Strategic Behaviour, Resource Valuation and Competition in Electricity Markets*

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Abstract

By means of a suitable Bayesian game we study spot electricity markets from a structural point of view. We address the problem of individual and aggregate eficciency and we show how to value water from market observables. We compare the former to engineering methods and apply our methodology to Colombian spot electricity market. Our results show that big gas and small hydro plants overbid, resources are undervalued by engineering costs and aggregate costs would have been considerably smaller if agents had played optimally. Revealed costs show a substantial gain in efficiency in the Vickrey auction compared to the actual uniform auction.

Keywords: Multi-unit auctions, Oligopoly, electricity markets.

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1 Introduction

Recent restructured electricity markets around the world (Wolak [2000a]) has called the attention of academic economists. Besides the importance of this industry in modern economies, there are many interesting issues regarding market design, the role of different allocating mechanisms, competition and market power and, in the case of hydroelectric energy, resource valuation among many others. In particular, there is a lot of interest in testing theoretical and econometric models with the hope that results will help improve the industry or at the very least, our modeling framework. Regarding the first issue, we show how to estimate aggregate efficiency compared to perfect competition. This also allows us to study individual productive efficiency, one of the main concerns in policy and academic circles. On the other hand, hydroelectric generated power adds additional difficulties to the above problems because it relies on a scarce and difficult resource to value. Moreover, because this type of generating technology plays a major role in many countries, it is important to develop techniques that allow to infer the true value of water. In this case, although marginal costs are approximately cero, the opportunity cost is not, and its difficult to identify. If energy is traded in an auction, as is mostly the case in modern energy markets, in general the true value of the resource is not equal to anything observed (bids nor equilibrium prices). Therefore there remains the difficult question about the opportunity in order to identify the true value of the resource based on market observables (bids, closing prices, etc.). In summary, we propose a general methodology that allows to study aggregate and individual productive efficiency and to identify marginal costs of competing generating units based on market observables. We apply this methodology to the Colombian spot electricity market and address the above issues.

Since auction mechanisms have played a key role in market design in the newly restructured industry, strategic behavior among agents play a major role in answering all of the above questions. This paper addresses the above issues within a structural framework. That is, we model electricity markets as a Bayesian game where observable data (bids, quantities and equilibrium prices) may or not , depending on the subject of study, be consistent with Bayesian Nash equilibrium or more generally, with rationalizable strategies. Our model relies on De Castro and Riascos [2009] where a very general multiunit auction game is considered and necessary first order conditions for Bayesian Nash equilibrium or rationalizable strategies are provided. We modify and adapt the model to the specific characteristics the Colombian electricity market where we address empirically the above questions raised. The approach is also indicative of how to model other markets and is not unique to our chosen application. Our methodology is in the in the same spirit of the structural econometric modelling approach to auctions (Hortacsu [2002], Paarsch and Young [2006] and in particular, to its applications to electricity markets (Hortacsu and Puller [2008] and Wolak [2000b]). More precisely, our approach is structural but rather than estimating structural parameters from market data, we construct and artificial economy that we simulate and compare to observable data. On the other hand, our model, we believe, is richer and better suited than some of the mentioned studies to capture important aspects of many electricity markets. For example, we explicitly model electricity auctions as discrete bid auctions. This might not be an important limitation in markets where many bids are allowed like in Texas balancing energy market (Hortacsu and Puller [2008]) but it can be important in markets as the British and Wales market where only three half hourly price bids are allowed. Also, as opposed to Hortacsu and Puller [2008] we make no assumption about the functional form of bidding strategies. We share an important distinguishing aspect of this literature which is the attempt to identify agents preferences from observable data in order to built counterfactuals to predict behavior under different market conditions (for example a different auction format). This is in sharp contrast to a purely econometric tradition in which counterfactuals are based on comparing observable outcomes in different market mechanisms but with no predictive power of outcomes in unobserved different hypothetical market structures.[1] Finally, we rely on the private values auctions paradigm. As we'll argue below, to the extend that a large amount of electricity is traded in private bilateral contracts (nowadays more than 100% of energy demand in some electricity markets) it is reasonable to assume that agents value privately electricity generation and care of their complete portfolio. That is, an electricity generator that has a net short position in electricity due to his private contracts the day before the auction, will value electricity very different than a generator that enters the auction with a net long position. Therefore, we believe that the private value paradigm is a reasonable approximation to the real valuation of electricity. In the case of hydroelectric energy, resource valuation is an issue, which makes a stronger case for considering these as private value auctions.

Our results suggest the following for the case of Colombian electricity market. By assuming that marginal costs for hydroelectric plants are those estimated by engineering methods (the industry standard), a dual programming approach known as MPODE, we show that agents bids are in general higher than the observed ones, suggesting a higher than optimal mark up and therefore a higher than optimal productive inefficiency (recall that the most one can get is a second best, given that there is a price to be paid for decentralization). When we assume that the true valuation of water is unobservable then under the hypothesis that agents play (and we observe) best reply strategies, we identify the true value of water and show that in general is higher than MPODE marginal costs. This suggests that, either agents base their bids on an underestimated

[1] One of the first studies in this purely econometric tradition for auctions of government bonds is Umlauf [1993].

piece of information (MPODE costs), or there is more than an optimal inefficiency in the market. Finally, by using a truthful telling mechanism such as the Vickrey auction mechanism, aggregate generating costs would follow the above pattern: If we assume MPODE marginal costs then aggregate costs would be lower than if we assume the revealed valuation and in turn, this would both be lower that the actual cost. More importantly, the former would be higher than those that are implied by our model if agents where closer to the optimal bids implied by best reply strategies under the assumption of marginal costs equal to MPODE compared to the actual uniform auction mechanism currently used.

We also explore the differences in strategic behavior among different types of generating plants and the introduction of private information based on bilateral contracts among agents. Interestingly the introduction of bilateral contracts allows for an intuitive validation of our empirical strategy. More precisely, when we take into account contracts we find no evidence, for coal powered generating plants, of differences between MPODE's marginal costs and revealed costs. Therefore, by using a similar strategy to Hortacsu et.al [2008] we back up in our model the level of contracts for coal plants and we show that there is negative correlation between contracts and the probability of being marginal. This we believe, is in line with the intuition that the higher the probability of being marginal, higher market power and less need to hedge through bilateral contracts.

2 Colombian Electricity Industry

Following the international trend, in the beginning of the nineties the Colombian government conducted a major reform in the electricity industry. This reform took the English electricity industry as a model.

The most important reform put forward during these years (1995) was the creation of a spot electricity market based on an hourly auction mechanism. This auction is a one sided auction where only generators participate, and in which with one day in advance each generator submits: (1) Their available generating capacity (*disponibilidad*) for the next day 24 hours (quantity), and (2) an hourly price at which they are willing to offer that quantity. In 2001 an important change to this mechanism was put forward and market participants were restricted to submit a unique price for the 24 hours. In this new format, in which we focus in this paper, bids are a unique price and in principle an hourly schedule of maximum generating capacities to supply energy the next day. The system administrator ranks bids based on their price and every hour determines which price/quantity balances supply and demand. All winning plants (i.e. lower bids than the equilibrium bid) are paid this equilibrium price making this a standard uniform auction. As we´ll show, generators rarely change between hours their available generating capacity therefore we approximate each plant offer by a unique price/quantity pair: an offer to supply the available capacity and that price for the next 24 hours. Therefore, since some firms (generating companies) own several plants, one can interpret this market as a multi-unit auction where market participants bid a vector of price/quantities one pair for each plant. This is an important issue that we incorporate into our model.

