Final Exam Solutions
Latin American Development in the Long Run

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Due: Tuesday, July 26, by 8.30am

Important instructions:

1. Work on the following questions individually.
2. Return your answers by the announced deadline. You may choose between ONE of the following. The first method is preferred, and the second is advised only for those leaving town after the course ends:
   (a) Leave your answers in the locker number W27, in the 8th floor of Edificio W, by 8.30am on Tuesday.
   (b) Send your answers via email to leopoldo@mit.edu by 8.30am on Tuesday. You should expect a confirmation of receipt sent by Leopoldo. If you don't, you must resend your original email.
3. Please be brief in all your answers.

1 State Capacity and Democratization

Consider an economy populated by λ rich agents who initially hold power, and $1-\lambda$ poor agents who are excluded from power, with $\lambda < 1/2$. All agents are infinitely lived and discount the future at the rate $\beta \in (0,1)$. Each rich agent has income θ/λ while each poor agent has income $(1-\theta)/(1-\lambda)$ where $\theta > \lambda$. The political system determines a linear tax rate, τ, the proceeds of which are redistributed lump-sum. Each agent can hide their money from the government and avoid paying taxes but in the process they lose a fraction ϕ of their income. Think of ϕ as a measure of state capacity. When the state is very weak, citizens can hide their income from the government at a very low cost. There are no other costs of taxation. The poor can undertake a revolution, and if they do so, in all future periods, they obtain a fraction $\mu(t)$ of the total income of the society (i.e., an income of $\mu(t)/(1-\lambda)$ per poor agent). The poor cannot revolt against democracy. The rich lose everything and receive zero payoff after a revolution. At the beginning of every period, the rich can also decide to extend the franchise to the poor, and this is irreversible. If the franchise is extended, the poor decide the tax rate in all future periods (as they would constitute the median voter in a democracy).

a) Suppose that the rich were guaranteed to hold power indefinitely. Find the preferred tax rate of a rich agent (τ^R). Show and explain carefully your derivations.

From the government budget constraint we know that the lump transfer t is equal to:

$$t = \tau(\theta) + \tau(1-\theta) = \tau$$

Hence, the net income of a rich individual is equal to:

$$y^R(\tau) = \frac{\theta}{\lambda}(1-\tau) + \tau = \frac{\theta}{\lambda} - \left(\frac{\theta - \lambda}{\lambda}\right)\tau$$

As $\theta > \lambda$ the rich always lose with redistribution and thus, their preferred tax rate is $\tau^R = 0$. Alternatively, notice that $\frac{\partial y^R(\tau)}{\partial \tau} = -\left(\frac{\theta-\lambda}{\lambda}\right) < 0$.

b) Suppose that the poor were guaranteed to hold power indefinitely. Find the preferred tax rate of a poor agent (τ^P). Show and explain carefully your derivations.

The net income of a poor individual is equal to:

$$y^P(\tau) = \frac{(1-\theta)}{(1-\lambda)}(1-\tau) + \tau = \frac{(1-\theta)}{(1-\lambda)} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\tau$$

In this case $\frac{\partial y^R(\tau)}{\partial \tau} = \left(\frac{\theta-\lambda}{1-\lambda}\right) > 0$ and thus the poor would like to set the highest tax rate possible as they always win with redistribution. However, notice that if the poor set a tax rate $\tau > \phi$ then all agents will prefer to hide their money and incur the cost ϕ in which case revenues would go to zero. Hence the maximum possible rate at which agents in the economy would not hide their income and decide to pay taxes is $\tau = \phi$ (at this rate they are indifferent). Hence, if the poor hold political power they will choose $\tau^P = \phi$.

c) Now suppose that $\mu(t) = \mu^l$ at all times. Also assume that $0 < \mu^l < 1-\theta$. What will be the Markov Perfect Equilibrium? (in particular, state clearly whether the poor agents will start a revolution, whether the rich agents will decide to extend the franchise and democratize and what will be the equilibrium tax rate in this economy). Give an intuition for the result.

