Human Capital Formation, Inequality, and Competition for Jobs*

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Abstract

This paper develops a model where heterogeneous agents compete for the best available jobs. Firms, operating with different technologies, rank job candidates in the human capital dimension and hire the best available candidate due to complementarities between the worker's human capital and technologies used in the production process. As a result, individuals care about their relative ranking in the distribution of human capital because this determines the firm they will be matched with and therefore the wage they will receive in equilibrium. The paper rationalizes a different channel through which peer effects and human capital externalities might work: competition between individuals for the best available jobs (or prizes associated with the relative position of individuals). We show that more inequality in the distribution of endowments negatively affects aggregate efficiency in human capital formation as it weakens the incentives to compete for the best available jobs. However, we find that, at least in many of the cases analysed, the opposite is true for wage inequality, namely, more wage inequality encourages competition and, as a result, agents exert more effort and accumulate more human capital in equilibrium.

Keywords: Human Capital, Inequality, Competition, Relative Ranking.

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1. Introduction

This paper develops a model of human capital accumulation and competition for jobs where there are strategic interactions between heterogeneous agents that compete for the best available jobs. We argue that higher inequality in the distribution of the endowments necessary to accumulate human capital negatively affects the equilibrium level of average human capital in the economy, which we will refer to as 'aggregate efficiency in human capital formation'. This effect is beyond the standard Jensen's inequality channel because inequality also affects individuals' incentives to accumulate human capital when they confront competition from their close peers for the best available jobs. Intuitively, as the mass of close competitors for any given job position increases, the incentives to differentiate from each other, by exerting higher effort and accumulating more human capital, also increases. However, we find that more wage inequality (i.e. more inequality in the returns to human capital accumulation) can have the opposite effect (at least for relatively low levels of wage inequality).[1] Namely, we find that more wage inequality can increase aggregate efficiency in human capital formation because it fosters competition for job positions between individuals, inducing them to exert more effort in the accumulation of human capital.[2] In equilibrium, individuals' optimal choices depend on both, the distribution of endowments that are complementary to time and effort invested in the accumulation of human capital (the distribution of opportunities), and on the distribution of wages (the distribution of returns to human capital accumulation). As we will show, changes in the degree of inequality in each of these distributions have opposite effects on individuals' choices and on aggregate efficiency in human capital formation.

On the demand side of the labor market, we will assume that firms, operating with different technologies, rank individuals in the human capital dimension and hire the best available candidate due to complementarities between human capital and technologies used in the production process. On the supply side, in choosing the optimal level of investment

[1] As will be clear below, we find a positive relationship between wage inequality and the average level of human capital in the economy in the simple model and in one of the simulations of the general model for a specific form of the distribution of wages. For the other form of the distribution of wages that we use we find that aggregate efficiency in human capital formation increases with wage inequality for low levels of wage inequality but decreases as wage inequality becomes too high.

[2] The results we obtain regarding wage inequality should be handled with care. Our model dos not include dynamic elements so we are only pointing out one potential positive effect of wage inequality on aggregate efficiency in human capital formation. However, there are other potential channels through which wage inequality might negatively affect human capital formation. See, for instance, footnote 38.

in the accumulation of human capital, individuals take two effects into account when evaluating the marginal benefit from exerting effort in the accumulation of human capital. The first effect is the usual direct marginal increase in income that results from a marginal increase in human capital (as in a standard model *à-la*-Becker (1964)). The second effect comes from the marginal change in the relative position of the individual that results when she invests one extra unit of time and effort in the accumulation of human capital, which, in turn, determines her relative position in the human capital distribution and, thus, the firm she will be matched with and the wage she will receive in equilibrium. As a result of this last effect, there is a so-called "rat-race" where individuals try to out-compete other individuals for the best available jobs. We will assume that individuals' human capital is remunerated according to its marginal product and thus, although more effort is exerted in equilibrium when individuals compete with each other for the best available job positions than in a standard model where there is no competition, this so-called 'excessive competition' is fully efficient. This is because the pricing of human capital in the labor market fully compensates individuals for their extra investment. In fact, when making the optimal decision on the amount of investment in human capital, individuals trade-off the disutility from exerting extra effort in the competition for jobs for the greater utility they obtain from being able to match with firms that operate with better technologies (that is, with firms that pay higher wages). If labor markets were not fully competitive and, for instance, wages were determined by Nash bargaining between firms and employees, then the excessive competition would not be fully efficient.[3,4]

Our model assumes that individuals' concerns for relative ranking are instrumental. That is, individuals care about their relative position in the distribution of human capital not because they derive utility from relative ranking *per se*, but because their relative position determines the firm they will be matched with and the wage they will receive in equilibrium. On the other hand, we will assume that firms care about the relative ranking (in the distribution of human capital) of the individual they hire because the technologies they use in production are complementary to the worker's human capital and, therefore, they would like to hire the worker with the highest human capital available in the market.[5]

[3] See Moen (1999).

[4] An interesting question that would arise if one does not assume that wages are determined competitively in the labor market is: under what conditions would there be too much investment in human capital when individuals have a concern for relative ranking? However, the focus of this paper is on the effects of inequalities of opportunities and wages on aggregate efficiency in human capital formation when individuals have concerns for relative ranking. The assumption of a perfectly competitive labor market is made to keep the analysis as simple as possible.

[5] We don't explicitly model the process by which firms choose the worker they hire, but, instead, assume

The literature on how inequality affects human capital formation has focused mostly on the role of credit market imperfections, wherein relatively poor individuals face financial constraints to pay for the costs associated with human capital accumulation, as they cannot use future earnings as collateral for the loans necessary to cover these costs. Furthermore, if there are decreasing returns to human capital accumulation, it is precisely these individuals (the relatively poor ones) who have the largest returns to resource investments in education. As a result, a redistribution of resources from rich to poor individual would increase aggregate efficiency in the accumulation of human capital because of the reallocation of resources towards more profitable investments. This theoretical idea has been extensively developed in the literature since the works by Galor and Zeira (1993) and Banerjee and Newman (1993). Other developments have been proposed by De Gregorio (1996) and Bénabou (1996, 2000).[6] Empirical evidence has been found in favor of the hypothesis that inequality affects human capital accumulation in the presence of credit constraints (see Flug et al., 1998 and De Gregorio, 1996). In a recent paper, Mejía and St-Pierre (2008) show that inequality in the endowments that are complementary to effort in the schooling process (inequality of opportunities) affects aggregate efficiency in the accumulation of human capital without relying on credit market imperfections. The argument in that paper is that there are crucial complementary factors to the schooling process that are non-purchasable when the time for making investment decisions in education comes (i.e. parental schooling levels, pre and post natal care, etc.). Because there are decreasing returns to time investment in human capital accumulation, and time investment in education is complementary to these factors, more inequality negatively affects aggregate human capital. Other papers in the literature have also explored political economy channels through which inequality affects human capital formation and economic growth. In particular, Glomm and Ravikumar (1992) and Ferreira (2001) emphasize the choice of public versus private schooling made through a political process as a key determinant of how inequality affects human capital formation.

The main contribution of this paper is to provide a rationale for a new, perhaps complementary, channel through which the inequalities of endowments and returns affect the incentives for human capital accumulation. An important difference with the existing literature is that the model we propose in this paper includes strategic interactions between individuals. That is, an individual's return from the accumulation of human capital depends not only on his own choices and on the production technologies, but also on the

that due to complementarities in production, all firms would like to hire the best available candidate in the labor market.

[6] See Aghion et al. (1999) for a throrough review of the literature.

entire distribution of endowments and returns. In other words, we argue that in deciding the optimal investment in human capital formation, there are strategic interactions between individuals. In this respect our model is also related to existing works on human capital externalities, and to the literature on peer effects in education. While most of the empirical literature on peer effects has focused on the effect of average education of peers on different measures of individual student's educational attainment (that is, on linear-in-means peer effects), two recent papers find that, in fact, the structure of peer effects is highly non-linear. That is, students benefit differently from the inclusion of a new student in the class depending on their relative position in the class and the relative position of the entering student. In particular, students benefit significantly more from the inclusion in their class of new students that are similar to them (see Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006), just as our model would predict. Human capital externalities have also been modeled in the literature as an average mean effect, that is, it is average human capital in the economy that affects each individual's marginal productivity in production (Lucas, 1988). Importantly, in the existing literature on peer effects and human capital externalities individuals benefit from being close to more educated students or colleagues because of close collaboration and spillovers in the classroom or in the workplace. Our paper departs from the existing literature in two important aspects in this respect. First, individuals are affected differently from an entering student in their cohort depending on their relative position and the relative position of the entering student. In particular, an individual is affected more by the choices made by those individuals close to her in the distribution than by the choices of individuals who are very different (as was shown empirically by Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006). And, second, we argue that individuals are affected by other individuals not because of close collaboration and spillover effects in the classroom or the workplace but because they are competing with each other for the best available jobs. While our model does not rule out important effects due to collaboration and cooperation, we propose another channel through which peer effects or human capital externalities might work: competition for the best available jobs (or other relevant prizes associated with relative position). Thus, our paper has important implications (predictions) for the empirical literature on peer effects and human capital externalities. Namely, we argue that in measuring human capital externalities or peer effects one should not only account for the mean human capital in the population but, also, for higher moments of the distribution of education. In particular, human capital externalities (due to competition) should be larger in societies with less inequality of opportunity and, also, in those parts of the distribution of endowments with a greater mass of individuals.

Also, the model predicts that peer effects and human capital externalities associated with competition should be larger in environments where the prizes associated with the relative position in the final dimension (grades, achievements, etc.) are more differentiated.

In addition to this introduction, the paper contains four sections. Section 2 discusses how concerns for relative position have been introduced in the economic literature and presents a short review of related contributions. In Section 3 we present the simple version of the model with two individuals and two firms. In section 4 we develop the general model. Section 5 concludes.

2. Concerns for relative ranking in the economics literature

Since the seminal work of Thorstein Veblen (1899), *A Theory of the Leisure Class*, several economists have argued that concerns for status (or the relative position in some relevant dimension(s)) have important economic consequences.[7] A central discussion in the literature that deals with concerns for relative ranking has to do with how we should understand such concerns, that is, whether they are direct or instrumental. While in the former case people have concerns for status because they obtain utility from having high status in its own sake, in the latter people care about status because status directly affects the goods and services that individuals ultimately consume (Postlewaite, 1998). While the strongest argument for incorporating direct concerns for relative position in the utility function is an evolutionary one,[8] the case for not incorporating direct concerns for status in the utility function is that economic models that incorporate them typically allow for very diverse behavior, there are almost no restrictions on equilibrium behavior and, as a result, the models lose predictive power.[9] In other words, differences in individual's preferences over status may directly account for differences in equilibrium choices.

Most of the contributions that have emphasized the importance of concerns for relative ranking have focus on conspicuous consumption. The idea is the following: because wealth is unobservable, the consumption of conspicuous goods serves as a signal of non observable ability. Furthermore, if there are complementary interactions between individuals (for instance, at the household level between men and women, or at the workplace between

[7] The reader is refered to Bastani (2007) for a thorough review of the literature on concerns for relative ranking.

[8] As Postlewaite (1998) explains, the desire to ascend to the top of the social hierarchy may have had selection value over the course of human evolution (and thus may be hardwire in humans) as high-ranked members usually enjoy access to better mates, more food, etc. which increases their survival probability and that of their offspring.

[9] See Postlewaite (1998).

employees and employers) conspicuous consumption might be welfare enhancing, even when the costs of conspicuous consumption[10] are taken into account, as they allow for a better (more efficient) matching (among others, see Cole et al., 1992 and 1995, Bagwell and Bernheim, 1996, and Rege, 2000). While concerns for status might generate excessive competition, this does not mean that excessive competition is inefficient (as has been argued by Frank, 1999 and others). In fact, when status can be purchased in a competitive market, the cost of acquiring status is simply a transfer payment that adds to the seller's wealth. For instance, Becker and Murphy (2000, ch. 4) show that competition for mates is fully efficient if the value that someone brings to the marriage is fully priced. In the same book, Becker, Murphy and Werning take Frank's (1999) example of wearing high heels and argue that "the demand for high heels is efficient, even when such shoes cause foot and back damage, if the marriage, or other, markets that match men and women compensates women fully for the utility gain to their husbands or other companions from their wearing high heels. This behavior is efficient even when it lowers the relative attractiveness of other women, including women who also wear high heels." (see Becker and Murphy, 2000, ch. 8). In fact, when women decide to wear high heels they trade-off the cost of wearing high heels for the utility gain they obtain from getting better husbands. Thus, wearing high heels can be understood as an equilibrium outcome of a game where women compete with each other for the best available partners.