An important characteristic of all plants and companies that participate in this market is their ability to do bilateral forward contracts (forward financial contracts). All information about these bilateral contracts is private and it is one of the most important pieces of information used by agents to design their bidding strategy in the spot market. Bilateral contracts are financial contracts, and all electricity demand and supply is actually allocated in the spot market where contracts are cleared financially. Therefore, as opposed to Texas electricity spot market, the Colombian spot market is not a balancing market (net of demand and supply of bilateral contracts) but an auction for all energy demanded. Finally, another important characteristic of Colombian electricity market are the expost payments due to differences between planed generating schedules and realized ones. That is, for different reasons, a plant called to generate at it's available capacity is unable to do so therefore, other plant not initially called to generate is called to do so and is paid a price which is a function of the equilibrium price and its bid. This rules are called "reconciliaciones" and affect the strategic behavior of plants (a company may want to lose in the auction expecting to be called to replace another generator). We are able to introduce contracts in our model but not this other important aspect to strategic behavior.

Finally a word on generating composition. In Colombia most energy is produced by hydroelectric plants (71%) but there are two other important actors. Gas and coal powered generating plants. Gas plants are responsible for 26% while coal plants for 2% of the generating capacity.

3 The Model

3.1 Setup

The model follows closely the general model in De Castro et.al [2009]. Important differences will be discussed below. Suppose that there are N strategic firms in the electricity industry. We denote these by $\mathbb{N} = \{1, 2, ..., N\}$. Each firm (agent) $i \in \mathbb{N}$ has $e^i \in \mathbb{N}$ generating plants which can be thermic (gas or coal) or hydroelectric. Firm $i \in \mathbb{N}$ receives a private signal, $t^i \in T^i$, where T^i is the information set of player i. We assume that each T^i has associated a $\sigma-$field of events which is denoted by $\mathcal{F}^i$. In particular, private information are the type and quantity of bilateral contracts which a firm has with other generating or commercialization firms. Additionally, this variable can also represent the information each player has about demand predictions, resource valuation or other important private information. Notice that individual signals may be dependent and of arbitrary dimension. Define by $T \equiv \prod_{i=1}^{N} T^i$, the type space, let $\Im^i$ be a σ-field of subsets of T^i and define by $\Im \equiv \prod_{i=1}^{N} \Im^i$ be product $\sigma-$field over the type space T. We denote by $t = \left(t^1, t^2, ..., t^N\right) = \left(t^{-i}, t^i\right)$ the vector of all firms' information, where $t^{-i} = \left(t^1, t^2, ..., t^{i-1}, t^{i+1}, ..., t^N\right)$. We also denote by $\Im^{-i} \equiv \prod_{j \neq i}^{N} \Im^j$ the product $\sigma-$field over all companies except for firm i. For each $i \in \mathbb{N}$ and $t^i \in T^i$ we denote by $\tau^{-i}(\cdot|t^i) : T^{-i} \rightarrow [0,1]$ the conditional distribution of firm i with information t^i, which represents the distribution that the firm i uses in order to evaluate all other's firms information. The j plant of agent i, is characterized by a pair $\left(\lambda_j^i, c_j^i\right)$, where λ_j^i, $c_j^i \in \Re_+$ and $j \in e^i$, $\forall i = 1, ..., N$, λ_j^i represents the available generating capacity of the $j-$plant and c_j^i the marginal cost. We denote by e^i, the number of plants of the i-th agent. We assume marginal costs are constant. Moreover, for each agent, let $c_1^i \leq c_2^i \leq ... \leq c_{e^i}^i$ $\forall i \in \mathbb{N}$. After observing his private information t^i agent i is to propose an e^i-dimensional vector of bids $\mathbf{b}^i$. Then a strategy for agent i is a bidding function $\mathbf{b}^i = \left(b_1^i, b_2^i, ..., b_{e^i}^i\right) : T^i \rightarrow \Re_+^{e^i}$. The bid is a vector of real numbers $\mathbf{b}^i \in B \subseteq \Re_+^{e^i}$ where B denotes the set of valid bids, that is, $B \equiv \left\{\mathbf{b}^i \in \Re_+^{e^i} : b_k^i \leq b_{k+1}^i \text{ for } k = 1, ..., e^i - 1\right\} \cap \left[\underline{b^i}, \overline{b^i}\right]$, where b_k^i is the minimum value that firm i is willing to bid for the k'th plant output; and $\left[\underline{b^i}, \overline{b^i}\right]$ denotes a e^i-rectangle that bounds the set of all bids. The bid profile is $\mathbf{b} : T \rightarrow \Re_+^{e^1} \times \Re_+^{e^2} \times ... \times \Re_+^{e^N}$. Let $e = \sum_{i=1}^{N} e^i$ be the total number of available generation plants in the industry. Then $\mathbf{b} : T \rightarrow \Re_+^{e}$. We denote by $\mathbf{b}^{-i}$ all others strategies except for agent i.

3.2 Allocation Rule

The allocation rule a takes the bid profile and determines how many and which players win and how much they win. Formally $a : \Re_+^{e^1} \times \Re_+^{e^2} \times ... \times \Re_+^{e^N} \rightarrow \{1, 2, ..., e\} \times [0,1]^{e^1} \times [0,1]^{e^2} \times ... \times [0,1]^{e^N}$ where $a = (a^0, a^1, a^2, ..., a^N)$, $a^0 \in \{1, 2, ..., e\}$ and $a^i = \left\{a_1^i, a_2^i, ..., a_{e^i}^i\right\} \in [0,1]^{e^i}$ $\forall i \in \mathbb{N}$. a^0 is the number of players which are allocated among the winner agents except the marginal player and a^i is the number of players who win according to the ideal dispatch. If $a_j^i = 1$ the player i wins with all his available capacity with his plant j. If $a_j^i \in (0,1)$ the plant wins with only a portion of his available capacity, and if $a_j^i = 0$ the plant does not win any right to sell. In order to define formally some properties of the allocation rule we need some notation. Let $(\mathbf{d}, \boldsymbol{\lambda}) : \Re_+^e \rightarrow \Re_+^e \times \Re_+^e$ be the function that from a bid profile $\mathbf{b} \in \Re_+^e$, orders the vector bids in ascending order, and $\boldsymbol{\lambda}(\mathbf{b})$ is the sorted vector of production availability according to $\mathbf{d}(\mathbf{b})$.