First we need to compute the value to the poor of starting a revolution. Their stream of payoffs will be:

$$V^P(R) = \frac{\mu^l}{(1-\lambda)} + \beta\frac{\mu^l}{(1-\lambda)} + \beta^2\frac{\mu^l}{(1-\lambda)} + ... = \frac{\mu^l}{(1-\lambda)(1-\beta)}$$

The value for the poor citizens of remaining in a non-democracy (N) where the rich have all the power and set a tax rate τ^N is given by:

$$V^P(N,\tau^N) = \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\tau^N\right)$$

If we compare the payoff to the citizens from starting a revolution or remaining in a non-democracy we get:

$$\begin{aligned} V^P(N,\tau^N) - V^P(R) &= \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\tau - \frac{\mu^l}{(1-\lambda)}\right) > \\ &> \frac{1}{1-\beta}\left(\frac{1-\theta-\mu^l}{1-\lambda}\right) > 0 \end{aligned}$$

where we used the fact that $\mu^l < 1-\theta$.

Notice that even for $\tau^N = 0$ (the most preferred tax rate of the rich) the poor will not revolt in this case and hence, in equilibirum we get that the rich do not democratize, they set their preferred tax rate $\tau^N = 0$ and the poor do not revolt. Intuitively, for very low values of μ revolutions are so costly/destructive that the poor only get to keep a very small fraction of total income after they revolt and hence they prefer to remain under non-democracy (i.e. revolution is not a credible threat). Alternatively, $\mu^l < 1-\theta$ may just reflect the fact that inequality (θ) is sufficiently low such that the poor are relatively well-off in a non-democracy such that a revolution becomes less attractive.

d) Now suppose that $\mu^l \in (1-\theta, (1-\phi)(1-\theta) + \phi(1-\lambda))$. Characterize the MPE in this case. Again, specify whether the poor will start a revolution, whether the rich will democratize and give an expression for the equilbrium tax rate. Why is the restriction $\mu^l <$

$(1-\phi)(1-\theta)+\phi(1-\lambda)$ **necessary? Give an intuition for how increases in state capacity (ϕ) affect this equilibrium.**

Now we are in the case in which $\mu^l > 1-\theta$ and thus, from part c), we know that the poor will prefer to revolt if they are not offered any redistribution. We need to check whether in this situation the rich can prevent a revolution by redistributing at a positive rate, or whether they have to give up their political power and democratize. The best the rich can do is redistribute at the preferred rate of the poor which is $\tau^P = \phi$ (in this context, redistributing at this rate is equivalent to democratizing as this will be the rate chosen by the poor in a democracy).

To find the parameter restrictions under which the best possible redistribution in a non-democracy can prevent a revolution the following condition must be satisfied:

$$V^P(N, \tau^N = \phi) - V^P(R) = \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\phi - \frac{\mu^l}{(1-\lambda)}\right) > 0$$

$$\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\phi - \frac{\mu^l}{(1-\lambda)} > 0$$

This is satisfied if the following restriction holds

$$\mu^l < (1-\theta) + \phi(\theta-\lambda)$$

or rewriting this condition we get

$$\mu^l < (1-\phi)(1-\theta) + \phi(1-\lambda)$$

If this is satisfied, there exists a $\overline{\tau}^N \in (0, \phi)$ such that the rich can make the no-revolution constraint binding and they can avoid democratizing or redistributing at a too large rate in order to prevent the revolution. Such rate is given by:

$$V^P(N, \tau^N = \phi) - V^P(R) = \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\overline{\tau}^N - \frac{\mu^l}{(1-\lambda)}\right) = 0$$

$$\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\overline{\tau}^N - \frac{\mu^l}{(1-\lambda)} = 0$$

$$\begin{aligned}\overline{\tau}^N &= \left(\frac{\mu^l}{(1-\lambda)} - \frac{1-\theta}{1-\lambda}\right)\left(\frac{1-\lambda}{\theta-\lambda}\right) \\ \overline{\tau}^N &= \frac{\mu^l-(1-\theta)}{\theta-\lambda}\end{aligned}$$

