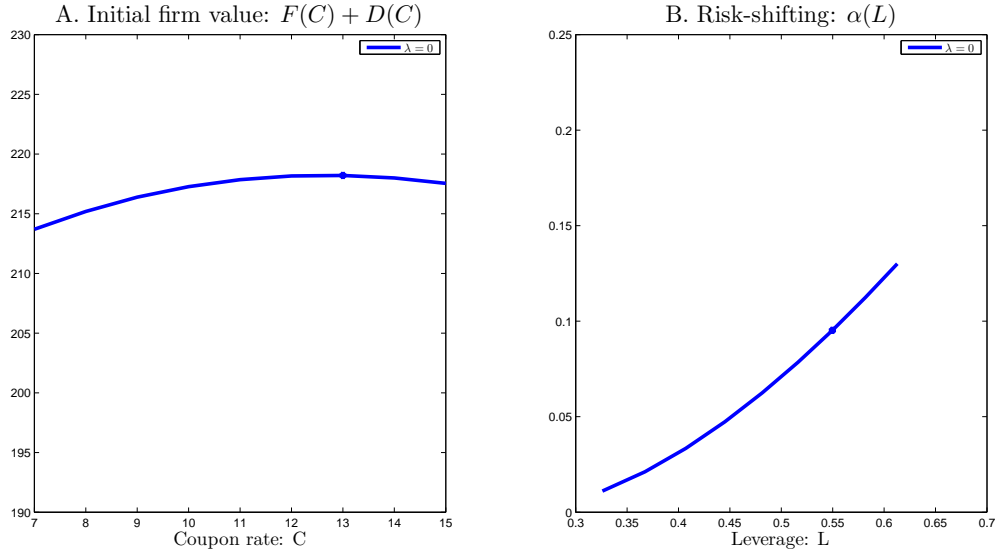


Figure 3.1: **Initial firm value, and risk-shifting.** Panel A plots initial firm value for different choices of the coupon C . Panel B traces the relation between risk-shifting and leverage as the coupon value changes. The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16$

where α^{SB} is the (second best) risk-shifting in the case without moral hazard. In first best shareholders would commit to zero risk-shifting. Since I have assumed that riskier investments have a lower return, there would be no incentive to engage in risk-shifting at all, thus $\alpha^{FB} = 0$.

Panel B in Figure 3.1 traces risk-shifting and leverage for different values of the coupon payment C . A larger coupon payment implies higher leverage, thus making shareholders more exposed to the upside, but with the possibility to default on the downside. That is, because of limited liability their losses are bounded, while their gains are unbounded. Alternatively, we can think that highly leveraged firms want to “gamble for resurrection” as they are closer to default. I refer to the mechanism by which leverage induces higher risk shifting as the *leverage effect*. As it can be seen from panel B of Figure 3.1 the most important observation from the benchmark case without moral hazard is that risk-shifting is governed by the *leverage effect*. In the next section, we will see that the presence of moral hazard creates a dynamic effect that amplifies the leverage effect and induces higher risk-shifting.

3.3 Optimal Capital Structure

In this section, I solve for the optimal coupon payment C . The initial shareholders need

to optimally split the firm into debt and equity as to get the largest possible value from the initial issuance. The value of equity post-issuance is given by (3.3) once the new shareholders take control of the firm. The value of debt satisfies (2.3), which implies that debt is fairly priced. The explicit formulas for the value of debt can be found in Appendix B. Formally the initial shareholder's problem is:

$$\max_C D(\mu_0; C) + F(\mu_0; C),$$

s.t (3.3) and (2.3).

The optimal coupon payment will tradeoff the tax benefit of debt and the cost of bankruptcy. Intuitively, for low values of C the firm can take on more debt as to benefit from the tax advantage of debt. However, as the leverage of the firm increases the expected costs of bankruptcy will pile up, and will balance out the tax benefits of debt. Panel A of Figure 3.1 shows a numerical example for the optimal choice of C .

4 Solution with Moral Hazard

In this section, I assume that the manager's effort is not observable. Therefore, the optimal contract needs to provide incentives for the manager to work. As in the previous section, I start by finding the optimal contract, and the value functions after the shock. Then, I use those expressions to solve the model before the shock. After that, I characterize the optimal amount of risk-shifting in the presence of moral hazard, describing the main features of the optimal contract. Finally, I calculate the optimal capital structure of the firm at time zero.

4.1 Solution After the Shock

Consider the case in which the shock has already taken place. The shareholder's problem consists of finding an optimal contract (P, τ_T) that maximizes shareholder's discounted cash-flows, subject to incentive compatibility and delivering the manager her required payoff W_{τ_S} .⁹ W_{τ_S} is the manager's continuation value immediately after the shock. The contract is incentive compatible if the manager's expected utility from τ_S onward given (P, τ_T) is maximized by choosing $a_t = 1$ at all times.

⁹Recall the after the shock there is no more risk-shifting incentive, thus the contract after τ_S will only specify payments to the manager, and a termination clause.

In order to characterize the optimal contract I write the problem recursively with the continuation value of the manager as the only state variable. For a given contract (P, τ_T) the manager's continuation value W_t given that she will follow effort choice a is given by:

$$W_t = E_t^a \left[\int_t^{\tau_T} e^{-\gamma(s-t)} (\mu_s \lambda (1 - a_s) ds + dP_s) \right]. \quad (4.1)$$

That is, W_t captures the manager's expected utility time t until termination provided that she follows effort choice a . The optimal contract is derived using the techniques developed by the seminal work of Sannikov (2008). Proposition 1 describes the dynamics of W_t and provides necessary and sufficient conditions for the contract to be incentive compatible.

Proposition 1. *Given a contract (P, τ) and an effort choice a there exists sensitivity β_t that is measurable with respect to F_t such that:*

$$dW_t = \gamma W_t - dP_t - \mu_s \lambda (1 - a_s) ds + \beta_t (dY_t - \mu_t dt), \quad (4.2)$$

for every $t \in (\tau_S, \tau_T)$. The contract is incentive compatible if and only if:

$$\beta_t \geq \lambda. \quad (4.3)$$

The first term in the evolution of W_t corresponds to the compensation required by the manager from her time preference. The second and third term correspond to the change in utility induced by the manager's consumption and the disutility of effort. The fourth term captures the sensitivity of the manager's continuation value to the change in output. Exposing the manager to the realizations of output provides her with incentives.

Condition (4.3) states that in order for the manager to work the sensitivity of her continuation value has to be sufficiently large. Intuitively, if the manager deviates and chooses to shirk ($a_t = 0$) for an instant dt , output decreases by μdt . Thus, the manager incurs a loss of $\beta_t \mu_t dt$ and gets private benefit $\lambda \mu_t dt$ by (4.2). Therefore, working is optimal for the manager if and only if

$$\beta_t \mu \geq \lambda \mu \quad \text{or} \quad \beta_t \geq \lambda.$$

Let $\hat{F}(W_t, \mu_t)$ denote the shareholders value function after the shock, when they have drawn profitability μ_t , and the promised utility to the manager is W_t . I suppress the

dependence of the value function on μ_t to ease notation. DeMarzo and Sannikov (2006) show that $\hat{F}(W)$ is strictly concave so that it is optimal to set $\beta = \lambda$. Intuitively, it is not optimal to make the manager bear more risk than the minimal required for her to work. Increasing the volatility of the manager's continuation value will increase the probability of inefficiently liquidating the firm. Moreover, since the shareholders can always make a lump-sum payment to the manager it must be the case that $\hat{F}'(W) \geq -1$ for all W . Let \bar{W} be the lowest value such that $\hat{F}'(\bar{W}) = -1$. \bar{W} will be a reflecting boundary at which $dP_t \geq 0$. Therefore, the manager's continuation value will always be between 0 and \bar{W} . Proposition 2 summarizes the optimal contract after the shock:

Proposition 2. *The shareholder's value function \hat{F} satisfies the following differential equation on the interval $[0, \bar{W}]$:*

$$r\hat{F}(W) = \max_{\beta \geq \lambda} \mu - (1 - \psi)C + \hat{F}'(W)\gamma W + \frac{\hat{F}''(W)}{2}\sigma^2\beta^2, \quad (4.4)$$

with boundary conditions:

$$\hat{F}(0) = \max\{\max_{W_{reset}} \hat{F}(W_{reset}) - M, 0\}, \quad \hat{F}'(\bar{W}) = -1, \quad \hat{F}''(\bar{W}) = 0.$$

When $W_t \in [0, \bar{W})$, the shareholders make no payments to the manager, and only pay her when W_t hits the boundary \bar{W} . The payment dP_t is such that the process W_t reflects on that boundary. If $W_{\tau_S} > \bar{W}$, the shareholders pay $W_{\tau_S} - \bar{W}$ immediately to the manager and the contract continues with the manager's new initial value \bar{W} . Once W_t hits 0 for the first time, the contract is terminated. At this point the shareholders can choose to default and get 0 or hire a new manager and optimally restart the contract. The optimal contract delivers a value of $\hat{F}(W_{\tau_S})$ to the shareholders.

Equation (4.4) says that the flow value of the shareholders value function is equal to the sum of the instantaneous expected cash-flow from the project net of debt payments, plus the capital gain induced by the change in the continuation value of the manager.

It is important to mention that in the cases when cash-flows are not large enough to cover debt payments, shareholders find it optimal to default on their debt obligations immediately. In the case with moral hazard that is equivalent to having the payout boundary equal to zero. More precisely, I can show that when $\mu_{\tau_S} \leq (1 - \psi)C$ the payout boundary \bar{W} equals

0. Thus, shareholders pay the manager her promised value W_{τ_S} and immediately default.

I end this subsection by providing a necessary and sufficient condition for the manager's high effort to be optimal for any $t \in [\tau_S, \tau_T]$. This condition is satisfied in the numerical simulations.

Proposition 3. *Implementing high effort is optimal at any time after the shock $t \in [\tau_S, \tau_T]$ if and only if:*

$$\hat{F}(W) \geq \frac{\gamma}{r} \hat{F}'(W) (W^S - W) + (1 - \psi) \frac{C}{r},$$

for all $W \in [0, \bar{W}]$, where $W^S = \frac{\lambda \mu}{\gamma}$ represents the utility of the manager if she shirks forever.