Only a few contributions in the economics literature on human capital and labor markets have incorporated concerns for relative ranking. In particular, Moen (1999) studies the incentives to invest in human capital in a model with labor market frictions and unemployment. In his model, an unemployed worker's chances of getting a job depends on his human capital relative to that of other unemployed workers because firms prefer to hire the most productive applicant due to rent sharing between them and the workers. Relative ranking affects the job finding rate and, as a result, there is a rat-race between unemployed individuals competing for job positions. Because wages are assumed to be determined by rent sharing between firms and workers (that is, the gains from education will not fully accrue to the workers in the form of higher wages) excessive competition might lead to inefficient overinvestment in human capital.

The most related contribution to this paper is a recent paper by Hopkins and Kornienko (2006). They study the effects of inequality in a tournament model where individuals compete for different rewards. Individuals, given their resources, make a simultaneous

[10] Conspicuous consumption (or "Veblen effects") exists when consumers are willing to pay a higher price for a functionally equivalent good (see Bagwell and Bernheim, 1996).

investment and output decision and then each individual is rewarded according to her relative position. The authors also emphasize the differential effect of inequality of resources and of inequality of rewards on individual equilibrium choices. While our main focus is on the relationship between inequalities of opportunities and wages and aggregate efficiency, theirs is on how changes in inequality of resources and rewards affect welfare for different segments of the population. In particular, they find that more inequality of resources lowers utility for agents in the middle and upper parts of the distribution, whereas an increase in the inequality of resources leads to lower utility for the relatively poor agents in society.[11]

3. The 2 individuals - 2 firms model.

This section presents a simple model with two individuals and two firms that captures some of the main results that will also arise in the general model presented in the next section of the paper.

3.1. Firms

Let us assume that there are two firms, $j = \{l, h\}$, that produce a single homogeneous good, q_j, using a production function that combines a technology level, a_j, and human capital, h_j, to produce the final good according to:

$$q_j = a_j * h_j, \tag{1}$$

where: $a_j > 0$ is the technology used by firm $j = \{l, h\}$. Assume, without loss of generality, that $a_h > a_l$. h_j is the human capital of the individual hired by firm j. Furthermore, we assume that each firm hires only one individual.[12]

We will assume that firms pay workers their marginal product per unit of human capital employed in production. That is, firm l pays the worker it hires $w_l = a_l$ per unit of human capital and firm h pays the worker it hires $w_h = a_h$ per unit of human capital employed in the production process.

[11] Galí and Fernandez (1999) also develop a tournament model of competition for places at college but their main interest was to compare the efficiency of two different mechanisms in allocating rewards: markets vs. tournaments.

[12] One can also think about one firm that has two job positions, each one operating with a different technology level.

In this framework job positions differ in their payments because different firms operate with different technologies. The assumption that the production technology is linear in human capital greatly simplifies the analysis and, also, allows us to isolate the standard effect of inequality in the distribution of human capital on aggregate production efficiency that works through Jensen's inequality (see Mejía and St-Pierre, 2008).[13] The standard Jensen's inequality arises if the amount of output produced is a concave function of human capital and, as a result, a more unequal distribution of this factor of production across individuals reduces aggregate production efficiency.

Because technologies are complementary to human capital in the production process, all firms would like to hire the individual with the highest human capital available in the labor market. That is, we assume that firms rank individuals in the human capital dimension and that they make job offers to the individual with the highest human capital who is available in the job market. In the matching process between firms and workers, the firm that operates with the advanced technology will hire the individual with the high human capital (it can offer her a higher wage than the firm that operates with the low technology so the individual with high human capital will accept this offer). On the other hand, the firm that operates with the low technology, although it would like to hire the individual with the high human capital, ends up hiring the individual with low human capital because the other individual is no longer available in the labor market. In other words, there is a perfectly assortative matching between firms and individuals in the labor market.

3.2. Individuals

There are two individuals with endowments of the complementary factors to the schooling process equal to θ_p and θ_r. θ_i with $i = \{p, r\}$ can be thought of as a measure of opportunities for human capital accumulation, where opportunities are a combination of all factors that complement individual's effort in the educational process, such as parental education, school and teacher quality, etc. Without loss of generality we assume that $\theta_r \geq \theta_p$. That is, individual r (the rich individual) has a larger (or equal) endowment of the complementary factors than individual p (the poor individual).

[13] This assumption also implies that the distribution of wages is independent of the distribution of human capital in the economy, which also simplifies the analysis and allows us to isolate changes in the distribution of returns to human capital accumulation from changes in the distribution of endowments. Nevertheless, the analysis that follows would go through with any production function where human capital and technology are complements.

Individuals accumulate human capital combining effort, e, and the complementary factors to the schooling process, θ, according to the following human capital production function:

$$h = h(e, \theta), \tag{2}$$

Assumption $A1 : h(\cdot, \cdot)$ is differentiable, $h_e(\cdot, \cdot) > 0$, $h_\theta(\cdot, \cdot) > 0$, $h_{ee}(\cdot, \cdot) < 0$, $h_{\theta\theta}(\cdot, \cdot) < 0$, $h_{e\theta}(\cdot, \cdot) > 0$ and $\lim_{e \to +\infty} h_e(e, \theta) = C > 0$ and $h(\gamma e, \theta) = \gamma^\kappa h(e, \theta)$ for all θ (homogeneity of degree κ in e)

According to $A1$, human capital is an increasing and strictly concave function of both effort and the complementary factors, and the marginal effect of effort on the accumulation of human capital is increasing in the complementary factors. In other words, effort is complementary to the endowment of the complementary factors in the production of human capital. Also, effort is strictly necessary for the accumulation of human capital.

Each individual i maximizes a utility function that depends positively on consumption and negatively on effort. Furthermore we assume that the utility function is separable in the two arguments.[14] Each individual's problem is:

$$\max_{\{e\}} U(c, e) = u(c) - v(e) \tag{3}$$

Assumption $A2 : u(c)$ and $v(\cdot)$ are differentiable, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{e \to +\infty} v'(e) = +\infty$, $\lim_{e \to 0} v'(e) = 0$.

Given that the model is static, consumption equals income which, in turn, is equal to the expected wage per unit of human capital times the stock of human capital that the individual brings to the labor market. That is, consumption equals the expected wage times the amount of human capital, $E(w) * h$.

We will assume that, from the individuals' perspective, the decision of the firm operating with the high technology to hire individual i includes a stochastic element. To allow for this stochastic element, let $p(h_i, h_j) \in [0, 1]$ be the probability, as perceived by individual i, that the firm that operates with the advances technology will hire her. This probability depends on individual i's human capital as well on individual j's human capital and is determined by the contest success function specified in Assumption $A3$.

[14]This is perfectly equivalent to a situation where consumption and leisure are the arguments in the utility function and where leisure time is sacrificed when time and effort are invested in the accumulation of human capital.

Assumption $A3 : p(h_i, h_j) = \dfrac{h_i^m}{h_i^m + h_j^m}$ for some $0 < m \leq 1$.

Notice that $A3$ implies that $p_{h_i} > 0$, $p_{h_j} < 0$, $p_{h_i h_i} < 0$ and that p is homogenous of degree 0 in (h_i, h_j). That is, the probability of being hired by the advanced technology firm for individual i increases as her human capital increases, decreases with the human capital of the other individual, and is strictly concave on h_i. Moreover, we assume that the probability is homogenous of degree zero meaning that only the relative level of human capital matters. Notice that the specification in $A3$ is more general than it seems as it is the only specification of a contest success function satisfying a very minimal set of assumptions including homogeneity of degree zero.[15]

Individual i's expected wage is given by:

$$E(w_i) = p(h_i, h_j) * w_h + (1 - p(h_i, h_j)) * w_l,$$

Assuming, again, without loss of generality, that $u(c) = c$, individual i takes individual j's effort as given and chooses her own effort to maximize her utility.[16] Individual i's problem is:

$$\max_{\{e_i\}} E(w_i)h(e_i, \theta_i) - v(e_i). \tag{4}$$

The first order condition of individual i's problem is:[17]

$$\frac{\partial p(h_i, h_j)}{\partial h_i} \frac{\partial h_i(\hat{e}_i, \theta_i)}{\partial e_i}(w_h - w_l)h(\hat{e}_i, \theta_i)) + E(w_i)h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) = 0 \tag{5}$$

The second and third term on the left hand side of equation 5 are the standard terms in models of human capital accumulation *à-la*-Becker - Ben-Porath: the direct marginal benefit and cost from exerting one extra unit of effort in the accumulation of human capital. The first term captures how an extra unit of time and effort allocated to the accumulation of human capital affects the probability of being hired by the firm that pays the high wage (that is, the firm operating with the advanced technology). In other words, when firms pay different wages because, for instance, they operate with different technologies, individuals

[15] See Skaperdas (1996).

[16] In other words we assume that the agents are playing a game of competition for the best available jobs by making simultaneous human capital investment decisions.

[17] Assumptions A1 through A3 guarantee that the maximization problem in equation 4 has a unique and interior solution and that the first order condition in equation 5 is sufficient.

have an extra incentive to invest time and effort in the accumulation of human capital to increase the probability of being hired for the best available job.[18]

3.3. Labor market equilibrium and comparative statics

A Nash Equilibrium (NE) of the game of competition for jobs is a pair of strategies $\{e_r, e_p\}$ that simultaneously satisfy the first order conditions for both agents, r and p, in equation 5. The reader may check that the assumptions above are sufficient to ensure that the reaction functions for both individuals satisfy the sufficient conditions for the existence of a fixed-point, hence a NE.

It is also worth noticing that for any given pair of endowments, θ_p and θ_r, the NE is unique and, moreover, the rich individual always exerts more effort than the poor one, provided her endowment is strictly larger than the one of the poor individual (as it is by assumption). The following proposition makes this point.

Proposition 1: For any pair of endowment (θ_p, θ_r), corresponds a unique NE (e_p, e_r) for which $e_r \geq e_p$ if $\theta_r \geq \theta_p$.

$Proof$: see the appendix.

Before proceeding further with the comparative statics, it is worth specifying a benchmark case, where individuals do not take into account the effect of effort on the probability of being hired by the firm operating with the high technology (the first term in equation (5). In the benchmark case individuals either take as given the probability of being hired by the firm operating with the advanced technology, or, alternatively, take as given the expected wage. The important point of setting up the benchmark case is that individuals are not able to affect the probability of being hired by the advanced technology firm by exerting more effort even though this probability is computed by assuming that they do. In the benchmark case the first order condition for individual i is:

$$E(w_i)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0, \tag{6}$$

Proposition 2: Effort and hence human capital accumulation are higher when individuals compete for job positions than in the benchmark case (where there is, by assumption, no competition between individuals for the job that pays the high wage).

[18] Standard models of human capital accumulation do not incorporate this effect because they implicitly assume that all available jobs operate with the same technology. As a result, there is no incentive for competition between applicants as the wage rate per unit of human capital is the same in all available jobs.

Proof : see the appendix.

Intuitively, when there is competition between individuals for the best available job positions they will exert more effort because they have an extra incentive to accumulate human capital beyond the standard marginal benefit (the second term in equation (5). This extra incentive is the marginal increase in the probability of being hired by the firm operating with the advanced technology that results form an extra unit of time and effort allocated to the accumulation of human capital.

For the next two results, we perform some comparative statics on the equilibrium level of effort. Let $(e_p(\theta_r, \theta_p, w_h, w_l), e_r(\theta_r, \theta_p, w_h, w_l))$ be the equilibrium mapping the exogenous variables $(\theta_r, \theta_p, w_l, w_h)$ to the (unique) equilibrium pair of efforts. Notice that this mapping is differentiable given assumptions A1 to A3.