Given that $\mu^l > 1-\theta$ and that $\theta > \lambda$ we know that $\overline{\tau}^N > 0$. We also know that $\overline{\tau}^N < \phi$. In sum, in the MPE, the rich do not democratize, they set a tax rate $\overline{\tau}^N = \frac{\mu^l-(1-\theta)}{\theta-\lambda}$ and the poor do not start a revolution. In this case, an increase in state capacity ϕ makes it more likely that the rich can use redistribution to prevent the revolution. If state capacity is very low then the rich can't commit to redistribute to the poor as too high tax rates will lead the citizens to hide their income from the government and the poor may then prefer to start a revolution.

e) Now suppose that $\mu(t) = \mu^l$ with probability $1-q$, and $\mu(t) = \mu^h$ with probability q, where $\mu^h > 1-\theta > \mu^l$. Construct a MPE where the rich extend the franchise, and from there on, a poor agent sets that tax rate. Determine the parameter values that are necessary for such an equilibrium to exist. Explain why extension of the franchise is useful for rich agents? Based on the the equilibrium you find, discuss the comparative statics with respect to inequality and

state capacity (i.e. whether increases in inequality or in state capacity make democratization more or less likely).

In this setting we know that whenever $\mu(t) = \mu^l$ then the poor cannot credibly threat with a revolution (as $\mu^l < (1-\theta)$) and hence in this case the rich do not democratize, they set $\tau = 0$ and the poor do not revolt. Whenever $\mu(t) = \mu^h$ we need to find restrictions on the parameters such that i) democratization is succesful in avoiding a revolution and ii) promises of redistribution at a positive rate whenever $\mu(t) = \mu^h$ are insufficient to prevent a revolution.

We know that Democracy is an absorbing state, and that in democracy the poor will constitute the median voter, choosing their most preferred rate of redistribution $\tau^P = \phi$. Then the value functions of the rich and the poor in democracy are

$$\begin{aligned} V^R(D) &= \frac{1}{1-\beta}\left(\frac{\theta}{\lambda} - \left(\frac{\theta-\lambda}{\lambda}\right)\phi\right) \\ V^P(D) &= \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\phi\right) \end{aligned}$$

For democratization to prevent a revolution we need:

$$\begin{aligned} V^P(D) &\geq V^P(R) \\ \frac{1}{1-\beta}\left(\frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\phi\right) &\geq \frac{1}{1-\beta}\frac{\mu^h}{(1-\lambda)} \end{aligned}$$

$$\mu^h \leq 1-\theta + (\theta-\lambda)\phi \equiv \mu^{**}$$

Next, we need to make sure that even the most attractive promise of redistribution from the rich to the poor whenever $\mu(t) = \mu^h$ is insufficient to prevent a revolution. The most attractive redistribution for the poor is at the rate $\tau = \phi$ (the one they would choose in a democracy). Hence, the value for the poor of remaining in a non-democracy that sets $\tau = \phi$ whenever $\mu(t) = \mu^h$ is given by:

$$V^p(N, \mu^h | \tau = \phi) = \frac{1-\theta}{1-\lambda} + \left(\frac{\theta-\lambda}{1-\lambda}\right)\phi + \beta(qV^p(N, \mu^h|\tau = \phi) + (1-q)V^p(N; \mu^l)) \tag{1}$$

Similarly,

$$V^p(N, \mu^l) = \frac{1-\theta}{1-\lambda} + \beta(qV^p(N, \mu^h|\tau = \phi) + (1-q)V^p(N; \mu^l)) \tag{2}$$

Combining (1) and (2) we solve for $V^p(N, \mu^h|\tau = \phi)$:

$$V^p(N, \mu^h|\tau = \phi) = \frac{(1-\theta) + (\theta-\lambda)\phi(1-\beta(1-q))}{(1-\beta)(1-\lambda)}$$

Then, promises of redistribution will not prevent a revolution whenever $\mu(t) = \mu^h$ if:

$$V^p(N, \mu^h|\tau = \phi) < V^P(R)$$

$$\frac{(1-\theta) + (\theta-\lambda)\phi(1-\beta(1-q))}{(1-\beta)(1-\lambda)} < \frac{1}{1-\beta}\frac{\mu^h}{(1-\lambda)}$$

$$\mu^h > (1-\theta) + (\theta-\lambda)\phi(1-\beta(1-q)) \equiv \mu^* < \mu^{**}$$