4.2 Solution Before the Shock

As in the previous section, I assume that debt is already in place. Later, I will find the optimal coupon chosen at the initial time. The contracting problem is to find an incentive compatible contract (α, P, τ_T) that maximizes the shareholder's utility subject to delivering the manager her initial required expected utility W_0 . Recall that prior to the shock the contract specifies a required amount of risk-shifting α , in addition to the manager's consumption P , and the termination of the contract τ_T .

Similar to proposition 1, I obtain the following proposition.

Proposition 4. *Given a contract (α, P, τ_T) and an effort choice a there exist sensitivities β_t and $\{\Delta(W, \hat{\mu})\}$ that are measurable with respect to F_t such that:*

$$dW_t = \gamma W_t - dP_t - \mu_t \lambda (1 - a_t) dt + \beta_t (dY_t - \mu_t dt) + \Delta(W_t, \hat{\mu}) dJ_t + \rho_t dt, \quad (4.5)$$

for every $t \in (0, \min\{\tau_S, \tau_T\})$, where $E_t[\Delta(W_t, \hat{\mu}) dJ_t] = \int_{\mathbb{R}} \alpha_t \Delta(W_t, \hat{\mu}) dU(\hat{\mu}) dt = -\rho_t dt$. Moreover, the contract is incentive compatible if and only if:

$$\beta_t \geq \lambda.$$

The first four terms are the same as in Proposition 1. The fifth term is new and captures the manager's exposure to the realized value of the shock. Once the Poisson shock occurs, if the realized value of profitability is $\mu_{t+} = \hat{\mu}$ the continuation value of the manager will be adjusted by an amount $\Delta(W, \hat{\mu})$. Because the contract specifies payments that are contingent

on the new profitability drawn, the adjustment to the continuation value is different for each realization of the draw. The final term $\rho_t dt$ is a compensating trend required to deliver the manager her promised value. In particular, ρ_t is calculated such that the last two terms have an expected value of zero. This condition comes from the martingale representation theorem and ensures that W_t represents the expected utility of the manager with respect to her information set at time t .

It is important to note that in principle the effort choice of the manager affects the magnitude of the adjustments to her continuation value $\Delta(W, \hat{\mu})$, from the perspective of the manager these adjustments have a zero effect on her expected utility. If the manager deviates and shirks, her continuation value will decrease. She will be faced with a different set of adjustments, and a different compensating trend ρ_t . However, as discussed above, the last two terms of (4.5) are a martingale difference with expected value zero. Thus her incentives to shirk are not affected these adjustments. Consequently, incentive compatibility of the contract only depends on the exposure of the manager to the realization of cash-flows, and follows the same intuition as before.

Let $F(W_t, \mu_0)$ denote the shareholders value function prior to the shock. In the sequel, I suppress the dependence of the value function on the profitability μ_0 to ease notation. Applying Ito's lemma to $F(W_t)$ using the dynamics of W_t given by (4.5) I find that the shareholder's expected cash-flow net of the cost of investment plus the expected appreciation in the value of the firm is given by

$$\begin{aligned}
E_t \left[dY_t - \frac{1}{2} \theta \alpha^2 dt + dF(W_t) \right] &= \left\{ \mu - (1 - \psi)C - \frac{1}{2} \theta \alpha^2 \right. \\
&\quad + F'(W)(\gamma W + \rho_t) + \frac{1}{2} F''(W) \sigma^2 \beta_t^2 \\
&\quad \left. + \alpha \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W)) dU(\hat{\mu}) \right) \right\}. \tag{4.6}
\end{aligned}$$

Shareholders want to maximize the RHS of (4.6) by choice of β, α, Δ provided that the contract is incentive compatible and satisfies the promise keeping constraint. Assuming that the value function is concave then it is optimal to set $\beta = \lambda$, as before. The inefficiency of liquidation provides the intuition why the manager should bear the minimum amount of risk required for her to choose work. A higher volatility would increase the probability of default

with no extra benefit to the shareholders. Moreover, since the shareholders can always make a lump-sum payment to the manager it must be the case that $F'(W) \geq -1$ for all W . Let the reflecting boundary \bar{W} be the lowest value such that $F'(\bar{W}) = -1$. I now characterize the optimal choices of $\{\Delta(W, \hat{\mu})\}_{\hat{\mu} \in \mathbb{R}}$. By concavity of $F(W)$ the optimal choices are given by:

$$F'(W_{\tau_S}) = \hat{F}'(W_{\tau_S} + \Delta(W_{\tau_S}, \hat{\mu}), \hat{\mu}), \quad \text{if } W_{\tau_S} + \Delta(W_{\tau_S}, \hat{\mu}) > 0$$

$$\Delta(W_{\tau_S}, \hat{\mu}) = -W_{\tau_S}, \quad \text{otherwise.}$$

The optimal adjustments $\{\Delta(W, \hat{\mu})\}$ to the manager's continuation value, which are applicable when there is a change in the profitability of the project, are such that the sensitivity of increasing the manager's continuation value by one unit are equalized before and after the shock. Because the shareholders have to deliver the manager her expected utility, the choice of adjustments $\{\Delta(W, \hat{\mu})\}$ have to be offset by the compensating trend ρ_t . The shareholders find it optimal to compensate the manager in the states in which it is cheapest for them, to the point in which the cost of compensation is equated across states. Intuitively, the adjustments are such that the continuation value of the manager is increased when the outcome is good (high $\hat{\mu}$), and it is decreased when the outcome is bad (low $\hat{\mu}$). If shareholders get a good outcome it is important for them to make sure that they can profit from it for a long time. Thus, they need a manager that has a large continuation value, and is far away from her liquidation boundary. In contrast, the benefit for shareholders of running a firm with low cash-flows is small, thus it is not critical to have a manager with a large continuation value. This feature of the optimal contract underspins the main result of the paper: the non monotonic relation between risk-shifting and leverage in the presence of managerial moral hazard. I call this mechanism internal hedging because it allows shareholders to minimize default (or costly replacement of the manager) when they get lucky, and thus benefit from the high cash-flows for an extended period of time.

Finally I turn to the choice of the optimal amount of risk-shifting. The first order condition with respect to α yields:

$$\alpha = \frac{1}{\theta} \left[\left(\int_{\mathbb{R}} (\hat{F}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W)) dU(\hat{\mu}) \right) - \int_{\mathbb{R}} F'(W) \Delta(W, \hat{\mu}) dU(\hat{\mu}) \right]. \quad (4.7)$$

Similar to what I found in the benchmark case without moral hazard (equation (3.2)) the amount of risk-shifting is proportional to the expected capital gain upon arrival of the shock. Notice that the capital gain depends directly on the realized value of the new profitability $\hat{\mu}$, but also on the amount by which the continuation value is adjusted $\Delta(W, \hat{\mu})$. In the next

section, I dissect in detail the role that the optimal choice of $\Delta(W, \hat{\mu})$ has in the choice of α .

The following Verification Theorem summarizes the optimal contract before the shock:

Proposition 5. *Suppose there exists a concave unique twice continuously differentiable solution $F(W)$ to the ODE*

$$rF(W) = \max_{\beta \geq \lambda, \alpha, \Delta(W, \hat{\mu})} \left\{ \mu - (1 - \psi)C + F'(W)(\gamma W + \rho(W)) + \frac{1}{2}F''(W)\sigma^2\beta(W)^2 + \alpha(W) \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W))dU(\hat{\mu}) \right) - \frac{1}{2}\theta\alpha(W)^2 \right\}, \quad (4.8)$$

where $\rho(W) = \int_{\mathbb{R}} \alpha(W)\Delta(W, \hat{\mu})dU(\hat{\mu})$, with boundary conditions:

$$F(0) = \max\{\max_{W_{reset}} F(W_{reset}) - M, 0\}, \quad F'(\bar{W}) = -1, \quad F''(\bar{W}) = 0.$$

Then $F(W)$ is the value function for the shareholders optimization problem (2.4). The optimal amount of risk-shifting $\alpha(W)$ is given by (4.7), the optimal adjustments to the manager continuation value after a shock $\Delta(W, \hat{\mu})$ are given by (??), and the optimal volatility to the manager's continuation value is given by $\beta(W) = \lambda$ by concavity of the value function. The continuation value of the manager follows (4) and the optimal payments to the manager are given by

$$P_t = (W_0 - \bar{W})^+ + \int_0^t \mathbf{1}_{\{W_s = \bar{W}\}} dP_s.$$

such that the process W_t reflects on the boundary \bar{W} . Once W_t hits 0 for the first time, the contract is terminated. At this point the shareholders can choose to default and get 0 or hire a new manager and optimally restart the contract. If $\tau_S > \tau_T$ then the remaining part of contract will be given by Proposition 5 starting at $W_{\tau_S} = W_{\tau_S^-} + \Delta(W_{\tau_S^-}, \hat{\mu}_{\tau_S})$.

Similar to the case in Proposition 2 the flow value of the shareholder's value function is equal to the firm's cash-flows, plus the expected capital gain of the firm. However, the last two terms in (4.8) are new. They correspond to the expected capital gain resulting from operating the firm under a new value of μ and optimally resetting the continuation value of the manager, net of the operating cost of the risky investment.

Panel A of Figure 4.1 shows an example of the value function $F(W)$, and two value functions after the shock: One in which profitability $\hat{\mu}$ is high and another for which it is low. The arrows show the adjustments $\Delta(W, \hat{\mu})$ to the continuation value in these two cases.