Proposition 3: Higher inequality in the distribution of the complementary factors decreases average human capital in the economy.

Proof : see the appendix.

Because the probability of being hired by the advanced technology firm is a strictly concave function of effort and the endowment of the complementary factors and effort are complements in the accumulation of human capital, a higher degree of inequality in the distribution of endowments reduces aggregate effort invested in the accumulation of human capital. That is, when inequality increases, the probability perceived by the poor individual of being hired by the firm with the advanced technology decreases more than the same probability perceived by the relatively rich individual increases.

Proposition 4: Higher inequality in the distribution of wages increases average human capital in the economy.

Proof: see the appendix.

Notice that inequality of endowments and inequality of returns (wages) affect differently the incentives to compete for the best available job positions. While more inequality of endowments disincentives competition, more wage inequality does the opposite.[19]

In order to have some sense of the magnitude of the effect of inequality in the endowments of the complementary factors, and of inequality in returns, on effort and human

[19] These results are in line with those obtained by Hopkins and Kornienko (2006).

capital accumulation, Figures 1(a) and (b) present the results obtained from the calibration of the model presented above.[20] Average effort and hence average human capital accumulation are higher when individuals compete for job positions than in the benchmark case (Proposition 1).[21] This is seen in Figure 1a by comparing the two lines for any given level of endowment inequality. Note also from this figure that as inequality increases average human capital in the economy decreases, but in the case of competition for jobs it decreases faster (Proposition 2). Figure 1b shows how average human capital changes as the difference between wages in the two available job positions increases for a given level of endowment inequality. As the wage difference becomes larger, individuals have higher incentives to compete for the high paying job position and, thus, they exert more effort and accumulate more human capital (Proposition 3).

[INSERT FIGURES 1 (a) AND (b) HERE]

4. The model with a continuum of individuals and firms

4.1. Firms

Suppose that there is a continuum of firms indexed by j that produce a homogeneous good according to the following production function:

$$q_j = a_j * h_j,$$

where, as in equation 1, $a_j > 0$ is the technology used by firm j and h_j is the human capital of the individual hired by firm j. Assume that each firm hires only one individual. Furthermore, assume that $a_j \sim H(a)$.[22] There is perfect competition in the labor market so firms remunerate human capital according to its marginal product. That is, the wage rate paid by firm j is equal to a_j. Wages, therefore, are distributed according to $H(w)$.

[20] We use the following functional forms for the calibration of the model: $h(e_i, \theta_i) = Ae_i^{\alpha}\theta_i^{1-\alpha}$, with $0 < \alpha < 1$, $p(h_i, h_j) = \frac{h_i}{h_i+h_j}$ (that is, we assume that $m = 1$ in A3), and $e_i = \frac{e_i^2}{2}$. Note that these functional forms satisfy assumptions $A1$ through $A3$. We set $A = 1$ and $\alpha = 3/4$. The results presented in Figure 1 (a) and (b) are robust to large variations of these parameters.

[21] For the benchmark case we take the probability of being hired by the advanced technology firm to be the probability that would arise if the two agents had engaged in a contest for the high paying position. Note that in this case individiduals take as given the probability that results in equilibrium but cannot affect it by exerting more effort.

[22] We assume that the CDF $H(.)$ is strictly increasing and continous.

4.2. Individuals

There is a continuum of individuals indexed by i. As in the two agents - two firms model, each individual has a given endowment, θ_i, of the factors that complement time and effort in the educational process. The endowments of the complementary factors are distributed in the population according to $G(\theta)$, with support in $[a, b]$. Human capital is accumulated (produced) using individuals' effort and the complementary factors, according to $h(e, \theta)$ (same as in equation 2). The human capital production function satisfies $A1$ above.

Individuals derive utility from consumption and disutility from effort according to:

$$U(c, e) = u(c) - v(e). \tag{7}$$

The utility function in equation 7 satisfies $A2$.

4.3. Matching between firms and workers in the labor market

Following the approach of Hopkins and Kornienko (2004), if we let $F(h)$ be the distribution of human capital across individuals, individual i's ranking in the distribution of human capital will be given by:

$$\gamma F(h(e, \theta) + (1 - \gamma) F^{-}(h(e, \theta)), \tag{8}$$

where $F^{-}(h) = \lim_{h' \to h} F(h)$ is the mass of individuals with human capital strictly less than h,[23] and $\gamma \in [0, 1)$ is a parameter that captures the decrease in the payoff from "ties".[24]

We will assume that, in hiring workers, firms rank individuals according to their human capital and hire the best candidate available in the market due to complementarities in production between the worker's human capital and technologies. Thus, the firm with the

[23] A simpler definiton of rank would be just $F(h)$ (as in Frank, 1985). The problem with this definition is that, if all agents accumulate the same level of human capital, $\widehat{h}$, then $F(\widehat{h}) = 1$, and all agents would have the highest ranking, and since there is a continumm of individuals, each having zero weight, an individual that increases her investment in human capital just above $\widehat{h}$ would see no increase in her ranking (see Hopkins and Kornienko, 2004).

[24] If all agents were to choose the same level of human capital, $\widehat{h}$, then they would all have ranking γ whereas if one individual chooses a level of human capital slightly greater than $\widehat{h}$ her ranking would be 1 ($> \gamma$) (see Hopkins and Kornienko, 2004).

most advanced technology would like to hire the individual with the highest human capital available in the market (the individual that ranks first in the distribution of human capital), the firm ranked second would like to hire the individual with the highest human capital available in the market (the individual who ranks second in the distribution of human capital), and so on and so forth. That is, there is a perfectly assortative matching between firms and individuals in the labor market.

Recalling that $H(a)$ denotes the distribution of technologies across firms and that the wage rate is equal to the marginal product of human capital, $w_j = a_j$, then individual i's ranking in the distribution of human capital exactly coincides with her ranking in the distribution of wages in the economy. That is:

$$\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta)) \;= H(w_i) \qquad \Rightarrow \tag{9}$$

$$R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] = w_i, \tag{10}$$

where w_i is the wage rate per unit of human capital that individual i receives in equilibrium and $R = H^{-1}$ is the inverse function (the quantile function) of the CDF of w.[25]

4.4. Individuals' optimization problem

Individuals take as given other individuals' effort and choose their own effort, e, to maximize $U(c,e)$.[26] In the following, we will assume that $u(c) = c$ without loss of generality[27]. As usual, the objective of an agent with endowment $\theta \in [a,b]$ is to solve the following problem:

$$\max_{e\,\in\,[e_a,+\infty]} R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) - v(e), \tag{11}$$

where $R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta)$ $(= w * h)$ is the level of income (and consumption) that an agent with an endowment θ will attain and where $e_a > 0$ is the minimum level of effort possible.[28]

[25] Recall that we have assumed before that the CDF $H(.)$ is strictly increasing and continous, so it has an inverse (quantile) function.

[26] That is, individuals play a simultaneous move game of competition for jobs.

[27] Our results only depend on the utility function being monotonically increasing in c.

[28] An alternative way to interpret this assumption is to suppose that the penalty for not exerting a minimum level of effort is arbitrarily large. This normalization does not affect the qualitative results.

Assumption $A4$**:**

(i) $v(\cdot)$ is differentiable, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{\theta \to +\infty} v'(\theta) = +\infty$ (from $A2$),

(ii) $h(e, \theta) = h(t(e, \theta))$ where[29] $t(e, \theta) = s(\theta)e$ such that: h and s are differentiable, $h > 0$, $h' > 0$, $h'' \leq 0$, $s > 0$, and $s' > 0$

(iii) $R(\cdot)$ is differentiable with $R'(\cdot) \geq 0$

4.5. Labor market equilibrium

A symmetric Nash Equilibrium(NE) is a mapping $e : [a, b] \to [e_a, +\infty$ that assigns a choice of effort $e(\theta)$ for any possible endowment level θ, where $e(\theta)$ is chosen to solve the problem in equation 11. By assumption $A4$ above, the NE can also be described by a mapping $t : [a, b] \to \mathbb{R}_+$ where $t(\theta) = s(\theta)e(\theta)$.

Let $h^{eq}(\theta) \equiv h(t(e(\theta), \theta))$ be the equilibrium human capital mapping. The next results provide a characterization of the solution mapping $t(\cdot)$.

Proposition 5: If the solution $t(\cdot)$ exists then:
(i) $t(\cdot)$ is strictly increasing
(ii) $t(\cdot)$ is continuous
(iii) $t(\cdot)$ is differentiable.

Proof: see the Appendix.

Proposition 6: Under $A4$, a solution function (symmetric Nash Equilibrium of the game of competition for jobs) $t(\cdot)$ exists, is unique, and is characterized by the following differential equation with the initial condition $t(a) = s(a)e_a$.

$$t'(\theta) = -\frac{R'(G(\theta)g(\theta)h(t)}{R(G(\theta))h'(t) - \frac{1}{s(\theta)}v'(\frac{t}{s(\theta)})} \tag{12}$$

Proof: see the Appendix.

4.6. Inequality of endowments, inequality of wages, and human capital accumulation.

As argued in the simple model presented in section 3, inequality of endowments (opportunities) and inequality of wages (returns) affect the incentives to invest time and effort

[29] With a slight abuse of notation.

in the accumulation of human capital in a different way. While more inequality of opportunities disencourages competition for the best available jobs and, as a result, individuals exert less effort and accumulate less human capital in equilibrium. However, more inequality of wages, by increasing the incentives to compete for the best available job positions, induces higher competition between agents and, hence, aggregate (and average) human capital accumulation is higher.

In order to evaluate these predictions in the general model we will use functional forms for the distribution of wages (technologies) in the economy and for the distribution of endowments across individuals. As we will explain below the functional forms that we will use for the two distributions will allow us to solve explicitly for the equilibrium level of effort for each agent in the Nash equilibrium of the game of competition for jobs. Importantly, these functional forms will also allow us to implement increases in inequality without affecting the mean of each distribution. In other words, we will study how effort and human capital accumulation respond as we implement a mean-preserving spread in the distribution of endowments, and, separately, a mean-preserving spread in the distribution of wages.

Assumption $A5$ (wage distribution): Let wages be distributed across firms (or job positions) according to the following CDF:

$$H(w;\kappa) = \begin{cases} 0 & for \quad w < 0 \\ Kw^{\kappa} & for \quad w \in \left[0, \frac{1+\kappa}{\kappa}\right] \\ 1 & for \quad w > \dfrac{1+\kappa}{\kappa} \end{cases}, \tag{13}$$

where $K = \left(\dfrac{\kappa}{1+\kappa}\right)^{\kappa}$ and with $\kappa \in (0, 1]$.

The Appendix describes in detail some of the main characteristics of the wage distribution function defined in equation 13. However, a few points are worth mentioning about this particular distribution: first, the mean wage is always equal to 1 ($E(w) = 1 \quad \forall \kappa$). Second, the parameter κ is an inverse measure of wage inequality. That is, as κ increases wage inequality decreases. In particular, as κ increases, two commonly used measures of inequality respond in the expected way. That is, the ratio of the median to the mean wage increases (i.e. there is less wage inequality), and the wage Gini coefficient decreases (which, again, means that there is less wage inequality).[30]

Assumption $A6$ (distribution of endowments): Let endowments be distributed across individuals according to the following CDF:

[30]See Mejia and St-Pierre (2008).

$$G(\theta;\phi) = \begin{cases} 0 & for \quad \theta < 0 \\ \Phi\theta^{\phi} & for \quad \theta \in \left[0, \frac{1+\phi}{\phi}\right] \\ 1 & for \quad \theta > \dfrac{1+\phi}{\phi} \end{cases}, \tag{14}$$

where $\Phi = \left(\dfrac{\phi}{1+\phi}\right)^{\phi}$ and with $\phi \in (0, 1]$.