In sum, in the MPE we get that whenever $\mu^h > \mu^{**}$ revolutions are not very costly, and the first time we observe $\mu(t) = \mu^h$ there is a revolution in equilibrium as even democratization cannot prevent it. Whenever $\mu^* < \mu^h < \mu^{**}$ then promises of redistribution are not sufficient to prevent a revolution and hence whenever

$\mu(t) = \mu^h$ the rich democratize, and from that point onwards the poor choose their preferred tax rate $\tau^p = \phi$. Whenever $\mu^h < \mu^*$ then revolutions are so costly that the rich do not have to democratize and they can prevent a revolution just by promising to redistribute every time $\mu(t) = \mu^h$ at a rate that makes the citizens indifferent between revolting and not revolting.

In the context of this model, the extension of the franchise is useful for the rich as it allows them to *credibly commit* to redistribute in every state of the world. The commitment problem emerges because whenever $\mu(t) = \mu^l$ and the threat of revolution disappears, the rich will always be tempted to deviate from the promise and choose $\tau = 0$ without being punished. Anticipating this, promises of redistribution will not be sufficient to prevent a revolution whenever $\mu^* < \mu^h < \mu^{**}$ and here, democratization becomes very useful for the rich as it allows them to prevent the revolution (which is the worst outcome from their point of view as they are fully expropriated).

The comparative statics with respect to inequality (parameterized by θ in the model) are as follows: very low levels of inequality make it less likely that we will be in a situation where $\mu^l, \mu^h > 1 - \theta$ and hence the poor are sufficiently well-off in a non-democracy that a revolution is never a credible threat even if the rich set their most preferred tax rate $\tau = 0$ in every state of the world. Hence whenever θ is too low, we never observe democratization. As θ increases, the revolution constraint becomes binding such that we are in a situation with $\mu^h > 1 - \theta$ as in the case above and the rich will have to either redistribute or democratize to prevent the revolution. Notice that both the μ^* and μ^{**} thresholds decrease when θ increases, but μ^{**} decreases at a faster rate. Hence, the size of the parameter space under which democratization takes place *become smaller* following an increase in inequality. The decrease in μ^{**} caused by increases in inequality may be such that we are in a situation where $\mu^h > \mu^{**}$ where not even democratization can prevent a revolution and we observe a revolution in equilibrium whenever $\mu(t) = \mu^h$. Intuitively, for very high levels of inequality the revolution becomes very attractive for the poor as they get to expropriate the rich (who hold a very large share of income at high levels of inequality). In sum, there is a non-linear relationship between inequality and democratization, but, conditional on revolution constraint binding, the interval of the parameter space where democratization takes place tends to shrink as θ increases.

The comparative statics with respect to state capacity (ϕ) are relatively straightforward. Notice that an increase in ϕ leads to an increase in both μ^* and μ^{**} but the latter increases a faster rate. Hence, the interval of the parameter space where democratization takes place increases as ϕ increases. This is intuitive: increases in state capacity increase the capacity of the government to redistribute and this makes it more likely that democratization will succeed at preventing a revolution. An increase in state capacity makes democracy particularly attractive for the poor as they will be able to redistribute a larger share of resources towards themselves in every state of the world.

f) In the context of the MPE you found in part e) discuss what will happen in a society with absolutely no state capacity, i.e. $\phi = 0$. What will determine the political regime in this society? Give an intuition for the result.

In the extreme, think of the case of a society with no state capacity where $\phi = 0$. In this case, democratization does not help solve any commitment problems as once in democracy, the poor will find it impossible to collect any revenues to redistribute (everyone will hide their income from the government). In this context it is impossible to redistribute and the level of inequality will determine whether we remain permanently in non-democracy with $\tau = 0$ (whenever both μ^l,$\mu^h < 1 - \theta$) or whether a revolution will eventually take place in equilibrium (whenever $\mu(t) = \mu^h$ if $\mu^h > 1 - \theta$).

2 The political economy of pensions

Consider a society with two groups of individuals young (j) and old (v). There is no income inequality; everyone earns an exogenous income y. We normalize population size to 1, so y is also total income. Old people also receive a pension (T).