Then, $\mathbf{d}_l(\mathbf{b})$ means the l-th lowest bid; and $\boldsymbol{\lambda}_l(\mathbf{b})$ is the quantity of such plants. We denote by $(\mathbf{d}^{-i}, \boldsymbol{\lambda}^{-i})$ $:T^{-i} \rightarrow \Re_{+}^{e-e^i} \times \Re_{+}^{e-e^i}$ the analogous function to $(\mathbf{d}, \boldsymbol{\lambda})$ without the i-th firm. For simplicity we will drop the strategy profile $\mathbf{b}$ from the vectors $a(\mathbf{b})$, $\mathbf{d}(\mathbf{b})$ and $\boldsymbol{\lambda}(\mathbf{b})$ (i.e. $a, \mathbf{d}$ and $\boldsymbol{\lambda}$)

Definition 1 *Let D be the energy demand forecast in an specific auction. An allocation rule is $a : \Re_{+}^{e^1} \times \Re_{+}^{e^2} \times ... \times \Re_{+}^{e^N} \rightarrow \{1, 2, ..., e\} \times [0,1]^{e^1} \times [0,1]^{e^2} \times ... \times [0,1]^{e^N}$ such that :*

1. *a^0 shows the number of plants called to generate:*

$$\begin{aligned} \sum_{k=1}^{a^0} \boldsymbol{\lambda}_k &< D \\ \sum_{k=1}^{a^0+1} \boldsymbol{\lambda}_k &\geq D \end{aligned}$$

Let $\overline{k} = a^0 + 1$, then $\overline{k}$ is the number of plants used.

2. *If $b_k^i > \mathbf{d}_{\overline{k}-k+1}^{-i}$ with $k \leq e^i$ then $a_k^i = 0$.*

3. *If $b_k^i < \mathbf{d}_{\overline{k}-k+1}^{-i}$ with $k \leq e^i$ then $a_k^i > 0$.*

4. *If for some $k \leq e^i$, $a_k^i = 1$, then, $\forall k' < k \leq e^i$, $a_{k'}^i = 1$.*

5. *The rule allocates at most D among the N agents.*

$$\sum_{i=1}^{N} \sum_{k=1}^{e^i} a_k^i \lambda_k^i = D$$

Remark 1 *Note that the allocation rule is completelly determined except in case of ties. We will assume that this is a cero probability event.*

Finally we define the following distribution functions [2] $F_{\mathbf{d}^{-i}}\left(\cdot|t^i\right)$ on $\Re_{+}^{e-e^i}$ as

$$F_{\mathbf{d}^{-i}}\left(b|t^i\right) \equiv \ \tau^{-i}\left(\left\{t^{-i} \in T^{-i} : b < \mathbf{d}^{-i}\left(t^{-i}\right)\right\}|t^i\right)$$

and let $f_{\mathbf{d}^{-i}}\left(b|t^i\right)$ be its density. The distribution denotes the probability that firm i has the lowest bid. Notice that $F_{\mathbf{d}^{-i}}$ is the distribution of the random variable $-\mathbf{d}^{-i}$. We define $F_{\mathbf{d}_j^{-i}}\left(\cdot|t^i\right)$ on $\Re_+$ as $F_{\mathbf{d}_j^{-i}}\left(b|t^i\right) \equiv \tau^{-i}\left(\left\{t^{-i} \in T^{-i} : b < d_j^{-i}\left(t^{-i}\right)\right\}|t^i\right)$ (strictly, the distribution of $-\mathbf{d}_j^{-i}$) and let $f_{d_j^{-i}}\left(b|t^i\right)$ be its density. The latter represents the probability that firm i has a lower bid than the j-th lowest bid of all other players' bids.

[2] Note that this definition implies that the function is decreasing.

3.3 Expected Payoff

The expected payoff for each firm is simply the sum of the expected payoffs of its plants given all players's bids and the information available. If the k-th plant of the i-th agent is released, his net additional payoff is denoted by $u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right)\right): T \times \Re_+^e \rightarrow \Re$. Notice that this payoff will depend on the firm being marginal or not.

Thus, if the bidding functions $\mathbf{b}^{-i}$ are fixed, the expected payoff of firm i of type t^i, when bidding b^i is:

$$\Pi^i\left(t^i, b^i, \mathbf{b}^{-i}\right) \equiv \int_{T^{-i}} \sum_{k=1}^{e^i} a_k^i\left(b^i, \mathbf{b}^{-i}\left(t^{-i}\right)\right) u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right)\right) \tau^{-i}\left(dt^{-i}|t^i\right)$$

Denote by $\xi_{\overline{k}}$ the conditional probability to t^i of $\overline{k}$ plants being released when firm i has private information t_i, bids b^i and all other firms bid $\mathbf{b}^{-i}$ (i.e $\xi_{\overline{k}} = \tau^{-i}(a^0 = \overline{k} - 1|t^i)$). Then $\Pi^i\left(t^i, b^i, \mathbf{b}^{-i}\right)$ is:

$$\Pi^i\left(t^i, b^i, \mathbf{b}^{-i}\right) = \int_{T^{-i}} \sum_{\overline{k}=1}^{e} \left(\sum_{k=1}^{e^i} a_k^i\left(b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) \xi_{\overline{k}} \right) \tau^{-i}\left(dt^{-i}|t^i\right)$$

where

$$a_k^i\left(b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) = a_k^i\left(b^i, \mathbf{b}^{-i}\left(t^{-i}\right); a^0 = \overline{k} - 1, t^i\right)$$

and

$$u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) = u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right); a^0 = \overline{k} - 1, t^i\right)$$

Hence the expected payoff is equivalent to:

$$\begin{aligned} = \quad & \sum_{\overline{k}=1}^{e} \left(\sum_{k=1}^{e^i} \int_{T^{-i}} 1_{\left[b_k^i < \mathbf{d}_{\overline{k}-k+1}^{-i}\right]} u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) \tau^{-i}\left(dt^{-i}|t^i\right) \right) \xi_{\overline{k}} + \\ & \sum_{\overline{k}=1}^{e} \left(\sum_{k=1}^{e^i} \int_{T^{-i}} a_k^i\left(b^i, \mathbf{b}^{-i}\left(t^{-i}\right)\right) 1_{\left[b_k^i = \mathbf{d}_{\overline{k}-k+1}^{-i}\right]} u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right); \overline{k}\right) \tau^{-i}\left(dt^{-i}|t^i\right) \right) \xi_{\overline{k}} \end{aligned}$$

Remark 2 *The main difference to De Castro and Riascos [2009] set up is the fact that we condition to the number of generating plants. This modelling device, under the hipothesis we make below allows for a direct application of the bidding characterization presented in that paper. In fact, it reduces the model to a standard multi-unit auction where in each auction demand is to be allocated to a fixed number of plants and since their supply is their available generating capacity, it is equivalent to an aution to win one or more rights to* *generate. That is, an auction for several fixed units. The set up below makes the point clearer.*

Example 1 *Consider the following auction formats.*

1. *The payoff in the multi-unit Vickrey auction is:*

$$u_k^i\left(t^i,t^{-i},b^i,\mathbf{b}^{-i}\left(t^{-i}\right)\right)=\begin{cases}\left(b^{-i}_{\overline{k}-k+1}-c_k^i\right)\lambda_k^i & \text{If the plant is inframarginal}\\ \left(D-\sum_{j=1}^{\overline{k}-1}\boldsymbol{\lambda}_j\right)\left(b^{-i}_{\overline{k}-k+1}-c_k^i\right) & \text{If the plant is marginal}\end{cases}$$

and each winner plant receives the highest competing bid that is removed by his bid.