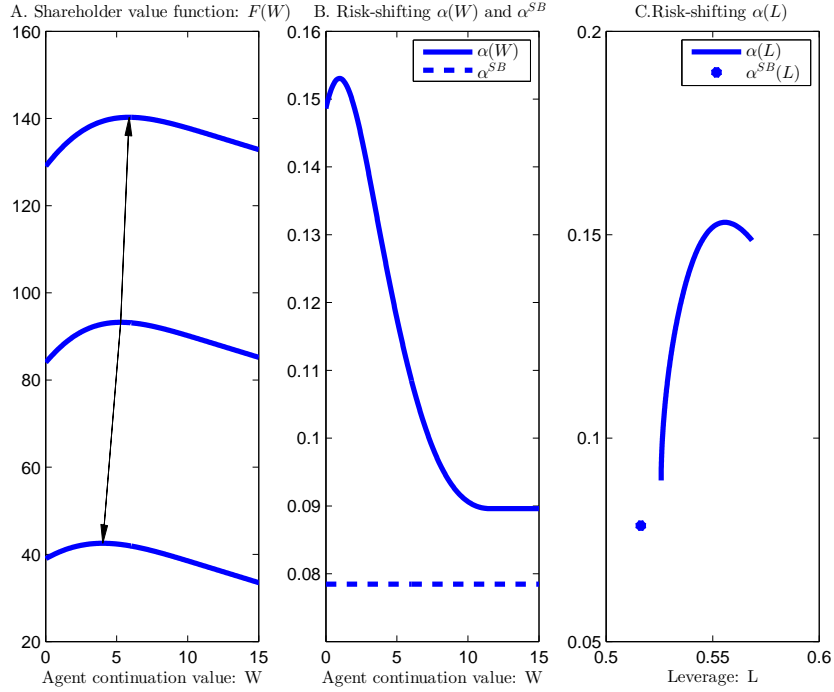


Figure 4.1: **Shareholder's value function, Risk-Shifting as a function of W , Risk-Shifting as a function of Leverage** Panel A plots the shareholder's value function before the jump, and two shareholder's value functions after the jump: one for the case of high $\hat{\mu}$, and one for the case of low $\hat{\mu}$. Panel B plots risk-shifting as a function of the continuation value of the agent. The dashed line corresponds to risk-shifting without moral hazard. Panel C traces the relation between risk-shifting and leverage as W changes. The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \lambda = 0.5, \kappa = 1, C = 12$

As seen from this example, the continuation value of the manager is increased after a good realization, and it is reduced after a bad realization. Panel B of Figure 4.1 depicts the optimal amount of risk-shifting $\alpha(W)$ as a function of the continuation value of the manager W , and compares it to the amount of risk-shifting in the case without moral hazard α^{SB} . Panel C of Figure 4.1 plots risk-shifting as a function of leverage $L = D(W)/(F(W) + E(W) + W)$, and compares it to the leverage and risk-shifting in the case without moral hazard. In the next section I will carefully discuss these results and provide intuition for the interaction between risk-shifting and moral hazard.

The following proposition shows that the shareholder's value function $F(W; \theta)$ decreases if it is more costly for the firm to engage in risk-shifting, i.e, if θ increases. Intuitively, once debt is in place, a lower value of θ makes it cheaper for the shareholders to implement higher risk-shifting, thus raising the option value of the equity as a result of the higher risk.

Proposition 6. *The shareholder's value function $F(W; \theta)$ is decreasing in θ for all $W \in [0, \bar{W}]$:*

$$\frac{\partial F(W; \theta)}{\partial \theta} = E \left[\int_t^{\min\{\tau_S, \tau_T\}} -e^{-r(s-t)} \frac{\alpha_s^2}{2} ds | W_t = W \right] \leq 0.$$

I end this subsection by providing a necessary and sufficient condition for the manager's high effort to be optimal for any $t \in [0, \tau_S]$. The parameter values used in the numerical examples satisfy this condition.

Proposition 7. *Implementing high effort is optimal at any time before the shock $t \in [0, \tau_S]$ if and only if:*

$$F(W) \geq \max_{\alpha, \Delta(W, \hat{\mu})} \left\{ (1 - \psi) \frac{C}{r} + \frac{\gamma}{r} F'(W) \left(W + \frac{r\rho(W)}{\gamma} - W^S \right) + \frac{\alpha(W)}{r} \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W)) dU(\hat{\mu}) \right) - \frac{1}{2r} \theta \alpha(W)^2 \right\},$$

for all $W \in [0, \bar{W}]$, where $W^S = \frac{\lambda\mu}{\gamma}$ represents the utility of the manager if she shirks forever, and $-\int_{\mathbb{R}} \alpha(W) \Delta(W, \hat{\mu}) dU(\hat{\mu}) = \rho(W)$.

4.3 Optimal Capital Structure

In this section I solve for the optimal coupon payment C . The initial shareholders need to optimally split the firm into debt and equity as to get the largest possible value from the initial issuance. The value of equity post-issuance is given by (4.4). The value of debt

satisfies (2.3), which implies that debt is fairly priced. The value of debt is calculated in the appendix, and can be calculated numerically as the solution to an ODE. At this point it is important to notice that the value of debt and of equity depend on W_0 as well as on C . Recall that to simplify the analysis, I assume shareholders have full bargaining power when negotiating the manager's initial compensation W_0 . Therefore, they will choose the manager's initial continuation value as to maximize their profits. The first order condition for this maximization is $F'(W_0) = 0$. Formally the initial shareholder's problem is to maximize the initial value of the firm subject to the assets being fairly priced:

$$\max_C D(\mu_0, W_0; C) + F(\mu_0, W_0; C),$$

s.t (3.3) and (2.3).

As in the case without moral hazard the optimal coupon will tradeoff the tax benefit of debt and the cost of bankruptcy. In general, the amount of risk-shifting is larger (see next section) than in the case without moral hazard, and thus the value of debt will reflect this increased probability of default. Hence the optimal ratio of debt to equity will be indirectly affected by the magnitude of the moral hazard problem.

5 Results

In this section I describe the results of the model. First, I explore how the presence of managerial moral hazard influences the amount of risk-shifting. I show that for a given amount of debt, firms in which managerial moral hazard is larger also engage in higher risk-shifting. Importantly, I find that risk-shifting is non-monotonic in leverage. Second, I study the optimal capital structure of firms with different levels of moral hazard. Firms in which moral hazard is prevalent issue less debt, and have lower leverage. Since moral hazard leads to more risk-shifting, bankruptcy costs will be greater in expectation. Thus, it is optimal for firms to reduce leverage as a way to lower expected bankruptcy costs. Third, I explore the implications of the model for managerial compensation. In agreement with empirical findings the model shows that managers are rewarded for events outside of their control. Fourth, I study firm survival probability and age effects of this model. The model implies that younger firms engage in more risk-shifting, and have lower survival probabilities than older firms. Finally, I discuss the property of counter-cyclical risk-shifting implied by the model, and how it contributes to the amplification and propagation of shocks in the

economy.

5.1 Risk-Shifting with and without Moral Hazard

In this section I discuss how managerial moral hazard influences the risk-shifting problem. Recall that λ represents the manager's cost of effort, and captures the severity of the moral hazard problem. I will explore this problem in two steps: First, I will focus on the amount of risk-shifting at the payout boundary \bar{W} .¹⁰ Under mild conditions, I show that $\alpha(\bar{W})$ converges from above to α^{SB} as λ goes to 0. Second, I will fix λ and show that $\alpha(W)$ is greater than α^{SB} . Moreover, I will show that the greater incidence of risk-shifting in the presence of moral hazard is not entirely explained by higher leverage, and I will elaborate on the role that the internal hedging effect plays.

5.1.1 Risk-shifting at the payout boundary

The model in section 4 converges to the model without moral hazard when the cost of moral hazard λ goes to 0. Therefore, it is intuitive that the amount of risk-shifting in the case with moral hazard will also converge to the amount of risk-shifting in the case without moral hazard as the cost of effort goes to 0. Proposition 8 below formalizes this intuition by showing that risk-shifting at the payout boundary in the presence of moral hazard converges to the risk-shifting without moral hazard. Moreover, I show that when the cost of replacing the manager is small, risk-shifting in the presence of moral hazard is greater than without moral hazard, and thus that the convergence is from above.

Proposition 8. *Let α^{SB} denote the amount of risk-shifting in the absence of moral hazard, and $\alpha(\bar{W})$ the amount of risk-shifting at the payout boundary for a given cost of effort λ . Then:*

1. $\alpha(\bar{W}) \rightarrow \alpha^{SB}$ as $\lambda \rightarrow 0$.
2. $\alpha(\bar{W}) \geq \alpha^{SB}$ if $\int \Delta(W, \hat{\mu}) dU(\hat{\mu}) \leq 0$.

Figure 5.1 shows an example in which $\alpha(\bar{W})$ converges from above to α^{SB} as the manager's cost of effort goes to 0. I turn next to the study of what drives this result, and how risk-shifting varies away from the payout boundary.

¹⁰As I will show below, \bar{W} is the attractive point of the system. Thus, focusing on the amount of risk-shifting near the payout boundary captures the risk-shifting that will be implemented a big proportion of the time.

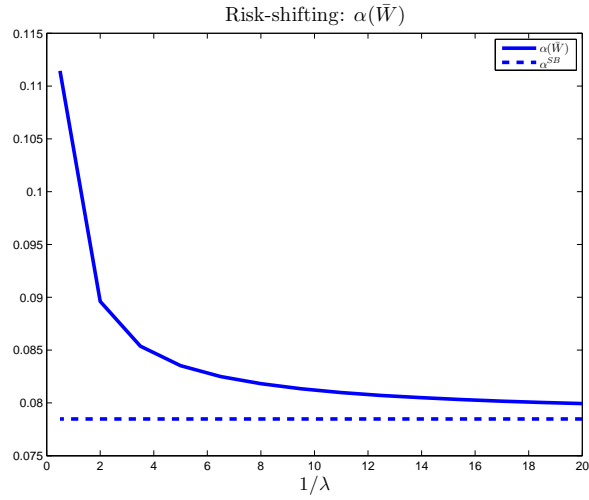


Figure 5.1: **Risk-Shifting at the payout boundary \bar{W}** This figure plots risk-shifting at the payout boundary for different values of λ . The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \kappa = 1, C = 12$