The utility of an individual in group k is given by:

$$u^k = c^k + \alpha^k H(T)$$

where c^k is private consumption, T is pension consumption, and $H(\cdot)$ is an utility function of pensions with standard properties ($H'(\cdot) > 0$ and $H''(\cdot) < 0$). α^k is a group-specific paramater ($k = \{j, v\}$) with $\alpha^j = 0$ and $\alpha^V = 1$. Pensions are financed with a tax τ on income (same tax for everyone), hence $c^k = (1-\tau)\, y$ for all k.

Finally the government cannot borrow or save, (this is a "pay-as-you-go" pension system) so its budget constraint is:

$$\tau \mathbf{y} = \mathbf{T}$$

2.1 Direct Democracy: Median Voter Theorem

a. **Show that preferences over policy are single-peaked and defined over a single dimension (e.g. T).**

Simplemente al sustitutir la restricción presupuestal del individuo y la del gobierno en la función de utilidad del individuo, queda claro que aunque hay dos variables de política (τ y T), las preferencias están definidas sobre una única dimensión. Esto, porque la resticción del gobierno reduce las dos dimensiones a una:

$$\begin{aligned} u^k &= c^k + \alpha^k H(T) = y(1-\tau) + \alpha^k H(T) \\ &= y\left(1 - \frac{T}{y}\right) + \alpha^k H(T) \end{aligned}$$

Llamamos ahora v^i la utilidad indirecta del individuo con respecto a la política T :

$$v^k(T) = \overline{y}\left(1 - \frac{T}{y}\right) + \alpha^k H(T)$$

Para verificar que las preferencias son de un sólo pico, tomemos cada tipo de individuo a la vez. En el caso de los jóvenes, tenemos que la utilidad depende monotónica y negativamente del gasto en pensiones. Por este motivo, las preferencias son de un sólo pico: se prefieren unas pensiones de cero a cualquier otra opción (pensiones negativas no son posibles) y cuanto más lejos de dicha política menor la utilidad de los jóvenes. En efecto, la derivada de la función indirecta de utilidad de los jóvenes con respecto a T es estrictamente negativa:

$$\begin{aligned} v^j(T) &= \overline{y}\left(1 - \frac{T}{y}\right) + \alpha^j H(T) \\ &= \overline{y}\left(1 - \frac{T}{y}\right) + (0)\, H(T) = \overline{y}\left(1 - \frac{T}{y}\right) \end{aligned}$$

$$\frac{\partial v^j(T)}{\partial T} = -1 < 0$$

En el caso de los individuos viejos, que sí se benefician con las pensiones, derive dos veces la función de utilidad indirecta para comprobar que es cóncava y por ende es de un solo pico:

$$\begin{aligned} v^v(T) &= \overline{y}\left(1 - \frac{T}{y}\right) + \alpha^j H(T) = \overline{y}\left(1 - \frac{T}{y}\right) + (1)\, H(T) \\ &= \overline{y}\left(1 - \frac{T}{y}\right) + H(T) \end{aligned}$$

$$\frac{dv^i(T)}{dT} = -1 + \alpha^i H'(T)$$

$$\frac{d^2v^i(T)}{dT^2} = \alpha^i H''(T) < 0$$

Por la concavidad de H obtenemos que las preferencias son de un solo pico. Al observar la primera derivada, observamos de hecho que inicialmente $\frac{dv^v(T)}{dT}$ será positivo puesto que las primeras unidades de pensiones generarán una utilidad marginal (H') muy alta, y que posteriormente, a medida que aumente la oferta de pensiones, dicha utilidad caerá y será superada por el término -1. El punto en donde estas dos fuerzas se compensan exactamente es el máximo único de la función; su único pico.

b. **What is the preferred level of pension expenditure by each group in society? Interpret the first order condition that determines such level, and calculate it for the case $H(T) = \log(T)$.**

A partir de lo discutido en el inciso anterior, vemos que el punto preferido de demanda de pensiones en la sociedad es cero para los individuos jóvenes: $T_j^* = 0$.