As in the case for the wage distribution, in this case the mean endowment is always equal to 1, and, for the endowment distribution described in equation 14, the parameter ϕ is an inverse measure of inequality in the distribution of endowments. That is, as ϕ increases endowments are more equally distributed across agents, the median to the mean endowment increases, and the Gini coefficient for the distribution of endowments decreases.[31]

Assumption $A7$ (human capital production function and cost function): Let us assume that the human capital production function takes the following functional form:[32]

$$h(e,\theta) = Ae^{\alpha}\theta^{1-\alpha}, \quad \text{with} \quad \alpha \in (0,1). \tag{15}$$

Also, we will assume that the function $v(e)$ takes the following functional form:[33]

$$v(e) = e^{\mu}, \quad \text{with} \quad \mu > 1. \tag{16}$$

4.6.1. Equilibrium effort, average human capital, and comparative static results

Recall from equation 9 that individual i's ranking in the distribution of wages coincides exactly with her ranking in the distribution of human capital in the population. From equation 9 we can obtain the wage rate that each individual gets in equilibrium by taking the inverse function of the wage distribution (see equation 10). Using the distribution functions 13 and 14, the function $R(G(\theta)) \equiv H^{-1}(G(\theta))$ from equation 12 is given by:

$$R(G(\theta)) = \left[\frac{G(\theta)}{K}\right]^{\frac{1}{\kappa}} = \left[\frac{\Phi}{K}\right]^{1/\kappa} \theta^{\frac{\phi}{\kappa}} \tag{17}$$

Using the distribution functions for wages (technologies) and for endowments in equations 13 and 14 respectively, and equations 15, 16, and 17, the solution to the differential equation that describes the Nash equilibrium of the game of competition for jobs is:[34]

[31] These results follow exactly in the same way as for the wage distribution (see the appendix for details).

[32] This functional form staisfies all conditions in Assumption $A1$.

[33] This functional form satisfies the conditions in Assumption $A2$.

[34] See the appendix for the derivation of this result.

$$\widehat{e} = \left[\left(\frac{\Phi}{K} \right)^{\frac{1}{\kappa}} \left(\frac{\alpha A}{\mu} \right) \frac{\mu + \mu \frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha \frac{\phi}{\kappa} - \mu\alpha} \right]^{\frac{1}{\mu - \alpha}} \theta^{\frac{1 + \frac{\phi}{\kappa} - \alpha}{\mu - \alpha}}. \tag{18}$$

Notice that the first term in the right hand side of equation 18 is increasing in ϕ.[35] However, the term $\theta^{\frac{1+\frac{\phi}{\kappa}-\alpha}{\mu-\alpha}}$ is decreasing in ϕ for $\theta < 1$. In other words, for a given value of the parameter κ, some individuals (specifically, those with a relatively high θ) will increase their optimal level of effort as inequality in endowments decreases (ϕ increases) whereas others (those with a relatively low θ) will decrease their optimal level of effort as inequality decreases. Recall that the simple model with two firms and two individuals had the same result. Namely, an increase in endowment inequality in the simple model decreases the optimal level of effort for the relatively poor individual and increases it for the rich individual. Furthermore, proposition 3 showed that average human capital decreases as endowment inequality increases because the decrease in human capital for the relatively poor individual is larger than the increase in human capital for the rich individual.

In order to study how average human capital changes as endowment inequality increases (that is, as ϕ decreases) in the general model, we need to find an expression for the average level of human capital in the economy. Using the human capital production function from equation 15, the distribution function for endowments from equation 14, and the expression for the equilibrium level of effort from equation 18, the average level of human capital in the economy is given by:

$$E(h) = \int_0^{\frac{1+\phi}{\phi}} h(\widehat{e}, \theta) g(\theta) d\theta = C^{\alpha} \Phi \phi \frac{\left(\frac{1+\phi}{\phi} \right)^{\alpha\left(\frac{1+\phi/\kappa - \alpha}{\mu - \alpha}\right) + \phi - \alpha + 1}}{\alpha \left(\frac{1 + \phi/\kappa - \alpha}{\mu - \alpha} \right) + \phi - \alpha + 1}, \tag{19}$$

where $C = \left[\left(\frac{\Phi}{K} \right)^{\frac{1}{\kappa}} \left(\frac{\alpha A}{\mu} \right) \frac{\mu + \mu \frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha \frac{\phi}{\kappa} - \mu\alpha} \right]^{\frac{1}{\mu - \alpha}}$ is the first term in the right hand side of equation 18. Note that average human capital depends on the parameters of the human capital production technology, preference parameters, as well as on the parameters of the distributions of endowments and wages. The next step is to understand how average human capital in the economy - our measure of aggregate efficiency in human capital formation -

[35] The third term inside the parenthesis of equation 18 depends positively on ϕ because, by assumption, $\mu > 1$, $\alpha < 1$, so $\mu > \alpha$.

changes as the two measures of inequalities in the economy change: endowment inequality and wage inequality. Given the complexity of the expression for average human capital found in equation 19 we conduct numerical simulations for a broad range of parameter values.

First, we study how the average level of human capital in the economy changes as inequality in endowments changes. In order to do this we do numerical simulations of equation 19 moving the parameter ϕ from 0.05 to 1 in the distribution function of endowments (equation 14). Recall that the distribution function that we are using has a constant average θ for any value of the parameter $\phi \in (0, 1]$. More precisely, $E(\theta) = 1 \ \forall \phi \in (0, 1]$. In other words, by changing the parameter ϕ we are inducing a mean-preserving spread in the distribution of endowments and, using equation 19, we can see how average human capital in the economy reacts to this change. In Figure 2, instead of using ϕ as the (inverse) measure of inequality in endowments we construct a commonly used (and direct) measure of inequality, namely, the Gini index for the distribution of endowments.[36] Figures 2a to 2c show the results of the numerical simulations for the average level of human capital as a function of the Gini index for endowments. In each of the figures we simulate the relationship between these two variables for wide variations in the other parameters of the model (α in Figure 2a, κ in Figure 2b, and μ in Figure 2c).

[INSERT FIGURES 2a TO 2c HERE]

In all cases we observe a negative relationship between aggregate efficiency in human capital formation (average human capital in the economy) and the measure of inequality in the distribution of endowments (inequality of opportunities). This result is, yet again, consistent with the results obtained with the simple model. As was explained before, while some agents (the relatively poor) reduce their effort when endowment inequality increases, others (the relatively rich) increase their effort as endowment inequality increases. At the aggregate level, however, we have a reduction of average human capital (and of average effort[37]) in the economy as endowments inequality (our measure of inequality of opportunities) increases, as it is shown in Figures 2a to 2c.

Second, we study how average human capital in the economy changes as inequality in the distribution of wages (that is, inequality in the distribution of technologies across firms) changes. In this case we vary the parameter κ from 0.05 to 1 in the distribution function of wages (equation 13). As in the case for the distribution of endowments, we have that

[36] The same results hold if we use the mean over median ratio as the measure of inequality of opportunities.

[37] We do not show the results for average effort as endowments inequality increases, but the reveal the same decreasing pattern observed for average human capital. The graphs for the change in average effort are available from the authors upon request.

$E(w) = 1 \; \forall \kappa \in (0, 1]$. In this case, by changing the parameter κ we are inducing a mean-preserving spread in the distribution of wages in the economy. Figures 3a to 3c present the main results of these simulations for a wide range of values for the other parameters of the model (α in Figure 3a, κ in Figure 3b, and μ in Figure 3c). In contrast with the results from the simple model, for the general model we obtain a non monotonic relationship between wage inequality and aggregate efficiency in human capital formation. As can be seen from Figures 3a to 3c, for relatively low levels of wage inequality more inequality in the distribution of wages increases the incentives to compete for the best available jobs and, thus, increases the average level of human capital in the economy. However, if wage inequality becomes too high, more inequality in the distribution of wages actually reduces aggregate efficiency in human capital formation.[38] This happens because, as κ goes down sufficiently relative to ϕ (that is, as wage inequality becomes high enough given a certain degree of endowment inequality) the objective function becomes too convex in effort, e, and, as a result, the relationship between wage inequality and average human capital in the economy reverses. That is, it becomes negative.

[INSERT FIGURES 3a TO 3c HERE]

4.6.2. Simulations with a different distribution function for endowments and wages

Despite the fact that closed form solutions to the Nash equilibrium of the game of competitions for jobs cannot be found for many other functional forms for distributions $G(.)$ and $H(.)$ one can, in principle, solve the model numerically with other distributions in order to check whether the main predictions of the model still hold.[39]

In order to evaluate the main qualitative results of the model with a different distribution we solve equation 12 numerically using the same functional forms for the human capital production technology (equation 15) and the disutility from exerting effort (equation 16) and assume, without loss of generality, that $e(a) = 1$, that is, the agent with the lowest endowment of the complementary factors exerts a level of effort equal to 1.[40] In the first

[38] Our model does not incorporate dynamic elements, so the results obtained regarding wage inequality should be handled with care. For instance, if wages in period t determine somehow the endowments in period $t+1$, then more wage inequality in period t would also mean more endowment inequality in period $t+1$ which would reduce average human capital in $t+1$. This is a natural extension of this paper that we leave for future work.

[39] We are also restricted to distribution functions where one can numerically implement mean-preserving spreads.

[40] Recall that we have been assuming, without loss of generality, that $u(c) = c$.

set of calibrations[41] we will fix the degree of inequality in the distribution of wages by assuming that $H(w) \sim U(0,1)$. That is, we assume that technologies (and therefore wages) are distributed according to a standard uniform. Also, we assume that $G(\theta) \sim U(1-\varepsilon, 2+\varepsilon)$ and, in doing the mean preserving spread in the distribution of endowments, we will increase the parameter ε from 0 to 0.5.

Figures 4 (a) and (b) present the results of the numerical solutions of the general model for two different values of the relative importance of effort in the accumulation of human capital, α (see equation 15). Each figure shows the result of the simulation of a mean preserving spread in the distribution of endowments (an increase in the parameter ε in the uniform distribution of endowments $G(\theta)$). As can be seen in these figures, more inequality in the distribution of endowments (higher ε) is associated with lower aggregate efficiency in human capital formation (as measured by average human capital in the population). In other words, the simulations of the equilibrium of the model using the uniform distribution also suggest that there is no trade-off between equality of opportunity and aggregate efficiency in human capital formation, just as in the case with 2 agents and 2 firms, and the case presented above where we were able to find a closed form solution of the equilibrium of the game.

[INSERT FIGURES 4a and 4b HERE].

Figures 5 (a) and (b) present the results of a similar exercise but this time we fix the degree of inequality in the distribution of endowments and do a mean preserving spread in the distribution of wages. In both cases (when α, the relative importance of effort in the accumulation of human capital, is 3/4 and 1/2) a higher degree of inequality in the distribution of wages is associated with higher aggregate efficiency in human capital formation. This result is consistent with the one obtained for the simple model and, at least partly, with the result obtained in the general model when we use the distribution functions is equations 13 and 14. Namely, more wage inequality (more inequality of returns) fosters competition for the best available job positions between individuals and, thus, induces individuals to exert more effort and accumulate more human capital in equilibrium.

[INSERT FIGURE 5 HERE].

[41] We use a 4(5) imbedded pair Runge-Kutta Scheme called the Dormand-Prince 4(5) (explicit) scheme to solve numerically the differential equation 12 (see Ascher and Petzold, 1998, ch. 4). We thank Lydia Boroughs for kindly helping us with this methodology.

5. Concluding remarks

This paper develops a model where heterogeneous agents compete for the best available job positions. One of the main working assumptions is that different firms operate with different technologies and, as a result, have open job positions with different remuneration. Because technologies are complementary to human capital in the production process, the firm operating with the most advanced technology is matched with the individual with the highest human capital in the job market, the second firm with the second individual in the distribution of human capital and so on and so forth. As a result of this, when individuals are choosing the optimal investment in human capital formation not only do they take into account the benefit of a marginal increase in their human capital and the marginal cost of one extra unit of investment, but, also, how that extra unit of time and effort invested in the accumulation of human capital affects their relative position in the distribution, which, in turn, affects the firm they will be matched with and, as a result, the wage they will receive in equilibrium.