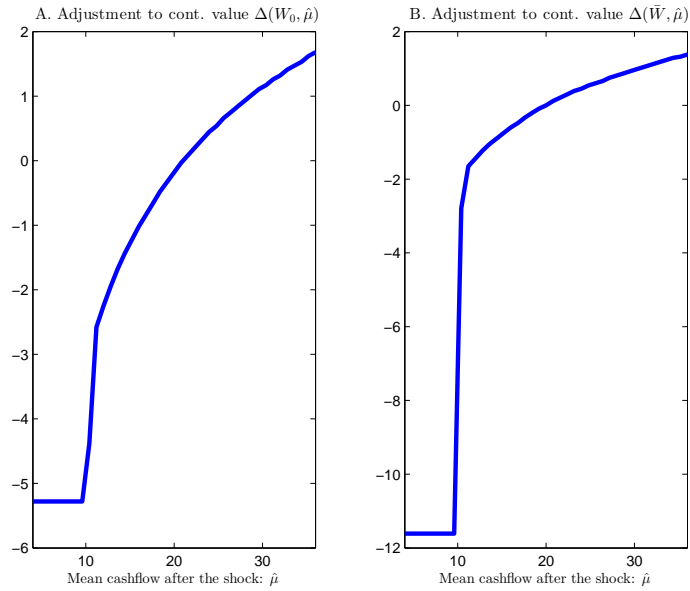


Figure 5.2: **Adjustment to the continuation value of the agent $\Delta(W, \hat{\mu})$** This figure plots the adjustment to the continuation value of the manager as a function of the realized profitability $\hat{\mu}$. Panel A plots these adjustments at the initial level of continuation value W_0 and Panel B plots it at the payout boundary \bar{W} . The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \lambda = 0.5, \kappa = 1, C = 12$

5.1.2 Risk-shifting: Leverage and Internal Hedging Effects

We had already seen from figure 4.1 that the amount of risk-shifting in the presence of moral hazard depends on the continuation value of the manager W . Importantly, risk-shifting $\alpha(W)$ is greater than in the case without moral hazard α^{SB} , and it does not vary monotonically with either the continuation value or leverage. This has important empirical implications because many models that try to estimate the magnitude of the risk-shifting problem often assume a linear and monotonic relation between leverage and risk-shifting. This model suggests that such models are misspecified. Rauh (2009) finds that firms pension funds tend to take on less risk when they are financially distressed. Similarly, Gilje (2013) using corporate investment risk measures available in SEC disclosures documents that firms reduce investment risk when leverage increases. In contrast, Eisdorfer (2009) finds indirect evidence that risk-shifting is higher for firms that are financially distressed. A similar finding by Landier et al. (2011) provides evidence of risk-shifting in the lending behavior of a large mortgage originator - New Century Financial Corporation - during a period of financial distress. Hence, my model has the potential to reconcile this seemingly contradictory evidence: risk-shifting initially increases as the firms become financially distressed (and leverage grows), but it tapers off and decreases for higher levels of financial distress (higher levels of leverage).

I will now explore why risk-shifting is greater in the presence of moral hazard and why it is non-monotonic in leverage. Notice from panel C of figure 4.1 leverage in the case with moral hazard is greater than in the case without it (for all values of W). This is not surprising as moral hazard decreases the overall value of the firm. Since I am holding the coupon payment C constant, the value of debt stays approximately unchanged, and leverage will increase. Highly leveraged firms benefit from having limited liability and profit from the convexity of the payoffs induced by the option to default. This is the standard leverage effect discussed in section 2 and it induces shareholders to engage in more risk-shifting.

However, the increment in risk-shifting is not completely explained by higher leverage. Recall that risk-shifting is increasing in the expected capital gain for shareholders after the shock. In the case with moral hazard, the optimal contract allows shareholders to make adjustments to the continuation value of the manager in response to the profitability drawn after the shock. On one hand, shareholders find it opportune to exercise the option to default in bad states. Therefore, in response to a bad shock the manager is immediately fired and her continuation value reduced to 0. The intuition for this result is that there is no use in having a highly incentivized manager when you have a project that is not worth continuing to operate. On the other hand, shareholders want to minimize the probability of costly

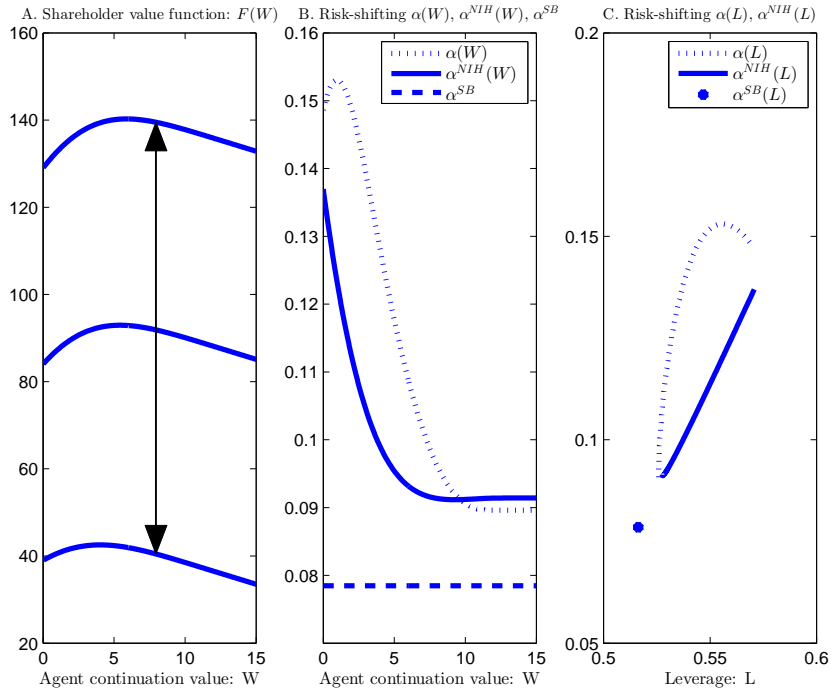


Figure 5.3: **Shareholder's value function, Risk-Shifting as a function of W , Risk-Shifting as a function of Leverage (Without Internal Hedging)** Panel A plots shareholder's value before the jump, and two shareholder's value after the jump: one for the case of high $\hat{\mu}$, and one for the case of low $\hat{\mu}$. Panel B plots risk-shifting as a function of the continuation value of the agent. The solid line corresponds to the case without internal hedging, the dotted line to the case with internal hedging, and the dashed line to the case without moral hazard. Panel C traces the relation between risk-shifting and leverage as W changes. The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \lambda = 0.5, \kappa = 1, C = 12$

replacing the manager (or defaulting) when they have drawn a good profitability. Thus, by reducing the manager's continuation value in the bad states, shareholders have leeway to increase W in the good states. The flexibility of the contract to make these adjustments is what I refer to as the internal hedging effect, because these adjustments allow the firm to internally hedge its optimal response to the profitability drawn. Consequently, the internal hedging effect allows shareholders to maximize their expected capital gain after the shock, increasing their appetite for risk-shifting.

Figure (5.2) depicts the adjustments to the continuation value of the manager $\Delta(W, \hat{\mu})$ in response to the realized profitability. Panel A depicts the adjustments when $W_{\tau_S} = W_0$ and Panel B shows the case when $W_{\tau_S} = \bar{W}$. In both cases, the size of the adjustment is increasing in $\hat{\mu}$. For very low values of $\hat{\mu}$ the continuation value of the manager is immediately reduced to 0 and her contract is terminated. For larger values of $\hat{\mu}$, shareholders find it optimal to continue with the project, and relax the incentive constraint of the manager in proportion to the new profitability of the project. As discussed above, the intuition for this result comes from the desire of shareholders to minimize costly replacement of the manager (or costly bankruptcy).

The internal hedging effect induces a non-monotonic relation between leverage and risk-shifting. Intuitively, for low values of W shareholders have very little room to punish the manager, and hence cannot really profit from the option to default. If the expected punishment in response to a bad shock is small, the optimal contract has little room to reward the manager in response to a good shock. Thus, the internal hedging effect is mainly inactive for low values of W . For high values of W there is no need to reward the manager in response to a good shock, since the current value of W is already sufficiently high and costly replacement is unlikely. Hence, the internal hedging effect is silent when W is close to the payout boundary \bar{W} . However, for values of W in the middle range, the internal hedging effect is strongest, since it allows shareholders to benefit from the option to default in bad states via punishment of the manager, and minimizing the probability of default by rewarding the manager in the good states.

To better highlight the role of the internal hedging effect, I will preclude the shareholders from making adjustments to the continuation value in response to the realized profitability. That is, I will impose the constraint $\Delta(W, \hat{\mu}) = 0$. Denote the solution to this problem $F_{NIH}(W)$, and the respective amount of risk shifting $\alpha_{NIH}(W)$, where the subscript *NIH* stands for No Internal Hedging. Formally $F_{NIH}(W)$ satisfies:

$$\begin{aligned}
rF_{NIH}(W) = \max_{\beta \geq \lambda, \alpha} & \left\{ \mu - (1 - \psi)C + F'_{NIH}(W)\gamma W + \frac{1}{2}F''_{NIH}(W)\sigma^2\beta^2 \right. \\
& \left. + \alpha \left(\int_{\mathbb{R}} (\hat{F}(W, \hat{\mu}) - F_{NIH}(W))dN(\hat{\mu}) \right) - \frac{1}{2}\theta\alpha^2 \right\}, \tag{5.1}
\end{aligned}$$

with boundary conditions:

$$F_{NIH}(0) = \max\{\max_{W_{reset}} F_{NIH}(W_{reset}) - M, 0\}, \quad F'_{NIH}(\bar{W}) = -1, \quad F''_{NIH}(\bar{W}) = 0.$$

Panel A of Figure 5.3 shows an example of the value function $F_{NIH}(W)$, and two value functions after the shock: one in which the value of the profitability $\hat{\mu}$ is high and another for which it is low. The arrows indicate that the continuation value of the manager will not change in response to the shock. Suppressing the flexibility of the contract to hedge against the shock by adjusting the continuation value shuts down the internal hedging effect. Panels B and C of Figure 5.3 indicate that risk-shifting $\alpha_{NIH}(W)$ is monotonic in the continuation value of the manager and in leverage. By suppressing the internal hedging effect the standard intuition that higher leverage should induce higher risk-shifting is restored. In other words, firms that are closer to default engage more in gambling for resurrection. However, that intuition is incomplete when the dynamic features of the optimal contract allow shareholders to make adjustments to the continuation value of the manager in response to profitability shocks. Hence, endowing the contract with this flexibility can reverse the usual relation between leverage and risk-shifting.

This analysis suggests that the compensation package of the manager is an important determinant of risk-shifting, even when risk-shifting is observable and contractible. Thus, attempts to regulate risk-shifting by means of restricting leverage are not optimal. Policies aimed at reducing risky activities need to look jointly at the leverage of the firms and the contract that binds the management and the shareholders of the firm.