2. *The payoff in the multi-unit uniform auction is:*

$$u_k^i\left(t^i,t^{-i},b^i,\mathbf{b}^{-i}\left(t^{-i}\right)\right)=\begin{cases}\left(\mathbf{d}_{\overline{k}}-c_k^i\right)\lambda_k^i & \text{If the plant is inframarginal}\\ \left(D-\sum_{j=1}^{\overline{k}-1}\boldsymbol{\lambda}_j\right)\left(b_k^i-c_k^i\right) & \text{If the plant is marginal}\end{cases}$$

and each winner plant receives the highest winner bid $\mathbf{d}_{\overline{k}}$.[3]

In order to characterize bidding behavior we use the following.

Condition 1 *Assume that:*

1. *The firms take* $\xi_{\overline{k}}$ *as an exogenous (given) variable.*
2. $F_{\mathbf{d}^{-i}_{\overline{k}-k+1}}\left(b|t^i\right)$ *has no atoms.*[4]

The first assumption is what is mentioned in the previous remark. This is the key assumption, and the fact that the maximum generating capacity is hardly a choice variable, that reduces the problem to a standard multi-unit auction for a fixed number of objects (in our case, for a fixed number of generating "slots" to attend demand).

Therefore we can write expected payoff as:

$$\Pi^i\left(t^i,b^i,\mathbf{b}^{-i}\right)=\sum_{\overline{k}=1}^{e}\left(\sum_{k=1}^{e^i}\int_{T^{-i}}1_{\left[b_k^i<\mathbf{d}^{-i}_{\overline{k}-k+1}\right]}u_k^i\left(t^i,t^{-i},b^i,\mathbf{b}^{-i}\left(t^{-i}\right);\overline{k}\right)\tau^{-i}\left(dt^{-i}|t^i\right)\right)\xi_{\overline{k}}$$

[3] We exclude the possibility that a specific agent has several marginal plants.
[4] This hypothesis eliminates the possibility of ties among plants.

Theorem 1 *Under condition 1, and assuming interior solutions, the first order conditions for the j-th plant of the i-th firm is:*

$$\begin{aligned} &\sum_{\overline{k}=1}^{e}\left(E_{\tau^{-i}}[u_j^i\left(\cdot;\overline{k}\right)|t^i,\mathbf{d}_{\overline{k}-j+1}^{-i}=b_j^i]\xi_{\overline{k}}\right)f_{\mathbf{d}_{\overline{k}-j+1}^{-i}}\left(b_j^i|t^i\right) \\ = & \\ &\sum_{\overline{k}=1}^{e}\left(E_{\tau^{-i}}[\partial_{b_j^i}u_j^i\left(b_j^i,\cdot;\overline{k}\right)1_{\left[b_j^i<\mathbf{d}_{\overline{k}-j+1}^{-i}\right]}|t^i]\xi_{\overline{k}}\right)+ \\ &\sum_{\overline{k}=1}^{e}\left(\sum_{k\neq j}^{e^i}E_{\tau^{-i}}[\partial_{b_j^i}u_k^i\left(b_j^i,\cdot;\overline{k}\right)1_{\left[b_k^i<\mathbf{d}_{\overline{k}-k+1}^{-i}\right]}|t^i]\xi_{\overline{k}}\right) \end{aligned}$$

Proof. See De Castro et.al [2009]. ■

The first term in this equation represents the marginal benefit of firm i winning the right to generate with an additional plant j. The second term is the marginal cost of winning with plant j and the third term is the marginal cost on all other winning plants.

Now we'll characterize optimum behavior for the two formats we are interested in: the uniform multi-unit auction and the Vickrey auction.

Proposition 1 *Under condition 1, and assuming interior solutions in the Vickrey and uniform auction, the first order conditions for these two auction formats are:*

1. *Vickrey auction. Since:* $\partial_{b_j^i}u_k^i(t,b)=0$ *then sincere bidding is optimal* $b_j^i=c_j^i$*.*

2. *Uniform auction. Since:*

$$\partial_{b_j^i}u_k^i(t,b)=\begin{cases}\lambda_k^i \quad \textit{for } j\neq k \\ 0 \quad \textit{for } j=k \end{cases} \quad \textit{If the plant is inframarginal}$$
$$\phantom{\partial_{b_j^i}u_k^i(t,b)=}\begin{cases}0 \quad \textit{for } j\neq k \\ \left(D-\sum_{j=1}^{\overline{k}-1}\boldsymbol{\lambda}_j\right) \quad \textit{for } j=k \end{cases} \quad \textit{If the plant is marginal}$$

it follows that:

$$b_j^i=c_j^i+e^i\left[\sum_{\overline{k}=1}^{e}\left(D-\sum_{j=1}^{\overline{k}-1}\boldsymbol{\lambda}_j\right)\frac{\Pr[M_{\overline{k},j}]}{f}\xi_{\overline{k}}+\sum_{\overline{k}=1}^{e}\frac{\lambda_k^i\Pr[M_{\overline{k},k}]}{f}\xi_{\overline{k}}\right] \tag{1}$$

where $\Pr[M_{\overline{k},j}]$ *is the probability of firm i being marginal with bid j conditional to the event that there* *will be $\overline{k}$ used and:*

$$f = \sum_{\overline{k}=1}^{e} \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j \right) f_{\mathbf{d}^{-i}_{\overline{k}-j+1}} \left(b^i_j | t^i \right) \xi_{\overline{k}}.$$

Proof. See De Castro et.al [2009]. ■

3.4 The Model with Bilateral Contracts

Latter we will want to introduce an important piece of private information used by agents. Bilateral contracts are an important source of uncertainty that firms face. The main incentive that a agents have (both sides) in order to sign contracts is to reduce future uncertainty of unexpected changes in electricity equilibrium prices at the auction due to variables such as rain-falls, coal and gas prices, demand and so on. Because contracts affect the bidding behavior of firms it is important to include them in the analysis.

According to Espinosa [2009] in the period 1995-2007 about 70% of the traded electricity in the Colombian Industry is done using bilateral agreements. We will consider the simplest kind of bilateral contract that in fact dominates the menu of available contracts.[5] These are standard forward contracts. Moreover, we assume that firms care only for the current expected payoff of this contracts and that, to a good approximation, we can assume a net representative position on forward contracts for the total amount of contracted energy for the next day at one single strike price. We denote this price of agent i and plant j, $p^{i,C}_j$. Now, to model this, recall that if a company does not get the right to sell (because it offers a price over the marginal price p) it should buy energy from the spot market. Additionally if the marginal price is lower than the price that the plant j of agent i signed the bilateral agreement (i.e. $p < p^{i,C}_j$), this firm is a net buyer in the spot market. Otherwise it is a net seller. We denote the number of all contracts for plant j of agent i by CC^i_j.

Example 2 *The payoffs when firm i and plant j has a net position in forward contract CC^i_j is:*

1. *The payoff in the multi-unit Vickrey auction is:*

$$u^i_k \left(t^i, t^{-i}, b^i, \mathbf{b}^{-i} \left(t^{-i} \right) \right) = \begin{array}{ll} \left(b^{-i}_{\overline{k}-k+1} - c^i_k \right) \lambda^i_k - (p - p^{i,C}_k) CC^i_k & \textit{If inframarginal} \\ b^{-i}_{\overline{k}-k+1} \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j - CC^i_k \right) - c^i_k \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j \right) + p^{i,C}_k CC^i_k & \textit{If marginal} \end{array}$$

[5] There are many types of contracts including short and long term, options and many exotic derivatives. Nevertherles, standard forwards contracts are the most important.