En el caso de los viejos, el monto preferido es aquel en el cual la primera derivada de $v^v(T)$ es igual a cero, es decir, donde:

$$1 = H'(T_v^*)$$

Intuitivamente, esta condición para una solución interior implica la igualdad entre el costo marginal (expresión del lado izquierdo) y beneficio marginal (lado derecho) de la ecuación. El costo asociado con incrementar en una unidad la oferta de pensiones es igual a 1 ya que al aumentar en una unidad la oferta de pensiones se debe reducir en uno el consumo de bien privado (pues dicho incremento se financia con impuestos). El individuo obtiene por este sacrificio un beneficio igual a $H'(T)$, la utilidad marginal de las pensiones. En el óptimo, costo y beneficio marginal deben igualarse, pues de lo contrario el individuo podría demandar más o menos pensiones aumentando su utilidad.

En el caso logarítmico lo anterior corresponde a $T_v^* = 1$.

c. **Suppose n is the share of young people (and $1-n$ the share of old). What would be the equilibrium policy in this society? Is single-peakedness important for this result?**

De acuerdo con lo descrito en (a), se cumplen las condiciones *suficientes* para el Teorema del Votante Mediano (preferencias de un solo pico y política definida sobre una única dimensión), por lo que la política de equilibrio sería la preferida por el votante mediano. Como aquí sólo hay dos tipos de individuos, quién es el votante mediano y el correspondiente equilibrio dependerá de si los jóvenes o los viejos son mayoría. Si los jóvenes son mayoría ($n > \frac{1}{2}$), la política de equilibrio sería $T^{pol} = 0$ y si los viejos son mayoría ($n > \frac{1}{2}$) la política de equilibrio sería $T^{pol} = T_v^*$.

Note que como el resultado viene determinado simplemente por lo preferido por el grupo mayoritario, la forma de sus preferencias, incluido el hecho de que sean de un sólo pico, realmente no importa para el resultado.

d. **Find the (utilitarian) socially optimal policy, and compare it to the political equilibrium (for a general H). Compute the policy for the case $H(T) = \log(T)$. What explanation could this model propose for the increase in pension expenditure in Colombia?**

Para hallar la política óptima, dese el punto de vista utilitarista, de provisión de pensiones, debemos maximizar la función de utilidad social benthamita:

$$\underset{\{T\}}{Max} \sum v^k = n\left[\left(1 - \frac{T}{y}\right)y\right] + (1-n)\left[\left(1 - \frac{T}{y}\right)y + H(T)\right]$$

Derivando la expresión, simplificando e igualando a 0:

$$\begin{aligned} -n + (1-n)\left[-1 + H'(T)\right] &= 0 \\ H'(T) &= \frac{1}{1-n} \end{aligned}$$

La provisión óptima de pensiones iguala el costo marginal de un incremento en las mismas (1, la reducción en la utilidad como consecuencia de un incremento en los impuestos para financiarlas) con el beneficio marginal de las pensiones ($H'(T)$, el beneficio marginal de las pensiones, multiplicado por $(1-n)$, la fracción de gente que las recibe).

Al comparar esta condición con la del equilibrio político vemos que la provisión política de pensiones sería inferior a la óptima si la mayoría de la población es joven (pues dicha población no internaliza los beneficios de las pensiones) y sería superior a la óptima si la mayoría de la población es vieja (pues no internaliza plenamente los costos).

En el caso logarítmico, vemos que la provisión óptima de pensiones sería:

$$T = 1 - n$$

Esto puede dar una explicación económica al incremento en el gasto en pensiones en Colombia: puede ser una respuesta óptima al incremento en el porcentaje de población vieja.

2.2 Representative democracy: probabilistic voting model

Now assume that citizens elect politicians who run for office. There are two candidates, A and B, and they receive some exogenous rents from power if they get elected. Denote their policy platforms with T^A and T^B. After announcing the platforms, citizens vote and the winner implements the announced platforms (that is, we assume that promises are fulfilled).