5.2 Capital Structure and Leverage

In this section, I discuss how managerial moral hazard influences the optimal capital structure of the firm, and the optimal leverage. Panel A of Figure 5.4 plots the initial value of the firm for different values of C for three different values of λ . As expected, the value of the firm decreases as the incidence of moral hazard increases (higher λ). Moreover, the optimal coupon chosen is decreasing in λ . The intuition for this result is that higher moral hazard induces higher risk-shifting. Bondholders anticipate higher rates of bankruptcy as a result,

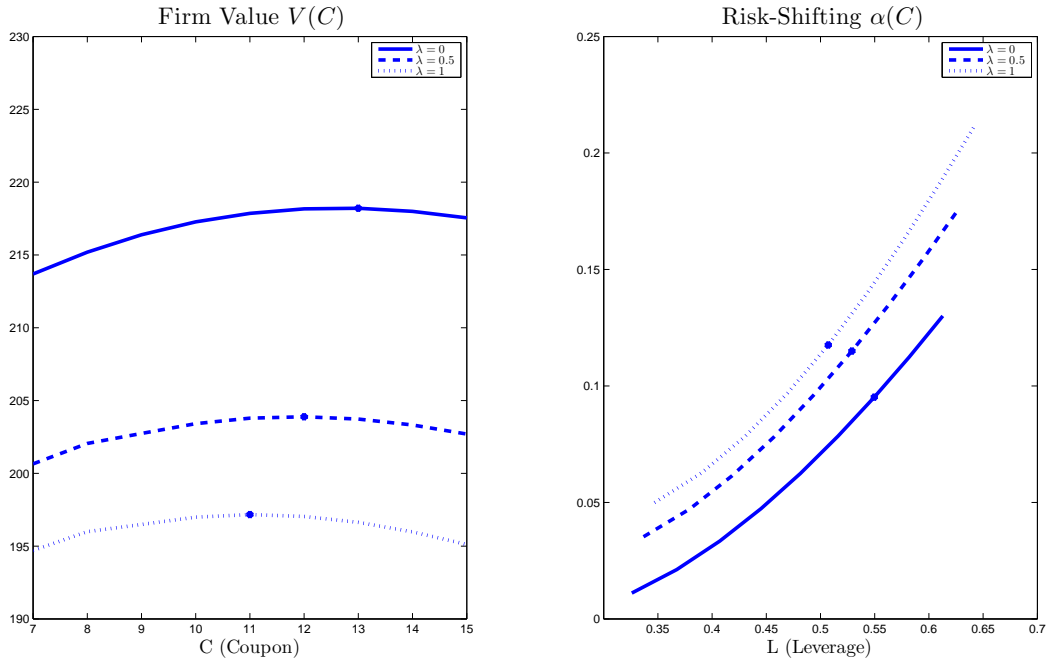


Figure 5.4: **Initial firm value, and risk-shifting** . Panel A plots initial firm value as a function of C for three different values of λ . Panel B traces the relation between risk-shifting and leverage as the coupon value changes. The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \kappa = 1$

Table 1: **Comparative statics: moral hazard.** This table reports the results from comparative statics on the parameter λ that captures the magnitude of the moral hazard problem. Other parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \kappa = 1, C = 12$

	coupon rate C	initial leverage L_0	initial firm value $F(W_0) +$ $D(W_0)$	initial risk- shifting $\alpha(W_0)$	change in risk-shifting $\alpha(W_0) -$ $\alpha(\bar{W})$	initial credit spread	change in credit spread
$\lambda = 0$	13	0.5497	218.20	0.0952	—	0.84%	—
$\lambda = 0.5$	12	0.5290	203.88	0.1148	0.0252	0.85%	0.04%
$\lambda = 1$	11	0.5072	197.16	0.1176	0.0361	0.72%	0.05%

and thus will only buy this debt at a discount. At time 0 the initial shareholders internalize the costs associated with higher bankruptcy, and decide to reduce the amount of debt issued. Moreover, the reduction in the initial issuance of debt dominates the reduction in firm value associated with higher moral hazard and leads to lower initial leverage $L_0 = \frac{D(W_0)}{D(W_0)+F(W_0)+W_0}$. The model implies that firms in which there is more prevalence of managerial moral hazard will choose a lower initial amount of leverage. Table 1 reports comparative statics for various values of λ .

Panel B of Figure 5.4 shows the relation between initial leverage L_0 and $\alpha(W_0)$. As expected, risk-shifting and leverage are positively related for a given value of λ . However, higher values of λ imply higher risk-shifting for the same value of leverage. This is consistent with the previous result that in a model with moral hazard, leverage is not the only determinant of the amount of risk-shifting undertaken by the firm.

5.3 Managerial Compensation

In this section, I discuss the implications for managerial compensation arising from the model. Recall the law of motion for the continuation value of the manager is given by

$$dW_t = \underbrace{\gamma W_t dt - dP_t}_{\text{Promise Keeping}} + \underbrace{\sigma \lambda dB_t}_{\text{Incentives}} + \underbrace{\Delta(W_t, \hat{\mu}) dJ_t + \rho_t}_{\text{Internal Hedging Channel}}$$

The first term reflects that the contract must be consistent with the promise made to the manager. In order for the continuation value W to appropriately capture the promise made to the manager, it should grow at the discount rate of the manager γ , net of the utility flow from the manager's consumption dP_t . The second term reflects the need to provide incentives for the manager to work. The extent to which the manager is exposed to the brownian shock has to be proportional to the cost of effort of the manager as to deter her from shirking. The third term corresponds to the exposure of the manager's continuation value to the Poisson shock. This term is what I have called the internal hedging effect and it represents the adjustments made to the continuation value of the manager in response to the profitability drawn. This term is similar to what Hoffmann and Pfeil (2010) call "reward for luck". In contrast to output, the manager has no influence over the profitability drawn upon realization of the shock,¹¹ yet her continuation utility is affected by it. In that sense, the manager is rewarded (or punished) by events that are outside of her control. This result is puzzling in light of Holmstrom's (1979) sufficient statistic result, as one would expect the optimal contract to filter out "noise" from the compensation package. However, in this dynamic setting luck shocks are informative about the future profitability of the firm, and hence about the cost of providing incentives for the manager in the future. In particular, by exposing the manager to this luck shock the contract minimizes costly replacement of the manager or inefficient liquidation of the firm.

Empirically, reward for luck is present in many types of compensation contracts. Bertrand and Mullainathan (2001) document that managerial compensation in the oil industry responds to "lucky shocks", such as increases in the price of oil, or changes in exchange rates for firms with operations in multiple countries. They posit that such reward for luck is the result of managerial discretion over their own compensation schemes, and suggest that the principal-agent framework fails to account for this phenomenon. In contrast, this paper offers an alternative explanation for this phenomenon when modeling the risk-shifting problem as a permanent shift in the profitability of the firm.¹²

I will now explore the optimal mixture of rewards and punishments implied by the optimal contract. Panel A in Figure 5.5 depicts the expected change in the manager continuation

¹¹ $\hat{\mu}$ is drawn from $U[\mu_0 - \delta, \mu_0 + \delta]$ irrespective of the manager's actions.

¹²Hoffman and Pfeil (2010) were the first to show that optimal dynamic contracting can capture the "reward for luck" effect. Their paper differs from mine in that the productivity shocks in their paper are exogenous, while in my paper they emerge endogenous as a result of the risk-shifting conflict between shareholders and bondholders.

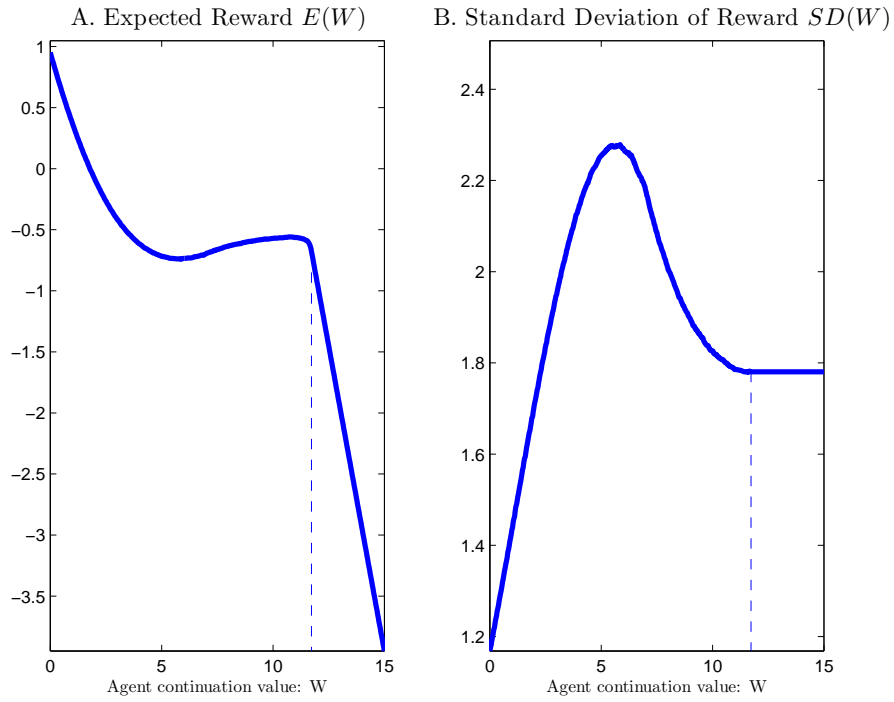


Figure 5.5: **Expected reward and standard deviation of the reward** . Panel A plots the expected value and the standard deviation of the “lucky” reward the manager will receive upon realization of a shock. The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \delta = 16, \lambda = 0.5, \kappa = 1$.

value upon realization of the shock

$$E(W) = \int_{\mathbb{R}} \Delta(W, \hat{\mu}) dU(\hat{\mu}).$$

For low values of W the manager is compensated with rewards. Because of the manager's limited liability, the optimal contract has little room to punish the manager, and thus on average the manager's continuation value increases. As the value of W grows the shareholders have more room to punish the manager in response to a bad shock. Therefore, in expectation the manager is punished upon arrival of the shock. An interesting implication of the model is that firms with a recent history of poor performance (low W) are more likely to reward their managers for events outside of their control. Hence, the model predicts a counter-cyclical reward for luck of managers: in good times managers are punished for events outside of their control, and in bad times they are rewarded.