2. *The payoff in the multi-unit uniform auction is:*

$$u_k^i\left(t^i, t^{-i}, b^i, \mathbf{b}^{-i}\left(t^{-i}\right)\right) = \begin{cases} \mathbf{d}_{\overline{k}}\left(\lambda_k^i - CC_k^i\right) - c_k^i \lambda_k^i + p_k^{i,C} CC_k^i & \text{If inframarginal} \\ b_k^i \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j - CC_k^i\right) - c_k^i \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j\right) + p_k^{i,C} CC_k^i & \text{If marginal} \end{cases}$$

Corollary 1 *Under condition 1 and assuming interior solutions in the Vickrey and uniform auctions with contract, the first order conditions are:*

1. *For the Vickrey auction with contracts the first order conditions doesn't change.*

2. *For the uniform with contracts:*

$$\begin{aligned} b_j^i &= \frac{1}{f} c_j^i \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j\right) + \\ & \frac{1}{f} e^i \left[\sum_{\overline{k}=1}^{e} \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j - CC_k^i\right) \Pr[M_{\overline{k},j}] \xi_{\overline{k}} + \sum_{\overline{k}=1}^{e} \lambda_k^i \Pr[M_{\overline{k},k}] \xi_{\overline{k}}\right] \end{aligned}$$

where:

$$f = \sum_{\overline{k}=1}^{e} \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j - CC_k^i\right) f_{\mathbf{d}^{-i}_{\overline{k}-j+1}}\left(b_j^i | t^i\right) \xi_{\overline{k}}.$$

Proof. Vickrey auction is immediate. For the uniform auction the only difference is when the plant is marginal and $j = k$. In this case $\partial_{b_j^i} u_k^i(t, b) = \left(D - \sum_{j=1}^{\overline{k}-1} \boldsymbol{\lambda}_j - CC_k^i\right)$. ■

4 Empirical Analysis

4.1 Data

The data used consist of daily observations from March 1, 2001 until June 30, 2007. Our database includes 25 plants, which belong to 5 companies.[6]

Although the spot market in Colombia started in 1995, we mentioned previously that an regulatory change was introduced to the rule of the auction. Therefore we restrict the analysis to the period where we can argue the auction format was the same.[7] On the other hand, we use the data until the 30th of June 2007 because starting on this date, some important variables for the analysis were not publicly available. In the Colombian spot market there are more than 25 plants producing energy. However, two reasons prompted us

[6] These are: EMGESA, ISAGEN, EEPPM, CHIVOR y CORELCA-GECELCA.

[7] In fact Espinosa [2009] report a structural change in the equilibrium price at the end of February 2001.

to choose only 25. First, those 25 plants are the most important ones because they are the marginal plants 98% of the time for the whole period (1995-2007). Second, most of the plants that were not included did not have enough information for the whole period. Some of them had only 2 years of available information, some others had no information at all. All the information on the generator's bids as well as other variables was obtained from NEON[8]. Simple summary statistics of the data are presented in Table 1. The third column shows the mean of each variable, the fourth one describes standard deviation between and within generating plants and the last one reflects the number of observations available. All the variables are deflated with the producers price index using year 2000 as the base year.

The first variable is "*Disponibiliad*" (i.e. λ_k^i) which shows the effective capacity of energy generation in KWh. Because this variable is reported hourly but has very low variance we've used the mean as a proxy for the daily λ_k^i in the model. That is, the assumption is that agents bid almost the same capacity for next day 24. The standard deviation between plants is high reflecting the very different different values of this variable for different plants.

The second variable used is plants' average bids. Although the mean of the marginal price (equilibrium price) is about 66 $/kWh, the mean of bids is much higher reflecting the presence of outliers and mark ups. Because in the Colombian electricity industry there are at least 3 different ways to produce energy, the variability on bids is high. The production of electricity through hydroelectric power is several times cheaper than through either gas or coal. Nevertheless, the variation in a given plant's bid over time is lower than the between plant's variation. Bids for a given plant depend on resource availability (water) or production costs (i.e. coal or gas price) among other things. On the other hand, the difference between plant's bids is a function of technology that hardly changes within the same plant.

We use the marginal costs reported by the MPODE program. This program is a stochastic dual dynamic programming model.[9] It is used by the industry to forecast prices in the short and long run. Roughly speaking, MPODE calculates the production cost for thermal and hydro plants by forecasting future rain precipitations.

The next variable, the mean of marginal costs is approximately 26 $/kWh. The variability between plants is lower than the deviation for a given plant over time. However, the variation between plants is about 62% of the mean, indicating an important difference across plants. The next variable is the difference between bids and marginal costs. For this variable the deviation between plants is bigger than the within variation suggesting that the variation in bids is more important in the mark-up variation than the variation of the

[8] NEON is the data base which is administred by XM, the market Colombian energy market operator. The data base contains among other variables, rain falls, demand, supply, bids submitted by the agents, and so on.

[9] *Modelo de Programación Dinámica Dual Estocástica* (MPODE).

marginal costs. The mark-up is about 84% of the price, suggesting that plants in the Colombian electricity industry may have high market power. One of the main objective of this paper is to measure and study more deeply these markups'.

The next two variables are very important in the analysis of the Colombian electricity industry. Rain falls is a good proxy of the supply side, because about the 90% of the marginal plants are hydropower plants. The mean for this variable is 113 millions of kWh per day, 15 million below the demands's mean. This shows that the hydropower plants are not able to supply all the energy demanded (only 88%). However, the variation of the rain falls are about four times higher than the demand's variation. The demand has been growing at a constant rate year by year, while the rainfalls does not have any parametric tendency and, although it is highly influenced by natural phenomena that are repeated every 4 to 6 years (el Niño phenomena), there is no obvious relation between different periods.

Variables		*Mean*	*Stand. Devi.*	*Obs.*
Disponibility kWh	Total	348208.8	329621	N=28104
	between		322919	n=25
	within		72591	T-bar=1124
Bid $/kWh	Total	160.6221	154	N=28104
	between		116	n=25
	within		98	T-bar=1124
Marginal Cost $/kWh	Total	26	29	N=281045
	between		16	n=25
	within		25	T-bar=1124
Mark Up $/kWh	Total	134	149	N=22852
	between		107	n=25
	within		100	T-bar=914
Rain Falls kWh per day	Total	1.13×10^8	5.43×10^7	N=28104
Demand kWh per day	Total	1.28×10^8	1.29×10^7	N=28104

Table 1:*Descriptive Statistics, Data from March 2001 to June 2007*.**Source:** *NEON*

4.2 Experiments

We make use of our characterization of bidding behavior in proposition 1 to make the following exercises.

Assume that MPODE´s marginal costs are the true costs of producing energy then, using the equation 1, we can estimate the optimal bids under the hypothesis that agents *could have played* a best reply strategy. We can then compare those optimal bids with the actual bids and address the issue of efficiency at an individual level. We call this the production efficiency problem at the individual level. Because there are a lot of different ways to compare the data we will restrict the comparative analysis in two ways, type of used resource (gas, coal and water) and size of the plant.

Assume next that we are unable to estimate the true valuation of electricity generation then, using proposition 1 we can back up the true valuation under the hypothesis that agents *played* a best reply strategy. We can then compare the identified valuations with those estimated by MPODE. We call this the resource valuation problem. Summing up, so far we have two good candidates for the true valuation of electricity.