Preferences are as in the previous section except that a new parameter σ^{ik} measures the ideological bias of individual i in gruop k ($k = \{j, v\}$) for party B, and δ captures an aggregate popularity shock for party B. Assume these shocks are uniformly distributed and centered at zero: σ^{ik} has density ϕ^k, so it is distributed uniformly over $\left[-\frac{1}{2\phi^k}, \frac{1}{2\phi^k}\right]$; and δ has density ψ, so it is uniformly distributed over $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$. Putting this together, the probability that an individual i in group k votes for A, $p^{ik}(T_A, T_B)$, is given by:

$$\mathbf{p}^{ik}(\mathbf{T}_A, \mathbf{T}_B) = \begin{array}{l} 1 \text{ if } V^k(T_A) - V^k(T_B) > \sigma^{ik} + \delta \\ \frac{1}{2} \text{ if } V^k(T_A) - V^k(T_B) = \sigma^{ik} + \delta \\ 0 \text{ if } V^k(T_A) - V^k(T_B) < \sigma^{ik} + \delta \end{array} \tag{3}$$

a. **Find π_k^A, the share of voters for A in each group.**

   El individuo "pendular" que en cada grupo dividirá a aquellos que prefieren votar por A y por B será aquel para el cual σ^{ik} tome el valor $\widetilde{\sigma}^{ik} = V^k(T_A) - V^k(T_B) - \delta$. Todos los individuos situados a la izquierda de dicho valor crítico votarán en el grupo k por el partido A, es decir:

$$\begin{aligned} \pi_A^k &= \int_{\frac{-1}{2\phi^k}}^{V^k(T_A) - V^k(T_B) - \delta} \phi^k d\sigma^{ik} \\ &= \frac{1}{2} + \phi^k \left[V^k(T_A) - V^k(T_B) - \delta\right] \end{aligned}$$

b. **Find π^A, the total number of votes for A. Simplify the expression and write it, when possible, in terms of the average "ideological density" in society, $\phi = \left[n\phi^j + (1-n)\,\phi^v\right]$.**

Agregando π_A^k para todos los grupos de la sociedad:

$$\begin{aligned}\pi^A &= \sum_k \pi_A^k = \\ &n\left\{\frac{1}{2}+\phi^j\left[V^j(T_A)-V^j(T_B)-\delta\right]\right\} \\ &+(1-n)\left\{\frac{1}{2}+\phi^v\left[V^v(T_A)-V^v(T_B)-\delta\right]\right\}\end{aligned}$$

Simplificando:

$$\begin{aligned}\pi^A &= \frac{1}{2}-\phi\delta+n\phi^j\left[V^j(T_A)-V^j(T_B)\right] \\ &+(1-n)\,\phi^v\left[V^v(T_A)-V^v(T_B)\right]\end{aligned}$$

c. **Find $P(T^A, T^B)$, the probability that A wins. Show your steps.**

Sólo requerimos que la cantidad de votos hallada en (b) supere a la mitad del electorado, $\frac{1}{2}$:

$$\begin{aligned}P\left(T^A,T^B\right) &= \\ P\left(\pi^A>\frac{1}{2}\right) &= P\left(\begin{array}{c}\frac{1}{2}-\phi\delta+n\phi^j\left[V^j(T_A)-V^j(T_B)\right] \\ +(1-n)\,\phi^v\left[V^v(T_A)-V^v(T_B)\right]>\frac{1}{2}\end{array}\right) \\ &= P\left(\delta<\frac{1}{\phi}\left(\begin{array}{c}n\phi^j\left[V^j(T_A)-V^j(T_B)\right] \\ +(1-n)\,\phi^v\left[V^v(T_A)-V^v(T_B)\right]\end{array}\right)\right)\end{aligned}$$

Usamos la distribución de δ para llegar a una epxresión para esta probabilidad.

$$\begin{aligned}P\left(T^A,T^B\right) &= \int_{\frac{-1}{2\psi}}^{\frac{n\phi^j\left[V^j(T_A)-V^j(T_B)\right]+(1-n)\left[V^v(T_A)-V^v(T_B)\right]}{\phi}}\psi d\delta \\ &= \frac{1}{2}+\frac{\psi}{\phi}\left\{\begin{array}{c}n\phi^j\left[V^j(T_A)-V^j(T_B)\right] \\ +(1-n)\,\phi^v\left[V^v(T_A)-V^v(T_B)\right]\end{array}\right\}\end{aligned}$$

d. **Now find the equilibrium policy proposals. Interpret and discuss the role of ϕ^k, and again compare with the utilitarian social optimum. What are the consequences of an increase in the share of old people $(1-n)$? How do these predictions compare with the ones in the Median Voter Theorem section?**