Panel B in Figure 5.5 depicts the standard deviation in the change of the manager's continuation value upon realization of the shock

$$SD(W) = \sqrt{\int_{\mathbb{R}} \Delta^2(W, \hat{\mu}) dU(\hat{\mu}) - [E(W)]^2}.$$

The inverted U-shape of this function shows that the adjustments to the continuation value of the manager in response to the lucky shock are more spread out for intermediate values of W . This is in agreement with the findings from panel A: because of limited liability for low values of W the punishment for the manager is at most W . Conversely, for high values of W there is no need to reward the manager in response to a good shock. However, for intermediate values of W the optimal contract induces large punishments in response to bad shocks, and large rewards in response to good ones. Importantly, spreading the continuation value of the manager in response to the profitability is the mechanism that activates the internal hedging effect. As a result, the non-monotonicity between leverage and risk-shifting resulting from the internal hedging effect is directly linked to the non-monotonicity in the variation of the adjustments made to the continuation value of the manager after the shock.

5.4 Business Cycle Implications

In this section, I discuss the model's implication for business cycle fluctuations. Recall

that under the optimal contract the continuation value of the manager $W \in [0, \bar{W}]$ follows

$$dW_t = \gamma W_t dt + \sigma \lambda dB_t + \{\Delta(W, \hat{\mu}) dJ_t + \rho_t dt\},$$

where $-\int_{\mathbb{R}} \alpha_t \Delta(W, \hat{\mu}) dU(\hat{\mu}) = \rho_t$. Since the compensation to the manager is deferred until \bar{W} , the evolution of W_t needs to appreciate at rate γ (first term), and from the martingale representation theorem expose the manager to the brownian shock (second term) and the Poisson shock (third term). These last two terms are zero in expectation. Therefore, on average the continuation value is drifting upwards towards the attractive point of the system \bar{W} .

On average the continuation value of the manager stays near the payout boundary, in which firms face low financial distress, and engage in little risk-shifting.¹³ However, a sufficiently bad sequence of output shocks erodes the continuation value of the manager and brings the firm into financial distress. Thus, shareholders find it optimal to engage in higher risk-shifting activities, which in turn raise the probability of bankruptcy. As a result, the initial negative sequence of output shocks is amplified by the aggregate deadweight cost of bankruptcy.

Importantly, in the benchmark model without moral hazard presented in section 3 output shocks have no persistent effect on the dynamics of the firm. In that case, the shock is fully absorbed by the shareholders, but has no impact on the amount of risk-shifting of the firm, nor on its expected probability of bankruptcy. Therefore, it is the counter-cyclical nature of risk-shifting induced by modeling jointly the moral hazard and the risk-shifting problem that underpins this amplification mechanism.

5.5 Firm Survival: Age Effects

Multiple studies have documented that young firms experience higher turnover rates than older firms. In this section, I study the implications of the model for firm survival. Recall that the outside option of the manager is sufficiently low that the initial continuation value of the manager W_0 is set to maximize shareholder value.¹⁴ The first order condition for this maximization is

$$F'(W_0; C) = 0. \tag{5.2}$$

¹³See Figure 4.1

¹⁴Other specifications are possible, in which I would need to specify the bargaining power of the shareholders and the manager. My results do not vary qualitatively as long as the initial continuation value of the manager $W_0 < \bar{W}$.

The optimal choice of W_0 implies a tradeoff for the shareholders. On the one hand, a high continuation value minimizes the costs of financial distress associated with liquidating the firm or costly replacement of the manager. On the other hand, a high continuation value implies greater payments to the manager in the future, which are costly to the shareholders. Intuitively, this will imply that the shareholders will choose a value $W_0 \in [0, \bar{W})$. More rigorously, combining the concavity of the value function, (5.2), and $F'(\bar{W}) = -1$ imply that $W_0 < \bar{W}$.

As discussed in the previous section, the manager's continuation value drifts upwards toward \bar{W} . This indicates that on average firms relax their financial constraints with the passage of time. Hence, as firms grow older they become less financially constrained, have lower risk-shifting, and higher survival rates. Column 5 in Table 1 calculates the difference in risk-shifting for new firms and for firms at the payout boundary. Similarly, column 7 compares the change in credit spreads, which reflect a higher probability of default for younger firms.

My mechanism bears resemblance to that in Albuquerque and Hopenhayn (2004). In their model, leverage goes down over time as firms reduce their long-term debt, thereby reducing the instances in which shareholders find it optimal to default. The key difference between these two models is that in my model higher survival rates for more mature firms results from lower risk-shifting, rather than from having reduced their debt obligations.

6 Concluding Remarks

This paper analyzes the interaction of the risk-shifting problem between shareholders and bondholders with the moral hazard problem between shareholders and the manager. I show the presence of managerial moral hazard induces shareholders to engage in higher risk-shifting activities. I break down this result into two effects: the leverage effect and the internal hedging effect. The leverage effect is standard, highly leveraged firms are closer to default, thus they have greater incentives to increase risk-shifting. The internal hedging effect is novel, the dynamic contract allows shareholders to compensate the manager contingent on the profitability drawn. Thus, relaxing the incentive constrain of the manager in the event that a high profitability is drawn. As a result, shareholders benefit more from the upside, hence choosing higher risk-shifting.

Moreover, the internal hedging effect induces a non-monotonic relation between risk-shifting and leverage. This non-monotonicity has the potential to reconcile seemingly contradictory empirical evidence on the sign of this relation. Importantly, policies aimed at regulating excessive risk-taking via capital requirements (effectively setting an upper bound

on leverage) are incomplete without looking at the structure of managerial compensation. In particular, regulating contracts that reward managers for luck can be a good complement to capital requirements.

An obvious shortcoming of the present work is the a priori structure of the debt contract. Endogenizing the form of the debt contract could produce further insights. Specifically, it would be interesting to study the role of the maturity structure of debt, and of performance sensitive debt in addressing the risk-shifting and moral hazard problems. It would also be interesting to consider the case in which the manager is risk-averse. In this case, the moral hazard problem will be compounded as it is costlier to expose the manager to risk. However, this may dampen the risk-shifting problem. I leave these questions for future work.

Appendices

A Proofs

Proof of Proposition 1: Fix an arbitrary contract (P, τ_T) . Define

$$W_t = E_t \left[\int_t^{\tau_T} e^{-\gamma(s-t)} (\mu_s \lambda (1 - a_s) ds + dP_s) \right],$$

as the manager's utility when she follows action a under this contract.

Let

$$M_t = E_t \left[\int_0^{\tau_T} e^{-\gamma s} (\mu_s \lambda (1 - a_s) ds + dP_s) \right] = \int_0^t e^{-\gamma s} (\mu_s \lambda (1 - a_s) ds + dP_s) + e^{-\gamma t} W_t,$$

which by construction is a martingale.

By the martingale representation theorem there exists measurable β_t such that $dM_t = \beta_t e^{-\gamma t} dB_t$. But we also know that

$$dM_t = e^{-\gamma t} (\mu_t \lambda (1 - a_t) dt + dP_t) - \gamma e^{-\gamma t} W_t + e^{\gamma t} dW_t,$$

Rearranging yields (4.2).

Moreover, since the manager is risk-neutral, if she shirks she receives λdt , but she loses $\beta_t dt$ via a lower continuation value. Applying the proofs of Propositions 1 and 2 in the Appendix in Sannikov (2008) completes the proof.

Proof of Proposition 2: The contract after the shock is identical to the hidden effort model of DeMarzo and Sannikov (2006, Section III). I prove this proposition by a similar procedure to the one in DeMarzo and Sannikov (2006).

Proof of Proposition 3: If the shareholders induce the manager to shirk, her continuation would evolve according to

$$dW_t = \gamma W_t dt - \lambda \mu dt - dP_t.$$

Optimality of implementing work between $t \in [\tau_S, \tau_T]$ implies that the expected gain for shareholders from letting the manager shirk is lower than under the existing contract for all $W \in [0, \bar{W}]$:

$$r\hat{F}(W) \geq -(1 - \psi)C + \hat{F}'(W)(\gamma W - \lambda\mu).$$

Defining $W^S = \frac{\lambda\mu}{\gamma}$ gives the result.

Proof of Proposition 4: Applying the martingale representation theorem in a similar manner to that in the proof of Proposition 1 shows (4.5). Moreover, incentive compatibility is proven applying the proofs of Propositions 1 and 2 in the Appendix in Sannikov (2008). This completes the proof.

Proof of Proposition 5: I verify that the shareholder value function and policy given are indeed optimal. Consider the case in which the manager cannot be replaced. It is not difficult to add replacement, but complicates notation substantially. Let $\Gamma = \Gamma(\alpha, P, \tau_T)$ be an arbitrary contract that implements high effort at all times, and define the shareholders objective function $J(W, \Gamma)$ as

$$J(W, \Gamma) = E \left[\int_0^{\min\{\tau_S, \tau_T\}} e^{-rt} (dY_t - c(\alpha)dt - (1 - \psi)C_t dt - dP_t) + 1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S} \hat{F}(W_{\tau_S}) \right].$$

Step 1. Define G_t^Γ :

$$G_t^\Gamma = \int_0^t e^{-rs} (dY_s - c(\alpha_s)ds - (1 - \psi)C_s ds - dP_s) + e^{-rt} F(W_t),$$

where W_t follows (4.5). Applying Ito's lemma and its generalization for point processes I obtain

$$\begin{aligned} e^{rt} dG_t^\Gamma &= \left\{ \mu_0 - c(\alpha_t) - (1 - \psi)C_t + F'(W_t) \left(\gamma W_t + \int_{\mathbb{R}} \alpha_t \Delta(W, \hat{\mu}) dU(\hat{\mu}) \right) \right. & (A.1) \\ &+ \frac{1}{2} F''(W_t) \beta_t^2 + \alpha_t \left(\int_{\mathbb{R}} (\hat{F}(W_t + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W)) dU(\hat{\mu}) \right) - rF(W_t) \left. \right\} dt \\ &+ \{F'(W_t) - 1\} dP_t + \{\sigma + F'(W_t)\beta_t\} dB_t \\ &+ \left\{ \hat{F}(W_t + \Delta(W_t, \hat{\mu}), \hat{\mu}) dJ_t - \alpha_t \left(\int_{\mathbb{R}} (F(W_t + \Delta(W, \hat{\mu}), \hat{\mu}) dU(\hat{\mu})) \right) dt \right\}. \end{aligned}$$

The first term is less than or equal to zero by (4.4), the second term is less than or equal

to zero since $F'(W) \geq -1$, and finally the last two terms are martingales and vanish in expectations. Thus

$$G_0^\Gamma = F(W_0) \geq E \left[\int_0^{\min\{\tau_S, \tau_T\}} e^{-rt} (dY_t - c(\alpha)dt - (1-\psi)C_t dt - dP_t) + 1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S} \hat{F}(W_{\tau_S}) \right].$$

Since Γ was arbitrary I conclude that $F(W)$ is an upper bound for the shareholder value function.