Now suppose that the regulator changed the auction format and rather than using a uniform auction they implemented a Vickrey auction. In this case, it is a well established fact, verified in previous examples that agents will bid their true valuation independently of the information structure of the game. Therefore, we are in a position to estimate four different aggregate generating costs. The two previous exercises allows us to estimate the aggregate generating costs of production for the case in which the true costs are the MPODE costs and for the case in which the true valuation is the identified by our second exercise. This is the aggregate efficiency problem or optimality problem for the auctioneer. The other two costs are the actual generating costs in the current uniform auction and the last one is the aggregate costs in the uniform auction if agents has played best reply strategies.

Finally we incorporate contracts into the model. Since bilateral contracts are private information, equation 1 does not allow us to identify marginal costs. Therefore, in this case we assume MPODE marginal costs are the true costs and we backup net contracts positions from observable data.

4.3 Empirical strategy

Our empirical strategy is based on proposition1. Clearly there are more variables than the ones discussed so far. In order to calculate optimal bids, we need to estimate $\boldsymbol{\lambda}_j$, $\Pr[M_{\overline{k},j}]$, $\xi_{\overline{k}}$ and $f_{\mathbf{d}^{-i}_{\overline{k}-j+1}}$ given c^i_j (when we assume c^i_j is the MPODE estimates), e^i , D and λ^i_j for every plant. We assume e^i , D and λ^i_j are exogenous and our main working hypothesis is that $\xi_{\overline{k}}$ is ex-ante exogenous. The rest of the variables $\boldsymbol{\lambda}_j$, $\Pr[M_{\overline{k},j}]$, and

$f_{\mathbf{d}^{-i}_{\overline{k}-j+1}}$ we need to estimate. Our approach is based on constructing an artificial economy in which agents will play repeatedly this auction and they will use previous observables data to infer their best estimate of the currently unknown variables. That is, assume that at date $t = 1$ agents observe the results of our first auction in our data set. Then for the next auction they estimate $\boldsymbol{\lambda}_j$ based on what happened in this first auction. To estimate $\Pr[M_{\overline{k},j}]$ for $t = 2$ we assume that $\mathbf{d}^{-i}_{\overline{k}-j}$ and $\mathbf{d}^{-i}_{\overline{k}-j+1}$ are the last ones then by taking all possible previous bids of agent i for plant j we estimate the probability that $\mathbf{d}^{-i}_{\overline{k}-j} < b^i_j < \mathbf{d}^{-i}_{\overline{k}-j+1}$ holds. That is, we estimate $\Pr[M_{\overline{k},j}]$. To estimate $\xi_{\overline{k}}$ we calculate the distribution of the number of plants which have won the auction in the past, that is the number of plants which were infra-marginal and marginal. The last variable $f_{\mathbf{d}^{-i}_{\overline{k}-j+1}}$, is calculated by first estimating the distribution $F_{\mathbf{d}^{-i}_{\overline{k}-j+1}}$. This is done by first calculating the number of times in the past that a given bid b^i_j was lower than the $\mathbf{d}^{-i}_{\overline{k}-j+1}$, that is, how many times $b^i_j < \mathbf{d}^{-i}_{\overline{k}-j+1}$. In order to calculate the density $f_{\mathbf{d}^{-i}_{\overline{k}-j+1}}$ we assume this may be approximated by a realization of: $\frac{\left[F_{\mathbf{d}^{-i}_{\overline{k}-j+1}}\left(b^i_j+\varepsilon\right)-F_{\mathbf{d}^{-i}_{\overline{k}-j+1}}\left(b^i_j\right)\right]}{\varepsilon}$ where $\varepsilon \sim U[0,\sigma]$ and σ is one standard deviation the set of previous bids b^i_j.

With this recursive algorithm we simulate auctions results in this artificial economy. This process results in a set of bids for every plant for the whole period. We start the process by assuming that the endogenous variables for the first period (that is, March 1, 2001) were the actual variables for the period before this date. We also studied different starting initial dates for the simulation. Our results are robust to initial conditions.

5 Results

In this section we report the results of the experiments.

5.1 Individual Efficiency

Assume MPODE costs are the true costs. Then using proposition 1 and the empirical strategy above we construct all the optimal bids that would have been observed if agents had played optimally according to the model. We the compare with the actual observed bids.

Our first result shows that on average, the difference between the actual and optimal bids for gas-plants are higher than coal plants, and the later have a higher difference than the hydroelectric plants.[10] Although we've found that sometimes the last conclusion does not hold, it holds on average for every year and for the

[10]Supporting Wolfram 98, who showed that higher production´s costs are associated with higher mark-ups.

mean of the whole period.

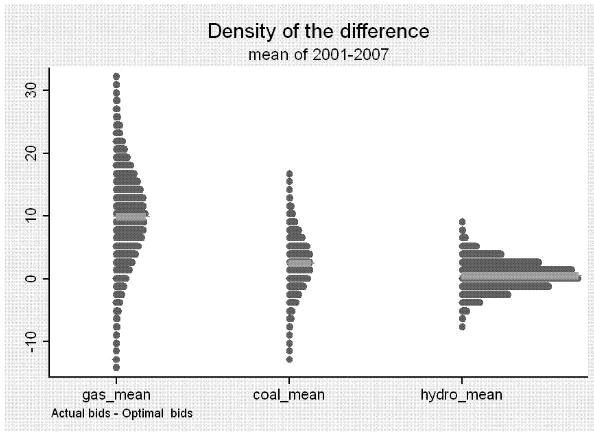


Figure 1

To see this, notice we have 9, 4 and 12 plants which use gas, coal and water, respectively as the generating resource. The next picture shows the distribution of the mean of the difference between the actual and optimal bids. We highlight three facts. First, the distribution for coal and gas plants have more area in the positive quadrant, showing that on average the actual bids are higher than the optimal bids. Second, hydroelectric plants average distribution is more leptokurtic than their counterparts, showing that the difference between the actual and optimal bids is lower. Third, the average difference for gas plants is 10$/kWh, while the difference for coal and hydro plants is near to zero. This result suggests that gas plants have higher markups than coal plants and this two higher than hydroelectric plants. Therefore, the result suggests that gas and coal plants are more distant from competitive equilibrium than what theory predicts. These facts, most likely, reflect market power of gas and coal plants during periods of high electricity prices.

Now consider dividing the sample of plants between small and big plants within each category (gas, coal or hydroelectric). We classify a plant as small if is below average *maximum* generating capacity.

5.1.1 Gas Plants

We have 9 plants which use gas. According to the classification scheme five plants are big, and the rest are small. We found that these five big plants have higher mark ups compared to the smaller plants. That is, the actual bids are statistically higher that the optimal bids. The next figure, shows the distribution of the difference between actual and optimal bids for gas plants, discriminating by the size of plants. We can see that the distributions are quite similar, however, the average distribution for small gas plants is almost zero.