We propose a new channel through which inequality affects aggregate efficiency in human capital formation. In particular we find that a more equal distribution of the endowments that are complementary to time and effort in the educational process increases aggregate efficiency in human capital formation. However, more inequality in the returns to human capital accumulation, by increasing the incentives to compete for the best available job positions, can, under certain circumstances, increase the incentives to compete for the best available jobs and, therefore, increase average human capital formation in the economy. With respect to this result we recognize that we are only pointing out one reason, perhaps important, as for why more wage inequality can provide more incentives for the accumulation of human capital. However, there are other channels through which wage inequality can be harmful for aggregate efficiency in the accumulation of human capital and, in general, aggregate production efficiency. In addition to the reason exposed in footnote 38, one can think that the accumulation of human capital also requires the investment of financial resources (and not only time and effort) and therefore more wage inequality might affect negatively human capital formation (as, for instance, in the channel proposed in Galor and Zeira, 1993).

Importantly, this paper proposes a different, perhaps complementary, explanation for the existence of peer effects and/or human capital externalities. While the explanation so far advanced in the literature for the existence of peer effects and human capital externalities is based on close collaboration, cooperation, and spillovers between individuals in the classroom or in the workplace, our explanation is based on a different story: individuals compete with each other for the best available job positions (or for differentiated prizes associated with relative ranking in the human capital dimension). That is, while in the traditional explanations for the existence of peer effects there are positive externalities from the accumulation of human capital between individuals (because of cooperation and collaboration), in the alternative rationale that we propose in this paper, an increase in human capital accumulation by one individual creates a negative externality on other individuals (specially on those that are very similar to her) forcing them to respond strategically with, also, more human capital accumulation so that they don't loose their relative position in the distribution of human capital. The proposed model rationalizes a mechanism for non-linear peer effects. In particular we argue that in estimating peer effects or human capital externalities one should not only take into account the first moment of the distribution of opportunities or human capital (as most of the empirical literature on peer effects has done) but also should account for higher moments of the distribution. Also, according to the intuition developed in the model about how peer effects and human capital externalities operate through competition, one should expect larger peer effects in environment where prizes associated with the relative position in a final dimension (grades, achievement, etc.) are more differentiated.

Although the paper was silent about the welfare effects (aggregate and individual) generated by competition between individuals for the best available jobs we recognize that there are important aspects to be studied in this respect. The paper's silence in this important respect is in part explained by the fact that this is precisely the main focus of attention of Hopkins and Kornienko (2006) in a similar and more general setting.

Appendix

Proof of Proposition 1.

If $\theta_p = \theta_r$ then the players are symmetric and will choose $e_r = e_p$. Uniqueness follows from the fact that $p(\cdot,\cdot)$ is homogenous of degree 0 so that the probability of getting the high wage w_h is constant ($= 1/2$) in any NE and that $E(w_i)h(e_i,\theta_i) - v(e_i)$ is (strictly) concave in e_i for $i = p, r$.

If $\theta_p < \theta_r$ then a proof by contradiction is appropriate. Indeed, suppose that $e_r < e_p$. Then the following inequality should hold:

$$E(w_p)h(e_p,\theta_p) - v(e_p) > \underline{E(w)}h(e_r,\theta_p) - v(e_r) > 0$$

$$E(w_r)h(e_r,\theta_r) - v(e_r) > \overline{E(w)}h(e_p,\theta_r) - v(e_p) > 0,$$

where $\underline{E(w)} = p(h(e_r,\theta_p), h(e_r,\theta_r))$ and $\overline{E(w)} = p(h(e_p,\theta_r), h(e_r,\theta_p))$

In turn, this implies that,

$$(E(w_p) - \underline{E(w)})h(e_p,\theta_p) > (\overline{E(w)} - E(w_r)\frac{h(e_r,\theta_r)}{h(e_p,\theta_r)})h(e_p,\theta_r)$$

Since $\frac{h(e_r,\theta_r)}{h(e_p,\theta_r)} < 1$ and $\underline{E(w)} = \overline{E(w)}$ by the assumption that $p(\cdot,\cdot)$ is homogenous of degree 0 in (e_p, e_r) and that $h_{e\theta} > 0$, this last inequality cannot hold. Therefore, $e_r > e_p$ if $\theta_r > \theta_p$. It remains to show uniqueness. By contradiction, suppose there exist two distinct NE (e_p^1, e_r^1) and (e_p^2, e_r^2) where $e_r^2 > e_r^1$. If $e_p^2 \geq e_p^1$ then (e_p^1, e_r^1) cannot be an equilibrium because player r would have a profitable deviation by choosing a higher level of effort $e' > e_r^1$. Otherwise, $e_p^2 < e_p^1$ and a similar argument holds for player p, this time eliminating the second NE. END OF PROOF

Proof of Proposition 2.

If $p_{e_i} = \frac{\partial p(h_i,h_j)}{\partial h_i}\frac{\partial h_i(e_i,\theta_i)}{\partial e_i} > 0$, that is, if the probability of individual i being hired by the firm operating with the advanced technology increases as her human capital increases (i.e. as his effort increases), then $p_{e_i}(w_h - w_l)h(\hat{e}_i,\theta_i)) > 0$ and, using equation 5, $E(w_i)h_{e_i}(\hat{e}_i,\theta_i) - v'(\hat{e}_i) < 0$. However, in the benchmark case, $E(w_i)h_{e_i}(e_i^*,\theta_i) - v'(e_i^*) = 0$ and so it must be that $\hat{e}_i > e_i^*$ since the function $E(w_i)h(e_i,\theta_i) - v(e_i)$ is strictly concave in e_i, as it is by assumptions $A1$ through $A3$. END OF PROOF.

Proof of Proposition 3.

Define the average endowment of the complementary factors as $\overline{\theta}$, and let $\theta_r = \overline{\theta} + \delta$, and $\theta_p = \overline{\theta} - \delta$. The parameter δ captures inequality in distribution of the complementary factors. With this definition, the larger is δ, the larger is inequality in the distribution of the complementary factors. Consistent with this notation, the equilibrium mapping can be written as $(e_p(\delta), e(\delta))$ given (w_l, w_h). Accordingly, let the $p_i(\delta) = p_i(h_i(\delta), h_j(\delta))$ be the probability of player i being hired by the firm operating with the advanced technology.

We now show that $p_r(\delta)$ is increasing in δ. First, $p_r(\delta)$ is a strictly monotonic function. Otherwise, there exists $\delta_1 < \delta_2$ such that $p_r(\delta_1) = p_r(\delta_2)$ and $p_p(\delta_1) = p_p(\delta_2)$. But then, necessarily, $e_p(\delta_2) < e_p(\delta_1)$ and $e_r(\delta_2) > e_p(\delta_1)$ leading to $h_p(\delta_2) < h_p(\delta_1)$ and $h_r(\delta_2) > h_r(\delta_1)$. Since $p(\cdot, \cdot)$ is homogenous of degree 0 in h_p, h_r then $p_r(\delta_1) \neq p_r(\delta_2)$ and $p_p(\delta_1) \neq p_p(\delta_2)$. Since $e_r(\delta) > e_p(\delta)$ for any $\delta > 0$ then $p_r(\delta)$ is increasing in δ.

Notice that $p_r(\delta)$ being increasing in δ implies that $e_r(\delta)$ is also (strictly) increasing in δ and that $e_p(\delta)$ is (strictly) decreasing in δ.

Finally, in equilibrium and under the assumptions $A1 - A4$, the first order condition in equation 5 can be rewritten as follows:

$$[((m+1)p_i(\delta) - mp_i^2(\delta))(w_h - w_l) + w_l]h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) = 0$$

Since the marginal benefit of increasing δ on the marginal benefit of effort, $\frac{d}{d\delta}([((m+1)p_r(\delta) - mp_r^2(\delta))(w_h - w_l) + w_l]h_{e_r}(\hat{e}_r, \bar{\theta} + \delta))$ is positive and smaller than the corresponding marginal loss of benefit for the poor individual, $-\frac{d}{d\delta}([((m+1)p_p(\delta) - mp_p^2(\delta))(w_h - w_l) + w_l]h_{e_p}(\hat{e}_p, \bar{\theta} - \delta))$, it follows that $\frac{de_r(\delta)}{d\delta} < \frac{de_p(\delta)}{d\delta}$. In turn, this implies that $\frac{dh_r(\delta)}{d\delta} < -\frac{dh_p(\delta)}{d\delta}$ or that $\frac{dh_r(\delta)}{d\delta} + \frac{dh_p(\delta)}{d\delta} < 0$. END OF PROOF.

Proof of Proposition 4.

Let $w_h = \bar{w} + \epsilon$ and $w_l = \bar{w} - \epsilon$ for some $\bar{w} > 0$ and $\epsilon \geq 0$. Under this set-up, an increase in ϵ corresponds to a mean preserving spread around the mean $\bar{w}$. This time, the equilibrium mapping can be written as $(e_p(\epsilon), e(\epsilon))$ given (θ_p, θ_r). Accordingly, let $p_i(\epsilon) = p_i(h_i(\epsilon), h_j(\epsilon))$ be the probability of player i winning the contest given any $\epsilon \geq 0$.

We now show that $p_r(\epsilon)$ is decreasing in ϵ. First, we show that $p_r(\epsilon)$ is (strictly) monotonic. By contradiction, suppose it exists $\epsilon_2 > \epsilon_1$ such that $p_r(\epsilon_2) = p_r(\epsilon_1)$. As in the proof of proposition 3, the assumptions $A1 - A3$ yields the following f.o.c.:

$$[((m+1)p_i(\epsilon) - mp_i^2(\epsilon))2\epsilon + \bar{w} - \epsilon]h_{e_i}(e_i, \theta_i) = v'(e_i)$$

Let $A_r(\epsilon) = (m+1)p_r(\epsilon) - mp_r^2(\epsilon)$ and $A_p(\epsilon) = (m+1)p_p(\epsilon) - mp_p^2(\epsilon)$. Notice that both $A_r(\epsilon)$ and $A_p(\epsilon)$ are strictly increasing and concave in p. Since $p_r(\epsilon) > 1/2$ for all ϵ by proposition 1 (and therefore $A_r(\epsilon) > 1/2$) then $e_r(\epsilon_2) > e_r(\epsilon_1)$. Let $\eta > 1$ such that $\frac{e_r(\epsilon_2)}{e_r(\epsilon_1)} = \eta$. We will show next that $\frac{e_p(\epsilon_2)}{e_p(\epsilon_1)} \neq \eta$.

Taking the ratio of the f.o.c. above for individual r, we obtain:

$$\frac{2\epsilon_2 A_r(\epsilon_2) + \bar{w} - \epsilon_2}{2\epsilon 1 A_r(\epsilon_1) + \bar{w} - \epsilon_1}\left(\frac{1}{\eta}\right)^{1-\alpha} = \frac{v'(e_r(\epsilon_2))}{v'(e_r(\epsilon_1))}$$

If $\frac{e_p(\epsilon_2)}{e_p(\epsilon_1)} = \eta$ then we have a similar expression for individual p, that is:

$$\frac{2\epsilon_2 A_p(\epsilon_2) + \bar{w} - \epsilon_2}{2\epsilon 1 A_p(\epsilon_1) + \bar{w} - \epsilon_1}\left(\frac{1}{\eta}\right)^{1-\alpha} = \frac{v'(e_p(\epsilon_2))}{v'(e_p(\epsilon_1))}$$

Observing that $\frac{2\epsilon_2 A_r(\epsilon_2) + \bar{w} - \epsilon_2}{2\epsilon 1 A_r(\epsilon_1) + \bar{w} - \epsilon_1} > \frac{2\epsilon_2 A_p(\epsilon_2) + \bar{w} - \epsilon_2}{2\epsilon 1 A_p(\epsilon_1) + \bar{w} - \epsilon_1}$ implies that $\frac{v'(e_r(\epsilon_2))}{v'(e_r(\epsilon_1))} > \frac{v'(e_p(\epsilon_2))}{v'(e_p(\epsilon_1))}$ which is impossible under the current assumptions. Thus, $\frac{e_p(\epsilon_2)}{e_p(\epsilon_1)} \neq \eta$ and therefore $p_r(\epsilon_2) \neq p_p(\epsilon_2)$. That is, $p_r(\epsilon)$ is a strictly monotonic function for $\epsilon \in [0, \bar{w}]$.