Step 2. Since the inequalities in (A.1) hold with equality for the policies in the proposition, I conclude that $F(W)$ is attained. Thus, $F(W)$ is the shareholders value function.

Proof of Proposition 6: I prove this proposition by adapting Lemma 6 in DeMarzo and Sannikov (2006). Rewrite (4.8) as:

$$rF(W) = \mu - (1-\psi)C + F'(W)\gamma W + \frac{1}{2}F''(W)\sigma^2\lambda^2 + \frac{1}{2}\theta\alpha(W)^2, \quad (\text{A.2})$$

where $\alpha(W)$, $\Delta(W, \hat{\mu})$, and $\beta(W) = \lambda$ represent the optimal policies from the maximization problem given by (??) and (4.7). Differentiating (A.2) with respect to θ yields

$$r \frac{\partial F(W)}{\partial \theta} = \frac{\partial F'(W)}{\partial \theta} \gamma W + \frac{\partial F''(W)}{\partial \theta} \frac{\sigma^2 \lambda^2}{2} + \sigma \alpha(W) \frac{\partial \alpha(W)}{\partial \theta} + \frac{\alpha(W)^2}{2},$$

where

$$\frac{\partial \alpha(W)}{\partial \theta} = -\frac{\alpha(W)}{\theta} + \frac{1}{\theta} \left[-\frac{\partial F(W)}{\partial \theta} - \frac{\partial F'(W)}{\partial \theta} \int \Delta(W, \hat{\mu}) dU(\hat{\mu}) \right].$$

Hence

$$r \frac{\partial F(W)}{\partial \theta} = -\frac{\alpha(W)^2}{2} + \frac{\partial F'(W)}{\partial \theta} (\gamma W - \rho_t) + \frac{\partial F''(W)}{\partial \theta} \frac{\sigma^2 \lambda^2}{2} + \alpha(W) \left[r \frac{\partial F(W)}{\partial \theta} \right],$$

with boundary conditions:

$$\frac{\partial F(W)}{\partial \theta} = \frac{\partial F(W_{reset})}{\partial \theta}, \quad \frac{\partial F'(\bar{W})}{\partial \theta} = 0. \quad (\text{A.3})$$

Applying the Feynman-Kac formula it follows that

$$\frac{\partial F(W; \theta)}{\partial \theta} = E \left[\int_t^{\tau_S} -e^{-r(s-t)} \frac{\alpha_s^2}{2} ds | W_t = W \right] \leq 0.$$

The boundary conditions in (A.3) have assumed that the manager is replaced when it's continuation value runs out. For the case in which termination equals default I would have obtained

$$\frac{\partial F(W; \theta)}{\partial \theta} = E \left[\int_t^{\tau_T} -e^{-r(s-t)} \frac{\alpha_s^2}{2} ds | W_t = W \right] \leq 0,$$

which completes the proof.

Proof of Proposition 7: I proceed in a similar fashion as in the proof of Proposition 3. If the shareholders induce the manager to shirk, her continuation would evolve according to

$$dW_t = \gamma W_t dt - \lambda \mu dt - dP_t + \Delta(W, \hat{\mu}) dJ_t + \rho_t dt.$$

Optimality of implementing work between $t \in [0, \tau_S)$ implies that the expected gain for the shareholders from letting the manager shirk is lower than under the existing contract for all $W \in [0, \bar{W}]$:

$$\begin{aligned} rF(W) \geq & \max_{\alpha, \Delta(W, \hat{\mu})} \left\{ (1 - \psi)C + F'(W)(\gamma W + \rho_t - \lambda \mu) \right. \\ & \left. + \alpha \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - F(W)) dU(\hat{\mu}) \right) - \frac{1}{2r} \theta \alpha^2 \right\}. \end{aligned}$$

Using $W^S = \frac{\lambda \mu}{\gamma}$ and reorganizing the terms gives the result.

Proof of Proposition 8: Evaluating (3.2) at $W = \bar{W}$ I obtain

$$\alpha(\bar{W}) = \frac{1}{\theta} \left[\left(\int_{\mathbb{R}} (\hat{F}(\bar{W} + \Delta(W, \hat{\mu}), \hat{\mu}) - F(\bar{W})) dU(\hat{\mu}) \right) - \int_{\mathbb{R}} F'(\bar{W}) \Delta(W, \hat{\mu}) dU(\hat{\mu}) \right],$$

substituting back into (4.4) I solve for $F(\bar{W})$, and plugging back in the above expression I obtain

$$\begin{aligned} \alpha(\bar{W}) = & \frac{1}{\theta} \left[-\theta r + \left(\theta^2 r^2 + 2\theta r \left(\int_{\mathbb{R}} \hat{F}(\bar{W} + \Delta(W, \hat{\mu}), \hat{\mu}) dU(\hat{\mu}) \right) \right. \right. \\ & \left. \left. + \int_{\mathbb{R}} \Delta(W, \hat{\mu}) dU(\hat{\mu}) - 2\theta(\mu_0 - (1 - \psi)C) + 2\theta \gamma \bar{W} \right) \right]^{1/2}. \end{aligned}$$

Recalling that

$$\alpha^{SB} = \frac{1}{\theta} \left[-\theta r + \sqrt{\theta^2 r^2 + 2\theta(r \int_{\mathbb{R}} \hat{F}(\hat{\mu}) dU(\hat{\mu}) - (\mu_0 - (1 - \psi)C))} \right],$$

and that $\hat{F}(\bar{W}_{\hat{\mu}}, \hat{\mu}) = \hat{\mu}/r - \gamma \bar{W}_{\hat{\mu}}/r$ I notice that as $\lambda \rightarrow 0$ then $\bar{W}_{\hat{\mu}} \rightarrow 0$ and thus $\int \Delta(W, \hat{\mu}) dU(\hat{\mu}) \rightarrow 0$. Moreover, when $\int \Delta(W, \hat{\mu}) dU(\hat{\mu}) \leq 0$ then $\alpha(\bar{W}; \lambda) \geq \alpha^{SB}$. Finally, when the cost of replacing the manager is constant I can show that the value functions are parallel shifts of each other for the cases in which the manager is replaced,¹⁵ and for the cases in which the firm defaults the payout boundary is lower. Therefore, the condition that $\int \Delta(W, \hat{\mu}) dU(\hat{\mu}) \leq 0$ will be satisfied.

B Pricing Formulas

In this appendix I calculate the pricing formulas for debt in the cases with and without moral hazard.

B.0.1 Debt price without moral hazard

The value of debt after the shock $\hat{D}(\mu)$ is given by:

$$\hat{D}(\mu) = \begin{cases} \frac{C}{r}, & (1 - \psi)C \leq \mu \\ \frac{(1 - \psi)\mu}{r}, & (1 - \psi)C > \mu \end{cases}$$

The value of debt before the shock $D(\mu_0)$ satisfies

$$rD(\mu_0) = C + \alpha \left[\int_{\mathbb{R}} \hat{D}(\hat{\mu}) dU(\hat{\mu}) - D(\mu_0) \right],$$

solving yields

$$D(\mu_0) = \frac{C}{r + \alpha} + \frac{\alpha \int_{\mathbb{R}} \hat{D}(\hat{\mu}) dU(\hat{\mu})}{r + \alpha}.$$

B.0.2 Debt price with moral hazard

The value of debt after the shock $\hat{D}(W; \mu)$ satisfies

¹⁵See Hoffmann and Pfeil (2012)

$$r\hat{D}(W; \mu) = C + \hat{D}'(W; \mu)\gamma W + \frac{\hat{D}''(W; \mu)\sigma^2\lambda^2}{2},$$

with boundary conditions

$$\hat{D}(0) = \frac{C}{r}, \quad \hat{D}'(\bar{W}) = 0,$$

when the manager is replaced. And with boundary conditions

$$\hat{D}(0) = \frac{(1-\phi)\mu}{r}, \quad \hat{D}'(\bar{W}) = 0,$$

when the firm defaults upon termination of the contract.

The value of debt before the shock $D(W; \mu_0) = D(W)$ satisfies

$$\begin{aligned} rD(W) &= C + D'(W)(\gamma W + \rho_t) + \frac{1}{2}D''(W)\sigma^2\lambda^2 \\ &+ \alpha \left(\int_{\mathbb{R}} (\hat{D}(W + \Delta(W, \hat{\mu}), \hat{\mu}) - D(W)) dU(\hat{\mu}) \right), \end{aligned}$$

with boundary conditions

$$D(0) = D(W_{Reset}), \quad D'(\bar{W}) = 0,$$

when the manager is replaced. And with boundary conditions

$$D(0) = \frac{(1-\phi)\mu_0}{r}, \quad D'(\bar{W}) = 0,$$

when the firm defaults upon termination of the contract.

B.0.3 Debt price without internal hedging (NIH)

The price of debt after the shock $\hat{D}(W; \mu)$ is the same as in the previous section. The price of debt before the shock when I shut down the internal hedging effect $D_{NIH}(W; \mu_0) = D_{NIH}(W)$ satisfies

$$\begin{aligned} rD_{NIH}(W) &= C + D'_{NIH}(W)\gamma W + \frac{1}{2}D''(W)\sigma^2\lambda^2 \\ &+ \alpha \left(\int_{\mathbb{R}} (\hat{D}(W, \hat{\mu}) - D_{NIH}(W)) dU(\hat{\mu}) \right), \end{aligned}$$

with boundary conditions

$$D_{NIH}(0) = D_{NIH}(W_{Reset}), \quad D'_{NIH}(\bar{W}) = 0,$$

when the manager is replaced, and with boundary conditions

$$D_{NIH}(0) = \frac{(1 - \phi)\mu_0}{r}, \quad D'_{NIH}(\bar{W}) = 0,$$

when the firm defaults upon termination of the contract.