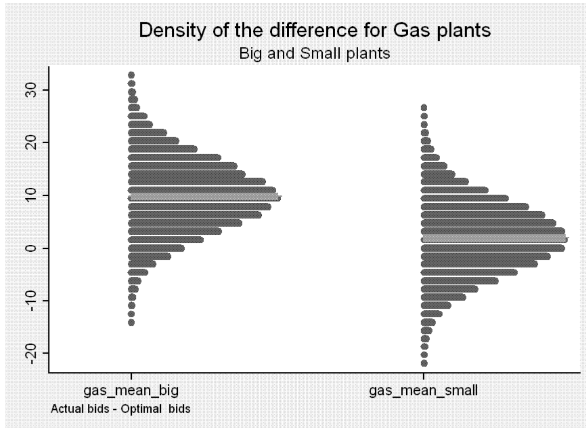


Figure 2

Therefore, the results suggest that big gas plants have higher markups than small gas plants.

5.1.2 Coal Plants

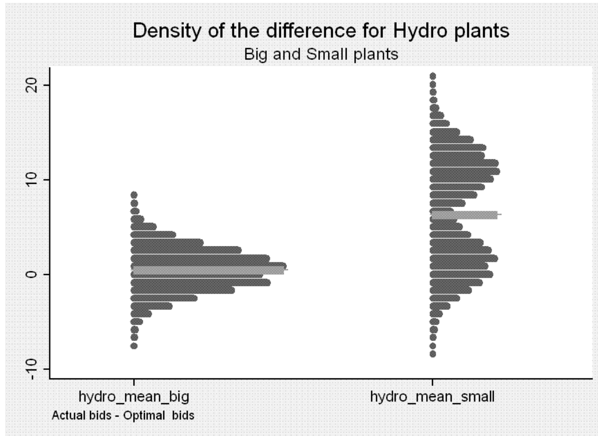
We have four plants which use coal, 50% of them are big. When we compare between size of the plants we found that one small and one big plant over report bids. However, there is not a statistically significant difference among small and big coal plants.

5.1.3 Hydroelectric Plants

We have 12 hydroelectric plants, six of them are classified as big and six as small. For the six big plants, only two have higher mark ups. On average, small plants have a bigger difference between the actual and optimal bid. The next figure shows the distribution of the differences between the actual and optimal bids. The hydroelectric big plants distribution is around 0, while the distribution for the small plants is a bimodal distribution showing that some of the small plants do not have any difference between the actual and optimal bids, but some of the small plants do.

The result suggests that at least some small plants overbid more than big plants. The result is in line with Hortacsu et.al [2008] where they show that small plants tend to overbid.

Figure 3



5.2 Resource Valuation

Now we assume that the actual bids are the optimal ones and we estimate the true valuation following the strategy described before. We then compare this costs with MPODE costs.

Recall that we have 9, 4 and 12 plants which use gas, coal and water, respectively. We found no significant difference between the true value and MPODE costs for thermic plants (gas and coal plants). Nevertheless, there is a significant and big difference for the hydroelectric plants. The next figure shows this difference.

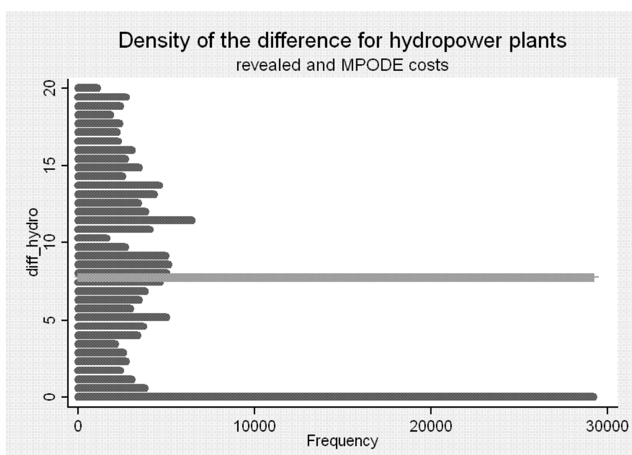


Figure 4

This figure shows two interesting facts: (1). There are a lot of cero values, which indicates that the MPODE costs are good predicting the true value of water in many plants and (2). There are many positive values showing that revealed costs are bigger than their counterpart MPODE´s costs. Furthermore, on average this difference is about 7$/kWh.

Now, looking closer at the difference for thermic plants we find some other interesting facts.

5.2.1 Gas Plants

Although we have not found any significant difference between both set of costs for gas plants, when we control by size, we find that there is a statistically difference for big and small plants. In fact, there is substantial dispersion in the difference between revealed and MPODE´s cost, sometimes even negative.

Figure *5* shows the distribution of the difference between actual and optimal bids for gas plants, discriminating by size of the plants. We can see that the distributions are quite different, however, the mean of the distribution for small plants of gas is almost zero while for big plants there is a big dispersion.

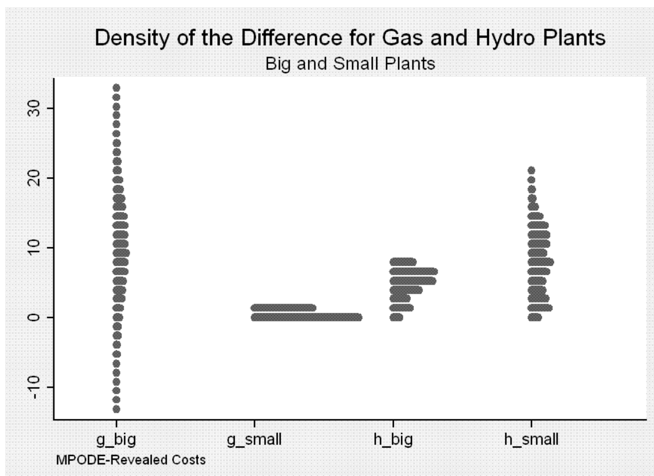


Figure 5

5.2.2 Coal Plants

We find the same result that in the last section. We cannot say anything about coal plants because one small and one big plants have a significant difference between the two set of costs. However, this difference is nearly zero. Controlling by size shows no significant difference among costs.

5.3 Aggregate Efficiency

An important question in auction theory is to what extend is one auction mechanism better than other from the point of view of aggregate revenue (or costs). We now compare the total expenditure under three scenarios: (1) In the actual uniform auction equilibrium prices should be those derived from our model (2). Equilibrium prices are those observed (3). In the Vickrey auction equilibrium prices are what they should be according to the model or alternatively they are MPODE´s costs. For each case we simulate auction results assuming demand and other exogenous variables as available generating capacity are the ones observed.