We now show that $p_r(\epsilon)$ is strictly increasing. To do so, let $\epsilon_2 > \epsilon_1$ with $\epsilon_2 = \bar{w}$ and $\epsilon_2 = 0$. Taking the ratio of the f.o.c. for individual r and p respectively, we obtain:

$$2\bar{w}A_r(\bar{w})\left(\frac{1}{\eta_r}\right)^{1-\alpha} = \frac{v'(e_r(\bar{w}))}{v'(e_r(0))}$$

$$2\bar{w}A_p(\bar{w})\left(\frac{1}{\eta_p}\right)^{1-\alpha} = \frac{v'(e_p(\bar{w})}{v'(e_p(0))}$$

where $\eta_i = \frac{e_i(\bar{w})}{e_i(0)}$

Since $A_r(\bar{w}) > A_p(\bar{w})$ and since $\frac{v'(e_i(\bar{w}))}{v'(e_i(0))}$ is non-decreasing in η_i then $\eta_p < \eta_r$. In turn, this implies that $\frac{e_r(\bar{w})}{e_r(0)} > \frac{e_p(\bar{w})}{e_p(0)}$ and $p_r(\bar{w}) > p_r(0)$. Therefore $p_r(\epsilon)$ is increasing.

To complete the proof, it remains to show that average human capital increases as ϵ increases. Again, consider $\epsilon_2 > \epsilon_1$ and the ratio of their f.o.c. in the same order:

$$\frac{2\epsilon_2 A_r(\epsilon_2)+\bar{w}-\epsilon_2}{2\epsilon 1 A_r(\epsilon_1)+\bar{w}-\epsilon_1}\left(\frac{1}{\eta_r}\right)^{1-\alpha}=\frac{v'(e_r(\epsilon_2))}{v'(e_r(\epsilon_1))}$$

$$\frac{2\epsilon_2 A_p(\epsilon_2)+\bar{w}-\epsilon_2}{2\epsilon 1 A_p(\epsilon_1)+\bar{w}-\epsilon_1}\left(\frac{1}{\eta_p}\right)^{1-\alpha}=\frac{v'(e_p(\epsilon_2))}{v'(e_p(\epsilon_1))}$$

Clearly, $\eta_r > 1$ and $\eta_r > \eta_p$ but there are two possibilities for η_p. Either $\eta_p > 1$ or $\eta_p < 1$. In the former case, the result is immediate as both individuals augment their human capital. When $\eta_p < 1$, what becomes crucial are the ratios $a_r \equiv \frac{2\epsilon_2 A_r(\epsilon_2)+\bar{w}-\epsilon_2}{2\epsilon 1 A_r(\epsilon_1)+\bar{w}-\epsilon_1}$ and $a_p \equiv \frac{2\epsilon_2 A_p(\epsilon_2)+\bar{w}-\epsilon_2}{2\epsilon 1 A_p(\epsilon_1)+\bar{w}-\epsilon_1}$. It is possible to show that $a_r a_p > 1$ so that $e_r(\epsilon_2) - e_r(\epsilon_1) > e_p(\epsilon_1) - e_r(\epsilon_2) > 0$ proving the result. END PROOF

Proof of Proposition 5.

(Part of this proof is adapted from similar proofs in Hopkins and Kornienko (2004)).

STEP 1: we show that $t(\cdot)$ is increasing.

The result follows directly from the observation that θ only affects the choice of t through the disutility of effort $v\left(\frac{t}{s(\theta)}\right)$. Moreover, since $\frac{\partial^2 v}{\partial t \partial \theta} = -\left[\frac{s'(\theta)}{s(\theta)^2}v'\left(\frac{t}{s(\theta)}\right) + \frac{ts'(\theta)}{s(\theta)^3}v''\left(\frac{t}{s(\theta)}\right)\right] <$ 0 and $v'' > 0$ then it follows that the solution $t(\cdot)$ is increasing, provided it exists.

STEP 2 We show that $t(\cdot)$ is continuous.

Notice that $t(\cdot)$ being strictly increasing implies the continuity of $R(\gamma F(h^{eq}(\cdot) + (1 - \gamma)F^{-}(h^{eq}(\cdot)))$, that is

$$\lim_{\theta\to\hat{\theta}} R(\gamma F(h^{eq}(\theta) + (1-\gamma)F^{-}(h^{eq}(\theta)))$$
$$= R(\gamma F(h^{eq}(\hat{\theta}) + (1-\gamma)F^{-}(h^{eq}(\hat{\theta}))) = R(F(h^{eq}(\theta)))$$

Furthermore, since $v(\cdot)$, $h(\cdot)$ are continuous then a standard continuity argument applies, which we briefly sketch in the following. By contradiction, suppose $t(\cdot)$ is not continuous, so there is a jump in the equilibrium solution at some endowment level of the complementary factors $\hat{\theta} \in [a, b]$ so that $\lim_{\theta\to\hat{\theta}} t(\theta) = \hat{t} \neq t(\hat{\theta})$. The rest is completed by showing that there exists an individual with level of endowment in a close neighborhood of $\hat{\theta}$ which would have a profitable unilateral deviation.

STEP 3 We show that $t(\cdot)$ is differentiable on $[a, b]$. From the previous steps, recall that $\gamma F(h^{eq}(\theta)) + (1-\gamma)F^{-}(h^{eq}(\theta)) = F(h^{eq}(\theta))$ for all $\theta \in [a, b]$. That is, there are no mass points.

Let $\hat{\theta} = \theta + \delta$ for some $\delta > 0$. We have,

$$R(F(h(t(\theta)))h(t(\theta)) - v\left(\frac{t(\theta)}{s(\theta)}\right) \geq R(F(h(t(\hat{\theta})))h(t(\hat{\theta})) - v\left(\frac{t(\hat{\theta})}{s(\theta)}\right)$$

Similarly, we have,

$$R(F(h(t(\hat{\theta})))h(t(\hat{\theta})) - v\left(\frac{t(\hat{\theta})}{s(\hat{\theta})}\right) \geq R(F(h(t(\theta)))h(t(\theta)) - v\left(\frac{t(\theta)}{s(\hat{\theta})}\right)$$

By the Mean Value theorem we have,

$$R(F(h(t(\hat{\theta})))h(t(\hat{\theta})) = R(F(h(t(\theta)))h(t(\theta))) +$$
$$(R'(F(h(t_1))f(h(t_1)h'(t_1))h(t_1) + R(F(h(t_1)))h'(t_1)))(t(\hat{\theta}) - t(\theta)) \text{ for some } t_1 \in [0, \infty$$

In turns, this implies that:

$$\left[v\left(\frac{t(\theta)}{s(\hat{\theta}))}\right) - v\left(\frac{t(\hat{\theta})}{s(\hat{\theta})}\right)\right] - [R'(F(h(t_1)))f(h(t_1))h'(t_1)h(t_1) + R(F(h(t_1)))h'(t_1)](t(\hat{\theta}) - t(\theta)) \geq 0$$

Similarly, another use of the mean value theorem yields

$$\left[v\left(\frac{t(\hat{\theta})}{s(\theta)}\right) - v\left(\frac{t(\theta)}{s(\theta)}\right)\right] - [R'(F(h(e_2)))f(h(t_2))h'(t_2)h(t_2) + R(F(h(t_2)))h'(t_2)](t(\theta) - t(\hat{\theta})) \geq 0$$

for some $t_2 \in [0, \infty$

Combining these two last inequalities, we obtain:

$$\frac{\left[v\left(\frac{t(\theta)}{s(\hat{\theta}))}\right) - v\left(\frac{t(\hat{\theta})}{s(\hat{\theta})}\right)\right]}{[R'(F(h(t_1)))f(h(t_1))h'(t_1)h(t_1) + R(F(h(t_1)))h'(t_1))]\delta} \leq \frac{t(\hat{\theta}) - t(\theta)}{\delta} \leq \frac{\left[v\left(\frac{t(\hat{\theta})}{s(\theta)}\right) - v\left(\frac{t(\theta)}{s(\theta)}\right)\right]}{[R'(F(h(e_2)))f(h(t_2))h'(t_2)h(t_2) + R(F(h(t_2)))h'(t_2))]\delta}.$$

By continuity, both the RHS and LHS of the expression converges to the same limit at δ approaches 0 ensuring that $\lim_{\delta\to 0} \dfrac{t(\hat{\theta}) - t(\theta)}{\delta}$ exists. By definition, this establishes that $t(\cdot)$ is differentiable at θ. END OF PROOF.

Proof of Proposition 6.

STEP 1

From the previous step and since the functions $R(\cdot)$, $F(\cdot)$, $h(\cdot,\cdot)$ and $v(\cdot)$ are continuously differentiable then the problem (11) can be characterized by the following (sufficient) first order condition[42]: for any $\theta \in (a, b]$:

$$R^{'}(F(h^{eq}(\theta)))f(h^{eq}(\theta))h'(t(\theta))h(t(\theta)) + R(F(h^{eq}(\theta)))h'(t(\theta)) - \frac{1}{s(\theta)}v'\left(\frac{t(\theta)}{s(\theta)}\right) = 0 \quad (20)$$

Alternatively, (20) can be rewritten as follows using the fact that $G(\cdot) \equiv F(h^{eq}(\cdot))$ and $g(\cdot) \equiv f(h^{eq}(\cdot))h'(\cdot)t'(\cdot)$.

$$\frac{R^{'}(G(\theta))g(\theta)h(t)}{t'(\theta)} + R(G(\theta))h'(t(\theta)) - \frac{1}{s(\theta)}v'\left(\frac{t(\theta)}{s(\theta)}\right) = 0 \quad (21)$$

STEP 2 The above first order condition can be rewritten as the following differential equation given the denominator does not vanish in the RHS of the following expression:

$$t'(\theta) = -\frac{R'(G(\theta)g(\theta)h(t)}{R(G(\theta))h'(t) - \frac{1}{s(\theta)}v'\left(\frac{t}{s(\theta)}\right)} \quad (22)$$

STEP 3 In equilibrium, the ranking of an individual with endowment $\theta = a$ is the lowest possible. Indeed, $R(G(a)) = 0$ and so her utility in equilibrium is: $-v\left(\dfrac{t(a)}{s(a)}\right)$. Clearly $e(a) = e_a$ or $t(a) = s(a)e_a$ dominates any other strategy for that player. We now show that this solution is continuous at $\theta = a$.

In equilibrium, an individual with endowment $\theta = a$ must not be able to increase her utility by increasing her effort and achieving a higher rank. This implies that in particular that:

[42] We implicitly assume that the choice of t is interior for all individuals but the poorest. Assumptions that can assure this result includes the INADA types of restrictions on $h(\cdot)$ and $v(\cdot)$.

$\lim_{\theta \to a^+} R(F(h(t(\theta))))h(t(\theta)) - v\left(\frac{t(\theta)}{s(a)}\right) \leq -v\left(\frac{t(a)}{s(a)}\right)$

Moreover, since $R(F(h(t(\theta))))h(t(\theta)) - v\left(\frac{t(\theta)}{s(\theta)}\right) \geq -v\left(\frac{t(a)}{s(a)}\right)$ for all θ we obtain the following by continuity:

$\lim_{\theta \to a^+} R(F(h(t(\theta))))h(t(\theta)) - v\left(\frac{t(\theta)}{s(\theta)}\right) = \lim_{\theta \to a^+} -v\left(\frac{t(\theta)}{s(\theta)}\right) = -v\left(\frac{t(a)}{s(a)}\right)$

The last equality follows from the fact that $\lim_{\theta \to a^+} R(F(h(e(\theta), \theta))) = 0$.

Since $v(\cdot)$ is strictly increasing and continuous, this implies $\lim_{\theta \to a^+} t(\theta) = t_a$. In other words, the equilibrium solution is continuous(from above) at $\theta = a$.

STEP 4

We now check that the first order conditions characterizing the differential equation (12) represent an optimal behavior for each agent. Although concavity and strict-concavity

cannot be easily guaranteed a priori because of the endogenous CDF F, we can check that pseudoconcavity holds.

Pseudococavity holds for an individual with endowment θ if $t(\theta)$ is increasing for $t < t(\theta)$ and decreasing for $t > t(\theta)$. Let $\hat{t} < t(\theta)$ and let $\hat{\theta}$ be such that $t(\hat{\theta}) = \hat{t}$. Notice that such $\hat{\theta}$ exists by properly choosing the $\hat{t}$ in the relevant range and that $\hat{\theta} < \theta$ since $t(\cdot)$ is strictly increasing.