La condición de primer orden para la elección de T_A (que concidirá con la política elegida por el partido B y será por ende la del equilibrio dada la simetría del problema) es la siguiente:

$$n\phi^j\frac{\partial V^j(T_A)}{\partial T_A}+(1-n)\,\phi^v\frac{\partial V^v(T_A)}{\partial T_A}=0$$

En este caso concreto ello equivale a

$$n\phi^j\,(-1)+(1-n)\,\phi^v\,(-1+H'\,(T))=0$$

Podemos escribir esta condición a partir de nuestra definición de la densidad promedio y tras simplificar como:

$$\frac{\phi}{(1-n)\,\phi^v}=H'\,(T)$$

A medida que crece el tamaño de la población vieja $(1-n)$ o la densidad ideológica de dicho grupo (ϕ^v) en relación con la densidad promedio, entonces aumenta el gasto en pensiones que ofrecen los candidatos (recuerde que $H(\cdot)$ es cóncava). En linea con la idea del modelo de votación probabilística según la cual, a pesar de que todos los individuos tienen un voto que nominalmente vale lo mismo, en realidad hay algunos votantes más poderosos que otros, cuanto más numerosos ($1-n$ más grande) y cuanto menos sesgado ideológcamente (ϕ^v más alto) sea la población vieja, más serán tenidas en cuenta sus preferencias por los políticos. De manera análoga, si los jóvenes son relativamente más numerosos y menos ideológicos, sus preferencias serán más tenidas en cuenta (el lado izquierdo de la ecuación aumentará) y caerá el monto de transferencias depensiones.

Note que en este contexto, el monto de transferencias cambia continuamente a medida que un grupo se vuelve políticamente más poderoso que otro, y no discontinuamente como en el caso del TVM, donde el grupo más numeroso lograba imponer exactamente su política preferida. Además, una vez sucedía esto, las pensiones aumentaban al nivel preferido por los viejos y permanecían en dicho nivel, en lugar de incrementarse continuamente ante mayores incrementos de la población vieja.

Para entender la importancia del parámetro que captura la densidad ideológica, imagine que hay un grupo más ideológico que el otro, en el sentido de que sus integrantes tienen una preferencia muy fuerte por el partido A o el B. Por lo tanto, la densidad ideológica será menor ya que dicho grupo contará con una mayor dispersión a lo largo del espacio $(-\frac{1}{2\phi^k}, \frac{1}{2\phi^k})$ de preferencias ideológicas. Entre tanto, el otro grupo, siendo poco "ideológicos" estará más concentrados en el centro de la distribución, inclinándose a votar por un partido u otro simplemente por los beneficios "económicos" que le entreguen. Los individuos en dichos grupos son votantes "pivotales" o "pendulares" (*swing voters*) ya que son los que más responden a los cambios en las plataformas de los partidos y son, por este motivo, los más tenidos en cuenta a la hora de ofrecer dichas plataformas.

Note además que en general, el equilibrio no será igual al obtenido bajo la maximización de una función de utilidad benthamita tradicional, excepto en el caso en el cual la densidad ideológica de cada grupo es igual ($\phi^k = \phi$ para todo k)

3 The Political Economy of Civil Wars

Using the papers on the reading list and the lectures, do you think that political or economic factors are more important in explaining the incidence of civil war?

Be brief. Write no more than 3 pages on 12pt font.

There is no right answer to this question. Here are key questions we had in mind when grading it:

- Are the claims made logically coherent and backed up with evidence? Are the claims convincing and the evidence sufficient?
- Are counterarguments explicitly acknowledged and is a rebuttal of these counterarguments attempted?
- Does the evidence cited show an understanding of the readings presented in class?
- Does the answer limit itself to the discussion of the reading connected with Civil Wars, or does it also connect to the overarching bigger ideas of the course?

You get a better grade if we can convincingly answer YES to the questions above.