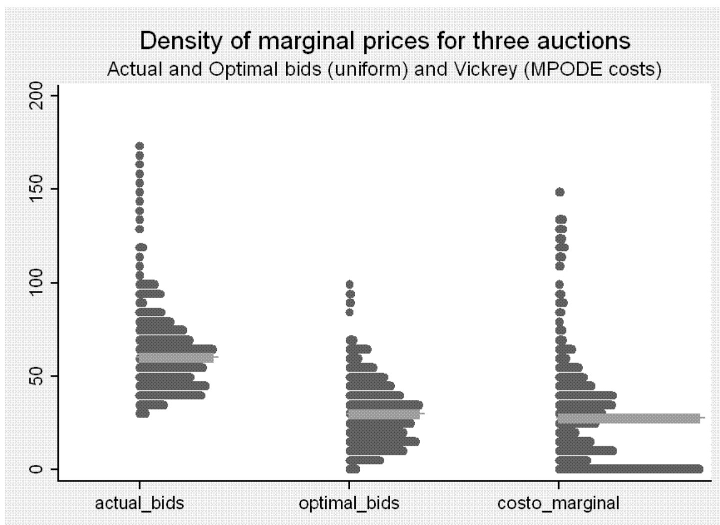
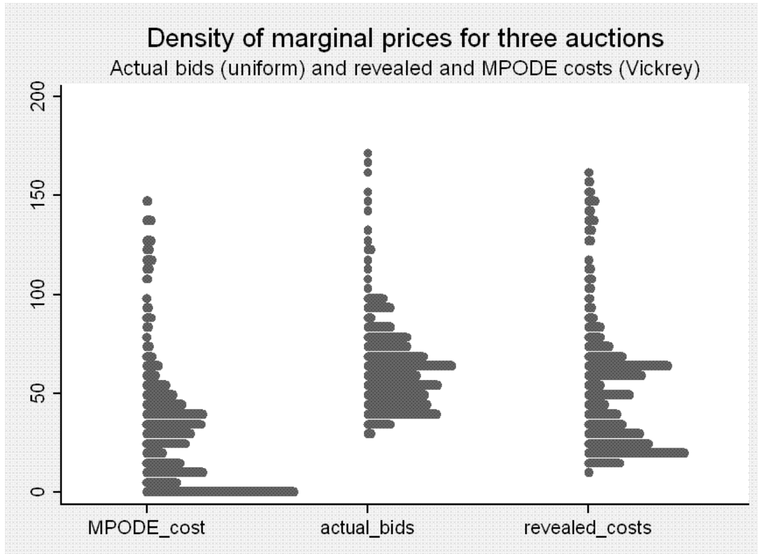


Figure 6

The next figure shows the distribution of the marginal prices for the three auctions for the whole period. We can see the actual bids' distribution is greater in means than the other two. On the other hand, optimal bids distribution have less outliers than the other two distributions. The total expenditure for the optimal bids simulation is \$572.785'360.896, while for the actual bids is \$678.005'990.126. When we calculate the expenditure for the Vickrey auction we find that is 47% of the optimal bids expenditure, that is \$269.143'130.112.

With this alternative we can compare the total expenditure for three sets of data: *1.* the actual marginal prices and *2.* The marginal prices for the Vickrey auction when the bids reported are the MPODE´s costs. *3.* The marginal prices for the Vickrey auction when the bids reported are the revealed costs.

Figure 7



This figure shows the distribution of the marginal prices for the three auctions for the whole period. The total expenditure for the actual bids is $678.005'990.126. When we calculate the expenditure for the Vickrey auction with MPODE´s cost is $269.143'130.112, while if we calculate the Vickrey auction with revealed costs, the total expenditure is $399.840'218.108.

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**Total**
Optimal Bids	91,645.6	97,373.5	74,462.1	85,917.8	68,734.2	80,189.9	74,462.1	572,785.3
Actual Bids	94,920.8	90,513.8	112,413.3	98,310.8	88,140.7	94,920.8	98,785.4	678,005.9
MPODE´s costs	33,642.8	36,603.4	39,833.1	44,408.6	33,373.7	40,371.4	40,909.7	269,143.1
Revealed´s costs	39,984.0	62,375.0	51,979.2	60,775.7	67,173.1	57,976.8	59,576.1	399,840.2

Table 2: *Total Expenditure for 4 Auctions, 2001-2007. Millions of pesos*

5.4 Bilateral Contracts

The last empirical exercise that we can make is to try to find out the contract's level of the plants for our period of study. It is clear from the first order conditions in the model with contract that in order to get the contract level it is necessary to know both sets of bids and marginal costs at the same time. We have a problem with gas and hydro-power plants, because we found reverse conclusion depending of assumption we use. If we suppose that the MPODE's costs are the real ones, then for plants which use gas and water as a primary resource in energy's production, the actual bids are different from the optimal bids. On the other hand, when we suppose that the actual bids are the real ones, then the MPODE's costs are not similar to the revealed costs. These conclusions, do not allow us to use a set of data which is consistent with the empirical results. However, those results are not true for the coal plants, because we did not find any significant difference between costs by plants size when we supposed that the optimal bids were the actual bids, or between prices when we supposed that the real cost were the MPODE's costs.

We use corollary 1 in order to obtain the contract level for coal plants[11]. We have four plants which use coal as primary resource. When we identify the contract level for those plants we found an interesting behavior by size plant. Table 3 shows the number of times that every one of the four plants were the marginal over the total number that the all coal plants submitted the marginal price. Then, 0,5 for this column (for a specific plant) means that for a given year, every two times that a coal plant was the marginal, this plant was once the marginal. The column represented by %c, means what percentage of the energy sold to the market was through contracts. The first two rows are the big coal plants.

Plants	**2001**		**2002**		**2003**		**2004**		**2005**		**2006**		**2007**	
	%p	**%c**	**%p**	**%c**	**%p**	**%c**	**%p**	**%c**	**%p**	**%c**	**%p**	**%c**	**%p**	**%c**
1 (b)	0,17	0,76	0,27	0,86	0,18	0,78	0,29	0,88	0,30	0,82	0,21	0,84	0,22	0,76
2 (b)	0,45	0,69	0,55	0,81	0,39	0,71	0,48	0,81	0,51	0,83	0,43	0,80	0,47	0,84
3	0,28	0,9	0,17	0,93	0,26	0,94	0,14	0,96	0,15	0,9	0,23	0,91	0,25	0,86
4	0,09	0,86	0,01	0,9	0,17	0,87	0,09	0,95	0,04	0,92	0,13	0,9	0,06	0,85

Table 3: *Contract Level for the Coal plants, 2001-2007*

There are three interesting findings: (1) On average big coal plants were the marginal ones, more than twice the number of times that the small plants were marginal. For the big plants this average is 35%, while for the small ones is only 15%. (2) . The average of contracted energy is higher for small plants that for their counterparts, big ones. In the case of the small plants, this average reaches 90%, 10% more than for the big coal plants. (3). There is a statistically, negative and significant correlation between these two variables (*i.e. -0,52*). This fact, shows that coal plants which have less probability to be marginal[12] contract more energy. That fact makes sense because if the plants do not win in the spot market, the only way in order to sell energy is through bilateral contracts.

[11] Notice that the sub-index of the contract levels for this equation is not on the sum operator.

[12] We have done the same exercise with the percentage of the times that the plants were infra-marginals plants. In both empirical exercises the conclusions hold.

6 Conclusions

In this paper we have proposed a model which is able to capture main features of the Colombian spot electricity market. We think the methodology is pretty general and that it could be used to study similar problems in other markets. Our main findings where: (1) . Big gas and small hydro plants overbid therefore they are more inefficient than what is optimal (2) . We find no evidence of overbidding in coal plants. (3). Valuing water may be tricky for hydro generating power; we show how to identify revealed valuations based on auction outcomes and we show that in many cases they are similar to engineering calculated costs except for big gas and small hydro plant for which they are usually higher. Therefore, the more than optimal inefficiency of these plants may be the result of an undervalued resource. (4) . At an aggregate level, we show that if engineering costs are correct and if agents had bid optimally according to the model, then aggregate generating costs would have been substantially smaller. Had a Vickrey auction been used, revealed costs imply that aggregate generating costs would have been even smaller than the previous two. If engineering costs are correct, in a Vickrey auction aggregate costs would have been even smaller than all of the previous. In fact, implausibly so, therefore it is hard to make a case for engineering costs.

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