Let $A(t, \theta) = R(F(h(t)))h(t) - v\left(\frac{t}{s(\theta)}\right)$, be the objective function of individual with endowment θ.

Conditional on $\frac{\partial}{\partial t}A(t, \theta) = 0$ and all other agents adopting their equilibrium strategies, we have the following:

$$\frac{\partial^2}{\partial t \partial \theta}A(t, \theta) = \frac{s'(\theta)}{s(\theta)^2}v'\left(\frac{t}{s(\theta)}\right) + \frac{ts'(\theta)}{s(\theta)^3}v''\left(\frac{t}{s(\theta)}\right) > 0$$

Therefore, for any $\hat{(t)} < t(\theta)$, $\frac{\partial}{\partial t}A(\hat{t}, \theta) > \frac{\partial}{\partial t}A(\hat{t}, \hat{\theta}) = 0$ which shows that $A(\cdot, \cdot)$ is strictly increasing for $t < t(\theta)$. Similarly, one can show that $A(\cdot, \cdot)$ is decreasing for $t > t(\theta)$. In other words, $A(t(\theta), \theta)$ is pseudoconcave at all $\theta \in (a, b]$. Moreover, this step establishes that the solution $t(\theta)$ is a singleton or that $t(\cdot)$ is a function.

STEP 5

The differential equation (12) with initial condition $t(a) = s(a)e_a$ has a unique (continuous and differentiable) solution $t(\cdot)$ by virtue of the fundamental theorem of differential equations if $e_a > 0$. Otherwise, $e_a = 0$ and the denominator of the RHS of (12) vanishes which may give rise to multiple solutions with the same boundary condition (This is a problem known in the analysis of auctions and which is also discussed in Hopkins and Kornienko's paper). In this case, uniqueness may be lost. In any case, a symmetric NE exists and uniqueness is guaranteed if $e_a > 0$. END OF PROOF.

Characteristics of the wage distribution function (equation 13).

Some of the characteristics of the wage cumulative distribution function, $H(w;\kappa)$, described in equation 13 are:

(i) *Mean*: $E(w) = 1 \quad \forall\kappa$

(ii) *Median*: $F(w_m) = \dfrac{1}{2} \;\Rightarrow\; w_m = \dfrac{1+\kappa}{\kappa 2^{1/\kappa}}$

(iii) As $\kappa \rightarrow 1$, the distribution function in equation 13 approaches the Uniform distribution.

(iv) Define the first measure of inequality in the distribution of w as: $\Omega_w = \dfrac{median}{mean}$. That is:

$$\Omega_w = \frac{1+\kappa}{\kappa 2^{\frac{1}{\kappa}}}. \tag{23}$$

A higher value of Ω_w corresponds to a lower degree of inequality of wages (because the median of the distribution is closer to the mean). Note also that:

$$\frac{\partial\Omega_w}{\partial\kappa} > 0 \text{ for } \quad \kappa \in (0,1].$$

Therefore, both κ and Ω_w are measures of inequality in the distribution of wages in the economy. that is, as κ and Ω_w increase, wage inequality decreases.

(v) Define the Gini coefficient of the distribution wages as:[43]

$$Gini_w = 2\int_0^{\frac{1+\kappa}{\kappa}} wH(w;\kappa)h(w;\kappa)dw - 1 \tag{24}$$

Solving the previous equation using the distribution given in equation 13 we have:

[43] See Lambert (2001), chapter 2.

$$Gini_w = \frac{2(1+\kappa)}{2\kappa+1} - 1$$

Using the last equation, note that: $\frac{\partial Gini_w}{\partial \kappa} < 0$. As the parameter that captures the degree of inequality in the distribution wages in the economy increases, the wage Gini coefficient, which is a measure of inequality of wages, decreases.

Derivation of equation 18 (solution to the differential equation 12).

Using equations 13, 14,15, 16, and 17, the differential equation 12 can be written as:

$$e'(\theta) = -\left[\frac{\phi}{\kappa}\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\theta^{\frac{\phi}{\kappa}-1}\frac{A\left(\frac{e}{\theta}\right)^{\alpha}}{\alpha A\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\theta^{\frac{\phi}{\kappa}-1}\left(\frac{e}{\theta}\right)^{\alpha-1} - \frac{\mu e^{\mu-1}}{\theta}} + \frac{1-\alpha}{\alpha}\left(\frac{e}{\theta}\right)\right] \tag{25}$$

Let´s assume that the solution to the differential equation in equation 25 is of the form $\widehat{e}(\theta) = \Lambda\theta^{\lambda}$, where Λ and λ are given constant terms that depend on the parameters of the model. If this is the case, then $\widehat{e}'(\theta) = \Lambda\frac{\widehat{e}}{\theta}$.Plugging this into the LHS of equation 25 and after some algebraic manipulation yields:

$$\widehat{e} = \left[\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\left(\frac{\alpha A}{\mu}\right)\frac{\mu + \mu\frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha\frac{\phi}{\kappa} - \mu\alpha}\right]^{\frac{1}{\mu-\alpha}}\theta^{\frac{1+\frac{\phi}{\kappa}-\alpha}{\mu-\alpha}}, \tag{26}$$

which confirms that the solution to the differential equation 18 is of the form $\widehat{e}(\theta) = \Lambda\theta^{\lambda}$, where:

$$\Lambda = \left[\left(\frac{\Phi}{K}\right)^{\frac{1}{\kappa}}\left(\frac{\alpha A}{\mu}\right)\frac{\mu + \mu\frac{\phi}{\kappa} - \mu\alpha}{\mu + \alpha\frac{\phi}{\kappa} - \mu\alpha}\right]^{\frac{1}{\mu-\alpha}},$$

and,

$$\lambda = \frac{1 + \frac{\phi}{\kappa} - \alpha}{\mu - \alpha}.$$

References

Aghion P., Caroli E., and Garcia-Peñalosa, C., 1999. Inequality and economic growth: The perspective of the new growth theories. Journal of Economic Literature 37 (4), 1615-1660.

Ascher, U., and L. Petzold, 1998. Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. Society for Industrial and Applied Mathematics.

Bagwell, L. and Bernheim, D., 1996. Veblen Effects in a Theory of Conspicuous Consumption. The American Economic Review 86 (3), 349-373.

Banerjee, A. and Newman A., 1993. Occupational choice and the process of development. Journal of Political Economy 101 (2), 274-298.

Bastani, S., 2007. Towards a Theory of Relative Preferences. Uppsala University, Mimeo.

Becker, G., 1964. Human Capital. Chicago: University of Chicago Press.

Becker, G. and Murphy, K., 2000. Social Economics: Market Behavior in a Social Environment. Cambridge, MA: Harvard University Press.

Ben-Porath, Y., 1967. The Production of Human Capital and the Life Cycle of Earnings. Journal of Political Economy 75 (4), 352-365.

Bénabou, R., 1996. Equity and efficiency in human capital investment: The Local Connection. Review of Economic Studies 63 (2), 237-264.

Bénabou, R., 2000. Meritocracy, redistribution, and the size of the pie, in: Arrow, K., Bowles, S. and Durlauf S. (Eds.), Meritocracy and Inequality, Princeton University Press, pp. 317-339.

Benabou, R., 2000. Unequal societies: Income distribution and the social contract. American Economic Review 90 (1), 96-129.

Cole, H., Mailath, G. and Postlewaite, A., 1992. Social Norms, Saving Behavior, and Growth. The Journal Political Economy 100 (6), 1092-1125.

Cole, H., Mailath, G. and Postlewaite, A., 1995. Incorporating Concern for Relative Wealth Into Economic Models. Federal Reserve Bank of Minneapolis Quarterly Review 19 (3), 12-21

Cole, H., Mailath, G. and Postlewaite, A., 1995. Response to "Aristocratic Equilibria". The Journal Political Economy 103 (2), 439-443.

De Gregorio, J., 1996. Borrowing constraints, human capital accumulation, and growth. Journal of Monetary Economics 37 (1), 49-71.

Ding, W., and Lehrer, S., 2006. Do Peer Effects Affect Student Achievement in China's

Secondary Schools?. NBER working paper # 12305.

Fernández, R. and Galí, J., 1999. To each according to...? Markets, tournaments and the matching problem with borrowing constraints. Review of Economic Studies 66, 799-824.

Ferreira, F., 2001. Education for the masses? The interaction between wealth, educational and political inequalities. Economics of Transition 9 (2), 533-552.

Flug, K., Spilimbergo A., and Wachteinheim, E., 1998. Investment in education: do economic volatility and credit constraints matter? Journal of Development Economics 55 (2), 465-481.Frank, 1999

Frank, R., 1999. Luxury Fever: Why Money Fails to Satisfy in an Era of Excess. Princeton University Press.

Galor, O. and Zeira J., 1993. Income distribution and macroeconomics. Review of Economic Studies 60 (1), 35-52.

Glomm, G. and B. Ravikumar, 1992. Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality. Journal of Political Economy 100 (4), 818-834.

Hopkins, E. and Kornienko, T., 2004. Running to Keep in the Same Place: Consumer Choice as a Game of Status. American Economic Review 94 (4), 1085-1107.

Hopkins, E. and Kornienko, T., 2006. Which Inequality? The Inequality of Resources Versus the Inequality of Rewards. Mimeo.

Hoxby, C., and Weingarth, C., 2007. School Reassignment and the structure of Peer Effects. Working paper, Harvard.

Lucas, R., 1988. On the mechanics of economic development. Journal of Monetary Economics 22 (1), 3-42.

Mejía, D. and St-Pierre, M., 2008. Unequal Opportunities and Human Capital Formation. Journal of Development Economics, in press.

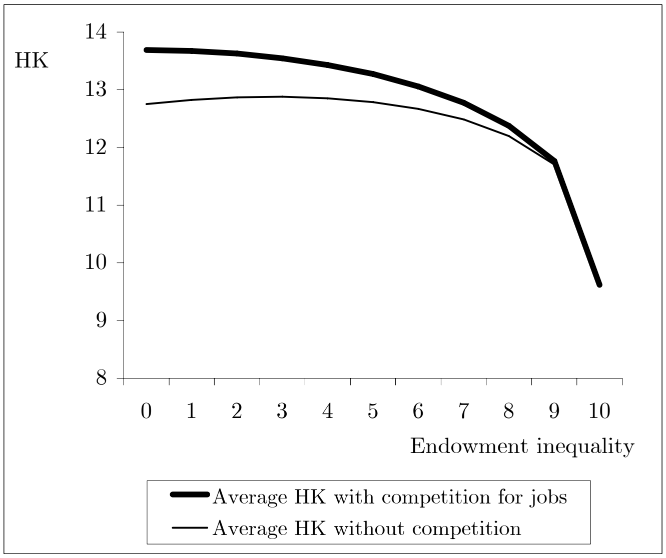
Moen, E., 1999. Education, Ranking, and Competition for Jobs. Journal of Labor Economics 17 (4), 694-723.

Postlewaite, A., 1998. The Social Basis of Interdependent Preferences. Economic European Review 42, 779-800.

Rege, M., 2000. Why Do People Care about Social Status?. Journal of Economic Behavior and Organization.

Skaperdas, S., 1996. Contest Success Functions. Economic Theory 7, 283-290.