References

- Albuquerque, Rui and Hugo Hopenhayn, 2004, Optimal Lending Contracts and Firm Dynamics, *Review of Economic Studies* 71, 285-315.
- Allen, Franklin, Richard A. Brealey, and Stewart C. Myers, 2006, Corporate finance.
- Atkeson, Andrew, and Robert E. Lucas, Jr., 1992, On Efficient Distribution with Private Information, *Review of Economic Studies* 59, 427-453.
- Bancel, Franck, and Usha R. Mittoo, 2004, Cross-country determinants of capital structure choice: a survey of European firms, *Financial Management* 103-132.
- Bertrand, Marianne, and Sendhil Mullainathan, 2001, Are CEOs rewarded for luck? The ones without principals are, *Quarterly Journal of Economics* 901-932.
- Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet, 2007, Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications, *Review of Economic Studies* 74, 345-390.
- Biais, Bruno, Thomas Mariotti, Jean-Charles Rochet, and Stephane Villeneuve, 2010, Large Risks, Limited Liability, and Dynamic Moral Hazard, *Econometrica* 78, 73-118.
- Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of Tobin's q, corporate investment, financing, and risk management, *The Journal of Finance* 66, 1545-1578.
- Bolton, Patrick, Hui Chen, and Neng Wang, 2013, Market timing, investment, and risk management, *Journal of Financial Economics* 109, 40-62.
- Cvitanic, Jaksza and Jianfeng Zhang, 2012, *Contract Theory in Continuous-Time Models*, Springer-Verlag, Berlin.
- Cvitančić, Jakša, Xuhu Wan, and Jianfeng Zhang, 2009, Optimal compensation with hidden action and lump-sum payment in a continuous-time model, *Applied Mathematics and Optimization* 59, 99-146.
- DeMarzo, Peter and Michael Fishman, 2007a, Agency and Optimal Investment Dynamics, *Review of Financial Studies* 20, 151-188.
- DeMarzo, Peter and Michael Fishman, 2007b, Optimal Long-Term Financial Contracting, *Review of Financial Studies* 20, 2079-2128.

- DeMarzo, Peter M., Michael Fishman, Zhiguo He, and Neng Wang, 2012, Dynamic Agency and the q Theory of Investment, *Journal of Finance* 67, 2295-2340.
- DeMarzo, Peter M. and Yuliy Sannikov, 2006, Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model, *Journal of Finance* 61, 2681–2724.
- DeMarzo, Peter, and Yuliy Sannikov, 2011, Learning, termination and payout policy in dynamic incentive contracts. mimeo.
- Eisdorfer, Assaf, 2008, Empirical evidence of risk shifting in financially distressed firms. *The Journal of Finance* 63, 609-637.
- Eisdorfer, Assaf, 2009, Convertible Debt and Risk-Shifting Incentives, *Journal of Financial Research* 32, 423-447.
- Ericsson, Jan, 2000, Asset substitution, debt pricing, optimal leverage and maturity, Debt Pricing, Optimal Leverage and Maturity.
- Fleming, Wendell H. and H. Mete Soner, 1993, *Controlled Markov Processes and Viscosity Solutions*, Springer-Verlag, New York.
- Gabaix, Xavier, and Augustin Landier, 2006, Why has CEO pay increased so much?, No. w12365. National Bureau of Economic Research.
- Glover, Brent, and Oliver Levine, 2013, Idiosyncratic Risk and the Manager. No. 2013-E25. Carnegie Mellon University, Tepper School of Business.
- Goldstein, Itay, and Dirk Hackbarth, 2014, Corporate Finance Theory: Introduction to Special Issue, *Journal of Corporate Finance*.
- Green, Edward, 1987, Lending and the Smoothing of Uninsurable Income, in *Contractual Arrangements for Intertemporal Trade*, ed. by E.C. Prescott and N. Wallace. Minneapolis: University of Minnesota Press, 3-25.
- Graham, John R., and Campbell R. Harvey, 2001, The theory and practice of corporate finance: evidence from the field *Journal of financial economics* 60, 187-243.
- Gryglewicz, Sebastian, 2011 A theory of corporate financial decisions with liquidity and solvency concerns, *Journal of Financial Economics* 99, 365-384.

- Gryglewicz, Sebastian, and Barney Hartman-Glaser, 2014, Dynamic Agency and Real Options, working paper.
- He, Zhiguo, 2009, Optimal Executive Compensation when Firm Size Follows Geometric Brownian Motion, *Review of Financial Studies* 22, 859-892.
- He, Zhiguo, 2011, A Model of Dynamic Compensation and Capital Structure, *Journal of Financial Economics* 100, 351-366.
- He, Zhiguo, Bin Wei, and Jianfeng Yu, 2012, Optimal Long-term Contracting with Learning, available at SSRN 1991518.
- Hoffmann, Florian, and Sebastian Pfeil, 2010, Reward for luck in a dynamic agency model, *Review of Financial Studies* 23.9 3329-3345.
- Hoffmann, Florian, and Sebastian Pfeil, 2013, Delegated Investment in a Dynamic Agency Model. Working Paper.
- Holmstrom, Bengt and Paul Milgrom, 1987, Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica* 55, 303–328.
- Hori, Keiichi, and Hiroshi Osano, 2013, Managerial incentives and the role of advisors in the continuous-time agency model *Review of Financial Studies* 26, 2620-2647.
- Jensen, Michael C., and William H. Meckling, 1976, Agency Costs and the Theory of the Firm, *Journal of Financial Economics* 3, 305-360.
- John, Teresa A., and Kose John, 1993, Top-management compensation and capital structure.", *The Journal of Finance* 48, 949-974.
- Ju, Nengjiu, and Xuhu Wan, 2011, Optimal Compensation and Pay-Performance Sensitivity in a Continuous-Time Principal-Agent Model, *Management Science*, forthcoming.
- Karatzas, Ioannis and Steven E. Shreve, 1991, *Brownian Motion and Stochastic Calculus*, 2nd Edition, Springer-Verlag, New York.
- Leland, Hayne E, 1994, Corporate debt value, bond covenants, and optimal capital structure, *The Journal of finance* 49, 1213-1252.
- Leland, Hayne E, 1998, Agency costs, risk management, and capital structure, *The Journal of Finance* 53, 1213-1243.

- Leung, Raymond CW, 2014, Continuous-Time Principal-Agent Problem with Drift and Stochastic Volatility Control: With Applications to Delegated Portfolio Management, working paper
- Lins, Karl V., Henri Servaes, and Peter Tufano, 2010, What drives corporate liquidity? An international survey of cash holdings and lines of credit, *Journal of financial economics* 98, 160-176.
- Miao, Jianjun, and Alejandro Rivera, 2013, Robust Contracts in Continuous Time. Boston University working paper.
- Opler, Tim, et al, 1999, The determinants and implications of corporate cash holdings, *Journal of financial economics* 52, 3-46.
- Ou-Yang, Hui, 2003, Optimal Contracts in a Continuous-Time Delegated Portfolio Management Problem, *Review of Financial Studies* 16, 173–208.
- Panousi, Vasia, and Dimitris Papanikolaou, 2012, Investment, idiosyncratic risk, and ownership, *The Journal of Finance* 67, 1113-1148.
- Piskorski, Tomasz, and Alexei Tchisty, 2010, Optimal Mortgage Design, *Review of Financial Studies* 23, 3098-3140.
- Piskorski, Tomasz, and Mark Westerfield, 2011, Optimal dynamic contracts with moral hazard and costly monitoring, Marshall School of Business Working Paper No. FBE, 36-09.
- Prat, Julien and Boyan Jovanovic, 2010, Dynamic Incentive Contracts under Parameter Uncertainty, NBER working paper 16649.
- Quadrini, Vincenzo, 2004, Investment and Liquidation in Renegotiation-Proof Contracts with Moral Hazard, *Journal of Monetary Economics* 51, 713-751.
- Rauh, Joshua D, 2009, Risk shifting versus risk management: investment policy in corporate pension plans, *Review of Financial Studies* 22, 2687-2733.
- Sannikov, Yuliy, 2008, A Continuous-Time Version of the Principal-Agent Problem, *Review of Economic Studies* 75, 957–984.

- Schattler, Heinz and Jaeyoung Sung, 1993, The First-Order Approach to the Continuous-Time Principal-Agent Problem with Exponential Utility, *Journal of Economic Theory* 61, 331–371.
- Spear, Stephen E. and Sanjay Srivastava, 1987, On Repeated Moral Hazard with Discounting, *Review of Economic Studies* 54, 599–617.
- Subramanian, Ajay, 2003, Carrots or sticks? Optimal compensation for firm managers, The 7th Annual International Real Options Conference Program, Washington DC.
- Subramanian, Ajay, and Baozhong Yang, 2013, Optimal Dynamic Risk-Taking and Asset Substitution. Available at SSRN 2076440.
- Szydlowski, Martin, 2012, Ambiguity in Dynamic Contracts, working paper, Northwestern University.
- Szydlowski, Martin, 2011 Incentives, project choice and dynamic multitasking. No. 1525. Discussion Paper, Center for Mathematical Studies in Economics and Management Science.
- Thomas, Jonathan and Tim Worrall, 1990, Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem, *Journal of Economic Theory* 51, 367-390.
- Taylor, Lucian, 2010, Why are CEOs rarely fired? Evidence from structural estimation, *The Journal of Finance* 65.6: 2051-2087.
- Williams, Noah, 2009, On Dynamic Principal-Agent Problems in Continuous Time, working paper, University of Wisconsin at Madison.
- Williams, Noah, 2011, Persistent Private Information, *Econometrica* 79, 1233-1274.
- Zhang, Yuzhe, 2009, Dynamic Contracting with Persistent Shocks, *Journal of Economic Theory* 144, 635-675.
- Zhu, John, 2012, Optimal Contracts with Shirking, *Review of Economic Studies*, rds038.