Veblen, T., 1899. The Theory of Leisure Class: An Economic Study of Institutions. New York: Macmillan



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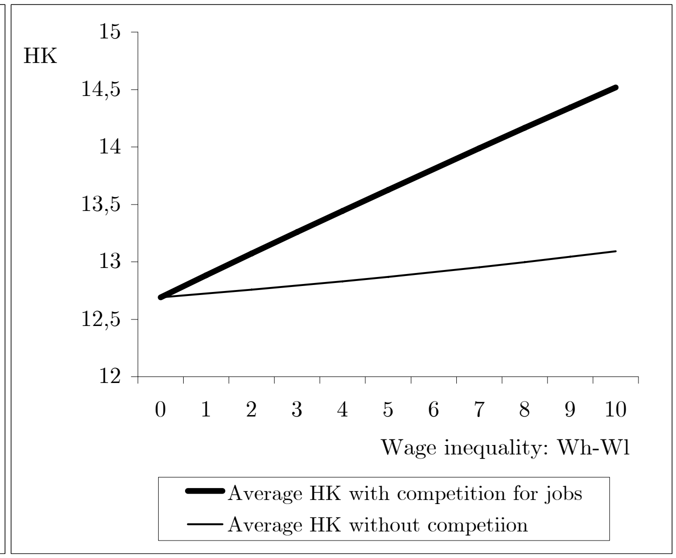

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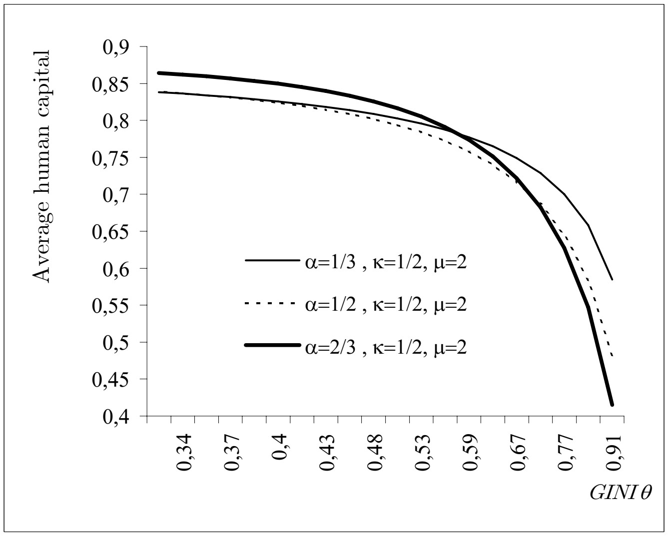
Figure 1: Calibration results for the 2 individuals - 2 firms model



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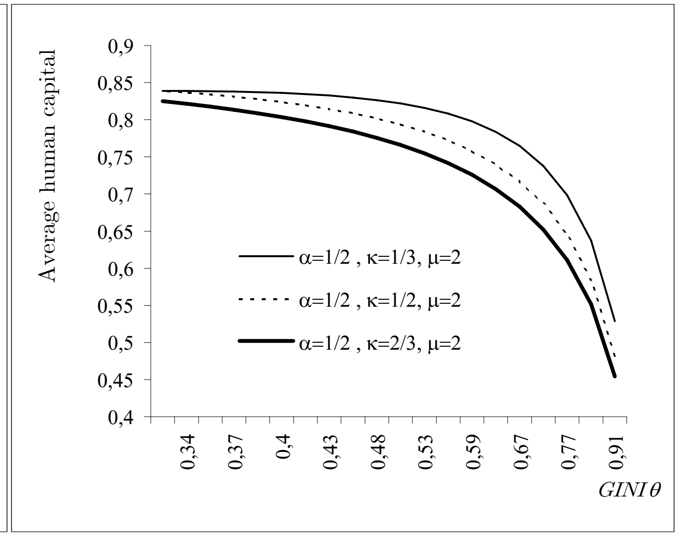

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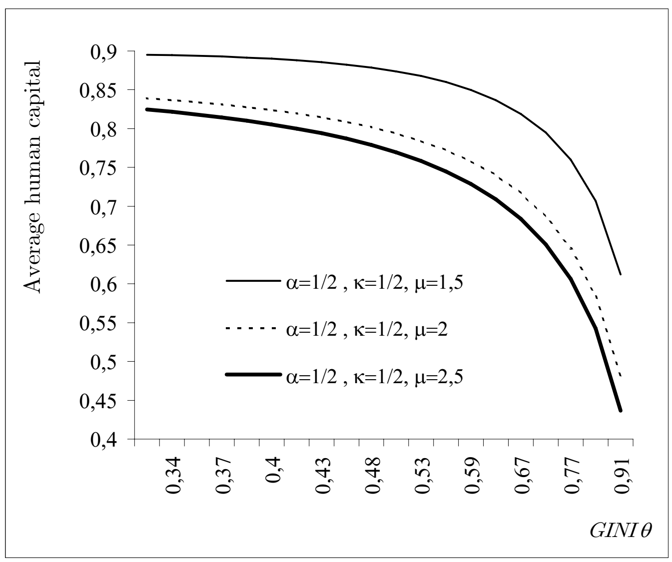

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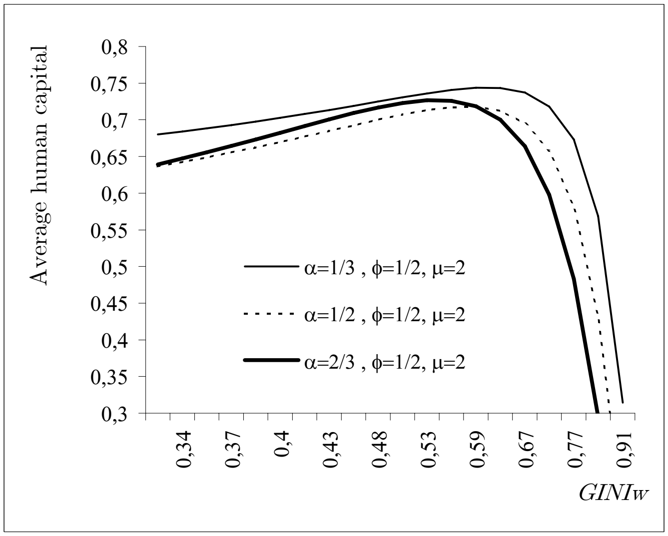
Figure 2: Endowment inequality vs. Average human capital in the General Model



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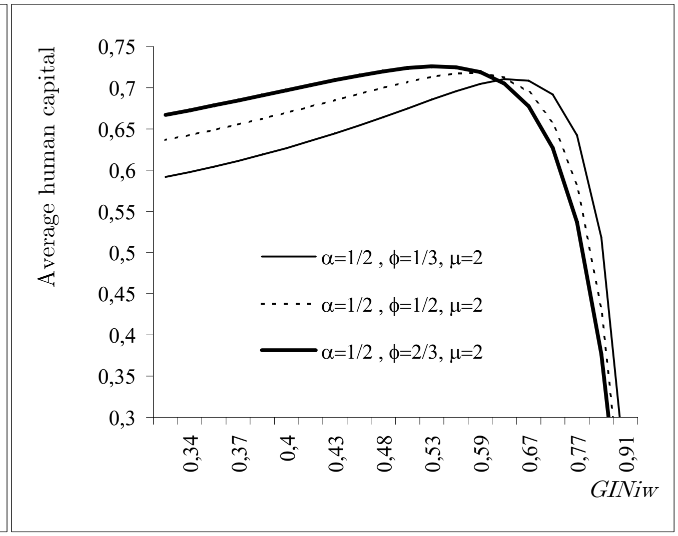
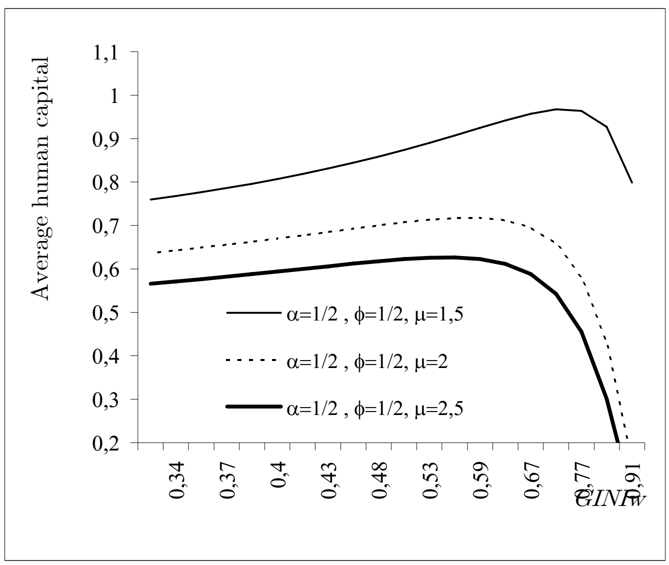


(b)



(c)

Figure 3: Wage inequality vs. Average human capital in the General Model

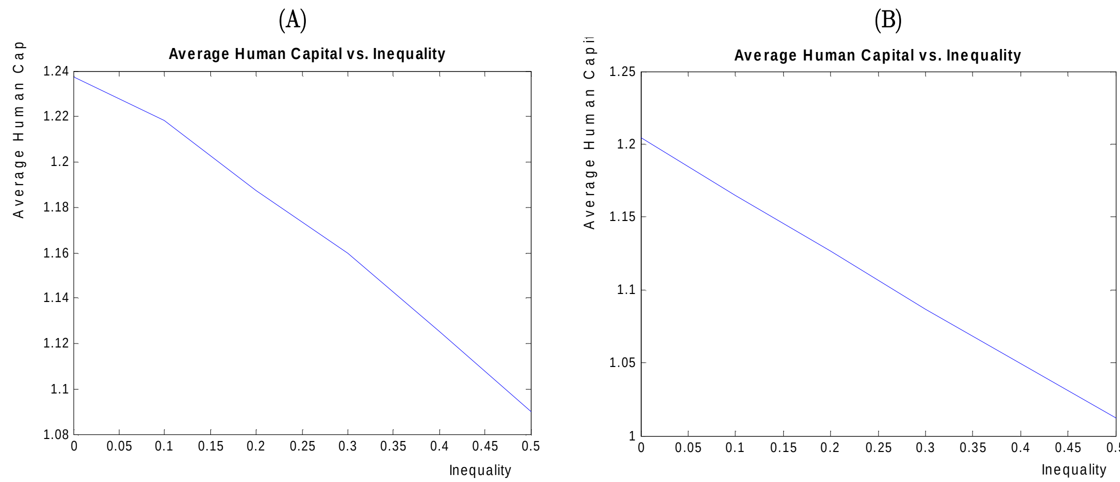


Figure 4: Endowment inequality vs. Average human capital in the General Model (using the Uniform distribution)
α=3/4 in panel (A) and α=1/2 in panel (B)

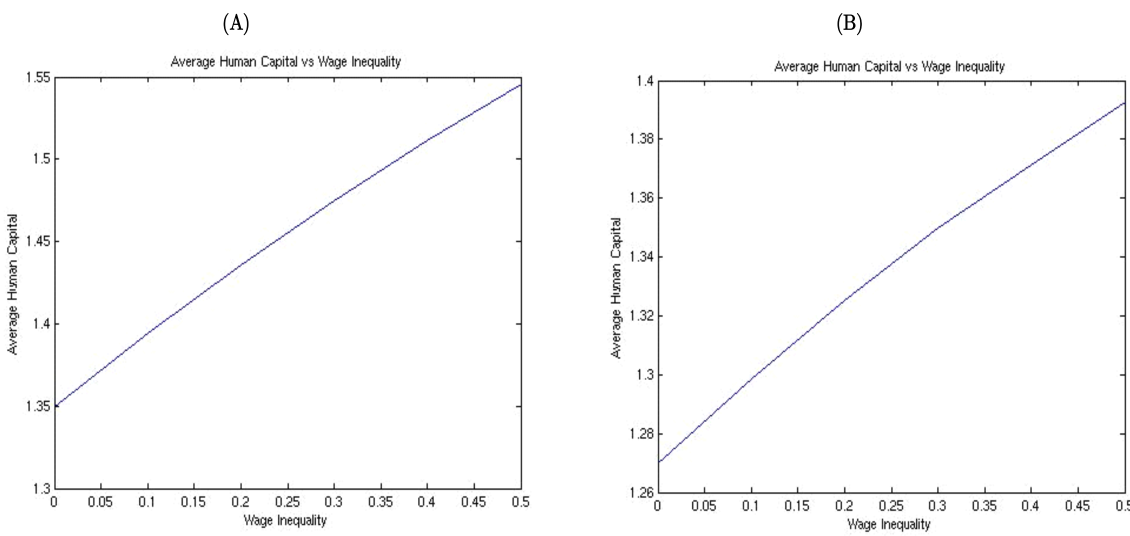


Figure 5: Wage inequality vs. Average human capital in the General Model (using the Uniform distribution)
α=3/4 in panel (A) and α=1/2 in panel (B)